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# METHOD OF MOMENTS ESTIMATORS FOR THE MOVING AVERAGE COEFFICIENTS IN AN ARMA (p,q) PROCESS

by

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## Introduction.

The standard procedure for computing the moving average method of moments parameter estimates for an ARMA process involves the numerical solution of a system of non-linear equations. In this paper it will be shown how the so-called inverse autocorrelations introduced by Cleveland [3] can be utilized to compute the moving average parameter estimates as the solution to a system of linear equations, and that that method is faster and more accurate than solving the non-linear equations alluded to above. In addition, the procedure described below ensures that an invertible solution is obtained. It also provides a natural procedure for obtaining estimates when the usual estimates do not exist. These "estimates" are shown to provide reasonable initializing values for calculating the maximum likelihood parameter estimates.

## Text.

The method of moments (or preliminary estimates, see Box and Jenkins [2] A.6.2) may be defined as follows:

<u>Definition 1</u>. Let $\hat{\rho}(j)$ be the sample autocorrelation function of a stochastic process and let $\hat{\phi}_1,\ldots,\hat{\phi}_n$ be the solution to

$$\begin{aligned}\hat{\rho}(m+1) &= \phi_1\hat{\rho}(m) + \cdots + \phi_n\hat{\rho}(m-n+1) \\ &\vdots \\ \hat{\rho}(m+1) &= \phi_1\hat{\rho}(m+n-1) + \cdots + \phi_n\hat{\rho}(m)\end{aligned} \tag{1}$$

Futher let $\hat{\theta}_1,\ldots,\hat{\theta}_m$ be the invertible solution (i.e., the solution which $1-\hat{\theta}z-\cdots-\hat{\theta}_m z_m$ has all of its roots outside the unit circle) to the system of equations

$$\hat{\rho}(j) = \rho(j;\hat{\phi}_1,\ldots,\hat{\phi}_n,\ \theta_1,\ldots,\theta_m),\ j=1,\ldots,m \tag{2}$$

(where $\rho(j;\alpha_1,\ldots,\alpha_n,\ \beta_1,\ldots,\beta_m)$ is the autocorrelation function of an ARMA(n,m) process with autoregressive parameters $\alpha_1,\ldots,\alpha_n$ and moving average parameters $\beta_1,\ldots,\beta_m$).

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We call $\hat{\phi}_1,\ldots,\hat{\phi}_n$, $\hat{\theta}_1,\ldots,\hat{\theta}_m$ the method of moments parameter estimators of an ARMA(n,m) process and

$$\hat{s}(\omega) = \frac{K|1 - \hat{\theta}_1 e^{2\pi i\omega} - \cdots - \hat{\theta}_m e^{2\pi i\omega m}|^2}{|1 - \hat{\phi}_1 e^{2\pi i\omega} - \cdots - \hat{\phi}_n e^{2\pi i\omega n}|^2}$$

is called the method of moments ARMA(n,m) spectral density estimator, where K is chosen so that

$$\int_{-.5}^{.5} \hat{s}(\omega)d\omega = 1.$$

Verbally $\hat{\phi}_1,\ldots,\hat{\phi}_n$ and $\hat{\theta}_1,\ldots,\hat{\theta}_m$ are those parameter values whose theoretical autocorrelation function $\rho(j;\hat{\phi}_1,\ldots,\hat{\phi}_n, \hat{\theta}_1,\ldots,\hat{\theta}_m)$ agrees with $\hat{\rho}(j)$ for $j = 1,\ldots,m+n$.

As is well-known, for $m > 0$, even if $\hat{\rho}(j)$ is a positive definite function, a solution for $\hat{\phi}_1,\ldots,\hat{\phi}_n$, $\hat{\theta}_1,\ldots,\hat{\theta}_m$ need not exist (e.g., for $n = 0$, $m = 1$, take $.5 < \hat{\rho}(1) < 1$). In that case we say that the method of moments parameter estimators and the method of moments spectral density estimator do not exist.

In Morton [7], a spectral density estimator called the G-spectral estimator was introduced (actually it was called the modified G-spectral estimator, it being a modification of a spectral estimator introduced by Gray [5] and studied by Gray, Houston and Morgan [6]).

The G-spectral estimator may be defined as follows.

Definition 2. Let $\hat{\rho}(j)$ be an estimator for the autocorrelation function $\rho(j)$ of a stochastic process. Further let $f_j = e^{2\pi i\omega j}\hat{\rho}(j)$ and $F_j = \sum_{v=-k}^{j} f_v$, where $k = \max\{1,n-m-1\}$ and m and n are given, non-negative integers. We then define

$$G_{n,m}(\omega) = 2\mathrm{Real}(e_n(F_m)-F_0) + 1,$$

where

$$e_n(F_m) = \begin{vmatrix} F_{m-n+1} & \cdots & F_{m+1} \\ f_{m-n+1} & \cdots & f_{m+1} \\ \vdots & & \vdots \\ f_m & \cdots & f_{m+n} \end{vmatrix} \div \begin{vmatrix} 1 & \cdots & 1 \\ f_{m-n+1} & \cdots & f_{m+1} \\ \vdots & & \vdots \\ f_m & \cdots & f_{m+n} \end{vmatrix}$$

for $n \geq 1$ and $e_0(F_m) = F_m$.

The following properties of the spectral estimator introduced above are proved in Morton [ 7].

Theorem 1. Let $G_{n,m}(\omega)$, $\hat{s}(\omega)$, and $\hat{\phi}_1,\ldots,\hat{\phi}_n$, $\hat{\theta}_1,\ldots,\hat{\theta}_m$ be as defined above. Define $\alpha_j = \hat{\phi}_j e^{2\pi i\omega j}$, $j = 1,\ldots,n$ .

Then

(i) if $\hat{s}(\omega)$ exists then

$$\hat{s}(\omega) \equiv G_{n,m}(\omega)$$

(ii) $$G_{n,m}(\omega) = 2\mathrm{Real}\left\{\frac{F_{m+1} - \alpha_1 F_m - \cdots - \alpha_n F_{m-n+1}}{1 - \alpha_1 - \cdots - \alpha_n} - F_0\right\} + 1$$

where the $F_j$ are as given in Definition 2.

The first result above shows that $G_{n,m}(\omega)$ is an ARMA spectral estimator which does not require calculation of the moving average parameter estimates. The second result provides a simple formula for calculating $G_{n,m}(\omega)$. The calculation formula we note does require the calculation of the autoregressive parameter estimates (from (1), for instance); however (1) is a linear system of equations which may be solved rather easily. Calculating $G_{n,m}(\omega)$ by (ii) provides a much more efficient means of calculating $G_{n,m}(\omega)$ than does direct evaluation of the above determinants at every desired frequency.

We are now in the position to give the result we will utilize in the calculation of $\hat{\theta}_1,\ldots,\hat{\theta}_m$.

<u>Theorem 2</u>. Let $\hat{\phi}_1,\ldots,\hat{\phi}_n$, $\hat{\theta}_1,\ldots,\hat{\theta}_m$ and $G_{n,m}(\omega)$ exist as defined above. If $G_{n,m}(\omega) > 0$ for all $\omega$ and

$$ci(k) = \int_0^{.5} \frac{\cos(2\pi i\omega k)}{|\hat{\phi}(e^{2\pi i\omega})|^2 G_{n,m}(\omega)}\, d\omega$$

then $\hat{\theta}_1,\ldots,\hat{\theta}_m$ is the solution to

$$\begin{aligned} ci(1) &= \theta_1 ci(0) + \cdots + \theta_m\, ci(1-m) \\ &\vdots \\ ci(m) &= \theta_1 ci(m-1) + \cdots + \theta_m\, ci(0)\ . \end{aligned} \qquad (3)$$

<u>Proof</u>. From the results above

$$\frac{1}{|\hat{\phi}(e^{2\pi i\omega})|^2 G_{n,m}(\omega)} = \frac{1}{K|\hat{\theta}(e^{2\pi i\omega})|^2} = f(\omega)\ \text{(say)}.$$

We note that $f(\omega)$ is proportional to the spectrum of a AR(m) process with parameters $\hat{\theta}_1,\ldots,\hat{\theta}_m$. Thus,

$$ci(j) = \int_0^{.5} \cos(2\pi i\omega)\, f(\omega)\, d\omega = \frac{1}{2}\int_{-.5}^{.5} e^{-2\pi i\omega j} f(\omega)\, d\omega\ ,$$

and we note that $ci(j)$ is proportional to the autocorrelation function of an AR(m) process with parameter values $\hat{\theta}_1,\ldots,\hat{\theta}_m$. So it follows that $\hat{\theta}_1,\ldots,\hat{\theta}_m$ is the solution to (3).

In this paper, we will refer to algorithms based on solving the system of equations in (3) as Method 1. We note from the above Theorem that Method 1 will always yield an invertible moving average operator (an algebraic proof that the operator resulting from (3) is invertible is given in Morton [ 7 ]).

The standard procedure for calculating the moving average parameter estimates is via an algorithm based on a solution to (2). The standard procedure for that calculation requires an iterative solution to a nonlinear system of equations (see Box and Jenkins [2], pp. 201 ff). Algorithms based on finding the solution to (2), we will call Method 2. The two methods may be compared by the computational speed and accuracy which they afford and as to whether or not an invertible solution is guaranteed. First we consider the question of invertibility.

It is well-known that, even if a solution to (2) exists, it is not unique. (See Box and Jenkins [2], pp. 198-199.) Uniqueness is then guaranteed, typically, by requiring that the polynomial

$$\hat{\theta}(z) = 1 - \hat{\theta}_1 z - \cdots - \hat{\theta}_m z_m$$

have all of its roots outside the unit circle (i.e., $\hat{\theta}(B)$ is required to be invertible). Thus a numerical algorithm which merely requires that the equations in (2) be satisfied does not guarantee an invertible moving average operator. In particular, one commonly used subroutine, FTMPS in IMSL, does not guarantee an invertible solution. As noted above, Method 1 always yields an invertible solution.

To compare the computational speed and accuracy of Method 1 and Method 2, we consider a comparison between the IMSL subroutine FTMPS and a program written by Morton which calculates the ci(j) by Simpson's rule with Richardson extrapolation (see, for instance, Dahlquist and Bjork [4], pp. 269 ff).

To make the comparison, the true autocorrelations were input for moving average orders $q = 2,3,4,5,6$ and the two methods were compared for accuracy of the calculated coefficients and for the CPU time required to make the calculation. Method 1 was applied using grid sizes of 20,40,60,80,100 in the numerical integration scheme. A summary of the results is given in Table 1. Note that, for these models, Method 1 is faster and more accurate in every case except for $q = 3$ with a grid size of 100 for which Method 2 was slightly faster (though much less accurate). For the $q = 5$ case, we note that Method 2 failed to yield an invertible operator, and, for the $q = 4$ case, we note that Method 2 failed to converge to a solution at all.

In the simulations above, a solution for $\hat{\theta}_1,\ldots,\hat{\theta}_m$ exists. However, it is well-known that that need not be the case. In fact, for the pure moving average case, a solution exists if, and only if,

$$f(\omega) = 1 + 2 \sum_{j=1}^{m} \hat{\rho}(j) \cos(2\pi\omega j) > 0, \quad -.5 \le \omega \le .5$$

(see, for instance, O. D. Anderson [1] pp. 137 ff.; for mixed processes, let $f(\omega) = |\hat{\phi}(e^{2\pi i\omega})|^2 G_{n,m}$ and add the condition that $\hat{\phi}(B)$ be a stationary operator). Thus, if $f(\omega) < 0$ for some $\omega$, no method of moments solution exists. One solution to that difficulty is to replace $f(\omega)$ by

$$g(\omega) = \frac{f(\omega) + c}{1 + c} = 1 + 2 \sum_{j=1}^{m} \frac{\hat{\rho}(j)}{1 + c} \cos(2\pi\omega j)$$

where $c$ is chosen so that $g(\omega) > 0$.

In the $m = 1$ case, for instance, an invertible solution exists if, and only if, $-.5 < \hat{\rho}(1) < .5$. Then, if $\hat{\rho}(1)$ is is outside that interval, the adjustment above simply "shrinks" $\hat{\rho}(1)$ to $\frac{\hat{\rho}(1)}{1+c}$, where c is chosen so that $\hat{\rho}(1)/(1+c)$ is in the admissible region.

The procedure described above for modifying the estimated autocorrelations in order to obtain a solution is not inherently restricted to either Method 1 or Method 2. However, it is not utilized by any of the commonly used algorithms which employ Method 2 (in particular, it is not utilized by FTMPS in IMSL); so its use will only be considered here in conjunction with Method 1. For the remainder of this paper, Method 1 will refer to calculating $\hat{\theta}_1,\ldots,\hat{\theta}_m$ by the equations in (3) where the function

$$f(\omega) = |\hat{\phi}(e^{2\pi i\omega})|^2 \, G_{n,m}(\omega)$$

is shifted upward if it takes on non-positive values. The precise statistical properties of the above estimator are unknown. However, as is illustrated below, it often provides "reasonable" initializing values for a maximum likelihood estimation procedure.

For many practitioners, the calculation of $\hat{\theta}_1,\ldots,\hat{\theta}_m$ is performed solely for the purpose of providing initializing values for an iteratively calculated, but more statistically efficient estimation routine (e.g., maximum likelihood). We, thus, now consider the effect of the above results on the computation time and the number of iterations required to calculate the maximum likelihood estimate using Method 1 as against the standard procedure of using Method 2 to calculate initializing values. To that end, 5 realizations of length 200 were simulated from each of the 5 models used above. Maximum likelihood estimates were calculated for each realization using both Method 1 and Method 2 to obtain the initializing values. Both the number of iterations required for convergence of the process and the total CPU time for the 5 realizations from each separate model were recorded.

To summarize the earlier results, we record the ways in which the two methods differ:

(1) Method 1 is typically both faster and more computationally accurate.

(2) Method 1 always yields a solution and Method 2 need not. Further it requires Method 2 longer to determine that no solution can be obtained than it does for Method 1 to adjust the equations and determine a solution to the adjusted set of equations.

(3) Method 1 ensures an invertible solution, while Method 2 does not.

We summarize the results of the simulation in Table 2. In every case, and, for each partition size used, initializing by Method 1 required less computation time than initializing by Method 2. The difference is primarily due to the fact that Method 1 is faster computationally and always gives a solution. We also note that in the case in which Method 2 yielded a non-invertible operator, the maximum likelihood procedure required many more iterations than the procedure required when initialized by an invertible operator.

In summary, then, we have introduced an alternative to the standard procedure for calculating the moving average parameter estimates. This procedure is computationally faster and more accurate than the standard procedure. It also ensures a solution, as well as ensuring that that solution be invertible. That was shown to be important, in speeding up the convergence of the maximum likelihood estimates calculation when used as initializing values for the iterative calculation of those estimates.

TABLE 1

Approximated Values of $\theta_i$ Using Various Partition Sizes in Simpson's Rule Compared to Standard Methods for Various MA(q) Processes

| | | | | | CPU TIME |
|---|---|---|---|---|---|
| | | q = 2 | | | |
| True Parameters($\theta_i$) | 1.3 | -.42 | | | |
| Method 1 | | | | | |
| Partitions | | | | | |
| 20 | 1.29977 | -.41950 | | | .014 |
| 40 | 1.29996 | -.41993 | | | .016 |
| 60 | 1.30000 | -.42000 | | | .019 |
| 80 | 1.30000 | -.42000 | | | .022 |
| 100 | 1.30000 | -.42000 | | | .026 |
| Method 2 | 1.28450 | -.41493 | | | .032 |
| | | q = 3 | | | |
| True Parameters | 1.38 | -1.15 | .6 | | |
| Method 1 | | | | | |
| Partitions | | | | | |
| 20 | 1.37519 | -1.14078 | .59026 | | .019 |
| 40 | 1.38013 | -1.15032 | .60027 | | .024 |
| 60 | 1.38006 | -1.15012 | .60013 | | .025 |
| 80 | 1.38001 | -1.15002 | .60003 | | .031 |
| 100 | 1.38000 | -1.15000 | .60000 | | .038 |
| Method 2 | 1.38109 | -1.14937 | .60074 | | .034 |
| | | q = 4 | | | |
| True Parameters | -.95 | -.9 | -.85 | | |
| Method 1 | | | | | |
| Partitions | | | | | |
| 20 | -.94726 | .89509 | -.84354 | -.79266 | .019 |
| 40 | -.94726 | -.89428 | -.84105 | -.78760 | .026 |
| 60 | -.94922 | -.89825 | -.84708 | -.79578 | .035 |
| 80 | -.94986 | -.89964 | -.84935 | -.79895 | .037 |
| 100 | -.95001 | -.90001 | -.84997 | -.79991 | .045 |
| Method 2 | No Convergence | | | | .081 |

TABLE 1 (Con't.)

q = 5

| True Parameters | -1.85 | -1.75 | -1.66 | -1.56 | -.72 | | CPU Time |
|---|---|---|---|---|---|---|---|
| Method 1 Partitions | | | | | | | |
| 20 | -1.840 | -1.733 | -1.640 | -1.536 | -.703 | | .022 |
| 40 | -1.846 | -1.740 | -1.644 | -1.538 | -.707 | | .030 |
| 60 | -1.849 | -1.748 | -1.655 | -1.553 | -.716 | | .041 |
| 80 | -1.8500 | -1.7497 | -1.6591 | -1.5586 | -.7192 | | .043 |
| 100 | -1.8501 | -1.7501 | -1.6600 | -1.5600 | -.7200 | | .053 |
| Method 2 | -2.031 | -1.917 | -1.822 | -1.711 | -.862* | | .071 |
| True Parameters | -1 | -.18 | -.17 | -.15 | .61 | .61 | |
| Method 1 Partitions | | | | | | | |
| 20 | -.995 | -.183 | -.175 | -.155 | .594 | .590 | .023 |
| 40 | -.997 | -.177 | -.166 | -.145 | .600 | .597 | .034 |
| 60 | -.996 | -.1792 | -.1684 | -.1479 | .6072 | .6062 | .044 |
| 80 | -1.0000 | -.1799 | -.1696 | -.1494 | .6094 | .6092 | .050 |
| 100 | -1.0001 | -.1800 | -.1699 | -.1499 | .6100 | .6100 | .059 |
| Method 2 | -.9995 | -.1791 | -.1712 | -.1489 | .6094 | .6101 | .082 |

*This operator is non-invertible.

TABLE 2

Table of the Number of Iterations Required for Method 1 and Method 2

q = 2

| | | Method 1 Partitions | | | | Method 2 |
|---|---|---|---|---|---|---|
| | | 20 | 40 | 100 | 200 | |
| Realization | 1 | 9 | 9 | 9 | 9 | 9 |
| | 2 | 11 | 11 | 11 | 11 | 24* |
| | 3 | 13 | 13 | 13 | 13 | 17* |
| | 4 | 9 | 9 | 9 | 9 | 9 |
| | 5 | 13 | 13 | 13 | 13 | 17* |
| Total CPU Time | | 2.775 | 2.780 | 2.859 | 2.935 | 4.149 |

q = 3

| | | Method 1 Partitions | | | | Method 2 |
|---|---|---|---|---|---|---|
| | | 20 | 40 | 100 | 200 | |
| Realization | 1 | 17 | 17 | 17 | 17 | 17* |
| | 2 | 21 | 25 | 22 | 21 | 19* |
| | 3 | 11 | 11 | 11 | 11 | 11 |
| | 4 | 15 | 13 | 13 | 13 | 15* |
| | 5 | 15 | 15 | 15 | 15 | 15 |
| Total CPU Time | | 5.552 | 5.659 | 5.558 | 5.698 | 6.177 |

q = 4

| | | Method 1 Partitions | | | | Method 2 |
|---|---|---|---|---|---|---|
| | | 20 | 40 | 100 | 200 | |
| Realization | 1 | 23 | 21 | 19 | 19 | 53** |
| | 2 | 15 | 15 | 15 | 15 | 15 |
| | 3 | 9 | 9 | 9 | 9 | 13* |
| | 4 | 15 | 19 | 17 | 17 | 19* |
| | 5 | 23 | 23 | 23 | 23 | 25* |
| Total CPU Time | | 7.940 | 8.279 | 7.961 | 8.143 | 12.714 |

TABLE 2 (Cont'd.)

q = 5

| | | Method 1 Partitions | | | | Method 2 |
|---|---|---|---|---|---|---|
| | | 20 | 40 | 100 | 200 | |
| Realization | 1 | 63 | 53 | 35 | 33 | 73* |
| | 2 | 45 | 45 | 57 | 43 | 41* |
| | 3 | 33 | 35 | 35 | 35 | 45* |
| | 4 | 23 | 35 | 21 | 21 | 43* |
| | 5 | 41 | 35 | 35 | 35 | 37* |
| Total CPU Time | | 24.296 | 24.290 | 22.163 | 20.454 | 31.477 |

q = 6

| | | Method 1 Partitions | | | | Method 2 |
|---|---|---|---|---|---|---|
| | | 20 | 40 | 100 | 200 | |
| Realization | 1 | 13 | 17 | 17 | 17 | 15* |
| | 2 | 13 | 13 | 11 | 13 | 11* |
| | 3 | 17 | 13 | 13 | 13 | 13* |
| | 4 | 25 | 21 | 21 | 21 | 15* |
| | 5 | 13 | 7 | 11 | 11 | 15* |
| Total CPU Time | | 12.905 | 11.610 | 11.521 | 12.500 | 16.689 |

Summary of the efficiencies for choosing starting values for the maximum likelihood estimation routine of Method 1 and Method 2. The values given in the Table are the number of iterations required for convergence to a solution.

*No solution exists to (2.2).

**A non-invertible solution to (2.2) was obtained.

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The standard procedure for computing the moving average method of moments parameter estimates for an ARMA process involves the numerical solution of a system of non-linear equations. In this paper it will be shown how the so-called inverse autocorrelations introduced by Cleveland [3] can be utilized to compute the moving average parameter estimates as the solution to a system of linear equations, and that the method is faster and more accurate than solving the non-linear equations alluded to above. In addition, the procedure described below ensures that an invertible solution is obtained.

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