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CLOSED FORM SOLUTION FOR THE BENDING STRESSES IN RING - STIFFEN--ETC(U)
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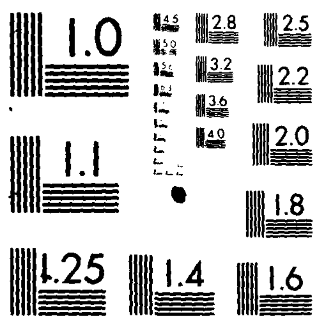

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CLOSED FORM SOLUTION FOR THE BENDING STRESSES IN RING - STIFFENED SHELLS WITH SHAPE IMPERFECTIONS (U)

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CLOSED FORM SOLUTION FOR THE BENDING STRESSES IN RING-STIFFENED SHELLS WITH SHAPE IMPERFECTIONS(U)

BY

D J CRESWELL

Summary (U)

A closed form solution for the displacements and stresses of a slightly non-circular, ring-stiffened shell is derived rigorously from thin shell theory. The solution may be implemented on programmable pocket calculators, providing an accurate, but easy to use, method for predicting bending stresses.

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December 1981

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NOTATION

d^r	displacement from rest (generic form)
d^i	initial imperfection (generic form)
$\bar{d}$	axisymmetric displacement (generic form)
E	Young's Modulus
ν	Poisson's ratio
R	shell radius
h	shell thickness
L	shell section length
z	distance from shell mid-surface (positive inwards)
e	distance from shell mid-surface to frame centroid (positive for internal frames)
e_θ, e_x, $\gamma_{x\theta}$	circumferential, axial and shear strains respectively
A_f	frame area
x	axial position; as a subscript denotes differentiation with respect to x when used in conjunction with u, v, w, w^i
θ	circumferential co-ordinate; as a subscript denotes differentiation with respect to θ when used in conjunction with u, v, w, w^i
u_I, v_I, w_I	eigen vector amplitudes associated with I^{th} basis function in the expansion of axial, circumferential and radial bifurcation displacements, respectively
$u(x,\theta)$, $v(x,\theta)$, $w(x,\theta)$	axial, circumferential and radial bifurcation displacements, respectively. (x, and θ often omitted). w positive inwards
$w^i(x,\theta)$	the radial out-of-circularity at x and θ. (x and θ often omitted)
$w_x{}^i$, $w_\theta{}^i$	derivatives of w^i with respect to x and θ, respectively
$\bar{u}(x)$	axial pre-buckling displacement
$\bar{w}(x)$	radial pre-buckling displacement
$\bar{w}_x(x)$	pre-buckling rotation

N_i	upper limit to the number of basis functions required in the expansion of bifurcation displacements
n	number of circumferential waves
e_{oc}	maximum departure from mean circle
$\underline{x}$	displacement vector
p	lateral pressure (positive external)
P	axial pressure (positive external)
I_{oc}	wave number of imperfection shape, $e_{oc} \sin I_{oc} \pi x/L$
N_f	number of frames
N_s	number of frame spaces
L_f	frame spacing
b	width of faying flange, or web thickness
R_g	centroidal radius of frame
A_{m2}	second moment of area of frame, for in-plane bending
A	frame area) in the Kendrick Frame expressions (Appendix)
I	second moment of area) in the Kendrick Frame expressions (Appendix)

CLOSED FORM SOLUTION FOR THE BENDING STRESSES IN RING-STIFFENED SHELLS WITH SHAPE IMPERFECTIONS

INTRODUCTION

An important consideration in the design of externally pressurised cylindrical shells is the allowance for the shape imperfections which inevitably occur during manufacture. Components of out-of-circularity which approximate to critical buckling mode shapes are considerably exaggerated as the applied pressure approaches the corresponding buckling pressure. Frame stresses may exceed the material yield strength, precipitating failure by one of several mechanisms.

2. Many methods are available for finding the buckling pressures of the perfect ring-stiffened shell, and while frame stresses may be found with simplifying assumptions concerning the way mode shapes will develop (1, 2, 3)*, comparatively few analyses exist which allow a completely satisfactory direct solution to the bending stresses.

3. The purpose of the analysis presented in this paper is to provide a closed form solution for the displacements and frame bending stresses in a nearly circular, uniformly framed, ring-stiffened cylindrical shell. The method is based rigorously on thin-shell theory, but is nevertheless simple enough to allow calculations to be made on a programmable pocket calculator.

THEORY

4. Kendrick has shown that the strains in a slightly non-circular shell may be written as follows (1):

$$e_\theta = (v_\theta - w)/R + (w_\theta^{\,2} + u_\theta^{\,2} - 2wv_\theta)/2R^2 + (w_\theta^{\,i} w_\theta - w^i w)/R^2$$

$$e_x = u_x + (1/2)(w_x^{\,2} + v_x^{\,2}) + w_x^{\,i} w_x$$

$$\gamma_x = u_\theta/R + v_x \qquad [1]$$

The strains contain quadratic products of the displacements, u, v, w and the imperfections w^i. From equations such as [1], a set of linear equations may be derived, the solution of which yields the displacements and stresses in the shell. An outline of the derivation is given below in general terms, applying to any set of strain expressions. A specific derivation, using the original Kendrick expressions (1) is given in the Appendix.

5. An expression for the strain of shell and frames is written in terms of the displacements from rest (d^r) and the initial imperfection (d^i). Parameters d^r and d^i represent in generic form terms of the following type:

---

*( ) = References on page 16

d^r : w^r, v^r, u^r and their derivatives with respect to axial position x and circumferential position θ

d^i : w^i, w^i_θ, w^i_x

where w, v and u are respectively radial, circumferential and axial displacements.

The strain expression contains terms proportional to d^r, $(d^r)^2$ and $d^r d^i$, the exact form of the expression depending to some extent on the basic formulation used: the Novazhilov formulation uses a Cartesian co-ordinate system; the method used by Kendrick uses cylindrical co-ordinates.

6. The total potential energy is given by the following expression

$$U_T = U_s + U_f - W$$

where the contributions from shell (U_s), frames (U_f) and external load (W) may be expressed as

$$U_s = \int_o^{2\pi} \int_o^L u_s\,(x,\, \theta)\, dx d\theta \quad . \qquad [2]$$

$$U_f = \int_o^{2\pi} \sum_{\text{frames}} u_f\,(x,\, \theta)\, d\theta \quad . \qquad [3]$$

$$W = \int_o^{2\pi} \int_o^L w\,(x,\, \theta)\, dx d\theta \quad . \qquad [4]$$

The shell and frame energy densities (u_s and u_f respectively) contain the following types of terms up to cubic order in d^r:

$$(d^r)^2 \qquad (d^r)^3 \qquad (d^r)^2 d^i \qquad (d^r)^3 d^i \quad .$$

The load potential term has terms in:

$$d^r \qquad (d^r)^2 \qquad \text{and} \qquad d^r\, d^i$$

7. Total displacements (d^r) are expressed as the sum of primary displacements ($\bar{d}$) and bifurcation or buckling displacements (d):

$$d^r = \bar{d} + d \quad . \qquad [5]$$

The following terms, of less than cubic order in d or $\bar{d}$, are then present in U_T:

$\bar{d}$	d		
$\bar{d}^2$	$\bar{d}d$	d^2	
$\bar{d}^3$	$\bar{d}^2 d$	$\bar{d}d^2$	d^3
$d^i \bar{d}^2$	$d^i \bar{d}d$	$d^i d^2$	
$d^i \bar{d}^3$	$d^i \bar{d}^2 d$	$d^i \bar{d}d^2$	$d^i d^3$

8. The primary displacements ($\bar{d}$) at any desired pressure p are evaluated first, including if necessary the effects due to an axisymmetric imperfection. It is assumed that non axisymmetric imperfections are sufficiently small so that $\bar{d}$ may be considered to be axisymmetric. The bifurcation displacements d are found by minimising U_T. Bifurcation displacements are assumed to be sufficiently small to enable coupling between different lobar modes to be ignored. The bifurcation mode shape is considered therefore to be a pure Fourier mode. With the simplifying assumptions concerning pre-buckling and buckling displacements, only five sets of terms need be considered:

From the strain energy densities:

1. d^2 (Eh/R)

2. $\bar{d}d$ (Eh/R^2)

3. $\bar{d}d^i d$ (Eh/R^2)

4. $d^i d^2$ (Eh/R^2)

and from the external work:

5. pd^2

where p is the applied pressure.

The typical order of magnitude of coefficients corresponding to each type is indicated in brackets. Terms 1 and 4 are independent of external load (p). Terms 2, 3 and 5 are directly dependent on p. The coefficients of type 4 terms are such that the contributions of the terms are smaller than the contributions from type 1 terms, by the ratio d^i/R. The effect of type 4 terms is to couple different lobar modes for large displacements. Provided the level of out-of-circularity is small in relation to the radius, and the gradient of the imperfection surface not too rapidly varying in the x-direction, type 4 terms may be ignored, maintaining compatibility with the assumption that the development of different lobar modes occurs independently.

9. Bifurcation displacements are represented in terms of Fourier functions.

$$u\,(x,\theta) = \cos n\theta \sum_{I=1}^{N_I} u_I \cos I\pi x/L$$

$$v\,(x,\theta) = \sin n\theta \sum_{I=1}^{N_I} v_I \sin I\pi x/L \qquad [6]$$

$$w\,(x,\theta) = \cos n\theta \sum_{I=1}^{N_I} w_I \sin I\pi x/L$$

On minimisation of U_T with respect to the amplitudes u_I, v_I, w_I, for $I = 1$ to N_I and scaling to a non-dimensional form, the following matrix problem arises

$$M\,\underline{x} = \underline{c} \qquad [7]$$

where
$$M = A + \psi B$$

$$\psi = pR\,(1 - \nu^2)\,/\,Eh \qquad [8]$$

Matrix A contains coefficients of type 1 terms (d^2). Matrix B comprises coefficients of terms from types 2 and 5. The vector $\underline{c}$ derives from group 3 terms, and $\underline{x}$ is the displacement vector whose terms are the amplitudes of the Fourier functions at any pressure p:

$$\underline{x}^T = (u_1\ v_1\ w_1\ \ldots\ldots\ u_I\ v_I\ w_I\ \ldots\ldots\ u_{N_I}\ v_{N_I}\ w_{N_I}) \qquad [9]$$

10. For the perfectly axisymmetric structure ($\underline{c} = 0$) the matrix problem of equation [7] reverts to the standard buckling eigen value problem. Matrices A and B are identical to the buckling matrices.

11. Prebuckling distributions required for the B matrix and $\underline{c}$ vector have, in this paper, been calculated as follows. The axisymmetric radial deformation of a frame in a uniformly framed, ring-stiffened cylinder, under unit hydrostatic load is given precisely by the following expression (4, 5)

$$\bar{w}_f = R^2\,(1 - \nu/2)\,/\,\{\,Eh\,[1 + A\,/\,(bh + 2Nh/\alpha)\,]\,\} \qquad [10]$$

where

$$A = A_f\,(R/R_g)^2$$

$$\alpha = [3\,(1 - \nu^2)]^{\frac{1}{4}}\,/\,\sqrt{Rh}$$

$$N = (\cosh \alpha L' - \cos \alpha L')\,/\,(\sinh \alpha L' + \sin \alpha L')$$

$$L' = L_f - b$$

To conserve radial load, the net circumferential stress in the shell must satisfy the following equation:

$$\bar{\sigma}_{\theta}^{sh} L_f h = \left(RL_f - EA_f \bar{w}_f \right) / R_g \qquad [11]$$

The longitudinal pre-stress $\bar{\sigma}_x^{sh}$ is given by

$$\bar{\sigma}_x^{sh} = -R/2h \qquad [12]$$

The axisymmetric circumferential strain ($\bar{e}_{\theta}^{sh}$) and the axial strain ($\bar{u}_x^{sh}$) in the shell are:

$$\bar{e}_{\theta}^{sh} = E^{-1} \left[\bar{\sigma}_{\theta}^{sh} - \nu R/2h \right] \qquad [13]$$

$$\bar{u}_x^{sh} = E^{-1} \left[\nu \, \bar{\sigma}_{\theta}^{sh} - R/2h \right] \qquad [14]$$

for the chosen sign convention, pressure positive external, displacements positive inwards.

12. The matrix M may now be defined very simply (details in Appendix). Shell and frame components are considered separately,

$$M = M_s + M_f .$$

For a ring-stiffened cylinder of uniform shell thickness the shell matrices are banded tri-diagonally,

$$\begin{aligned}
M_s(IU, IU) &= m_0^{uu} + I^2 m_2^{uu} \\
M_s(IU, IV) &= I m_1^{uv} \\
M_s(IU, IW) &= I m_1^{uw} + I^3 m_3^{uw} \\
M_s(IV, IV) &= m_o^{vv} + I^2 m_2^{vv} \\
M_s(IV, IW) &= m_o^{vw} + I^2 m_2^{vw} \\
M_s(IW, IW) &= m_o^{ww} + I^2 m_2^{ww} + I^4 m_4^{ww}
\end{aligned} \qquad [15]$$

where

$$\begin{aligned}
IU &= 3I - 2 \\
IV &= 3I - 1 \qquad\qquad I = 1, 2, \ldots\ldots\ldots N_I \\
IW &= 3I
\end{aligned} \qquad [16]$$

to be compatible with the displacement vector definition, equation [9]. Expressions for m_o^{uu} etc specific to the Kendrick formulation are given in the Appendix.

13. Satisfactory solution of overall buckling problems may be obtained if out-of-plane bending stiffness of frames is ignored in both A and B matrices (3). The frame terms of the M matrix then contain neither x-derivatives nor terms in axial displacement u.

$$
\begin{aligned}
M_f\,(IV\,,\,JV) &= f^{v}\,S(I,J) \\
M_f\,(IV\,,\,JW) &= f^{vw}\,S(I,J) \\
M_f\,(IW\,,\,JW) &= f^{w}\,S(I,J)
\end{aligned}
\qquad [17]
$$

where

$$
S(I,J) = (2/N_s) \sum_{i=1}^{N_f} \sin\,(I\pi x_i/L)\,\sin\,(J\pi x_i/L) \qquad [18]
$$

14. The $\underline{c}$ vector may be similarly decomposed into shell and frame terms

$$
\underline{c} = \underline{c}_s + \underline{c}_f
$$

If the imperfection is of the form $e_{oc}\cos n\theta \sin\,(I_{oc}\pi x/L)$ the shell components of the $\underline{c}$ vector may be written as follows (equations [13] and [23] of the Appendix):

$$
\begin{aligned}
c_s\,(IU) &= I\,c_1^{u}\delta_{II_{oc}} \\
c_s\,(IV) &= c_o^{v}\delta_{II_{oc}} \\
c_s\,(IW) &= \left[c_o^{w} + c_2^{w}\,I^2\right]\delta_{II_{oc}}
\end{aligned}
\qquad [19]
$$

The frame terms are given by

$$
\begin{aligned}
c_f\,(IV) &= c_f^{v}\,c(I) \\
c_f\,(IW) &= c_f^{w}\,c(I)
\end{aligned}
\qquad [20]
$$

where

$$
c(I) = (2/N_s) \sum_{i=1}^{N_f} \sin\,(I\pi x_i/L)\,\sin\,(I_{oc}\pi x_i/L) \qquad [21]
$$

15. For a uniformly framed structure (identical, uniformly spaced frames) only sinusoidal components obeying the following relationship for some integral value of K are coupled (3):

$$I = (\tfrac{1}{2}) \left\{ \left[1 - (-1)^{I'}\right] \left[(I'-1)N_s + K\right] \pm \left[1 + (-1)^{I'}\right] \left[I'N_s - K\right] \right\}$$

$$\text{for } I' = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots N_I \qquad [22]$$

Functions satisfying equation [22] with either " " or "-" in the centre provide a complete orthogonal set. Selecting the "-" option, it may be shown straightforwardly that

$$S(I,J) = 1 \qquad [23]$$

for all I,J satisfying equation [22]. Only the long wavelength shapes will be required,

$$e_{oc} \sin(K\pi x/L) \qquad \text{for } K = 1, 2, 3, \ldots\ldots\ldots\ldots,$$

representing the fundamental modal components of equation [22]. If $I_{oc} = K$, $K < N_f$, then

$$c(I) = 1 \qquad [24]$$

for all I satisfying equation [22]. It follows immediately that the fundamental matrix, equation [7], may be stated in the following form:

$$M_s(IU,IV)u_I + M_s(IV,IV)v_I + M_s(IV,IW)w_I + f^v\Sigma v_I + f^{vw}\Sigma w_I$$

$$= c_0^v \delta_{II_{oc}} + c_f^v \qquad [25]$$

$$M_s(IU,IW)u_I + M_s(IV,IW)v_I + M_s(IW,IW)w_I + f^{vw}\Sigma v_I + f^w\Sigma w_I$$

$$= (c_0^w + c_2^w I^2)\delta_{II_{oc}} + c_f^w \qquad [26]$$

$$M_s(IU,IU)u_I + M_s(IU,IV)v_I + M_s(IU,IW)w_I = I\, c_1^u \delta_{II_{oc}} \qquad [27]$$

the summations being performed for a set of I given by [22].

16. From equation [27], u_I can be written explicitly in terms of v_I and w_I, allowing the matrix dimension to be reduced by a factor of 2/3:

$$S^v(I)v_I + S^{vw}(I)w_I = c^v(I) - f^v\Sigma v_I - f^{vw}\Sigma w_I$$

$$S^{vw}(I)v_I + S^w(I)w_I = c^w(I) - f^{vw}\Sigma v_I - f^w\Sigma w_I \qquad [28]$$

where

$$S^{v}(I) = M_s(IV,IV) - M_s(IU,IV)^2 / M_s(IU,IU)$$

$$S^{vw}(I) = M_s(IV,IW) - M_s(IU,IV)\, M_s(IU,IW) / M_s(IU,IU) \qquad [29]$$

$$S^{w}(I) = M_s(IW,IW) - M_s(IU,IW)^2 / M_s(IU,IU)$$

$$c^{v}(I) = c_f^{v} + c_{sh}^{v}$$

$$c^{w}(I) = c_f^{w} + c_{sh}^{w} \qquad [30]$$

$$c_{sh}^{v} = \delta_{II_{oc}} \left[c_0^{v} - I\, c_1^{u}\, M_s(IU,IV) / M_s(IU,IU) \right]$$

$$c_{sh}^{w} = \delta_{II_{oc}} \left[c_0^{w} + c_2^{w} I^2 - I\, c_1^{u}\, M_s(IU,IW) / M_s(IU,IU) \right] \qquad [31]$$

17. Equations [28] show that v_I and w_I, for any I, can be expressed in terms of summations over v_I and w_I:

$$v(I) = s^{w}(I) \left[c^{v}(I) - \eta \right] - s^{vw}(I) \left[c^{w}(I) - \xi \right]$$

$$w(I) = s^{v}(I) \left[c^{w}(I) - \xi \right] - s^{vw}(I) \left[c^{v}(I) - \eta \right] \qquad [32]$$

where

$$\eta = f^{v}\, \Sigma^{v} + f^{vw}\, \Sigma^{w}$$

$$\xi = f^{vw}\, \Sigma^{v} + f^{w}\, \Sigma^{w} \qquad [33]$$

Σ^{v} and Σ^{w} denoting Σv_I and Σw_I respectively,

$$s^{v}(I) = S^{v}(I) / D(I)$$

$$s^{vw}(I) = S^{vw}(I) / D(I)$$

$$s^{w}(I) = S^{w}(I) / D(I) \qquad [34]$$

$$D(I) = S^{v}(I)\, S^{w}(I) - S^{vw}(I)^2$$

18. Substituting equations [32] into equations [28], it may be seen that

$$\Sigma^{v} = \left\{\left[1 + \bar{s}_{v} f^{w} - \bar{s}_{vw} f^{vw}\right] C_1 - \left[\bar{s}_{w} f^{vw} - \bar{s}_{vw} f^{w}\right] C_2\right\} / \left\{\left[1 + \bar{s}_{w} f^{v} - \bar{s}_{vw} f^{vw}\right]\left[1 + \bar{s}_{v} f^{w} - \bar{s}_{vw} f^{vw}\right] - \left[\bar{s}_{v} f^{vw} - \bar{s}_{vw} f^{v}\right]\left[\bar{s}_{w} f^{vw} - \bar{s}_{vw} f^{w}\right]\right\} \quad [35]$$

$$\Sigma^{w} = \left[C_1 - (1 + \bar{s}_{w} f^{v} - \bar{s}_{vw} f^{vw}) \Sigma^{v}\right] / (\bar{s}_{w} f^{vw} - \bar{s}_{vw} f^{w}) \quad [36]$$

where

$$\bar{s}_{v} = \Sigma s^{v}(I) \quad \bar{s}_{vw} = \Sigma s^{vw}(I) \quad \bar{s}_{w} = \Sigma s^{w}(I) \quad [37]$$

$$C_1 = \bar{s}_{w} c_{f}^{v} - \bar{s}_{vw} c_{f}^{w} + s^{w}(1) c_{sh}^{v} - s^{vw}(1) c_{sh}^{w}$$

$$C_2 = \bar{s}_{v} c_{f}^{w} - \bar{s}_{vw} c_{f}^{v} + s^{v}(1) c_{sh}^{w} - s^{vw}(1) c_{sh}^{v} \quad [38]$$

19. Equations [32] to [38] represent the closed form solution for the circumferential (v) and radial (w) displacements of a hydrostatically loaded, slightly non-circular, uniformly framed, ring-stiffened cylinder, with shape imperfection $e_{oc} \sin(K\pi x/L) \cos(n\theta)$. Equations [35] to [38] may be evaluated directly in terms of known shell and frame coefficients (equations [21] and [25] of Appendix).

20. Equations [32] give v(I) and w(I) from which the shell displacements at any position x using equations [6] and [22]. In this derivation the central sign of equation [22] has been taken to be minus. If the 'plus' option is used the final expressions take on the slightly modified, but numerically equivalent, form given in (3).

21. The stresses, at position z on a frame section, are given by the following expression

$$\sigma(z) = \sigma_{ax}(z) + \sigma_{b}(z) \quad [39]$$

where the axisymmetric stress $\sigma_{ax}(z)$ is given by

$$\sigma_{ax}(z) = E\, \bar{w}_{f} / (R - S_{i} z) , \quad [40]$$

and the bending stress $\sigma_{b}(z)$ is given by

$$\sigma_{b}(z) = (E/R) \left[n\, v(x) - w(x) + (z/R)(n^2 - 1)\, w(x)\right] \quad [41]$$

NUMERICAL EXAMPLES

22. Displacements and stresses have been evaluated on the two rather different structures given in Table 1.

23. Displacements and stresses in the closely frame-spaced submersible structure are given in Tables 2-4. The predictions of closed form and matrix method are almost identical. The closed form method is particularly useful for convergence tests (Table 4).

24. In the widely frame-spaced VAC type structure severe mixed-mode effect occurs for $n > 2$. The closed form method provides therefore a very simple method of allowing for mixed mode effect in ring-stiffened cylinders. Displacements and stresses predicted by closed form and matrix methods agree almost exactly (Figure 1, Table 5).

CONCLUSIONS

25. By means of analytical simplification, it is possible to derive rigorously from thin-shell-theory, a closed form expression for the bending stresses in a uniformly framed ring-stiffened cylinder with non-axisymmetric shape imperfection.

26. The analytic method has been implemented on a programmable pocket calculator and results are shown to agree with the predictions of more usual matrix methods which require quite large computers for accurate solution.

27. It is felt that the 'closed form' method provides a useful means of checking the prediction of:

a. more sophisticated methods, such as finite difference and finite element, and

b. approximate methods used in design codes, such as the effective breadth approach of BS5500.

The method is particularly appropriate for carrying out convergence studies to assess the fine-ness of mesh required in more sophisticated analyses.

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NOTE: Not all of the References referred to above are automatically available to the general public.

TABLE 1

CASE 1 - PV4 SUBMERSIBLE

19 equispaced frames

$R = 2000$ mm	$d = 180.92$ mm
$h = 17.879$ mm	$h_w = 9.046$ mm
$L = 12000$ mm	$f = 90.46$ mm
$E = 207000$ N/mm^2	$h_f = 15.07$ mm
$\nu = 0.3$	INTERNAL

CASE 2 - PV4 VAC TYPE

1 central frame

$R = 2500$ mm	$d = 192$ mm
$h = 20$ mm	$h_w = 12$ mm
$L = 14000$ mm	$f = 0$
$E = 207000$ N/mm^2	$h_f = 0$
$\nu = 0.3$	INTERNAL

R	shell radius	d	web depth
h	shell thickness	h_w	web thickness
L	shell length	f	flange width
E	Young's modulus	h_f	flange thickness
ν	Poisson's ratio		

TABLE 2

x/L	v Closed Form	v Matrix	w Closed Form	w Matrix	$\lvert\sigma_b^{fl}\rvert$ Closed Form	$\lvert\sigma_b^{fl}\rvert$ Matrix	$\lvert\sigma_b^{ms}\rvert$ Closed Form	$\lvert\sigma_b^{ms}\rvert$ Matrix
0.05	0.6434	0.6323	1.3687	1.3445	35.18	34.63	8.36	8.15
0.1	1.2720	1.2503	2.7046	2.6560	69.43	68.41	16.52	16.10
0.15	1.8685	1.8365	3.9720	3.9020	102.09	100.51	24.27	23.65
0.2	2.4195	2.3781	5.1426	5.0519	132.16	130.1[illegible]	31.43	30.61
0.25	2.9107	2.8603	6.1866	6.0775	159.01	156.54	37.81	36.82
0.3	3.3301	3.2752	7.0782	6.9534	181.33	179.10	43.26	42.13
0.35	3.6676	3.6049	7.7956	7.6581	200.37	197.25	47.64	46.40
0.4	3.9146	3.8479	8.3209	8.1742	213.87	210.55	50.85	49.52
0.45	4.0656	3.9961	8.6414	8.4890	222.11	218.66	52.81	51.43
0.5	4.1163	4.0459	8.7432	8.5948	224.89	221.36	53.47	52.07

FV4 Submersible n = 2 displacements and bending stresses at 3 N/mm^2 for shape 20 cos 2θ sin πx/L. Comparison of results from the closed form method, evaluated on a pocket calculator (M = 5) and a matrix method (30 x 30, IIAC = 6).

TABLE 3

x/L	v Closed Form	v Matrix	w Closed Form	w Matrix	$\lvert\sigma_b^{fl}\rvert$ Closed Form	$\lvert\sigma_b^{fl}\rvert$ Matrix	$\lvert\sigma_b^{ms}\rvert$ Closed Form	$\lvert\sigma_b^{ms}\rvert$ Matrix
0.05	0.3137	0.3096	0.6660	0.6563	17.19	17.05	3.99	3.84
0.1	0.5967	0.5890	1.2669	1.2484	32.70	32.43	7.60	7.30
0.15	0.8213	0.8107	1.7437	1.7183	45.01	44.63	10.46	10.05
0.2	0.9655	0.9530	2.0500	2.0200	52.91	52.47	12.30	11.81
0.25	1.0152	1.0020	2.1553	2.1239	55.64	55.17	12.93	12.42
0.3	0.9655	0.9530	2.0500	2.0200	52.91	52.47	12.30	11.81
0.35	0.8213	0.8107	1.7437	1.7183	45.01	44.64	10.46	10.05
0.4	0.5967	0.5890	1.2669	1.2484	32.70	32.43	7.60	7.30
0.45	0.3137	0.3096	0.6660	0.6563	17.19	17.05	3.99	3.84
0.5	0	$0.36\ 10^{-8}$	0	$0.68\ 10^{-8}$	0	0	0	0

PV4 Submersible n = 2 displacements and bending stresses at 3 N/mm^2 for shape 20 cos 2θ sin 2πx/L. Comparison of results from the closed form method (M = 5) and a matrix method (30 x 30 matrix, IFAC = 0).

TABLE 4

M	N	V	W	σ_b^{fl}	σ_{TOT}^{fl}
1	6	3.854	8.060	220	417.0
2	12	4.080	8.653	224	421.5
3	[illegible]	4.106	8.723	225	422.0
5	30	4.116	8.749	225	423.0

PV4 Submersible n = 2, shape 20 cos 2θ sin πx/L.

Convergence test: M convergence parameter,

N equivalent (IFAC = 0) matrix size.

Pressure 3 N/mm^2.

TABLE 5

FRAME DISPLACEMENTS - PV4 VAC TYPE STRUCTURE

pressure = 0.3 N/mm^2 e_{oc} = 25 mm I_{oc} = 1

	n = 2	n = 3	n = 4	n = 5	n = 6
Circumferential Displacements (v)					
Closed Form	1.3725	5.0266	2.4920	1.1397	0.44280
Matrix	1.3724	5.0292	2.5007	1.1502	0.44616
Radial Displacements (w)					
Closed Form	2.7460	15.470	10.412	6.0950	2.9730
Matrix	2.7449	15.475	10.444	6.1474	2.9940

APPENDIX

In equations [1] e_θ, e_x, $\gamma_{x\theta}$ are respectively the circumferential, axial and shear strains. The displacements w, v, etc are measured from rest, the superscript "r" being omitted for convenience. The extensional shell energy is then given by the standard plane-stress, thin shell expression:

$$U_e = \int_o^{2\pi} \int_o^L \left[ERh/2\ (1 - \nu^2)\right] \{e_\theta{}^2 + e_x{}^2 + 2\nu e_\theta e_x + (1/2)(1 - \nu)\gamma_{x\theta}{}^2\}\ d\theta dx \qquad [A1]$$

2. The following expression for the shell bending strain energy was used by Kendrick (6):

$$U_b = \int_o^{2\pi} \int_o^L \left[ERhk/2\ (1 - \nu^2)\right] \{R^2 w_{xx}{}^2 + (w_{\theta\theta} + w)^2/R^2 + (1/2)(1 - \nu)(w_{x\theta} - u_\theta/a)^2 + (3/2)(1 - \nu)(v_x + w_{x\theta})^2 + 2\nu w_{xx}\ (w_{\theta\theta} + v_\theta) + 2Ru_x w_{xx}\}\ dxd\theta \qquad [A2]$$

where $k = h^2/12R^2$. The shell energy contribution $U_s = U_e + U_b$. Frame extensional strains are expressed in terms of the mid-shell strain e_θ and the change of curvature due to in-plane bending, giving the following expression for the extensional frame energy F_e,

$$F_e = \sum_{r=1}^{N_f} (EAR/2) \int_o^{2\pi} \{e_\theta - (w_{\theta\theta} + w)(e/R^2)\}^2\ d\theta \qquad [A3]$$

3. Frame bending energy is given by:

$$F_b = \sum_{r=1}^{N_f} (EI/2R^3) \int_o^{2\pi} (w_{\theta\theta} + w)^2\ d\theta \qquad [A4]$$

4. The total frame energy $U_f = F_e + F_b$. The work done by external load on an initially imperfect cylinder is found to be given by the following expression:

$$W = (p/2)\int_0^{2\pi}\int_0^L \{2Rw - 2w^i w - w^2 + 2R(w^i + w)(u_x + v_\theta/a) - Ru_x v_\theta\}\, d\theta dx + (PR^2/2)\int_0^{2\pi}\int_0^L - u_x\, dx d\theta \qquad [A5]$$

5. In all the expressions given above the superscript "r" denoting displacements from rest has been omitted. On making the transformation:

$$u^r(x,\theta) = \bar{u}(x) + u(x,\theta)$$

$$v^r(x,\theta) = v(x,\theta) \qquad [A6]$$

$$w^r(x,\theta) = \bar{w}(x) + w(x,\theta)$$

the total potential energy U_T is found to be given by the following expression:

$$U_T = U_A + U_B + U_C \qquad [A7]$$

where

$$U_A = \int_0^{2\pi}\int_0^L \left[EhR/2\,(1 - \nu^2)\right] \{u_x{}^2 + (v_\theta - w)^2/R^2 + 2\nu u_x\,(v_\theta - w)/R + (1/2)(1 - \nu)(u_\theta/R + v_x)^2 + k\left[R^2 w_{xx}{}^2 + (w_{\theta\theta} + w)^2/R^2 + (1/2)(1 - \nu)(w_{x\theta} - u_\theta/R)^2 + (3/2)(1 - \nu)(v_x + w_{x\theta})^2 + 2\nu w_{xx}\,(w_{\theta\theta} + v_\theta) + 2Ru_x w_{xx}\right]\}\, d\theta dx + \sum_{r=1}^{N_f} (EAR/2) \int_0^{2\pi} \{\left[(w_{\theta\theta} + w)(e/R^2) + (w - v_\theta)/R\right]^2 + (I/R^4 A)(w_{\theta\theta} + w)^2\}\, d\theta \qquad [A8]$$

$$U_B = \int_0^{2\pi}\int_0^L \left[Eh/2R^2(1-\nu^2)\right] \{\bar{w}\left[-2v_\theta^{\ 2} - u_\theta^{\ 2} - w_\theta^{\ 2} + 4wv_\theta - R^2\nu\,(w_x^{\ 2} + v_x^{\ 2}) - 2R\nu u_x v_\theta\right] + \bar{u}_x\left[R^3(v_x^{\ 2} + w_x^{\ 2}) + R\nu\,(u_\theta^{\ 2} + w_\theta^{\ 2} - 2wv_\theta)\right] + \bar{w}_x\left[2R^3u_xw_x - 2\nu R^2w_x(w - v_\theta)\right]\}\, d\theta dx + \sum_{r=1}^{N_f}(EA/2R^2)\bar{w}\int_0^{2\pi}\{-(1+e/R)u_\theta^{\ 2} - 2v_\theta^{\ 2} - (1+e/R)w_\theta^{\ 2} + 4(1+e/R)wv_\theta + (2e/R)w_{\theta\theta}v_\theta\}\, d\theta + (p/2)\int_0^{2\pi}\int_0^L \{w^2 - 2w\,(v_\theta + Ru_x) + Ru_xv_\theta\}\, d\theta dx \qquad \boxed{A9}$$

$$U_C = \int_0^{2\pi}\int_0^L \left[Eh/2R^2(1-\nu^2)\right] \{\bar{w}\left[-2\nu Ru_xw^i - 2v_\theta w^i + 4ww^i - 2w_\theta w_\theta^{\ i} - 2R^2\nu w_xw_x^{\ i}\right] + \bar{u}_x\left[-2R\nu ww^i + 2R\nu w_\theta w_\theta^{\ i} + 2R^3w_xw_x^{\ i}\right] + \bar{w}_x\left[2R^2\nu(v_\theta - w)w_x^{\ i} + 2R^3u_xw_x^{\ i}\right]\}\, d\theta dx + \sum_{r=1}^{N_f}(EA/2R^2)\bar{w}\int_0^{2\pi}\{-2v_\theta w^i + 4(1+e/R)ww^i - 2(1+e/R)w_\theta w_\theta^{\ i} + (2e/R)\,w_{\theta\theta}w^i\}\, d\theta + p\int_0^{2\pi}\int_0^L w^i\,(w - v_\theta - Ru_x)\, d\theta dx \quad . \qquad \boxed{A10}$$

Terms proportional to the end loads (P) do not appear explicitly in the final expressions. Effects due to end loads are transferred through $\bar{w}$, $\bar{u}_x$ and $\bar{w}_x$.

6. On minimization of the energy expressions, the A and B matrices and the c-vector may be written as follows:

$$A_s(IU,IU) = n^2(1 + k)(1 - \nu)/2 + I^2\lambda^2$$

$$A_s(IU,IV) = -nI\lambda(1 + \nu)/2$$

$$A_s(IU,IW) = \nu I\lambda - n^2(1 - \nu)kI\lambda/2 + k(I\lambda)^3$$

$$\boxed{A11}$$

$$A_s(IV,IV) = n^2 + (1 - \nu)(1 + 3k)(I\lambda)^2/2$$

$$A_s(IV,IW) = -n\left[1 + (3 - \nu)k\ (I\lambda)^2/2\right]$$

$$A_s(IW,IW) = (1 + k) + n^2k\left[n^2 - 2 + 2(I\lambda)^2\right] + k(I\lambda)^4$$

$$B_s(IU,IU) = -n^2\ b_s\ \bar{s}_\theta$$

$$B_s(IU,IV) = -nI\lambda/2 + n\ b_s\nu I\lambda\bar{e}_\theta^{\ sh}$$

$$B_s(IU,IW) = I\lambda$$

$$\boxed{A12}$$

$$B_s(IV,IV) = b_s\ \bar{s}_x(I\lambda)^2 - 2n^2\ b_s\bar{e}_\theta^{\ sh}$$

$$B_s(IV,IW) = -n\left[1 - 2b_s\bar{e}_\theta^{\ sh} + b_s\nu\bar{u}_x^{\ sh}\right]$$

$$B_s(IW,IW) = 1 + b_s\left[(I\lambda)^2\ \bar{s}_x - n^2\ \bar{s}_\theta\right]$$

$$C_s(IU) = \psi e_{oc}I\lambda\left[1 + b_s\nu\bar{e}_\theta^{\ sh}\right]\delta_{II_{oc}}$$

$$C_s(IV) = -\psi e_{oc}n\left[1 + b_s\bar{e}_\theta^{\ sh}\right]\delta_{II_{oc}}$$

$$\boxed{A13}$$

$$C_s(IW) = \psi e_{oc}\left[1 + b_s\left\{(2 - n^2)\bar{e}_\theta^{\ sh} + \nu(n^2 - 1)\bar{u}_x^{\ sh} + \bar{s}_x(I\lambda)^2\right\}\right]\delta_{II_{oc}}$$

$$A_f(IV,JV) = n^2 f\ S(I,J)$$

$$A_f(IV,JW) = A_f(IW,JV) = -nf\left[1 - (n^2 - 1)e_r\right] S(I,J) \qquad [A14]$$

$$A_f(IW,JW) = \left\{f\left[(1 + e_r)^2 - 2n^2 e_r + n^2(n^2 - 2)e_r^2\right] + g(n^2 - 1)^2\right\} S(I,J)$$

$$B_f(IV,JV) = 2n^2 b_f\ S(I,J)$$

$$B_f(IV,JW) = B_f(IW,JV) = -nb_f\left[2(1 + e_r) - n^2 e_r\right] S(I,J) \qquad [A15]$$

$$B_f(IW,JW) = n^2 b_f\ (1 + e_r)\ S(I,J)$$

$$C_f(IV) = -\psi n c_f\ C(I)$$

$$C_f(IW) = -\psi c_f \left[2(n^2 - 1)e_r + (n^2 - 2)\right] C(I) \qquad [A16]$$

where:-

$$IU = 3I-2 \qquad JU = 3J-2$$

$$IV = 3I-1 \qquad JV = 3J-1 \qquad [A17]$$

$$IW = 3I \qquad JW = 3J$$

$$k = h^2/12R^2 \qquad \lambda = \pi R/L \qquad b_s = Eh/R(1 - \nu^2) \qquad f = A_f N_s(1 - \nu^2)/hL$$

$$g = A_{m2} N_s (1 - \nu^2)/hLR^2 \qquad e_r = e/R \qquad b_f = -EA_f \bar{w}_f N_s/R^2 L \qquad c_f = -e_{oc} b_f \qquad [A18]$$

$$S(I,J) = (2/N_s) \sum_{i=1}^{N_f} \sin\,(I\pi x_i/L)\ \sin\,(J\pi x_i/L) \qquad [A19]$$

$$C(I) = (2/N_s) \sum_{i=1}^{N_f} \sin\,(I\pi x_i/L)\ \sin\,(I_{oc}\pi x_i/L) \qquad [A20]$$

7. Calculations are simplified if matrix M is cast in the form of equation [15] with m_o^{uu} etc defined as follows:

$$m_o^{uu} = n^2(1 + k)(1 - \nu)/2 - \psi n^2 b_s \bar{s}_\theta$$

$$m_2^{uu} = \lambda^2$$

$$m_1^{uv} = -n\lambda(1 + \nu)/2 + \psi\left[b_s \nu\lambda n \bar{e}_\theta^{sh} - n\lambda/2\right]$$

$$m_1^{uw} = \nu\lambda - n^2(1 - \nu)k\lambda/2 + \psi\lambda$$

$$m_3^{uw} = k\lambda^3$$

$$m_o^{vv} = n^2 - \psi 2n^2 b_s \bar{e}_\theta^{sh} \qquad [A21]$$

$$m_2^{vv} = (1 - \nu)(1 + 3k)\lambda^2/2 + \psi\bar{s}_x\lambda^2 b_s$$

$$m_o^{vw} = -n - \psi n\left[1 - 2b_s\bar{e}_\theta^{sh} + b_s\nu\bar{u}_x^{sh}\right]$$

$$m_2^{vw} = -n(3 - \nu)k\lambda^2/2$$

$$m_o^{ww} = (1 + k) + kn^2(n^2 - 2) + \psi\left[1 - n^2 b_s\bar{s}_\theta\right]$$

$$m_2^{ww} = 2n^2k\lambda^2 + \psi b_s\lambda^2\bar{s}_x$$

$$m_4^{ww} = k\lambda^4$$

where:-

$$\bar{s}_\theta = \bar{\sigma}_\theta^{sh}(1 - \nu^2)/E \qquad [A22]$$

$$\bar{s}_x = \bar{\sigma}_x^{sh}(1 - \nu^2)/E$$

8. With reference to equations [30] and [31], the c-vector coefficients appropriate to the Kendrick expressions are as follows:

$$c_1^u = -\psi e_{oc}\lambda\left[1 + b_s\nu\bar{e}_\theta^{sh}\right]$$

$$c_o^v = \psi e_{oc}n\left[1 + b_s\bar{e}_\theta^{sh}\right] \qquad \boxed{A23}$$

$$c_o^w = -\psi e_{oc}\left[1 + b_s\{(2 - n^2)\bar{e}_\theta^{sh} + \nu(n^2 - 1)\bar{u}_x^{sh}\}\right]$$

$$c_2^w = -\psi e_{oc}\bar{s}_x\lambda^2 b_s$$

$$c_f^v = \psi n c_f \qquad \boxed{A24}$$

$$c_f^w = \psi c_f\left[2(n^2 - 1)e_r + (n^2 - 2)\right]$$

9. Frame coefficients f^v, f^{vw} and f^w are given by:

$$f^v = n^2\left[f + \psi 2b_f\right]$$

$$f^{vw} = -n\left[f\{1 - (n^2 - 1)e_r\} + \psi b_f\{2(1 + e_r) - n^2 e_r\}\right] \qquad \boxed{A25}$$

$$f^w = f\left[(1 + e_r)^2 - 2n^2 e_r + n^2(n^2 - 2)e_r^2\right] + g(n^2 - 1)^2 + \psi b_f n^2(1 + e_r)$$

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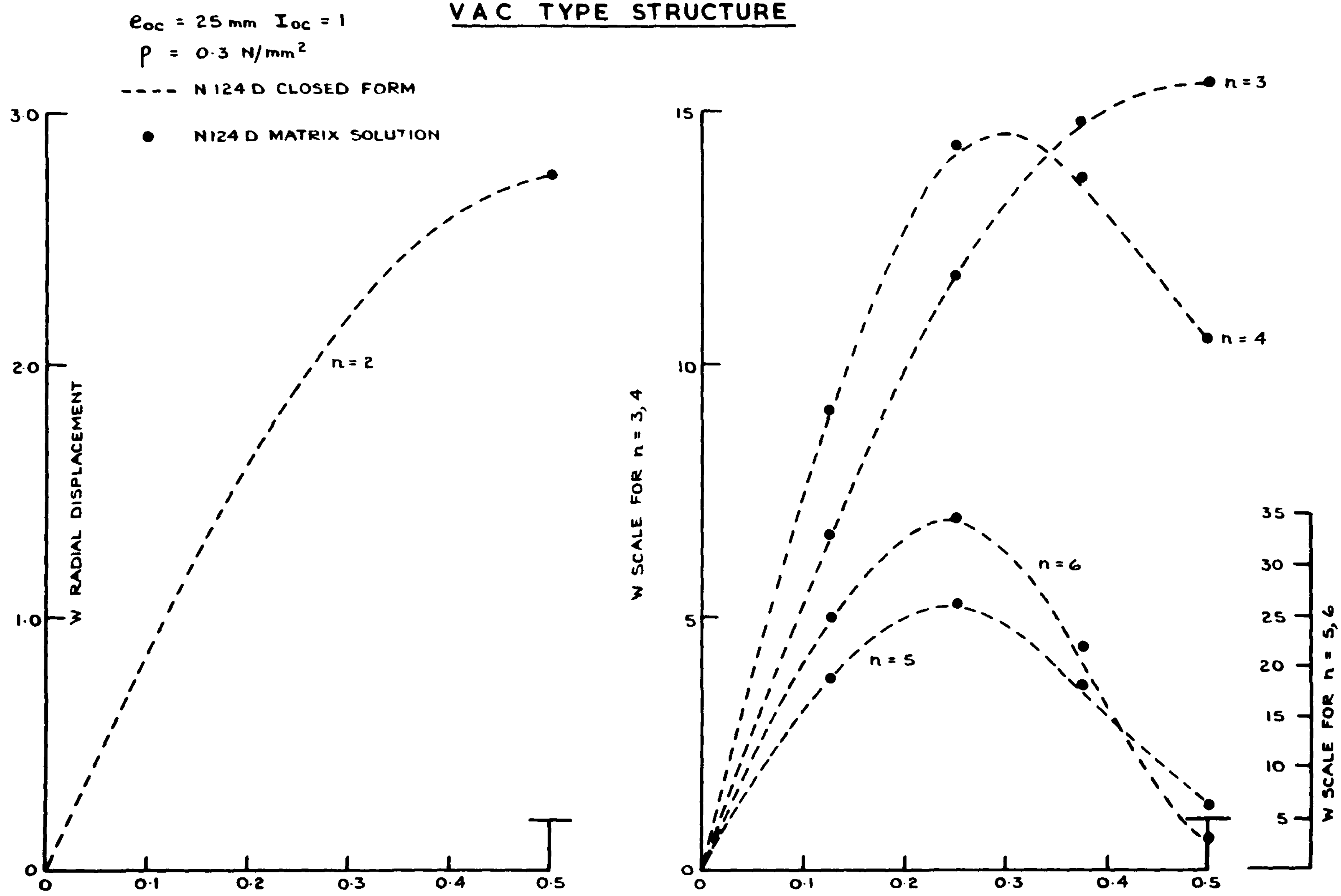

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Ring Stiffened Cylinders
Imperfect Shells
Shell Theory
Hydrostatic Loading

Summary

A closed form solution for the displacements and stresses of a slightly non-circular, ring-stiffened shell is derived rigorously from thin shell theory. The solution may be implemented on programmable pocket calculators, providing an accurate, but easy to use, method for predicting bending stresses.

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