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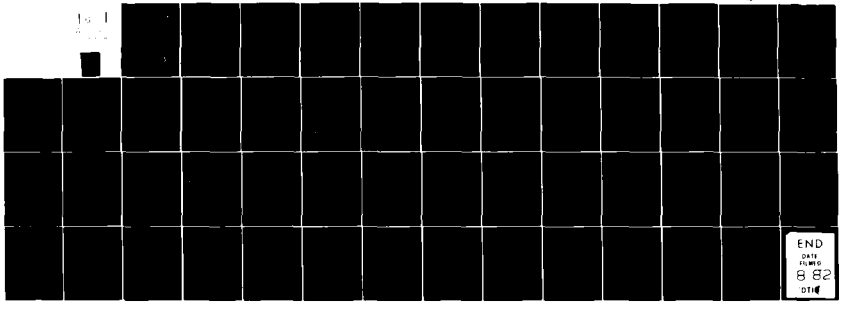

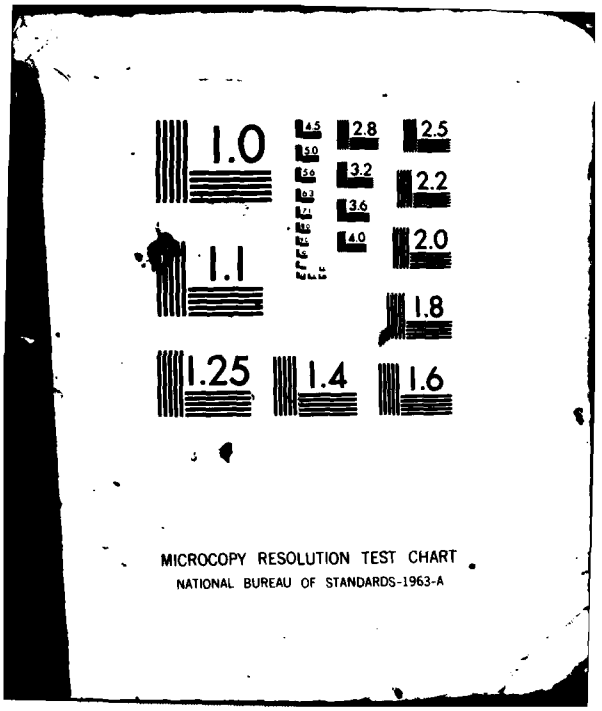
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NASTRAN AND FEARS ANALYSES OF TWO PROBLEMS OF PLANE ELASTICITY

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DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

NASTRAN AND FEARS ANALYSES OF TWO PROBLEMS OF PLANE ELASTICITY

BY

Donald A. Gignac and Ivo Babuska

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COMPUTATION, MATHEMATICS, AND LOGISTICS DEPARTMENT
DEPARTMENTAL REPORT

JULY 1982

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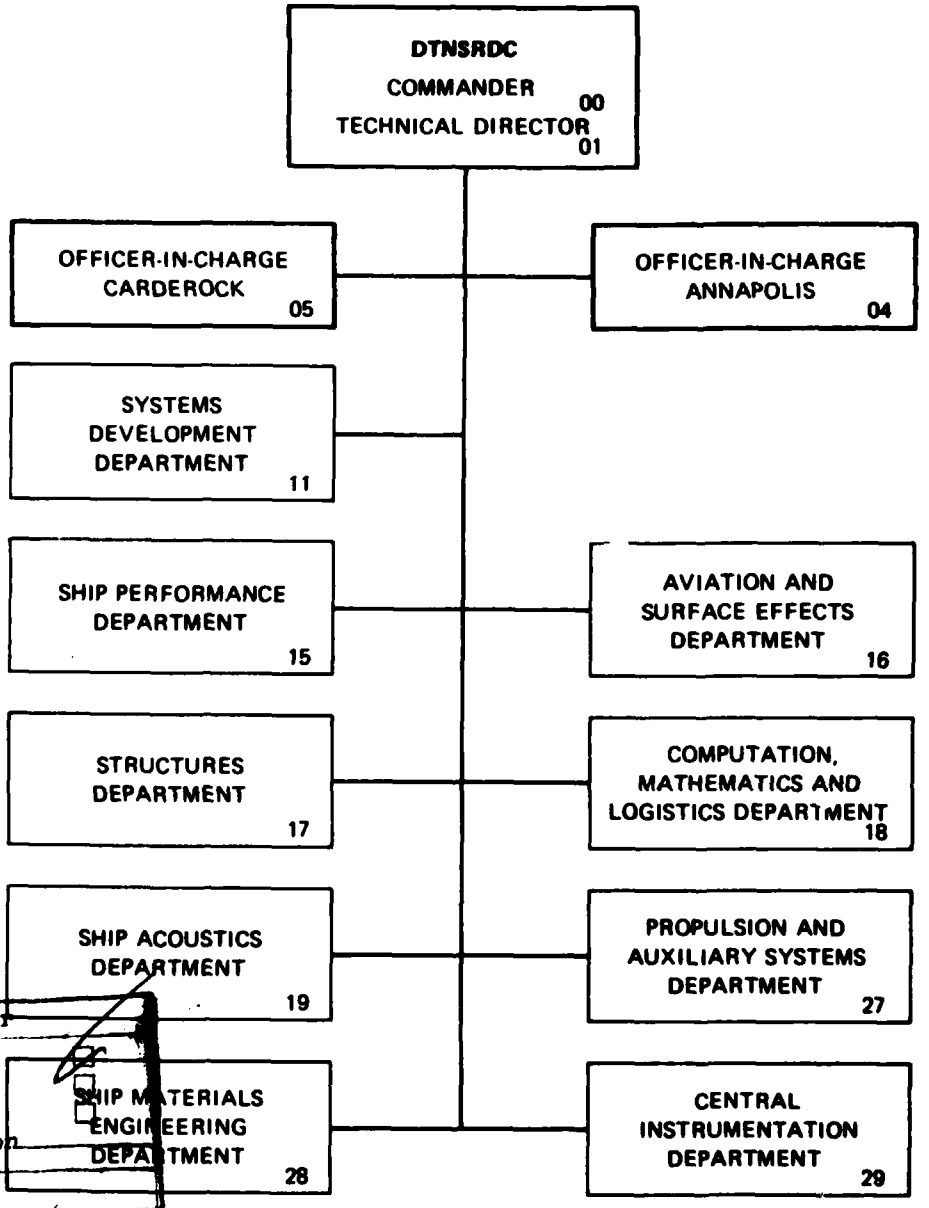

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This report investigates the plane stress capability of the FEARS (Finite Element Adaptive Refinement Solver) program which features both adaptive meshing and a posteriori error bounding. Two benchmark plane stress problems were solved by both the FEARS and NASTRAN (NASA Structural Analysis) programs for comparison. This report presents only the basic computational results. Other reports to follow shortly will motivate the experimental procedures, analyze the results, and provide the conclusion.

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ABSTRACT

This report investigates the plane stress capability of the FEARS (Finite Element Adaptive Refinement Solver) program which features both adaptive meshing and a posteriori error bounding. Two benchmark plane stress problems were solved by both the FEARS and NASTRAN (NASA Structural Analysis) programs for comparison. This report presents only the basic computational results. Other reports to follow shortly will motivate the experimental procedures, analyze the results, and provide the conclusion.

ADMINISTRATIVE INFORMATION

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1. INTRODUCTION

This report presents the basic results of the numerical solutions of two benchmark problems in plane elasticity. The purpose of this computation was to compare the performance of the NASTRAN (NASA Structural Analysis) and FEARS (Finite Element Adaptive Refinement Solver) programs. A subsequent report will analyze the results and present conclusions.

Section 2 of this report describes the two benchmark problems. Section 3 briefly describes the use of the NASTRAN program to solve the two problems, and also describes the data generator program, DGNEW, used to prepare data cards for NASTRAN. Section 4 presents the results obtained by NASTRAN for both benchmark problems. Section 5 briefly describes the computation with the FEARS program and Section 6 presents the basic results of the computation.

2. THE BENCHMARK PROBLEMS

The two plane stress benchmark problems are illustrated in Figures 1 and 2. Figure 1 shows the domain and the load on the sides of Problem 1. Because the domain is symmetric only the upper right hand shaded portion need be considered. We shall give the values of the parameters later.

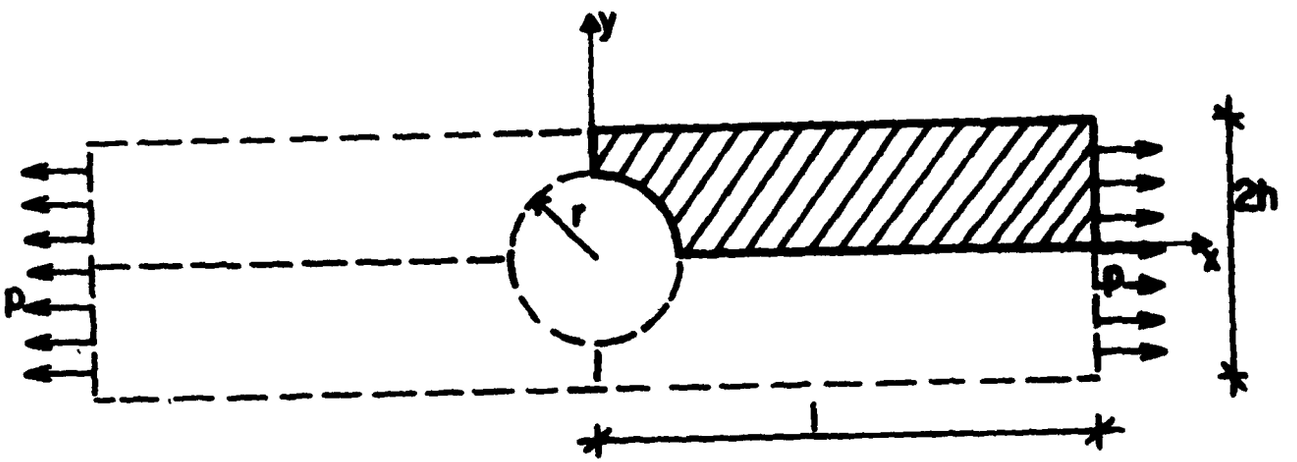

Figure 1 - Problem 1

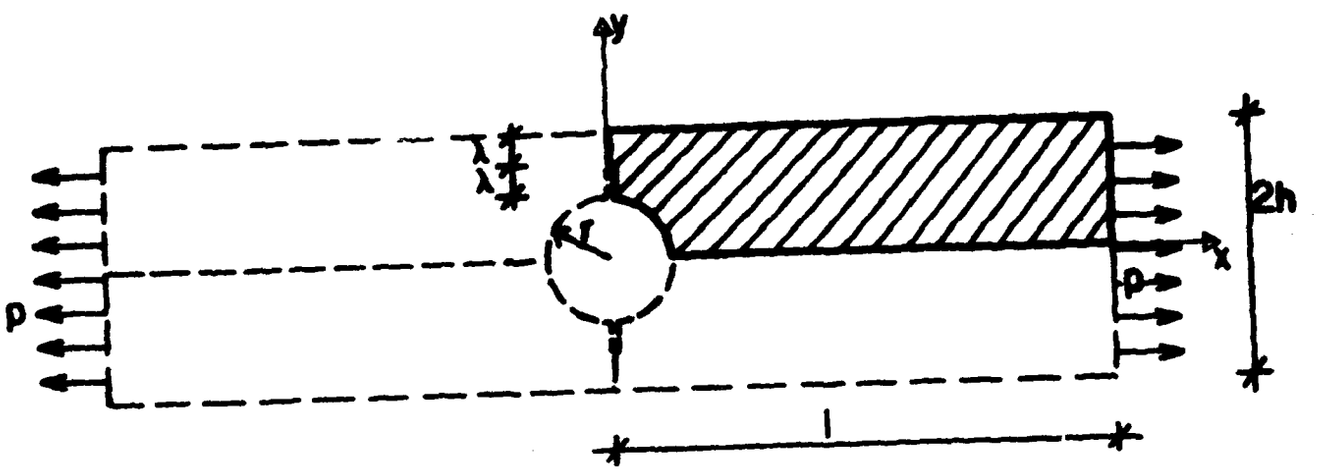

Figure 2 - Problem 2

Problem 2 uses the same configuration as problem 1 but with a crack in the vertical direction. The length of the crack, λ, is $\frac{1}{2}(h-r)$. See Figure 2. A detailed justification for the selection of these two problems will be given in a forthcoming report. The present investigation discusses the FEARS program with respect to the presence of singularities in the geometry and the solution. Problem 1 clearly degenerates as r approaches h, and Problem 2 has a singular solution at the tip of the crack. Both Problems 1 and 2 are plane stress problems where E, Young's modulus of elasticity, is 3.0×10^7 and ν, Poisson's ratio, is 0.3. These two problems are discussed individually in more detail.

Problem 1.

The following parameters were used

$h = 1$

$\ell = 6$

$p = 1$ (normal stress)

The solution to problem 1 consists of the displacement vector

$$(u,v)^T$$

and the tensor of stresses

$$S = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}$$

The problem is to find the solution $(u,v)^T$ with sufficient accuracy with respect to the energy norm and the stress concentration factor Σ of the stress σ_{xx} where

$$\Sigma = \frac{\max \sigma_{xx}}{\frac{ph}{h-r}}$$

Problem 2.

Problem 2 has the same geometry as Problem 1, but with a crack of length

λ. The following parameters were used

$$h = 1$$
$$\ell = 6$$
$$p = 1 \text{ (normal stress)}$$
$$r = .7$$
$$\lambda = .15$$

The problem is to find the solution $(u,v)^T$ and the stress intensity factor, K_1, with sufficient accuracy. K_1 is the coefficient of the principal singular part of the solution.

As before the problem involves the energy ε and a constant, K_1, the stress intensity factor, defined to be the coefficient of the first singular term of the solution at the tip of the crack.

For more information on the following formulas the reader is referred to Pu, Hussain, et al. [2] and chapter 2 of Morosov and Nikischov [3]. The solution is singular at the tip of the crack. If we introduce polar coordinates as shown in Figure 3 we obtain

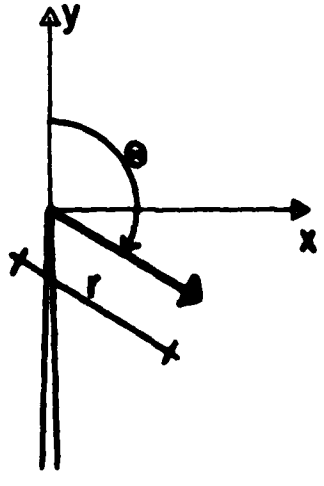

Figure 3. Polar Coordinate Scheme for Problem 2

$$u = \frac{K_1}{2G}\sqrt{\frac{r}{2\pi}}\, F_u(\theta) \tag{1}$$

$$v = \frac{K_1}{2G}\sqrt{\frac{r}{2\pi}}\, F_v(\theta) \tag{2}$$

where

$$F_u(\theta) = \sin\frac{\theta}{2}(\kappa + 1 - 2\cos^2\frac{\theta}{2}) \tag{3}$$

$$F_v(\theta) = \cos\frac{\theta}{2}(\kappa - 1 + 2\sin^2\frac{\theta}{2}) \tag{4}$$

and $G = \frac{E}{2(1+\nu)}$ is the shear modulus.

In the case of plane stress

$$\kappa = \frac{3-\nu}{1+\nu}$$

These formulas allow us to compute K_1 in different ways. If O is (0,0) and A = $(L\sin\theta, L\cos\theta)$ then we can appropriate K_1 by

$$K_1 = \frac{(2G\sqrt{2\pi})[u(A) - u(O)]}{\sqrt{L}\, F_u(\theta)} \tag{5a}$$

or

$$K_1 = \frac{(2G\sqrt{2\pi})[v(A) - v(O)]}{\sqrt{L}\, F_v(\theta)} \tag{5b}$$

Here (u(0),v(0)) is the displacement of the tip of the crack and (u(L),v(L)) is the displacement of the grid point A located a distance L from the tip of the crack and where the displacement vector makes an angle θ with the y-axis, as shown in Figure 3.

Alternatively, another term can be added to the expansion of (u,v), that is

$$u = C_1 + C_2\sqrt{r} + C_3 r \tag{6a}$$

$$v = C_4 + C_5\sqrt{r} + C_6 r \tag{6b}$$

where the C_1 are functions of Θ. If two collinear grid points make an angle Θ with the y-axis, and they are located at distances L and ξL (where $0<\xi<1$)

$$K_1 = \frac{(2G\sqrt{2\pi})[u(\xi L) - \xi u(L) - (1-\xi)u(0)]}{(\sqrt{\xi} - \xi)\sqrt{L}\ F_v(\Theta)} \tag{7a}$$

or

$$K_1 = \frac{(2G\sqrt{2\pi})[v(\xi L) - \xi v(L) - (1-\xi)v(0)]}{(\sqrt{\xi} - \xi)\sqrt{L}\ F_v(\Theta)} \tag{7b}$$

Finally K_1 can also be computed by the energy method. When the energy ε of the solution is a function of the length of the crack λ then

$$2\frac{d\varepsilon}{d\lambda} = \frac{K_1^2}{E} \tag{8}$$

However, this computation requires a second finite element model where the crack length has been changed by an amount $d\lambda$.

3. DESCRIPTION OF NASTRAN COMPUTATION

A plane stress problem is input to NASTRAN via the "bulk data deck" consisting of the grid, connection, force, constraint, and material properties cards. The geometry of the problem is specified on the grid and connection cards which must adhere to a rigid format decreed by NASTRAN. Since preparing these cards by hand is both tedious and laborious a computer program, DGNEW,

(a data generator) was written to generate the mesh from which these grid and connection cards are produced. Often the data generator is a fairly complicated program in its own right as in the present case.

The data generator DGNEW generates the meshes for both problems 1 and 2. The parameters N_1, N_2, r, h, ℓ, α, and KK are read from tape 5 in free format. N_1 is the number of partitions of side AB of subdomain Ω_1 (Figure 4b), and N_2 is the number of partitions of side HG of subdomain Ω_3 (Figure 4d). r is the radius of the arc of the circle in subdomain Ω_1 (Figure 4a). h is the length of side HG of subdomain Ω_3 (Figure 4a). ℓ is the length of side BDH (Figure 4a). α is the desired aspect ratio. If the value of the last parameter KK is zero then it is assumed there is no crack. Otherwise the crack begins at the node specified by the value of KK. DGNEW generates card images of the appropriate grid, connection, force, and constraint cards. A listing of DGNEW which utilizes quarter points for the crack tip of problem 2, is given in the appendix to this report.

The NASTRAN data deck also contains control cards pertaining to the computation and to the printing of output. These cards must also adhere to a specified NASTRAN format.

The NASTRAN IS2D8 element was used for these computations. It is the usual two-dimensional, quadratic, isoparametric, plane stress element with eight nodes, a so-called "serendipity" element. Stresses at the nodes are extrapolated from stress computed on 3 x 3 array of Gauss integration points. Elements of this type are of degree 2, that is, for a smooth solution and uniform meshes whose elements are squares with side h the rate of convergence in the energy norm is $O(h^2) = O\left(\frac{1}{N^2}\right)$ where N is the number of degrees of freedom.

For Problem 1 only elements of this "serendipity" type were used. For Problem 2 two different meshes were used. The first mesh consists entirely of "serendipity" elements as described above. The second mesh consists of the same "serendipity" elements except for those two elemets which have had the "midpoints" of their two sides adjacent to the vertex (which is the tip of the crack) changed into "quarter points". That is, on the two sides of these two elements which intersect at the tip of the crack, the "midpoints" are now located only a quarter of a length of their side away from the tip of the crack, instead of half the length of their side. We shall refer to these elements as "quarter-point 'serendipity'" elements.

The mesh generator constructs a mesh for Ω using the physical domensions (r,ℓ,h) of the domain Ω pictured in Figure 4a, the prescribed number of left and right partitions, N_1 (Figure 4b) and N_2 (Figures 4c and 4d), and the specified aspect ratio α for the elements. In the course of generating this mesh, two other integers (N_3, N_4) and a real number q ($r<q<\ell$) are computed from this input data. N_3 is the number of partitions of the side BD of subdomain Ω_1 (Figure 4b). N_4 is the number of columns of elements in subdomain Ω_3 (Figure 4d). q is the magnitude of side BD of subdomain Ω_1. These nine parameters (r, ℓ, h, N_1, N_2, α, N_3, N_4. and q) completely describe the mesh constructed for Ω. The original domain Ω is then subdivided into three subdomains Ω_1, Ω_2, and Ω_3 as shown in Figure 4a. The subdomains are Ω_1 = ABCD, Ω_2 = CDFE, and Ω_3 = FDHG.

The mesh of Ω_1 is shown in Figure 4b. The side BD is divided into N_3 intervals, thus determining N_3+1 grid points. (The method for determining N_3 and q is described later in this section.) The arc AC is divided into N_3 non-uniform intervals determined by the N_3 angles ψ_1, ψ_2,...ψ_{N_3} and has N_3+1 grid points along it determined by the intervals. The respective grid points on BD and AC are connected by straight lines which in turn, are divided into N_1 equal intervals thus determining N_1+1 grid points along each line. The mesh of Ω_1 is determined by these $(N_1+1)(N_3+1)$ grid points.

The angles $\psi_1,\psi_2,\ldots\psi_N$ and N_3 are chosen such that the aspect ratio of any element is approximately α. The procedure is as follows:

$$r\tilde{\psi}_i = \left(\frac{\overline{AB}}{N_1}\right)\alpha = \frac{\ell_o\alpha}{N_1}$$

$$r\tilde{\psi}_i = \frac{\ell_i\alpha}{N_1}$$

where ℓ_i is the approximate length of the line segment connecting the respective grid points on AC and BD. ℓ_i is given by the formula

$$\ell_i = \ell_o + \frac{\left(\sum_{j=1}^{i}\tilde{\psi}_j\right)(\ell-\ell_o)}{\beta}$$

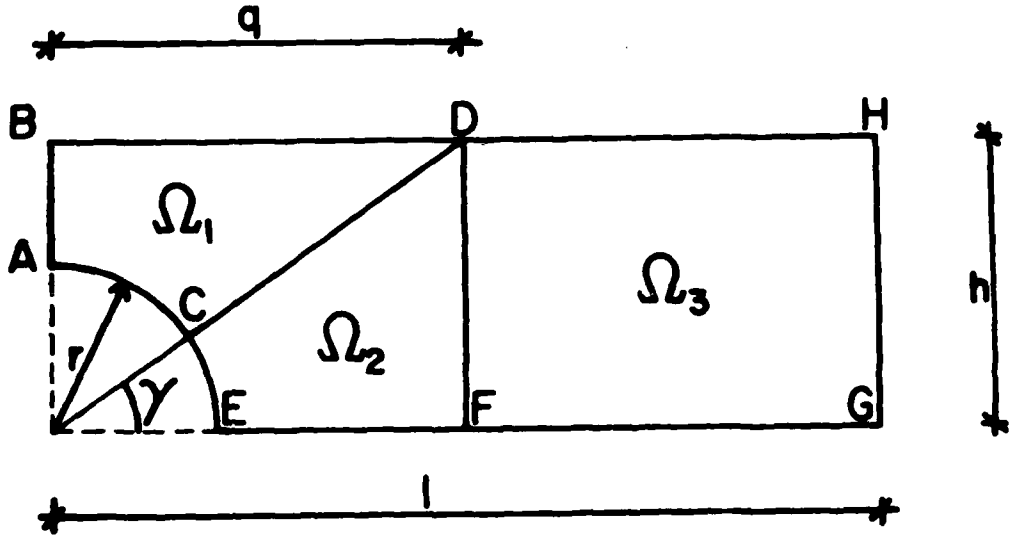

Figure 4a - Partition of Domain Ω

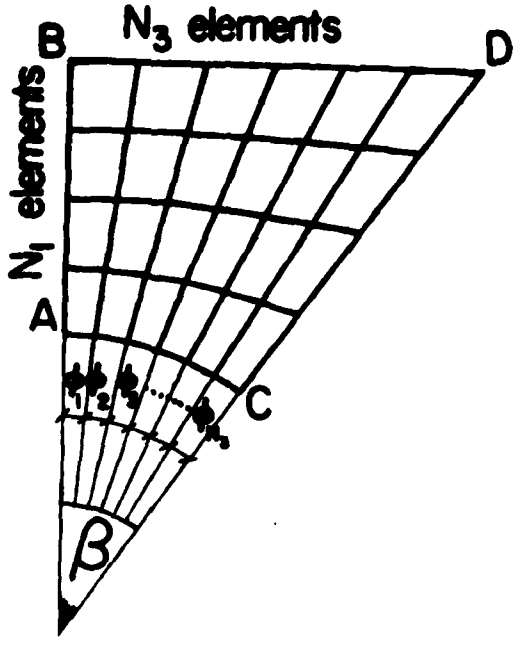

Figure 4b - The Mesh of Subdomain Ω_1

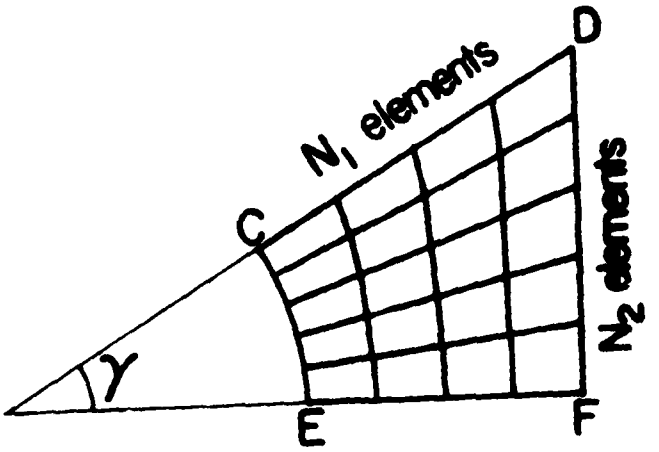

Figure 4c - The Mesh of Subdomain Ω_2

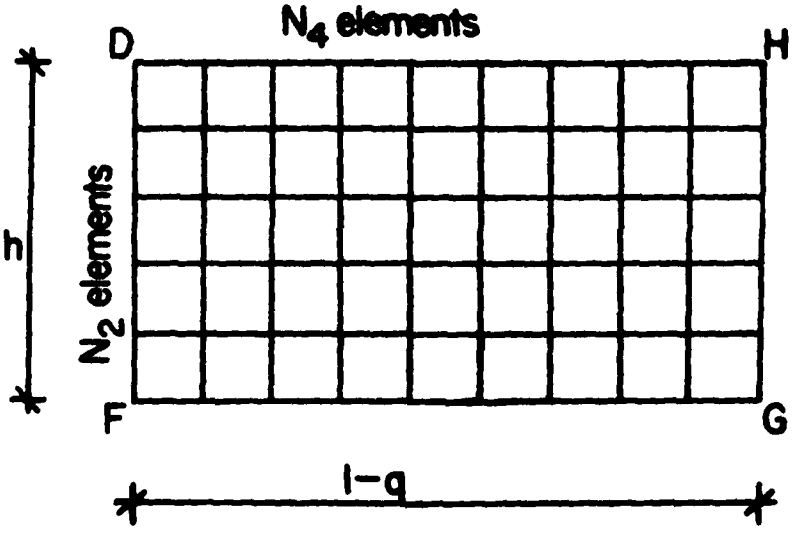

Figure 4d - The Mesh of Subdomain Ω_3

where $\ell = CD$ is defined by

$$(r + \bar{\ell})\cos\beta = h$$

and

$$\ell_o = \overline{AB} = h-r.$$

If $\tilde{\Psi}_i$ is defined as $\tilde{\psi}_1 + \tilde{\psi}_2 + \ldots \tilde{\psi}_i$ then for some integer j_o there will $\tilde{\Psi}_{jo}$ which will be $\geq \beta$ where β is the radial angle of subdomain Ω_1 in Figure 4b. If N_3 is the smallest integer j_o for which this is true, the angles ψ_i are obtained by scaling down the $\tilde{\psi}_i$ thus

$$\psi_i = \psi_i \zeta$$

The mesh in Ω_2 is shown in Figure 4c. The sides EC and FD are divided into N_2 equal segments. The respective grid points on each side are connected by straight lines each of which, in turn, is divided into N_1 equal parts, giving $(N_1+1)(N_2+1)$ grid points for the mesh of Ω_2. The required aspect ratio of the elements in Ω_2 determines q. The angle γ is obtained by solving

$$\tan\gamma = \frac{h}{r + (r\gamma)\left(\frac{N_1}{N_2}\right)\alpha}$$

The mesh of Ω_3 is shown in Figure 4d. N_4, the number of equal partitions of both sides FG and DH, is determined by the aspect ratio α thus

$$\alpha\left(\frac{h}{N_2}\right)\left(\frac{N_4}{N_4+1}\right) < \Delta \leq \alpha\left(\frac{h}{N_2}\right)$$

$$\Delta = \frac{(\ell-q)}{N_4}$$

Since the sides FD and GH are both divided into N_2 equal parts, there are $(N_4+1)(N_2+1)$ grid points for Ω_3 which, in turn, determine a uniform rectangular mesh for Ω_3.

The elements determined by the grid points of the mesh of Ω are isoparametric with eight grid points; the mid-points are located in the middle of the sides. When the side of an element is a circular arc, arc length is used to locate the mid-point.

Table 1 presents the various meshes (number of elements and grid points) generated for one value of r (.999) with different aspect ratios and several choices of N_1 and N_2 to show the dependence of the number of elements and DOF on the parameters. The Table 1 column headings are identified as follows:

I is the line number of the table

K is the microfiche identification number for the particular NASTRAN run

α is the aspect ratio

N_1 is the number of the left hand partitions

N_2 is the number of the right hand partitions

N_{Ω_1} is the number of elements in the first subdomain Ω_1

N_{Ω_2} is the number of elements in the second subdomain Ω_2

N_{Ω_3} is the number of elements in the third subdomain Ω_3

N_Ω is the total number of elements in the domain

GP is the total number of grid points (nodes)

DOF is the number of degrees of freedom

TABLE 1 - MESHES GENERATED FOR R = 0.999 IN PROBLEM 1

I	K	α	N_1	N_2	N_{Ω_1}	N_{Ω_2}	N_{Ω_3}	N_Ω	GP	DOF
1	8	1.5	4	3	96	12	24	132	475	916
2	9	1.5	6	4	198	24	44	266	907	1766
3	10	1.5	8	5	336	40	70	446	1477	2892
4	17	1.0	12	8	1068	96	264	1428	4569	9022
5	18	1.0	8	5	480	40	100	620	2047	4020
6	19	1.0	4	3	132	12	362	186	645	1248
7	20	.75	12	8	1188	96	352	1636	5235	10332
8	21	.5	8	5	792	40	200	1032	3401	6688
9	22	.3	8	5	792	40	330	1162	3843	7520

Figure 5 shows a sample mesh for Problem 1.

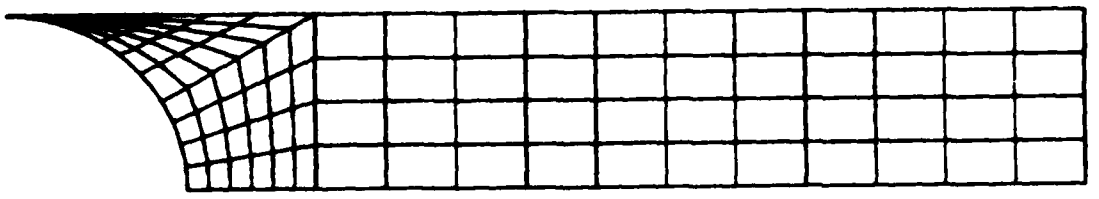

Figure 5 - A Simple Mesh for Problem 1

4. NASTRAN RESULTS

In the solution of the two benchmark problems using NASTRAN, as shown schematically in Figures 1 and 2, h = 1, ℓ = 6, p = 1, and r was varied. The results for Problem 1 are given in Table 2. The meanings of the column headings are as previously defined with the following additions:

R is the radius of the circle whose arc is a side of domain Ω.

SCF is the stress concentration factor Σ previously defined in the text.

CP is the CYBER 7400 central processor time. (The CYBER 7400 is essentially a CDC 6600.)

The asterisks on some of the stress concentration factors in Table 2 mean that the maximum stress did not occur at the expected place (0,r) for that NASTRAN solution.

The scheme of Problem 2 is given in Figure 2, and the parameters used were h = 1, ℓ = 6, p = 1, r = .7 and λ = .15. As noted before, Problem 2 was solved in two different ways, that is, with and without quarter-point elements. These results are presented in Tables 3a (all "serendipity" elements - no quarter-point elements) and 3b (all "serendipity" elements except for the two elements that have the tip of the crack for a vertex - these are quarter-point elements). The column heading KK refers to the grid point (node) at which the tip of the crack is located. σ_{MAX} is the maximum stress which is located at the tip of the crack. The other headings were previously defined. It should be noted that the exact stresses are infinite at the tip of the crack. Table 3c presents the results when the tip of the crack for run 3S in Table 3b is perturbed from (0.,.85) to (0.,.868750).

As stated in Section 2, the stress intensity factor K_1 can also be computed by the energy release method of Equation 8. To make use of this procedure the y-coordinate of the tip of the crack was perturbed from y = .85 (run 3S) to .868750 (run 4S) and Problem 2 was solved with NASTRAN for N_1 = 16, N_2 = 12. This perturbation of the y-coordinate was easily effected by taking the value of the KK parameter of the data generator DGNEW to be 19, thus putting the tip of the crack on the extreme left hand grid-point common

TABLE 2 - PROBLEM 1 RESULTS

I	K	R	N_1	N_2	α	SCF	SE X 10^7	GP	N_Ω	DOF	CP
1	61	0.05	8	6	1.0	2.431	1.0009	899	266	1694	206
2	62	0.05	16	12	1.0	2.743	1.0009	3343	1048	6480	974
3	55	0.1	4	3	1.0	2.312	1.0037	252	67	452	70
4	56	0.1	8	6	1.0	2.624	1.0039	879	260	1656	202
5	57	0.1	16	12	1.0	2.746	1.0040	3267	1024	6332	951
6	52	0.2	4	3	1.0	2.437	1.0162	241	64	6432	68
7	53	0.2	8	6	1.0	2.545	1.0164	859	254	1618	197
8	54	0.2	16	12	1.0	2.544	1.0165	3241	1016	6284	941
9	49	0.3	4	3	1.0	2.384	1.0391	244	65	440	68
10	50	0.3	8	6	1.0	2.391	1.0394	845	250	1594	195
11	51	0.3	16	12	1.0	2.373	1.0394	3227	1012	6262	934
12	46	0.4	4	3	1.0	2.286	1.0761	244	65	440	68
13	47	0.4	8	6	1.0	2.264	1.0764	877	260	1660	201
14	48	0.4	16	12	1.0	2.254	1.0764	3301	1036	6414	959
15	37	0.5	4	3	1.0	2.198	1.1339	258	69	468	70
16	38	0.5	8	6	1.0	2.184	1.1341	903	268	1712	206
17	39	0.5	16	12	1.0	2.177	1.1341	3451	1084	6714	1013
18	40	0.6	4	3	1.0	2.145	1.2247	261	70	476	71
19	41	0.6	8	6	1.0	2.134	1.2249	935	278	1778	214
20	42	0.6	16	12	1.0	2.129	1.2249	3575	1124	6966	1009
21	32	0.7	4	3	1.0	2.107	1.3746	289	78	532	76
22	44	0.7	8	6	1.0	2.100	1.3748	993	296	1896	227
23	45	0.7	16	12	1.0	2.097	1.3748	3749	1180	7318	1124
24	1	0.8	4	3	1.5	2.078	1.6499	234	63	432	72
25	23	0.8	4	3	1.0	2.076	1.6501	306	83	568	88
26	2	0.8	6	4	1.5	2.074	1.6503	421	120	792	126
27	24	0.8	8	6	1.0	2.072	1.6504	1077	320	2066	307
28	25	0.8	16	12	1.0	2.071	1.6505	4073	1284	7970	1796
29	26	0.85	4	3	1.0	2.061	1.8943	320	87	596	91
30	27	0.85	8	6	1.0	2.058	1.8948	1155	346	2222	327
31	28	0.85	16	12	1.0	2.057	1.8948	4335	1368	8496	1919
32	3	0.9	4	3	1.0	2.045	2.3146	348	95	652	101
33	14	0.9	6	4	1.5	2.044	2.3149	467	134	886	139
34	29	0.9	8	6	1.0	2.043	2.3155	1239	372	2392	353
35	30	0.9	16	12	1.0	2.042	2.3156	4647	1468	9122	2105

TABLE 2 (continued)

I	K	R	N_1	N_2	α	SCF	SE x 10^7	GP	N_Ω	DOF	CP
36	4	0.95	4	3	1.5	2.024	3.2750	293	80	552	86
37	31	0.95	4	3	1.0	2.024	3.2794	404	111	764	110
38	13	0.95	6	4	1.5	2.024	3.2802	547	158	1046	162
39	32	0.95	8	6	1.0	2.024	3.2824	1395	420	2704	403
40	33	0.95	16	12	1.0	2.024	3.2827	5209	1648	10248	2464
41	5	0.98	4	3	1.5	1.996	5.1580	335	92	636	97
42	34	0.98	4	3	1.0	2.006	5.1904	449	124	856	119
43	12	0.98	6	4	1.5	2.006	5.1939	627	182	1206	187
44	15	0.98	8	5	1.0	2.010	5.2105	1388	417	2700	459
45	35	0.98	8	6	1.0	2.010	5.2113	1583	478	3082	465
46	36	0.98	16	12	1.0	2.010*	5.2130	6021	1908	11874	2087
47	6	0.99	4	3	1.5	1.950	7.1511	363	100	292	104
48	58	0.99	4	3	1.0	1.983	7.2817	491	136	940	113
49	7	0.99	6	4	1.5	1.984	7.2923	687	200	1326	204
50	11	0.99	8	5	1.5	1.997	7.3459	1113	334	2164	356
51	59	0.99	8	6	1.0	2.004	7.3789	1765	534	3446	417
52	16	0.99	12	8	1.0	2.004*	7.3853	3467	1080	6818	1600
53	8	0.999	4	3	1.5	1.319*	17.2194	475	123	916	135
54	19	0.999	4	3	1.0	1.484	18.5617	645	180	1248	182
55	9	0.999	6	4	1.5	1.365	18.5919	907	266	1766	273
56	10	0.999	8	5	1.5	1.635*	19.9331	1477	446	2892	492
57	18	0.999	8	5	1.0	1.807	21.5183	2047	620	4020	618
58	21	0.999	8	5	0.5	1.915*	22.6040	3401	1032	6688	1200
59	22	0.999	8	5	0.3	1.940*	22.5819	3843	1162	7520	1400
60	17	0.999	12	8	1.0	1.928*	22.8096	4569	1428	9022	1876
61	20	0.999	12	8	0.75	1.950*	23.0445	5235	1636	10332	2295

*For these NASTRAN solutions the maximum stress did not occur at the expected place (o,R).

TABLE 3 - PROBLEM 2 RESULTS

a. "SERENDIPITY" ELEMENTS

K	KK	N_1	N_2	α	σ_{MAX}	SE x 10^7	GP	N_Ω	DOF	CP
1	5	4	3	1	17.345	1.61776	289	78	536	76
2	9	8	6	1	23.853	1.63167	993	296	1904	227
3	17	16	12	1	33.832	1.64043	3749	1180	7334	1123

b. TWO QUARTER-POINT "SERENDIPITY" ELEMENTS*

K	KK	N_1	N_2	α	σ_{MAX}	SE x 10^7	GP	N_Ω	DOF	CP
1S	5	4	3	1	73.936	1.64582	289	78	536	76
2S	9	8	6	1	94.836	1.64769	993	296	190	227
3S	17	16	12	1	131.71	1.64869	3711	1168	7260	1106

c. TWO QUARTER-POINT "SERENDIPITY" ELEMENTS AND PERTURBED TIP OF CRACK

K	KK	N_1	N_2	α	σ_{MAX}	SE x 10^7	GP	N_Ω	DOF	CP
4S	19	16	12	1	46.123	1.71764	3711	1168	7262	1106

* A very minor change in the algorithm for α in the data generator added an extra tier of 12 elements in the second subdomain for the third solution of Table 3b.

to elements 9 and 10. Runs 3S and 4S gave 4.69723 as the value of K_1 from Equation (8).

The stress intensity factor K_1 has also been computed by fitting the singular behavior of the solution at the tip of the crack. Here Equations (1) and (2) were used to obtain Equations (5) and (6) which can be used for various choices of grid points and angles θ. Figure 6 shows the numbering scheme used for these grid points in the computation of K_1.

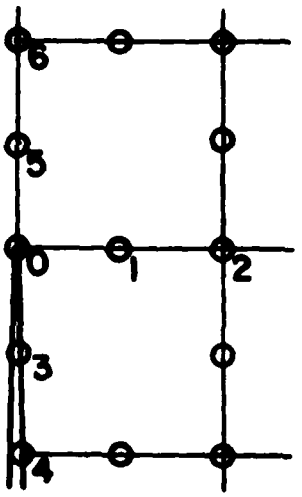

Figure 6 - Numbering Scheme for Computation of K_1

Table 4 gives the coordinates (in columns headed X and Y) and the displacements (in columns headed U_1 and U_2) of the grid points (in column headed PT) required to compute K_1.

The results from Problem 2 are presented in Tables 5 (all "serendipity" elements) and 6 (two quarter-point elements). In these tables the column headings I and θ refer to $F_u(\theta)$(I = 1) and $F_v(\theta)$(I = 2) where θ is the angle of Figure 3. The grid point or points used are listed under the PT or PTS heading, depending on whether the one-point formula, Equation (5), or two-point formula, Equation (7), is used. For the two-point formula the ξ column heading gives the ratio of the distance of the closest point to the furthest point. The values of K_1 are listed under the K_1 heading.

TABLE 4 - COORDINATES AND DISPLACEMENTS REQUIRED TO COMPUTE THE RESULTS OF PROBLEM 2

a. ALL "SERENDIPITY" ELEMENTS, $N_1 = 4$, $N_2 = 3$

PT	X	Y	U x 10^8	V x 10^7
4	0.0000	0.7750	6.5910	-2.2387
3	0.0000	0.8125	4.0986	-2.2318
0	0.0000	0.8500	0.0000	-2.2095
5	0.0000	0.8875	0.0000	-2.1496
6	0.0000	0.9250	0.0000	-2.0995
1	1.0424	0.8494	1.8248	-2.0111
2	2.0847	0.8488	3.2685	-1.8624

b. ALL "SERENDIPITY" ELEMENTS, $N_1 = 8$, $N_2 = 6$

PT	X	Y	U x 10^8	V x 10^7
4	0.0000	0.8125	4.6015	-2.3021
3	0.0000	0.8313	2.8135	-2.2906
0	0.0000	0.8500	0.0000	-2.2710
5	0.0000	0.8688	0.0000	-2.2245
6	0.0000	0.8875	0.0000	-2.1784
1	0.0265	0.8498	1.5205	-2.1175
2	0.0530	0.8495	2.6420	-2.0149

c. ALL "SERENDIPITY" ELEMENTS, $N_1 = 16$, $N_2 = 12$

PT	X	Y	U x 10^8	V x 10^7
4	0.0000	0.8313	3.2419	-2.3407
3	0.0000	0.8406	1.9829	-2.3298
0	0.0000	0.8500	0.0000	-2.3133
5	0.0000	0.8594	0.0000	-2.2770
6	0.0000	0.8688	0.0000	-2.2395
1	0.0138	0.8499	1.1071	-2.2058
2	0.0276	0.8499	1.9128	-2.1407

TABLE 4 (Continued)

d. TWO QUARTER-POINT "SERENDIPITY" ELEMENTS, $N_1 = 4$, $N_2 = 3$

PT	X	Y	U x 10^8	V x 10^7
4	0.0000	0.7750	7.2988	-2.3260
3	0.0000	0.8313	3.4097	-2.3460
0	0.0000	0.8500	0.0000	-2.3551
5	0.0000	0.8688	0.0000	-2.2649
6	0.0000	0.9250	0.0000	-2.1944
1	0.0216	0.8497	1.7230	-2.1774
2	0.0863	0.8488	3.7172	-1.9273

e. TWO QUARTER-POINT "SERENDIPITY" ELEMENTS, $N_1 = 8$, $N_2 = 6$

PT	X	Y	U x 10^8	V x 10^7
4	0.0000	0.8125	5.0238	-2.3529
3	0.0000	0.8406	2.3685	-2.3590
0	0.0000	0.8500	0.0000	-2.3700
5	0.0000	0.8594	0.0000	-2.2995
6	0.0000	0.8875	0.0000	-2.2354
1	0.0135	0.8499	1.3565	-2.2297
2	0.0540	0.8495	2.9315	-2.0542

f. TWO-QUARTER-POINT "SERENDIPITY" ELEMENTS, $N_1 = 16$, $N_2 = 12$

PT	X	Y	U x 10^8	V x 10^7
4	0.0000	0.8313	3.4982	-2.3673
3	0.0000	0.8453	1.6639	-2.3673
0	0.0000	0.8500	0.0000	-2.3755
5	0.0000	0.8547	0.0000	-2.3234
6	0.0000	0.8688	0.0000	-2.2711
1	0.0070	0.8500	0.9697	-2.2740
2	0.0281	0.8499	2.0931	-2.1623

TABLE 5 - PROBLEM 2 RESULTS (ALL "SERENDIPITY" ELEMENTS)

a. ONE POINT FORMULA RESULTS (5)

PT	I	θ	K_1		
			$N_1=4, N_2=3$	$N_1=8, N_2=6$	$N_1=16, N_2=12$
3	1	3.14159	3.86277	3.97900	3.85003
4	1	3.14159	4.46716	4.52453	4.45891
1	1	1.57080	3.67993	3.49180	3.71401
2	1	1.57080	4.52145	4.42238	4.53724
1	2	1.57080	3.71509	3.79737	3.60747
2	2	1.57080	4.38273	4.69706	4.09439
5	2	0.00000	1.82550	1.66087	2.01358
6	2	0.00000	2.56794	2.15704	2.89432

b. TWO POINT FORMULA RESULTS (7)

PTS	I	θ	ξ	K_1		
				$N_1=4, N_2=3$	$N_1=8, N_2=6$	$N_1=16, N_2=12$
3,4	1	3.14159	0.50000	2.40364	2.66199	2.39939
1,2	1	1.57080	0.50000	1.64831	1.24517	1.72656
1,2	2	1.57080	0.50000	2.10324	1.62532	2.43196
5,6	2	0.00000	0.50000	0.03308	0.4630	-.11251

TABLE 6 - PROBLEM 2 RESULTS (TWO QUARTER-POINT "SERENDIPITY" ELEMENTS)

a. ONE POINT FORMULA RESULTS (5)

PT	I	θ	K_1		
			$N_1=4, N_2=3$	$N_1=8, N_2=6$	$N_1=16, N_2=12$
3	1	3.13159	4.68128	4.59875	4.56916
4	1	3.14159	5.81036	4.87721	4.80282
1	1	1.57080	4.62042	4.59608	4.55982
2	1	1.57080	4.98413	4.96620	4.92133
1	2	1.57080	4.76799	4.81492	4.77368
2	2	1.57080	5.73636	5.34991	5.01356
5	2	0.00000	3.53653	3.91476	4.08655
6	2	0.00000	3.15213	3.73316	4.09673

b. TWO POINT FORMULA RESULTS (7)

PTS	I	θ	ξ	K_1		
				$N_1=4, N_2=3$	$N_1=8, N_2=6$	$N_1=16, N_2=12$
3,4	1	3.14159	0.25000	4.35219	4.32029	4.33553
1,2	1	1.57080	0.25000	4.2567L	4.22597	4.19831
1,2	2	1.57080	0.25000	3.79162	4.27994	4.53379
5,6	2	0.00000	0.25000	3.92093	4.09636	4.07637

5. DESCRIPTION OF FEARS COMPUTATION

Because, unlike NASTRAN, the FEARS program has a built-in data generator, the input to FEARS consists merely of the basic data of the particular problem to be solved:

- the description of the domain and the type of boundary conditions
- the coefficients of the pertinent partial differential equation of plane elasticity
- the type of error norm used
- the type and amount of output desired.

The error norm used was the simplest one possible, i.e., the energy error norm L_q for $q = 2$ and $q = \infty$. The input parameter p set to 1.0 specifies the L_2 norm; p set to 0.0 specifies the L_∞ norm. In the present case of plane stress the strain energy density is defined as

$$U = \frac{1}{2E}\left[\sigma_{xx}^2 + \sigma_{yy}^2 - 2\gamma\sigma_{xx}\sigma_{yy} + 2(1+\gamma)\sigma_{xy}^2\right] \tag{9}$$

where

$$\sigma_{xx} = \frac{E}{1-\gamma^2}\left[\frac{\partial u}{\partial x} + \gamma\frac{\partial v}{\partial y}\right] \tag{10a}$$

$$\sigma_{yy} = \frac{E}{1-\gamma^2}\left[\frac{\partial v}{\partial y} + \gamma\frac{\partial u}{\partial x}\right] \tag{10b}$$

$$\sigma_{xy} = \frac{E}{1-\gamma^2}\left[\frac{\partial u}{\partial y} + \gamma\frac{\partial v}{\partial x}\right] \tag{10c}$$

The energy norm of the solution $s = (u,v)^T$ with respect to the L_2 norm is defined by

$$||s||_{E,2} = ||U^{\frac{1}{2}}||_{L_2} \tag{11}$$

and the energy norm of s with respect to the L_∞ norm by

$$||s||_{E,\infty} = ||U^{\frac{1}{2}}||_{L^\infty} \tag{12}$$

If $e = (e_1, e_2)^T$ is the difference between the exact and the finite element solution, the two energy error norms are given by Equations (11) and (12) respectively.

FEARS uses elements of degree 1, that is for uniform meshes of size h and a smooth solution the rate of convergence in the energy norm is $O(h) = O(\frac{1}{N})$ where N is the number of degrees of freedom.

6. FEARS RESULTS

In Problem 1 the user partitions the domain Ω into a set of two-dimensional subdomains which are "curvilinear rectangles". The particular choice of the 2-D subdomains can influence the solution in some way because it can affect the aspect ratios of the elements of the mesh. This partition is characterized by the coordinates of the vertices, the curvature of the lines joining the vertices, and the numbering of these vertices, lines, and subdomains. Figure 7 shows the partition of the domain Ω used for r = .98. The vertices are identified by numbers in circles, the lines connecting the vertices by ordinary numbers, and the subdomains by numbers in squares. The coordinates of the vertices are given in Table 7.

The FEARS program then constructs adaptively a series of meshes with respect to the error norm selected. The FEARS program at present provides the stresses in the middle of the elements, and the stress concentration factor, SCF, is computed according to the choice of the L_2 (p=1.0) and L_∞ (p=0.0) energy error norms, respectively. In Table 8 the data for the next-to-last and the last partitions are listed under the column headings "FIRST MESH" and "SECOND MESH". The partition described under column heading "THIRD MESH" was adaptively constructed but not computed due to program limitations.

Table 9 presents the energy error norm and the error in the stress concentration factor (SCF) for solutions computed in the L_2 adaptive mode, the L_∞ adaptive mode, and the solutions computed using uniform meshes. Again the

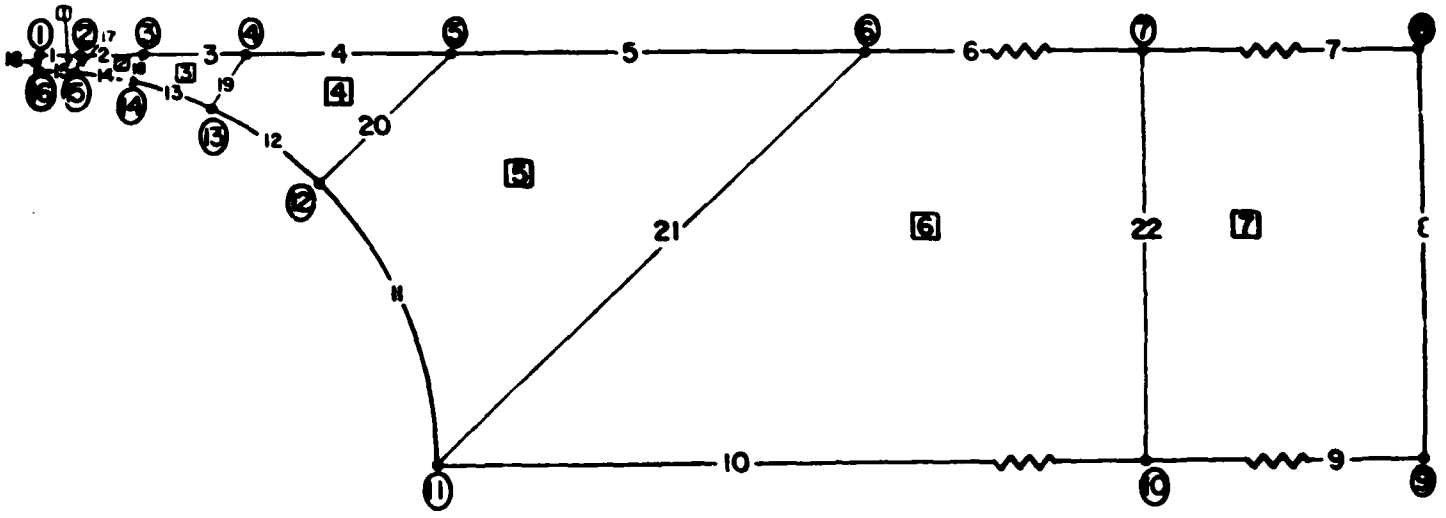

Figure 7 - The FEARS Partition of the Domain Ω for Problem 1

TABLE 7 - COORDINATES OF THE PARTITION OF DOMAIN Ω FOR PROBLEM 1

I	X	Y
1	0.0000	1.0000
2	0.1000	1.0000
3	0.2500	1.0000
4	0.5000	1.0000
5	1.0000	1.0000
6	2.0000	1.0000
7	3.5000	1.0000
8	6.0000	1.0000
9	6.0000	0.0000
10	3.5000	0.0000
11	0.9800	0.0000
12	0.6930	0.6930
13	0.4383	0.8765
14	0.2377	0.9507
15	0.0975	0.9751
16	0.0000	0.9800

TABLE 8 - NUMBER OF ELEMENTS IN THE PARTITIONED DOMAIN WITH RESPECT TO BOTH THE ADAPTIVE L_2 AND L_∞ MODES

Index of 2-D Subdomain	Number of Elements in That 2-D Subdomain					
	FIRST MESH		SECOND MESH		THIRD MESH*	
	L_2	L_∞	L_2	L_∞	L_2	L_∞
1	55	148	202	154	751	559
2	52	31	139	76	511	76
3	64	37	256	73	682	103
4	67	16	238	16	841	16
5	61	16	148	16	349	16
6	31	16	67	16	202	16
7	16	16	16	16	16	16
Total number of Elements	346	280	1066	367	3352	802
DOF	782	636	2242	792		

* Both of the meshes under the third mesh heading were adaptively constructed but solutions were not computed due to program limitations.

TABLE 9 - ERRORS IN THE ENERGY NORM AND THE STRESS CONCENTRATION FACTORS

a. FOR MESHES CONSTRUCTED IN THE ADAPTIVE L_2 MODE

NUMBER OF ELEMENTS	DOF	$\|\|e\|\|_{E,2}$ x 10^4	$\frac{\|\|e\|\|_{E,2}}{\|\|u\|\|_{E,2}}$ x 100	ERROR IN SCF	$\frac{\text{SCF ERROR}}{\text{SCF}}$ x 100
346	782	1.1119	15.39	0.0922	4.58%
1066	2242	0.6476	8.96%	0.0325	1.62%

b. FOR MESHES CONSTRUCTED IN THE ADAPTIVE L_∞ MODE

NUMBER OF ELEMENTS	DOF	$\|\|e\|\|_{E,2}$ x 10^4	$\frac{\|\|e\|\|_{E,2}}{\|\|u\|\|_{E,2}}$ x 100	ERROR IN SCF	$\frac{\text{SCF ERROR}}{\text{SCF}}$ x 100
136	342	1.8876	26.14	0.1625	8.08%
280	636	1.6874	23.36	0.0931	4.63%
367	792	1.5875	21.98	0.0501	2.49%

c. FOR UNIFORM MESHES

NUMBER OF ELEMENTS	DOF	$\|\|e\|\|_{E,2}$ x 10^4	$\frac{\|\|e\|\|_{E,2}}{\|\|u\|\|_{E,2}}$ x 100	ERROR IN SCF	$\frac{\text{SCF ERROR}}{\text{SCF}}$ x 100
112	290	1.9647	27.21%	0.3208	15.96%
448	1026	1.0866	15.05%	0.1077	5.36%

stress concentration factors are computed by extrapolation from stresses in the center of the elements.

The energy error norm $\| e \|_{E,2}$ can be easily computed if the exact energy is known. If ε_{EXACT} is the exact energy and ε is the energy of the Finite Element solution, then

$$||e||^2_{E,2} = \varepsilon_{EXACT} - \varepsilon$$

When the NASTRAN and FEARS results are extrapolated to the limit $\tilde{\varepsilon}_{EXACT} = 5.2130150 \times 10^{-7}$. Similarly by extrapolation to the limit we obtain $\widetilde{SCF}_{EXACT} = 2.0106$. (The tilde notation indicates the extrapolated values of ε_{EXACT} and SCF_{EXACT}.)

Table 10 gives the error estimators for the energy norm $\| e \|_E$ with respect to the adaptive L_2 mode and for uniform meshes.

Table 11 gives the error estimators, scaled in the same way as the stress concentration factors, for the stress concentration factors with respect to the L_∞ norm and for the stress concentration factors for uniform meshes.

For consistent comparison the error extimator for the L_∞ energy norm must be compared with the actual error measured in the norm. Table 12 shows the actual errors in the energy at the point (0,.98) and the error estimators with respect to the adaptive L_∞ mesh and for uniform meshes, although the maximal error has not necessarily occurred at this point.

Figure 8 shows a partition of the domain Ω for Problem 2. As in Problem 1, the vertices are identified by numbers in circles, the lines by ordinary numbers,and the subdomains by numbers in squares. The coordinates of the vertices are given in Table 13. The FEARS program requires that a vertex be placed at the top of the crack.

Because the stresses are infinite at the tip of the crack, the only adaptive mode that could be used is that of the L_2 energy norm. By extrapolation as before the exact energy of the solution was found to be 1.649701×10^{-7}. The energy release procedure for computing the stress intensity factor K_I, using Equation (8) with respect to the L_2 norm, requires adaptivity with respect to that norm.

TABLE 10 - ERROR ESTIMATORS AND THE ACTUAL ERROR IN THE ENERGY NORM

a. FOR MESHES CONSTRUCTED IN THE ADAPTIVE L_2 MODE

Number of Elements	DOF	$\|\|e\|\|_{E,2}$ x 10^4	Estimator x 10^4	$\theta = \frac{\text{Estimator}}{\|\|e\|\|_{E,2}}$
346	782	1.1119	1.0657	0.9584
1066	2242	0.6476	0.6461	0.9977

b. FOR UNIFORM MESHES

Number of Elements	DOF	$\|\|e\|\|_{E,2}$ x 10^4	Estimator x 10^4	$\theta = \frac{\text{Estimator}}{\|\|e\|\|_{E,2}}$
112	290	1.9647	1.6878	0.8591
448	1026	1.0866	1.0293	0.9472

TABLE 11 - ERROR ESTIMATORS FOR THE STRESS CONCENTRATION FACTOR AND THE ACTUAL ERROR

a. FOR MESHES CONSTRUCTED IN THE ADAPTIVE L_∞ MODE

Number of Elements	DOF	Error in SCF	Estimator	$\theta = \frac{\text{Estimator}}{\text{Error in SCF}}$
136	342	0.1625	0.3251	2.0006
280	636	0.0931	0.2090	2.2448
367	792	0.0502	0.1886	2.6617

b. FOR UNIFORM MESHES

Number of Elements	DOF	Error in SCF	Estimator	$\theta = \frac{\text{Estimator}}{\text{Error in SCF}}$
112	290	0.3208	0.5251	1.6369
448	1026	0.1077	0.3462	3.2144

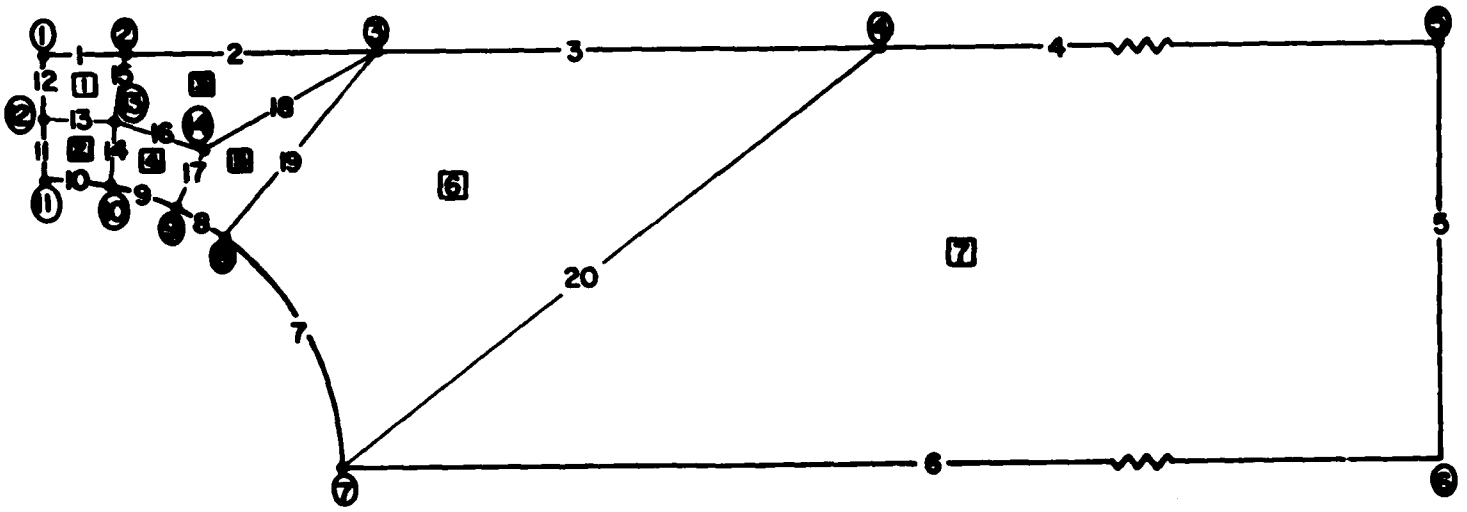

Figure 8 - The FEARS Partition of the Domain Ω for Problem 2

TABLE 12 - MAXIMAL ERRORS IN THE ENERGY AND THE ERROR ESTIMATES

a. FOR MESHES CONSTRUCTED IN THE ADAPTIVE L_∞ MODE

Number of Elements	DOF	Actual Error x 10^3	Estimator x 10^3	$\Theta = \frac{\text{Estimator}}{\|\|e\|\|_{E,\infty}}$
136	342	2.2618	2.0987	0.9279
280	636	1.2365	1.3493	1.0913
367	792	1.1976	1.2172	1.0164

b. FOR UNIFORM MESHES WITH RESPECT TO L_∞

Number of Elements	DOF	Actual Error x 10^3	Estimator x 10^3	$\Theta = \frac{\text{Estimator}}{\|\|e\|\|_{E,\infty}}$
112	290	3.9371	3.3896	0.8609
448	1026	2.4662	2.2349	0.9062

TABLE 13 - COORDINATES OF THE PARTITION OF THE DOMAIN Ω FOR PROBLEM 2

I	X	Y
1	0.0000	1.0000
2	0.2000	1.0000
3	0.8000	1.0000
4	2.0000	1.0000
5	6.0000	1.0000
6	6.0000	0.0000
7	0.7000	0.0000
8	0.4373	0.5466
9	0.3131	0.6261
10	0.1373	0.6864
11	0.0000	0.7000
12	0.0000	0.8500
13	0.1667	0.8335
14	0.3801	0.7603

TABLE 14 - ERROR ESTIMATORS FOR MESHES CONSTRUCTEE IN THE ADAPTIVE L_2 MODE

Step	Number of Elements in 2-D's							Number of Elements	DOF	$\varepsilon \times 10^6$	$\|\|e\|\|_{E,2} \times 10^4$	$\frac{\|\|e\|\|_{E,2}}{\|\|u\|\|_{E,2}} \times 100$	$\theta = \frac{\text{Estimator}}{\|\|e\|\|_{E,2}}$
	1	2	3	4	5	6	7						
1	16	16	16	16	16	16	16	112	274	0.1569	0.8975	22.09%	0.6881
2	16	19	16	16	16	19	19	121	286	0.1579	0.8405	20.69%	0.8019
3	19	19	16	16	16	19	19	124	292	0.1583	0.8152	20.07%	0.7953
4	22	25	16	16	16	43	34	172	376	0.1604	0.6796	16.73%	0.8776
5	25	25	16	16	16	45	34	175	382	0.1606	0.6631	16.33%	0.8756
6	40	37	34	16	31	64	46	268	566	0.1623	0.5177	12.74%	0.9323
7	43	37	34	16	31	64	46	271	572	0.1624	0.5063	12.46%	0.9350
8	55	46	46	16	43	67	58	331	674	0.1628	0.4611	11.35%	0.9473
9	58	46	46	16	43	67	58	334	680	0.1629	0.4547	11.19%	0.9493
10	187	112	160	16	145	238	166	1024	2024	0.1643	0.2647	6.51%	0.9626
11	187	118	160	16	145	238	166	1030	2034	0.1643	0.2599	6.40%	0.9878
12	190	118	160	16	145	238	166	1033	2040	0.1643	0.2568	6.33%	0.9900

Several versions of Problem 2 were solved. In the basic case, the tip of the crack was placed on the nodal point with coordinates (0.,.85). Other cases were obtained by varying the y-coordinate of the tip of the crack from .85. (In the FEARS input this perturbation is effected by changing only one number.)

The presence of the singularity causes a special situation in the following sense. The construction of a completely optimal mesh sometimes involves the refinement of a very small number of elements (possible only one) at one step of the FEARS procedure. This situation could be avoided by using another FEARS command which increases the number of elements to be refined essentially without additional computer cost.

Table 14 shows how the sequence of meshes was created when complete optimality of the meshes was desired. Table 15 gives the basic results for the Finite Element solutions of Table 14.

Tables 15a and 15b show other sequences of meshes adaptively constructed by strategies which avoid a small increase in the number of degrees of freedom during mesh refinement. These strategies consist of inserting two (Table 15a) or three (Table 15b) so-called "short passes" as required into the usual adaptive mesh refinement procedure. Table 16 shows the performance of the uniform meshes.

The stress intensity factor is computed by the energy method which requires the meshes to be the same, topologically speaking. The special features of the FEARS program provide a means for obtaining consistent meshes. The length of the crack can be increased very slightly by altering the y-coordinate of the vertex at the tip of the crack. The meshes can be so constructed in an adaptive manner as to produce the same mesh (topologically speaking) as before. The number of the adaptive steps used to construct this second mesh depends on how much the crack length is increased. A very small increase in crack length extends the number of admissible steps and decreases the error caused by approximating the derivative by a difference, but it also could increase the effect of round-off error.

Table 17 shows the maximal number of elements in the topologically identical meshes for the basic case, the energy of the Finite Element solution, and the growth of the error estimators.

TABLE 15 - MESHES CONSTRUCTED IN THE ADAPTIVE L_2 MODE WITH CURTAILMENT OF SMALL REFINEMENTS

a. BY USE OF TWO SHORT PASSES BETWEEN LONG PASSES

Step	Number of Elements in 2-D's							Number of Elements	DOF	$\varepsilon \times 10^6$	$\|\|e\|\|_{E,2} \times 10^4$	$\frac{\|\|e\|\|_{E,2}}{\|\|u\|\|_{E,2}} \times 100$	$\theta = \frac{\text{Estimator}}{\|\|e\|\|_{E,2}}$
	1	2	3	4	5	6	7						
1	46	46	46	16	46	64	46	310	640	0.1616	0.5841	14.38%	0.7806
2	49	58	46	16	46	64	55	334	688	0.1624	0.5079	12.50%	0.8979
3	70	73	55	16	52	91	79	445	880	0.1632	0.4190	10.31%	0.9357

b. BY USE OF THREE SHORT PASSES BETWEEN LONG PASSES

Step	Number of Elemnts in 2-D's							Number of Elements	DOF	$\varepsilon \times 10^6$	$\|\|e\|\|_{E,2} \times 10^4$	$\frac{\|\|e\|\|_{E,2}}{\|\|u\|\|_{E,2}} \times 100$	$\theta = \frac{\text{Estimator}}{\|\|e\|\|_{E,2}}$
	1	2	3	4	5	6	7						
1	46	34	16	16	25	55	37	229	474	0.1613	0.6094	15.00%	0.9019
2	157	151	97	22	112	223	157	919	1740	0.1640	0.3099	7.63%	0.9713

TABLE 16 - ERRORS AND ERROR ESTIMATORS FOR THE UNIFORM MESHES

Step	Number of Elements in 2-D's							Number of Elements	DOF	$\varepsilon \times 10^6$	$\|\|e\|\|_{E,2} \times 10^4$	$\frac{\|\|e\|\|_{E,2}}{\|\|u\|\|_{E,2}} \times 100$	$\theta = \frac{\text{Estimator}}{\|\|e\|\|_{E,2}}$
	1	2	3	4	5	6	7						
1	16	16	16	16	16	16	16	112	274	0.1569	0.8975	22.09%	0.6881
2	64	64	64	64	64	64	64	448	994	0.1616	0.5809	14.29%	0.7197

TABLE 17 - ENERGY AND ESTIMATED ERROR IN THE ENERGY WITH REFERENCE TO THE LOCATION OF THE TIP OF THE CRACK FOR TOPOLOGICALLY IDENTICAL MESHES.

Y	Number of Elements	DOF	$\varepsilon \times 10^6$	Error Estimator of the Error in the Energy $\times 10^8$
0.8600	112	274	0.1601	0.4131
0.8400	124	292	0.1552	0.3900
0.8450	124	292	0.1567	0.4049
0.4860	124	292	0.1571	0.4079
0.8540	172	376	0.1617	0.3675
0.8492	271	572	0.1621	0.2224
0.8501	1030	2034	0.1643	0.0660

TABLE 18 - ENERGY AND COMPUTED ESTIMATE OF THE ERROR IN THE ENERGY FOR THE CASE OF y = .85

Number of Elements	DOF	$\varepsilon \times 10^6$	Error Estimator of the Error in the Energy $\times 10^8$
112	274	0.1569	0.3814
124	292	0.1583	0.4206
172	376	0.1604	0.3557
271	572	0.1624	0.2240
1030	2034	0.1643	0.0659

TABLE 19 - STRESS CONCENTRATION FACTOR K_1 COMPUTED BY THE ENERGY RELEASE APPROACH

Y	Number of Elements	DOF	K_1	Estimator for the Error in K_1	$\frac{\text{Estimator}}{K_1} \times 100$
0.8600	112	274	4.3928	0.2116	4.81%
0.8400	124	292	4.3036	0.2082	4.83%
0.8450	124	292	4.3479	0.2116	4.86%
0.8460	127	292	4 3569	0.2122	4.87%
0.8540	172	376	4.5175	0.1916	4.24%
0.8492	271	572	4.5961	0.1337	2.90%
0.8501	1030	2034	4.7224	0.0365	.77%

Table 18 gives the energy and the error in the energy for the meshes of Table 17 for the basic case of y = .85.

Equation (8) is now used to compute the stress intensity factor K_1 by replacing the derivative with the one-sided difference. Table 19 gives the K_1 factors and the computed estimators for various values of y.

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APPENDIX - PROGRAM LISTING OF DATA GENERATOR

```
      PROGRAM DATA51(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1,TAPE2,
     1      TAPE3,TAPE4,TAPE5)
      DIMENSION M(8)
C
C           HOLE PROBLEM DATA GENERATOR FOR NASTRAN
C
C          TAPE1=GRID CARDS
C          TAPE2=ELEMENT (CIS2D8) CARDS
C          TAPE3=CONSTRAINT (SPC1) CARDS
C          TAPE4=FORCE CARDS
C
      DIMENSION>N2+1
      COMMON BETA(31),SIDEY(31)
C
C      DIMENSION >N3 (SEE FORMAT NO. 696)
C       N3=NUMBER OF ELEMENTS IN X-DIRECTION IN THIRD SUB-DOMAIN
      COMMON XDOT(2,3,3)
C
C       DIMENSION>2*N2+1 (SEE FORMAT NO. 254)
      COMMON IIA(61),IBB(61),F(61)
C
C      DIMENSION>N (SEE FORMAT NO.257)
C       N=(NUMBER  OF ELEMENTS AROUND ARC IN FIRST SUB-DOMAIN) + 1
      COMMON TOPX(1,3,3),THETA(1,3,3),XLS(1,3,3),S(1,6,3)
C
C      GRID POINT ARRAYS
      COMMON X(10000),Y(10000),NEW(10000)
C
C       ARRAY DIMENSIONS TO BE CHECKED
      DATA  NN2,NN,NIF,NLIMS/30,1000,10000,2000/
C
C       INPUT DATA
C      N1=NUMBER OF ELEMENTS ALONG Y-AXIS OF FIRST SUB-DOMAIN
C      N2=NUMBER OF ELEMENTS IN Y-DIRECTION IN THIRD SUB-DOMAIN
C      R=RADIUS OF CIRCLE
C      H=HEIGHT OF RIGHT SIDE
C      XL=LENGTH OF TOP SIDE
C      AR=ASPECT RATIO
C
      READ(5,*)N1,N2,R,H,XL,AR,KK
      IF(KK.LT.1)KK=1
      WRITE(6,49)N1,N2,R,H,XL,AR,KK
   49 FORMAT(1X,*N1,N2,R,H,XL,AR,KK=*,2I10,4E15.8,I10)
C
C      CHECK N2
      IF (N2.LE.NN2) GO TO 255
      WRITE(6,254)
  254 FORMAT(1X,*N2 TOO BIG*)
      GO TO 999
C
  255 IPID=1
      IEID=0
      ID1=3
      ISID1=11
      COS1=1.0
      ISID2=21
      LIM2=2*N2+1
```

```
      X2N1=2*N1
      XN1=N1
      XN2=N2
      N1P1=N1+1
      N2P1=N2+1
      LIM12=2*N1+1
      X2N2=2*N2
      PI=3.14159265358979 3
C
C     ITERATE FOR ALPHA
C
      ANGIT=(PI/180.0)*27.5
      DO 150 I=1,1000
      STORE=AR*(XN1/XN2)*ANGIT+1.0
      STORE=STORE*(R/H)
      ANGNEW=ATAN2(1.0,STORE)
      J=I
      IF (ABS((ANGIT-ANGNEW)/ANGIT).LE..001) GO TO 152
  150 ANGIT=ANGNEW
      WRITE(6,151)
  151 FORMAT(1H1//40X,*ALPHA DID NOT CONVERGE!!!!!!!*)
      GO TO 999
  152 WRITE(6,153)ANGNEW,J
  153 FORMAT(30X,*ALPHA,NO ITERATIONS =*,E16.8,I10)
C
C     DETERMINE SUBDOMAIN I
C
      GAMMA=ANGNEW
      DIST=(XN1/XN2)*R*GAMMA*AR
      RDIST=R+DIST
      ALPHA=ATAN2(H,R+DIST)
      XLZERO=H-R
      XLN=RDIST/COS(ALPHA)-R
      XL_IFF=XLN-XLZERO
      THETAM=.5*PI-ALPHA
      THETA(1)=0.0
      XLS(1)=XLZERO
      DO 437 I=2,100
      S(I)=(XLS(I-1)/XN1)*AR
      THETA(I)=S(I)/R+THETA(I-1)
      N=I
      IF (THETA(I).LT.THETAM) GO TO 436
      GO TO 438
  436 XX=THETA(I)/(THETAM*TAN(ALPHA))
      XLS(I)=SQRT((XX-R*SIN(THETA(I)))**2+(H-R*COS(THETA(I)))**2)
  437 CONTINUE
C
  438 IF (N.LE.NN) GO TO 256
      WRITE(6,257)
  257 FORMAT(1X,*TOPX,THETA,XLS, AND S DIMENSION TOO SMALL*)
      GO TO 999
C
  256 SCALEF=THETAM/THETA(N)
      DO 439 I=1,N
  439 THETA(I)=SCALEF*THETA(I)
      TOPXN=RDIST
      DO 440 I=1,N
```

```
  440 TOPX(I)=(THETA(I)/THETAM)*TOPXN
C
C           SUBDOMAIN I
C
      IP=0
      NEWIP=0
      DO 500 L=1,N
      XIP=SIN(THETA(L))*R
      YIP=COS(THETA(L))*R
      XINC=(TOPX(L)-XIP)/X2N1
      YINC=(H-YIP)/X2N1
      DO 499 I=1,LIM12
      IP=IP+1
      NEWIP=NEWIP+1
      NEW(IP)=NEWIP
      X(IP)=XIP+FLOAT(I-1)*XINC
      Y(IP)=YIP+FLOAT(I-1)*YINC
      IF (KK.GT.1 .AND. IP.EQ.KK-1) GO TO 323
      GO TO 324
  323 Y(IP)=Y(IP)+YINC/2.
      GO TO 499
  324 IF (KK.GT.1 .AND. IP.EQ.KK+1) GO TO 325
      GO TO 499
  325 IF (IP.NE.KK+1) GO TO 499
      Y(IP)=Y(IP)-YINC/2.
  499 CALL WRTPT1(I,IP,X(IP),Y(IP))
      NEWIP=NEWIP+N1PO
  500 CONTINUE
      NEWIP=LIM12
      DO 550 L=2,N
      ANGLE=0.5*(THETA(L)+THETA(L-1))
      IP=IP+1
      NEWIP=NEWIP+1
      NEW(IP)=NEWIP
      X(IP)=SIN(ANGLE)*R
      Y(IP)=COS(ANGLE)*R
      ISTORE=IP
      CALL WRTPT1(1,IP,X(IP),Y(IP))
      DO 525 I=3,LIM12,2
      IP=IP+1
      NEWIP=NEWIP+1
      NEW(IP)=NEWIP
      IA=(L-2)*LIM12+I
      IB=(L-1)*LIM12+I
      X(IP)=0.5*(X(IA)+X(IB))
      Y(IP)=0.5*(Y(IA)+Y(IB))
      IF (KK.GT.1 .AND. L.EQ.2 .AND. I.EQ.N1PO) GO TO 350
      GO TO 351
  350 X(IP)=(3.0*X(IA)+X(IB))/4.
      Y(IP)=(3.0*Y(IA)+Y(IB))/4.
  351 CALL WRTPT1(I,IP,X(IP),Y(IP))
      IG1=IA
      IEND=IEND+1
      IG2=IA-2
      IG3=IB-2
      IG4=IB
      IG5=IA-1
```

```
      IG6=ISTORE
      IG7=IB-1
      IG8=IP
      WRITE(6,202)IB,IP,IP1C,IG1,IG2,IG3,IG4,IG5,IG6,IEIC,IEID,IG7,IG8,
     1   IJ1
  202 FORMAT(8HCIS2D8  ,8I8,1H+,I7/1H+,I7,3I8)
  525 ISTORE=IP
      NEWIP=NEWIP+LIM12
  550 CONTINUE
      NPTSI=IP
C
C      PREPARE PART OF SPC1 CARDS
      IC=1
      WRITE(3,303)((ISID1,IC,IPP),IPP=KK,LIM12)
  303 FORMAT(6HSPC1,,4X,3I8)
C
C     SUBDOMAIN II
C
      ANGLE=ALPHA/XN2
      YINC=H/XN2
      DO 596 I=1,N2PC
      BETA(I)=FLOAT(I-1)*ANGLE+THETAM
  596 SIDEY(I)=H-FLOAT(I-1)*YINC
      NEWIP=NPTSI+N1PC
      DO 600 L=2,N2PC
      XIP=SIN(BETA(L))*R
      YIP=COS(BETA(L))*R
      XINC=(RDIST-XIP)/X2N1
      YINC=(SIDEY(L)-YIP)/X2N1
      IF (L.NE.N2PC) GO TO 597
      YIP=0.0
      YINC=0.0
  597 DO 599 I=1,LIM12
      IP=IP+1
      NEWIP=NEWIP+1
      NEW(IP)=NEWIP
      X(IP)=XIP+FLOAT(I-1)*XINC
      Y(IP)=YIP+FLOAT(I-1)*YINC
  599 CALL WRTPT1(I,IP,X(IP),Y(IP))
      NEWIP=NEWIP+N1PC
  600 CONTINUE
      IASTRT=(N-1)*LIM12
      NEWIP=NPTSI
      DO 625 L=2,N2PC
      ANGLE=0.5*(BETA(L)+BETA(L-1))
      IP=IP+1
      X(IP)=SIN(ANGLE)*R
      NEWIP=NEWIP+1
      NEW(IP)=NEWIP
      Y(IP)=COS(ANGLE)*R
      CALL WRTPT1(1,IP,X(IP),Y(IP))
      ISTORE=IP
      IBSTRT=NPTSI+(L-2)*LIM12
      DO 624 I=3,LIM12,2
      IP=IP+1
      NEWIP=NEWIP+1
      NEW(IP)=NEWIP
```

```
      IA=IASTRT+I
      IB=IBSTRT+I
      X(IP)=0.5*(X(IA)+X(IB))
      Y(IP)=0.5*(Y(IA)+Y(IB))
      CALL WRTPT1(I,IP,X(IP),Y(IP))
      IG1=IA
      IEID=IEID+1
      IG2=IA-2
      IG3=IB-2
      IG4=IB
      IG5=IA-1
      IG6=ISTORE
      IG7=IB-1
      IG8=IP
      WRITE(2,202)IEID,IPID,IG1,IG2,IG3,IG4,IG5,IG6,IEID,IEID,IG7,IG8,
     1     ID1
  624 ISTORE=IP
      NEWIP=NEWIP+LIM12
  625 IASTPT=IBSTRT
C
C           SUBDOMAIN III
C
      NPTSII=IP
      D=XL-RDIST
      N3=D/((H/XN2)*AR)
      XN3=N3
      LIM3=N3+1
C
      IF (LIM3.LE.NLIM3) GO TO 695
      WRITE(6,696)
  696 FORMAT(1X,*X3DT DIMENSION TOO SMALL*)
      GO TO 999
C
  695 X3DT(1)=RDIST
      XINC=D/XN3
      NEWIP=NPTSII+N2PO
      DO 700 L=2,LIM3
      XIP=RDIST+FLOAT(L-1)*XINC
      X3DT(L)=XIP
      YIP=0.0
      YINC=H/X2N2
      DO 699 I=1,LIM2
      IP=IP+1
      NEWIP=NEWIP+1
      NEW(IP)=NEWIP
      X(IP)=XIP
      Y(IP)=YIP+FLOAT(I-1)*YINC
  699 CALL WRTPT1(699,IP,X(IP),Y(IP))
      NEWIP=NEWIP+N2PO
  700 CONTINUE
      ISTORE=NPTSI+N2*LIM12
      DO 710 I=1,LIM2,2
      IAA(I)=ISTORE
  710 ISTORE=ISTORE-LIM12
      IAA(LIM2)=N*LIM12
      ISTORE=NPTSII
      DO 711 I=2,LIM2,2
```

```
      IAA(I)=ISTORE
  711 ISTORE=ISTORE-N1P0
      NEWIP=NPTSII
      DO 750 L=2,LIM3
      IP=IP+1
      NEWIP=NEWIP+1
      NEW(IP)=NEWIP
      STORE=0.5*(XBOT(L)+XBOT(L-1))
      X(IP)=STORE
      Y(IP)=0.0
      CALL WRTPT1(999,IP,X(IP),Y(IP))
      ISTORE=IP
      DO 770 I=1,LIM2
  770 IBB(I)=NPTSII+(L-2)*LIM2+I
      DO 775 I=3,LIM2,2
      IP=IP+1
      NEWIP=NEWIP+1
      NEW(IP)=NEWIP
      IA=IAA(I)
      IB=IBB(I)
      X(IP)=STORE
      Y(IP)=Y(IA)
      CALL WRTPT1(999,IP,X(IP),Y(IP))
      IG1=IA
      IEID=IEID+1
      IG2=IAA(I-2)
      IG3=IBB(I-2)
      IG4=IB
      IG5=IAA(I-1)
      IG6=ISTORE
      IG7=IBB(I-1)
      IG8=IP
      WRITE(2,202)IEID,IPID,IG1,IG2,IG3,IG4,IG5,IG6,IEID,IEID,IG7,IG8,
     1          ID1
  775 ISTORE=IP
      DO 749 J=1,LIM2
  749 IAA(J)=IBB(J)
      NEWIP=NEWIP+LIM2
  750 CONTINUE
C
C       CHECK GRID POINT ARRAYS
      IF (IP.LE.NIP) GO TO 748
      WRITE(6,747)
  747 FORMAT(1X,*X,Y DIMENSIONS TOO SMALL*)
      GO TO 999
C
C       PREPARE REST OF SPC1 CARDS
C
  748 IC=2
      DO 785 IQ=LIM12,ISTORE
      IF (Y(IQ).NE.0.0) GO TO 785
      WRITE(3,303)ISID1,IC,IQ
  785 CONTINUE
C
C       PREPARE FORCE CARDS
      DO 786 I=1,LIM2,2
  786 F(I)=2.0
```

```
      F(1)=1.)
      F(LIM2)=1.0
      DO 707 I=2,LIM2,2
  707 F(I)=4.0
      FNUM=6.0*KN2/M
      DO 709 I=1,LIM2
  709 F(I)=F(I)/FNUM
      ISTORE=NPTSII+N3*LIM2
      LIMF=ISTORE-LIM2+1
      I=0
      DO 708 IQ=LIMF,ISTORE
      I=I+1
  708 WRITE(4,404)ISIG2,IQ,F(I),COS1
  404 FORMAT(3HFORCE    ,2I4,3X,F8.7,F8.3)
C
C       NORMAL TERMINATION OF DATA GENERATOR
      REWIND 2
      IF (IP.LE.9000) GO TO 850
      WRITE(6,349)
  349 FORMAT(1H1/40X,*TOO MANY GRID POINTS (>9000)*)
      GO TO 999
  850 CONTINUE
      WRITE(6,613)IP,IEID
      WRITE(6,175)(I,NEW(I),I=1,IP)
  175 FORMAT(9HSEQGP    ,8I8)
C       TEMPORARY REGENERATION OF CIS2D8 CARDS
      DO 105 I=1,IEID
      READ(2,100)M
  100 FORMAT(24X,6I8,8X/8X,2I8)
      DO 102 K=1,8
      J=M(K)
  102 M(K)=NEW(J)
  105 WRITE(3,110)M
  110 FORMAT(8HCIS2D8  ,16X,6I8/1H+,7X,2I8)
      REWIND 1
      REWIND 2
      REWIND 3
      REWIND 4
      REWIND 8
      STOP
C
C       ENSURE RUN WILL ABORT BECAUSE DATA IS BAD
C
  999 REWIND 1
      REWIND 2
      REWIND 3
      REWIND 4
      ENDFILE 1
      ENDFILE 2
      WRITE(6,613)IP,IEID
  613 FORMAT(1H1/40X,*NO GRID,NO ELMT=*,2I10)
      STOP
C
      END
```

```
      SUBROUTINE WRTPT1(I,IP,X,Y)
C
C       WRITES A GRID CARD ON TAPE1 USING ONE OF THREE FORMATS
C
      ICOR=0
      IF (I.GT.1) GO TO 441
      ICOR=1
      R=SQRT(X*X+Y*Y)
      PI=3.1+159265358979
      T=(180./PI)*ATAN2(Y,X)
      WRITE(1,104)IP,ICOR,R,T,IP,IP
C 101 FORMAT(4HGRID,4X,I4,3X,2H1,4X,F8.7,F8.4)
      RETURN
  441 IF (X.GE.1.) GO TO 442
      WRITE(1,104)IP,ICOR,X,Y,IP,IP
C 102 FORMAT(4HGRID,4X,I8,3X,F8.7,F8.6)
      RETURN
  442 WRITE(1,104)IP,ICOR,X,Y,IP,IP
C 103 FORMAT(4HGRID,4X,I8,3X,F8.6,F8.6)
      RETURN
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