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PRODUCTION LOT SIZING AND THE POWER OF 2

Research Report No. 82-8

by

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June, 1982

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PRODUCTION LOT SIZING AND THE POWER OF 2

Thom J. HODGSON
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I. INTRODUCTION

One of the most basic problems in production management is that of determining the size of the production batches to be run in the production facility. There are many different methodologies that have been developed over the years for approaching this problem. In fact, it is fair to say that production lot sizing remains one of the most studied (and difficult) production problems (both in industry and academics) today.

The problem can take on many different forms. The form depends on :

1. The number of different products to be produced;
2. the design of the production process (number of stages of production, number of machines, in-process inventory limitations, etc.);
3. special operating rules imposed;
4. the eventual use that the resulting production plan will be put to (long range versus day to day scheduling);
5. the form of demand forecasts; and
6. other factors.

It is not the intent of this paper to review all the different approaches to this problem. The interested reader is refered to the reference list for a sample of approaches. Rather we will outline an approach to production lot sizing that we have observed in use in industry, but which does not appear to be well known either in the academic community or in industry.

Two of the authors observed this approach when they were employed by a large Automotive Company in the late 1960's. Recently, other researchers have observed the approach at an even larger Automotive Company [3]. In the following sections, first, the power of 2 lot sizing approach observed in industry is described. Then, a discussion of the effect of planning horizons is provided. Finally, a mathematical programming approach is presented for implementing the approach under one of many possible scenarios when computer facilities are available.

II. POWER OF 2 LOT SIZING

In choosing production lot sizes for various items to be produced, one typically tries to pick lot sizes such that forecast (and/or actual) demand is met, the resulting machine schedule is feasible, and sum of production costs, machine setup costs, and inventory carrying costs is minimized. This is, both mathematically and practically, a very difficult problem. We refer the reader to references [5] and [8] for an overview of this classical approach. What power of 2 lot sizing does is limit the number of possible solutions to the lot sizing problem to a set of solutions which are most apt to be implementible in the production process, and by doing so (limiting) it reduces the complexity of the decision process itself (see [6, 7]).

For purposes of this exposition a simple senario (one of many possible) will be developed in order to introduce power of 2 lot sizing. Consider a set of N products which are to be produced on a single production facility. There is a forecast of demand extending more than eight weeks into the future for each of the N products. Production data is available for setup times (and costs) production rates (and costs), inventory carrying costs for finished inventories, and the amount of initial on-hand finished inventory. The problem is to develop an eight week production schedule which is feasible, i.e., the production facility is scheduled to produce only one product at a time, and the production schedule will result in the forecast demand being satisfied (no stockouts). Note that the length of the production schedule (horizon) may be considerably shorter than the length of the forecast (forecast horizon). This is normally the case in industry. If not, it is generally easy to extend the demand forecast with rough estimates.

Under the present scenario, a possible production schedule for a given product might be to produce that product in periods 1, 3, 4, 7 and 8. In other words the production schedule might be very irregular. What is being proposed here is to limit the set of possible production schedules for each product to a small set of regular schedules only. Specifically, the set of possible production schedules is as follows :

1. Produce the given product once a week, producing enough to satisfy the demand for that week.
2. Produce the given product once every two weeks, producing enough to satisfy the demand for the week of production and the following week. Of course, within this alternative, it is necessary to choose to produce in either the odd or even weeks.

4. Produce the given product once every four weeks, producing enough to satisfy the demand for the week of production and the following three weeks. Within this alternative, it is necessary to choose to start production in either week one, two, three, or four.

8. Produce the given product one every eight weeks, producing enough to satisfy demand for the week of production and the following seven weeks. Within this alternative there are eight sub-alternatives with respect to the time production is started.

In all, for each product, there are 1 + 2 + 4 + 8 = 15 alternative schedules. In actuality this number may be somewhat smaller. For example, if a product has no on hand inventory and demand in the first week is positive, then there are only four alternatives :

1. produce every week;
2. produce every other week starting in week 1 (i.e., the odd weeks);
3. produce every four weeks starting in week 1; and
4. produce every eight weeks starting in week 1.

Any other alternative will result in a stockout in week 1, which is not allowed. If there is enough on hand inventory so that it is not necessary to start production until the second week then the number of feasible alternatives increases to seven :

1. produce every week (except there is no production in week 1);

2-3. produce every other week (odd or even);

4-5. produce every four weeks, starting in week 1 or 2;

6-7. produce every eight weeks, starting in week 1 or 2.

Note that the alternatives of producing every 1, 2, 4, or 8 weeks is, in fact, just the powers of two from zero to three (i.e. $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$).

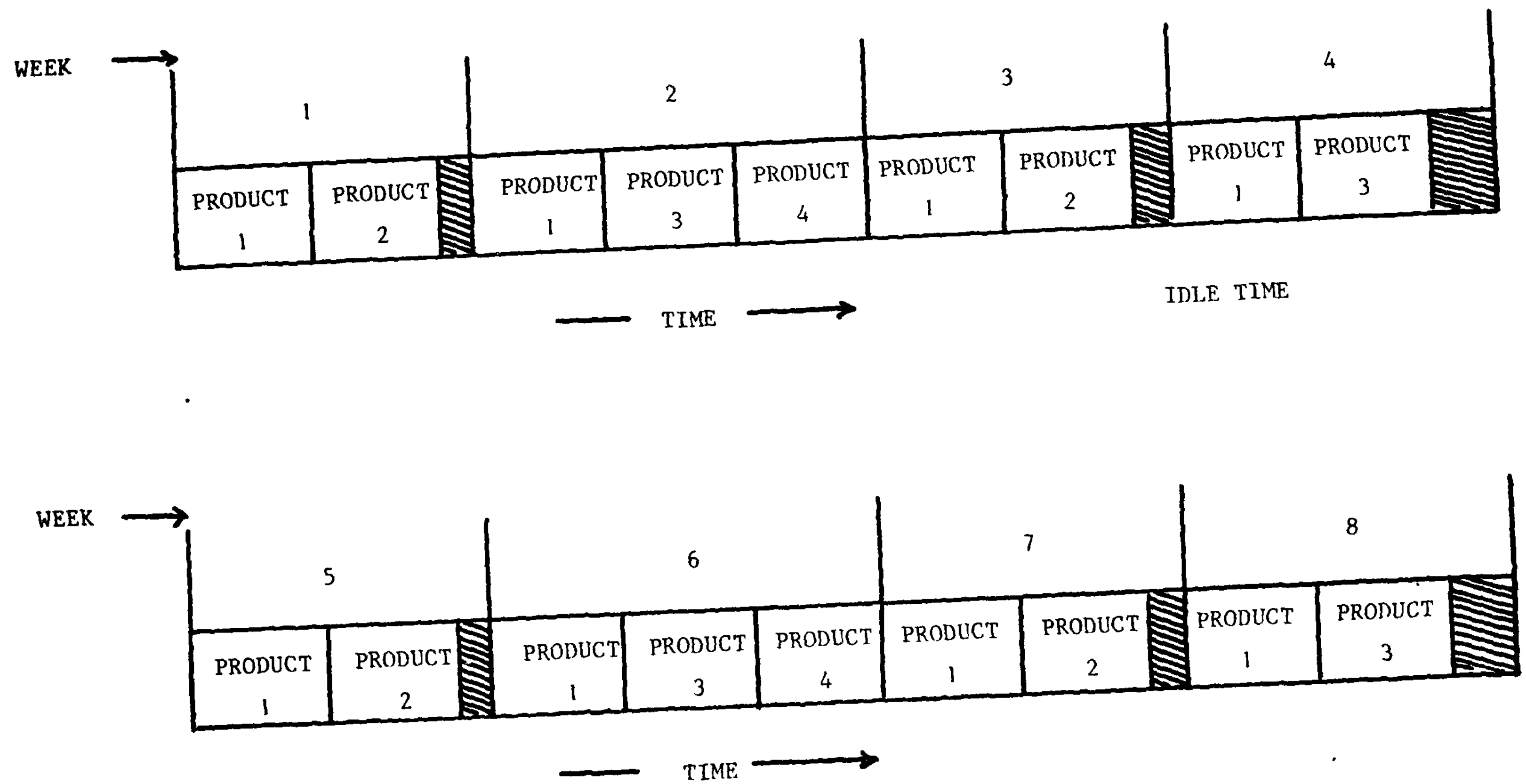
Theoretically, there is a penalty for imposing limitations on the set of possible schedules. Our experience, with production planning models using this concept, is that the penalty, in terms of setup and inventory costs is something less than 3 % [6] (and, in fact, probably less than 1 %). One should note that inventory and setup costs are normally a small percentage of the total costs, production cost (material plus labor) being the major factor.

The benefit for using 1, 2, 4, 8 (power of 2) scheduling comes from ease of schedule implementation on the shop floor. The most obvious benefit, in terms of schedule implementation, is that the schedule for a given set of items is easy for the foreman to remember. It is our experience that schedules which appear straightforward to the man on the shop floor have the best chance of successful implementation. A second benefit is that power of two schedules tend to repeat themselves. In figure 1 is a block diagram of a schedule for four products. Product 1 is produced every week. Products 2 and 3 are produced every two weeks (in the odd and even weeks, respectively). Product 4 is produced every four weeks (starting in week 2). Note that the schedule for the second four weeks is exactly the same as the first four weeks. Although the production runs are drawn exactly the same each time a given product is produced, in fact, they would vary according to the demand forecast for that product. Repeatable schedules tend to be easier to implement with respect to production management, and, particularly, are easier to implement with machine setup and maintenance management.

In terms of hand or computer calculated schedules, this approach limits the number of possibilities which must be evaluated. In the late 1960's, the Metal Stamping Division of a large Automotive Company used power of 2 lot sizing for production planning and feasibility analysis. This exercise was performed twice a year. Since the demand forecasts were constant, it was necessary only to determine the feasibility of a schedule for any pressline over an eight week period (horizon). Reasonable schedules could be obtained by hand. In the 1970's, this process was completely computerized affording management a much wider range of decision choices and "What if ...?" type of analysis, which resulted in considerable cost savings. At a larger Automotive Company CAIE and MAXWELL [3] have implemented computerized systems using power of 2 lot sizing. There are other companies currently that also use power of 2 scheduling systems, but little is known as to what extent the approach has been formalized.

III. THE SCHEDULING HORIZON

Returning to our discussion, in the present case it can be seen that the choice of a fixed decision (power of 2) structure provides some advantages in terms of the effect of the planning horizon on the development of a schedule. Feasibility is calculated over an eight week horizon. However, the affect of the production schedule may extend well beyond eight weeks. For



Example power of two schedule

FIGURE 1

example, assume a decision for a given product is to produce every four weeks but starting in the third week (assuming that demand in weeks 1 and 2 can be satisfied out of on-hand inventory). Then costs and resource utilization for the production run made in the third week are calculated based on satisfying forecast demand for weeks 3, 4, 5, and 6. Costs and resource utilization for the next production run (made in the seventh week) are calculated based on forecast demand for weeks 7, 8, 9 and 10. (i.e. beyond the eight week "planning" horizon). Costs, however are calculated only to the end of the planning horizon (8 weeks), i.e., the costs of carrying the inventory for periods 9 and 10 are calculated only to the end of week 8, and the carrying costs incurred in weeks 9 and 10 are not included.

A feature of the model is that it takes into account demand beyond the planning horizon, thereby minimizing the effect of the limited planning horizon on the schedule. Most other approaches to production scheduling consider forecast demand up to the end of the planning horizon only (in this case, eight weeks). The normal use of a model of this sort in a production control (or Material Requirements Planning) system would be as a rolling schedule (Baker [1] , Blackburn and Millen [2]. That is, each week (or possibly every other week) the model would be used to generate a schedule. The results of the first week would be implemented. Then the next week the model would be rerun with updated demand forecast and on hand inventories from the previous weeks production. The first week of that schedule would be implemented (i.e., the second week of the original schedule). This process would be repeated every week with only the first week of the schedule being implemented. It should be pointed out that the effect on the actual lot size decision by the choice of an eight week horizon is not as critical as might be expected since : 1. demand beyond the horizon is taken into account if it affects production within the horizon; and 2. only the first week (or two) of any scheduling run would be implemented.

The reader should note that the choice of an eight week horizon is abitrary and taken for purposes of this scenario. For particular applications, other horizon lengths may be appropriate.

IV. A MATHEMATICAL PROGRAMMING APPROACH FOR POWER OF THE SCHEDULING

In this section an approach for integrating the lot size decision with the question of production capacity limitations is presented. It is an approach which is intended for computer implementation, but, in fact, has the same requirements for data generation as hand calculation approaches. It is necessary to define a set of decision variables for the mathematical program. Each decision variable can take on the values zero and one. One, if the particular schedule possibility is selected, and zero if it is not.

Let

X_{ijk} = 1 if product "i" is to be produced every "j" weeks starting in week "k"

= 0 otherwise.

For each product "i" there are fifteen possible decision variables.

X_{i11} (produce every week)

X_{i21}, X_{i22} (produce every other week in either the odd (k=1) or even (k=2) weeks)

X_{i41}, X_{i42}, X_{i43}, X_{i44} (produce every four weeks starting in weeks k=1, 2, 3, or 4)

X_{i81},..., X_{i88} (produce every eight weeks starting in weeks k=1,...,8.

Clearly, for each product "i", only one decision variable (one schedule) can be chosen from the set of possibilities. The following constraints enforce that requirement on each product i

$$\sum_{j=1,2,4,8} \sum_{k=1}^{j} X_{ijk} = 1 \qquad i = 1,2, \ldots, N \qquad (1)$$

Before proceeding futher with the development of the mathematical program, it is worthwhile to consider a small example which indicates how initial (on hand) inventories are handled within the decision structure.

Product "1", whose forecast demand is in Table 1, is to be produced every other week in the even weeks (i.e. $X_{122} = 1$, all other $X_{1jk} = 0$). The on hand inventory for product 1 is 205. The means that there is enough on hand to satisfy the demand in week 1 and part of week 2. Therefore, for this decision, the production schedule is as shown in Table 1 (line two). If the decision were to produce once a week (i.e., $X_{111} = 1$), the production schedule would be as shown in Table 1 (line three). With demand for week 1 satisfied

TABLE 1 : Forecast demand, example schedules, cost comutations, facility utilization.

		1	2	3	4	5	6	7	8	9	10
1	forecast demand	100	200	150	250	175	200	0	225	150	200
2	production schedule X_{122}	0	245	0	425	0	200	0	375		
3	production schedule X_{111}	0	95	150	250	175	200	0	225		
4	setup and inventory costs X_{122}	0	100 150 250	0	100 175 275	0	100 0 100	0	100 150 250		
							Total cost = $ 875				
5	setup and inventory costs X_{111}	0	100 0 100	100 0 100	100 0 100	100 0 100	100 0 100	0	100 0 100		
							Total costs = $ 600				
6	setup and production time X_{122}	0	4 24.5 28.5	0	4 42.5 46.5	0	4 20 24	0	4 37.5 41.5		
7	setup and production time X_{111}	0	4 9.5 13.5	4 15 19	4 25 29	4 17.5 21.5	4 20 24	0	4 22.5 26.5		

from on hand inventory, there is no production in week 1. Producing every four weeks, starting in week 3 (X_{43}=1) is one example of a decision which is not possible and must be zero since to choose that decision possibility would result in a stockout in week 2. Consequently, decision variable X_{143} can be eliminated all together from the mathematical program (lowering the computational requirements).

Assume (for purposes of the example) that it costs $100.00 to set up the production facility to produce product 1. In addition, assume that it costs $1.00 to carry a unit of product 1 in inventory for one week. Production which is shipped immediately to the customer (i.e., shipped in the same week it is produced) does not incur any carrying cost. The total cost of setup and inventory carrying for producing in the even weeks (X_{122} = 1) is just $875 over the eight week horizon. (see line 4, Table 1). The total cost of setup and inventory carrying for producing every week (X_{111} = 1) is just $600 over the eight week horizon, since there is no inventory carrying cost and no setups in weeks 1 and 7. The setup and inventory carrying costs for each of the possible decisions can be calculated in a straightforward fashion as illustrated above.

It is now appropriate to define a cost constant which will be used in the mathematical program.

Let

C_{ijk} : the total setup and inventory carrying cost associated with producing product "i" every "j" weeks, starting in week "k" over the entire 8 week horizon.

With the definition of C_{ijk}, it is possible to state, quantitatively, an objective for building a schedule.

$$\text{minimize} \sum_{i=1}^{N} \sum_{j=1,2,4,8} \sum_{k=1}^{j} C_{ijk} X_{ijk} \tag{2}$$

If $X_{ijk} = 1$, then the cost C_{ijk} is incurred. The objective is to minimize the sum of those costs. Certainly the minimization of (2) is still subject to the constraints of equation (1). In addition, the minimization also is subject to feasibility constraints on the production facility. That is, in any week it is not possible to assign more work than the production facility has capacity to perform. Therefore, the selection of a set of schedules (one for each product) must not result in more production hours being scheduled in any week than are available.

Let

T_w = the number of setup and production hours available on the production facility in week "w", and

Let

A^w_{ijk} = the number of setup and production hours used on the production facility in week "w" by product "i" if it is produced every "j" weeks starting in week "k".

Assume (for purposes of the example) that setup time for product 1 is 4 hours. In addition, assume that it takes 6 minutes (1/10 hour) to produce a unit of product 1. Returning once more to table 1, the set of the setup plus production times appear in line 6 for X_{122} and in line 7 for X_{111}.

Since it is necessary to limit the choices of schedules to the set which are feasible with respect to the production facility availability, the following constraints are necessary

$$\sum_{i=1}^{N} \sum_{j=1,2,4,8} A^{w}_{ijk^*} X_{ijk^*} \leq T_w \qquad w = 1, \ldots, 8 \tag{3}$$

k^* is defined to be that k which results in production in week "w" when the decision is to produce once every "j" weeks. Table 2 contains the index of k^*'s for all j and w. The mathematical programming model can now be stated fully (in reality, it is an integer, 0-1, linear program).

$$\text{Minimize} \quad \sum_{i=1}^{N} \sum_{j=1,2,4,8} \sum_{k=1}^{j} C_{ijk} X_{ijk} \tag{2}$$

Subject to

$$\sum_{j=1,2,4,8} \sum_{=1}^{j} X_{ijk} = 1 \qquad i = 1, \ldots, N \tag{1}$$

$$\sum_{i=1}^{N} \sum_{j=1,2,4,8} A^{w}_{ijk^*} X_{ijk^*} \leq T_w \qquad w = 1, \ldots, 8 \tag{3}$$

$$X_{ijk} = 0,1 \qquad i = 1, \ldots, N$$

$$j = 1,2,4,8$$

$$k = 1, \ldots, j$$

TABLE 2

Index of k^*'s

j w	11111111 12345678	22222222 12345678	44444444 12345678	88888888 12345678
k^*	11111111	12121212	12341234	12345678

V. A NUMERICAL EXAMPLE

Consider a single production facility producing four different products. Each product has a the same production rate of one piece per hour and the same inventory carrying cost of one dollar (BF) per week. Initial inventories, demand forecasts, and net forecasts (demand forecasts reduced by initial inventories) are given in table 3. Setup times for the four products are 2, 2, 2, and 1 hours respectively, and the setup costs are 5, 7, 3, and 18 dollars (BF) respectively. There are 40 hours/per week available for setup and production time, and the scheduling horizon is eight weeks. Each product is limited to four decision variables (X_{ijk}'s) so that there are a total of sixteen decision variables for the problem. For each product, only the best (least cost) four decisions are considered. The problem, in tableau form, appears in table 4.

The optimal (least cost, feasible) solution is $X_{121} = X_{222} = X_{321} = X_{411} = 1$, all other $X_{ijk} = 0$. Product 1 is produced every other week in the odd weeks. Product 2 is produced every other week in the even weeks. Product 3 is produced every other week in the odd weeks (except week 1 because demand for the first two weeks is covered by initial inventory). Product 4 is produced every week (except weeks 1 and 2 because demand for the first two weeks is covered by initial inventory).

It should be noted that solution techniques for problems of this type are not necessarily straightforward. Although the problem is stated as a linear program, the requirement that the decision variables (X_{ijk}) take on the values 0 and 1 only eliminates the simplex method as a solution technique. However, IBM's MPSX-MIP (Mixed Integer Linear Programming Package) is one of several commercial codes available to solve problems of this type. In addition, our personal experience has been that the implicit enumeration scheme of FERREIRA and HODGSON [6] is quite efficient also.

WEEK		1	2	3	4	5	6	7	8	9	10	11
	INITIAL INVENTORY	DEMAND FORECAST										
PRODUCT 1	10	0* / 8	5* / 7	6	6	12	7	6	5	6	8	5
PRODUCT 2	15	0* / 10	2* / 7	2	13	8	12	11	8	7	10	2
PRODUCT 3	9	0* / 2	0* / 3	0* / 4	11	10	2	2	2	0	11	1
PRODUCT 4	8	0* / 4	0* / 3	10* / 11	14	4	4	3	8	9	9	1

* Demand forecast reduced by initial inventory

DEMAND FORECAST DATA FOR EXAMPLE PROBLEM

TABLE 3

	PRODUCT 1				PRODUCT 2				PRODUCT 3				PRODUCT 4				RIGHT HAND SIDE
Variables	X_{111}	X_{121}	X_{122}	X_{141}	X_{211}	X_{222}	X_{221}	X_{242}	X_{311}	X_{322}	X_{321}	X_{343}	X_{421}	X_{422}	X_{411}	X_{443}	
one decision per product EQ (1)	1	1	1	1													= 1
					1	1	1	1									= 1
									1	1	1	1					= 1
													1	1	1	1	= 1
facility constraints EQ (3)	0	7	0	19	0	0	4	0	0	0	0	0	0	0	0	0	≤ 40
	7	0	13	0	4	6	0	27	0	0	0	0	0	11	0	0	≤ 40
	8	14	0	0	4	0	17	0	0	0	13	0	25	0	11	33	≤ 40
	8	0	20	0	15	23	0	0	13	23	0	27	0	19	15	0	≤ 40
	14	21	0	32	10	0	22	0	12	0	14	0	9	0	5	0	≤ 40
	9	0	15	0	14	25	0	40	4	6	0	0	0	8	5	0	≤ 40
	8	13	0	0	13	0	21	0	4	0	6	0	12	0	4	30	≤ 40
	7	0	13	0	10	17	0	0	4	4	0	16	0	18	9	0	≤ 40
Objective EQ(2)	35	43	50	79	49	56	63	114	15	21	24	38	80	98	108	114	
		*				*					*				*		

* optimal decision

TABLEAU LAYOUT OF EXAMPLE 0-1 LINEAR PROGRAM

TABLE 4

EXTENSIONS TO THE MODEL

In the senario developed in previous sections only one production facility was considered. In many applications it would be desirable to deal with a multi-stage production facility (some (or all) of the products must go through two or more stages of production). As an example, a two-stage production facility will be considered. With two production stages, more complex decisions must be made. The decision structure for the single-stage facility can be expanded in the following manner. Let

$X_{ijkj'k'}$ = 1 if product "i" is to be produced on the first stage every "j" weeks starting in week "k" and produced on the second stage every "j'" weeks starting in weeks "k'".

= 0 otherwise.

$C_{ijkj'k'}$ = the total setup and inventory carrying cost associated with producing product "i" on the first stage every "j" weeks starting in week "k" and on the second stage every "j'" weeks starting in week "k'".

$A^{ws}_{ijkj'k'}$ = the number of setup and production hours used on stage "s" in week "w" by product "i" if it is produced on the first stage every "j" weeks starting in week "k", and on the second stage every "j'" weeks starting in week "k'".

$T_{w,s}$ = the number of setup and production hours available on the production facility in week "w", facility "s".

Paralleling the single stage formulation the two-stage problem can be stated as a 0-1 integer linear program.

$$\text{minimize} \sum_{i=1}^{N} \sum_{j=1,2,4,8} \sum_{k=1}^{j} \sum_{j'=1,2,4,8} \sum_{k'=1}^{j'} C_{ijkj'k'} X_{ijkj'k'} \qquad (4)$$

Subject to

$$\sum_{j=1,2,4,8} \sum_{k=1}^{j} \sum_{j=1,2,4,8} \sum_{k'=1}^{j'} X_{ijkj'k'} = 1 \qquad i=1,\ \ldots,N \qquad (5)$$

$$\sum_{i=1}^{N} \sum_{j=1,2,4,8} \sum_{j'=1,2,4,8} A_{ijk^*j'k'^*} < T_{ws} \qquad w = 1,\ldots,8 \quad s = 1,2 \qquad (6)$$

$$X_{ijkj'k'} = 0,1 \qquad \begin{aligned} i &= 1,\ldots,N \\ j &= 1,2,4,8 \\ k &= 1,\ldots,j \\ j' &= 1,2,4,8 \\ k' &= 1,\ldots,j' \end{aligned}$$

Clearly the number of decision variables grows considerably as the production system grows more complex. However, we have solved a number of 2-stage, 10-product, 16 decisions per product (160 decision variables) problems using a simple enumeration scheme [6]. The longest computation time was approximately .1 second on an IBM 3033. The average computation time was less than .02 seconds (our work in this area is still in progress).

It should be noted that this approach lends itself to much more complex production systems. While straightforward analysis of such systems is beyond the scope of this paper.

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