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Time-varying systems; crossed products; representation theory; stability; Floquet theory; transform theory.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research conducted for the past two years into linear time-varying systems is outlined. Crossed product algebras make precise the intuitive idea of a class of systems which can be synthesized from the usual delay elements and a fixed class of time-varying gains. It has been shown that crossed products appear to be a most appropriate setting for input-output analysis of linear time-varying dynamical systems; they also always admit a bounded decomposition of a Hermitian operator into the sum of a causal operator and its adjoint. Finally it has been shown that in the context of crossed products (CONTINUED)

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~~ITEM #20, CONTINUED:~~ it is possible (with a restriction on the class of time-varying gains) to formulate a generalized transform theory which closely parallels that for time-invariant systems, and yields previously known results for periodic systems.

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The Application of Crossed Products to the Stability

and Design of Time-Varying Systems

During the past two years, research has been conducted into some theoretical aspects of linear time-varying system theory from the input-output viewpoint, using the concept of a crossed product algebra as a framework. The objectives of this work were the following:

a) To provide a framework for the extension (to various classes of time-varying systems) of as much as possible of the methods and intuition provided for time-invariant systems by the classical transforms and frequency-domain theory. This was motivated in part by the fact that for some classes of time-varying systems (notably periodic) there was a generalized transform theory.

b) To shed light on some of the problems which arise in the general operator theory of systems; it was felt that some of these were due to the fact that the theory admits very general operators, including some which do not appear to have physical relevance, and that the problems might be alleviated if attention were restricted to a physically more realistic class of operators.

Status of the Research

Although neither of the objectives outlined above was attained entirely, some progress was made towards both. The situation with respect to the second problem is probably the more satisfactory of the two; we will describe this first.

Since the primary constraint on signal magnitude in most real systems is a limitation on instantaneous size (e.g. clipping, overflow) rather than a limitation on the mean-square value or total energy content, of a signal, it is usually felt that the L^∞, (BIBO, supremum) norm is a more realistic norm that the L^2, or Hilbert space, norm. The latter, however, is the more commonly used, especially in the time-varying case, for two reasons: firstly, it is far easier to work with, and secondly, in the time-varying case, there is no adjoint operation on the set of bounded linear operators on an L^∞-space, so that such fundamental system-theoretic problems as spectral factorization can not even be formulated. In the light of these considerations, the significance of the following result should be clear:

I) A crossed product algebra (of a certain type) can be regarded as a subalgebra, with an adjoint operation, of the algebra of all linear bounded-input bounded-output (BIBO) operators on $L^\infty(Z)$ or $L^\infty(R)$. Further, the natural norm on an element in the crossed product is not less than the operator norm of the corresponding operator on L^∞.

Thus a crossed product is an algebra which possesses an adjoint, and in which boundedness (in the crossed product norm) and causality imply BIBO stability.

This indicates that crossed products are a natural setting for a physically realistic input-output system theory. This view is bolstered by the following:

II) Every asymptotically stable finite-dimensional linear dynamical system with bounded coefficients (which are also assumed continuous in the continuous-time case) has a transfer operator which is an element of a crossed product algebra; conversely, the set of such transfer operators is dense in a crossed product algebra.

This result indicates that crossed product algebras are precisely the right class of Banach algebras in which to study input-output properties of finite-dimensional linear dynamical systems.

With regard to solving the problems which arise in the Hilbert-space theory, the results are mixed. The specific results in mind are the additive and multiplicative decompositions of Hermitian and positive operators respectively. In the Hilbert space theory, the former may not exist even in the time-invariant case, while the latter is known to exist in the discrete-time case and to fail sometimes in continuous time.

In the case of crossed products, we have the following results:

III) In a crossed product, every Hermitian element can be decomposed into the sum of a causal element and its adjoint; the mapping thus defined from Hermitians to causals has norm equal to one half. Every positive element whose distance from the identity is less than one can be factored as the product of a causal and its adjoint; and the set of positive elements having such a factorization is open in the set of positive elements.

These results show that the situation for additive decomposition is much better in crossed products than in the Hilbert-space theory, while for spectral factorization it is not as good, except for the fact that in continuous time we can show that the set of elements having spectral factorizations is open in the crossed product theory.

Our inability to prove the existence of spectral factorizations in general is one of the major ways in which we fall short of achieving our second objective. Another less obvious way (which is actually more closely related to our first objective) is in our failure to prove that the crossed product algebras (over $\mathbb{Z}$ and $\mathbb{R}$) are symmetric — that is, that the sum of the identity and any positive element is invertible.

We turn next to the first objective—the formulation of a transform theory for linear time-varying systems. Here far more work remains to be done. At the intuitive level, the crossed product approach makes precise the idea of a time-varying convolution; one may also view a crossed product algebra as the algebra of operators which can be synthesized from the usual time-invariant memory elements and a specified commutative algebra A of time-varying gains. We then have the following basic results.

IV) If the Gelfand carrier space, X, of A has a finite, positive Borel measure μ which is invariant under the action of the group $\mathbb{Z}$ or $\mathbb{R}$ on X, then the following statements hold:

a) For each point on the imaginary axis or the unit circle, there is a representation of the crossed product on the space $L^2(X,\mu)$.

b) For the causal subalgebra of the crossed product, there is a representation on $L^2(X,\mu)$ for each point in the left half-plane or the unit disk, and the mapping which assigns to each point in the left half-plane or the unit disk the representation of a fixed causal element of the crossed product at that point is analytic (in the operator norm).

c) An element of the crossed product is invertible if and only if its image in the representation corresponding to each point on the unit circle or imaginary axis is invertible; a causal element is causally invertible if and only if its image in the representation corresponding to each point in the unit disk or left half-plane is invertible. Since stability is essentially equivalent to causal invertibility, this is precisely analogous to the classical Hurwitz-type stability criteria.

Although these results form the basis for a theory analogous to the classical frequency-domain theory, there are some problems, and much work remains to be done. The first problem is the assumption of a finite, positive invariant measure. Although this is satisfied in many situations, there are some where it is not. The second problem has already been alluded to; since we do not know that the crossed product is symmetric, we can conclude only that the inverses described above exist in the enveloping C*-algebra of the crossed product, and not necessarily in the crossed product itself. Finally, in many situations, each of the representations described above is faithful, and so the process yields no algebraic simplification. There are two other approaches which it opens up, however. The first of these depends on the existence of a densely defined functional on the representing algebras which has many of the properties of a trace. Although its existence has not been verified in general, it is believed that this will follow from the existence of the invariant measure; this opens the possibility of defining a winding number, or index, on the crossed product, and thereby formulating an analog of the Nyquist test.

The second approach has to do with the fact that even when there is no algebraic simplification, the representations take us from the study of a system on $\mathbb{Z}$ or $\mathbb{R}$, which are noncompact sets, to a system on X, which is compact. This is highly analogous to Floquet theory for periodic systems, and we have shown that for certain almost-periodic systems with two incommensurable frequencies, there is a procedure analogous to Floquet theory which will determine stability.

Activities Under AFOSR Grant 80-0156

Papers:

"Some Comments on Lumped-Distributed Networks and Differential-Delay Systems," J. Murray, in Applications of Algebra and Algebraic Geometry to Linear System Theory, Providence, AMS.

"Time-Varying Systems and Crossed Products," J. Murray, submitted to Mathematical Systems Theory.

Conference Paper:

"A *-Norm for Uniform Stability of Discrete-Time Systems," J. Murray, MTNS Symposium, Santa Monica, August, 1981.

Conferences Attended:

1980 IEEE International Symposium on Circuits and Systems, Houston, April 1980.

Twelfth Southeastern Symposium on System Theory, and Workshop on the Mathematical Theory of Networks and Systems, Virginia Beach, May 1980.

Nineteenth IEEE Conference on Decision and Control, Albuquerque, December 1980.

1981 IEEE International Conference on Acoustics, Speech and Signal Processing, Atlanta, March-April 1981.

24th Midwest Symposium on Circuits and Systems, Albuquerque, June 1981.

International Symposium on Mathematical Theory of Networks and Systems, Santa Monica, August 1981.

Second IEEE ASSP Workshop on Two-Dimensional Signal Processing, Lake Mohonk, October 1981.

20th IEEE Conference on Decision and Control, San Diego, December 1981.