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NONLOCAL EFFECTS OF CRACK CURVING

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Princeton University

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NONLOCAL MECHANICS OF CRACK CURVING*

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ABSTRACT

The nonlocal elasticity solution is obtained for the Griffith crack problem under combined loadings, Modes I and II. It is shown that the fracture begins at a critical point in the neighborhood of the crack tip. The location of the critical point is determined and the critical angle at which the crack begins to deviate from its straight line path is determined. These results are in good agreement with atomic lattice theory, and classical predictions.

INTRODUCTION

Presently, there exists several criteria for the static and dynamic curving of a line crack under combined loadings. These are based on either a maximum circumferential stress (cf. Erdogan and Sih [1], Cotterell and Rice [2]), or the Griffith energy release rate (cf. Hussain, et al [3]). Experimental assessment of these criteria was carried out by Kobayashi, et al, in a series of papers of which we mention [4]. Because of the usual crack tip singularity, these criteria are to be applied at a critical distance r_c from the crack tip. It is speculated that r_c is a material property.

The main purpose of the present paper is: (i) to determine r_c, theoretically; (ii) to obtain the direction of crack curving under combined loadings, Modes I and II; (iii) to give a crack curving criterion based on the maximum circumferential stress field.

We note that such a program cannot be carried out by means of classical elasticity theory since according to this theory, the maximum stress is infinite and it is located at the crack tip.

In several previous papers (cf. [5], [6], [7]), it was shown that the nonlocal elasticity solutions of crack problems do not contain a stress singularity. Moreover, the maximum stress occurs not at the tip, but at an exterior point to the crack surfaces — in the close vicinity of the crack tip.

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For brittle solids, a fracture criterion based on the maximum tensile stress, was established (cf. [5], [6], [8]). Accordingly, when the maximum tensile stress exceeds the cohesive stress that holds bonds together, fracture will occur. Calculations based on this hypothesis proved to be in accordance with the Griffith fracture criterion with the additional dividend that the Griffith constant is fully determined. Cohesive stress calculations showed excellent agreement with the results known to metalurgists (see [5] to [10]).

Motivated by these findings, we proceed to examine here the problem of crack instability and curving for the plane problem under combined loads.

RESUME OF BASIC EQUATIONS

Basic equations of nonlocal, linear, homogeneous, isotropic elastic solids consist of (cf. [5], [11], [12]):

$$t_{k\ell,k} + \rho(f_\ell - \ddot{u}_\ell) = 0 \tag{1}$$

$$t_{k\ell}(\underset{\sim}{x},t) = \int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|,\varepsilon)\,\sigma_{k\ell}(\underset{\sim}{x}',t)\,dv(\underset{\sim}{x}') \,, \tag{2}$$

$$\sigma_{k\ell} = \lambda\, e_{rr}\,\delta_{k\ell} + 2\mu\, e_{k\ell} \,, \tag{3}$$

$$e_{k\ell} = \frac{1}{2}(u_{k,\ell} + u_{\ell,k}) \tag{4}$$

where $t_{k\ell}$, ρ, f_ℓ and u_ℓ are respectively, the stress tensor, mass density, body force and the displacement field. We employ a superposed dot to indicate partial derivative with respect to time and a comma to indicate partial derivative with respect to rectangular coordinates x_k, i.e.

$$\dot{u}_k = \frac{\partial u_k(\underset{\sim}{x},t)}{\partial t} \,, \qquad u_{k,\ell} = \frac{\partial u_k}{\partial x_\ell}$$

As usual, repeated indices indicate summation.

Except for the constitutive equations (2), these equations are identical to those of the classical (local) elasticity theory. Equation (2) replaces the classical Hooke's law. It expresses the physical fact that the stress at a point $\underset{\sim}{x}$ depends on strains at <u>all</u> other points $\underset{\sim}{x}'$ in the body. Naturally, the influence of strain at $\underset{\sim}{x}$ is expected to be greatest in the stress at $\underset{\sim}{x}$ and the influence of strains at any other point $\underset{\sim}{x}'$, at a distance $|\underset{\sim}{x}'-\underset{\sim}{x}|$ from $\underset{\sim}{x}$, must diminish with distance. Hence the nonlocal modulus $\alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|,\varepsilon)$ must die out with $|\underset{\sim}{x}'-\underset{\sim}{x}|$ having a maximum at $\underset{\sim}{x}' = \underset{\sim}{x}$.

The nonlocal attenuation modulus α has the dimension of length^{-3} so that it depends on an internal characteristic length a. This is indicated by $\varepsilon = \beta a/\ell$ where ℓ is an external characteristic length (e.g. wave length, crack length), a is an internal characteristic length (e.g., lattice parameter, granular distance) and β is a non-dimensional constant appropriate to each material.

Nonlocal elasticity reduces to the classical theory in the limit

$$(5) \qquad \lim_{\varepsilon\to 0} \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|,\varepsilon) = \delta(|\underset{\sim}{x}'-\underset{\sim}{x}|)$$

where δ is the Dirac delta measure. Based on these and other considerations, Eringen [11], [12], proposed that α must be a Dirac delta sequence, and obtained several kernels which produce excellent agreement with the dispersion curves of plane waves in lattice dynamics, in the entire Brillouin zone.

Here we give one of these kernels, suitable for the treatment of two-dimensional problems, Ari and Eringen [7]:

$$(6) \qquad \alpha(|\underset{\sim}{x}|,\varepsilon) = (2\pi\ell^2\varepsilon^2)^{-1} K_0(\sqrt{\underset{\sim}{x}\cdot\underset{\sim}{x}}/\ell\varepsilon), \qquad \varepsilon = \beta a/\ell$$

where K_0 is the modified Bessel's function. We note that equation (6) satisfies the differential equation

$$(7) \qquad (1 - \varepsilon^2\ell^2\nabla^2)\alpha = \delta(|\underset{\sim}{x}'-\underset{\sim}{x}|)$$

a property which is useful in the treatment of boundary-value problems. If we apply the operator (7) to equation (2), we obtain

$$(8) \qquad (1 - \varepsilon^2\ell^2\nabla^2)\, t_{k\ell} = \sigma_{k\ell}$$

From this, by taking the divergence of both sides, it follows that

$$(9) \qquad \sigma_{k\ell,k} + (1 - \varepsilon^2\ell^2\nabla^2)(\rho f_\ell - \rho\ddot{u}_\ell) = 0$$

where we have used equation (1). If we further substitute from equations (3) and (4), we will have

$$(10)\qquad (\lambda+\mu)\, u_{k,\ell k} - \mu \nabla^2 u_\ell + (1-\epsilon^2 \ell^2 \nabla^2)(\rho \ddot{u}_\ell - \rho f_\ell) = 0$$

valid in rectangular coordinates. In this way, the integro-differential equations (1) to (4) are reduced to singularly perturbed partial differential equations.

Particularly simple results are obtained for the *static case* and vanishing body forces. In this case, we have

$$(11)\qquad \sigma_{k\ell,k} = 0$$

which is identical to the equilibrium equations of classical elasticity theory.

GRIFFITH CRACK, MODE I AND II

A line crack $|x_1| < c\,,\ x_2 = 0$ located in an infinite plane subject to a uniform tensile loading, perpendicular to the crack line at infinity, is known as the Griffith crack problem, Mode I. We consider the superposition to Mode I a constant shear loading which is known as the Mode II. The solution of both problems in nonlocal elasticity were given previously by Eringen [8] and his coworkers [6]. Here, we employ an alternative method of solution using a different kernel, namely equation (6).

For the static case with vanishing body forces, equation (10) reduces to the classical Navier's equation

$$(12)\qquad (\lambda + \mu)\, u_{k,\ell k} + \mu \nabla^2 u_\ell = 0$$

whose solution is well-known, c.f., [13]. The classical stress field $\sigma_{k\ell}$ in the neighborhood of the crack tip, is of the form

$$(13)\qquad \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \begin{bmatrix} \frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{5\theta}{2} \\ \frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{5\theta}{2} \\ -\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{5\theta}{2} \end{bmatrix} + \frac{K_{II}}{\sqrt{2\pi r}} \begin{bmatrix} -\frac{7}{4}\sin\frac{\theta}{2} - \frac{1}{4}\sin\frac{5\theta}{2} \\ -\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{5\theta}{2} \\ \frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{5\theta}{2} \end{bmatrix}$$

where K_I and K_{II} are classical stress intensity factors and (r,θ) are plane polar coordinates with the origin at the right crack tip.

In nonlocal theory, $\sigma_{k\ell}$ given by equation (13) is not the stress field. The stress field $t_{k\ell}$ is obtained by solving equation (8), subject to regularity conditions, i.e., $t_{k\ell}$ must be bounded at the crack tip and at infinity. This is borne out also from the previous solution given in [5] and [7].

We expect that, at large distances from the crack tip, the classical solution will approximate the stress field well. Moreover, as $\varepsilon \to 0$, equation (8) gives $t_{k\ell} \to \sigma_{k\ell}$. Therefore, there exists a boundary layer in the neighborhood of the crack. This then suggests that we may obtain an inner solution of equation (8) and match it to the outer solution $\sigma_{k\ell}$. In fact, this is why the approximate expressions (equation (13)), which are valid in the vicinity of the crack tip, are adequate for the determination of $t_{k\ell}$ in the vicinity of the crack tip.

Introducing the complex stress field for any second-order symmetric tensor, $\tau_{k\ell}$ by

$$\text{(14)} \qquad \Theta_{\underset{\sim}{\tau}} = \tau_{11} + \tau_{22} , \qquad \Phi_{\underset{\sim}{\tau}} = \tau_{22} - \tau_{11} + 2i\, \tau_{12}$$

The differential equations (8) may be replaced by equivalent equations

$$\text{(15)} \qquad \begin{aligned} (1 - \varepsilon^2 \ell^2 \nabla^2)\, \Theta_{\underset{\sim}{t}} &= \Theta_{\underset{\sim}{\sigma}} , \\ (1 - \varepsilon^2 \ell^2 \nabla^2)\, \Phi_{\underset{\sim}{t}} &= \Phi_{\underset{\sim}{\sigma}} \end{aligned}$$

where, by using equations (14 and (13),

$$\text{(16)} \qquad \begin{aligned} \Theta_{\underset{\sim}{\sigma}} &= \frac{1}{\sqrt{2\pi r}} [(K_I + i\, K_{II})\, e^{i\theta/2} + (K_I - i\, K_{II})\, e^{-i\theta/2}] , \\ \Phi_{\underset{\sim}{\sigma}} &= \frac{1}{2\sqrt{2\pi r}} [(K_I + 3i\, K_{II})\, e^{-i\theta/2} + (- K_I + i\, K_{II})\, e^{-i5\theta/2}] \end{aligned}$$

Consequently, the integration of equation (15) requires finding the solution of a differential equation of the form

$$\text{(17)} \qquad \frac{\partial^2 g_n}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial g_n}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 g_n}{\partial \theta^2} - g_n = - \rho^{-\frac{1}{2}}\, e^{in\theta/2} , \qquad n = \pm 1, \pm 5$$

where

$$g_n(\rho,\theta) = f_n(\rho)\, e^{in\theta/2} \qquad \rho = r/\varepsilon\ell \tag{18}$$

The general solution of equation (17) is

$$f_n(\rho) = A\, I_{n/2}(\rho) + B\, K_{n/2}(\rho) + \int^{\rho} [I_{n/2}(z)\, K_{n/2}(\rho) - I_{n/2}(\rho)\, K_{n/2}(z)]\, z^{\frac{1}{2}}\, dz \tag{19}$$

where I_ν and K_ν are modified Bessel's functions. Constants of integrations A and B are determined by using the regularity conditions at $r=0$ and $r=\infty$, namely f_n must be bounded at $\rho=0$ and as $\rho\to\infty$.

$$f_n = \int_0^{\rho} I_{n/2}(z)\, K_{n/2}(\rho)\, z^{\frac{1}{2}}\, dz + \int_{\rho}^{\infty} I_{n/2}(\rho)\, K_{n/2}(z)\, z^{\frac{1}{2}}\, dz \qquad n = \pm 1,\ \pm 5 \tag{20}$$

Employing well-known expressions of $I_{n/2}$ *and* $K_{n/2}$ *[14], we find that*

$$\begin{aligned} f_{\pm 1} &= \rho^{-\frac{1}{2}}\,(1 - e^{-\rho})\,, \\ f_{\pm 5} &= \rho^{-\frac{1}{2}}\, e^{-\rho}\left(1 + \frac{3}{\rho} + \frac{3}{\rho^2}\right) \int_0^{\rho} \left[\left(1 + \frac{3}{z^2}\right)\sinh z - \frac{3}{z}\cosh z\right] dz \\ &\quad + \rho^{-\frac{1}{2}}\left[\left(1 + \frac{3}{\rho^2}\right)\sinh\rho - \frac{3}{\rho}\cosh\rho\right] \int_{\rho}^{\infty} e^{-z}\left(1 + \frac{3}{z} + \frac{3}{z^2}\right) dz \end{aligned} \tag{21}$$

Consequently,

$$\begin{aligned} \Theta_{\underset{\sim}{t}} &= (2\pi\varepsilon\ell)^{-\frac{1}{2}}\,[(K_I + iK_{II})\, e^{i\theta/2} + (K_I - iK_{II})\, e^{-i\theta/2}]\, f_1(\rho)\,, \\ \Phi_{\underset{\sim}{t}} &= \frac{1}{2}\,(2\pi\varepsilon\ell)^{-\frac{1}{2}}\,[(K_I + 3iK_{II})\, f_1(\rho)\, e^{-i\theta/2} + (-K_I + iK_{II})\, f_5(\rho)\, e^{-5i\theta/2}] \end{aligned} \tag{22}$$

But, in polar coordinates, we have

$$\begin{aligned} t_{rr} + t_{\theta\theta} &= \Theta_{\underset{\sim}{t}}\,, \\ t_{\theta\theta} - t_{rr} + 2i\, t_{r\theta} &= \Phi_{\underset{\sim}{t}}\, e^{2i\theta} \end{aligned} \tag{23}$$

from which we determine the stress field.

$$t_{rr} = (2\pi\varepsilon\ell)^{-\frac{1}{2}} \Big\{[K_I(\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}) + K_{II}(-\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2})]f_1(\rho) + \frac{1}{4}(K_I\cos\frac{\theta}{2} - K_{II}\sin\frac{\theta}{2})f_5(\rho)\Big\},$$

(24) $$t_{\theta\theta} = (2\pi\varepsilon\ell)^{-\frac{1}{2}} \Big\{[K_I(\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}) - K_{II}(\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2})]f_1(\rho) + \frac{1}{4}(-K_I\cos\frac{\theta}{2} + K_{II}\sin\frac{\theta}{2})f_5(\rho)\Big\},$$

$$t_{r\theta} = \frac{1}{4}(2\pi\varepsilon\ell)^{-\frac{1}{2}}[(K_I\sin\frac{3\theta}{2} + 3K_{II}\cos\frac{3\theta}{2})f_1(\rho) + (K_I\sin\frac{\theta}{2} + K_{II}\cos\frac{\theta}{2})f_5(\rho)]$$

These results are valid in the vicinity of the crack tip.

FRACTURE AND CRACK CURVING

Based on the physics of matter, the fracture must occur when the maximum tensile stress exceeds the cohesive stress which holds bonds together, Eringen [5,8]. Consequently, fracture will begin at a point (r_c, θ_c) which are the roots of

(25) $$\frac{\partial t_{\theta\theta}}{\partial r} = 0, \quad \frac{\partial t_{\theta\theta}}{\partial \theta} = 0$$

provided that $t_{\theta\theta}(r_c, \theta_c)$ is the maximum tensile stress.

Note that unlike classical elasticity, $t_{\theta\theta max}$ is not at the crack tip Thus, fracture begins ahead of the crack tip at some location, determined by equation (25).

First, consider the case of Mode I only. In this case, $K_{II} = 0$ and equation (25) gives

(26) $$\theta_c = 0, \quad 5\frac{df_1}{d\rho} = \frac{df_5}{d\rho}$$

so that the fracture is along the crack line at a point ρ_c satisfying (26). Computations give

$$\rho_c = r_c/\varepsilon\ell = 1.095076 \tag{27}$$

This result is in excellent agreement with that calculated by means of atomic lattice theory by Elliot [15] (see Table 1).

Next, we consider the combined Modes I and II. From equations (24) and (25), it is clear that

$$\rho_c = r_c/\varepsilon\ell = f(K_{II}/K_I) \tag{28}$$

$$\theta_c = g(K_{II}/K_I)$$

Following common practice, these functions are plotted against crack angle γ, defined by

$$\cot\gamma = K_{II}/K_I \tag{29}$$

In Figure 1, critical distance r_c is given as a function of the crack angle γ. We notice that r_c decreases with the crack angle. The closest distance to the crack tip is obtained when $K_I = 0$ and farthest when $K_{II} = 0$. The change between these two cases is almost a straight line. In Figure 2, values of θ_c are compared with classical results obtained from [13, p. 99]. The agreement is, in general, excellent.

TABLE I

HOOP STRESS ALONG THE CRACK LINE

	t_{max}/t_0	x_c/ℓ
Elliot [15]	27.62	$1 + 0.2(a/\ell)$
Nonlocal (Present) $\varepsilon\ell = 0.22a$*	25.41	$1 + 0.2409(a/\ell)$
$\varepsilon\ell = 0.31a$	21.40	$1 + 0.3394(a/\ell)$

*Values of $\varepsilon\ell/a$ are from Ref. [7].

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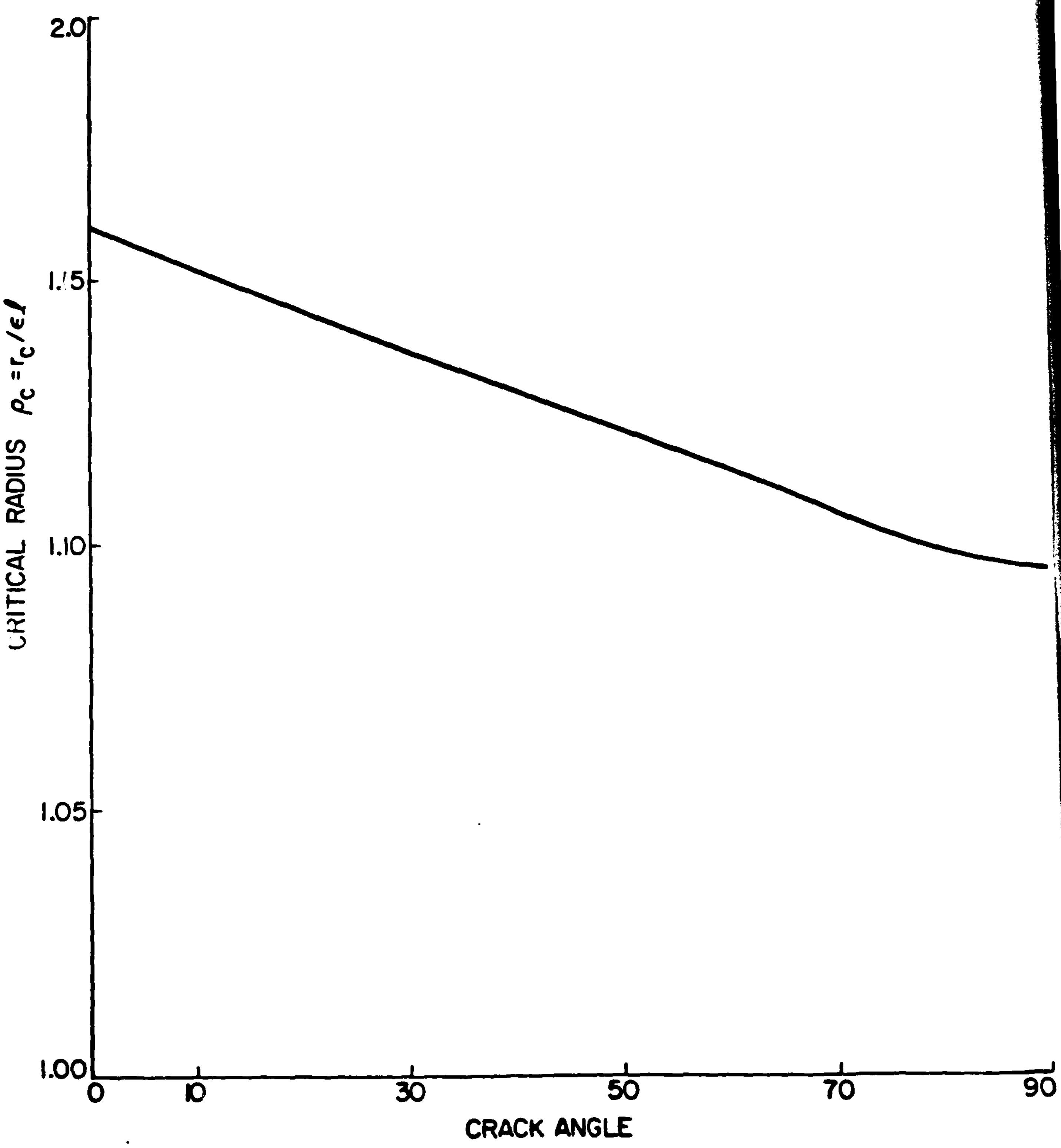
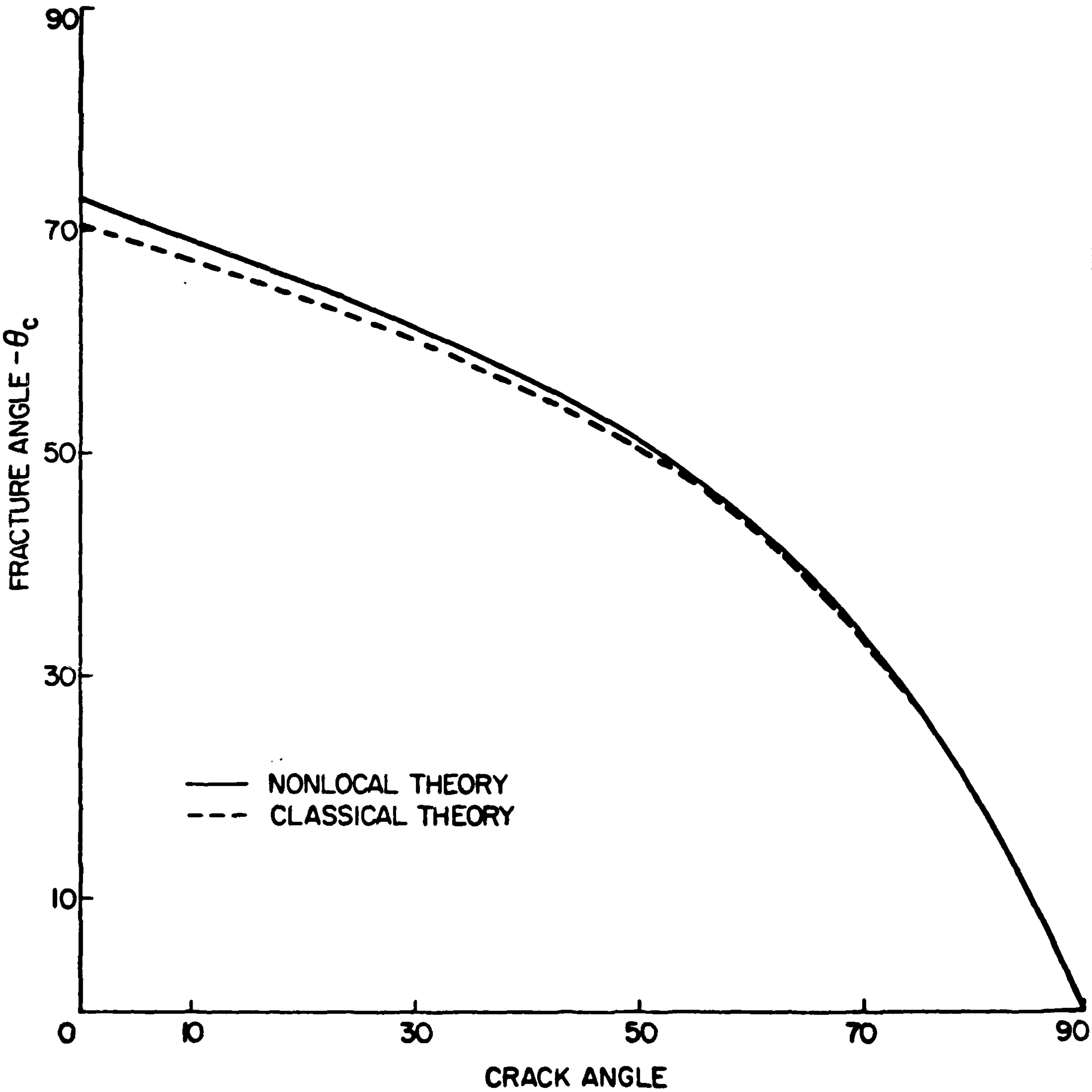


CRITICAL RADIUS vs CRACK ANGLE

FIGURE 1



FRACTURE ANGLE vs CRACK ANGLE
FIGURE 2

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Air Force Office of Scientific Research
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Department of the Air Force
Air University Library
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