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THE HORIZONTAL WIRE OVER EARTH AS A TRANSMISSION LINE: A MODEL --ETC(U)
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The Horizontal Wire Over Earth as a Transmission Line: A Simple
Surface-Impedance Approach

by Momi R. Wilson



U.S. Army Electronics Research
and Development Command
Harry Diamond Laboratories

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER HDL-TM-81-22	2. GOVT ACCESSION NO. AD-A117 892	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) The Horizontal Wire Over Earth as a Transmission Line: A Model Surface- Impedance Approach		5. TYPE OF REPORT & PERIOD COVERED Technical Memorandum
7. AUTHOR(s) Monti R. Wilson		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Harry Diamond Laboratories 2800 Powder Mill Road Adelphi, MD 20783		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Materiel Development and Readiness Command Alexandria, VA 22333		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Ele: 62120A
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE July 1982
		13. NUMBER OF PAGES 80
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES HDL Project: X751E7 DRCMS Code: 612120H25001 DA Project: 1L162120AH25		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Electromagnetic coupling Transmission line Surface impedance		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A simple model formulation and exact solution is given for the per-unit-length surface impedance that characterizes the finite conducting earth in a transmission-line approximation of the response of a horizontal wire over earth to incident electromagnetic fields. The two-dimensional inhomogeneous plane waves in the model are discussed, and a transparent formulation and solution of the wave-vector dispersion relation is presented in terms of a		

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20. Abstract (Cont'd)

complex refraction-angle formalism. Finally, we consider a TM (transverse magnetic) purely vector-potential model of the wire over ground in order to show that the TM surface-impedance model explicitly excludes TE (transverse electric) waves and is, in fact, identical to the familiar transmission line.

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1. INTRODUCTION AND SUMMARY

An enduring problem in electromagnetic theory is that of calculating the response of a horizontal wire over earth to incident electromagnetic fields. An approximate solution of the problem that is widely used is obtained by regarding the wire over earth as a transmission line.

The basic unknown of the transmission-line approach is the approximate relationship (from field theory) between the line waves and the electromagnetic fields in the earth; this relationship enters the transmission-line formulation as a surface impedance for the ground plane.

The purpose of this paper is to present a heretofore overlooked model formulation and exact solution to the wire-over-ground surface-impedance problem. The surface-impedance model of section 2 results from the joining of transmission-line waves to purely TM (transverse magnetic) electromagnetic soil fields. More precisely, we also show (sect. 4) that the TM surface impedance arises from a purely vector-potential formulation of the wire-over-ground problem, if the model assumed in section 2 is granted. It is then possible to derive (1) the line-inductance parameter, (2) the "charge" distribution, (3) the finite width of dominant wave localization over the ground plane (derived

consistently with the line-capacitance parameter), and (4) the surface-impedance boundary relations, all from the vector-potential solution. The zero possibility of certain TE (transverse electric) field components (i.e., all omitted field components) follows automatically from the vector-potential solution. The vector-potential model is indistinguishable from the commonly accepted definition of a surface-impedance loaded transmission line. This paper does not consider the extent to which higher order waves, excluded from the TM model, compromise the TM line response (predominantly at high frequencies, where the transmission-line "radiation resistance" is large).

Our surface impedance does not agree with either the low-frequency surface impedance in the work of Carson¹ or the essentially identical low-frequency surface impedance that is a limiting case of the Wait² formulation of the wire-over-ground problem. At higher frequencies we find a guided surface wave and surface impedance, unlike the results in the paper of Kikuchi.³

Section 3 is devoted to the two-dimensional inhomogeneous plane waves resulting in the surface-impedance model. The connection between the waves and the complex refraction angle formalism is discussed.

¹J. R. Carson, *Bell Sys. Tech. J.*, 5 (October 1926), 539-554.

²James R. Wait, *Radio Science*, 7, No. 6 (June 1972), 675-679.

³H. Kikuchi, *Electrotech. J. Japan*, 2, 3/4 (1956), 73-78.

2. CALCULATION OF Z_g , PER-UNIT-LENGTH SURFACE IMPEDANCE

I first describe the situation at hand and give the central results. The derivation and discussion of the surface-impedance model is then taken up.

A horizontal wire of radius a , lying at height h over planar earth, is regarded as a transmission line in essentially the same sense as the Beverage or wave antenna.⁴ The wire is highly conducting and characterized by a prescribed (per-unit-length) internal impedance, Z_i , where $Z_i = R_i + i\omega\ell_i$; the resistance, R_i , and inductance, ℓ_i , are functions of radian frequency ($\omega = 2\pi f$), and f is the usual cw frequency parameter in hertz.

The earth is characterized by a scalar, frequency-dependent complex dielectric-response function,⁵ $\epsilon(\omega)$, where $\epsilon(\omega) = \epsilon_1(\omega) - i\epsilon_2(\omega)$. The earth conductivity is $\sigma(\omega) = \omega\epsilon_2(\omega) = \sigma_0 + \omega\epsilon_{2r}(\omega)$, where σ_0 is a constant dc conductivity and ϵ_{2r} is the remaining loss part of ϵ_2 .

I take here an $e^{i\omega t}$ Fourier convention; therefore, the line propagation constant⁶ is $\gamma = \alpha + i\beta$, $\alpha(\omega)$ is the attenuation part, and $v(\omega) = \omega/\beta(\omega)$ relates β to the phase velocity, v . The square of γ is

⁴See, for example, J. A. Schelkunoff and H. T. Friis, *Antennas (Theory and Practice)*, John Wiley and Sons, Inc. (1952).

⁵J. D. Jackson, *Classical Electrodynamics*, second edition, John Wiley and Sons, Inc. (1975), p 309.

⁶R. W. P. King, *Transmission Line Theory*, Dover Publications, Inc. (1965), p 91.

$$\gamma^2 = i\omega C(Z_g + Z_i + i\omega l_e) , \quad (1)$$

where l_e is per-unit-length external inductance and C is per-unit-length capacitance. The values of C and l_e are exactly the values calculated for a wire over a perfectly conducting ground plane (sect. 4). I assume no shunt conductance ($G = 0$) for the wire.

The wire as a transmission line is complete once we know the per-unit-length surface impedance for the earth presence, namely

$$Z_g = R_g + iX_g , \quad (2)$$

where R_g is resistance and X_g is reactance. I find for Z_g the result

$$Z_g = 0.5c_1 \left[-1 + \left(1 + \frac{4c_2}{c_1^2} \right)^{1/2} \right] \exp[i(\phi_1 - 0.5 \phi_2)] . \quad (3)$$

The parameters are given by the following seven expressions ($\mu_0 = 4\pi 10^{-7}$ H/m):

$$c_1 = C/\omega w^2(\epsilon_1^2 + \epsilon_2^2) , \quad (4)$$

$$\phi_1 = \tan^{-1} \left(0.5(\epsilon_2/\epsilon_1)[1 - (\epsilon_1/\epsilon_2)^2] \right) , \quad (5)$$

$$c_2 = (F_r^2 + F_i^2)^{1/2}/w^2(\epsilon_1^2 + \epsilon_2^2)^2 , \quad (6)$$

$$\phi_2 = \tan^{-1} (F_i/F_r) , \quad (7)$$

$$F_i = (R_i C \omega^{-1} - \mu_0 \epsilon_2)(\epsilon_1^2 - \epsilon_2^2) + 2\epsilon_1 \epsilon_2 [\mu_0 \epsilon_1 - (\lambda_e + \lambda_i)C] , \quad (8)$$

$$F_r = [\mu_0 \epsilon_1 - (\lambda_e + \lambda_i)C](\epsilon_1^2 - \epsilon_2^2) - 2\epsilon_1 \epsilon_2 (R_i C \omega^{-1} - \mu_0 \epsilon_2) , \quad (9)$$

$$w = 2\pi h . \quad (10)$$

Thus, if we substitute Z_g of equation (3) into (1) and extract the positive root⁶ $\gamma \approx \alpha + i\beta$, we completely define the uniform transmission-line parameters with respect to the basis waves from which the well-known response solutions^{7,8} for the terminated, finite-length line with arbitrary excitation are constructed.

The transmission-line voltage is V with a current I on the upper wire. It is helpful to regard the line response as a guided surface

⁶R. W. P. King, *Transmission Line Theory*, Dover Publications, Inc. (1965), p 91.

⁷R. F. Gray, *Nuclear Electromagnetic Pulse Simulation by Point Source Injection Techniques for Shielded and Unshielded Penetrations*, Harry Diamond Laboratories, HDL-TR-1737 (December 1975).

⁸See, for example, A. A. Smith, Jr., *Coupling of External Electromagnetic Fields to Transmission Lines*, Wiley Interscience (1977).

wave and to first focus attention on the earth-surface magnetic field. For brevity and compactness of expression I have decided, in most of what follows, to write down only the frequency-domain representatives of responses (denoted by underlined symbols; all responses not underlined denote real responses) and to suppress the explicit conversion of the underlined quantities to real responses (unless confusion would arise). In our right-hand coordinate system, x is upward from the ground and z is along the transmission line. For calculating per-unit-length Z_g in the frequency domain, it is convenient to suppose an unbounded line with a Dirac impulse point-source line-voltage generator at $z = 0$ and to single out the $+z$ -directed Fourier transmission-line traveling waves.

The TM Z_g follows from a single model assumption of simultaneous contributions from all y locations of surface magnetic field to the transmission-line return current $-I$, namely at $x = 0$, and some fixed z ,

$$-I(t) = \int_{-\infty}^{\infty} dy H_y(0, y, t) \quad .$$

Hence, in general, the y integral of H_y must exhibit common time dependence or

$$H_y(0, y, t) = h_1(t)h_2(y) \quad ,$$

where h_1 and h_2 are two real functions. It follows from $\int_{-\infty}^{\infty} dy h_2$ immediately that space frequency k_y is zero in the line waves. We may then restrict ourselves again to $h_1(\omega, z)$ in

$$-I = \int_{-\infty}^{\infty} dy H_y(0, y) = w H_{yg} \exp(-\gamma z) \quad , \quad (11)$$

where H_{yg} is a frequency-dependent constant and width parameter $w = \int_{-\infty}^{\infty} h_2(y) dy$.

The soil fields in the limit $x = 0^-$ must also have $k_y = 0$ and share common $h_2(y)$ and h_1 , since $H_y(0, y)$ is the prescribed soil field boundary H_y distributed at $x = 0$ over all y instantaneously, and all soil fields are derivable from H_y . In (11) we could prescribe heuristically from magnetostatics the results

$$h_1 = H_{yg}$$

where

$$H_{yg} = \frac{H_y(0, 0)}{2}$$

and

$$h_2(y) = \frac{2hw}{2\pi(h^2 + y^2)} \quad ;$$

hence, $\int_{-\infty}^{\infty} h_2(y) dy = w$ and $h_2(0) = 2w/2\pi h = 2$ from equation (10). In section 4 we show that the prescription is not necessary since, if the prescription is a bit further elaborated, it has the same results as the

vector-potential calculations. In the earth only the TM electromagnetic fields E_x , E_z , and H_y are kept, whence

$$E_x = - \frac{1}{i\omega\epsilon} \frac{\partial H_y}{\partial z} , \quad (12)$$

$$E_z = \frac{1}{i\omega\epsilon} \frac{\partial H_y}{\partial x} , \quad (13)$$

$$H_y = \frac{1}{i\omega\mu_0} \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) . \quad (14)$$

On taking the exponential dependence $e^{-i\vec{k} \cdot \vec{x}}$, if we substitute equations (12) and (13) into (14), we obtain

$$\omega^2 \mu_0 \epsilon H_y = (k_x^2 + k_z^2) H_y$$

or

$$k^2 = \mu_0 \epsilon \omega^2 ,$$

where $k_y = 0$.

Since ϵ is a scalar, $\vec{k} \cdot \vec{E} = 0$ is the statement of $\text{div } \vec{D} = 0$ in the soil. Normal $\vec{D}$ must be continuous at $x = 0$; thus, the connection at $x = 0$ of line wave to soil waves involves only the integrated (defined) charge per unit length $q = \int_{-\infty}^{\infty} dy \vec{D} \cdot \hat{x}$ taken with respect to upward normal $\hat{x}$. Or, if we use equation (12) to evaluate q , we obtain

$$q(x = 0) = - \frac{1}{i\omega} \int_{-\infty}^{\infty} dy \frac{\partial H_y}{\partial z} = - \frac{\partial}{\partial z} (-I) \frac{1}{i\omega} , \quad (15)$$

which is just the continuity equation

$$i\omega \underline{q}(x=0) = -\frac{\partial}{\partial z}(-\underline{I}) = -i\omega C\underline{V} \quad , \quad (15a)$$

where equation (11) has been used in (15) (the right side is also the time derivative of eq (72)). (Remember that $\underline{q}_{\text{wire}} = -\underline{q}(x=0)$.)

We need now only superpose Fourier waves in

$$\underline{H}_{-y}(\vec{x}) = \frac{h_2(y)}{2\pi} \int_{-\infty}^{\infty} dk_z \underline{A}_0(k_z) \exp(-ik_x x) \exp(-ik_z z) \quad , \quad (16)$$

where

$$k_x = -(\mu_0 \epsilon \omega^2 - k_z^2)^{1/2} \quad , \quad (17)$$

constrained by $k^2 = \mu_0 \epsilon \omega^2$; the square root in equation (17) taken in quadrant 4 yields downward-traveling damped waves in the earth.

We first place $x = 0$ in equation (16), integrate (16) over dy from $-\infty$ to ∞ , and then equate it to (11); we then multiply by $e^{ik_z' z}$ and integrate over all z , using the following representation of the Dirac delta function δ ,

$$1/2\pi \int_{-\infty}^{\infty} dz e^{-i(k_z - k_z')z} = \delta(k_z - k_z') \quad .$$

All this produces

$$A_0(k_z) = H_{yg} \int_{-\infty}^{\infty} dz e^{-\gamma z} e^{ik_z z} \theta(z) = \frac{H_{yg}}{-i(k_z + i\gamma)} \quad (18)$$

In (18), equation (11) is restricted to $z > 0$ by the Heaviside step function $\theta(z) = 1$ ($z > 0$), $\theta(z) = 0$ ($z < 0$). Finally, if we insert equation (18) into (16), and close the contour in the lower half plane, we find, from the pole at

$$k_z = -i\gamma \quad (19)$$

and equation (17), the basic solution

$$H_y(x) = h_2(y) H_{yg} \exp[i(\mu_0 \omega^2 \epsilon + \gamma^2)^{1/2} x] \exp(-\gamma z), \quad x \leq 0, \quad (20)$$

from which $\vec{E}$ follows, according to equations (12) and (13).

The per-unit-length surface impedance satisfies equation (13) at $x = 0$ written as

$$\vec{E}_z(0, y) = e^{-\gamma z} \vec{E}_{zg} h_2(y) = w Z_g H_{yg} h_2(y) e^{-\gamma z},$$

where $e^{-\gamma z} \vec{E}_{zg} = \vec{E}_z(0, 0)/2$, analogous to H_{yg} ; hence, using equations (20), (13), (11), and $h_2(y)$ (below eq (11)), and integrating the above equation over y , we find

$$Z_g = \frac{e^{-\gamma z} \vec{E}_{zg}}{-I_z} = \frac{(\omega^2 \mu_0 \epsilon + \gamma^2)^{1/2}}{w \omega \epsilon} \quad (21)$$

Therefore, also, $wZ_g = E_z(0,0)/H_y(0,0) = -k_x/\omega\epsilon$. The Z_g solution follows from inserting equation (1) into (21) squared. The details are given in appendix A.

We must clarify an important point concerning the total (reaction) electric field at $x = 0$. A derivation of the transmission-line voltage equation for the case at hand gives

$$\frac{\partial V}{\partial z} = -I(Z_g + Z_i + i\omega l_e) + E_z^{inc}(h,z) - E_z^{inc}(0,z) , \quad (22)$$

which includes, for generality, a distributed electric-field excitation (the wire is assumed perfectly thin). Equation (22) follows from the well-known

$$\oint (\vec{E} + \vec{E}^{inc}) \cdot d\vec{\ell} = 0 \quad (23)$$

taken about a rectangular path, of thickness Δz , over $x = 0$ to h . We first ignore l_e and proceed with the usual Taylor expansion by taking from (23)

$$\oint \vec{E} \cdot d\vec{\ell} = -\oint \vec{E}^{inc} \cdot d\vec{\ell} = - \text{"applied voltage"} . \quad (24)$$

For each path segment, we also define a reaction voltage for path $a \rightarrow b$ as $V_{ab} = -\int_a^b \vec{E} \cdot d\vec{\ell}$. Thus, by invoking only the sign of $d\vec{\ell}$, we correctly add to equation (24) as yet unspecified electric fields along the paths. We ignore E_x^{inc} and invoke $Z_i I = E_z(h,z)$ and $Z_g(-I) = E_z(0,z)$, according

to my current sense convention on choosing a clockwise traverse of the paths. Finally, we add to the right-hand side of (22) (shown completed) the contribution

$$-i\omega l_e I(\Delta z) = -\oint \vec{E}' \cdot d\vec{l} = \frac{\partial V'}{\partial z} (\Delta z) , \quad (25)$$

where V' is the voltage and $\vec{E}'$ is the electric field contributions in linear superposition due solely to the external inductance l_e . The important point is that the expression

$$E'_z = 0 \quad (26)$$

in the horizontal paths of (25) is the correct approximation in general, regardless of Z_i and Z_g being nonzero. For this reason, E_z in equation (21) is the correct superposition component to total reaction E_z ($x = 0$).

To show explicitly that equation (26) is correct, we need only examine the signless quantity

$$\begin{aligned} |N'| &= \Delta z \int_0^t dt \vec{S}' \cdot \frac{\vec{S}'}{|\vec{S}'|} \\ &= \left| \int_z^{z+\Delta z} dz \int_0^t dt \int_{-\infty}^{\infty} dy \int_0^h dx \frac{\partial E'_x}{\partial z} H_y(0, y) \right| \end{aligned} \quad (27)$$

for an infinitesimally thick transverse-line slice, where $\vec{S}'$ is a Poynting flux contribution due solely to the $\vec{E}'$ of (25). Using the

earlier voltage definition, substituting equations (11) and (25) into (27), and integrating by parts with $I(0-) = 0$, we find

$$|N'| = \frac{\ell_e I^2(t) \Delta z}{2} ;$$

this shows that $|N'/\Delta z|$ is indeed the correct magnetic energy per unit length for the line in the presence of full conductor impedance. Of course, we redefine $t = 0-$ for any plane selected, but the result is true in complete generality for the arbitrarily excited and terminated line, since total $\vec{S}'$ is linear in the superposition voltage component V' .

Because of equation (25) it is clear that in a consistent approximation, the product $\ell_e w$ must act instantaneously; hence, both ℓ_e and w must be real and constant (independent of frequency). Parenthetically, C (through the upper medium dielectric constant ϵ_{up}) can depend on frequency ($\ell_e C = \mu_{up} \epsilon_{up}(\omega)$, where μ_{up} is a real constant), but, for an air medium, the effect of $\epsilon_{up}(\omega)$ is insignificant.

Qualitatively, the earth in this model is infinite and spatially homogeneous, so that no upward-going waves originate in the soil volume. In the air the dominant waves are the one-dimensional line waves with $\vec{k} = k_z \hat{z}$. Higher order waves bring in $\vec{k}_\perp$, wave-vector components perpendicular to $k_z \hat{z}$.

We can roughly estimate when these higher order waves containing $\vec{k}_\perp$ in the air will be important. To do this we can borrow an argument from Kompaneys⁹ which originally applied to limits of accuracy in radio-location. The argument is that one attempts to confine these higher order waves roughly in a half cone flared about $k_z \hat{z}$, so that the waves contribute to a forward directivity of the "antenna." Similar isosceles triangles are applicable with height ℓ_0 or β and half base length $w_0/2$ or $k_\perp$; hence, the wave confinement is estimated at $w_0/2\ell_0 \approx k_\perp/\beta$, and the space-frequency wave-uncertainty relation is $4(w_0/2)k_\perp > 2\pi$. Eliminating $k_\perp$ we find

$$(\beta w_0)^2 > 4\pi^2(\ell_0/\lambda) ,$$

where $\beta = 2\pi/\lambda$, or

$$(\beta h)^2 > \pi^2(\ell_0/\lambda) , \quad (28)$$

if $w_0/2 = h$ is taken as the appropriate estimate. For fixed ℓ_0 versus ℓ_0/λ , $\ell_0/\lambda \approx 0.25$ is roughly the onset of the higher order waves, and $\ell_0/\lambda \approx 1$ roughly the fully developed higher order waves. Thus equation (28) gives about

$$(\beta h)^2 > 2 \rightarrow 10 ,$$

which indicates roughly the buildup region of the higher order waves.

⁹A. S. Kompaneys, *Theoretical Physics*, Dover Publications, Inc. (1962), p 179.

Usually, in transmission-line applications one attempts to have $(\beta h)^2 \ll 1$. For the infinite line, of course, l_0 is any line section of length $l_0 \approx \lambda$, which is instantaneously full of the waves of interest (steady waves, now).

Any $\epsilon(\omega)$ that exhibits $\epsilon_2/\epsilon_1 < 1$ at high frequency will yield a reactance crossover ($X_g = 0$), followed by increasingly negative X_g , at some high frequency. It is easy to see directly from the Maxwell equations that the impedance must be a pure negative reactance if $\alpha = 0$ results from soil with $\epsilon_2 = 0$ (sect. 3). Hence, the reactance X_g is also negative when ϵ_2/ϵ_1 is small. The X_g crossover can be calculated from $\phi_1 - 0.5\phi_2 = 0$, using equations (5) and (7). This equation results in

$$\eta^6 + \eta^4(2r - 1) + \eta^2(4r - 5) + 2r - 3 = 0 ,$$

$$r = (l_e + l_i)C/\mu_0\epsilon_1 ,$$

where $R_i \approx 0$ and $\eta = \epsilon_2/\epsilon_1$. Usually, r is small, so we can solve $\eta_0^6 - \eta_0^4 - \eta_0^2 - 3 = 0$ resulting in $\eta_0 \approx 1.73$ or $\epsilon_2/\epsilon_1 \approx 1.7$. Solving the $r \neq 0$ equation perturbatively ($\eta = \eta_0 + \lambda$), we find roughly $\lambda \approx -r/3\eta_0$ to order η_0^{-1} . The zeroth solution is accurate enough for most purposes, though easily made exact numerically.

3. TWO-DIMENSIONAL INHOMOGENEOUS PLANE WAVES

From equations (17) and (19), the wave vectors k_x and k_z are

$$k_x = -(A - iB) , \quad (29)$$

$$k_z = \beta - i\alpha .$$

Thus, A and B are positive definite for the quadrant 4 square root in equation (21). We can solve equations (17), (21), and (29) for A and B (the only practical way to obtain these quantities):

$$A = \omega w (\epsilon_1 R_g + \epsilon_2 X_g) , \quad (30)$$

$$B = \omega w (\epsilon_2 R_g - \epsilon_1 X_g) . \quad (31)$$

On the other hand, we can take the square root in equation (21) directly, producing

$$A - iB = (P_1^2 + P_2^2)^{1/4} [\cos (\pi/2 + \phi_c/2) - i \sin (\pi/2 + \phi_c/2)] \quad (32)$$

where

$$\phi_c = \tan^{-1} [P_1/(-|P_2|)] ,$$

with

$$P_1 = \mu_0 \omega^2 \epsilon_2 - \omega C(R_i + R_g) = 2AB ,$$

$$P_2 = \mu_0 \omega^2 \epsilon_1 - \omega C[X_g + \omega(\ell_e + \ell_i)] = A^2 - B^2 .$$

The root $A - iB$ of equation (32) lies always in octant 7 of quadrant 4, and A and B satisfy the phase identity

$$-\pi/2 + \frac{1}{2} \tan^{-1}[2AB/(B^2 - A^2)] = \tan^{-1}(-B/A) ,$$

which holds also for the other root (octant 3) $-A + iB$. This root follows from the previous identity under $A \rightarrow -A$, $B \rightarrow -B$; the angles are then reckoned in the opposite sense (positive clockwise from the $-x$ axis).

The unique situation here is that this model excludes octant 8 of quadrant 4 since the $\epsilon_2/\epsilon_1 \gg 1$ limit corresponds to the $A = B$ limit between octants 7 and 8, approaching from octant 7 or $A < B$. As $\epsilon_2 \rightarrow 0$ the other boundary of octant 7 is reached.

In the lossless limit ($\epsilon_2 = 0$, $R_i = 0$), one finds $R_g = 0$, $\alpha = 0$, $A = 0$, $\phi_1 = -\pi/2$, $\phi_2 = 0$, and lossless $B = B_0$, or

$$B_0 = \left\{ \left(\omega C[X_g + \omega(\ell_e + \ell_i)] - \mu_0 \epsilon_1 \omega^2 \right)^2 \right\}^{1/4} . \quad (33)$$

This limit is correct because the divergence of the real part of the complex Poynting vector must vanish in the absence of σ losses; hence, R_g must vanish so that α vanishes and $\alpha\beta + AB = 0$.

Clearly, the branch is unique since Z_g is correct* for $\epsilon = \epsilon_1$, and the analytic continuation of Z_g to $\epsilon = \epsilon_1 - i\epsilon_2$ is the desired answer. In order for AB to be always positive, there is a formal restriction that $R_i < \sigma l_e / \epsilon_0$. Normally, the inequality is never violated.

The real soil at high frequency is in the region of $\epsilon_2 / \epsilon_1 < 1$, and the lossless solution just given is merely an idealization of the real case with A small but nonzero. The qualitative point here is that the damping in the soil tends to be purely reactive (pure being $\epsilon_2 = 0$) when ϵ_2 / ϵ_1 is small, yet the high-frequency solution still exhibits a pronounced skin effect with B^{-1} small. Thus, the real soil naturally favors this octant 7 solution at both high and low frequency.

It is evident that the derivatives $\partial H_y / \partial x$ and $\partial E_z / \partial x$ do not enter explicitly the joining of the transmission-line solution to soil fields. Tangential $\underline{E}_z = -Z_g \underline{I}$ is known and is sufficient to predict the purely transmitted fields into the soil. It is of interest now to see

*It is readily shown that our Z_g solution for $\epsilon_2 = 0$ is the one of two possibilities that exhibits X_g real for unrestricted ϵ_1 .

how the wave-vector formalism relates to complex refraction-angle formalism and also to surface waves of the more familiar kind. We first need to collect a few results.

First of all, we can write the complex wave vector as

$$\vec{k} = \beta_0 \vec{m} - i\alpha_0 \vec{n} ,$$

where the normal $\vec{m}$ to the constant-phase planes and normal $\vec{n}$ to the constant-amplitude planes are

$$\vec{m} = -\cos \psi_1 \hat{x} + \sin \psi_1 \hat{z} ,$$

$$\vec{n} = -\cos \psi_2 \hat{x} + \sin \psi_2 \hat{z} ,$$

where $\hat{x}$ and $\hat{z}$ are unit vectors, and angles ψ_1 and ψ_2 are positive counterclockwise from the $-x$ axis. The real wave number β_0 is

$$\beta_0 = \text{Re}(\vec{k} \cdot \vec{m}) .$$

We must demand that $-A + iB = \beta_0 m_x - i\alpha_0 n_x$ and $-i\alpha + \beta = \beta_0 m_z - i\alpha_0 n_z$; hence, the following four relations must all hold simultaneously:

$$\beta = \beta_0 m_z ,$$

$$\alpha = \alpha_0 n_z ,$$

$$\beta_0 m_x = -A ,$$

$$\alpha_0 n_x = -B .$$

The true phase velocity is therefore

$$v_m = \omega/\beta_0 = (\omega/\beta) \sin \psi_1 .$$

We also find that

$$\beta_0 = (A^2 + \beta^2)^{1/2} ,$$

$$\alpha_0 = (\alpha^2 + B^2)^{1/2} ,$$

$$\alpha^2 + \beta^2 + A^2 + B^2 = \alpha_0^2 + \beta_0^2 , \quad (34)$$

$$A^2 + \beta^2 - \alpha^2 - B^2 = \beta_0^2 - \alpha_0^2 , \quad (35)$$

$$\psi_1 = \tan^{-1} (\beta/A) ,$$

$$\psi_2 = \tan^{-1} (\alpha/B) .$$

Consider now a complex refraction angle θ_1 . The vector analogous to $\vec{m}$ is

$$\vec{n}_1 = -\cos \theta_1 \hat{x} + \sin \theta_1 \hat{z} .$$

The phase of the refracted wave is

$$\vec{k} \cdot \vec{x} = k_1 \vec{n}_1 \cdot \vec{x}$$

where

$$k_1 = (\mu_0 \epsilon \omega^2 \Omega)^{1/2} = \omega \mu_0^{1/2} (n - i\kappa) .$$

Here, n and κ are given by the octant 8 square root

$$n = \left[\frac{1}{2} (|\epsilon| \Omega + \Omega \epsilon_1) \right]^{1/2} ,$$

$$\kappa = \left[\frac{1}{2} (|\epsilon| \Omega - \Omega \epsilon_1) \right]^{1/2} ,$$

where $|\epsilon| = (\epsilon_1^2 + \epsilon_2^2)^{1/2}$. Hence,

$$n^2 - \kappa^2 = \epsilon_1 \Omega ,$$

$$n\kappa = \epsilon_2 \Omega / 2 ,$$

where Ω is a real positive definite quantity, to be determined momentarily.

The $\vec{k} = (-A + iB)\hat{x} + (\beta - i\alpha)\hat{z}$ form of $\vec{k}$ is identical to the $k_1\vec{n}_1 = \vec{k}$ form if

$$\cos \theta_1 = (\epsilon^*\Omega)^{1/2} (A - iB)/\mu_0^{1/2} |\epsilon|\omega\Omega, \quad (36)$$

$$\sin \theta_1 = (\epsilon^*\Omega)^{1/2} (\beta - i\alpha)/\mu_0^{1/2} |\epsilon|\omega\Omega \quad (37)$$

(where $\epsilon^* = \epsilon_1 + i\epsilon_2$), since $(\epsilon^*\Omega)^{1/2} = n + i\kappa$ and

$$n^2 + \kappa^2 = \Omega|\epsilon|.$$

The formal Snell law is

$$k_1\vec{n}_1 \cdot \hat{z} = k_z$$

or

$$\sin \theta_1 = k_z/k_1.$$

One evaluates the θ_1 expressions, equations (36) and (37), to find

$$\cos \theta_1 = [(nA + \kappa B) - i(nB - \kappa A)]/\mu_0^{1/2} |\epsilon|\omega\Omega, \quad (38)$$

$$\sin \theta_1 = [(n\beta + \kappa\alpha) + i(\kappa\beta - n\alpha)]/\mu_0^{1/2} |\epsilon|\omega\Omega. \quad (39)$$

The factors in parentheses are positive definite since $\beta/\alpha > 1$ and $n/\kappa > 1$ (for $\epsilon_1 > 0$).

Note now that the dispersion relation is

$$k^2 = k_1^2 n_1^2 = \mu_0 \epsilon \omega^2 \Omega \quad (40)$$

since $n_1^2 = 1$. Using (38) and (39) above, the condition $n_1^2 = 1$ in real and imaginary equality gives the equation pair

$$\epsilon_1 \Omega (A^2 + \beta^2 - \alpha^2 - B^2) + 2\epsilon_2 \Omega (\alpha\beta + AB) = \mu_0 |\epsilon|^2 \omega^2 \Omega^2, \quad (41)$$

$$\epsilon_1 \Omega (\alpha\beta + AB) = (\epsilon_2/2) \Omega (A^2 + \beta^2 - \alpha^2 - B^2), \quad (42)$$

if we use $n^2 - \kappa^2$ and $n\kappa$ above to reduce. In unknowns $2(\alpha\beta + AB)$ and $A^2 + \beta^2 - \alpha^2 - B^2$, the solution to the equation pair is

$$A^2 + \beta^2 - \alpha^2 - B^2 = \mu_0 \epsilon_1 \omega^2 \Omega, \quad (43)$$

$$2(\alpha\beta + AB) = \mu_0 \epsilon_2 \omega^2 \Omega. \quad (44)$$

We see immediately that if $\alpha\beta + AB$ (and ϵ_1) is positive definite, Ω must be the positive definite

$$\Omega = |A^2 + \beta^2 - \alpha^2 - B^2|/\mu_0 \epsilon_1 \omega^2 . \quad (45)$$

Hence, the real equality equation (43) of the dispersion relation solution simply defines

$$A^2 + \beta^2 - \alpha^2 - B^2 = |A^2 + \beta^2 - \alpha^2 - B^2|$$

or elimination to the identity (i.e., $x = |x|$ is true on eliminating the left-hand side, resulting in $|x| = |x|$). A proof of this absolute condition is given in equation (60) below. Note that one can write equivalently

$$\vec{n}_1 = \vec{n}_r - i\vec{n}_I$$

with pure imaginary $i\vec{n}_I$ and pure real $\vec{n}_r$:

$$\vec{n}_r = [-(nA + \kappa B)\hat{x} + (n\beta + \kappa\alpha)\hat{z}]/\mu_0^{1/2} |\epsilon| \omega \Omega , \quad (46)$$

$$\vec{n}_I = [-(\kappa\beta - n\alpha)\hat{z} - (nB - \kappa A)\hat{x}]/\mu_0^{1/2} |\epsilon| \omega \Omega . \quad (47)$$

Then $\vec{n}_1^2 = 1$ becomes

$$\vec{n}_r^2 - \vec{n}_I^2 = 1 ,$$

$$\vec{n}_r \cdot \vec{n}_I = 0 .$$

These equations are identical to equations (41) and (42) when reduced with $n^2 - \kappa^2$ and $n\kappa$. We see that vector orthogonality between $\vec{n}_r$ and $\vec{n}_i$ always holds; however, this does not imply that $\text{Re}(\vec{k}) \cdot \text{Im}(\vec{k}) = 0$ in general, since

$$-\text{Re}(\vec{k}) \cdot \text{Im}(\vec{k}) = \alpha\beta + AB \quad .$$

The meaning of the other dispersion relation, equation (44), arises from forming both the complex Poynting vector and the divergence relation (for simplicity, let us suppose $y = 0$ with field amplitudes specified at $x = 0$). The Poynting vector is as follows:

$$\vec{S} = \frac{1}{2}(\vec{E} \times \vec{H}^*) \quad ,$$

which we evaluated from only

$$\vec{E} = -(\vec{k} \times \vec{H})/\omega\epsilon \quad ;$$

the divergence relation is

$$-\nabla \cdot \text{Re}(\vec{S}) = (\sigma/2)|\vec{E}|^2 \quad , \quad (49)$$

where $|\vec{E}|^2 = \vec{E} \cdot \vec{E}^*$. From (49) we obtain two results: first, using equation (34),

$$|\vec{E}|/|\vec{H}| = (\alpha_0^2 + \beta_0^2)^{1/2}/\omega|\epsilon|, \quad (50)$$

and second, also using $\alpha\beta + AB = \alpha_0\beta_0\vec{m}\cdot\vec{n}$,

$$\vec{m}\cdot\vec{n} = \cos(\psi_1 - \psi_2) = \sigma(\beta_0^2 - \alpha_0^2)/2\alpha_0\beta_0\omega\epsilon_1. \quad (51)$$

On imposing $\beta_0^2 - \alpha_0^2 = |\beta_0^2 - \alpha_0^2|$, equation (51) is the same as equation (44) of the dispersion relation. That $\cos(\psi_1 - \psi_2)$ is even means that the power spectrum of the ohmic heating density is the same for left-going line waves, just as for right-going waves. The left-going waves are obtainable by $\gamma \rightarrow -\gamma$ ($z < 0$); hence, $\vec{m}$ and $\vec{n}$ are mirror symmetric across the $-x$ axis (ψ_1 and ψ_2 are then negative and clockwise from the $-x$ axis). It is evident that the wave-creation origin can be placed at any z . The validity of the result Z_g with respect to the terminated line is due to phenomenological terminations being local (extensionless) and hence not changing $\gamma(\omega)$ on reflection of the elementary basis waves. The terminations introduce purely temporal delays (as well as amplitude changes) between incoming and outgoing basis waves in the finite line solution.

It is appropriate now to solve for Ω and explain the apparent medium $\epsilon\Omega$ aspect of the previous work. No matter what k^2 is (see app B), one must have the identity (using eq (50))

$$|\underline{\vec{E}}|^2/|\underline{\vec{H}}|^2 = (\omega\mu_0)^2/|\underline{\vec{k}}|^2 = (\alpha_0^2 + \beta_0^2)/\omega^2|\epsilon|^2, \quad (52)$$

where $|\underline{\vec{k}}|^2 \equiv |\underline{\vec{k}} \cdot \underline{\vec{k}}|$. On substituting $|\underline{\vec{k}}|^2 = \mu_0|\epsilon|\omega^2\Omega$ into equation (52), and using the definition of Ω in (45) and also in (35), we find the ratio R on squaring to be

$$R = (\beta_0^2 - \alpha_0^2)^2/(\beta_0^2 + \alpha_0^2)^2 = \mu_0^4\omega^8\epsilon_1^2|\epsilon|^2/(\alpha_0^2 + \beta_0^2)^4. \quad (53)$$

We can also complete the square using $\cos(\psi_1 - \psi_2)$, from equation (51), resulting in

$$(\beta_0^2 - \alpha_0^2)^2 = (\beta_0^2 + \alpha_0^2)^2 - (\sigma^2/\omega^2\epsilon_1^2)(\beta_0^2 - \alpha_0^2)^2 \sec^2(\psi_1 - \psi_2). \quad (54)$$

Forming R from equation (54) and equating it to (53) gives

$$(\alpha_0^2 + \beta_0^2)^4 = \mu_0^4\omega^8|\epsilon|^4[1 + (\epsilon_2^2/|\epsilon|^2) \tan^2(\psi_1 - \psi_2)]; \quad (55)$$

hence, from (52) we have

$$|\underline{\vec{E}}|/|\underline{\vec{H}}| = \Omega^{-1/2}(\mu_0/|\epsilon|)^{1/2}. \quad (56)$$

Using (55) we find explicitly that

$$\Omega^{-1/2} = [1 + (\epsilon_2^2/|\epsilon|^2) \tan^2(\psi_1 - \psi_2)]^{1/8}. \quad (57)$$

The vector norms are now

$$(\alpha_0^2 + \beta_0^2)^{1/2} = (|\vec{k}|^2)^{1/2} = \Omega^{-1/2} \omega (\mu_0 |\epsilon|)^{1/2} \quad (58)$$

$$|\vec{k}^2|/|\vec{k}|^2 = \Omega^2 ;$$

the ratio is 1 for a homogenous plane wave having $\text{Re}(\vec{k})$ parallel to $\text{Im}(\vec{k})$ (for example, Ω approaches 1 arbitrarily closely as ϵ_2/ϵ_1 tends to infinity).

From the standpoint of equation (56), the transmission-line surface wave transmits into an apparent medium with spatial anisotropy due to $\psi_1 - \psi_2$ in equation (57), despite the fact that the bulk medium is isotropic with respect to ϵ . The deviation of Ω from unity is typically such that the right-hand side of (57) exceeds unity by about 5 percent or so at the higher frequencies.

Clearly, the bulk medium ϵ is still ϵ and not $\Omega\epsilon$. To see this we can use σ as an example and examine heating per unit volume or

$$\int_0^\infty \vec{J} \cdot \vec{E} \, dt = 1/2 \int_0^\infty \frac{d\omega}{2\pi} \sigma |\vec{E}|^2 .$$

Using the $\vec{k}^2$ norms above, however, along with (48) we find

$$|\underline{\vec{E}}|^2 = |\underline{\vec{k}}|^2 |\underline{\vec{H}}|^2 / \omega^2 |\epsilon|^2 = (\mu_0 / \Omega |\epsilon|) |\underline{\vec{H}}|^2$$

and also

$$|\underline{\vec{E}}|^2 = |\underline{\vec{k}}|^2 |\underline{\vec{H}}|^2 / |\epsilon|^2 \omega^2 \Omega^2 = \underline{(|\underline{\vec{k}}|^2 / \mu_0 |\epsilon| \omega^2 \Omega)} (\mu_0 / \Omega |\epsilon|) |\underline{\vec{H}}|^2$$

with the underlined factor equal to 1 in the last equation. Thus, substituting $|\underline{\vec{E}}|^2$ into the integral exhibits only the partial apparent medium aspect of equation (56).

The partial quadrature solution, entirely in terms of α_0 , β_0 , ψ_1 , and ψ_2 proceeds from the norm relation

$$\alpha_0^2 + \beta_0^2 = \mu_0 |\epsilon| \omega^2 \Omega^{-1} ,$$

and the absolute value of equation (51), which becomes

$$\alpha_0 \beta_0 = \mu_0 \sigma \omega \Omega \sec (\psi_1 - \psi_2) / 2$$

solved by square completion. We find

$$\alpha_0 + \beta_0 = Q_0 = [\Omega^{-1} \mu_0 |\epsilon| \omega^2 + \mu_0 \sigma \omega \Omega \sec (\psi_1 - \psi_2)]^{1/2} .$$

We obtain α_0 and β_0 by solving the last equation simultaneously with

$$\beta_0^2 - \alpha_0^2 = \mu_0 \epsilon_1 \omega^2 \Omega$$

(which is equivalent to $\beta_0^2 - \alpha_0^2 + |\beta_0^2 - \alpha_0^2|$), once for $\alpha_0 = Q_0 - \beta_0$ and once for $\beta_0 = Q_0 - \alpha_0$ (I shall not write down the derivation). This quadrature is redundant, practically speaking, since the most feasible solution seems to be solving A , B , α , and β , as stated previously, and then obtaining ψ_1 , ψ_2 , Ω , α_0 , and β_0 , successively.

If we now consider a different problem and prescribe independently of the ϵ medium

$$\alpha = 0 \quad ,$$

$$\beta \rightarrow \beta \sin \theta_0 \quad ,$$

$$\Omega \rightarrow 1 \quad ,$$

for an infinite solution plane (where θ_0 is the real incidence angle from vertical), then the usual Fresnel equations determine the transmitted amplitudes. We can solve the θ_1 equations given above by way of

$$\cos \theta_1 = \rho_0 e^{-i\gamma_0}$$

(using also $\sin \theta_1$) for ρ_0 and γ_0 . Thus, in this case we obtain

$$A = \rho_0 \mu_0^{1/2} \omega (n \cos \gamma_0 - \kappa \sin \gamma_0) , \quad (59)$$

$$B = \rho_0 \mu_0^{1/2} \omega (\kappa \cos \gamma_0 + n \sin \gamma_0) ,$$

and this solution (trivially allowing for a different $e^{-i\omega t}$ convention) is identical to that given by Stratton,¹⁰ for the well-known problem just stated. (My B , A , n , κ , γ_0 , and β correspond to Stratton's p , q , $\alpha_1/\omega\mu_0^{1/2}$, $\beta_1/\omega\mu_0^{1/2}$, γ , and α_2 .) This problem is a homogeneous plane wave ($\Omega = 1$) for which the limit $\epsilon_2 = 0$ yields $\gamma_0 = \kappa = B = 0$, $A \neq 0$, and real refraction angle $\psi_1 = \theta_1$. In this case, reactive damping ($A = 0$, $B \neq 0$, $\epsilon_2 \rightarrow 0$) is not possible.

Returning now to the transmission-line surface-wave problem, equations (59) for A and B remain true but α and β are no longer prescribed, and $\Omega \neq 1$ has been restored. This exact special case in which $\epsilon_2 = 0$ in the refraction formalism requires that A now vanish; hence, we require that

$$n \cos \gamma_0 - \kappa \sin \gamma_0 = 0$$

$$\tan \gamma_0 = n/\kappa + \infty$$

¹⁰J. S. Stratton, *Electromagnetic Theory*, McGraw Hill, Inc. (1941), p 502.

as $\kappa \rightarrow 0$; hence, $\alpha = 0$, $\gamma_0 = \pi/2$, $A = 0$, and

$$\rho_0 = B_0/\mu_0^{1/2}\omega(\epsilon_1\Omega)^{1/2} ,$$

$$\cos \theta_1 = -iB_0/\omega(\mu_0\epsilon_1\Omega)^{1/2} ,$$

$$\sin \theta_1 = \beta/\omega(\mu_0\epsilon_1\Omega)^{1/2} ,$$

$$\tan \theta_1 = -\beta/iB_0 = \underline{E}_x/\underline{E}_z ,$$

where B_0 is $B = B_0$ given by (33). In this case $\psi_1 = \pi/2$ (pure forward) and $\psi_2 = 0$. Defining the complex refraction angle as

$$\theta_1 = -i\theta_r + \frac{\pi}{2} ,$$

where θ_r is a real angle, one finds $\sin \theta_1 = \cosh \theta_r$, $\cos \theta_1 = i \sinh \theta_r$, and

$$\theta_r = -\coth^{-1} (\beta/B_0) ;$$

hence, $\cos^2 \theta_1 + \sin^2 \theta_1 = 1$ is just $\beta^2 - B_0^2 = \mu_0\epsilon_1\omega^2\Omega$, true under $\beta^2 - \beta_0^2 \rightarrow |\beta^2 - B_0^2|$ because $1 \equiv |1|$.

I shall digress for just a moment to give a general proof of the absolute condition in the dispersion-relation solution. The claim is that, in general (for any ϵ_1), equation (41) is

$$\operatorname{Re}(\cos^2 \theta_1 + \sin^2 \theta_1) = 1 \quad (60)$$

which is identical to

$$|\cos^2 \theta_1 + \sin^2 \theta_1| = 1 \quad (61)$$

The proof is immediate from (46) and (47) with $\cos \theta_1 = -n_{rx} + in_{Ix}$, $\sin \theta_1 = n_{rz} - in_{Iz}$. We can verify easily that (60) is identical to (61) because $(n_{rz}n_{Iz} + n_{rx}n_{Ix})^2 = 0$ as a result of using (43) and (44) in elimination. Thus, for $\epsilon_1 > 0$, the absolute elimination of (43) means only that Ω is positive.

To obtain Ω and finish the lossless solution we must avoid taking the limit $\epsilon_2 \rightarrow 0$ directly in Ω (57). The convenient additional equation is the $|\vec{k}|^2$ norm from equation (58) or

$$\beta^2 + B_0^2 = \mu_0 \epsilon_1 \omega^2 \Omega^{-1} \quad (62)$$

Thus, since $|\beta^2 - B_0^2| = \mu_0 \epsilon_1 \omega^2 \Omega$, one has

$$\rho_0 = B_0 / |\beta^2 - B_0^2|^{1/2} .$$

On expressing the left-hand side of the last equation exactly as ρ_0 was first given above (between eq (59) and (60)) and then eliminating B_0 using (62), we find

$$|2\beta^2 - \mu_0 \epsilon_1 \omega^2 \Omega^{-1}|^{1/2} = (\mu_0 \epsilon_1 \omega^2 \Omega)^{1/2} .$$

Solving uniquely $-2\beta^2 + \mu_0 \epsilon_1 \omega^2 \Omega^{-1} = \mu_0 \epsilon_1 \omega^2 \Omega$ one obtains

$$\Omega = -\beta^2 / \mu_0 \epsilon_1 \omega^2 + [1 + \beta^4 / (\mu_0 \epsilon_1 \omega^2)^2]^{1/2} \quad (63)$$

(since Ω is real and the plus sign is unique to Ω positive definite). In the $|x_g|$ negligible high-frequency regime, $\beta \approx \omega(\mu_0 \epsilon_0)^{1/2}$ and equation (63) becomes

$$\Omega \approx 1 - (\epsilon_0 / \epsilon_1) + (\epsilon_0 / \epsilon_1)^2 / 2 . \quad (64)$$

The Ω of equation (64) is close to 1 for real soil having $\epsilon_0 / \epsilon_1 \approx 1/6$ or so at high frequency.

In real soil, however, high-frequency α and β can have a slight effect on the soil fields (as if $\alpha \approx 0$) yet α can be still appreciable unless ϵ_2 / ϵ_1 is sufficiently small. In order to see what constitutes

small ϵ_2/ϵ_1 , the following high-frequency approximations to R_g and X_g are useful:

$$R_{\text{high}} = w^{-1}(\mu_0/\epsilon_1)^{1/2}(\epsilon_2/\epsilon_1)(3/2), \quad \omega/c > 1, \quad \epsilon_2/\epsilon_1 < 1, \quad \epsilon_1/\epsilon_0 > 5, \quad (65)$$

$$X_{\text{high}} = -w^{-1}(\mu_0/\epsilon_1)^{1/2}. \quad (66)$$

The $\omega/c > 1$ stipulation is essential so that B is sufficiently large. These rough estimates are good to about 20 percent or better if

$$(3/2)(\epsilon_1/\epsilon_0)^{-3/2}w^{-1}(\mu_0/\epsilon_0)^{1/2}(\omega\ell_e)^{-1} < 0.1. \quad (67)$$

At high frequencies, these estimates improve to a few percentage points (factor (68) should be used in X_g). As frequency increases, ϵ_2/ϵ_1 becomes smaller.¹¹ Hence, $\beta \approx \omega/c$, and $\alpha \approx (\omega/c) \sin(R_g/2\omega\ell_e)$. The estimates come from equation (44), dropping α and β , and using $A^2 + B^2 = \mu_0\epsilon_1\omega^2$ (where $\Omega \approx 1$). This yields

$$A_{\text{high}} = 0.5 \sigma(\mu_0/\epsilon_1)^{1/2},$$

$$B_{\text{high}} = \omega(\mu_0\epsilon_1)^{1/2}, \quad |\epsilon| \approx \epsilon_1;$$

¹¹C. L. Longmire and K. S. Smith, *A Universal Impedance for Soils*, Defense Nuclear Agency, Topical Report DNA-3788T (October 1975).

thus equations (65) and (66) can be obtained from equations (30) and (31). Incidentally, taking $\psi_2 = 0$, $\tan \psi_1 = \beta/A$, $\beta \approx \omega/c$, and using A_{high} for A , we find directly from (57) that $\Omega^{-1/2} \approx (1 + 4 \epsilon_0/\epsilon_1)^{1/8} \approx 1 + \epsilon_0/2\epsilon_1$, in agreement with $\Omega^{-1/2}$ from (64) to leading order in ϵ_0/ϵ_1 . Since (66) is entirely independent of ϵ_2 , X_g should come from $-w\omega\epsilon_1 X_g = B_0$. In (33) we should retain the l_e term, however, as β^2 is not quite negligible with respect to calculating X_g . The result is simply an additional factor of

$$(1 - \epsilon_0/\epsilon_1)^{1/2} + 1 - \epsilon_0/2\epsilon_1 \quad (68)$$

on the right-hand side of equation (66). Finally, with (67) well satisfied, R_g is essentially entirely due to ohmic loss. The tendency at high frequency as ϵ_2/ϵ_1 diminishes is for α to be purely ohmic while B is purely reactive. The surprising feature of real soil is that σ can be appreciable and yet ϵ_2/ϵ_1 can be small in a "lossless" situation.

As for low-frequency approximations, we again drop α and β in (44), taking $A = B$. The familiar result for $R_g = X_g$ is

$$R_{\text{low}} = X_{\text{low}} = (\mu_0 \omega / 2\sigma)^{1/2} \omega^{-1} \quad (69)$$

The approximation can be good (better than ~10 percent) only for $\epsilon_2/\epsilon_1 > 10$.

4. PURELY VECTOR-POTENTIAL MODEL OF TM TRANSMISSION LINE

It is of considerable qualitative interest to obtain the section 2 Z_g and soil field model from a purely vector-potential TM formulation of the wire-over-ground problem. In our region of macroscopic electrodynamics, $\nabla \cdot \vec{D} = 0$ everywhere; hence, no scalar potential need be introduced explicitly (it is zero).

Because $\nabla \cdot \vec{A} = 0$, we first consider a TM vector potential $\vec{A}$ in region 1 (air) volume that is purely radial. The radial electric field $\vec{E} \cdot \hat{r} = E_r$ (where $\hat{r}$ is outward from the wire) is

$$-i\omega A_{r1} = \frac{C\underline{V}}{2\pi\epsilon_0 r} = E_r, \quad (70)$$

where

the wave dependence is $e^{-ik_z z} e^{i\omega t}$, implicit in $\underline{V}$ of equation (15a),

$$r = [(x - h)^2 + y^2]^{1/2}, \text{ and}$$

A_{r1} is the incident vector potential due solely to the current $\underline{I}$ on the wire.

The boundary condition at the wire surface is satisfied by equation (70), which yields normal $\underline{D}$ or free CV waves at the wire; $\underline{V}$ is specified as in section 2. This must be so since $k_z \approx 0$ and $(\text{curl } \vec{H})_r \approx 0$ in the wire (TM); hence, $\underline{E}_r$ at the wire surface jumps abruptly to an exceedingly small value inside the metal.

The reaction $\vec{A}$ at $x = 0$ must be added to equation (70). Because the reaction must instantaneously yield only $\underline{D}_x$ at $x = 0$ and every y point, the reflection or reaction symmetry of the divergenceless axial vector $\vec{A}_r$ must be opposite that of an optical mirror reflection of the vector arrow, so that y projections cancel in the sum and $E_y = 0$ identically over $x = 0$. Then the normal projection of equation (70) is doubled at $x = 0$:

$$\underline{A}_{-x1} = \frac{-2h\underline{A}_{-r1}(r_0)}{r_0}, \quad (71)$$

where $r_0 = (h^2 + y^2)^{1/2}$. If we integrate normal $\underline{D}_x$ over y at $x = 0$, from equation (71), we obtain

$$-CV = \int_{-\infty}^{\infty} \frac{2h\varepsilon_0 i \omega \underline{A}_{-r1} dy}{r_0}, \quad (72)$$

which is exactly satisfied on inserting (70).

The integrand of (72) is a transverse magnetostatic y distribution, so that

$$\underline{D}_x(0,y) = \frac{2h(-CV)}{2\pi(h^2 + y)^2} \quad (73)$$

Hence, $\underline{H}_y(0,y)$ has a distribution like that of (73), with $-CV$ replaced by $-\underline{I}$. To obtain this result, we substitute $-CV = -k_z \underline{I}/\omega$ into equation (73), and use $\underline{D}_x(0,y) = k_z \underline{H}_y(0,y)/\omega$ from (12). We can use equation (73) to verify (10), since $0.5 \underline{D}_x(0,0)w = -CV$, where $0.5 \underline{D}_x(0,0)$ is the "conserved mean planar field," that is, $w^{-1} \int_{-\infty}^{\infty} dy \underline{D}_x(0,y)$. A similar mean value definition applies to $\underline{E}_z$ and $\underline{H}_y$ also. The consistency check here is that in the contact limit $h = a$, the following conditions hold: $w = 2\pi a$ and $\underline{D}_x(0,0) \approx -2CV/w$, as shown by equation (73); thus, $\underline{D}_x(0,0)/2 = -CV/w$ is the negative ($\hat{r} \cdot \hat{x} = -1$) of the radial w uniform $\underline{D}_r$ around the wire ($\underline{D}_r$ is also the mean value). The line wave connection to the region 2 soil fields of section 2 is now complete from the air-side approach to $x = 0$. One can obtain all field amplitudes from $\underline{H}_{yg}$ (described in (20) and below (11)), since we find from equation (10), equation (73), and $\underline{H}_y(0,0) = \omega \underline{D}_x(0,0)/k_z$ that

$$\underline{H}_{yg} = \omega \epsilon_0 k_z^{-1} [0.5 \underline{E}_{x1}(0,0)] = -\omega k_z^{-1} CV w^{-1} ,$$

where $\epsilon_0 \underline{E}_{x1}(0,0) = \underline{D}_x(0,0)$. The essential explicit minus sign of $\underline{H}_{yg}$ results in a Poynting flux density $\underline{E}_z(0,y) \underline{H}_y^*(0,y)$ into the ground.

Let us now show that ℓ_e , Z_i , and Z_g are already included in the TM model in a "consistently exact" way. At the wire and conductor surfaces, two kinds of z-component vector-potential surface A_{zs} are present: (1) A_{zs1} , which is the $\ell_e I$ contribution, and (2) A_{zs2} , for which $-i\omega A_{zs2}$ is the surface electric field in the surface-impedance boundary condition.

At the wire surface we have surface $\vec{H} = H_i \hat{\phi}$, where

$$\mu_0 H_i = \frac{\partial A_{r1}}{\partial z} - \frac{\partial A_{zs2}}{\partial r}, \quad r \rightarrow a, \quad (74a)$$

and

$$\mu_0 H_i = \frac{\partial A_{zs1}}{\partial r}, \quad r \rightarrow a. \quad (74b)$$

We may obtain our results from equations (74a) and (74b) with r locally free and then pass to $r = a$. The radial A_{r1} derivative from equation (70) is

$$-i\omega \frac{\partial A_{r1}}{\partial z} = \frac{C}{2\pi\epsilon_0 r} (-i\omega \ell_e I - Z_i I), \quad r \rightarrow a, \quad (75)$$

where the parentheses in equation (75) contain the part (linear superposition) of $\partial V/\partial z$ that contributes locally at the wire surface. The first term in equation (75), along with (70), contributes $\mu_0 H_i$, where

$$H_{-i} = \frac{I}{2\pi r} , \quad r = a .$$

As a Taylor expansion shows, A_{-zs2} cannot change H_{-} locally--nor hence anywhere else--inside an air-medium pillbox at the surface that collapses to zero radial thickness. The second term in equation (75) thus must cancel $-i\omega(\partial A_{-zs2}/\partial r)$ from equation (74a). This yields

$$E_{-z\text{wire}} = -i\omega A_{-zs2} = \frac{Z_i I (-\ln r + D)}{f_g} , \quad r = a , \quad (76)$$

where $C = 2\pi\epsilon_0/f_g$. The correct result (independent of f_g) in equation (76) follows if constant D is adjusted so that $f_g = \ln(2h/a)$. The remaining A_{-zs1} must necessarily satisfy (74b). This result, where H_{-i} is as given above and constant D is as in (76), gives

$$A_{-zs1} = \ell_e \frac{I}{e} , \quad (77)$$

where $\ell_e = \mu_0 f_g / 2\pi$. Clearly, A_{-zs1} has an ignorable (zero) local induction contribution to ℓ_e in the pillbox argument, and the sum of equations (74a) and (74b) is obviously not meaningful ($H_{-wire} \neq 2H_{-i}$).

At the ground surface $x = 0$, the surface vector potential must be $A_{-zs1} = 0$; this condition is required so that local A_{-zs1} yields a zero contribution to equation (77) from a surface integral over $x = 0$.

Similarly, at $x = 0$ we have

$$\mu_{0-y} = - \frac{\partial A_{zs2}}{\partial x} + \frac{\partial A_{-x}}{\partial z} \quad (78a)$$

and

$$\mu_{0-y}(0,y) \equiv - \frac{\partial A_{zs1}}{\partial x} \quad (78b)$$

The A_{-x} derivative is, in general, for arbitrary y ,

$$-i\omega \frac{\partial A_{-x}}{\partial z} = \frac{-hC}{\pi \epsilon_0 r_0^2} (-i\omega \ell e_{-} I - Z_g I) \quad (79)$$

Equation (79) follows from A_{-x1} above, (71), (70) and $\partial V / \partial z$ acting locally at $x = 0$.

The $\partial A_{-x} / \partial z$ contribution from the second right-hand term of equation (79) must cancel with $-(\partial A_{zs2} / \partial x)$ in (78a). Actually, our inclusion of the cancelling terms in the equations is not necessary, since only the one-line boundary condition (Δz in eq (23)) at $x = 0$, $y = 0$ must result. The "cancellation at a point" condition results in $\partial A_{zs2} / \partial x = -(2\pi f_g) H_{-y}(0,y) Z_g$ from the air side, where $E_{zs2} = -i\omega A_{zs2}$ and $y \rightarrow 0$ are implied. Assuming that $H_{-y}(0,y) Z_g = E_z(0,y) w^{-1}$ and solving the last derivative equation radially by way of $\partial / \partial x = (-h/r)(\partial / \partial r)$ on $E_{zs2}(r)$, where $E_z(0,y) = E_z(0,0) h^2 / r^2$, we find

$$\underline{E}_{zs2}(r) = \frac{\underline{E}_z(0,0)}{f_g} (\ln r + D) \quad . \quad (80)$$

Thus equation (80) is the identity $\underline{E}_z(0,0)$ as $r \rightarrow r(0,0) = h$, where $D = \ln 2 - \ln a$. The remaining $\partial \underline{A}_x / \partial z$ contribution from equation (79) yields $\mu_{0-y} H(0,y)$, thus entirely satisfying equation (78a).

Analogous to (74b), equation (78b) simply defines the local normal derivative of $\underline{A}_{zs1}$. $\underline{A}_{zs1}$ is again ignorable.

It is obvious that $f_g = \ln(2h/a)$ is not required for model consistency. In fact we must replace this ℓ_e with the ℓ_e resulting from the more exact

$$f_g = \ln\left(\frac{h}{a} + \left[\left(\frac{h}{a}\right)^2 - 1\right]^{1/2}\right) \quad . \quad (81)$$

The origin of air volume ℓ_e can be attributed to a linearly superposed vector potential $\underline{A}_{z1} \hat{z}$ that contributes the surface $\underline{A}_{zs1}$ considered previously. Let subscript t denote transverse plane vector components. Then $\nabla^2 = \nabla_t^2 + (\partial^2 / \partial z^2)$. The vector potential $\underline{A}_{z1}$ carries the suppressed $e^{-ik_z z} e^{i\omega t}$ dependence. $\underline{A}_{z1}$ satisfies the everywhere instantaneous Faraday law

$$\nabla_t \times (-i\omega \underline{A}_{z1} \hat{z}) = -i\omega \mu_0 \vec{H}_{0t} \quad ,$$

where $\underline{H}_{0t} = \mu_0^{-1} [\text{curl}(\underline{A}_{z1} \hat{z})]_t$. Evidently the wave equation $(\nabla^2 + k_z^2) \underline{F} = 0 = \nabla_t^2 \underline{F}$ is satisfied for $\underline{F} = \underline{A}_{z1} \underline{H}_{0t}$ and $\nabla \cdot (\epsilon_0 \underline{E}_{0t})$ where $\underline{E}_{0t} = -i\omega \underline{A}_{z1} \hat{z}$. The last example shows that $\underline{A}_{z1} \hat{z}$ is a source-free, ignorable divergence axial vector; hence, the "incident plus reaction" boundary condition $\underline{A}_{zs1} = 0$ necessarily is required at $x = 0$ in solving $\nabla_t^2 \underline{A}_{z1} = 0$. We may note that $\underline{A}_{z1}$ creates zero $\underline{H}_z$; hence $\nabla \cdot \underline{H}_{0t} = \nabla_t \cdot \underline{H}_{0t} = 0$. Thus, $\nabla \cdot \underline{H} = 0$ is satisfied completely in the TM model. The meaning of $\nabla_t \cdot \underline{H}_{0t} = 0$ is that $\partial^2 \underline{A}_{z1} / \partial x \partial y$ is the identity, placing no restriction on $\underline{A}_{z1}$.

A useful boundary relation at wire surface is

$$\frac{\oint_{\hat{t}} \underline{E}}{f_g} = \frac{a [\text{curl}(\underline{A}_{z1} \hat{z})] \cdot \hat{t}}{f_g}, \quad (82)$$

where $\hat{t}$ is the tangent vector along a wire cross-section perimeter ($\underline{A}_{z1}$ is TM uniform in ϕ at the wire surface). A numerical solution is not necessary, because the standard circular equipotential approximate solution (vector-potential version¹²) of $\nabla_t^2 \underline{A}_{z1} = 0$ here results in a surface potential value $\underline{A}_{z1} = \underline{D}_0 f_g$ constant on the circle of wire surface (f_g is defined in eq (81)). We may construct a locally Laplacian wire coordinate-system solution for $r \approx a$ of $\underline{A}_{z1} = \underline{D}_0 f_g a/r$, and with $\hat{\phi} \cdot [\text{curl}(\underline{A}_{z1} \hat{z})] \rightarrow -(\partial \underline{A}_{z1} / \partial r)$, the boundary relation (82) yields $\underline{D}_0 = \mu_0 \underline{I} / 2\pi$ as $r \rightarrow a$.

¹²See, for example, S. Ramo and J. R. Whinnery, *Fields and Waves in Modern Radio*, second edition, John Wiley and Sons, Inc. (1953), p 138.

Equations (78b) and (74b), where $\underline{H}_{0t\phi} = \underline{H}_i$, reveal that $\underline{H}_{0t}$ alone must yield the correct $\vec{H}$ in the air volume that is continuous in the tangential component at the wire surface and at the $x = 0$ surface, on approach to the surfaces from the air. Clearly, then, $\underline{H}_i(r)\hat{\phi} = \hat{\phi}/2\pi r$, where $\underline{H}_i(r) = \mu_0^{-1}(\partial \underline{A}_{-r1}/\partial z)$. This result is nonzero only at $r = a$ and $r = r_0$. The total $\underline{H}_i$ at $x = 0$ is $2\underline{H}_i(r_0)$, and $\underline{H}_y(0,y)$ is indeed the y projection $2\underline{H}_i(r_0)\hat{\phi} \cdot \hat{y} \equiv \mu_0^{-1}(\partial \underline{A}_{-x1}/\partial z)_1$, where $\hat{\phi} \cdot \hat{y} = -h/r_0$, and where $(\partial \underline{A}_{-x1}/\partial z)_1$ comes entirely from the noncancelled first right-hand term of (79). That $\underline{H}_i$ and hence $\underline{A}_{-r1}$ must vanish in the air volume follows from $\text{curl curl}(\underline{H}_i\hat{\phi}) = \partial^2 \underline{H}_i\hat{\phi}/\partial z^2 = \omega^2 \mu_0 \epsilon_0 \underline{H}_i\hat{\phi}$ and $\text{curl curl}(\underline{A}_{-r1}\hat{r}) = -\partial^2 \underline{A}_{-r1}\hat{r}/\partial z^2 = 0$. The propagating k_z dependence of $\underline{H}_{0t}$ is essential.

In the TM vector-potential model, $\underline{A}_{-r1}$, as well as $\partial \underline{A}_{-r1}/\partial z$, is actually zero in the air volume, with $\underline{A}_{-r1}$ in normal $\underline{D}$ also entering the model only as wire and $x = 0$ surface values. The air volume vanishing of $\underline{A}_{-r1}$ is absolutely essential physically; otherwise, at $x = 0 + \epsilon$ (ϵ is infinitesimal) there is an additional (to eq (73)) $\underline{D}_x$ contribution, $k_z^2(-i\omega \underline{A}_{-x1})/\mu_0 \omega^2$ where $\underline{A}_{-x1} = -h\underline{A}_{-r1}/r_0$ in $\partial \underline{A}_{-x1}/\partial z$. This false $\underline{D}_x$, which is made continuous (suppose superposition) with a $\underline{D}_x$ from the soil, requires common air-soil $k_z = \omega(\mu_0 \epsilon_0)^{1/2}$. The surface curl, $(\text{curl } \underline{H}_{0t})_x = i\omega \underline{D}_x(0,y)$, is continuous in $\partial \underline{H}_y/\partial z$ at $x = 0$ when $\underline{H}_{0t} = \underline{H}_y(0,y)$. The remaining $x = 0$ curls of $\underline{H}_{0t}$ are $(\text{curl } \underline{H}_{0t})_z = 0$ (where $\underline{A}_{-zs1} = 0$) and $(\text{curl } \underline{H}_{0t})_y = 0$ (normal $\underline{H}_{0t}$ vanishes). At the

wire, taking $H_{0t\phi}$ only, we similarly find $(\text{curl } H_{0t})_r$ continuous, $(\text{curl } H_{0t})_\phi = 0$ and $(\text{curl } H_{0t})_z = (1/r)(\partial/\partial r)(rH_{0t\phi})$ zero with local $H_{0t\phi} = I/2\pi r$, $r \approx a$.

In the vector-potential model, the Neumann analog tangential H_{-1} is redundantly equal to the Dirichlet H_{0t} at wire surface and $x = 0$. We can conclude from the uniqueness theorem that the H_{0t} resulting from $\nabla_{t-z1}^2 A_{-1} = 0$ must yield the identical A_{-1} if the Dirichlet H_{0t} is prescribed as a surface-tangential H condition, thus defining the Neumann problem. The Neumann problem then becomes a straw man that need not be considered.

There is, however, still an unresolved question concerning the exact $D_x(0,y)$ distribution that is dependent on approximations made in solving the Dirichlet A_{-1} problem. It must be true that equation (73) is correct since the traveling-wave vector-potential solution is physically unique already. A complementary (to the l_e solution) standard approximate capacitance solution¹² can be devised that yields what we call the exact capacitance C , based on a circular equipotential at the wire surface; however, this solution cannot predict the distribution equation (73) as $h/a \rightarrow 1$, since both static formulations result in equation (73) with h replaced by $d_0 = a[(h/a)^2 - 1]^{1/2}$ and $w = 2\pi d_0$. A vanishing

¹²See, for example, S. Ramo and J. R. Whinnery, *Fields and Waves in Modern Radio*, second edition, John Wiley and Sons, Inc. (1953), p 138.

where $h/a \rightarrow 1$ is, of course, absurd. One has to give up $h/a \rightarrow 1$ to recover $d_0 \rightarrow h$ and suppose that $a \rightarrow 0$ for h/a fixed infinitesimal.

Although this is not a proof of the exactness of equation (73) with respect to f_q , we can show formally in a special case that f_q of (81) arises from equation (73) and the unit-voltage line-charge Green function that vanishes at the observer's location where the source is at the origin, namely

$$-G_0 = - \frac{\ln \left(\left[\frac{\rho^2 + \rho_0^2 - 2\rho\rho_0 \cos(\phi - \phi_0)}{h^2} \right]^{1/2} \right)}{f_g},$$

where ρ is the observer ρ, ϕ source coordinate with respect to a wire-based coordinate system. We calculate the (vanishing) transverse derivative at $x = 0, y = 0$,

$$\frac{\partial G_0}{\partial y}(0,0) = - \int_{-\infty}^{\infty} dy' \frac{\partial G_0}{\partial y} D_x(0,y') \quad , \quad (84)$$

by taking equation (73) for both $\frac{\partial G_0}{\partial y}$ and D_x . Taking $\rho = h, \phi = 0$, and $\frac{\partial G_0}{\partial y} = \nabla G_0 \cdot \hat{y}$, we may replace the last equation with $1 = - \int d\rho_0 d\phi_0 \rho_0 \cos^2 \phi_0 \sin \phi \frac{\partial G_0}{\partial \rho} [\delta(\phi_0) \rho_0^{-1}]$. If $\sin \phi_0$ is substituted for $\sin \phi$ and coordinate ρ_0 for ρ in the G_0 derivative, the result is

$$f_g = \int_1^1 du (u^2 - 1)^{-1/2} = 0 \quad , \quad .$$

where $u = h/a$; hence, f_g is the limit $h/a \rightarrow 1$ of equation (81), because of $\ln(1/1)$.

The TM model predicts that the TE-related amplitudes of $\underline{H}_x$, $\underline{E}_y$, and $\underline{H}_z$ are exactly zero. A formal continuous y-integrated matching of assumed soil $\underline{H}_x$ and $\underline{H}_z$ at $x = 0$ (with air fields arising from Cartesian components due solely to surface $\underline{A}_{r1}$) results in soil field amplitudes $\underline{H}_x$ and $\underline{H}_z$ being dependent solely on the $\underline{E}_y$ amplitude (since $k_y = 0$). Thus, the formal antisymmetry in y of $\underline{H}_x$ and $\underline{H}_z$ in the air from $\underline{A}_{r1}$ is not the actual reason that the y-integrated amplitudes vanish. The vanishing is due to the stronger condition of pointwise vanishing required of $\underline{E}_y$. The vector-potential $\underline{A}_{z1}$ produces zero $\underline{H}_x$ because of the vanishing of $\partial \underline{A}_{z1} / \partial y$ at $x = 0$.

It is natural that the TM model can create no TE waves, since the y antisymmetry in $\underline{E}_y$ and $\underline{H}_x$ is in fact just the opposite of the true symmetry in $\underline{E}_y$ and $\underline{H}_x$ at $x = 0$ that would arise from ground reaction of the circularly uniform $\underline{A}_\phi$ vector-potential component. This symmetry yields a forward Poynting flux down the line from $\underline{E}_y$ and $\underline{H}_x$. Observe, however, that if we examine $(\text{curl } \vec{H})_\phi$ inside the metal wire, $k_z \approx 0$ again forces $\underline{H}_r$ to jump as $\underline{E}_r$ did in the TM case. The wire surface

value of H_r must then be exceedingly small. $H_r \hat{r}$ is an axial vector; hence, H_y from such an $H_r \hat{r}$ is also opposite the TM symmetry in y at $x = 0$.

At the same time, $(\text{curl } \vec{H})_r$ in the metal wire cannot yield "zero" E_r unless $\partial H_z / \partial \phi$ is "zero" or circularly uniform H_z . But then E_ϕ must, from $(\text{curl } \vec{H})_\phi$, also have the circular uniformity of H_z at the metal surface. Now E_ϕ comes from vector potential A_ϕ , which thus contributes consistently zero divergence to $\nabla \cdot \vec{A} = 0$. Consequently, the H_z from A_ϕ is "zero" because ϕ is uniform.

If H_z and H_r are "zero," the E_ϕ is also "zero" at the wire surface; hence, ϕ -uniform TE excitations should be negligibly produced by the wire. Non- ϕ -uniform excitations should be negligibly excited under electrically thin wire conditions, in the absence of line discontinuities.

It is peculiar in the Wait wire-over-ground formulation² that the field components H_x , E_y , and H_z are present, yet the only wire boundary condition is that $E_z = 0$ along the wire. How then can the TM excitation from the wire excite these TE components by way of ground interaction? The answer to the question is that the Wait formulation should not allow

²James R. Wait, *Radio Science*, 7, No. 6 (June 1972), 675-679.

nonzero $\underline{H}_x$, $\underline{E}_y$, and $\underline{H}_z$. We can find no error in the following procedure. Take the y derivatives given in Wait's paper² under the sign and integrate fields over dy from $-\infty$ to ∞ ; this yields a δ function to be integrated over λ , that is, $\delta(k_y)$, $k_y = \lambda = 0$. Assume that $\mu_1 = \mu_2 = \mu_0$. The field continuity at $x = 0$ then yields the result $M(0) = -N(0)$ (from three equations stating the $\underline{H}_x$, $\underline{H}_z$, and $\underline{E}_y$ continuity). Hence, it follows immediately that $M(0) = N(0) = 0$ and that $\underline{H}_x$, $\underline{H}_z$, and $\underline{E}_y$ are identically zero pointwise over $x = 0$. The normal $\underline{D}_x$ continuity yields (redundantly)

$$\beta^2 = \frac{\mu_0 \omega^2 (\underline{\epsilon}_1^3 - \underline{\epsilon}_2^3)}{\underline{\epsilon}_1^2 - \underline{\epsilon}_2^2},$$

where $\underline{\epsilon}_1 = \epsilon_{\text{air}}$ and $\underline{\epsilon}_2 = \epsilon_{\text{soil}}$ and β^2 is the negative of our γ^2 . If we require that $(\underline{\epsilon}_1^3 - \underline{\epsilon}_2^3) = K(\underline{\epsilon}_1^2 - \underline{\epsilon}_2^2)$, the solution of the last equation is $K = \underline{\epsilon}_1 = \underline{\epsilon}_2$ and $\beta^2 = \mu_0 \omega^2 \underline{\epsilon}_1 = k_1^2$. But then $\underline{E}_z = (k_1^2 - \beta^2)\underline{u} = 0$ everywhere. This result can be understood in the implied lossless case ($\underline{\epsilon}_1 = \underline{\epsilon}_2$) since a lossless TE solution has zero axial electric field.

The above $\lambda = 0$ modification of the Wait formulation yields (before we set $\underline{\epsilon}_1 = \underline{\epsilon}_2$) the result that

²James R. Wait, *Radio Science*, 7, No. 6 (June 1972), 675-679.

$$\frac{\int_{-\infty}^{\infty} dy \underline{E}_z(0,y)}{\int_{-\infty}^{\infty} dy \underline{H}_y(0,y)} = \frac{-(\mu_0 \omega^2 \underline{\epsilon}_1 - \beta^2)^{1/2} [1 + R(0)]}{\underline{\epsilon}_1 \omega [1 - R(0)]} = 0 .$$

But we require $R(0) = 0$ (actually $1 - R(0) = T(0) + 1$), so that $\underline{H}_{1y} = \underline{H}_{2y}$ at $x = 0$ results* in $\int_{-\infty}^{\infty} \underline{H}_{1y} dy = -\underline{I}$; hence, the Hertz potential $\underline{\pi}$ in Wait's formulation can result formally in our Z_g , except that ϵ_{soil} in our Z_g is replaced by ϵ_{air} .

A final remark concerning the vector-potential model: the additive ground interaction of $\vec{H}$ with respect to the incident wire $\vec{H}$ will not allow what is sometimes called an "antenna mode" with co-directional wire current $\underline{I}$ and return current $\int_{-\infty}^{\infty} dy \int_{-\infty}^0 dx (i\omega \underline{\epsilon} \underline{E}_z)$. So long as $k_{\perp}$ is zero in the air, the transmission-line solution seems to be the solution to the free-wave wire-over-ground problem.

5. CONCLUDING REMARKS

Although there is, at present, a lack of suitable cw data for $\alpha(\omega)$ and $\beta(\omega)$ concerning wires close to earth, we have some limited evidence that our Z_g is realistic. Gray⁷ has reported propagation velocities versus wire height over earth, experimentally deduced from timing of

⁷R. F. Gray, *Nuclear Electromagnetic Pulse Simulation by Point Source Injection Techniques for Shielded and Unshielded Penetrations*, Harry Diamond Laboratories, HDL-TR-1737 (December 1975).

*The coefficient of the Hertz potentials given by Wait (ref 2) following equation (8) is incorrect and should be multiplied by a factor of 2.

reflection pulses at cable center on an open 40-ft-long cable of radius 1 cm. Using a semi-empirical $\epsilon(\omega)$,¹¹ ignoring dispersion, and taking frequency $f \approx 10^7$ Hz, we find an $\omega/c\beta$ of 0.59, compared to measured 0.55 at $h = 0.05$ m with a soil moisture content of 50 percent. Our $\omega/c\beta$ result closely follows the measured result upward in height (≤ 1 m) to within 10 percent, the high-line agreement being (always) accurately 1.

¹¹C. L. Longmire and K. S. Smith, A Universal Impedance for Soils, Defense Nuclear Agency, Topical Report DNA-3788T (October 1975).

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APPENDIX A.--DERIVATION OF EQUATION (3)

APPENDIX A

The body of this report addresses the problem of calculating the response of a horizontal wire over earth to incident electromagnetic fields. In an approximate solution of this problem, the wire over earth is regarded as a transmission line; the basic unknown in this solution is the relationship between the line waves and the electromagnetic fields in the earth: that is, the surface impedance of the ground plane.

Equation (3) in the body of the report describes Z_g , the per-unit-length surface impedance of the finite conducting earth. In this appendix we derive equation (3) by solving equation (21) self-consistently with γ^2 , where γ is the line propagation constant and a function of Z_g . See section 2 in the body of the report for a detailed description of the model and definitions of symbols.

Inserting equation (1) into equation (21) and squaring gives

$$Z_g^2 + c_1 e^{i\phi_1} Z_g - c_2 e^{i\phi_2} = 0 \quad . \quad (A-1)$$

The parameters c_1 , ϕ_1 , c_2 , and ϕ_2 are given in equations (4) through (10) in the body of the paper. Basically, we can solve (A-1) at the point $\phi_1 = \phi_2 = 0$ (this is a physical solution for $\epsilon_1 = \epsilon_2$ and $(\ell_i + \ell_e)C = \mu_0 \epsilon_1$). We can then analytically continue the resulting

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z_g to the desired parameters, having verified that the $\epsilon_2/\epsilon_1 \gg 1$ limit of z_g is physically correct.

Proceeding algebraically, with $\phi_1 = \phi_2 = 0$ understood, we solve (A-1) and find

$$z_g = 0.5c_1 e^{i\phi_1} [-1 + (1 + 4c_2 e^{i\phi/c_1^2})^{1/2}] , \quad (A-2)$$

where

$$\phi = \phi_2 - 2\phi_1 . \quad (A-3)$$

(The positive root sign is correct, as later shown.) Now equation (A-2) gives

$$z_g = -0.5c_1 \cos \phi_1 + 0.5c_1 A_0 \cos [\phi_1 + 0.5 \tan^{-1}(r)] \quad (A-4)$$

$$+ i(-0.5c_1 \sin \phi_1 + 0.5c_1 A_0 \sin [\phi_1 + 0.5 \tan^{-1}(r)]) ,$$

with

$$A_0 = [(1 + 4c_2 \cos \phi/c_1^2)^2 + 16c_2^2 \sin^2 \phi/c_1^4]^{1/4}$$

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and

$$r = 4c_2 \sin \phi / (c_1^2 + 4c_2 \cos \phi) .$$

From (A-4) we verify that

$$R_g(\phi_1 - \pi/2) = X_g(\phi_1) ,$$

which is the identical phase-following behavior of

$$Z_g = |Z_g| \{ \cos [\phi_1 + \delta(\phi_2)] + i \sin [\phi_1 + \delta(\phi_2)] \} . \quad (A-5)$$

At $\phi_1 = 0$ we can determine the functional form $\delta(\phi_2)$ of the phase shift δ , if we require that

$$\delta + \phi_1 = 0, \quad X_g = 0 . \quad (A-6)$$

From (A-6) and (A-3) we find immediately

$$\delta = -0.5\phi_2 . \quad (A-7)$$

Furthermore, with $\cos(\phi) = 1$ (where $\phi = 0$) we have from (A-2), just

$$|Z_g| = 0.5c_1 [-1 + (1 + 4c_2/c_1^2)^{1/2}] , \quad (A-8)$$

APPENDIX A

which, together with equations (A-5) and (A-7), yields the general solution (3) given in the text.

If $\epsilon_2/\epsilon_1 \gg 1$, $\phi_1 = \phi_2 = \pi/2$, and $c_2 = \omega\mu_0/w^2\sigma_0$ (where $c_2/c_1 \gg 1$), we see that equation (3) gives (with negligible error) just

$$wZ_g + (\pi f \mu_0 / \sigma_0)^{1/2} (1 + i) ;$$

this is the familiar textbook one-dimensional result which can also be obtained (to the same approximation) by simply dropping γ^2 in equation (21), given that $\epsilon_2/\epsilon_1 \gg 1$.

APPENDIX B.--DERIVATION OF EQUATION (52)

APPENDIX B

In the body of the text we showed that the electromagnetic fields in the earth are inhomogeneous plane waves with wave vector $\vec{k} = \beta_0 \vec{m} - i\alpha_0 \vec{n}$. The square $\vec{k}^2$ factors to $\vec{k}^2 = \mu_0 \omega^2 \epsilon \Omega_u$, where Ω_u is the unsigned Ω such that $\Omega = |\Omega_u|$. The imaginary part of the $\vec{k}^2$ dispersion relation is $-2\alpha_0 \beta_0 \vec{m} \cdot \vec{n} i = \mu_0 \omega^2 (-i\epsilon_2) \Omega_u$; this requires that $\Omega_u \rightarrow \Omega$ since, for example, $\beta_0^2 < \alpha_0^2$ and $\Omega_u < 0$, because $A < B$ in the metal limit where $\alpha \approx \beta \approx 0$. We showed that $\vec{k}^2 = \mu_0 \omega^2 \epsilon \Omega$ has the correct solution because $n_1^2 = 1$ in $\vec{k}^2 = k_1^2 n_1^2$; this results in the other half of the dispersion relation solution being $\beta_0^2 - \alpha_0^2 = \mu_0 \omega^2 \epsilon_1 \Omega$. If we take the absolute value of $\beta_0^2 - \alpha_0^2$, this last equality allows $\Omega_u \rightarrow \Omega$ to have the same consequence as the real part of $\vec{k}^2 = \mu_0 \omega^2 \Omega_u$ taken as an absolute-value equality.

In order to solve explicitly for Ω , however, it is necessary to observe that equation (52) holds; also, equation (50) follows from the Poynting divergence relation.

To derive equation (52) we note that the factorization of Ω_u in $\vec{k}^2$ changes no physical result. Thus we must first factor out Ω_u^{-1} from $\vec{k}^2$ in the equation of squared amplitudes that is formed from the squared Maxwell curl $\vec{E}$ equation, namely

$$\vec{H}^2 = \frac{\Omega_u^{-1} \vec{k}^2 \vec{E}^2}{\mu_0^2 \omega^2}, \quad (B-1)$$

APPENDIX B

so that $\Omega_u^{-1}(\Omega_u \epsilon \mu_0 \omega^2)$ in equation (B-1) gives

$$\vec{H}^2 = \frac{\epsilon \vec{E}^2}{\mu_0} . \quad (B-2)$$

Equation (B-2) also results from the squared Maxwell curl $\vec{H}$ equation with factored $\Omega_u^{-1} k^2$ again substituted for k^2 . In the homogeneous wave equation solutions for $\vec{E}$ and $\vec{H}$, the vanishing factor $k^2 - \mu_0 \epsilon \omega^2$ becomes $\Omega_u^{-1} k^2 - \mu_0 \epsilon \omega^2 = 0$ if $k^2 = \mu_0 \epsilon \omega^2 \Omega_u$. It is then true that

$$|\vec{E}^2| = \Omega |\vec{H}^2| , \quad (B-3)$$

since substituting equation (B-3) into the absolute value of either equation (B-1) or equation (B-2) yields the same factorization-independent result, namely,

$$\frac{|\vec{E}^2|}{|\vec{H}^2|} = \frac{(\omega \mu_0)^2}{|k^2|} ,$$

which is the first equality in equation (52); $|\vec{H}^2| = |\vec{H}|^2$.

From the two Maxwell equations and equation (B-3) we also find

$$\Omega^{-1}(\mu_0^2 \omega^2 |\vec{H}^2| / \Omega^{-1} |k^2|) = |k|^2 |\vec{H}|^2 / \omega^2 |\epsilon|^2 ,$$

APPENDIX B

in which the curl $\vec{H}$ equation (curl $\vec{E}$ equation) is used in the right (left) side of the last equation. Thus, the consequence is

$$|\vec{k}|^2 |\vec{k}|^2 = \omega^4 |\epsilon|^2 \mu_0^2, \quad (B-4)$$

which is truly independent of Ω from the vector norms of equation (58).

We then use the $\vec{n}_r, \vec{n}_I$ formalism to evaluate (B-4), i.e.,

$$\vec{k} = \vec{k}_r + i\vec{k}_I,$$

$$\vec{k}_r = \mu_0^{1/2} \omega (\vec{n} \vec{n}_r - \kappa \vec{n}_I),$$

$$\vec{k}_I = \mu_0^{1/2} \omega (\kappa \vec{n}_r + \vec{n} \vec{n}_I),$$

where $\vec{n}_r$ and $\vec{n}_I$ are equations (46) and (47), respectively. We then find

$$\vec{n}_r^2 + \vec{n}_I^2 = \Omega^{-2}. \quad (B-5)$$

Further evaluating equation (B-5), we find

$$1 = (A^2 + B^2 + \beta^2 + \alpha^2)(n^2 + \kappa^2)/\mu_0 \omega^2 |\epsilon|^2. \quad (B-6)$$

This result proves, in fact, our assertion (52), since the norm is unique to equations (B-6) and (58). We saw from the beginning, in the

APPENDIX B

complex index of refraction development, that Ω cancels out formally, giving an algebraic identity between $\vec{k}$ forms.

The point is that equation (B-5) must evidently be true independently; therefore Ω^{-2} factors out in (B-5) leaving the remaining explicit Ω dependence entirely in $n^2 + \kappa^2 = |\epsilon|\Omega$ in equation (B-7). The squared $\vec{E}$ norm relation above shows that $\Omega = 1$ can only be true if $\vec{E}$ has one vector component.

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