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EFFECT OF MAGNETIC SHEAR ON THE ELECTROSTATIC ION CYCLOTRON WAV--ETC(U)

JUN 82 G GANGULI, P PALMADESSO

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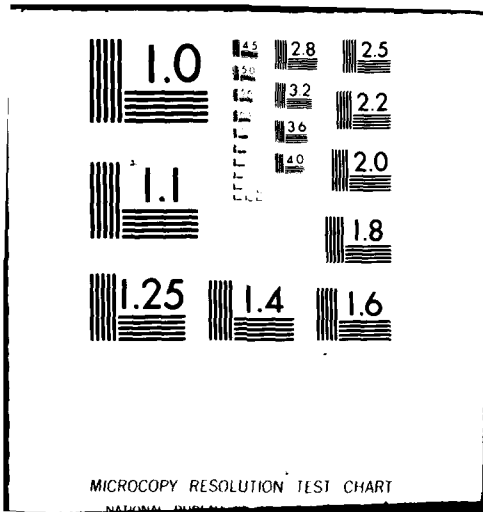
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CONTENTS

I. INTRODUCTION ..... 1

II. THEORY ..... 3

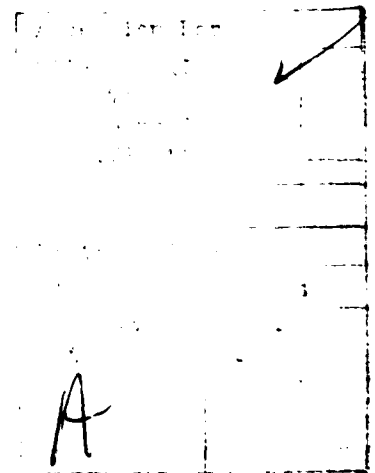
III. RESULTS ..... 9

IV. ENHANCED COHERENCE DUE TO MAGNETIC SHEAR ..... 14

V. CONCLUSION ..... 16

ACKNOWLEDGMENTS ..... 17

REFERENCES ..... 18



## EFFECT OF MAGNETIC SHEAR ON THE ELECTROSTATIC ION CYCLOTRON WAVES IN MAGNETOSPHERIC PLASMAS

### I. INTRODUCTION

The current driven ion-cyclotron instability (CDICI) has been of considerable interest to space plasma physicists. Recent observations by Kintner (1981) and Kintner et al. (1978) have made it very topical. Drummond and Rosenbluth (1962) were first to examine this instability analytically while Kindel and Kennel (1971) studied it in the context of space plasmas in the earth's magnetosphere. The results from Kindel and Kennel have frequently been applied to observations involving the electrostatic CDICI in space. It should be noted, however, that the analysis of both Drummond and Rosenbluth and Kindel and Kennel is a local analysis. They consider a uniform zeroth order magnetic field (i.e.,  $\underline{B} = B_0 \hat{z}$ ), neglecting the self-consistent magnetic field generated by field aligned currents. This magnetic field (usually small) will give rise to a shear in the zero order magnetic field (see Figure 1) and consequently make the zero order field space dependent (i.e.,  $\underline{B}(x) = B_y(x) \hat{y} + B_z \hat{z}$ , where shear is in the x-direction).

Magnetic shear in general is a damping agent and can significantly alter the local mode structure (e.g., it was shown by Ross and Mahajan, (1978) that an infinitesimal shear can completely damp the universal drift instability). Recently Waltz and Dominguez (1981), motivated by the TFR (1978) experiment, have provided numerical results for the behavior of CDICI in a sheared magnetic field pertinent to the TFR (1978) parameters. Ganguli and Bakshi (1981) have given a detailed analytical treatment of the CDICI in a sheared magnetic field and have concluded that even a small shear can give a significant reduction in the growth rates. It also greatly reduces the region of unstable perpendicular wavelengths.

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In this brief report we illustrate the importance of the non-local treatment of the CDICI for the space plasmas. We shall show that the nonlocal theory leads to enhanced coherence in the electrostatic CDICI, consistent with the observations of Kintner (1981).

## II. THEORY

### a. Assumptions

We consider a slab geometry where both the electrons and the ions have a Gaussian distribution function. The plasma has negligible  $\beta$  ( $\beta \equiv nkT/8\pi B^2 \ll 1$ ) and the electrons drift with respect to the ions along the magnetic field with a velocity  $V_d$ . Both species have a finite non-zero temperature  $T_\sigma = m_\sigma v_\sigma^2/2 k_B$ ,  $\sigma$  denoting the species,  $m_\sigma$  is the mass,  $v_\sigma$  is the thermal velocity and  $k_B$  is the Boltzmann's constant. We neglect  $(k\lambda_\sigma)^2 \ll 1$  where  $k$  is the wavevector in arbitrary direction while  $\lambda_\sigma$  is the Debye length. Since for most space plasmas of interest  $T_e \gtrsim T_i$  we shall restrict the analysis here for the temperature ratio  $\tau = T_i/T_e \lesssim 1$ . The ambient magnetic field is in the  $\hat{y}$ - $\hat{z}$  plane and contains a shear in the  $\hat{x}$ -direction. This field is given by

$$\underline{B} = B_0 \{ \hat{z} + (x/L_s) \hat{y} \}, \quad (1)$$

with  $x/L_s \ll 1$  and  $L_s = (\partial\theta/\partial x)^{-1}$  where  $\theta = \tan^{-1}(B_y/B_z)$ . Thus  $L_s$  is the scale length characterizing the magnetic shear. In the absence of shear the field configuration is  $\underline{B} = B_0 \hat{z}$ . The plasma just described is unstable to the CDICI when  $\omega - k_\parallel V_d < 0$  for  $\omega$  and  $k_\parallel$  consistent with the dispersion relation. The waves at maximum growth (for  $T_e \approx T_i$ ) are characterized by  $\omega_r \sim \Omega_i$ ,  $\Omega_i = eB_0/m_i c$ , the ion Larmor frequency;  $\gamma \ll \omega_r$ ; and  $k\rho_i \approx 0(1)$  where  $\rho_i = (2k_B T_i/m_i)^{1/2}/\Omega_i$ , is the ion Larmor radius. Electron Landau and ion-cyclotron dampings reduce the growth rate depending on the magnitude of  $k_\parallel$  (see Ganguli and Bakshi (1981)).

The above description of the plasma gets significantly modified by the introduction of a shear in the magnetic field as given in (1). The magnetic

field rotates in the  $\vartheta$ - $z$  plane (see Figure 1) as a function of  $x$ . If at  $x = 0$  we have  $k_{\parallel} = 0$  then at  $x = x_1$  we see that  $k_{\parallel} \neq 0$  since at  $x_1$  the magnetic field has rotated by an angle  $\theta_1 = x_1/L_s$ . Thus the dispersive properties of the plasma are also a function of  $x$ . It is this fact that changes the boundary condition (from plane waves to outgoing energy flux condition) which the local theory fails to account for, thereby giving dubious results. We introduce shear (i) locally, by replacing  $k_{\parallel}$  by  $k_{\parallel}^0 + s k_y x$ , where  $s = 1/L_s$  and (ii) globally by replacing  $ik_x$  by " $\frac{\partial}{\partial x}$ ". Since the magnitudes of the magnetic shear of interest to us is quite small, we neglect the orbital effects of shear [Bakshi, Bellew, Ganguli and Satyanarayana (1977), Bellew (1978), Ganguli (1980) and Linsker (1981)] arising mainly out of the shear kinematic drift of the particles in the sheared magnetic field.

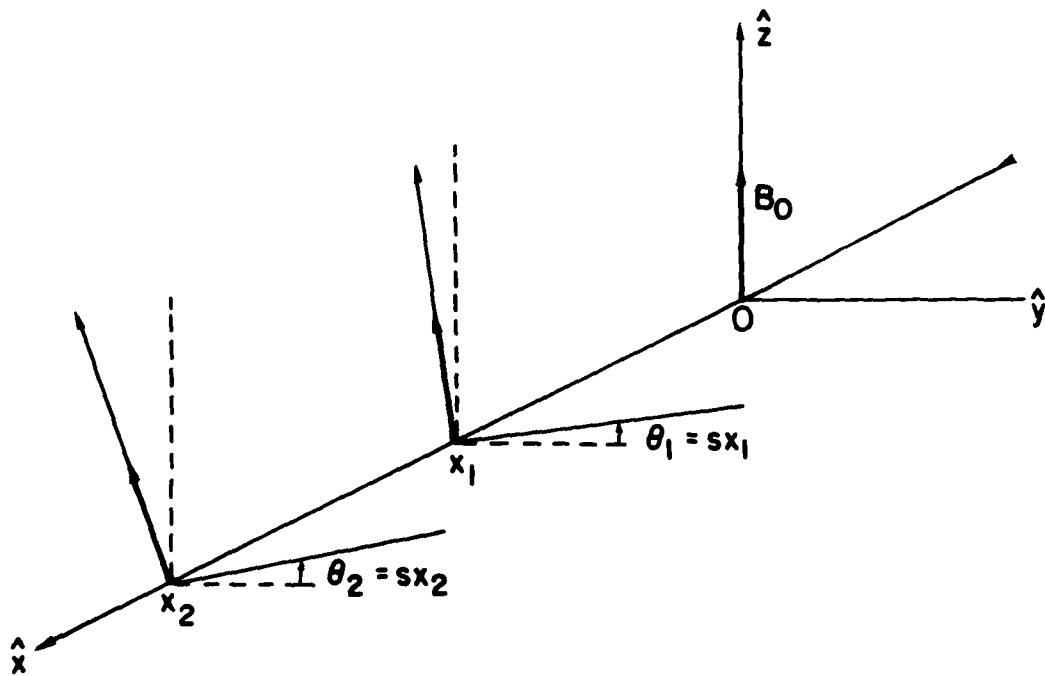


Fig. 1 — A sketch of a sheared magnetic field. At  $x = 0$  the magnetic field  $B_0$  is in the  $\hat{z}$ -direction. At  $x = x_1$  the magnetic field rotates by an angle  $\theta_1 = sx_1$ , where  $s = 1/L_s$ . A wave perpendicular to the magnetic field  $B_0$  at  $x = 0$  is no longer so at  $x = x_1$ .

b. Dispersion Relation

The general dispersion relation for CDICI in the absence of shear is given by

$$k^2 + \sum_{n,\sigma} \frac{\Gamma_n(b_\sigma)}{\lambda_\sigma^2} \left[ 1 + \left( \frac{\omega - k_\parallel V_{d\sigma}}{|k_\parallel| v_\sigma} \right) Z \left( \frac{\omega - n\Omega_\sigma - k_\parallel V_{d\sigma}}{|k_\parallel| v_\sigma} \right) \right] = 0, \quad (2)$$

where  $b_\sigma = k_\perp^2 \rho_\sigma^2 / 2$ ,  $\Gamma_n(b) = I_n(b) e^{-b}$  and  $I_n(b)$  are modified Bessel's function.

As previously described we study the effects of the magnetic shear by replacing  $k_\parallel$  by  $k_y u$ , where  $u = sx$  ( $\equiv k_\parallel(x) / k_y$ ) and  $ik_x$  by  $\frac{\partial}{\partial x}$  in equation (2) (we have assumed  $k_\parallel^0 = 0$ ). Transforming to the ion frame (i.e., setting  $V_{di} = 0$ ), retaining the  $n = 0$  term for the electrons and  $n = 0, \pm 1, \pm 2$  terms for the ions (since we wish to study the first harmonic only) and expanding  $\Gamma_n(b - (\rho_i^2/2) \partial^2 / \partial x^2)$  to  $O(\partial^2 / \partial x^2)$  (here  $b = k_y^2 \rho_i^2 / 2$ ) we obtain a second order differential equation

$$\left\{ -A(x) \frac{\partial^2}{\partial x^2} + Q_1(x) \right\} \phi(x) = 0. \quad (3)$$

The equation can be reduced to (see Ganguli and Bakshi (1981)),

$$\left\{ (\rho_i s)^2 \frac{\partial^2}{\partial u^2} + Q(u) \right\} \phi(u) = 0, \quad (4)$$

where,

$$Q(u) = - \frac{Q_1(u)}{A(u)},$$

$$Q_1(u) = 1 + \tau + \zeta_1 \Gamma_0 Z(\zeta_1) + \zeta_1 \Gamma_1 \{ Z((1-p)\zeta_1) + Z((1+p)\zeta_1) \}$$

$$+ \zeta_1 \Gamma_2 \{ Z((1-2p)\zeta_1) + Z((1+2p)\zeta_1) \}$$

$$+ \tau \zeta_e \left( 1 - \frac{k_{\parallel} v_d}{\omega} \right) Z \left( \zeta_e \left( 1 - \frac{k_{\parallel} v_d}{\omega} \right) \right),$$

$$A(u) = \left( \frac{\zeta_1}{2} \right) [ Z(\zeta_1) \Gamma'_0 + \{ Z((1-p)\zeta_1) + Z((1+p)\zeta_1) \} \Gamma'_1 \\ + \{ Z((1-2p)\zeta_1) + Z((1+2p)\zeta_1) \} \Gamma'_2 ],$$

and

$$\Gamma'_n = \frac{\partial \Gamma_n}{\partial b}.$$

We have assumed  $k_x^2 \ll k_y^2$  in neglecting higher order derivatives in the expansion of  $\Gamma_n$ .

Following Ganguli and Bakshi (1981) we expand the "potential",  $Q(u)$  of the differential equation (4) around  $\xi = (u - u_0) = 0$  to  $O(\xi^2)$ . Here  $u_0$  is the angle of propagation for the maximum growth rate in the absence of shear. We thus obtain,

$$\{ (\rho_1 s)^2 \frac{\partial^2}{\partial \eta^2} + a + \frac{Q_0''}{2} \eta^2 \} \phi(\eta) = 0, \quad (5)$$

where,

$$\eta = \xi + Q_0' / Q_0'',$$

$$a = Q_0 - Q_0'^2 / 2Q_0'',$$

and

$$Q_0 = Q(u_0), \quad Q_0' = \frac{d}{du} Q(u)|_{u=u_0}, \quad Q_0'' = \frac{d^2}{du^2} Q(u)|_{u=u_0}.$$

The dispersion relation under outgoing energy flux boundary condition similar to Pearlstein and Berk (1969) is,

$$Q(\omega, u_0) = (2\ell + 1)(\rho_i s)(-Q_0''/2)^{1/2} + Q_0'^2/2Q_0''. \quad (6)$$

Note  $Q(\omega, u_0) = 0$  yields the local dispersion relation maximized over  $k_{\parallel}$ , while the right hand side of the equation (6) arises out of the non-local treatment. The first term in the right hand side is proportional to the magnetic shear and vanishes in the zero shear limit. The second term, is not explicitly shear dependent. This term can contribute significantly even for infinitesimal shear thereby making the nonlocal dispersion relation solutions much different from those of the local dispersion relation.

### III. RESULTS

The non-local dispersion relation (6) is solved numerically by iterative technique. We simplify the "potential" by expanding all the ion Z-functions except  $n=1$  for large arguments, retain terms to  $O(1/\tau_1^2)$  and treat  $A(u)$  as a constant evaluated at  $u_0$ . [For details see Ganguli and Bakshi (1981)].

Figure 2 shows a plot of  $\gamma/\Omega_i$  against  $b$  for various values of the shear and  $\tau = .5$ ,  $\mu = 1837$  and  $V_d/v_e = .25$ . Note the difference between the local solution and the zero shear limit of the non-local solution. This large reduction in the growth rate is due to the second term in equation (6) arising out of the non-local treatment. Figure 2 also shows that the value of  $b$  for which maximum growth rate occurs remains essentially unaltered from the local theory, even when shear is increased. In addition the expected reduction of the maximum growth rate, the primary effect of increased shear is to narrow the band of unstable perpendicular wavelengths. This result is responsible for enhanced coherence of the wave as we explain in the following section. The uneven spacing of the  $\rho_{i,s} = 0, .005$  and  $.01$  curves shows that non-linearity in the shear parameter (through the implicit dependence of  $\omega$  on  $(\rho_{i,s})$ ) already sets in at  $\rho_{i,s} = .01$ .

In Figure 3 and 4 we provide a  $\gamma/\Omega_i$  versus  $b$  plots for realistic space plasma parameters relevant to the S3-3 data. In Figure 3 we consider a hydrogen plasma ( $\mu = 1837$ ),  $\tau = .75$ ,  $V_d/v_e = .3$  and  $(\rho_{i,s}) = 10^{-6}$ . Note once again the big drop in the local growth rate as well as the significant reduction of the unstable perpendicular wavevector band. Figure 4 illustrates the same features for  $\tau = 1$  and  $V_d/v_e = .35$ . The consequence of the non-local treatment is even more drastic in this case. The nonlocal treatment completely damps the instability.

We have also examined an oxygen plasma and see that the threshold is much lower compared with the hydrogen plasma. The effect of the non-local term on a multispecies plasma is an interesting question which is now being investigated and will be reported elsewhere.

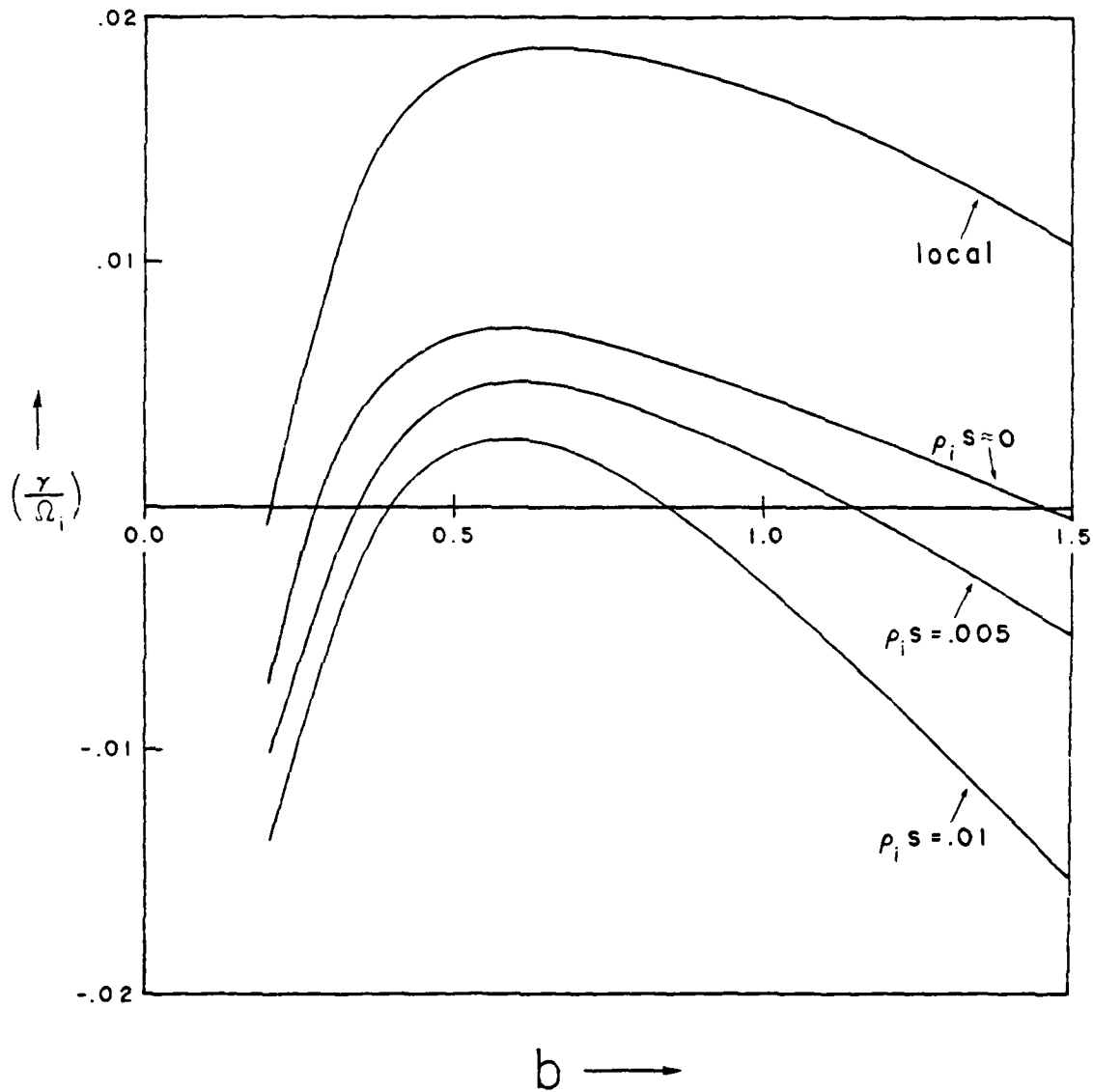


Fig. 2 — A plot of the growth rate  $(\gamma/\Omega_i)$  against  $b$  for various shear values. Here  $\tau = .5$ ,  $\mu = 1837$ , and  $V_d/v_e = .25$ . The big difference between the local solution and the  $(\rho_i s) = 0$  limit of the non-local solution is due to a shear independent term arising from the non-local treatment. The curves also indicate that the effect of the shear is more pronounced for larger  $b$ .

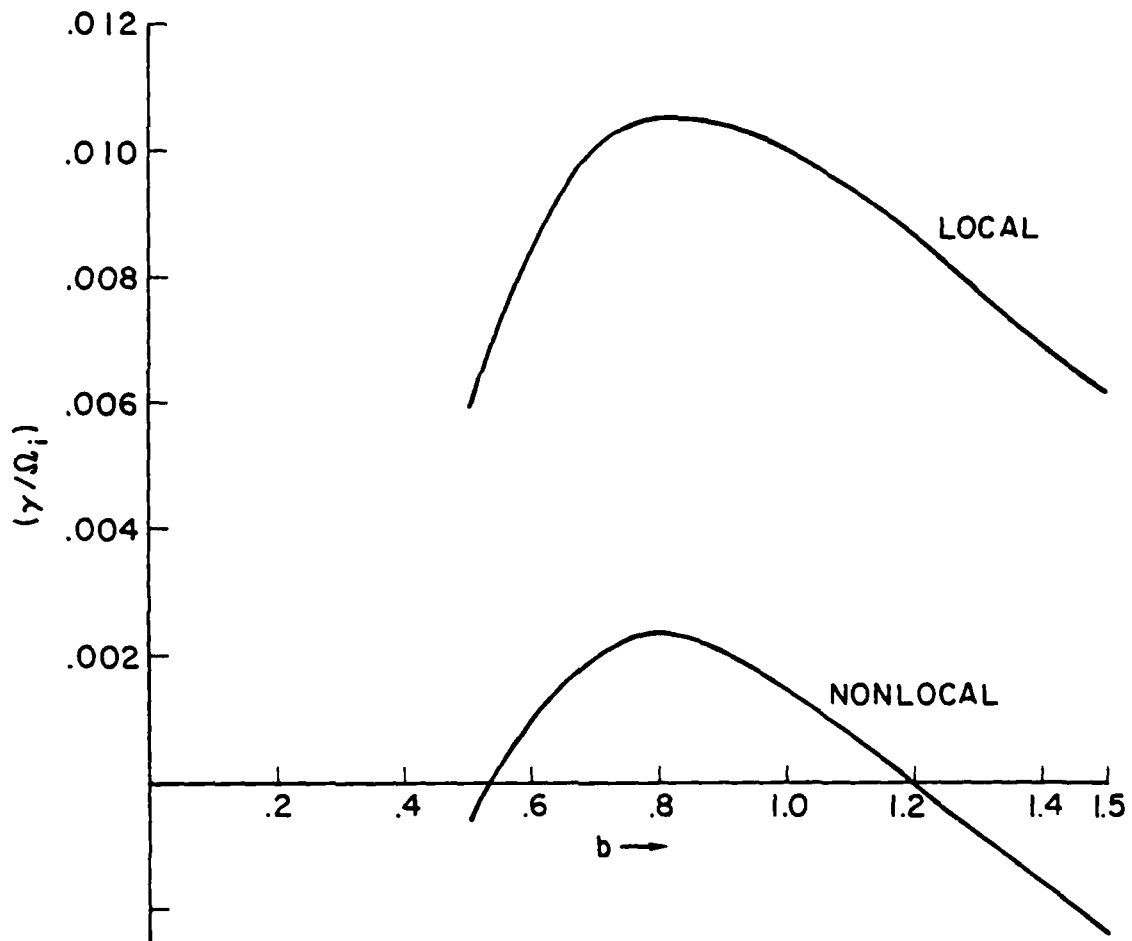


Fig. 3 — A plot similar to Figure 2. Here  $\tau = .75$ ,  $\mu = 1837$  and  $V_d/v_e = .3$ . We provide only the local and the non-local spectrum. The non-local curve has a very small shear,  $(\rho_1 s) = 10^{-6}$ . Note the significant reduction in the unstable perpendicular wavevector band.

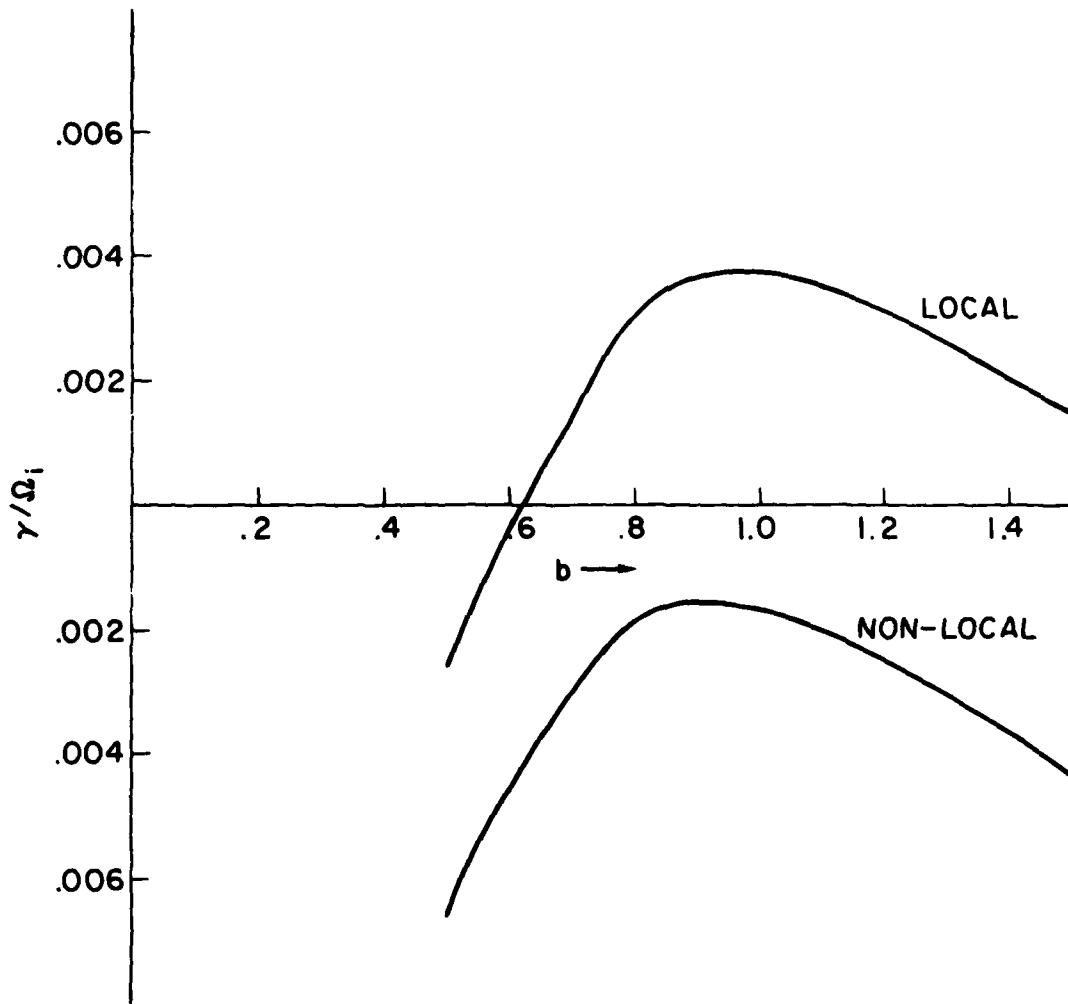


Fig. 4 — Growth rate versus  $b$  plot for  $\mu = 1837$ , and  $V_d/V_e = .35$ . The effect of the non-local treatment is very drastic in this case. The instability is totally suppressed.

#### IV. ENHANCED COHERENCE DUE TO MAGNETIC SHEAR:

Kintner (1981) and Kintner et al. (1978) have reported observations of the electrostatic CDICI for  $\tau \lesssim 1$  with the electron temperatures in the range of a few eV. In this temperature range the instability is observed with a high degree of coherence. Since the measurements were made in the satellite frame (a frame moving with respect to the plasma) the frequencies are Doppler shifted [Frederick and Coroniti (1976)], i.e.,

$$\omega' = \omega + \underline{k} \cdot \underline{v}_s, \quad (7)$$

where  $\omega'$  is the observed frequency in the satellite frame,  $\underline{k}$  is the wavevector and  $\underline{v}_s$  is the satellite velocity. It can be seen from (7) that if the second term is comparable to the first for a large band of unstable  $\underline{k}$  the observed frequency will be highly incoherent. In the case of S3-3 data the satellite orbit is almost perpendicular to the magnetic field and hence  $\underline{k}$  in (7) is just  $\underline{k}_1$ .

Consider Figure 3 where  $T_e = (4/3)T_i$ ,  $v_d/v_e = .3$  and  $\mu = 1837$ . We have plotted  $(\gamma/\Omega_i)$  versus  $b = (k_y^2 \rho_i^2 / 2)$ . We ignore the contribution due to  $k_x$  in  $b$  since  $k_x \ll k_y$  was assumed earlier.

By definition we have,

$$k_y^2 \rho_i^2 = 2b,$$

thus

$$k_y = (2b)^{1/2} \Omega_i / v_i.$$

For  $T_e = T_i$  the real part of the frequency is roughly  $1.2 \Omega_i$  at the maximum

growth. With these substitutions in (7) we see that

$$\omega' = 1.2 \Omega_i \left( 1 + \frac{(2b)^{1/2}}{1.2} \left( \frac{\mu}{\tau} \right)^{1/2} \frac{v_s}{v_e} \right). \quad (8)$$

As given by Kintner et al. (1978) we consider the electron temperature to be around one eV which means that the electron speed is 600 km/s and using  $v_s$  to be 7 km/s and for the parameters of Figure 3 we get,

$$\omega' = 1.2 \Omega_i (1 + .7(b)^{1/2}).$$

Thus, at  $b \sim .8$  where the growth rate maximizes for  $\tau = .75$  the second term is already comparable to the first and gets larger for larger  $b$ . At  $b = 2$  they are both of the same order. However the non-local curve in the Figure 3 shows that for the same parameters the wave grows in a narrower wavelength band beyond which the second term vanishes thus enhancing coherence. A comparison between the local and the non-local curves shows that the band of unstable wavelength is reduced by more than half. For higher shear (see Figure 2) this band can be reduced to less than a quarter of the local value, thereby giving an even higher degree of coherence.

## V. CONCLUSION

The effects of magnetic shear on electrostatic CDICI for the parameters relevant to the S3-3 satellite data have been examined. Shear produces a large reduction in the local growth rate and significantly reduces the band of unstable perpendicular wavelengths. The latter effect leads to enhanced coherence as observed in the S3-3 data.

Although we have shown that a shear in the magnetic field can lead to enhanced coherence in the electrostatic CDICI, there can be other mechanisms contributing to this phenomenon along with the magnetic shear. Recently Okuda and Ashour-Abdalla (1981) have shown that non-linear saturation can also lead to coherence. However, it should be noted that in their treatment they have used a plane wave boundary condition corresponding to the local analysis. From our studies we find that the marginal stability can be significantly altered by non-local effects. This may lead to alterations in the inferences of Okuda and Ashour-Abdalla (1981) and in other numerical simulations of the electrostatic CDICI using a plane wave boundary condition.

We have also examined the effect of the magnetic shear on heavier species and have found it to be equally important. An interesting situation of considerable practical importance is the effect of the magnetic shear on a multispecies plasma. Furthermore, the effect of magnetic shear on the Current Driven Ion Acoustic Instability, and hence, the effect of shear on the relationship of the critical velocity on the temperature ratio [Kindel and Kennel (1971)] is an important question. These problems are now being investigated and will be reported elsewhere.

#### ACKNOWLEDGMENTS

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