

AD A116949

(12)

RADC-TR-82-49

In-House Report

February 1982



AN ITERATIVE PHASE-ONLY NULLING METHOD

Robert A. Shore

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

DTIC FILE COPY

**ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, New York 13441**

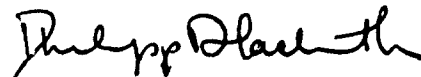
**DTIC
ELECTE
S JUL 16 1982
E**

82 07 16 003

This report has been reviewed by the RADC Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

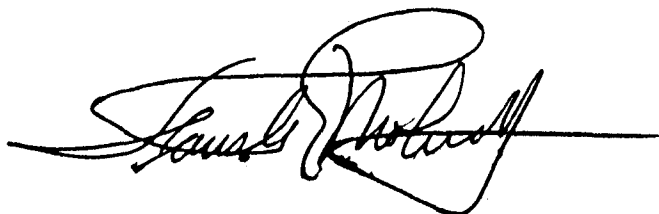
RADC-TR-82-49 has been reviewed and is approved for publication.

APPROVED:



PHILIPP BLACKSMITH
Chief, EM Techniques Branch

APPROVED:



FAUSTO E. MOLINET, Lt Colonel, USAF
Ass't Chief, Electromagnetic Sciences Division

FOR THE COMMANDER:



JOHN P. HUSS
Acting Chief, Plans Office

If your address has changed or if you wish to be removed from the RADC mailing list, or if the addressee is no longer employed by your organization, please notify RADC (EECS) Hanscom AFB MA 01731. This will assist us in maintaining a current mailing list.

Do not return copies of this report unless contractual obligations or notices on a specific document requires that it be returned.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RADC-TR-82-49	2. GOVT ACCESSION NO. AD-A116949	3. RECIPIENT'S CATALOG NUMBER 82-080
4. TITLE (and Subtitle) AN ITERATIVE PHASE-ONLY NULLING METHOD		5. TYPE OF REPORT & PERIOD COVERED In-House
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Robert A. Shore		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Deputy for Electronic Technology (RADC/EEC) Hanscom AFB Massachusetts 01731		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 23051408
11. CONTROLLING OFFICE NAME AND ADDRESS Deputy for Electronic Technology (RADC/EEC) Hanscom AFB Massachusetts 01731		12. REPORT DATE February 1982
		13. NUMBER OF PAGES 21
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE N/A
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release, distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES RADC Project Engineer - Robert A. Shore, EEC		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Linear arrays Null synthesis Phase-only Numerical method		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An iterative numerical method is presented for calculating the minimum phase-only perturbations of the element weights of a linear array to impose pattern nulls at prescribed locations. The method is based on repeated linearizations of the equations for the imposed nulls.		

DD FORM 1 JAN 73 1473

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	



Contents

1. INTRODUCTION	5
2. ANALYSIS	6
3. NUMERICAL RESULTS	15
4. CONCLUSIONS	20
REFERENCES	21

Illustrations

1. Geometry of Array	7
2. Maximum of Perturbed Pattern Power at Imposed Null Locations for 1, 2, 3 and 4 Nulls as a Function of Number of Iterations (40 dB Chebyshev amplitude taper)	16
3. Maximum of Perturbed Pattern Power at Imposed Null Locations for 1, 2, 3 and 4 Nulls as a Function of Number of Iterations (20 dB Chebyshev amplitude taper)	17

Tables

1. Cancellation Ratio (dB) for the Sector $15.23^\circ \leq \theta \leq \theta_2$ With
Equispaced Imposed Nulls

19

An Iterative Phase-Only Nulling Method

1. INTRODUCTION

In a recent report¹ we investigated in some detail the problem of perturbing a given set of array element weights to impose nulls in the pattern at specified locations. Two types of perturbations were examined: perturbations of both the amplitudes and phases of the element coefficients, and perturbations of the phases alone. When both the amplitudes and the phases of the weights are allowed to vary, the requirement of nulls at the specified locations in the pattern leads to a system of linear equations for the weight perturbations which can be solved exactly subject to an additional constraint involving minimization of the perturbations. If only the phases of the element coefficients are allowed to vary, the system of equations for the imposed nulls in the pattern is non-linear and cannot be solved exactly. Under the assumption that the phase perturbations are small, however, the system of equations for the nulls can be linearized and then solved in the same way as the equations for combined amplitude and phase perturbations. Because of the approximation involved in the linearization, the resulting phase perturbations do not give perfect nulls as do the combined amplitude and phase perturbations. The depth of null obtained depends on how well the small angle approximation used to linearize the equations for the

(Received for publication 18 February 1982)

1. Shore, R. A., and Steyskal, H. P. (1982) Nulling in Linear Array Patterns With Minimization of Weight Perturbations, RADC-TR-82-32.

nulls is actually satisfied by the solutions to the linearized equations. It was found that, even for a low sidelobe array in which the linearized phase-only nulling procedure gave reasonably deep nulls for one or two constrained null locations, the effectiveness of cancellation in a sector of the pattern achieved with phase-only nulling deteriorated as the size of the sector was increased by adding equispaced null locations. This behavior contrasted sharply with that found when combined amplitude and phase perturbations were used. The combination resulted in increasingly effective cancellation throughout a sector as the width of the sector was increased by adding more nulls.

The failure of the linearized phase-only nulling procedure to give a satisfactory solution to the nulling problem for several closely spaced nulls, even in a low sidelobe structure, makes it of interest to consider alternate procedures for solving the phase-only nulling problem. In this report, I describe a simple scheme of phase-only nulling in which the linearization procedure is used iteratively. This scheme is found to be extremely effective in low sidelobe structures although it fails to work as a general method. The method is described in the next section. Some numerical results are then presented, comparing the iterative phase-only nulling method with the simple (that is, non-iterative) linearization method and with nulling using combined amplitude and phase perturbations.

2. ANALYSIS

Consider a linear array of equispaced isotropic elements (Figure 1). The spacing between the elements is d and the phase reference center is assumed to be the center of the array. Let w_n , $n = 1, 2, \dots, N$, be the complex weight of the n^{th} array element. Then the array field pattern, $p(u)$ is

$$p(u) = \sum_{n=1}^N w_n e^{j d_n u}$$

where

$$d_n = \frac{N-1}{2} - (n-1), \quad n = 1, 2, \dots, N$$

and

$$u = k d \sin \theta$$

with

$$k = \frac{2\pi}{\lambda}$$

and θ the angle measured from broadside to the array. Note that the d_n are odd symmetric with respect to the phase reference center; that is,

$$d_n = -d_{N-n+1}.$$

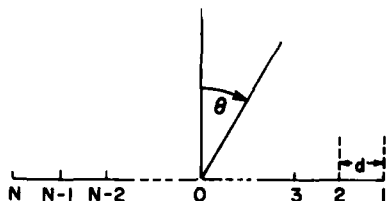


Figure 1. Geometry of Array

The general problem in which we are interested is as follows. Let a_n , $n = 1, 2, \dots, N$, be a given taper, assumed to be symmetric with respect to the phase reference center, of the amplitudes of the element excitations, and let u_s be the direction for the peak of the array pattern. Then the array coefficients are

$$w_{on} = a_n e^{-j d_n u_s}, \quad n = 1, 2, \dots, N.$$

We wish to find the set of perturbations, ϕ_n , $n = 1, 2, \dots, N$ of the phases of the element weights that will (a) result in a perturbed pattern with nulls at a set of prescribed locations, u_k , $k = 1, 2, \dots, M$, and (b) be "as small as possible." What does it mean to be "as small as possible?" The perturbed coefficients are

$$\begin{aligned} w_n &= a_n e^{-j d_n u_s} e^{j \phi_n} \\ &= a_n e^{-j d_n u_s} + a_n e^{-j d_n u_s} (e^{j \phi_n} - 1). \end{aligned} \quad (1)$$

The first term on the RHS of Eq. (1) is the original weight, and the second term is the total perturbation of the weight. Following Reference 1, the discussion will consider two forms of minimizing the perturbations. The first is to minimize

$$\sum_{n=1}^N |e^{j \phi_n} - 1|^2 = 4 \sum_{n=1}^N \sin^2 \left(\frac{\phi_n}{2} \right)$$

and the second is to minimize

$$\sum_{n=1}^N \left(a_n |e^{j\phi_n} - 1| \right)^2 = 4 \sum_{n=1}^N \left[a_n \sin \left(\frac{\phi_n}{2} \right) \right]^2.$$

The first form corresponds to minimizing the perturbations relative to the original weights, while the second corresponds to minimizing the total perturbations.

From Eq. (1) it follows that we can represent the perturbed array pattern as

$$p(u) = p_0(u) + \sum_{n=1}^N a_n e^{-j d_n u_s} (e^{j\phi_n} - 1) e^{j d_n u}$$

where $p_0(u)$ is the original array pattern

$$p_0(u) = \sum_{n=1}^N a_n e^{-j d_n u_s} e^{j d_n u}.$$

Hence the equation system for the nulls is

$$\sum_{n=1}^N a_n e^{-j d_n u_s} (e^{j\phi_n} - 1) e^{j d_n u_k} = -p_0(u_k), \quad k = 1, 2, \dots, M. \quad (2)$$

Separating the real and imaginary parts of Eq. (2) we obtain

$$\sum_{n=1}^N a_n \cos[d_n(u_s - u_k) - \phi_n] = 0 \quad \left. \vphantom{\sum_{n=1}^N} \right\} \quad k = 1, 2, \dots, M. \quad (3a)$$

$$\sum_{n=1}^N a_n \sin[d_n(u_s - u_k) - \phi_n] = 0 \quad \left. \vphantom{\sum_{n=1}^N} \right\} \quad k = 1, 2, \dots, M. \quad (3b)$$

The problem can then be stated as follows: find the set of phase perturbations, ϕ_n , $n = 1, 2, \dots, N$, that satisfy the equation system (3) and that minimize either

$$\sum_{n=1}^N \sin^2 \left(\frac{\phi_n}{2} \right) \quad (4a)$$

or

$$\sum_{n=1}^N \left[a_n \sin \left(\frac{\phi_n}{2} \right) \right]^2. \quad (4b)$$

This problem is an example of what is known in the operations research literature as a nonlinear programming problem (see for example References 2 and 3). The problem in general has no exact solution. Furthermore, it is not at all obvious that the equation system for the nulls has any solution at all and indeed an example can easily be found when it does not (consider an array with an odd number of elements for which the amplitude of the central element is greater than the sum of all the other amplitudes). However, we will not attempt here to answer the question of under what circumstances solutions exist to Eqs. (3). Rather, we will simply assume that the given amplitude distribution is such that the equation system for the ϕ_n is underdetermined if $2M < N$, that it has an infinity of possible solutions, and that a minimization criterion is used to select a particular one of these solutions. Since an explicit solution to the problem cannot be found, an efficient numerical method for finding the solution is desired.

The scheme we propose here is based on the assumption that the sidelobes of the original pattern are sufficiently low that only small phase perturbations are required to place nulls at the specified locations. Under this assumption we can expand Eqs. (3a, 3b) and neglect all terms quadratic and higher in ϕ_n thus obtaining

$$\left. \begin{aligned} \sum_{n=1}^N a_n \phi_{1,n} \sin [d_n (u_s - u_k)] &= -p_0(u_k) \\ \sum_{n=1}^N a_n \phi_{1,n} \cos [c_n (u_s - u_k)] &= 0 \end{aligned} \right\} \quad k = 1, 2, \dots, M. \quad (5a)$$

$$(5b)$$

Under the same assumption of small ϕ_n the functions to be minimized can be approximated by

$$\sum_{n=1}^N \phi_{1,n}^2 \quad (6a)$$

and

$$\sum_{n=1}^N (a_n \phi_{1,n})^2. \quad (6b)$$

We have written $\phi_{1,n}$ instead of ϕ_n here, not only to indicate that the solution to Eqs. (5) is an approximation to the solution to Eqs. (3) we are seeking, but also, as will be seen shortly, because the $\phi_{1,n}$ are to be the first stage in an iterative process leading to the desired solution.

2. Himmelblau, D. M. (1972) Applied Nonlinear Programming, McGraw-Hill, New York.
3. Schittkowski, K. (1980) Nonlinear Programming Codes, Springer-Verlag, New York.

The solution to Eq. (5a) that minimizes Eq. (6a) or Eq. (6b) was found in our earlier report¹ and the result is merely stated here: let

$$c_n = \begin{cases} a_n, & \sum_{n=1}^N \phi_{1,n}^2 = \min \\ 1, & \sum_{n=1}^N (a_n \phi_{1,n})^2 = \min, \end{cases} \quad (7)$$

$$t_n = \frac{c_n^2}{\sum_{n=1}^N c_n^2}, \quad (8)$$

and

$$\underline{y} = - [p_o(u_1), p_o(u_2), \dots, p_o(u_m)]^T;$$

then

$$\phi_{1,n} = \frac{2t_n}{a_n} \sum_{m=1}^M b_m \sin [d_n (u_s - u_m)] \quad (9)$$

where the vector of coefficients b_m is obtained by inverting the matrix equation

$$A \underline{b} = \underline{y}$$

with the elements of the matrix A given by

$$[A]_{km} = \sum_{n=1}^N 2t_n \sin [d_n (u_s - u_k)] \sin [d_n (u_s - u_m)] \quad k, m = 1, 2, \dots, M.$$

Equation (5b) is automatically satisfied by this solution because of the antisymmetry of the $\phi_{1,n}$ with respect to the phase reference center. The weights corresponding to the phase perturbations $\phi_{1,n}$ are

$$w_{1,n} = a_n e^{-j d_n u_s} e^{j \phi_{1,n}}$$

and the array pattern is then

$$p_1(u) = \sum_{n=1}^N w_{1,n} e^{j d_n u}.$$

Now, if the phase perturbations $\phi_{1,n}$ are fairly good approximations to the desired perturbations ϕ_n , then the values of $p_1(u)$ at the specified null locations $u = u_k$, $k = 1, 2, \dots, M$, will be smaller in magnitude than the original values $p_0(u_k)$. Hence it is reasonable to suppose that the coefficients $w_{1,n}$ can be perturbed in turn with a set of still smaller phase perturbations so as to place nulls in the pattern at the specified locations. This idea leads us to the following iterative scheme, similar to that used by Cheng and Raymond,⁴ and by Cheng.⁵ Let the weights at the end of the i^{th} iteration, $i \geq 1$, be

$$w_{i,n} = a_n e^{-j d_n u_s} e^{j \phi_{i,n}}, \quad n = 1, 2, \dots, N$$

and the corresponding array pattern

$$p_i(u) = \sum_{n=1}^N w_{i,n} e^{j d_n u}.$$

For $i = 0$, define

$$\phi_{0,n} \equiv 0$$

$$w_{0,n} \equiv a_n e^{-j d_n u_s}.$$

Let $\phi_{i+1,n} - \phi_{i,n}$ be the phase perturbation introduced at the $(i+1)^{\text{th}}$ iteration, $i \geq 0$. Then similarly to Eqs. (5a and 5b), we have

$$\left. \begin{aligned} \sum_{n=1}^N a_n (\phi_{i+1,n} - \phi_{i,n}) \sin [d_n (u_s - u_k) - \phi_{i,n}] &= -p_i(u_k) \\ \sum_{n=1}^N a_n (\phi_{i+1,n} - \phi_{i,n}) \cos [d_n (u_s - u_k) - \phi_{i,n}] &= 0 \end{aligned} \right\} \quad k = 1, 2, \dots, M$$

or

4. Cheng, D.K., and Raymond, Jr., P.D. (1971) Optimization of array directivity by phase adjustments, Electron. Letts. 7:552-554.
5. Cheng, S.D. (1971) Optimization techniques for antenna arrays, Proc. IEEE, 90:1664-1674.

$$\sum_{n=1}^N a_n \phi_{i+1,n} \sin[d_n(u_s - u_k) - \phi_{i,n}] = -p_i(u_k) + \sum_{n=1}^N a_n \phi_{i,n} \sin[d_n(u_s - u_k) - \phi_{i,n}] \quad (10a)$$

$$\sum_{n=1}^N a_n \phi_{i+1,n} \cos[d_n(u_s - u_k) - \phi_{i,n}] = \sum_{n=1}^N a_n \phi_{i,n} \cos[d_n(u_s - u_k) - \phi_{i,n}]. \quad (10b)$$

The functions to be minimized are

$$\sum_{n=1}^N \phi_{i+1,n}^2$$

or

$$\sum_{n=1}^N (a_n \phi_{i+1,n})^2.$$

Eqs. (10a and 10b) are of the same form as Eqs. (5a and 5b) except for the RHS of Eq. (10b). But because of the antisymmetry of the $\phi_{1,n}$ noted above, the RHS of Eq. (10b) is zero when $i=1$, and hence by induction the same will be true for succeeding iterations as well. Thus, similarly to Eq. (9), the $\phi_{i+1,n}$ are given by

$$\phi_{i+1,n} = \frac{2t_n}{a_n} \sum_{m=1}^M b_{i+1,m} \sin[d_n(u_s - u_m) - \phi_{i,n}]. \quad (11)$$

The vector of coefficients $b_{i+1,m}$ is obtained by inverting the matrix equation

$$A_{i+1} b_{i+1} = y_{i+1} \quad (12)$$

where

$$y_{i+1} = - \left\{ p_i(u_1) - \sum_{n=1}^N a_n \phi_{i,n} \sin[d_n(u_s - u_1) - \phi_{i,n}], \dots, p_i(u_M) - \sum_{n=1}^N a_n \phi_{i,n} \sin[d_n(u_s - u_M) - \phi_{i,n}] \right\}^T \quad (13)$$

and the elements of the matrix A_{i+1} are given by

$$[A_{i+1}]_{k,m} = \sum_{n=1}^N 2 t_n \sin[d_n(u_s - u_k) - \phi_{i,n}] \sin[d_n(u_s - u_m) - \phi_{i,n}]. \quad (14)$$

Testing for convergence and termination of the iterative procedure can be based either on the phase perturbations themselves (for example, termination when $\max_n \{ |\phi_{i+1,n} - \phi_{i,n}| \} < \epsilon$ or when $\sum_n |\phi_{i+1,n} - \phi_{i,n}| < \epsilon$), or on the values of the beam coefficients. That convergence of the phase perturbations implies convergence of the beam coefficients follows easily from Eq. (12) in conjunction with Eqs. (13) and (14), since

$$\underline{b}_{i+1} = A_{i+1}^{-1} \underline{y}_{i+1}$$

and A_{i+1} and y_{i+1} approach limiting values if the $\phi_{i,n}$ converge to limiting values ϕ_n .

It is also worth noting that if the limiting values of the phase perturbations are small, the resulting cancellation pattern can be interpreted as the sum of pairs of slightly distorted beams, one pair for each imposed null, in a way that is completely analogous to the interpretation of the non-iterative phase-only cancellation pattern as the sum of pairs of beams (see Reference 1). For from Eq. (11), we see that if the $\phi_{i,n}$ converge to limiting values ϕ_n , then

$$\phi_n = \frac{2 t_n}{a_n} \sum_{m=1}^M b_m \sin[d_n(u_s - u_m) - \phi_n]. \quad (15)$$

Also, from Eq. (1), if ϕ_n is small then the change in the n^{th} weight can be approximated by

$$\begin{aligned} \Delta w_n &= a_n e^{-j d_n u_s} (e^{j \phi_n} - 1) \\ &\approx j a_n \phi_n e^{-j d_n u_s} \end{aligned} \quad (16)$$

so that, substituting Eq. (15) in Eq. (16),

$$\begin{aligned} \Delta w_n &\approx t_n \sum_{m=1}^M b_m e^{-j d_n u_s} \left\{ e^{j[d_n(u_s - u_m) - \phi_n]} - e^{-j[d_n(u_s - u_m) - \phi_n]} \right\} \\ &= t_n \sum_{m=1}^M b_m \left\{ e^{-j(d_n u_m + \phi_n)} - e^{-j[d_n(2u_s - u_m) - \phi_n]} \right\} \end{aligned}$$

and hence the cancellation pattern is given approximately by

$$\Delta p(u) \approx \sum_{m=1}^M b_m \sum_{n=1}^N t_n \left\{ e^{j[d_n(u - u_m) - \phi_n]} - e^{j[d_n(u - \{2u_s - u_m\}) + \phi_n]} \right\}.$$

If the phase perturbations, ϕ_n , were zero, the cancellation pattern would thus be represented as the sum of M pairs of beams, one member of each pair directed at an imposed null location, and the other member, of opposite sign, directed at the symmetric location with respect to the axis of the mainlobe of the original pattern. Indeed, this is the approximate representation of the cancellation pattern for the noniterative phase-only nulling scheme (see Reference 1). If the phase perturbations are non-zero but small, it is to be expected that the phase perturbations introduce a small amount of distortion in the shape of the component beams but that the beams representation of the cancellation pattern still holds. Hence, for small perturbations, the limiting values of the beam coefficients obtained with the iterative nulling method will not differ greatly from the values of the coefficients obtained with the noniterative nulling scheme. As with the non-iterative method, the nulls produced at the prescribed locations will be accompanied by a comparable raising of the pattern at the symmetric locations with respect to the mainlobe axis.

If the iterative procedure converges, the limiting values of the $\phi_{i,n}$ should satisfy the system of Eqs. (3) for the pattern nulls. If the limiting values are small they should also be reasonable approximations to the values of the ϕ_n that not only satisfy Eqs. (3) but which also minimize either expression (4a) or (4b).^{*} Further work is needed, however, to obtain a quantitative estimate on how close the limiting values of the $\phi_{i,n}$ will be to the ideal solution. A direct comparison of the values obtained by a more powerful non-linear programming method with the values obtained here, would, of course, serve this purpose. The solution obtained by the iterative scheme proposed here has value not only because it provides a useful approximation to the desired answer, but also because it can serve as a starting point for further refinement by a more powerful non-linear optimization method. Such methods often must be started at a feasible solution—that is, a solution to the constraint equations.

^{*}On first glance it might appear that the solution to Eqs. (3) that minimizes, say $\sum \phi_n^2$ also minimizes (4a). For if we let $\{\psi_n\}$ be the set of phase perturbations that satisfy Eqs. (3) and minimize (4a), and $\{\phi_n\}$ be the set of phase perturbations that satisfy Eqs. (3) and minimize $\sum \phi_n^2$ then the $\{\psi_n\}$ and $\{\phi_n\}$ must satisfy the following set of inequalities, assuming $0 < |\phi_n| \leq \pi/2$, $0 < |\psi_n| < \pi/2$:

3. NUMERICAL RESULTS

A computer program was written to implement the iterative nulling scheme described in the previous section, and run on a CDC 6600 computer. All computations were performed for an array with 41 elements with half wavelength spacing. Unless otherwise stated, the original or unperturbed pattern corresponds to a 40 dB Chebyshev taper of the element excitations. Figure 2 shows results obtained when the iterative program was run to impose nulls at one (15.23°), two (15.23°, 15.78°), three (15.23°, 15.78°, 16.33°), and four (15.23°, 15.78°, 16.33°, 16.88°) locations. We have plotted the maximum of the values of the perturbed pattern power at the locations at which nulls were desired (that is, the least deep null) vs number of iterations. Figure 2a gives the results obtained when $\sum (a_n \phi_n)^2$ was minimized,

$$4 \sum_{n=1}^N \sin^2 \left(\frac{\psi_n}{2} \right) \leq 4 \sum_{n=1}^N \sin^2 \left(\frac{\phi_n}{2} \right) < \sum_{n=1}^N \phi_n^2 \leq \sum_{n=1}^N \psi_n^2. \quad (\dagger)$$

If in general

$$\sum_{n=1}^N \psi_n^2 \geq \sum_{n=1}^N \phi_n^2 \Rightarrow \sum_{n=1}^N \sin^2 \left(\frac{\psi_n}{2} \right) \geq \sum_{n=1}^N \sin^2 \left(\frac{\phi_n}{2} \right)$$

as it does when $N=1$, it would follow that the first "<" in Eq. (†) must be strictly "=". Unfortunately this is not true, as can be seen by letting $\psi_1 = 1 + \epsilon$, $\psi_n = 0, n > 1$ $\phi_n = 1/\sqrt{N}, n = 1, 2, \dots, N$. We then have, for $0 < \epsilon < 1$, $\sum \psi_n^2 > \sum \phi_n^2$ and $\sum \sin^2 \left(\frac{\psi_n}{2} \right) \approx \sin^2 (1/2) = 0.230$

but

$$\sum_{n=1}^N \sin^2 \left(\frac{1}{2\sqrt{N}} \right) > N \left[\left(\frac{1}{2\sqrt{N}} \right)^2 - \frac{1}{3} \left(\frac{1}{2\sqrt{N}} \right)^4 \right] = \frac{1}{4} - \frac{1}{48N} > 0.240$$

for $N > 1$.

and Figure 2b gives the results corresponding to minimizing $\sum o_n^2$. The starting point of all curves is the maximum value of the power of the original pattern at the null locations. The value for one iteration is the least null depth achieved with the simple (that is, non-iterative) linearized nulling procedure that coincides with the first step of the iterative procedure. We see that in all cases the iterative procedure gives null depths (< -250 dB) that greatly exceed the null depths obtained with the non-iterative nulling procedure. As a number of null locations increases, however, more iterations are required to achieve the same depth of null. There is also a tendency (as is shown, for example, in the four-null curve of Figure 2b) for the maximum depth of null achievable by the iterative procedure to slowly decrease as the number of null locations increases.

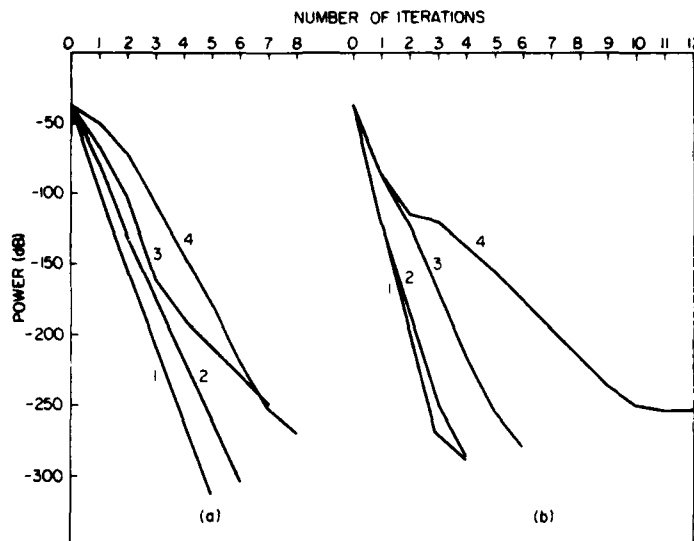


Figure 2. Maximum of Perturbed Pattern Power at Imposed Null Locations for 1, 2, 3 and 4 Nulls as a Function of Number of Iterations. 40 dB Chebyshev amplitude taper. Figure 2a: $\sum (a_n \phi_n)^2 = \min$; Figure 2b: $\sum \phi_n^2 = \min$

Figure 3 shows corresponding results obtained starting with a 20 dB Chebyshev taper of the element excitations, when the iterative program was run to impose nulls at one (14.70°), two (14.70° , 15.28°), three (14.70° , 15.28° , 15.86°), and four (14.70° , 15.28° , 15.86° , and 16.44°) locations. We see that for both minimization criteria the iterative procedure was able to produce deep nulls for the one and two

null cases, although considerably more iterations were required compared with the 40 dB calculations of Figure 2. (Note the curious feature of Figure 3b in which fewer iterations were required for the two-null case than for the one-null case.) For three nulls, the iterative procedure was ineffective for the case of minimizing

$\sum (a_n \phi_n)^2$ but worked well for the case minimizing $\sum \phi_n^2$. For four nulls, the iterative procedure was unable to achieve any nulling for either minimization criterion. These results point out the limitations of the iterative method when the sidelobes of the original pattern are relatively high. The phase perturbations required to produce nulls in the pattern are then larger in magnitude than they are for low sidelobe patterns, especially when multiple closely spaced nulls are prescribed, with the result that the iterative procedure (consisting of repeated linearizations based on the assumption of small phase perturbations) fails to work.

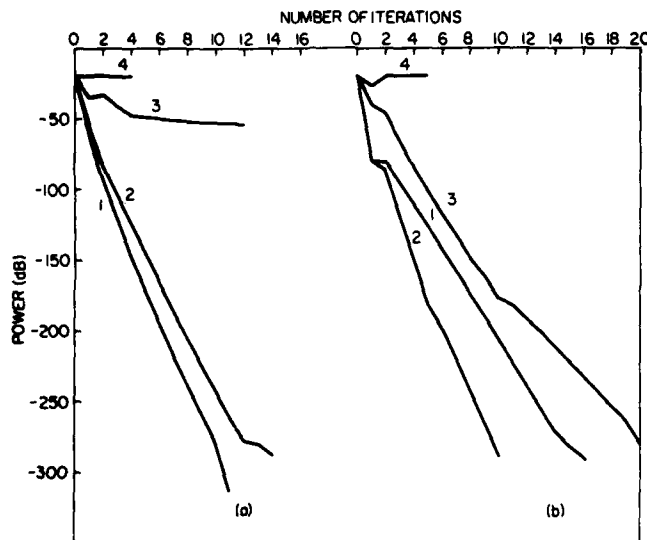


Figure 3. Maximum of Perturbed Pattern Power at Imposed Null Locations for 1, 2, 3 and 4 Nulls as a Function of Number of Iterations. 20 dB Chebyshev amplitude taper. Figure 3a: $\sum (a_n \phi_n)^2 = \min$; Figure 3b: $\sum \phi_n^2 = \min$

As an application of the iterative nulling method calculations were performed to study the variation of cancellation effectiveness within a sector of the pattern as

the width of the pattern is increased by adding equispaced nulls. As in our earlier report¹ the power cancellation ratio in the sector $\Delta\theta = \theta_1 \leq \theta \leq \theta_2$ is defined

$$C = \frac{\max_{\theta \in \Delta\theta} [p(\theta)]^2}{\max_{\theta \in \Delta\theta} [p_0(\theta)]^2}$$

where $p_0(\theta)$ is the original pattern and $p(\theta)$ is the perturbed pattern. In Table 1 we have tabulated the cancellation ratios for various sector widths and different nulling methods. The entries in the i^{th} row of the table are the cancellation ratios obtained by the methods listed across the top of the table for the sector $15.23^\circ \leq \theta \leq (\theta_2)_i$ with nulls placed at 15.23° and at the locations $\theta = (\theta_2)_j$, $j \leq i$. For example, the entry of -68.0 dB in the fourth row for iterative phase-only nulling minimizing $\sum (a_n \phi_n)^2$ means that when nulls were imposed by this method at the locations 15.23° , 15.78° , 16.33° , 16.88° , and 17.44° , the cancellation ratio in the sector $15.23^\circ \leq \theta \leq 17.44^\circ$ was -68.0 dB. The sequence of null locations $15.23^\circ, \dots, 18.00^\circ$ is equispaced with respect to $\sin \theta$. For comparison purposes we have included in Table 1 not only the cancellation ratios obtained with the two iterative phase-only nulling methods but also, taken from Reference 1, the cancellation ratios obtained with the noniterative phase-only methods and the exact combined amplitude and phase perturbation methods.

The outstanding feature of Table 1 is the remarkable improvement in cancellation effectiveness resulting from the use of the iterative phase-only methods as compared with the non-iterative phase-only methods. For six nulls, cancellation ratios of -82 dB and -78 dB were obtained using the iterative methods as compared with ≈ -1 dB for the non-iterative phase-only methods. This improvement is, of course, attributable to the fact that the iterative methods are able to produce far deeper nulls at the prescribed locations than are the non-iterative methods, with the result that the entire pattern within the sector is pulled down much further. Indeed, the iterative phase-only methods give sector cancellation comparable to the exact combined amplitude and phase perturbation methods for all values of θ_2 through 17.44° . For $\theta_2 = 18.00^\circ$, the iterative phase-only methods give slightly less effective sector cancellation than do the combined amplitude and phase perturbation methods. This is probably attributable to the fact that, as noted above, the depth of null achievable by the iterative methods slowly degrades as the number of imposed nulls increases. For $\theta_2 = 18.00^\circ$, that is, six imposed nulls, the shallowest of the six nulls produced by the iterative phase-only methods was -238 dB for minimized $\sum (a_n \phi_n)^2$ and -221 dB for $\sum \phi_n^2$.

Table 1. Cancellation Ratio (dB) for the Sector $15.23 \leq \theta \leq \theta_2$ With Equispaced Imposed Nulls

θ_2 ($^\circ$)	$\sum (a_n \phi_n)^2 = \min.$ iterative	$\sum \phi_n^2 = \min.$ iterative	$\sum (a_n \phi_n)^2 = \min$ non-iterative	$\sum \phi_n^2 = \min.$ non-iterative	$\sum \Delta w_n ^2 = \min.$	$\sum \left(\frac{ \Delta w_n }{a_n} \right)^2 = \min.$
15.78	-29.8	-27.8	-31.4	-27.8	-30.0	-27.8
16.33	-46.6	-43.1	-27.3	-45.5	-47.7	-43.5
16.88	-62.9	-56.7	-11.4	-51.8	-60.6	-54.8
17.44	-68.0	-62.5	-12.2	-29.5	-67.9	-62.5
18.00	-82.0	-77.9	-0.7	-1.3	-87.1	-85.3

Even though the iterative phase-only methods studied in this report are not as effective in certain multiple null situations and in high sidelobe applications as are combined amplitude and phase perturbation methods, nevertheless it is important to recognize that these limitations are not limitations so much of phase-only nulling per se as they are of the particular iterative scheme proposed here. Based on the results obtained and discussed in this report, it would appear that in many situations phase-only methods are in principle as effective in nulling and sector cancellation as are combined amplitude and phase perturbation methods. Further work is needed, however, to refine the numerical procedures involved in phase-only nulling. The principal drawback of phase-only nulling is that the nulls and sector cancellation achieved on one side of the mainlobe of the original pattern are accompanied by a comparable raising of the pattern at the locations and sectors symmetrically placed with respect to the mainlobe. This, rather than limited nulling effectiveness, appears to be the main price one pays for using phase-only nulling.

4. CONCLUSIONS

In this report, an iterative numerical method is described for calculating the minimum phase-only perturbations of the element excitations of a linear array required to impose nulls in the pattern at prescribed locations. The method is based on repeated linearizations of the equations for the imposed nulls. The iterative scheme is found to be extremely effective in low sidelobe applications when the required phase perturbations are small, but fails to work as a general method when the required phase perturbations are large.

References

1. Shore, R. A., and Steyskal, H. P. (1982) Nulling in Linear Array Patterns With Minimization of Weight Perturbations, RADC-TR-82-32.
2. Himmelblau, D. M. (1972) Applied Nonlinear Programming, McGraw-Hill, New York.
3. Schittkowski, K. (1980) Nonlinear Programming Codes, Springer-Verlag, New York.
4. Cheng, D. K., and Raymond, Jr., P. D. (1971) Optimization of array directivity by phase adjustments, Electron. Letts. 7:552-554.
5. Cheng, S. D. (1971) Optimization techniques for antenna arrays, Proc. IEEE, 99:1664-1674.