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               BEARING ESTIMATION USING RANDOM ARRAYS.(U)
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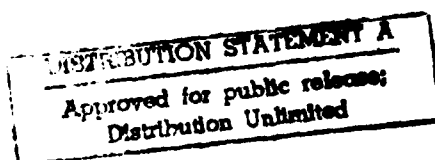
Bearing Estimation using Random Arrays

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Abstract

A sonobuoy field is a random two dimensional array if the signals from the sensors are coherently processed. The sensor position coordinates must be estimated to a fraction of a wavelength in order to coherently process the signals, which is a separate processing task since the field has a changing random pattern due to drifting of the buoys. This paper reviews the statistical properties of the maximum likelihood bearing estimator computed from coherent summation of output signals from randomly deployed sensors. It is shown that the approximate mean-square bearing error for a small random array favorably compares with that for a square array of equally spaced sensors with about one-half the aperture. The random array is not subject to the aliasing (grating lobes) problem of an equally spaced array, and a great deal of array gain can be achieved from a large array of randomly placed sensors. Serious consideration should be given to tracking the locations of drifting sonobuoys so as to coherently process them as an array.

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Bearing Estimation using Random Arrays

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Introduction

A sonobuoy field is a random two dimensional array if the signals from the sensors are coherently processed (Thorn, Booth, and Lockwood [1]). The sensor position coordinates must be estimated to a fraction of a wavelength in order to coherently process the signals, which is a separate processing task since the field has a changing random pattern due to drifting of the buoys. It is technically feasible to track the pattern, but the processing gain from such an effort must be perceived as sufficiently great to justify such an effort. This paper reviews the statistical properties of the maximum likelihood bearing estimator computed from coherent summation of output signals from randomly deployed sensors. Stochastic properties of the array response function are discussed by Thorn, et. al. and Steinberg [2].

The signal received at a sensor from a distant acoustic source in a horizontally homogeneous ocean is a plane wave plus ambient noise (Clay and Medwin, [3]). Using complex variable notation, a single frequency plane wave at point (x_1, x_2) and time t is

$$p(t,x) = A \exp[i2\pi f_0(t-(x_1 \cos\theta_0 + x_2 \sin\theta_0)/c)]$$

where θ_0 is the horizontal direction of propagation with respect to the x-axis, c is the phase velocity, f_0 is the wave's frequency (the wavelength is $\lambda_0 = c/f_0$) and A is the complex amplitude. This amplitude will be assumed to be a constant for the expositon of coherent

processing, but actual signals will have some amplitude and phase modulation which reduces the signal coherence across the array. A reduction of coherence results in an increase in the mean square error of bearing computed from any of the coherent processing methods discussed next.

1. Coherent Processing

The most widely used coherent processing technique is delay-and-sum beamforming. Suppose that the signal received by the kth sensor in a horizontal planar array is $s(t, \underline{x}_k) = p(t, \underline{x}_k) + \epsilon(t, \underline{x}_k)$ where $\epsilon(t, \underline{x}_k)$ is a Gaussian noise field at time t and point (x_{1k}, x_{2k}). The usual approach for beamforming is to compute the beam output signal $B(t, \theta_\ell) = \sum_{k=1}^{M} s(t + \tau_{k\ell})$ for a set of delays $\tau_{k\ell} = (x_{1k}\cos\theta_\ell + x_{2k}\sin\theta_\ell)/c$ for sensors $k = 1, \ldots, M$ and beam (look) angles $\ell = 1, \ldots, N$. The energy in beam θ_ℓ over a sampling window of length T is $|B(\theta_\ell)|^2$ where $B(\theta_\ell)$ is the filtered beam signal in a band about f_o of bandwidth $1/T$. If the plane wave signal is broadband, then the filter passband is adjusted to the bandwidth of the signal.

Levin [4] shows that the maximum likelihood estimator of the propagation angle θ_o is the $\hat{\theta}_o$ in the interval $0 < \theta < 2\pi$ that maximizes $|B(\theta)|^2$. Define

$$\hat{\sigma}_1^2 = M^{-1} \sum_{k=1}^{M} (x_{1k} - \bar{x}_1)^2, \qquad \hat{\sigma}_2^2 = M^{-1} \sum_{k=1}^{M} (x_{2k} - \bar{x}_2)^2 \qquad (1.1)$$

where $\bar{x}_1 = M^{-1}\sum_{k=1}^{M} x_{1k}$ and $\bar{x}_2 = M^{-1}\sum_{k=1}^{M} x_{2k}$. Set $\sum_{k=1}^{M}(x_{1k} - \bar{x})(x_{2k} - \bar{y}) = 0$

by a proper rotation of the coordinate system. Then if $\rho M \hat{\sigma}_1^2$ and $\rho M \hat{\sigma}_2^2$ are large, where ρ denotes the power signal-to-noise ratio in a narrow band about $\hat{f}_0$, then the mean-square error of $\hat{\theta}_0$ is given by the approximation (Hinich [5])

$$E(\hat{\theta}_0 - \theta_0)^2 \simeq (\lambda_0/2\pi)^2(2\rho M)^{-1}(\hat{\sigma}_1^{-2}\sin^2\theta_0 + \hat{\sigma}_2^{-2}\cos^2\theta_0)\ \text{rad}^2. \qquad (1.2)$$

This approximation also holds for a discrete set of beam angles and an estimator $\hat{\theta}_N$ that maximizes $|B(\theta_\ell)|^2$, provided that the error $|\theta_{\ell+1}-\theta_\ell|$ is small for all ℓ. In many applications, however, the bias due to the discreteness of the θ_ℓ is larger than the square-root of the right hand side of (1.2). It is important in these applications to interpolate between $\hat{\theta}_N$ and its adjacent angles.

A simple way to interpolate is to use the frequency-wavenumber equivalence with beamforming (Hinich [6]). Suppose that $s(t,\underline{x}_k)$ is passed through the same narrow band filter as was used to produce $B(\theta)$. For example, the fast Fourier transform (FFT) can be used to filter discretely sampled signals to yield the filtered output

$$s(\underline{x}_k) = N^{-1}\Sigma_{n=0}^{N-1} s(n\tau,\underline{x}_k)\exp(-i2\pi mn/N), \qquad (1.3)$$

where $T = N\tau$ is the sampling period, τ is the sampling interval, and m is the integer part of $f_0 T$. Then

$$B(\theta_\ell) = \sum_{k=1}^{M} s(\underline{x}_k)\exp[i(\kappa_{1\ell}x_{1k} + \kappa_{2\ell}x_{2k})] \qquad (1.4)$$

where $\kappa_{1\ell} = (2\pi/\lambda_0)\cos\theta_\ell$ and $\kappa_{2\ell} = (2\pi/\lambda_0)\sin\theta_\ell$. The FFT can be used to compute (1.4) for as fine a grid as desired by imbedding $\{\underline{x}_k\}$ in a square

lattice of equally spaced points, and adding zeros to expand the two dimensional array.

From now on it will be assumed that the bearing interpolation is sufficiently refined and ρM is sufficiently large so that expression (1.2) holds for $\hat{\theta}_0$. This expression will now be used to analyze the ex-ante root mean-square error of $\hat{\theta}_0$, rmse $\hat{\theta}_0$, when the sensor locations are randomly determined.

2. Gaussian Random Arrays

Suppose that the x_{1k} and x_{2k} are M independent realizations from two independent Gaussian random variables X_1 and X_2. With no loss of generality center the origin of the coordinate system so that $X_1 \sim N(0,\sigma^2_1)$ and $X_2 \sim N(0,\sigma^2_2)$, where σ^2_n is the variance of X_n for n=1,2. To further simplify the analysis, let $\sigma_1 = \sigma_2 = \sigma$.

It then follows that $\sigma^{-2}\Sigma^M_{k=1}(x_{1k} - \bar{x}_1)^2$ and $\sigma^{-2}\Sigma^M_{k=1}(x_{2k} - \bar{x}_2)^2$ are independent chi-square χ^2_{M-1} variates with M-1 degrees of freedom. Denote $\beta_T = Pr(\chi^2_{M-1} > T^2)$, e.g. $0.99 = Pr(\chi^2_{15} > 5.2)$ using a standard chi-square distribution table. By independence, the probability that

$$\Sigma^M_{k=1}(x_{1k} - \bar{x}_1)^2 > (\sigma T)^2 \quad , \quad \Sigma^M_{k=1}(x_{2k} - \bar{x}_2)^2 > (\sigma T)^2$$

is β^2_T, and thus

$$Pr\{M^{-1}(\hat{\sigma}^{-2}_1 \sin^2\theta + \hat{\sigma}^2_2 \cos^2\theta) < (\sigma T)^{-2}\} > \beta^2_T . \tag{2.1}$$

Applying (2.1) to (1.2) and ignoring the approximation error,

$$\text{mse } \hat{\theta}_o < (\lambda_o/8\pi\rho^{1/2} \sigma T)^2 \quad \text{rad}^2 \tag{2.2}$$

with probability greater than β_T^2. For example if $\beta_T = 0.99$, $M = 16$, $\sigma = 6\lambda_o$, and $\rho = 1/10$ (-10 db), then $T^2 = 5.2$ and from (2.2),

rmse $\hat{\theta}_o < 0.53°$ with probability greater than 0.98.

It is useful to compare this result with the mean-square error of $\hat{\theta}_o$ for a square lattice array whose $M=N^2$ sensors are at the points $\{md,nd:m,n=1,\ldots,N\}$. Setting $L = (N-1)d$ for the length of the square's sides,

$$\sum_{k=1}^{M} (x_{1k} - \overline{x}_1)^2 = \sum_{k=1}^{M} (x_{2k} - \overline{x}_2)^2 = ML^2/12. \tag{2.3}$$

Thus from (1.2) the mse $\hat{\theta}_o$ for the _random array is less than_ that for the square lattice array (with probability greater than β_T^2) if

$$L < (12/M)^{1/2} T\sigma. \tag{2.4}$$

For example if $M=16$ and $\beta_T = 0.99$, then the bound is $L < 1.98 \sigma$. If $M = 36$ and $\beta_T = 0.99$, $T^2 = 18.5$ and $L < 2.48 \sigma$.

If the square is centered on the origin rather than at $((N+1)/d,(N+1)/d)$, the probability that a sensor is in the square is $[Pr(|Y| < L/2\sigma)]^2$ where $Y \sim N(0,1)$. Thus if $L = 4\sigma$, this probability is 0.91. Since $(0.99)^2 = 0.98$, it follows from the examples above that the small Gaussian random array has approximately the same bearing accuracy as a square lattice array of about one-half the aperture.

This comparison with the lattice array has been made very conservative by the selection of $\beta_T = 0.99$. The rmse $\hat{\theta}_o$ for a small random array will often be as small as a lattice array of equal aperture.

3. Conclusion

It is shown that the approximate mean-square bearing error for a random array favorably compares with that for a square array of equally spaced sensors with about one-half the aperture. The random array is not subject to the aliasing (grating lobes) problem of an equally spaced array, and a great deal of array gain can be achieved from a large array of randomly placed sensors. Serious consideration should be given to tracking the locations of drifting sonobuoys so as to coherently process then as an array.

7

References

1. J. V. Thorn, N. Booth, and J. C. Lockwood, "Random and Partially Random Acoustic Arrays," J. Acoust. Soc. Am. 67, 1277-1285 (1980).

2. B. D. Steinberg, Principles of Aperture and Array System Design (Wiley, New York, 1976), Chap. 8.

3. C. S. Clay and H. Medwin, Acoustical Oceanography (Wiley, New York, 1977), Chap. 9.

4. M. J. Levin, "Least-Squares Array Processing for Signals of Unknown Form," Radio Electron. Eng. 29, 213-222 (1965).

5. M. J. Hinich, "Estimating Signal and Noise using a Random Array, J. Acoust. Soc. Am. 71, 97-99 (1982).

6. M. J. Hinich, "Frequency-Wavenumber Array Processing," J. Acoust. Soc. Am. 69, 732-737 (1981).

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