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THE INFLUENCE OF SURFACE TENSION ON CAVITATING FLOW PAST A CURV--ETC(U)

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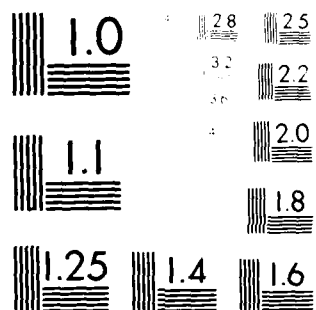
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THE INFLUENCE OF SURFACE TENSION ON
CAVITATING FLOW PAST A CURVED OBSTACLE

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ABSTRACT

The classical solution for cavitating flow past a curved obstacle leaves the position of the separation points undetermined. It is shown that this degeneracy is removed by introducing surface tension. A unique solution is obtained by requiring the flow to leave the obstacle tangentially. As the surface tension tends to zero this solution tends to the classical solution satisfying the Brillouin-Villat condition. A perturbation solution for small values of the surface tension is derived. Graphs of the results for the cavitating flow past a circular cylinder are presented.

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Key Words: Surface tension, Cavitation

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SIGNIFICANCE AND EXPLANATION

The classical Helmholtz-Kirchhoff solution for cavitating flow past a flat plate yields infinite curvature of the free-surface at the edges of the plate. Ackerberg (1975) and Cumberbatch and Norbury (1979) attempted to remove this singularity in the curvature by including surface tension. Although they obtain a solution in the neighborhood of the separation point they did not match it with any acceptable outer solution. The problem was solved by Vanden Broeck (1981) who provided conclusive analytical and numerical evidence that the slope is not continuous at the separation points. Thus the inclusion of surface tension in the Helmholtz-Kirchhoff solution does not remove the infinite curvature singularity at the separation points. On the contrary it makes the problem more singular by introducing a discontinuity in slope and therefore an infinite velocity at these points.

In the present paper we generalize Vanden Broeck's results to the cavitating flow past a curved obstacle. (See Figure 1.) The position of the separation point may be either fixed if it is at a pointed corner of the body, or free if it is at a certain location of a smoothly curved obstacle. The classical solution without surface tension is computed numerically by collocation. This solution is then used to construct an asymptotic solution for small values of the surface tension. It is found that for most positions of the separation point, the slope is not continuous at the separation points. The velocity is infinite or equal to zero there. However, for a given value of the surface tension there exists a particular position of the separation points for which the slope is continuous. This solution tends to the classical solution satisfying the Brillouin-Villat condition as the surface tension tends to zero. Graphs of the results for the flow past a circular cylinder are included.

The results presented are obtained from a local solution in the vicinity of the separation point. They are therefore useful for any problem in which a free streamline separates from a rigid obstacle.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

THE INFLUENCE OF SURFACE TENSION ON CAVITATING FLOW
PAST A CURVED OBSTACLE

Jean-Marc Vanden Broeck

1. Introduction.

In recent years important progress has been achieved in the understanding of the influence of surface tension on cavitating flow past a flat plate. The classical Helmholtz-Kirchhoff solution yields infinite curvature of the free surface at the edges of the plate. Ackerberg (1975) constructed an asymptotic solution for small values of the surface tension in which the slope and the curvature of the free surface at the edges are both equal to those of the plate. Ackerberg's solution contains capillary waves downstream. Cumberbatch and Norbury (1979) observed that these waves are not physically acceptable because they require a supply of energy from infinity. They suggested that solutions without waves could be obtained by forcing the slope of the free surface at the edges to be equal to the slope of the plate and allowing the curvature to be different from zero at the edges. Although they obtained a local solution, they did not succeed in matching it with any outer solution. The problem was solved by Vanden Broeck (1981) who provided conclusive analytical and numerical evidence that the slope is not continuous at the separation points. Both velocity and curvature are infinite there.

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In the present paper we generalize Vanden Broeck's results to the cavitating flow past a curved obstacle (see Figure 1). The position of the separation point may be either fixed if it is at a pointed corner of the body, or free if it is at a certain location of a smoothly curved obstacle. An example of fixed detachment is provided by the cavitating flow past a flat plate in which the flow leaves the plate at the edges. Similarly the flow sketched in Figure 1 corresponds to fixed detachment if the obstacle is cut along the straight line AB. In the case of free detachment the classical solution leaves the position of the separation points A and B undetermined. This degeneracy is usually resolved by imposing the Brillouin-Villat condition which requires the curvature of the free surface to be finite at the separation points. (Birkhoff and Zarantonello (1957)).

The problem is formulated in Section 2 and the classical solution without surface tension is computed numerically in Section 3. The scheme is similar in philosophy if not in details to the scheme derived by Brodetsky (1923) and later extended by Birkhoff et. al. (1953, 1954). Explicit results are presented for the cavitating flow past a circular cylinder.

In Section 4 the numerical solution of Section 3 is used to construct an asymptotic solution for small values of the surface tension. It is found that for most positions of the separation points, the slope is not continuous at A and B. The velocity is infinite or equal to zero there. However for a given value of the surface tension there exists a particular position of the separation points A and B for which the slope is continuous at A and B. This solution tends to the classical solution satisfying the Brillouin-Villat condition as the surface tension tends to zero.

2. Formulation.

We consider the cavitating flow past a curved obstacle (see Figure 1). We denote by L a typical dimension of the obstacle. At infinity we have a flow with constant velocity U . The fluid is assumed to be inviscid and incompressible. We restrict our attention to obstacles which are symmetrical with respect to the direction of the velocity at infinity. Flows past non-symmetrical obstacles can be treated similarly. It is convenient to introduce dimensionless variables by choosing L as the unit length and U as the unit velocity.

We introduce the dimensionless potential ϕ and stream function ψ . The constant b is chosen such that $\phi = 1$ at the separation points. Without loss of generality we choose $\phi = 0$ at $x = y = 0$. The free surface, the obstacle and the negative x -axis are portions of the stream-line $\psi = 0$.

We denote the complex velocity by $u - iv$ and we define the function $\tau - i\theta$ by the relation

$$u - iv = e^{\tau - i\theta}. \quad (2.1)$$

We shall seek $\tau - i\theta$ as an analytic function of $f = \phi + i\psi$ in the half plane $\psi \leq 0$. The complex potential plane is sketched in Figure 2. At infinity we require the velocity to be unity in the x -direction so that the function $\tau - i\theta$ vanishes at infinity in view of (2.1).

On the surface of the cavity the Bernoulli equation and the pressure jump due to surface tension yield

$$\frac{1}{2}q^2 - \frac{T}{\rho}K = \frac{1}{2}U^2. \quad (2.2)$$

Here q is the flow speed, K the curvature of the cavity surface counted positive when the center of curvature lies inside the fluid regions, T the surface tension and ρ the density. In dimensionless variables this

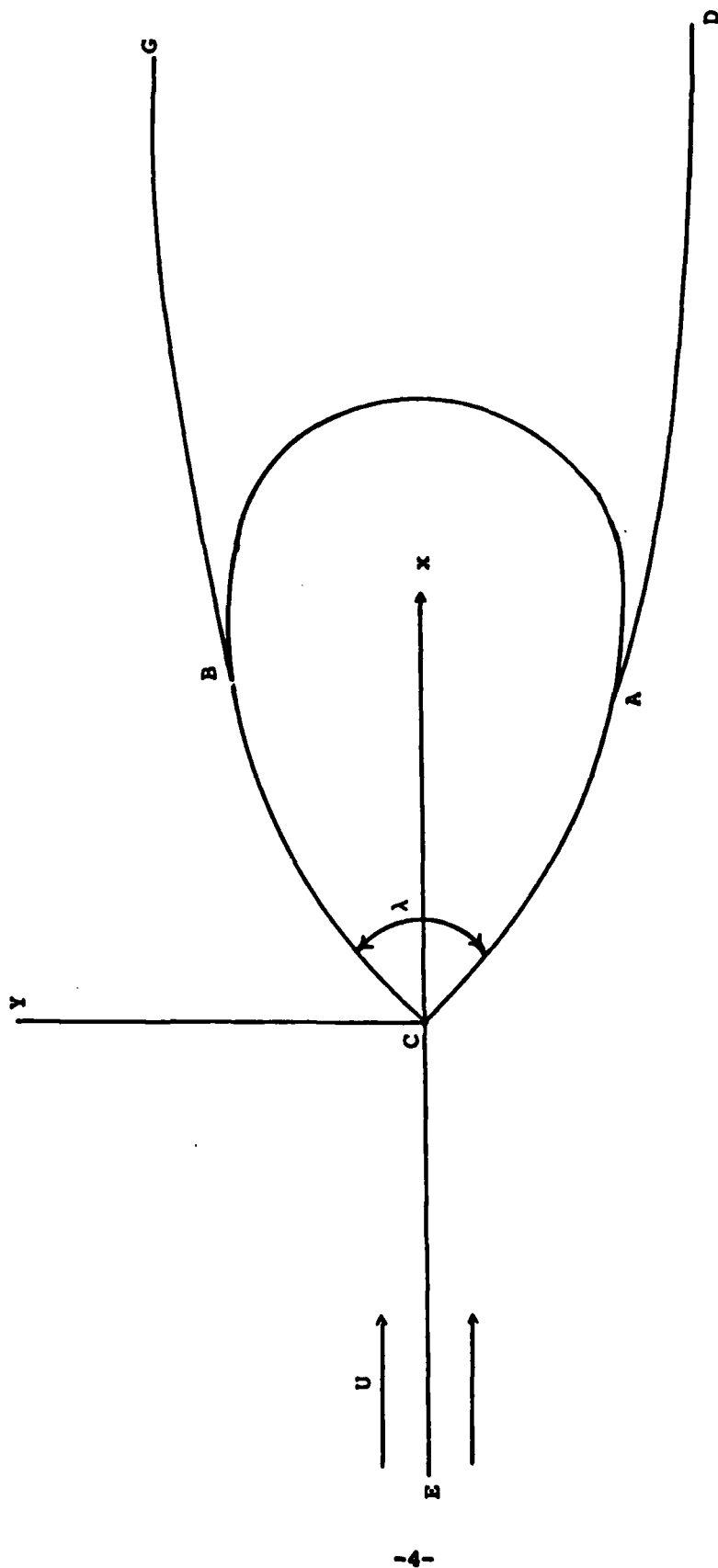


Figure 1
Sketch of the flow and the coordinates.

becomes (see Ackerberg (1975) for details)

$$\frac{e^{\tau}}{b} \frac{\partial \theta}{\partial \phi} = \frac{\alpha}{2}(e^{2\tau} - 1), \quad 1 < \phi < \infty. \quad (2.3)$$

Here α is the Weber number defined by

$$\alpha = \frac{\rho U^2 L}{T} \quad (2.4)$$

The symmetry of the problem and the kinematic condition on the obstacle yield

$$\theta(\phi) = 0, \quad \psi = 0, \quad \phi < 0 \quad (2.5)$$

$$F[x(\phi), y(\phi)] = 0, \quad \psi = 0, \quad 0 < \phi < 1. \quad (2.6)$$

Here $F(x, y) = 0$ is the equation of the shape of the obstacle and the functions $\theta(\phi)$, $x(\phi)$ and $y(\phi)$ denote respectively $\theta(\phi, 0-)$, $x(\phi, 0-)$ and $y(\phi, 0-)$.

This completes the formulation of the problem of determining the function $\tau - i\theta$ and the constant b . For each value of α , $\tau - i\theta$ must be analytic in the half plane $\psi \leq 0$ and satisfy the boundary conditions (2.3), (2.5) and (2.6).

3. Solution without surface tension.

When surface tension is neglected, the Weber number is infinite and the condition (2.3) reduces to the free-streamline condition $\tau = 0$.

We define the new variable t by the transformation

$$\sqrt{f} = (t - \frac{1}{t}) \frac{1}{2i}. \quad (3.1)$$

The problem in the complex plane t is illustrated in Figure 3. Following Brodetsky (1923) we introduce the function $\Omega'(t)$ by the relation

$$\tau - i\theta = -\frac{\lambda}{\pi} \log \frac{1+t}{1-t} - \Omega'(t) \quad (3.2)$$

where the angle λ is defined in Figure 1. The conditions (2.3) and (2.5) show that $\Omega'(t)$ can be expressed in the form of a Taylor expansion in odd powers of t . Hence

$$\tau - i\theta = -\frac{\lambda}{\pi} \log \frac{1+t}{1-t} - \sum_{n=1}^{\infty} A_n t^{2n-1}. \quad (3.3)$$

The function (3.3) satisfy the conditions (2.3) and (2.5). The coefficients A_n have to be determined to satisfy the condition (2.6) on the surface $A C B$ of the obstacle. We use the notation $t = r e^{i\sigma}$ so that points on $A C B$ are given by $r = 1$, $-\frac{\pi}{2} < \sigma < \frac{\pi}{2}$. Using (3.1) and (2.1) we have

$$\frac{\partial x}{\partial \sigma} = b \sin 2\sigma e^{-\tau} \cos \theta, \quad \rho = 1, \quad -\frac{\pi}{2} < \sigma < \frac{\pi}{2} \quad (3.4)$$

$$\frac{\partial y}{\partial \sigma} = b \sin 2\sigma e^{-\tau} \sin \theta, \quad \rho = 1, \quad -\frac{\pi}{2} < \sigma < \frac{\pi}{2}. \quad (3.5)$$

We solve the problem approximately by truncating the infinite series in (3.3) after N terms. We find the N coefficients A_n and the constant b by a hybrid method involving collocation and finite differences. Substituting $t = e^{i\sigma}$ into (3.3) we have

$$\theta(\sigma) = \frac{\lambda}{2} + \sum_{n=1}^N A_n \sin[(2n-1)\sigma] \quad (3.6)$$

$$\tau(\sigma) = -\frac{\lambda}{\pi} \log \frac{\sin \sigma}{1 - \cos \sigma} - \sum_{n=1}^N A_n \cos[(2n-1)\sigma] \quad (3.7)$$

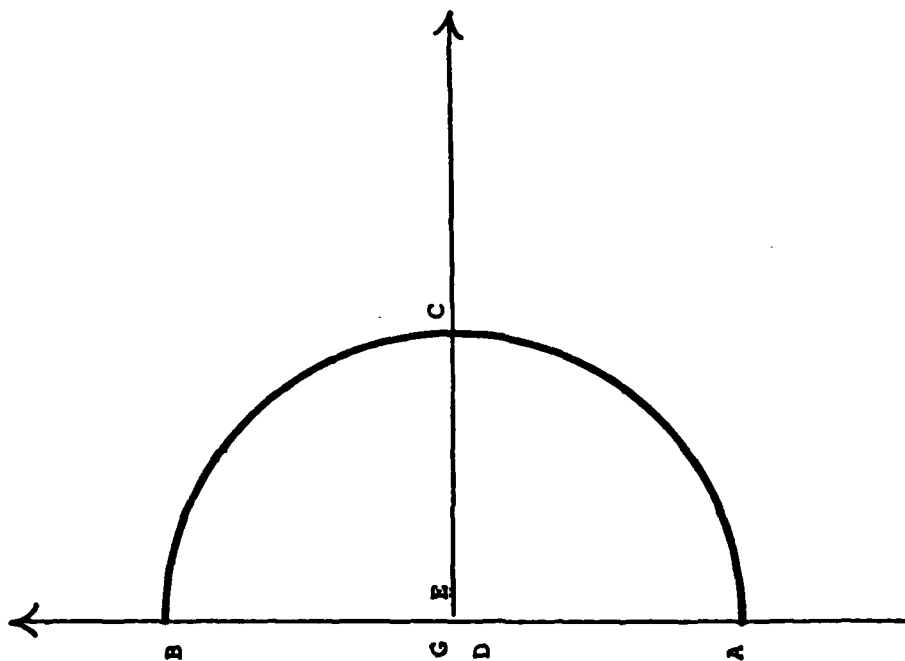


Figure 3

The t-plane

We now introduce the N mesh points

$$\sigma_I = -\frac{\pi}{2N} I, \quad I = 1, \dots, N \quad (3.8)$$

and the N intermediate mesh points

$$\sigma_I^M = -\frac{\pi}{2N} (I - \frac{1}{2}), \quad I = 1, \dots, N. \quad (3.9)$$

Using (3.4) - (3.7) and (3.9) we obtain $(\frac{\partial x}{\partial \sigma})_{\sigma=\sigma_I^M}$ and $(\frac{\partial y}{\partial \sigma})_{\sigma=\sigma_I^M}$ in terms of the coefficients A_n and the constant b . These expressions enable us to evaluate $x(\sigma_I)$ and $y(\sigma_I)$ by the trapezoidal rule. Then (2.6) provides N algebraic equations for the $N+1$ unknowns A_n and b , namely

$$F[x(\sigma_I), y(\sigma_I)] = 0, \quad I = 1, \dots, N. \quad (3.10)$$

The last equation is obtained by specifying the abscissa w of the separation point A . Thus

$$x(-\frac{\pi}{2}) = w. \quad (3.11)$$

The system (3.10)-(3.11) is easily solved by Newton's method. Explicit computations were performed for the cavitating flow past a circular cylinder. The unit length L was chosen as the radius of the cylinder. The scheme converges rapidly and the solutions obtained were found to agree with the numerical results given by Birkhoff and Zarantonello (1957).

Profiles of the cavity for various values of the angular position γ of the separation points are presented in Figure 4. For $\gamma < \gamma^* \approx 55^\circ$ the free surface enters the body. These solutions are acceptable if the body is cut along the straight line AB . For $\gamma > \gamma^{**} \approx 124^\circ$, the free surfaces cross over and the corresponding solutions are not physically acceptable. Physically acceptable solutions for $\gamma > \gamma^{**}$ can be obtained by using the method presented by Vanden-Broeck and Keller (1980) to prevent overlapping in capillary waves of large amplitude. These solutions are found to be the cusped cavities considered before by Southwell and Vaisey (1946), Lighthill (1949) and others (see Figure 4). The pressure in the cavity is found as part of the

solution. Similarly in the work of Vanden-Broeck and Keller (1980) the pressure in the trapped bubble was found as part of the solution. As γ tends to γ^{**} the pressure in the cavity tends to zero. As γ tends to 180° the cavity shrinks to a point, and the solution reduces to the classical potential flow past a circle. Thus the family of cusped cavities is the physical continuation for $\gamma > \gamma^{**}$ of the family of open cavities.

The curvature of the free surface in the neighborhood of the separation point A is given by the formula (Brodetsky (1923))

$$\frac{1}{b} \frac{\partial \theta}{\partial \phi} \sim -\frac{1}{2} C (\phi - 1)^{-1/2} \text{ or } \phi \rightarrow 1 \quad (3.12)$$

where

$$C = -\frac{\lambda b^{-1/2}}{\pi} - b^{-1/2} \sum_{n=1}^N (-1)^{n+1} (2n-1) A_n. \quad (3.13)$$

These formula are true for the cavitating flow past any curved obstacle.

A graph of C versus the angular position of γ of the separation points for the flow past a circular cylinder is shown in Figure 5. The constant C vanishes for $\gamma = \gamma^*$. Thus (3.12) shows that the curvature at the separation points is infinite unless $\gamma = \gamma^*$. If we impose the Brillouin-Villat condition, the problem with free detachment has a unique solution corresponding to $\gamma = \gamma^*$.

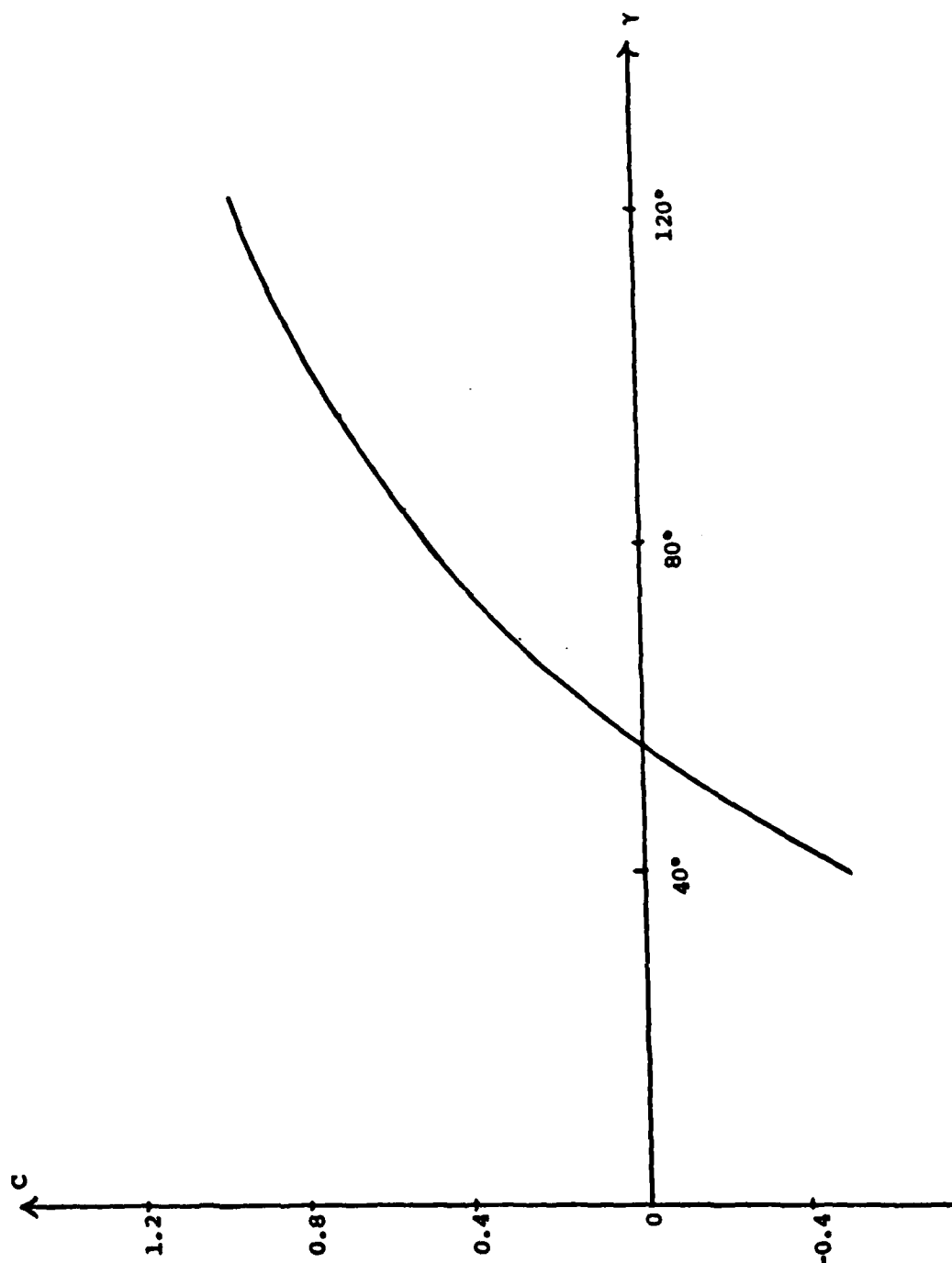


Figure 5
Computed values of the parameter C as a function of the angle γ

4. Perturbation solution for small values of the surface tension.

We seek a solution in the vicinity of the separation point A. Following Ackerberg (1975) we introduce the following scaling of the variables

$$f^* = \alpha(bf - b) \quad (4.1)$$

$$\tau^* - i\theta^* = \alpha^{1/2}(\tau - i\theta - i\frac{\pi}{2} + i\gamma). \quad (4.2)$$

The function τ^* satisfies Laplace's equation in the lower half plane

$\psi^* < 0$. Thus

$$\frac{\partial^2 \tau^*}{\partial \phi^{*2}} + \frac{\partial^2 \tau^*}{\partial \psi^{*2}} = 0 \quad \text{in} \quad \psi^* < 0. \quad (4.3)$$

The boundary conditions (2.3) and (2.6) linearize in the limit $\alpha \rightarrow \infty$ so that the boundary conditions on $\psi^* = 0$ are (see Ackerberg (1975) for details)

$$\frac{\partial \tau^*}{\partial \psi^*} = 0 \quad \text{on} \quad \psi^* = 0 \quad \text{for} \quad \phi^* < 0 \quad (4.4)$$

$$\frac{\partial \tau^*}{\partial \psi^*} = \tau^* \quad \text{on} \quad \psi^* = 0 \quad \text{for} \quad \phi^* > 0. \quad (4.5)$$

Relation (3.12) gives the behavior

$$\tau^* \sim \text{Im } C(f^*)^{1/2} \quad \text{as} \quad |f^*| \rightarrow \infty. \quad (4.6)$$

Cumberbatch and Norbury (1979) noticed that the problem (4.3) - (4.5) had been treated by Friedrichs and Levy (1948). The solution of (4.3) - (4.6) not containing waves and having the weakest singularity at A is given on the free surface by

$$\theta^*(\phi^*) = -\frac{C}{2} \left(\frac{\pi}{\alpha}\right)^{1/2} - \frac{C}{2(\pi)^{1/2}} (\phi^* \ln \phi^* - \phi^*) \quad (4.7)$$

$$\tau^*(\phi^*) = \frac{1}{2\sqrt{\pi}} C \ln \phi^*. \quad (4.8)$$

The leading order terms in (4.7) and (4.8) correspond to flow past a corner of angle

$$\delta = \pi - \frac{C}{2} \left(\frac{\pi}{\alpha}\right)^{1/2}. \quad (4.9)$$

However the solution (4.7), (4.8) is not valid near $\phi = 1$ because τ^* is unbounded at $\phi = 1$. Following Vanden-Broeck (1981) we seek a local solution

which corresponds to a flow past a corner of angle δ . Thus we write

$$e^{\tau} \sim E(\phi - 1)^{\pi/(2\pi-\delta) - 1}. \quad (4.10)$$

Here E is a constant to be determined as part of the solution. Substituting

(4.10) into (2.3) we have

$$\frac{\partial \theta}{\partial \phi} \sim \frac{\alpha b}{2} \{E(\phi - 1)^{\pi/(2\pi-\delta)-1} - E^{-1}(\phi - 1)^{1-\pi/(2\pi-\delta)}\}. \quad (4.11)$$

Matching (4.11) and (4.7) we find

$$E = 1. \quad (4.12)$$

Thus we have succeeded in matching the solution (4.7), (4.8) with a local solution corresponding to the flow past a corner of angle δ . In particular these results imply that

$$\theta(1) = -\frac{\pi}{2} + \gamma - \frac{C}{2} \left(\frac{\pi}{\alpha}\right)^{1/2}. \quad (4.13)$$

Relation (4.9) shows that $\delta > \pi$ for $C < 0$ and $\delta < \pi$ for $C > 0$.

Therefore the velocity at the separation points is infinite for $C < 0$ and equal to zero for $C > 0$.

Graphs of $\theta(1)$ versus $\alpha^{-1/2}$ for the circular cylinder are shown in Figure 6. The velocity at the separation points is infinite for $\gamma < \gamma^*$ and equal to zero for $\gamma > \gamma^*$.

Although we did only compute an asymptotic solution for α large, we have every reason to believe that a solution exists for all values of α . As α tends to zero, the free surfaces must approach two horizontal straight lines. Therefore

$$\lim_{\alpha \rightarrow 0} \theta(1) = 0. \quad (4.14)$$

Providing $\theta(1)$ is a continuous function of α , Figure 6 and (4.14) imply the existence for each value of $\gamma^* < \gamma < 90^\circ$ of one value of $0 < \alpha < \infty$ for which $\theta_1 = -\frac{\pi}{2} + \gamma$. We describe this relation between α and γ by the function

$$\gamma = g(\alpha). \quad (4.15)$$

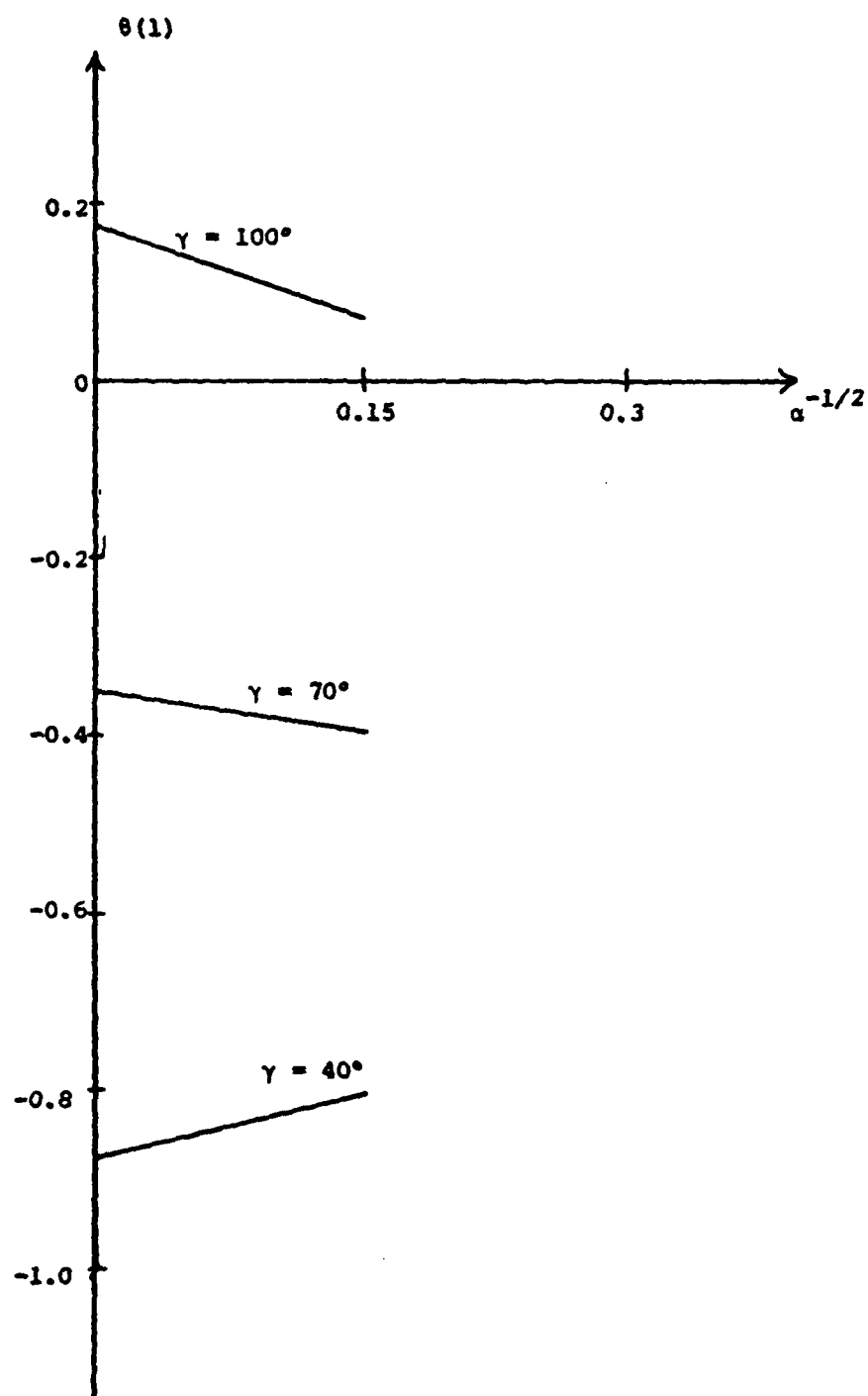


Figure 6
 Computed values of $\theta(1)$ as a function of $\alpha^{-1/2}$ for
 $\gamma = 40^\circ, 70^\circ$ and 100° .

This result can be reformulated as follows. For each value of the Weber number α there exists an angular position $\gamma = g(\alpha)$ of the separation points for which the flow leaves the obstacle tangentially.

As α tends to zero the free surfaces tend to two horizontal straight lines. This solution leaves the cylinder tangentially only if $\gamma = 90^\circ$.

Therefore

$$\lim_{\alpha \rightarrow 0} g(\alpha) = 90^\circ. \quad (4.16)$$

As α tends to infinity, the solution is described by the asymptotic solution (4.7) and (4.8). This solution leaves the obstacle tangentially only if $C = 0$ (see formula (4.9)). Therefore Figure 6 implies

$$\lim_{\alpha \rightarrow \infty} g(\alpha) = \gamma^*. \quad (4.17)$$

Relation (4.17) shows that the family of solution defined by (4.15) tends to the classical solution satisfying the Brillouin-Villat condition, as $\alpha \rightarrow \infty$.

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