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COMPARISON THEOREMS FOR REACTION-DIFFUSION SYSTEMS DEFINED IN A--ETC(U)
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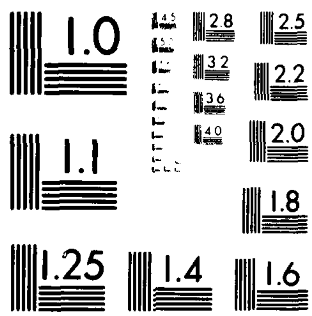

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COMPARISON THEOREMS FOR REACTION-DIFFUSION SYSTEMS DEFINED IN AN UNBOUNDED DOMAIN

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ABSTRACT

Comparison theorems are proved for systems of equations of the form

$$u_t = d_1 u_{xx} + f(x,t,u,v)$$
$$v_t = d_2 v_{xx} + g(x,t,u,v) \quad .$$

Here u and v are defined in $\mathbb{R} \times [0,T]$ for some positive time T, d_1 and d_2 are positive constants, and f and g are uniformly Lipschitz continuous functions defined for $(x,t) \in \mathbb{R} \times [0,T]$, $(u,v) \in \mathbb{R} \times \mathbb{R}$. It is assumed that f is a monotone, increasing or decreasing, function of v and g is a monotone, increasing or decreasing, function of u.

AMS (MOS) Subject Classification: 35K55

Key Words: Comparison theorems, Reaction-Diffusion equations

Work Unit Number 1 - Applied Analysis

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SIGNIFICANCE AND EXPLANATION

Comparison theorem techniques have played a central role in the study of scalar, nonlinear, parabolic differential equations. These techniques have proven less successful in the study of systems of equations for several reasons. Usually a very strong monotonicity condition must be imposed on the nonlinear terms of the equations. This severely restricts the applicability of the comparison theorems. Furthermore, there are technical difficulties associated with unbounded domains for systems of equations which are not present for scalar equations. In this report we demonstrate how these difficulties can be overcome for certain systems of reaction-diffusion equations. These systems have numerous applications including nerve conduction and mathematical ecology.

COMPARISON THEOREMS FOR REACTION-DIFFUSION SYSTEMS
DEFINED IN AN UNBOUNDED DOMAIN

David Terman

Section 1: Introduction

Comparison theorem techniques have played a central role in the study of scalar, nonlinear, parabolic differential equations (see [1], [2] for example). These techniques have proven less successful in the study of systems of equations for several reasons. Usually a very strong monotonicity condition must be imposed on the nonlinear terms of the equations. This severely restricts the applicability of the comparison theorems. Furthermore, there are technical difficulties associated with unbounded domains for systems of equations which are not present for scalar equations. In this report we demonstrate how these difficulties can be overcome for certain systems of reaction-diffusion equations. Since the main purpose of this report is to demonstrate how to treat unbounded domains we do not attempt to present the main theorem in its most general form. In fact, to better motivate the results we only consider systems of the form:

$$
\begin{aligned}
u_t &= d_1 u_{xx} + f(x,t,u,v) \\
v_t &= d_2 v_{xx} + g(x,t,u,v) \quad .
\end{aligned}
\tag{1.1}
$$

In a later report we shall show how to extend the results presented here to more general systems.

We assume throughout that $(x,t) \in \bar{\Omega}_T$ where $\Omega_T = R \times (0,T)$ for some positive T. The functions f and g are assumed to be uniformly Lipschitz continuous functions defined for $(x,t) \in \bar{\Omega}_T$ and $(u,v) \in R \times R$. The constants d_1 and d_2 are assumed to be positive. We must also assume that f is either an increasing or decreasing function of v, and g is either an increasing or decreasing function of u. We shall, therefore, consider the following three cases:

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(1.2)
(a) f is an increasing function of v and g is an increasing function of u,

(b) f is a decreasing function of v and g is a decreasing function of u,

(c) f is a decreasing function of v and g is an increasing function of u.

Systems of equations which satisfy one of the above conditions arise quite often in the Physical Sciences. For example, ecological models for two interacting species are often of this form. In fact, systems which satisfy (1.2a), (1.2b), or (1.2c) are sometimes referred to as, respectively, models for symbiosis, competition, or predator prey. The FitzHugh-Nagumo equations, which model the conduction of electrical impulses in the nerve axon, is another example of a system of equations which satisfies (1.2c). (See [4], [5]).

The comparison theorems are presented in the spirit of sub and supersolutions. We assume throughout that there exist functions (u,v), $(\underline{u},\underline{v})$, and $(\bar{u},\bar{v})$ which satisfy the following conditions:

(1.3)
(a) $u, v, \underline{u}, \underline{v}, \bar{u}$, and $\bar{v}$ are all bounded continuous functions in Ω_T,

(b) the first derivative with respect to t and second derivative with respect to x of $u, v, \underline{u}, \underline{v}, \bar{u}$, and $\bar{v}$ are all bounded continuous functions in Ω_T,

(c) $(\underline{u}(x,0),\underline{v}(x,0)) \leq (u(x,0),v(x,0) \leq (\bar{u}(x,0),\bar{v}(x,0))$ for $x \in \mathbb{R}$,

(d) $(\underline{u}(x,t),\underline{v}(x,t)) \leq (\bar{u}(x,t),\bar{v}(x,t))$ in Ω_T.

Here $(a,b) \leq (c,d)$ means that $a \leq c$ and $b \leq d$. To simplify the statement of the comparison theorems we define the following operators:

$$F(x,t,u,v) \equiv u_t - d_1 u_{xx} - f(x,t,u,v)$$
(1.4)
$$G(x,t,u,v) \equiv v_t - d_2 v_{xx} - g(x,t,u,v) \quad .$$

We are now ready to state the main theorems.

<u>Theorem 1</u>. Assume that (1.2a) and (1.3) are both satisfied. If

$$(F(x,t,\underline{u},\underline{v}),\ G(x,t,\underline{u},\underline{v})) \leq (F(x,t,u,v),\ G(x,t,u,v))$$

in Ω_T, then $(\underline{u},\underline{v}) \leq (u,v)$ in Ω_T.

Note that this theorem also implies that if (1.2a) and (1.3) are satisfied, and

$$(F(x,t,u,v),\ G(x,t,u,v)) \leq F(x,t,\bar{u},\bar{v}),\ G(x,t,\bar{u},\bar{v}))$$

in Ω_T, then $(u,v) \leq (\bar{u},\bar{v})$ in Ω_T.

<u>Theorem 2</u>. Assume that (1.2b) and (1.3) are both satisfied. If

$$F(x,t,\underline{u},\bar{v}) \leq F(x,t,u,v) \quad \text{and} \quad G(x,t,u,v) \leq G(x,t,\underline{u},\bar{v})$$

in Ω_T, then $\underline{u} \leq u$ and $v \leq \bar{v}$ in Ω_T.

Of course this theorem also implies that if (1.2b) and (1.3) are satisfied, and

$$F(x,t,u,v) \leq F(x,t,\bar{u},\underline{v}) \quad \text{and} \quad G(x,t,\bar{u},\underline{v}) \leq G(x,t,u,v)$$

in Ω_T, then $u \leq \bar{u}$ and $\underline{v} \leq v$ in Ω_T.

<u>Theorem 3</u>. Assume that (1.2c) and (1.3) are both satisfied. If

$$(F(x,t,\underline{u},\bar{v}),\ G(x,t,\underline{u},\underline{v})) \leq (F(x,t,u,v),\ G(x,t,u,v)) \leq (F(x,t,\bar{u},\underline{v}),\ G(x,t,\bar{u},\bar{v}))$$

in Ω_T, then $(\underline{u},\underline{v}) \leq (u,v) \leq (\bar{u},\bar{v})$ in Ω_T.

Note that in the predator-prey case (Theorem 3) one must have a lower bound and an upper bound at the same time. In the other two cases one is able to obtain one sided estimates. For this reason the predator-prey case is usually the most difficult to treat.

A number of authors have proved similar results under the additional assumption that the spacial domain of u and v is bounded. See, for example, Walter [6].

The main tool in the proof of Theorem 1 is the following comparison theorem for scalar nonlinear parabolic equations. The proof of this theorem, with slight modifications since, here, f and g depend on (x,t) may be found in [1].

<u>Theorem 4</u>. Suppose that u and v are bounded continuous functions in $\bar{\Omega}_T$ and u_t, v_t, u_{xx}, v_{xx} are bounded continuous functions in Ω_T. Suppose that $h(x,t,u)$ is a uniformly Lipschitz continuous function defined in $\bar{\Omega}_T \times \mathbb{R}$. Finally, suppose that

$$u_t - u_{xx} - h(x,t,u) \leq v_t - v_{xx} - h(x,t,v) \quad \text{in} \quad \Omega_T ,$$
$$u(x,0) \leq v(x,0) \qquad \text{in} \quad \mathbb{R} .$$

Then $u \leq v$ in Ω_T.

Theorems 1, 2, and 3 are proved by constructing a sequence of functions $(u_n(x,t), v_n(x,t))$, $n = 1,2,\ldots,$ which are solutions of scalar equations. It is shown that the sequence of functions (u_n, v_n) converge uniformly on bounded subsets of Ω_T to the solution (u,v). Hence, the proof of the above theorems also gives us the existence of a solution to System 1.1.

In this report we only prove Theorem 3 since the proofs of Theorem 1 and Theorem 2 are quite similar.

Section 2: Proof of Theorem 3

Theorem 3 is proved by approximating the solution of System (1.1) by a sequence of functions $(u_n(x,t), v_n(x,t))$, $n = 1,2,\dots$, which are solutions of scalar differential equations. We show, using Theorem 4, that for each n, $(\underline{u},\underline{v}) \le (u_n,v_n) \le (\bar{u},\bar{v})$ in Ω_T. We then show that the sequence of functions (u_n,v_n) converge uniformly on bounded subsets of Ω_T to the solution (u,v).

The functions $u_n(x,t)$ and $v_n(x,t)$ are defined as follows. Let $u_1(x,t)$ be the solution of the equations

$$F(x,t,u_1,\underline{v}) = F(x,t,u,v) \quad \text{in } \Omega_T ,$$
$$u_1(x,0) = u(x,0) \quad \text{in } \mathbb{R} .$$

Let $v_1(x,t)$ be the solution of the equations:

$$G(x,t,u_1,v_1) = G(x,t,u,v) \quad \text{in } \Omega_T ,$$
$$v_1(x,0) = v(x,0) \quad \text{in } \mathbb{R} .$$

Assuming that the functions $(u_1,v_1),\dots,(u_k,v_k)$ have been defined we let $u_{k+1}(x,t)$ be the solution of the equations:

$$F(x,t,u_{k+1},v_k) = F(x,t,u,v) \quad \text{in } \Omega_T ,$$
$$u_{k+1}(x,0) = u(x,0) \quad \text{in } \mathbb{R} .$$

We then let $v_{k+1}(x,t)$ be the solution of the equations:

$$G(x,t,u_{k+1},v_{k+1}) = G(x,t,u,v) \quad \text{in } \Omega_T ,$$
$$v_{k+1}(x,0) = v(x,0) \quad \text{in } \mathbb{R} .$$

In what follows it will be convenient to set $(u_0(x,t), v_0(x,t)) \equiv (\underline{u}(x,t), \underline{v}(x,t))$ and $(u_{-1}(x,t), v_{-1}(x,t)) = (\bar{u}(x,t), \bar{v}(x,t))$. We show, using induction, that for $n \ge 1$,

$$(2.1) \qquad (-1)^{n+1}u_{n-1}(x,t) \le (-1)^{n+1}u_n(x,t) \le (-1)^{n+1}u_{n-2}(x,t) \quad \text{in } \Omega_T ,$$

and

$$(2.2) \qquad (-1)^{n+1}v_{n-1}(x,t) \le (-1)^{n+1}v_n(x,t) \le (-1)^{n+1}v_{n-2}(x,t) \quad \text{in } \Omega_T .$$

First suppose that $n = 1$. We wish to show that $\underline{u}(x,t) \leq u_1(x,t) \leq \bar{u}(x,t)$ in Ω_T. This is proven using Theorem 4. Note that, by assumption, $\underline{u}(x,0) \leq u_1(x,0) \leq \bar{u}(x,0)$ in $\mathbb{R}$, and $F(x,t,\bar{u},\underline{v}) \geq F(x,t,u,v) = F(x,t,u_1,\underline{v})$ in Ω_T. Theorem 4 now implies that $\bar{u} \geq u_1$ in Ω_T. Moreover, since f is a decreasing function of v, it follows that $F(x,t,\underline{u},\underline{v}) \leq$ $\leq F(x,t,\underline{u},\bar{v})$ in Ω_T. Hence, $F(x,t,\underline{u},\underline{v}) \leq F(x,t,\underline{u},\bar{v}) \leq F(x,t,u,v) = F(x,t,u_1,\underline{v})$. Theorem 4 now implies that $\underline{u} \leq u_1$ in Ω_T.

Now suppose that (2.1) holds for some $n \geq 1$. We show that (2.2) holds. If n is even then

$$u_{n-2}(x,t) \leq u_n(x,t) \leq u_{n-1}(x,t) \quad \text{in} \quad \Omega_T \ .$$

Since $g(x,t,u,v)$ is an increasing function of u, it follows that

$$G(x,t,u_{n-1},v_n) \leq G(x,t,u_n,v_n) \leq G(x,t,u_{n-2},v_n) \quad \text{in} \quad \Omega_T \ .$$

Furthermore, since $G(x,t,u,v) = G(x,t,u_{n-1},v_{n-1}) = G(x,t,u_n,v_n) = G(x,t,u_{n-2},v_{n-2})$ in Ω_T it follows that

$$G(x,t,u_{n-2},v_{n-2}) \leq G(x,t,u_{n-2},v_n)$$

and

$$G(x,t,u_{n-1},v_n) \leq G(x,t,u_{n-1},v_{n-1}) \quad \text{in} \quad \Omega_T \ .$$

Using Theorem 4 and the assumptions that $v_{n-2}(x,0) = v_{n-1}(x,0) = v_n(x,0)$ in $\mathbb{R}$ we now conclude that

$$v_{n-2}(x,t) \leq v_n(x,t) \leq v_{n-1}(x,t) \quad \text{in} \quad \Omega_T \ .$$

Hence, (2.2) is satisfied. A similar argument shows that (2.2) holds if n is odd.

We now assume that (2.2) holds and show that (2.1) holds with n replaced by $n + 1$. This will complete the induction argument. If n is even, then

$$v_{n-2}(x,t) \leq v_n(x,t) \leq v_{n-1}(x,t) \quad \text{in} \quad \Omega_T \ .$$

Since $f(x,t,u,v)$ is a decreasing function of v it follows that

$$F(x,t,u_{n+1},v_{n-2}) \leq F(x,t,u_{n+1},v_n) \leq F(x,t,u_{n+1},v_{n-1}) \quad \text{in} \quad \Omega_T \ .$$

Moreover, since $F(x,t,u,v) = F(x,t,u_{n+1},v_n) = F(x,t,u_n,v_{n-1}) = F(x,t,u_{n-1},v_{n-2})$ in Ω_T it follows that

$$F(x,t,u_{n+1},v_{n-2}) \leq F(x,t,u_{n-1},v_{n-2})$$

and

$$F(x,t,u_{n+1},v_{n-1}) \geq F(x, ,u_n,v_{n-1}) \quad \text{in } \Omega_T \ .$$

Theorem 4 and the assumption that $u_{n+1}(x,0) = u_n(x,0) = u_{n-1}(x,0)$ in $\mathbb{R}$ now imply that

$$u_n(x,t) \leq u_{n+1}(x,t) \leq u_{n-1}(x,t) \quad \text{in } \Omega_T \ .$$

Hence, (2.1) holds with n replaced by $n + 1$ if n is even. A similar argument shows that this is true if n is odd.

We have now shown that

$$\underline{u} \leq u_2 \leq u_4 \leq \cdots \leq u_{2n} \leq \cdots\cdots \leq u_{2n+1} \leq \cdots \leq u_3 \leq u_1 \leq \bar{u}$$

and

$$\underline{v} \leq v_2 \leq v_4 \leq \cdots \leq v_{2n} \leq \cdots\cdots \leq v_{2n+1} \leq \cdots \leq v_3 \leq v_1 \leq \bar{v}$$

in Ω_T. Hence, there exist pairs of functions $(\underline{U},\underline{V})$ and $(\bar{U},\bar{V})$ such that (u_{2n},v_{2n}) converges to $(\underline{U},\underline{V})$ and (u_{2n+1},v_{2n+1}) converges to $(\bar{U},\bar{V})$ uniformly on bounded subsets of Ω_T as $n \to \infty$. Clearly, $(\underline{u},\underline{v}) \leq (\underline{U},\underline{V}) \leq (\bar{U},\bar{V}) \leq (\bar{u},\bar{v})$ in Ω_T. To complete the proof of Theorem 3 we show that $(u,v) \equiv (\underline{U},\underline{V}) \equiv (\bar{U},\bar{V})$ in Ω_T.

Let $K(x,t)$ be the fundamental solution for the heat equation. That is,

$$K(x,t) = \frac{1}{2\pi^{1/2} t^{1/2}} e^{-x^2/4t} \ .$$

Then, setting $\lambda_1 = d_1^{-1/2}$ and $\lambda_2 = d_2^{-1/2}$, we have for each $n \geq 1$,

$$u_{2n}(x,t) = \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t)u(\xi,0)d\xi +$$

$$+ \int_0^t \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t-\tau)[f(\xi,\tau,u_{2n},v_{2n-1}) - F(\xi,\tau,u,v)]d\xi d\tau$$

(2.3)

$$v_{2n}(x,t) = \int_{-\infty}^{\infty} K(\lambda_2 x-\xi,t)v(\xi,0)d\xi +$$

$$+ \int_0^t \int_{-\infty}^{\infty} K(\lambda_2 x-\xi,t-\tau)[g(\xi,\tau,u_{2n},v_{2n}) - G(\xi,\tau,u,v)]d\xi d\tau \ .$$

Passing to the limit, $n \to \infty$, in (2.3) it follows that

(2.4a)
$$\underline{U}(x,t) = \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t)u(\xi,0)d\xi +$$

$$+ \int_0^t \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t-\tau)[f(\xi,\tau,\underline{U},\bar{V}) - F(\xi,\tau,u,v)]d\xi d\tau$$

(2.4b) $$\underline{V}(x,t) = \int_{-\infty}^{\infty} K(\lambda_2 x-\xi,t)v(\xi,0)d\xi + $$
$$+ \int_0^t \int_{-\infty}^{\infty} K(\lambda_2 x-\xi,t-\tau)[g(\xi,\tau,\underline{U},\underline{V}) - G(\xi,\tau,u,v)]d\xi d\tau \quad .$$

Similarly, $\bar{U}$, $\bar{V}$ satisfy the equations

(2.5a) $$\bar{U}(x,t) = \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t)u(\xi,0)d\xi + $$
$$+ \int_0^t \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t-\tau)[f(\xi,\tau,\bar{U},\underline{V}) - F(\xi,\tau,u,v)]d\xi d\tau$$

(2.5b) $$\bar{V}(x,t) = \int_{-\infty}^{\infty} K(\lambda_2 x-\xi,t)v(\xi,0)d\xi + $$
$$+ \int_0^t K(\lambda_2 x-\xi,t)[g(\xi,\tau,\bar{U},\bar{V}) - G(\xi,\tau,u,v)]d\xi d\tau \quad .$$

'Subtracting' (2.5a) from (2.4a) and (2.5b) from (2.4b) one finds that in Ω_T,

(2.6a) $$\underline{U}(x,t) - \bar{U}(x,t) = \int_0^t \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t-\tau)[f(\xi,t,\underline{U},\bar{V}) - f(\xi,\tau,\bar{U},\underline{V})]d\xi d\tau$$

(2.6b) $$\underline{V}(x,t) - \bar{V}(x,t) = \int_0^t \int_{-\infty}^{\infty} K(\lambda_2 x-\xi,t-\tau)[g(\xi,\tau,\underline{U},\underline{V}) - g(\xi,\tau,\bar{U},\bar{V})]d\xi d\tau \quad .$$

Let $w(x,t) = \underline{U}(x,t) - \bar{U}(x,t)$ and $z(x,t) = \underline{V}(x,t) - \bar{V}(x,t)$. Then from (2.6) and the assumptions on f and g, there exist bounded functions $\beta_1(x,t)$, $\beta_2(x,t)$, $\gamma_1(x,t)$, and $\gamma_2(x,t)$ such that in Ω_T,

(2.7a) $$w(x,t) = \int_0^t \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t-\tau)[\beta_1(\xi,\tau)w(\xi,\tau) + \gamma_1(\xi,\tau)z(\xi,\tau)]d\xi d\tau$$

(2.7b) $$z(\xi,\tau) = \int_0^t \int_{-\infty}^{\infty} K(\lambda_2 x-\xi,t-\tau)[\beta_2(\xi,\tau)w(\xi,\tau) + \gamma_2(\xi,\tau)z(\xi,\tau)]d\xi d\tau \quad .$$

Let

$$B = \sup_{(x,t)\in\Omega_T} \{|\beta_1(x,t)| + |\beta_2(x,t| + |\gamma_1(x,t)| + |\gamma_2(x,t)|\}$$

and

$$p(t) = \sup_{x\in\mathbb{R}} \{|w(x,t)| + |z(x,t)|\} \quad .$$

Then, adding (2.7a) and (2.7b) we conclude that

$$p(t) \leq 2B \int_0^t p(\tau)d\tau \quad \text{for} \quad t \in (0,T) \quad .$$

From Gronwall's inequality it follows that $p(t) \equiv 0$ in $(0,T)$. Hence,

$$(\underline{U},\underline{V}) \equiv (\bar{U},\bar{V}) \quad \text{in} \quad \Omega_T \quad . \tag{2.8}$$

It remains to show that $(u,v) \equiv (\underline{U},\underline{V})$ in Ω_T. Since this follows from an argument similar to the one just given we shall only sketch the proof. First note that (2.5) and (2.8) imply that $(\underline{U},\underline{V})$ satisfy the equations

$$\begin{aligned} \underline{U}(x,t) &= \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t)u(\xi,0)d\xi + \int_0^t \int_{-\infty}^{\infty} K(\lambda_1 x-\xi,t-\tau)]f(\xi,\tau,\underline{U},\underline{V}) - F(\xi,\tau,u,v)]d\xi d\tau \\ \underline{V}(x,t) &= \int_{-\infty}^{\infty} K(\lambda_2 x-\xi,t)v(\xi,0)d\xi + \int_0^t \int_{-\infty}^{\infty} K(\lambda_2 x-\xi,t-\tau)[g(\xi,\tau,\underline{U},\underline{V}) - G(\xi,\tau,u,v)]d\xi d\tau \quad . \end{aligned} \tag{2.9}$$

However, from (1.4), it follows that (u,v) is also a solution of the equations (2.9). Therefore, the proof of Theorem 3 will be completed once we show that the solution of the equations (2.9) is unique. Because this follows from a Gronwall type argument similar to the one just given we do not give the details.

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Comparison theorems, Reaction-Diffusion equations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Comparison theorems are proved for systems of equations of the form

$$u_t = d_1 u_{xx} + f(x,t,u,v)$$

$$v_t = d_2 v_{xx} + g(x,t,u,v) \quad .$$

Here u and v are defined in $\mathbb{R} \times [0,T]$ for some positive time T, d_1 and d_2 are positive constants, and f and g are uniformly Lipschitz continuous

(continued)

ABSTRACT (continued)

functions defined for $(x,t) \in \mathbb{R} \times [0,T]$, $(u,v) \in \mathbb{R} \times \mathbb{R}$. It is assumed that f is a monotone, increasing or decreasing, function of v and g is a monotone, increasing or decreasing function of u.

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