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SOLID BODY EQUATIONS TO CALCULATE THE TRAJECTORY OF RAMJET

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Six-degree-of-freedom trajectory equations for a ramjet propelled, gun launched projectile are formulated. An outline for FORTRAN computer program flow charts also appear in the report. Special emphasis is given to the effect of wind on trajectory errors.

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Notation

C_D : Drag Coefficient

C_{L_α} : Lift differential coefficient

C_{m_α} : Static moment differential coefficient

C_ε : Resultant force of perturbing influences

d : Diameter

F : Force

g : Acceleration of gravity

I : Moment of inertia around lateral axis of projectile

I_x : Moment of inertia around longitudinal axis of projectile

m : Mass

M : Moment

M_ε : Resultant moment of perturbing influences

p : Angular velocity around x_B axis

q : Angular velocity around y_B axis

r : Angular velocity around z_B axis

r_t : Distance between center of gravity and nozzle throat

v : Velocity

w : Wind velocity

x,y,z : Ground axis coordinates

α : Angle of attack

β : Side slip angle

θ,δ : Angles defined in Chapt. 3 (2)

ϕ,ξ : Angles defined in Chapt. 3 (3)

η : Angle rotated around x_B axis

μ	:	Factor angle of trim of canard
ν	:	Trim angle of canard
π	:	Factor angle caused by attitude of projectile
ρ	:	Density of air

Suffix :

a	:	Wind axis coordinate
B	:	Body axis coordinate
v	:	Velocity axis coordinate
x,y,z	:	Rectangular axis

1. INTRODUCTION

Solid body equations to calculate the trajectory of a projectile which is flying with thrust of ramjet under hypersonic Mach number are formulated. The projectile is gun launched. This projectile has an axi-symmetric configuration and is fin stabilized with four pop-out wings. The projectile has canards to control attitude. Ideally the projectile has no spin; however, due to effects from miscellaneous forces and moments which work on the body, it may acquire spin motion.

In order to calculate the trajectory considering such conditions, solid body equations with 6 degrees of freedom are induced. One of the main problems which will occur during flight motion of the projectile is trajectory errors which are caused by numerous origins especially due to the wind.

Total range of these projectiles are rather great. Consequently, perturbing influences from these origins are important to notice when long range trajectories are calculated. Some of the primary origins of error are mass unbalance due to impreciseness of construction and from shock of launching, misalignment of jet, misalignment of intake air flow, and so on.[1]

In order to minimize these effects, it is useful to set the value of thrust of the ramjet equal and opposite to that of aerodynamic drag. The trajectory is equivalent to projectile flight in a vacuum and is known as thrust-equal-drag trajectory. The effect of wind, when the direction of wind is the same as that of the trajectory, can be readily calculated. In case of a side wind, the projectile will be carried laterally.

In order to cancel the effect of a side wind which will carry the projectile laterally and to restore the planar motion of the trajectory, active control can be used. For example, canards can be employed. Due to active aerodynamic control, total range will be affected. Active control modulates aerodynamic drag. The boundary and initial conditions will influence whether total range will be expanded or shortened by the effect of canards.

In this paper, effects due to winds are considered; however, perturbations of other origins are concentrated in one term in the equations of motion as constants. In order to calculate these data by electronic computer, flow diagrams for FORTRAN are indicated.

2. Coordinate Systems

In order to construct solid body equations, coordinate systems are considered as follows:[2]

(1) Ground axis coordinate system (0-xyz).

The origin is on the center of gravity of the projectile at the time of departure of projectile. The x axis is toward the direction of horizontal line of shot. The y axis is vertically upright, and the z axis is horizontally rectangular in a right hand sense.

(2) Velocity axis coordinate system ($0\text{-}x_v y_v z_v$).

The origin is at the center of gravity, CG, of the projectile. The x_v axis is along the velocity vector of the projectile, i.e., x_v is tangent to the trajectory. The transformation to create the (x_v, y_v, z_v) coordinate system from the (x,y,z) coordinate system is, first, a rotation about the z axis by an angle θ. A new coordinate system is formed and is identified as the (x_1, y_1, z) coordinate system.

The velocity vector, $\vec{v}$ and x_v are located in the x_1-z plane. Now rotate the (x_1, y_1, z) coordinate system about the y_1 axis until the x_1 axis is parallel with $\vec{v}$. The angle of rotation about the y_1 axis is σ. The resulting coordinate system is the (x_v, y_v, z_v) system. Refer to Figure 1 for an illustration of the axes.

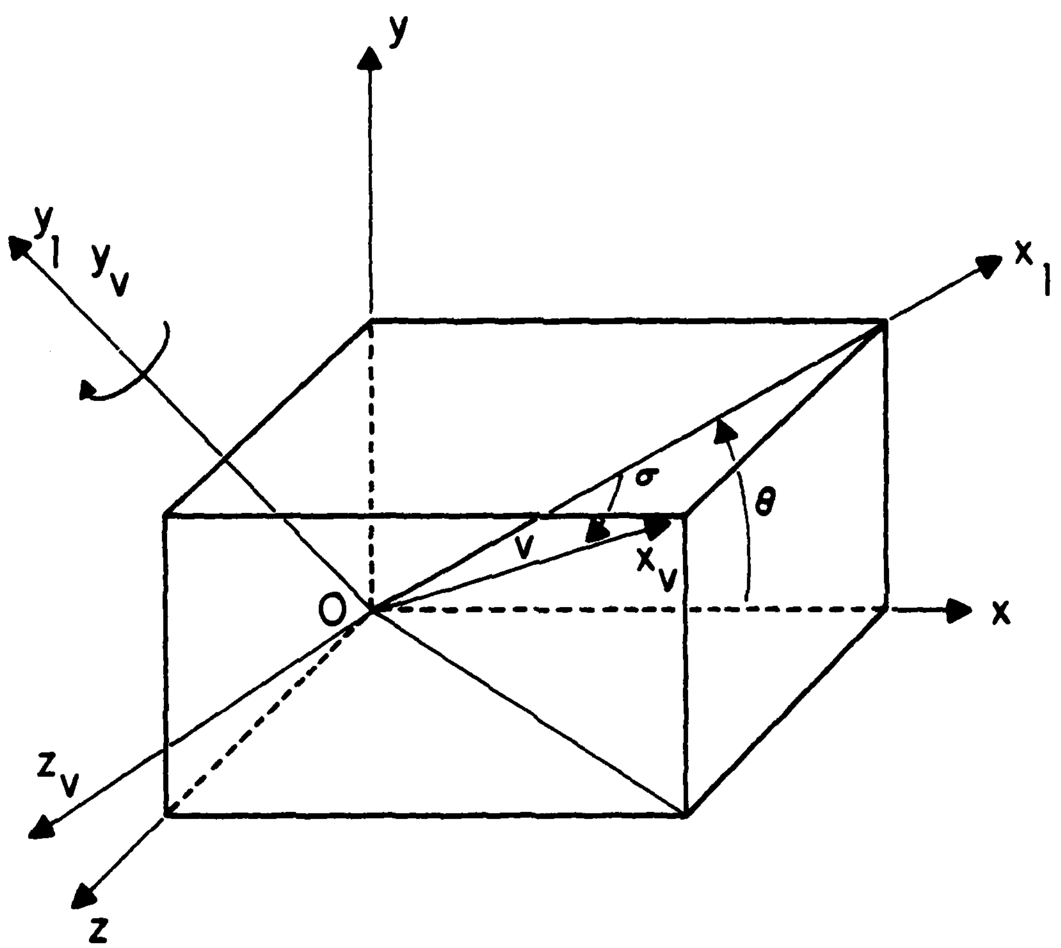


Figure 1. Illustration of geometry for the velocity axis coordinate system.

(3) Nonrotating body axis coordinate system $(0 - x_B y_B z_B)$.

The origin is at the center of gravity of projectile: the x_B axis coincides with the longitudinal axis of symmetry. The y_B and z_B axis are transformed as the same manner as that of velocity axis coordinate system by ϕ instead of θ, and by ξ instead of σ.

(4) Wind axis coordinate system $(0 - x_{va} y_{va} z_{va})$. The system moves with relative wind speed similar to velocity axis coordinate system. The relevant angles are (θ_a, σ_a) instead of (θ, σ) with the x_{va} axis along the relative velocity vector.

3. Equations

Solid body equations with six degrees of freedom are developed by the method of coordinate transformation in order to calculate the trajectory of the projectile with ramjet propulsion as follows:

$$\dot{x} = v \cos\theta \cos\sigma \tag{1}$$

$$\dot{y} = v \sin\theta \cos\sigma \tag{2}$$

$$\dot{z} = -v \sin\sigma \tag{3}$$

$$\dot{v} = \frac{1}{m} F_{xv} \tag{4}$$

$$\dot{\theta} = \frac{1}{mv \cos\sigma} F_{yv} \tag{5}$$

$$\dot{\sigma} = -\frac{1}{mv} F_{zv} \tag{6}$$

$$\dot{\phi} = r / \cos\xi \tag{7}$$

$$\dot{\xi} = q \tag{8}$$

$$\dot{\eta} = r \tan\xi \tag{9}$$

$$\dot{b} = M_{xB}/I_x \tag{10}$$

$$\dot{q} = (1 - I_x/I)pr + M_{yB}/I \tag{11}$$

$$\dot{r} = -(1 - I_x/I)pq + M_{zB}/I \tag{12}$$

$$F_{xv} = -mg \sin\theta \cos\sigma + T\cos(\varphi - \theta) - \tfrac{1}{2}\rho v^2 SC_D \qquad (13)$$

$$F_{yv} = -mg \cos\theta - T\alpha + \tfrac{1}{2}\rho v^2 SC_{L_\alpha}\alpha + C_\varepsilon \cos\pi \qquad (14)$$

$$F_{zv} = -mg \sin\theta \sin\sigma - T\beta + \tfrac{1}{2}\rho v^2 SC_{L_\alpha}\beta + C_\varepsilon \sin\pi \qquad (15)$$

$$M_{xB} = 0 \qquad (16)$$

$$M_{yB} = \tfrac{1}{2}\rho v^2 SdC_{m_\alpha}\beta - Tr_t\beta + M_\varepsilon \cos\pi \qquad (17)$$

$$M_{zB} = \tfrac{1}{2}\rho v^2 SdC_{m_\alpha}\alpha - Tr_t\alpha + M_\varepsilon \sin\pi \qquad (18)$$

$$\alpha = \tan^{-1}\frac{\cos\xi \sin(\varphi-\theta)}{\cos\sigma\cos\xi\cos(\varphi-\theta) + \sin\sigma\sin\xi} \qquad (19)$$

$$\beta = \tan^{-1}\frac{\sin\sigma\cos\xi\cos(\varphi-\theta) - \cos\sigma\sin\xi}{\cos\sigma\cos\xi\cos(\varphi-\theta) + \sin\sigma\sin\xi} \qquad (20)$$

For a projectile with ramjet propulsion, eq. (16) is true usually. At launch, $\dot{p} = 0$, and there is no initial spin so that $p = 0$. For case of $p = 0$, equations (11) and (12) become

$$\dot{q} = M_{yB}/I \qquad (11)'$$

$$\dot{r} = M_{zB}/I \qquad (12)'$$

Furthermore, if thrust has a value near to drag, then dispersion becomes a minimum so that the last terms of equations (14), (15), (17) and (18) are deleted from these equations.

Effect of Wind

When a wind blows, a wind axis coordinate system is considered. Using these coordinates, the x axis is along the relative velocity between projectile and air. The motion of the projectile can be calculated the same way as in the case of no wind. For this case, we use a subscript (a) which is applied to the notations of the case of no wind.

In order to obtain velocities which were observed from the ground, we must add distances transported by wind during the time interval to values calculated on the wind axis coordinate. The relations between velocities on wind axis coordinate and velocity axis coordinate are as

$$\begin{cases} v_a \cos\theta_a \cos\sigma_a = v\cos\theta\cos\sigma - w_x & (21) \\ v_a \sin\theta_a \cos\sigma_a = v\sin\theta\cos\sigma - w_y & (22) \\ v_a \sin\gamma_a = v\sin\sigma + w_z & (23) \end{cases}$$

From these equations, we can calculate v_a, θ_a, σ_a knowing data without wind, v, θ, σ and the component of wind relative to the ground $w_x w_y w_z$.

In the case of computer calculation by a step-by-step method we can use (21), (22), (23) to get initial value of v, θ, σ in each successive step. These values of v, θ, σ are relative to the ground axis coordinates, so that the trajectory is obtained from them.

4. Effect of Canard

From mentioned above, it is clear that projectile is not within the plane of launching but flies in a different direction from the first schedule when wind is blowing. In order to cancel the miss distance, we can use a canard giving some suitable trim angle. Trim angle will be given on body axis coordinate and whose effect will act as sinusoidal form such as ν sin μ where μ is factor angle measured from y_B axis right wise around x_B axis. In order to cancel the effect of sidewise dispersion $\dot{z} = 0$ so $\sigma = 0$, $\dot{\sigma} = 0$ must be satisfied.

therefore

$$F_{zv} = 0 \qquad (24)$$

In order to demand (24) we must add a term for the canard on the right terms of eq. (15)

$$-mg\sin\theta\sin\sigma - T\beta + \tfrac{1}{2}\rho v^2 S C_{L_\alpha}\beta + C_\varepsilon \sin\pi = \nu\sin\mu \qquad (25)$$

There are many factors to give effect to the left side terms of this equation, so it is difficult to discuss analytically. However, the value of the terms are calculated on each step so that we can decide a suitable value of canard angle for each step of the calculation.

5. Program Flowcharts

In order to calculate these equations, a computer program will be built.[3] It is illustrated by program flowcharts. This computer program consists of a main program and subroutines.

Inputs which are used for calculation are ready and stored in increment address by orders in the program to use for calculation. Data calculated are provided printed on sheets and so on. Program flowcharts of main program and subroutines are shown on figures, and they are explained as follows:

a. Main Program

Main program has a job to store and supply data from increment address of computer and to make initial condition in shape suitable for calculation. It gives these data of subroutines to compute and to get results of calculation. Data returned from subroutine are printed periodically. Calculation is carried by one step of variables.

These data are accumulated step-by-step until they come to some objective point. When the objective point is satisfied, the calculation is stopped. The point where the projectile touched the ground, point of print and points to change initial conditions are detected and ordered to computer. It contains FORMAT to print these results.

b. Subroutine COMPUTE

In order to calculate eq. (1) ∿ (12) there need data of eq. (13) ∿ (20) which are in the state suitable for input. In this subroutine these data are prepared using three subroutines of ATOM, FORCE and MOMEN. These data are sent to subroutine RKG. There eq. (1) ∿ (12)

are calculated and results are sent back to this routine. Additional data are calculated here. These data are sent back to main program.

t. Subroutine RKG

This subroutine has a job to calculate each of the differential equations in a Runga-Kutta-Gill method. Suitable data for input are sent from subroutine COMPUTE. This subroutine calls differential equations one by one from subroutine DANDO.

d. Subroutine DANDO

This subroutine stores each equations whwch composes the trajectory equations in state of FORTRAN form.

e. ATOM

This subroutine provides air temperature and air density for the standard state of heights from sea level. Air temperature is used to calculate Mach number.

f. FORCE

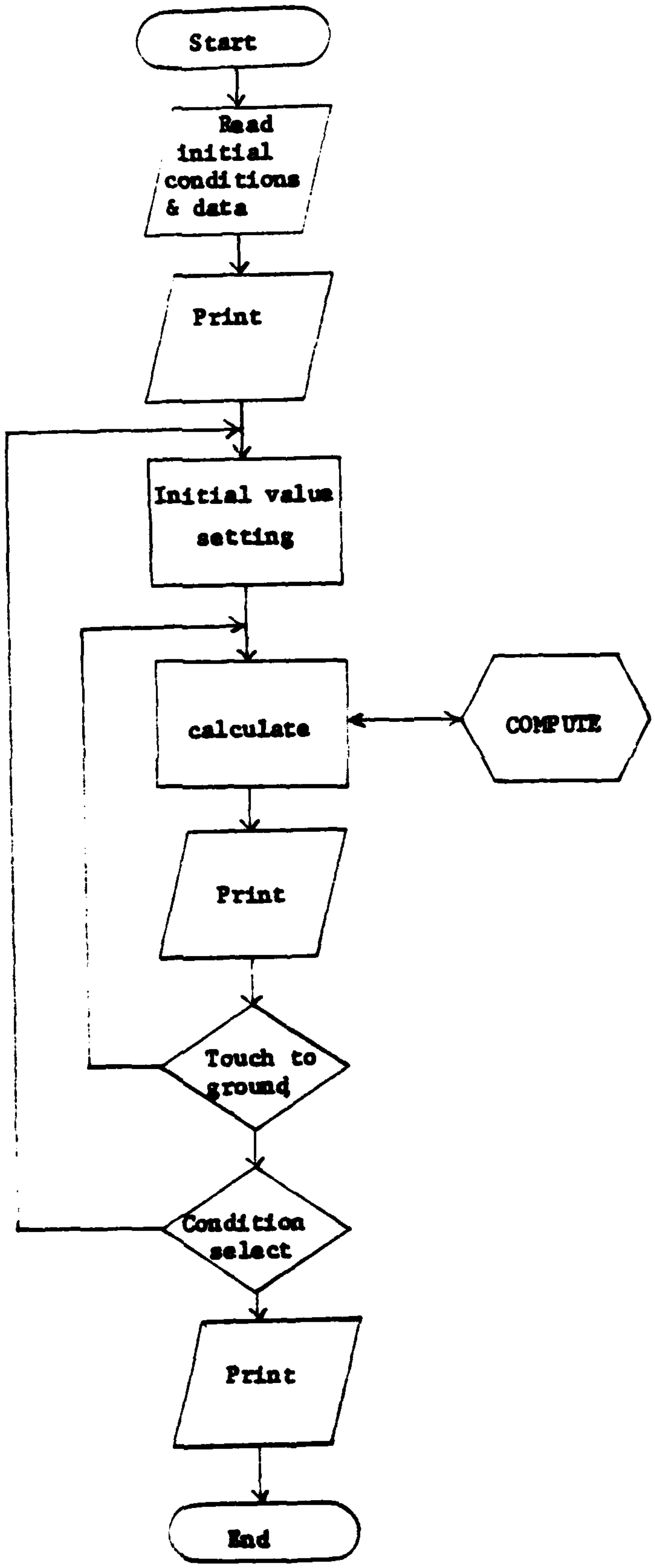
Data of forces are stored in increment address. On this subroutine forces that are arranged suitably to the state of input of the instance are selected and prepared for supply. This subroutine has a function to make linear interpolation.

g. MOMEN

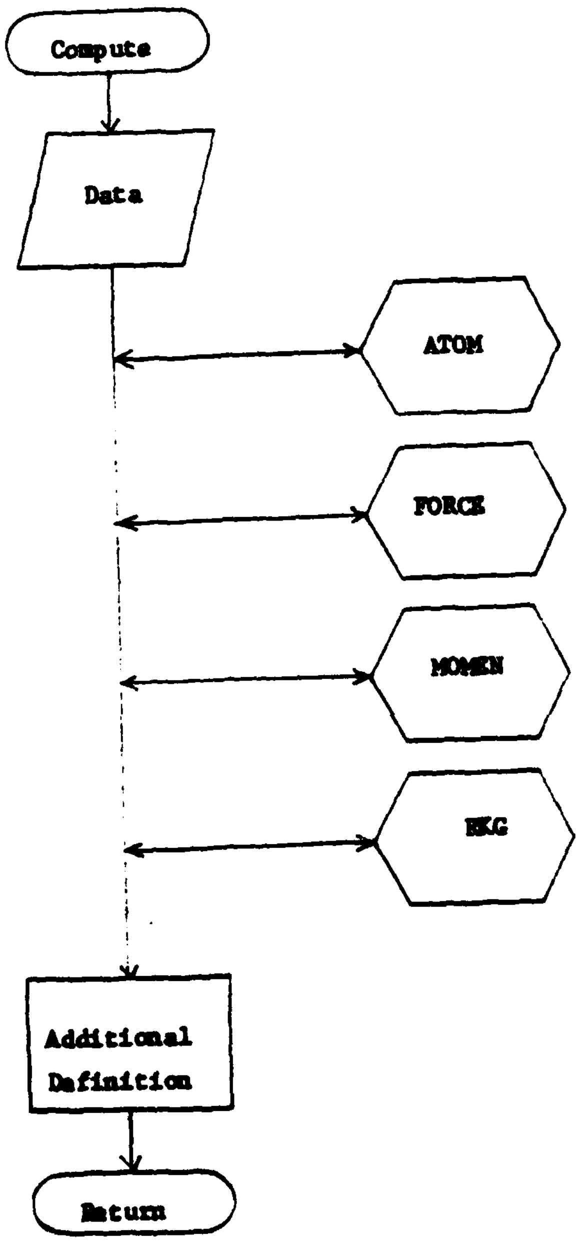
Data of moments are stored in increment address. Moments that are arranged suitably to the state of input of the instance are selected and prepared to supply. This subroutine has a function to make linear interpolation.

Program Flow Charts

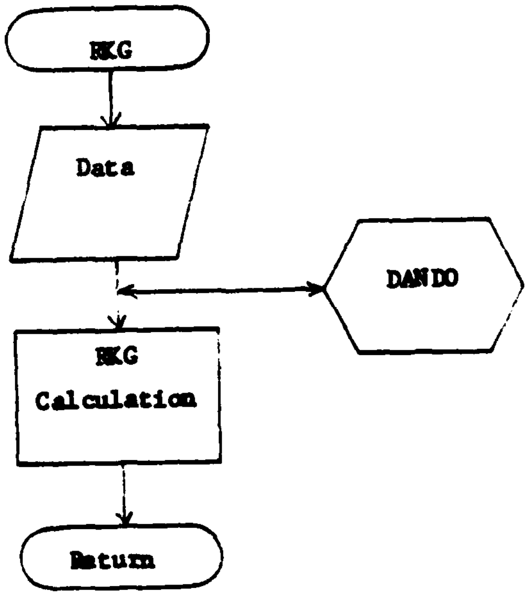
(a) Main Program



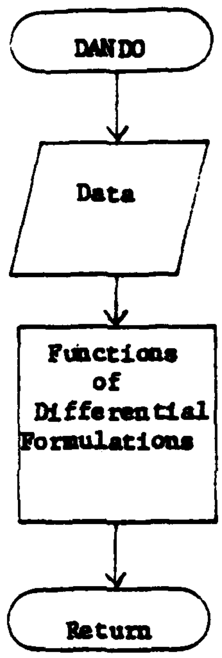
(b) Subroutine COMPUTE



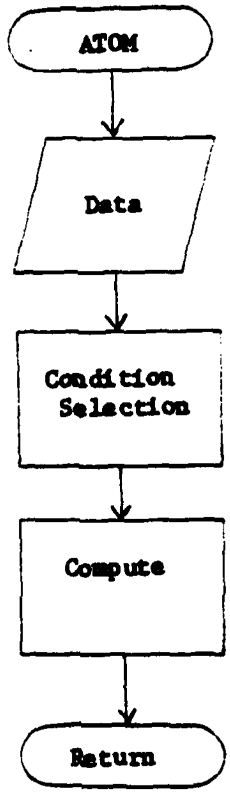
(C) Subroutine HKG



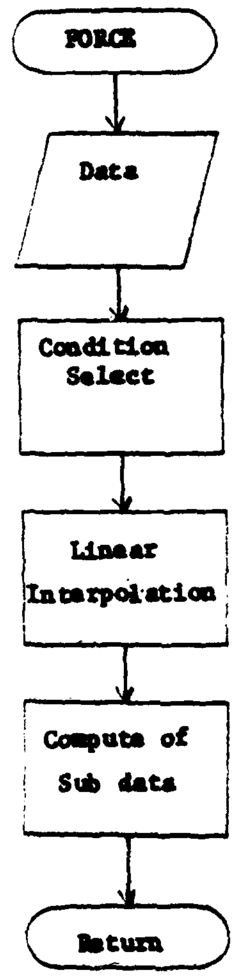
(d) Subroutine DANDO



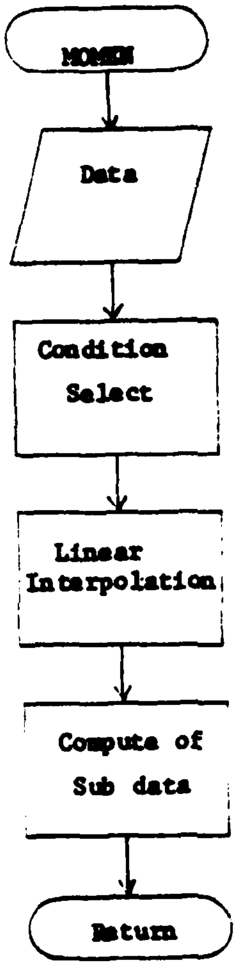
(e) Subroutine ATOM



(f) Subroutine FORCE



(g) Subroutine MOMEN



6. Conclusions

Solid body equations with 6-degree-of-freedom are formulated in order to calculate the trajectory of a projectile which is flying with the thrust of a ramjet.

In order to calculate by electronic digital computer; program flow charts are developed. FORTRAN programs will be made for each subroutine and program. Trajectory errors due to the effect of the wind are considered mainly. Dispersion due to the wind will be calculated by these programs. If there need to cancel the effect of a side wind, trim angle of canard will be given step by step in the procedure of calculation.

7. Acknowledgment

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