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TECHNICAL REPORT

TR - 134

# ALGORITHMS FOR DISTRIBUTED DETECTION GIVEN WAVEFORM OBSERVATIONS

by


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In this report the authors consider the problem of detection using geographically distributed sensors and limited communications. It is assumed that the sensor observations consist of Gaussian processes, the statistics of which depend upon the hypothesis. The cases considered include known and unknown signal corrupted by noise which may be correlated between sensors. A variety of numerical examples are given to illustrate the features of different detection algorithms.



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# ABSTRACT

In this report we consider the problem of detection using geographically distributed sensors and limited communications. It is assumed that the sensor observations consist of Gaussian processes, the statistics of which depend upon the hypothesis. The cases considered include known and unknown signal corrupted by noise which may be correlated between sensors. A variety of numerical examples are given to illustrate the features of different detection algorithms.

CONTENTS

Section 1

# INTRODUCTION AND SUMMARY

## 1.1 INTRODUCTORY REMARKS

This document describes research carried out by ALPHATECH, Inc. for the Air Force Office of Scientific Research under contract F49620-81-C-0015. We describe a study into the problem of detection using geographically distributed sensors and limited communication. This research lead to a deeper understanding of this problem and to the development of good suboptimal distributed detection algorithms.

In this section we summarize the work on this project to date. This is done by briefly outlining the contents of three interim reports (included as appendices).

## 1.2 DECENTRALIZED DETECTION GIVEN WAVEFORM OBSERVATIONS

In this interim report (Appendix A) we formulated a decentralized detection problem in which the observations at each sensor are known waveforms corrupted by Gaussian noise processes (assumed independent between sensors). In this problem formulation an agent associated with each sensor must make a decision based only on the observations available from that sensor. These decisions are then either used as the basis for some local action or are transmitted to a fusion center where they are combined to form a global estimate. In either case a cost is associated with each combination of true and false decisions and the problem is then to determine the optimal decision law for each agent.

Because we assume that the signal being observed is known and that the noise is uncorrelated between agents we can determine the optimal decision laws. By using a Karhunen-Loeve expansion we can reduce the problem to a sequence of problems involving a finite number of observations each of which can be solved using the techniques of Tenney and Sandell [1]. The solutions of these problems form a sequence, the limit of which is the solution to the waveform observation problem.

Two cases are considered in this interim report. The first is that of detection when the noise is white and the second is that of detection when the noise may be any Gaussian process with known mean and covariance. In both cases the solution takes the following form. Each agent forms a likelihood ratio based on his own observations. This statistic is then compared to a threshold and a decision is made based on whether the statistic is above or below the threshold. The thresholds are determined so as to minimize the global cost (i.e. the agents' separate thresholds are optimized jointly).

Examples are given illustrating both the case in which local decisions are combined in a fusion center and in which they are penalized directly by the global cost function.

### 1.3 DISTRIBUTED DETECTION OF KNOWN SIGNALS IN CORRELATED NOISE

In this interim report (Appendix B) we investigated the effect on the distributed detection problem of noise which is correlated between agents. The solution to this problem is shown to require solving difficult coupled functional equations. Since these equations are not analytically tractable and since they are not amenable to numerical solution techniques, we consider a suboptimal detection policy termed the <u>decentralized likelihood ratio test</u> (DLRT).

The DLRT is motivated by the optimal results for the uncorrelated noise case. Each agent forms a likelihood ratio test and compares it to a threshold to determine that agent's decision. The thresholds are those which minimize the expected value of the global cost.

We explore the behavior of this suboptimal law by considering a variety of numerical problems. We find that, in general, the performance degrades as the correlation increases. This is in complete contradistinction to the centralized case where performance as a function of correlation is highly problem dependent.

This leads us to explore whether the DLRT is a poor suboptimal estimator or whether its performance is indicative of that which could be expected from an optimal distributed detection algorithm. For the case in which the signals received by each sensor vary only in magnitude we can show that a sufficient statistic for optimal detection is in fact the local likelihood ratio. The optimal decision law must then be some binary function of this statistic. Binary functions can be parameterized by the "thresholds" dividing the different regions (DLRT corresponds to the special case of one threshold per sensor). A variety of numerical examples were studied: in all the DLRT was optimal. This seems to indicate that the DLRT is a good suboptimal detection law and that its performance with respect to centralized laws reflects the constraint of decentralization rather than severe suboptimality.

## 1.4 DISTRIBUTED DETECTION OF AN UNKNOWN SIGNAL IN NOISE

In this interim report (Appendix C) we consider the problem of distributed detection when the noise between agents is uncorrelated but in which the signal is assumed to be a Gaussian random process of known mean and covariance. As in the problem considered in Section 1.3, the optimal detection laws must be found by solving coupled functional equations. As we cannot solve these equations we again consider the performance of DLRT algorithms.

This problem is more difficult than that considered earlier as the likelihood ratio is a nonlinear function of the observations. This means that the statistics computed are not Gaussian and thus the optimal thresholds and the corresponding performance are very difficult to determine exactly.

However, for the case in which the observation time is long in comparison to the dynamics of the signal process, the likelihood ratio is nearly Gaussian. We therefore determine the first and second moments of the likelihood ratio and compute the thresholds as if this statistic were Gaussian. Several numerical examples provide insight into the performance of the DLRT law for this problem.

Unfortunately, the Gaussian approximation used above is not valid in the "tails" of the distribution and thus cannot be used to determine low probability of false alarm detection laws. Approximations based on Chernov bounds ([2] and [3]) have been developed which are quite accurate and will be reported on subsequently.

## 1.5 SUMMARY

In this project we have characterized the performance of distributed detection algorithms based on the DLRT algorithm. While this algorithm was selected on an ad hoc basis and is thus suboptimal a priori, evidence indicates that it may be optimal in certain cases, and thus we expect the insights obtained are indicative of the requirement of decentralization and not that of local likelihood ratio tests.

We have examined the behavior of the DLRT on a wide variety of cases: known and unknown signal, correlated and uncorrelated noise, decentralized and hierarchical decision laws, etc. In each of these cases we have deepened the understanding of how well distributed detection laws can perform.

## REFERENCES


[1] Tenney, R.R. and N.R. Sandell, Jr., "Detection with Distributed Sensors," IEEE Transactions on Aerospace and Electronic Systems, Volume AES-17, Number 4, 1981, pp. 501-509

[2] Van Trees, H.L., Detection, Estimation, and Modulation, Part I, J. Wiley and Sons, New York, 1968

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# APPENDIX A

## DECENTRALIZED DETECTION GIVEN WAVEFORM OBSERVATIONS

# DECENTRALIZED DETECTION GIVEN WAVEFORM OBSERVATIONS

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## ABSTRACT

In this paper we solve a decentralized hypothesis testing problem in which the observations are waveforms corrupted by Gaussian noise processes. This paper extends the results of Tenney and Sandell [1] which allowed only discrete observations.

## 1. INTRODUCTION

In this paper we extend the distributed detection results derived by Tenney and Sandell [1] for discrete observations to the case of continuous waveform observations. This extension is accomplished (as in the classical centralized case) by the use of a Karhunen-Loeve expansion [2] of the received waveform. The cases we consider are those in which a collection of distributed agents observes known deterministic signals corrupted by Gaussian white noise either with or without a Gaussian-colored noise component. For these cases we can explicitly compute the statistics of the likelihood functions, thus reducing the waveform observation case to that of a scalar observation. The outline of this paper is the following.

In Section 2 we briefly describe the types of situations which give rise to distributed detection problems. In Section 3 we briefly review the results of Tenney and Sandell [1], and in Section 4 we extend these results to the case of waveform observations in white Gaussian noise. In Section 5 we modify this extension to account for observations corrupted by a combination of white and colored Gaussian noise. Finally, in Section 6 we discuss various extensions of these results that we are currently investigating.

## 2. MOTIVATION

Consider a situation in which geographically distributed sensors observe a common region for the purpose of determining the presence or absence of a certain phenomenon.* Because of such considerations as cost, reliability, survivability, communications bandwidth, etc., we assume that the observations of the various sensors cannot be transmitted to a common node for centralized processing. Rather, a local decision must be made at each sensor which can then either be transmitted to some common node or be used as the basis for some local action.

If the local decisions are transmitted to a central node then we assume that they are combined to form a "global" decision by some prespecified decision function f (e.g., voting). Further, we assume that costs can be associated with the various combinations of truth and global decision (corresponding

*Think of multistatic radar for a concrete example.

to false alarms, missed detections, etc.). This leads to the situation depicted in Fig. 2-1 in which two agents associated with distributed sensors make local decisions $u_1$ and $u_2$. These decisions are combined to produce a global decision

$$u = f(u_1, u_2) \quad . \tag{2-1}$$

Note that the global cost $J'(u,H)$ can be rewritten so as to depend only on the local decisions by defining

$$J(u_1, u_2, H) = J'(f(u_1, u_2), H) \quad . \tag{2-2}$$

If the local decisions are not transmitted to a central node but rather are used as the basis of local actions then we assume that a cost can be associated with each combination of truth and local decisions (e.g., one agent detects a target and another does not). This situation is illustrated in Fig. 2-2 and might arise, for example, if weapons are colocated with distributed sensors and are to be used based on local sensor decisions. One could then associate costs with having several agents detect and fire on a single target, having only one agent detect and fire on a target, etc.

For our purposes the key observation is to note that in both cases (hierarchical and distributed decisionmaking) the cost can be written as a function of the local decisions. Thus we can consider both of these cases by formulating a problem in which agents interact only through a common cost function $J(u_1, u_2, H)$.

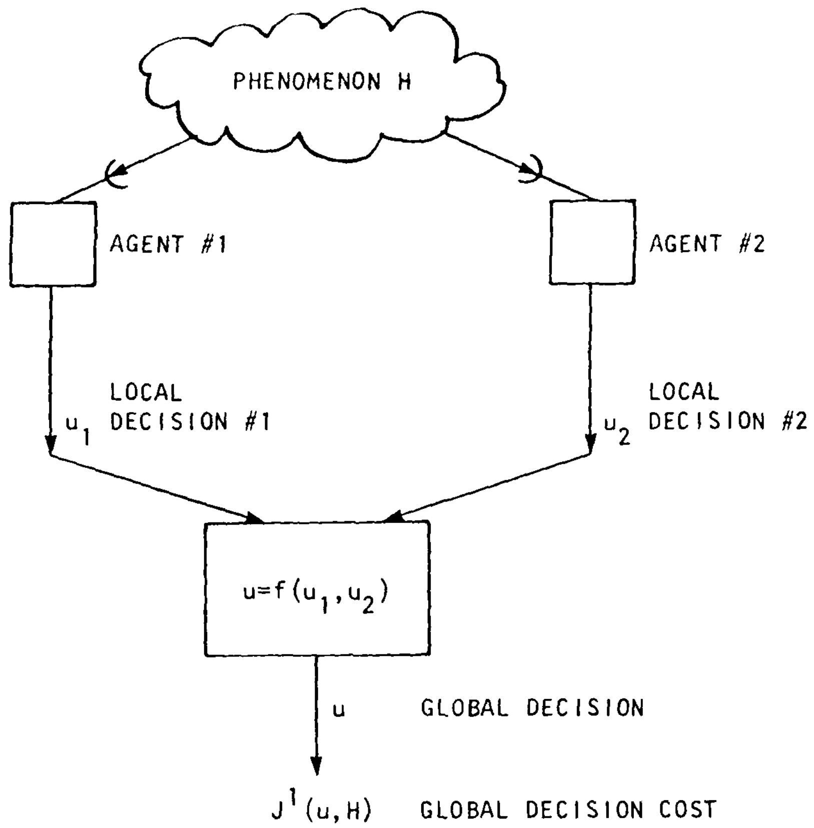


Figure 2-1. Hierarchical Decisionmaking.

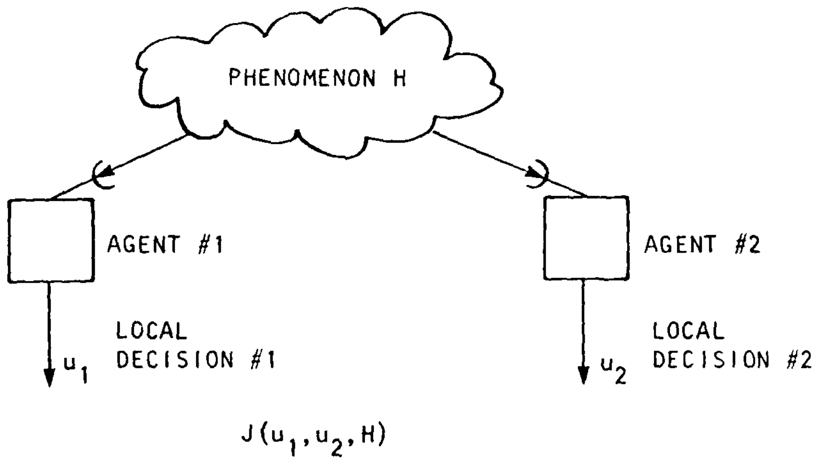


Figure 2-2. Distributed Decisionmaking.

## 3. REVIEW

In [1] Tenney and Sandell solved the following problem. Assume that two hypotheses, $H^0$ and $H^1$, are possible and occur with a priori probabilities

$$p(H^0) = p^0 \qquad p(H^1) = p^1 \quad . \tag{3-1}$$

For each hypothesis the sensor observations $y_i$ (which may be random vectors) have known probability distributions

$$p(y_1, y_2 | H^i) = p(y_1 | H^i) p(y_2 | H^i) \quad , \quad i=0,1 \quad . \tag{3-2}$$

Note that we assume the observations $y_1$ and $y_2$ are statistically independent when conditioned on the hypotheses. Only decision rules of the form

$$u_i = \begin{cases} 0, & H^0 \text{ is declared to have been detected} \\ 1, & H^1 \text{ is declared to have been detected} \end{cases} ,$$

where $u_i$ is a function only of $y_i$, are allowed. The decision rule can be defined by the conditional probability distribution function

$$p(u_i = 0 | y_i) \tag{3-3}$$

which defines a randomized decision rule. The objective is to choose the (randomized) decision rules so as to minimize the expected cost, i.e.,

$$\min E[J(u_1, u_2, H)] \quad . \tag{3-4}$$

For this problem formulation Tenney and Sandell proved that the optimum decision rules are deterministic and are given by*

$$\ell r(y_1) \underset{1}{\overset{0}{\gtrless}} t_1 \tag{3-5}$$

$$\ell r(y_2) \underset{1}{\overset{0}{\gtrless}} t_2 \tag{3-6}$$

where

$$\ell r(y_i) = \frac{p(y_i|H^0)p^0}{p(y_i|H^1)p^1} \tag{3-7}$$

The thresholds $t_1$ and $t_2$ satisfy the following equations.

$$t_1 = \frac{\Delta J_1+\Delta J_2 \displaystyle\int_{\{y_2|\ell r(y_2)>t_2\}} p(y_2|H^1)dy_2}{\Delta J_3+\Delta J_4 \displaystyle\int_{\{y_2|\ell r(y_2)>t_2\}} p(y_2|H^0)dy_2} \tag{3-8}$$

$$t_2 = \frac{\Delta J_5+\Delta J_2 \displaystyle\int_{\{y_1|\ell r(y_1)>t_1\}} p(y_1|H^1)dy_1}{\Delta J_6+\Delta J_4 \displaystyle\int_{\{y_1|\ell r(y_1)>t_1\}} p(y_1|H^0)dy_1} \tag{3-9}$$

where

$$\Delta J_1 = J(0,1,H^1) - J(1,1,H^1) \tag{3-10a}$$

---

*The notation $f(x) \underset{1}{\overset{0}{\gtrless}} y$ indicates choose u=0 if $f(x)>y$, u=1 if $f(x)<y$, and make either choice if $f(x)=y$.

$$\Delta J_2 = J(0,0,H^1) + J(1,1,H^1) - J(1,0,H^1) - J(0,1,H^1) \tag{3-10b}$$

$$\Delta J_3 = J(1,1,H^0) - J(0,1,H^0) \tag{3-10c}$$

$$\Delta J_4 = J(1,0,H^0) + J(0,1,H^0) - J(0,0,H^0) - J(1,1,H^0) \tag{3-10d}$$

$$\Delta J_5 = J(1,0,H^1) - J(1,1,H^1) \tag{3-10e}$$

and

$$\Delta J_6 = J(1,1,H^0) - J(1,0,H^0) \quad . \tag{3-10f}$$

Equations 3-8 and 3-9 define agent-by-agent optimal decision strategies (i.e., ones which cannot be improved by the unilateral action of an agent). If more than one solution to these equations exists then the costs corresponding to these solutions must be compared to determine the globally optimal strategy.

## 4. WAVEFORM OBSERVATIONS IN WHITE NOISE

In this section we consider the problem posed in Section 3 modified so that the observations are waveforms rather than vectors. The solution of the previous section is not directly applicable since $p(y_j|H^i)$ need no longer be well defined thus invalidating Eqs. 3-6 through 3-9. The approach we take to extending the results of [1] to the waveform case requires that we assume the received waveforms are Gaussian processes. In this section we assume the received waveform is either white noise or a known deterministic signal plus white noise; in the next section we allow the noise waveforms to be arbitrary Gaussian processes.

The problem formulation is as given in Section 3 except that instead of specifying the conditional probability distributions of Eqs. 3-2 we define the observations under the two hypotheses by

$$H^1: \quad y_i(t) = \sqrt{E_i}\, s_i(t) + n_i(t) \qquad 0 \le t \le T \tag{4-1a}$$

$$H^0: \quad y_i(t) = n_i(t) \qquad 0 \le t \le T \ . \tag{4-1b}$$

The $n_i(t)$ are zero-mean Gaussian white noise processes with spectral height $N_i(t)$ and we assume $n_1(t)$ and $n_2(t)$ are statistically independent. Furthermore, without loss of generality, we assume

$$\int_0^T s_i^2(t)dt = 1 \ . \tag{4-2}$$

The approach we take to solving this distributed detection problem is to represent the received waveforms as a countably infinite sequence of coefficients. Any truncation of this sequence produces a problem that is readily solved by the techniques of Section 3. We solve the waveform problem by showing that the limiting solution obtained by considering more and more coefficients is well defined and solves the waveform problem.

The received waveforms can be expressed as*

$$y_i(t) = \underset{K\to\infty}{\text{l.i.m.}} \sum_{k=1}^{K} y_i^k \phi_i^k(t) \quad ; \quad 0 \le t \le T \tag{4-3}$$

*An arbitrary orthonormal set is allowed here because we have assumed that the observation noise is white.

where l.i.m. is interpreted as limit in the mean and Eq. 4-3 is equivalent to

$$\lim_{K\to\infty} E\{\| y_i(t) - \sum_{k=1}^{K} y_i^k \phi_i^k(t) \|^2\} = 0 \quad . \tag{4-4}$$

The functions $\phi_i^k(t)$ are assumed to be such that $\{\phi_i^k(t)\}_{k=1}^{\infty}$ are complete orthonormal sets for i=1,2. For convenience in what follows we require that

$$\phi_i^1(t) = s_i(t) \quad , \quad 0 \le t \le T \quad . \tag{4-5}$$

The $y_i^k$ are defined by

$$y_i^k = \int_0^T y_i(t)\phi_i^k(t)dt \tag{4-6}$$

and are clearly Gaussian random variables. Since $\{\phi_i^k\}$ are orthonormal sets we have that

$$y_i^1 = \begin{cases} \int_0^T s_i(t)n_i(t)dt \triangleq w_i^1 : & H^0 \\ \int_0^T s_i(t)[\sqrt{E_i}\, s_i(t)+n_i(t)]dt \triangleq \sqrt{E_i} + w_i^1 : & H^1 \end{cases} \tag{4-7}$$

and for k≥2

$$y_i^k = \begin{cases} \int_0^T \phi_i^k(t)n_i(t)dt \triangleq w_i^k : & H^0 \\ \int_0^T \phi_i^k(t)[\sqrt{E_i}\, s_i(t)+n_i(t)]dt \triangleq w_i^k : & H^1 \end{cases} \quad . \tag{4-8}$$

Furthermore,

$$E\{y_i^\ell y_i^k | H^j\} = 0 \quad , \quad \ell \neq k \quad , \quad i=1,2 \quad , \quad j=1,2 \tag{4-9}$$

and

$$E\{(y_1^\ell - \bar{y}_1^\ell)(y_2^k - \bar{y}_2^k) | H^j\} = 0 \tag{4-10}$$

for all $\ell$ and k. This implies that only the $y_i^1$ depend upon the hypothesis and that the $y_1^\ell$ and $y_2^k$ are independent.*

Let us now consider the problem in which, instead of receiving a waveform $y_i(t)$, the i-th agent instead receives the vector

$$\underline{y}_i^K = [y_i^1, y_i^2, \ldots y_i^k] \tag{4-11}$$

where the $y_i^k$ are defined in Eqs. 4-7 and 4-8. We know from [1] that for agent 1 the optimum decision law is given by

$$\frac{p(\underline{y}_1^K | H^0) p^0}{p(\underline{y}_1^K | H^1) p^1} \underset{1}{\overset{0}{\gtrless}} t \tag{4-12}$$

where

$$t_1 = \frac{\Delta J_1 + \Delta J_2 \displaystyle\int_{\ell r(\underline{y}_2^K) > t_2} p(\underline{y}_2^K | H^1) d\underline{y}_2^K}{\Delta J_3 + \Delta J_4 \displaystyle\int_{\ell r(\underline{y}_2^K) > t_2} p(\underline{y}_2^K | H^0) d\underline{y}_2^K} \quad . \tag{4-13}$$

---

*Without the Gaussian assumption on the noise we only have that the $y_1^\ell$ and $y_2^k$ are uncorrelated.

Because the $y_i^k$ are statistically independent* and because only $y_i^1$ depends upon the hypothesis

$$\frac{p(\underline{y}_1^K|H^0)p^0}{p(\underline{y}_1^K|H^1)p^1} = \frac{p(y_1^1|H^0) \prod_{k=2}^{K} p(y_1^k)p^0}{p(y_1^1|H^1) \prod_{k=2}^{K} p(y_1^k)p^1} \tag{4-14a}$$

$$= \frac{p(y_1^1|H^0)p^0}{p(y_2^1|H^1)p^1} \quad . \tag{4-14b}$$

Similarly for agent 2 we have that

$$\ell r(\underline{y}_2^K) = \frac{p(y_2^1|H^0)p^0}{p(y_2^1|H^1)p^1} \tag{4-15a}$$

$$= \ell r(y_2^1) \quad . \tag{4-15b}$$

Equation 4-13 thus can be written as

$$t_1 = \frac{\Delta J_1+\Delta J_2 \int_{\ell r(y_2^1)>t_2} p(y_2^1|H^1)dy_2^1 \prod_{k=2}^{K} \int_{-\infty}^{\infty} p(y_2^k)dy_2^k}{\Delta J_3+\Delta J_4 \int_{\ell r(y_2^1)>t_2} p(y_2^1|H^0)dy_2^1 \prod_{k=2}^{K} \int_{-\infty}^{\infty} p(y_2^k)dy_2^k} \tag{4-16a}$$

---

*Here is where the Gaussian noise assumption is used. If the $y_i^k$ are only uncorrelated we cannot expand $p(\underline{y}_1^k|H^i)$.

$$t_1 = \frac{\Delta J_1 + \Delta J_2 \displaystyle\int_{\ell r(y_2^1) > t_2} p(y_2^1 | H^1)\, dy_2^1}{\Delta J_3 + \Delta J_4 \displaystyle\int_{\ell r(y_2^1) > t_2} p(y_2^1 | H^0)\, dy_2^1} \quad . \tag{4-16b}$$

Note that neither $t_1$ nor $\ell r(\underline{y}_1^K) = \ell r(y_1^1)$ depend on K. We can thus let K to go to infinity and the decision laws remain unchanged. The solution to the waveform observation problem is thus given by

$$\ell r(y_i^1) \underset{1}{\overset{0}{\gtrless}} t_i \tag{4-17}$$

where $t_1$ is given by Eq. 4-16b and $t_2$ is given by

$$t_2 = \frac{\Delta J_5 + \Delta J_2 \displaystyle\int_{\ell r(y_1^1) > t_1} p(y_1^1 | H^1)\, dy_1^1}{\Delta J_6 + \Delta J_4 \displaystyle\int_{\ell r(y_1^1) > t_1} p(y_1^1 | H^0)\, dy_1^1} \quad , \tag{4-18}$$

and where

$$y_i^1 = \int_0^T s_i(t) y_i(t)\, dt \quad . \tag{4-19}$$

Because we have assumed Gaussian statistics for the observation noise we can obtain a more compact expression for $\ell r(y_1^1)$ and for the equations defining the thresholds. Let us compute the mean and variance of $y_i^1$ under the hypotheses:

$$E\{y_i^1 | H^0\} = E\{\int_0^T s_i(t) n_i(t)\, dt\} = 0 \tag{4-20}$$

$$E\{y_i^1|H^1\} = E\{\int_0^T s_i(t)[\sqrt{E_i}\, s_i(t)+n_i(t)]dt\} = \sqrt{E_i} \tag{4-21}$$

$$E\{(y_i^1-\bar{y}_i^1)^2|H^i\} = E\{\int_0^T \int_0^T s_i(t)s_i(\tau)n_i(t)n_i(\tau)dtd\tau\} \tag{4-22a}$$

$$= \int_0^T \int_0^T s_i(t)s_i(\tau)N_i(t)\delta(t-\tau)dtd\tau \tag{4-22b}$$

$$= \int_0^T s_i^2(t)N_i(t)dt \triangleq \tilde{N}_i^2 \quad . \tag{4-22c}$$

If we rewrite Eq. 4-17 as

$$\ln[\ell r(y_i^1)] \underset{1}{\overset{0}{\gtrless}} \ln(t_i) = \tau_i \tag{4-23}$$

then the decision rules become

$$-\frac{\sqrt{E_i}}{\tilde{N}_i^2}\, y_i^1 + \frac{E_i}{2\tilde{N}_i^2} \underset{1}{\overset{0}{\gtrless}} \tau_i - \ln(\frac{P^0}{P^1}) \tag{4-24}$$

or equivalently

$$\int_0^T s_i(t)y_i(t)dt \underset{0}{\overset{1}{\gtrless}} T_i \tag{4-25}$$

where the thresholds $T_i$ satisfy

$$T_1 = \frac{1}{2}\sqrt{E_1} - \frac{\tilde{N}_1^2}{\sqrt{E_1}} \ln\left\{\frac{\Delta J_1+\Delta J_2 \ \mathrm{erf}[(T_2-\sqrt{E_2})/\tilde{N}_2]}{\Delta J_3+\Delta J_4 \ \mathrm{erf}[T_2/\tilde{N}_2]} \ \frac{P^0}{P^1}\right\} \tag{4-26}$$

$$T_2 = \frac{1}{2}\sqrt{E_2} - \frac{\tilde{N}_2^2}{\sqrt{E_2}} \ln \left\{ \frac{\Delta J_5 + \Delta J_2 \operatorname{erf}[(T_1 - \sqrt{E_1})/\tilde{N}_1]}{\Delta J_6 + \Delta J_4 \operatorname{erf}[T_1/\tilde{N}_1]} \right\} \quad . \tag{4-27}$$

Note that, as in classical detection theory, the waveform case is attacked by determining a finite set of sufficient statistics and writing the decision law in terms of these statistics. In the case just considered $y_i^1$ is a scalar sufficient statistic for agent i and this leads to the test of $y_i^1$ against a threshold. The sufficient statistic computed in Eq. 4-19 can also be found as the output of a matched filter designed for the i-th signal. Thus the optimal decision law can be interpreted as comparing the outputs of "local" matched filters against thresholds which are determined "globally."

As in Section 3, the equations defining the thresholds are necessary but not sufficient and thus if multiple solutions exist, the associated costs must be computed and compared in order to determine a globally optimal solution. Note too that since the waveform case reduces to a scalar problem if the underlying system is a fusion system the ROC curves computed in [1] (or similar ones) can be used to analyze the tradeoff between probability of detection and false alarm.

## 5. GENERAL GUASSIAN WAVEFORM OBSERVATIONS

In this section we extend the results of the previous section by allowing the observations of each agent to be a known deterministic signal corrupted by an arbitrary zero-mean Gaussian process,* where the signal depends upon the hypothesis. Thus we have that the observations are

*The assumption of zero mean is not restrictive as any nonzero mean can be incorporated into the signal. We shall require that the noise have a white component so as to avoid the unrealistic possibility of perfect detection.

$$H^1: \quad z_i(t) = \sqrt{E_i^1}\, s_i^1(t) + n_i(t) \quad , \quad T_o \le t \le T_f \tag{5-1a}$$

$$H^0: \quad z_i(t) = \sqrt{E_i^0}\, s_i^0(t) + n_i(t) \quad , \quad T_o \le t \le T_f \tag{5-1b}$$

where

$$E\{n_i(t)n_i(\tau)\} \triangleq K_i^n(t,\tau) \tag{5-2}$$

and, as usual, we assume that $n_1(t)$ and $n_2(t)$ are independent.

We assume that the signals $s_i^0(t)$ and $s_i^1(t)$ are exactly zero outside the interval $0 \le t \le T$. We allow the observation period to be different from [0,T] since with nonwhite noise an extended observation period can allow better performance.*

Rather than work with $z_i(t)$ as given by Eq. 5-1 we define $y_i(t)$ as a reversible function of $z_i(t)$ and work with $y_i(t)$. We choose $y_i(t)$ to make the derivation more straightforward. Since $y_i(t)$ is obtained from $z_i(t)$ by a reversible operation working with $y_i(t)$ yields the same detection performance as working with $z_i(t)$ [2]. We define

$$y_i(t) = z_i(t) - \sqrt{E_i^0}\, s_i^0(t) \quad , \quad T_o \le t \le T_f \tag{5-3}$$

so that

$$H^1: \quad y_i(t) = \sqrt{E_i}\, s_i(t) + n_i(t) \quad , \quad T_o \le t \le T_f \tag{5-4a}$$

---

*Consider the case where $s_i(t)$ is constant on [0,1] and zero elsewhere and $n_i(t)$ is constant on [-1,1]. Clearly perfect detection is possible if $T_o < 0$.

$$H^0: \quad y_i(t) = n_i(t) \quad , \quad T_o \leq t \leq T_f \tag{5-4b}$$

where

$$s_i(t) = \frac{\sqrt{E_i^1}\, s_i^1(t) - \sqrt{E_i^0}\, s_i^0(t)}{\sqrt{E_i}} \quad , \quad T_o \leq t \leq T_f \tag{5-5}$$

$$E_i = \int_0^T [\sqrt{E_i^1}\, s_i^1(t) + \sqrt{E_i^0}\, s_i^0(t)]^2 dt \quad . \tag{5-6}$$

Clearly we have that

$$\int_0^T s_i^2(t)dt = 1 \tag{5-7}$$

for $s_i(t)$ defined by Eq. 5-5.

As in the last section, to solve the waveform problem we consider a sequence of detection problems formed by truncating a Karhunen-Loeve (K-L) expansion. To avoid the unrealistic possibility of perfect detection we assume that the noises $n_i(t)$ have white components. Thus, formally, we can write

$$K_i^n(t,\tau) = K_i^c(t,\tau) + N_i(t)\delta(t-\tau) \quad . \tag{5-8}$$

We now expand the observation $y_i(t)$ in a K-L expansion

$$y_i(t) = \underset{K\to\infty}{\mathrm{l.i.m.}} \sum_{k=1}^{K} y_i^k \phi_i^k(t) \quad , \quad T_o \leq t \leq T_f \tag{5-9}$$

where the $\phi_i^k(t)$ satisfy the integral equation

$$\tilde{\lambda}_i^k \phi_i^k(t) = \int_{T_o}^{T_f} K_i^c(t,u)\phi_i^k(u)\,du \quad , \quad T_o \le t \le T_f \quad . \tag{5-10}$$

If the solutions to Eq. 5-10 do not form a complete orthonormal set (i.e., if $K_i^c(t,\tau)$ is not positive definite), we augment it to make it complete.

Under $H^1$ we have that

$$y_i^k = s_i^k + n_i^k \tag{5-11}$$

where

$$s_i^k \triangleq \int_{T_o}^{T_f} \sqrt{E_i}\; s_i(t)\phi_i^k(t)\,dt \tag{5-12}$$

$$n_i^k \triangleq \int_{T_o}^{T_f} n_i(t)\phi_i^k(t)\,dt \quad . \tag{5-13}$$

Under $H^0$ we have that

$$y_i^k = n_i^k \quad . \tag{5-14}$$

Furthermore

$$E\{n_i^k\} = 0 \tag{5-15}$$

$$E\{(n_i^k)^2\} = \tilde{\lambda}_i^k + \tilde{N}_i^2 \triangleq \lambda_i^k \tag{5-16}$$

$$E\{n_i^\ell n_i^k\} = 0 \qquad \ell \neq k \tag{5-17}$$

and

$$E\{(y_1^\ell-\bar{y}_1^\ell)(y_2^k-\bar{y}_2^k)\Big| H^i\} = 0 \quad . \tag{5-18}$$

Consider the problem where each agent receives the first K coefficients in the K-L expansion of $y_i(t)$. As before we define

$$\underline{y}_i^K = [y_i^1,\ldots,y_i^k] \quad . \tag{5-19}$$

The decision law for agent 1 is

$$\ell r(\underline{y}_1^K) \underset{1}{\overset{0}{\gtrless}} t_1^K \tag{5-20}$$

where

$$\ell r(\underline{y}_1^K) = \frac{\prod_{k=1}^{K} p(y_1^k|H^0)}{\prod_{k=1}^{K} p(y_1^k|H^1)} \tag{5-21}$$

$$t_1^K = \frac{\Delta J_1+\Delta J_2 \displaystyle\int_{\ell r(\underline{y}_2^K)>t_2^K} p(\underline{y}_2^K|H^1)\,d\underline{y}_2^K}{\Delta J_3+\Delta J_4 \displaystyle\int_{\ell r(\underline{y}_2^K)>t_2^K} p(\underline{y}_2^K|H^0)\,d\underline{y}_2^K}\;\frac{p^1}{p^0} \quad . \tag{5-22}$$

Equation 5-21 can be written as

$$\ell r(y_1^K) = \frac{\prod_{k=1}^{K} \frac{1}{\sqrt{2\pi\lambda_i^k}} \exp\left[-\frac{1}{2}\frac{(y_i^k)^2}{\lambda_i^k}\right]}{\prod_{k=1}^{K} \frac{1}{\sqrt{2\pi\lambda_i^k}} \exp\left[-\frac{1}{2}\frac{(y_i^k-s_i^k)^2}{\lambda_i^k}\right]} . \tag{5-23}$$

Canceling common terms and letting $K\to\infty$ we have

$$\ell n[\ell r(y_i(t))] = -\sum_{k=1}^{\infty} \frac{y_i^k s_i^k}{\lambda_i^k} + \frac{1}{2}\sum_{k=1}^{\infty}\frac{(s_i^k)^2}{\lambda_i^k} \tag{5-24}$$

Using the definition of $s_i^k$ and $y_i^k$ we can rewrite Eq. 5-24 as

$$\ell n[\ell r(y_i(t))] = -\int_{T_o}^{T_f} \left\{ \int_{T_o}^{T_f} y_i(t) \sum_{k=1}^{\infty} \frac{\phi_i^k(t)\phi_i^k(u)}{\lambda_i^k} \sqrt{E_i}\, s_i(u)du \right\} dt$$

$$+\frac{E_i}{2}\int_{T_o}^{T_f} \left\{ \int_{T_o}^{T_f} s_i(t) \sum_{k=1}^{\infty} \frac{\phi_i^k(t)\phi_i^k(u)}{\lambda_i^k} s_i(u)du \right\} dt . \tag{5-25}$$

Defining

$$Q_i(t,u) = \sum_{k=1}^{\infty} \frac{\phi_i^k(t)\phi_i^k(u)}{\lambda_i^k} \tag{5-26}$$

and

$$g_i(t) = \sqrt{E_i}\int_{T_o}^{T_f} Q_i(t,u)s_i(u)du \tag{5-27}$$

we can rewrite Eq. 5-25 as

$$\ell_i \triangleq \ell n[\ell r(y_i(t))] = -\int_{T_o}^{T_f} (y_i(t) + \frac{1}{2}\sqrt{E_i}\, s_i(t))g_i(t)dt \quad . \tag{5-28}$$

The decision rule for agent 1 thus becomes

$$-\int_{T_o}^{T_f} (y_1(t) - \frac{1}{2}\sqrt{E_1}\, s_1(t))g_1(t)dt \overset{0}{\underset{1}{\gtrless}} \ell n(t_1) = \tau_1 \tag{5-29}$$

where

$$t_1 = \lim_{K\to\infty} t_1^K \tag{5-30}$$

and $t_1^K$ is given by Eq. 5-22. If we define

$$\ell_i^K = \ell n[\ell r(\underline{y}_i^K)] \tag{5-31}$$

then Eq. 5-22 can be rewritten as

$$t_1^K = \frac{\Delta J_1 + \Delta J_2 \displaystyle\int_{\ell n(t_2^K)}^{\infty} p(\ell_2^K|H^1)d\ell_2^K}{\Delta J_3 + \Delta J_4 \displaystyle\int_{\ell n(t_2^K)}^{\infty} p(\ell_2^K|H^0)d\ell_2^K} \; \frac{p^1}{p^0} \quad . \tag{5-32}$$

The limit of $t_i^K$ can now be obtained

$$t_I = \frac{\Delta J_1 + \Delta J_2 \int_{\tau_2}^{\infty} p(\ell_2|H^1)\,d\ell_2}{\Delta J_3 + \Delta J_4 \int_{\tau_2}^{\infty} p(\ell_2|H^0)\,d\ell_2} \; \frac{P^1}{P^0} \quad . \tag{5-33}$$

To determine $t_1$ we need the statistics of $\ell_2$. But $\ell_2$ is a linear function of $y_2(t)$ so it is a Gaussian random variable. We have

$$E\{\ell_2|H^0\} = \frac{\sqrt{E_2}}{2} \int_{T_o}^{T_f} s_2(t)g_2(t)dt \triangleq m_2 \tag{5-34}$$

$$E\{\ell_2|H^1\} = -\frac{\sqrt{E_2}}{2} \int_{T_o}^{T_f} s_2(t)g_2(t)dt = -m_2 \tag{5-35}$$

and

$$E\{(\ell_2-\bar{\ell}_2)^2|H^i\} = \int_{T_o}^{T_f} \left\{ \int_{T_o}^{T_f} g_2(t)K_2^n(t,u)g_2(u)du \right\} dt \quad . \tag{5-36}$$

Recalling the definition of $g_2(t)$ from Eq. 5-27 we have

$$E\{(\ell_2-\bar{\ell}_2)^2|H^i\} = \sqrt{E_2} \int_{T_o}^{T_f} \left\{ \int_{T_o}^{T_f} g_2(t) \int_{T_o}^{T_f} K_2^n(t,u)Q_2(u,v)s_2(v)dvdu \right\} dt \quad . \tag{5-37}$$

It can be shown [2] that $Q_i(u,v)$, as defined by Eq. 5-26 satisfies

$$\int_{T_o}^{T_f} K_n(t,u)Q(u,v)du = \delta(t-v) \tag{5-38}$$

so that Eq. 5-37 becomes

$$E\{(\ell_2-\bar{\ell}_2)^2|H^i\} = \sqrt{E_2}\int_{T_o}^{T_f} g_2(t)s_2(t)dt = 2m_2 \quad . \tag{5-39}$$

The threshold $\tau_i$ thus satisfy

$$\tau_1 = \ell n \frac{\Delta J_1+\Delta J_2[1-\mathrm{erf}((\tau_2+m_2)/\sqrt{2m_2})]}{\Delta J_3+\Delta J_4[1-\mathrm{erf}((\tau_2-m_2)/\sqrt{2m_2})]}\ \frac{p^1}{p^0} \tag{5-40}$$

$$\tau_2 = \ell n \frac{\Delta J_5+\Delta J_2[1-\mathrm{erf}((\tau_1+m_1)/\sqrt{2m_1})]}{\Delta J_6+\Delta J_4[1-\mathrm{erf}((\tau_1-m_1)/\sqrt{2m_1})]}\ \frac{p^1}{p^0} \tag{5-41}$$

The decision rules are thus given by

$$\int_{T_o}^{T_f} y_i(t)g_i(t)dt \underset{0}{\overset{1}{\gtrless}} \frac{1}{2}\sqrt{E_i}\int_{T_o}^{T_f} s_i(t)g_i(t) - \tau_i \tag{5-42}$$

where $g_i(t)$ are determined from Eqs. 5-27 and 5-38, the $\tau_i$ solve Eqs. 5-40 and 5-41 and $y_i(t)$ is determined from Eq. 5-3.

As before the equations defining the $\tau_i$ are necessary but not sufficient. Thus to determine a globally optimal solution, the costs corresponding to all solutions to Eqs. 5-40 and 5-41 must be computed and compared.

## 6. SUMMARY

In this paper we have extended the results of Tenney and Sandell [1] to the case in which each agent makes waveform observations in Gaussian noise.

In these cases the optimal decision laws are the same as in the centralized case (i.e., we perform certain linear operations on the signal and compare the resultant scalar to a threshold), however, as in [1] the agents thresholds satisfy coupled nonlinear equations. We have given explicit formulae for determining these thresholds for both the case in which the noise is purely white and in which the noise is colored (with a white component).

In ongoing research we are considering problems in which the observations of the different agents need not be conditionally independent. For these cases we are able to use many of the techniques of this paper, though we find that globally optimal results cannot usually be determined. The results of this paper motivate a suboptimal decentralized detection scheme in which signal processing is only locally optimal but the thresholds are determined globally.

## *REFERENCES*


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# APPENDIX B

## DISTRIBUTED DETECTION OF KNOWN SIGNALS IN CORRELATED NOISE

Technical Paper 131

# DISTRIBUTED DETECTION OF KNOWN SIGNALS IN CORRELATED NOISE

by

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ABSTRACT

In this paper we extend the results of Lauer and Sandell [1] for distributed detection of known signals in uncorrelated noise to the case in which the noise may be correlated. Optimal decision laws cannot be obtained so a class of suboptimal laws is considered. Numerical examples are considered which indicate the tradeoffs inherent in using distributed detection laws.

## SECTION 1

## INTRODUCTION

In this paper we extend the distributed detection results derived by Lauer and Sandell [1] and Tenney and Sandell [2] for uncorrelated sensor noise to the case of noise which is correlated between the distributed sensors. Determining the optimal decentralized detection law for the correlated noise problem requires the solution of sets of functional equations. Since these equations are not analytically tractable and since they are not amenable to numerical solution techniques, we consider a suboptimal detection policy termed the Decentralized Likelihood Rato Test (DLRT) and motived by the uncorrelated noise results.

In section 2 we introduce the correlated noise problem and briefly describe physical situations which give rise to it. In section 3 we show that determining the optimal distributed detection policy requires solving coupled sets of functional equations and we introduce the class of suboptimal detection policies with which we will be concerned for the remainder of the paper. In section 4 we give some numerical results illustrating the behavior and performance of the suboptimal laws. In section 5 we briefly consider whether the loss in performance of the decentralized as compared to the centralized detection law associated with the example of section 4 is intrinsic in the constraint to decentralized signal processing or is due to the fact that only a suboptimal law is being considered.

Finally, in section 6 we discuss various extensions of the results of this paper that are currently under investigation.

## SECTION 2

## PROBLEM FORMULATION AND MOTIVATION

As in [1] we assume that there are two distributed sensors, indexed by $i = 1, 2$, and two hypotheses to be tested based on the sensor observations, indexed by $j = 0, 1$. The observations under the two hypotheses are assumed to be generated by*

$$H^1: \quad y_i(t) = \sqrt{E_i}\, s_i(t) + n_i(t) \qquad T_o \le t \le T_f \tag{2-1a}$$

$$H^0: \quad y_i(t) = \qquad\qquad n_i(t) \qquad T_o \le t \le T_f. \tag{2-1b}$$

Here we assume that $s_i(t)$ is a known signal with unit energy which is zero outside the interval $[0, T]$ where $T_o \le 0 < T \le T_f$. The $n_i(t)$ are assumed to be zero-mean Gaussian processes where

$$E\left\{n_i(t) n_k(\tau)\right\} = K_{ik}^n(t, \tau) \tag{2-2}$$

and we assume (to avoid the possibility of singular detection)

$$K_{ii}^n(t, \tau) = K_i^c(t, \tau) + N_i^2 \delta(t - \tau) \tag{2-3}$$

with $N_i \neq 0$. Note that, unlike [1] we do not assume that the noise processes

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* Recall from [1] that assuming no signal is present under $H^0$ entails no loss of generality.

are uncorrelated, i.e. we allow $K_{ik}^{n}(t_1\tau)$ to be non zero for $i \neq k$.

As in [1] we assume that the objective is to minimize the expected global cost, i.e.,

$$\min_{u_1,u_2} E[J(u_1,u_2,H)] \tag{2-4}$$

where only decentralized decision rules of the form

$$u_i = \begin{cases} 0 & , H^0 \text{ is declared to have occured} \\ 1 & , H^1 \text{ is declared to have occured} \end{cases} ,$$

in which $u_i$ is a function of $y_i(t)$ alone are allowed. Recall from [1] that this formulation of the cost allows us to consider both cases in which decisions $u_i$ are made strictly by local agents and cases in which tentative local decisions are made which are sent to a global decision maker for a final decision.

The assumption that the sensor noise is correlated arises realistically in many detection problems. For example, in passive detection (e.g., passive sonar or passive electromagnetic signal detection) noise is likely to be correlated between nearby sensors (e.g. ambient sea or atmospheric noise). Even in active detection correlated noise may be a realistic assumption. For example, the radar returns from a fluctuating target are likely to be correlated if the sensors are located close together.

## SECTION 3

## SUBOPTIMAL DECISION POLICY

Let us attack the problem of section 2 using the technique and notation of ([1], sections 4 and 5). This simply involves expanding the observations in a Karhunen-Loeve (K-L) expansion [3] and considering the problems formed by truncating the infinite series of K-L coefficients.

Recall that we expand $y_i(t)$ via

$$y_i(t) = \underset{K \to \infty}{\text{l.i.m.}} \sum_{k=1}^{K} y_i^k \phi_i^k(t) \quad , \quad T_o \leq t \leq T_f \tag{3-1}$$

where the $\phi_i^k(t)$ satisfy

$$\tilde{\lambda}_i^k \phi_i^k(t) = \int_{T_o}^{T_f} K_i^c(t,u)\phi_i^k(u)du \quad , \quad T_o \leq t \leq T_f \tag{3-2}$$

Under $H^1$ we have

$$y_i^k = s_i^k + n_i^k \tag{3-3}$$

where

$$s_i^k \triangleq \int_{T_o}^{T_f} \sqrt{E_i}\, s_i(t)\phi_i^k(t)dt \tag{3-4}$$

$$n_i^k \triangleq \int_{T_o}^{T_f} n_i(t)\phi_i^k(t)dt, \tag{3-5}$$

while under $H^0$ we have that

$$y_i^k = n_i^k \ . \tag{3-6}$$

As in [1]

$$E\{n_i^k\} = 0 \tag{3-7}$$

$$E\{(n_i^k)^2\} = \tilde{\lambda}_i^k + N_i^2 \tag{3-8}$$

$$E\{n_i^\ell n_i^k\} = 0 \qquad \ell \neq k \tag{3-9}$$

however, unlike [1], the K-L coefficients are generally correlated. That is,

$$E\left\{(y_1^\ell - E\{y_1^\ell | H^j\})(y_2^k - E\{y_2 | H^j\}) | H^j\right\} = \int_{T_o}^{T_f} \int_{T_o}^{T_f} \phi_1^\ell(t) K_{12}^n(t,\tau) \phi_2^k(\tau) dt d\tau \tag{3-10a}$$

$$\triangleq c^{\ell k} \tag{3-10b}$$

which is not generally zero.

From [1] the optimal decision law based on the first K coefficients of the K-L expansion is given by the solution of the following two equations:

$$\ell r_1(\underline{y}_1^K) \underset{1}{\overset{0}{\gtrless}} \frac{\Delta J_1 + \Delta J_2 \displaystyle\int_{\underline{y}_2^K | u_2 = 0} p(\underline{y}_2^K | \underline{y}_1^K, H^1) d\underline{y}_2^K}{\Delta J_3 + \Delta J_4 \displaystyle\int_{\underline{y}_2^K | u_2 = 0} p(\underline{y}_2^K | \underline{y}_1^K, H^0) d\underline{y}_2^K} \tag{3-11}$$

$$\ell r_2(\underline{y}_2^K) \underset{1}{\overset{0}{\gtrless}} \frac{\Delta J_5 + \Delta J_2 \displaystyle\int_{\underline{y}_1^K | u_1 = 0} p(\underline{y}_1^K | \underline{y}_2^K, H^1) d\underline{y}_1^K}{\Delta J_6 + \Delta J_4 \displaystyle\int_{\underline{y}_1^K | u_1 = 0} p(\underline{y}_1^K | \underline{y}_2^K, H^0) d\underline{y}_1^K} \tag{3-12}$$

Note that the decision law for each sensor is required to determine the region of integration for the right hand side (r.h.s.) of equations 3-11 and 3-12. Unlike the uncorrelated noise case the structure of the optimal distribution decision law is not of the form

$$\ell r_i(\underline{y}_i^K) \underset{1}{\overset{0}{\gtrless}} t_i^K \tag{3-13}$$

since the r.h.s of equations 3-11 and 3-12 depend on $\underline{y}_1^K$ and $\underline{y}_2^K$. Thus we cannot convert the functional equations 3-11 and 3-12 into equivelent algebraic equations for $t_i^K$.

No analytic solution to these equations has been determined and numerical techniques do not seem to be computationally tractable. We are thus motivated to examine suboptimal decision laws for the problem of distributed detection with correlated sensor noise.

The approach we take is to note that equations 3-11 and 3-12 can be written as

$$\ell r_i(\underline{y}_i^K) \underset{1}{\overset{0}{\gtrless}} t_i^K(\underline{y}_i^K) \tag{3-14}$$

and that by replacing $t_i^K(\underline{y}_i^K)$ with a non data dependent threshold $t_i^K$, the functional equations reduce to algebraic equations of the form considered in [1] and [2]. This is equivalent to assuming locally optimized signal processing (i.e., signal processing that would be optimal if only one sensor at a time were considered) but with a globally optimized threshold.

This approach has the further advantage of being simple to implement: the limit as $K \to \infty$ of the likelihood ratio $\ell_i(\underline{y}_i^K)$ is a scalar computed by integrating $y_i(t)$ against a deterministic function $g_i(t)$. The optimal solution need not necessarily be a finite dimensional test and thus considerations of implementability might motivate examination of this <u>decentralized likelihood ratio test</u> (DLRT) even if equations 3-11 and 3-12 could be solved.

The limit of $\ln[\ell r_i(\underline{y}_i^K)]$ as $K \to \infty$ is given by

$$\ell_i \triangleq \ln[\ell r_i(y_i)] = -\int_{T_o}^{T_f}\left(y_i(t) - \frac{1}{2}\sqrt{E_i}s_i(t)\right)g_i(t)dt \tag{3-15}$$

where

$$g_i(t) \triangleq \sqrt{E_i} \int_{T_o}^{T_f} Q_i(t,u)s_i(u)du \tag{3-16}$$

and $Q_i$ satisfies

$$\int_{T_o}^{T_f} K_{ii}^n(t,u)Q_i(u,v)du = \delta(t-v). \tag{3-17}$$

The detection laws we shall consider thus are of the form

$$\ell_i \underset{1}{\overset{0}{\gtrless}} \tau_i \tag{3-18}$$

To determine the optimal $\tau_i$ we note that the $\ell_i$ are Gaussian, and determine the mean, variance and correlation of the $\ell_i$ under each hypothesis. We compute

$$E\{\ell_i|H^o\} = +\frac{\sqrt{E_i}}{2} \int_{T_o}^{T_f} s_i(t)g_i(t)dt \triangleq +m_i, \tag{3-19}$$

$$E\{\ell_i|H^1\} = -\frac{\sqrt{E_i}}{2}\int_{T_o}^{T_f} s_i(t)g_i(t)dt \overset{\Delta}{=} -m_i, \tag{3-20}$$

$$E\left\{(\ell_i - E\{\ell_i|H^j\})^2|H^j\right\} = \sqrt{E_i}\int_{T_o}^{T_f} g_i(t)s_i(t)dt = 2m_i, \tag{3-21}$$

$$E\left\{(\ell_1 - E\{\ell_1|H^j\})\ (\ell_2 - E\{\ell_2|H^j\}|H^j\right\} = \int_{T_o}^{T_f} g_1(t)\left\{\int_{T_o}^{T_f} K_{12}^n(t,u)g_2(u)du\right\}dt \overset{\Delta}{=} c. \tag{3-22}$$

The cost associated with any pair of thresholds $\tau_1$ and $\tau_2$ is given (to within an constant independent of $\tau_1$ by

$$\Delta J_1 \int_{\tau_1}^{\infty} p(\ell_1|H^1)d\ell_1 p(H^1) - \Delta J_3 \int_{\tau_1}^{\infty} p(\ell_1\ H^0)d\ell_1 p(H^0)$$

$$+\Delta J_2 \int_{\tau_1}^{\infty}\int_{\tau_2}^{\infty} p(\ell_1,\ell_2|H^1)d\ell_2 d\ell_1 p(H^1) - \Delta J_4 \int_{\tau_1}^{\infty}\int_{\tau_2}^{\infty} p(\ell_1,\ell_2|H^0)d\ell_2 d\ell_1 p(H^0). \tag{3-23}$$

One half of a necessary condition for optimality of the $\tau_i$ can be obtained by differentiating 3-23 with respect to $\tau_1$ and setting the result to zero. This yields*

$$-\Delta J_1 p(t_1|H^1) + \Delta J_3 p(\tau_1|H^0) - \Delta J_2 \int_{\tau_2}^{\infty} p(\tau_1,\ell_2|H^1)d\ell_2 + \Delta J_4 \int_{\tau_2}^{\infty} p(\tau_1,\ell_2|H^0)d\ell_2 = 0\ . \tag{3-24}$$

Using equations 3-19 through 3-22 we can rewrite 3-24 as

---

* We assume $p(H^0) = p(H^1) = 1/2$ for ease in notation.

$$\tau_1 = \ell n \left\{ \frac{\Delta J_1 + \Delta J_2 \left(1 - \mathrm{erf}\left[\left(\tau_2 + m_2 - \rho \sqrt{m_2/m_1}\,(\tau_1 + m_1)\right) / \sqrt{2m_2(1-\rho^2)}\,\right]\right)}{\Delta J_3 + \Delta J_4 \left(1 - \mathrm{erf}\left[\left(\tau_2 - m_2 - \rho \sqrt{m_2/m_1}\,(\tau_1 - m_1)\right) / \sqrt{2m_2(1-\rho^2)}\,\right]\right)} \right\} \tag{3-25}$$

Similar computations yield the second half of the necessary condition:

$$\tau_2 = \ell n \left\{ \frac{\Delta J_5 + \Delta J_2 \left(1 - \mathrm{erf}\left[\left(\tau_1 + m_1 - \rho \sqrt{m_1/m_2}\,(\tau_2 + m_2)\right) / \sqrt{2m_1(1-\rho^2)}\,\right]\right)}{\Delta J_6 + \Delta J_4 \left(1 - \mathrm{erf}\left[\left(\tau_1 - m_1 - \rho \sqrt{m_1/m_2}\,(\tau_2 - m_2)\right) / \sqrt{2m_1(1-\rho^2)}\,\right]\right)} \right\} . \tag{3-26}$$

Equations 3-25 and 3-26 are necessary conditions for the optimality of $\tau_1$ and $\tau_2$. In general these equations have multiple solutions, in which case one must evaluate the cost associated with each solution to determine the best detection law in this class.

## SECTION 4

## EXAMPLE

In this section we consider an example involving a scalar observation with correlated noise. First we introduce the observation model and rewrite the necessary conditions, then in subsections 4.1 and 4.2 we consider two different global cost functions.

Assume that the observations are given by

$$H^1: \; y_i(t) = \sqrt{2E_i}\,\sin(2\pi t)+n_i(t) \qquad 0\le t\le 1 \tag{4-1a}$$

$$H^0: \; y_i(t) = \qquad n_i(t) \qquad 0\le t\le 1 \tag{4-1b}$$

where $n_i(t)$ is zero-mean unit variance white Gaussian noise with

$$E\{n_1(t)n_2(t)\} = \rho\,\delta(t-\tau). \tag{4-2}$$

We assume $p^0 = p^1 = 1/2$ and note that, from 3-15,

$$\ell_i = -\int_0^1 y_i(t)\,\sqrt{2E_i}\,\sin(2\pi t)dt+E_i/2. \tag{4-3}$$

The detection law can thus be rewritten as

$$y_i^1 \triangleq \sqrt{2}\int_0^T y_i(t)\sin(2\pi t)dt \underset{0}{\overset{1}{\gtrless}} T_i \triangleq \frac{\sqrt{E_i}}{2} - \frac{1}{\sqrt{E_i}}\,\tau_i. \tag{4-4}$$

Defining

$$a_i = \sqrt{E_i} \tag{4-5}$$

and noting that

$$m_i = E_i/2, \tag{4-6}$$

we have from equations 3-25 and 3-26

$$T_1 = \frac{a_1}{2} - \frac{1}{a_1} \ln\left\{\frac{\Delta J_1 + \Delta J_2 \ \mathrm{erf}[(T_2 - a_2 - \rho(T_1 - a_1))/\sqrt{1-\rho^2}]}{\Delta J_3 + \Delta J_4 \ \mathrm{erf}[(T_2 - \rho T_1)/\sqrt{1-\rho^2}]}\right\} \tag{4-7}$$

$$T_2 = \frac{a_2}{2} - \frac{1}{a_2} \ln\left\{\frac{\Delta J_5 + \Delta J_2 \ \mathrm{erf}[(T_1 - a_1 - \rho(T_2 - a_2))/\sqrt{1-\rho^2}]}{\Delta J_6 + \Delta J_4 \ \mathrm{erf}[(T_1 - \rho T_2)/\sqrt{1-\rho^2}]}\right\} \tag{4-8}$$

## 4.1 BAYESIAN FORMULATION

We assume that the thresholds $T_i$ are to be selected so as to minimize the expected Bayes cost defined by equation 2-4 where

$$J = \begin{cases} 0 & \text{if } u_1 = u_2 = H \\ 1 & \text{if } u_1 \neq u_2 \\ k & \text{if } u_1 = u_2 \neq H \end{cases} \tag{4-9}$$

This type of cost criterion may arise if a decisionmaker is associated with each sensor and must act on the basis of that sensor's data alone. For example, if the sensors are radars and the decisions determine whether or not missiles are fired at enemy targets, then having two missed detections may be much more serious than having one missed detection. This situation would be modeled by choosing $k > 2$, so that a double error is more than twice as costly as two single errors.

For J defined by 4-9, equations 4-7 and 4-8 become

$$T_1 = \frac{a_1}{2} - \frac{1}{a_i} \ln \left\{ \frac{1+(k-2)\,\mathrm{erf}\left[\dfrac{T_2-a_2-\rho(T_1-a_1)}{\sqrt{1-\rho^2}}\right]}{(k-1)-(k-2)\,\mathrm{erf}\left[-\dfrac{T_2-\rho T_1}{\sqrt{1-\rho^2}}\right]} \right\} \tag{4-10}$$

$$T_2 = \frac{a_2}{2} - \frac{1}{a_2} \ln \left\{ \frac{1+(k-2)\,\mathrm{erf}\left[\dfrac{T_1-a_1-\rho(T_2-a_2)}{\sqrt{1-\rho^2}}\right]}{(k-1)-(k-2)\,\mathrm{erf}\left[\dfrac{T_1-\rho T_2}{\sqrt{1-\rho^2}}\right]} \right\} \tag{4-11}$$

and we note that the "locally optimal" decision laws* ($T_1 = a_1/2$, $T_2 = a_2/2$) satisfy equations 4-10 and 4-11. This can be seen by noting that

$$\ln \left\{ \frac{1+(k-2)\,\mathrm{erf}\left[-\dfrac{a_2-\rho a_1}{2\sqrt{1-\rho^2}}\right]}{(k-1)-(k-2)\,\mathrm{erf}\left[\dfrac{a_2-\rho a_1}{\sqrt{1-\rho^2}}\right]} \right\} = \ln \left\{ \frac{1+(k-2)\left(1-\mathrm{erf}\left[\dfrac{a_2-\rho a_1}{2\sqrt{1-\rho^2}}\right]\right)}{(k-1)-(k-2)\,\mathrm{erf}\left[\dfrac{a_2-\rho a_1}{\sqrt{1-\rho^2}}\right]} \right\} \tag{4-12a}$$

$$= \ln \left\{ \frac{(k-1)-(k-2)\,\mathrm{erf}\left[\dfrac{a_2-\rho a_1}{2\sqrt{1-\rho^2}}\right]}{(k-1)-(k-2)\,\mathrm{erf}\left[\dfrac{a_2-\rho a_1}{2\sqrt{1-\rho^2}}\right]} \right\} \tag{4-12b}$$

---

*That is the laws which would be optimal for each sensor in isolation with respect to a minimum probability of error criterion.

$$= \ell n(1) = 0 \tag{4-12c}$$

Equations 4-10 and 4-11 are symmetric; if $(\hat{T}_1, \hat{T}_2)$ is a solution, then $(a_1-\hat{T}_1, a_2-\hat{T}_2)$ is also a solution. This is seen by noting that if

$$\hat{T}_1 = \frac{a_1}{2} - \frac{1}{a_1} \ell n \left\{ \frac{1+(k-2)\,\mathrm{erf}\left[\dfrac{\hat{T}_2-a_2-\rho(\hat{T}_1-a\ )}{\sqrt{1-\rho^2}}\right]}{(k-1)-(k-2)\,\mathrm{erf}\left[\dfrac{\hat{T}_2-\rho\hat{T}_1}{\sqrt{1-\rho^2}}\right]} \right\} \tag{4-13a}$$

then

$$a_1-\hat{T}_1 = \frac{a_1}{2} - \frac{1}{a_1} \ell n \left\{ \frac{(k-1)-(k-2)\,\mathrm{erf}\,[\hat{T}_2-\rho\hat{T}_1/\sqrt{1-\rho^2}]}{1+(k-2)\,\mathrm{erf}\,[\hat{T}_2-a_2-\rho(\hat{T}_1-a_1)/\sqrt{1-\rho^2}]} \right\} \tag{4-13b}$$

$$= \frac{a_1}{2} - \frac{1}{a_1} \ell n \left\{ \frac{(k-1)-(k-2)\left[1-\mathrm{erf}\left[\dfrac{(a_2-\hat{T}_2)-a_2-\rho\left((a_1-\hat{T}_1)-a_1\right)}{\sqrt{1-\rho^2}}\right]\right]}{1+(k-2)\left[1-\mathrm{erf}\,[(a_2-\hat{T}_2)-\rho(a_1-\hat{T}_1)/\sqrt{1-\rho^2}]\right]} \right\} \tag{4-13c}$$

$$= \frac{a_1}{2} - \frac{1}{a_1} \ell n \left\{ \frac{1+(k-2)\,\mathrm{erf}\left[\dfrac{(a_2-\hat{T}_2)-a_2-\rho(a_1-\hat{T}_1)-a_1)}{\sqrt{1-\rho^2}}\right]}{(k-1)-(k-2)\,\mathrm{erf}\left[\dfrac{(a_2-\hat{T}_2)-\rho(a_1-\hat{T}_1)}{\sqrt{1-\rho^2}}\right]} \right\} . \tag{4-13d}$$

The costs associated with these two symmetric solutions are identical and thus, if a solution with $T_i \neq a_i/2$ exists, only one of this pair need be evaluated.

Graphical solution of equations 4-10 and 4-11 shows that there are at most three solutions to these equations, and for certain values of k and $\rho$

there is only one solution (the "locally optimal" solution). In Figures 4-1 and 4-3 we plot the Bayes cost associated with the decentralized likelihood ratio test (DLRT) and with the "locally optimal" (LO) solutions for $a_1 = 1$, $a_2 = 2$ and various $\rho$ and k. In Figures 4-2 and 4-4 we plot the associated thresholds.

Figure 4-1 is a plot of the Bayes cost as a function of k for $\rho = 0$ and $\rho = .5$ . Note that, as $\rho$ increases, the optimal DLRT solution becomes much better than the LO solution. This is because the optimal solution skews the thresholds to avoid the double errors which are more common for larger $\rho$ and more costly for larger k (see Figure 4-2).

Note that for the LO solution the cost increases as an affine function of k since the probability of a double error is independent of k. The optimal DLRT solution never exceeds a Bayes cost of 1 since this cost can be obtained by setting the thresholds so that one detector always decides 0 while the other always decides 1.

Figure 4-3 is a plot of the Bayes cost as a function of $\rho$ for $k = 5$. Again we see that as $\rho$ increases the optimal DLRT solution becomes much better than the LO solution. This occurs because, as $\rho \rightarrow 1$, the probability of a double error (for fixed thresholds) increases. The optimal DLRT solution skews the thresholds (Figure 4-4) to decrease the probability of a double error. This yields a 27% decrease in cost over the LO solution.

These results indicate that in some cases there is a significant gain to be had by using the optimal decentralized likelihood ratio test rather than a naive approach which ignores the correlation between sensors. Since we cannot determine the globally optimal decision rule, it is not possible

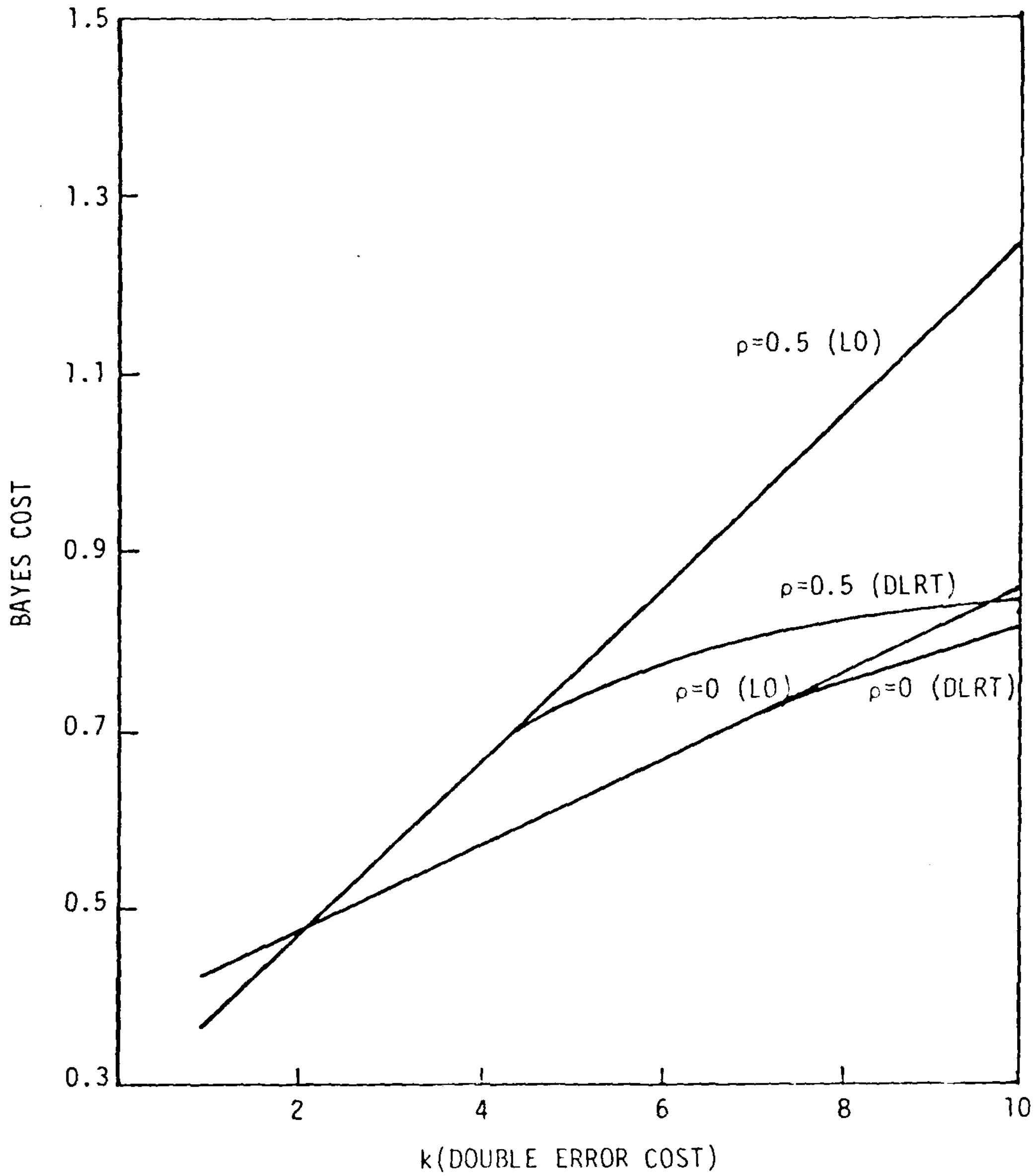


Figure 4-1. Bayes Cost as a Function of k for DLRT and LO Laws with $E_1=1$, $E_2=4$ and $\rho=0$ or 0.5.

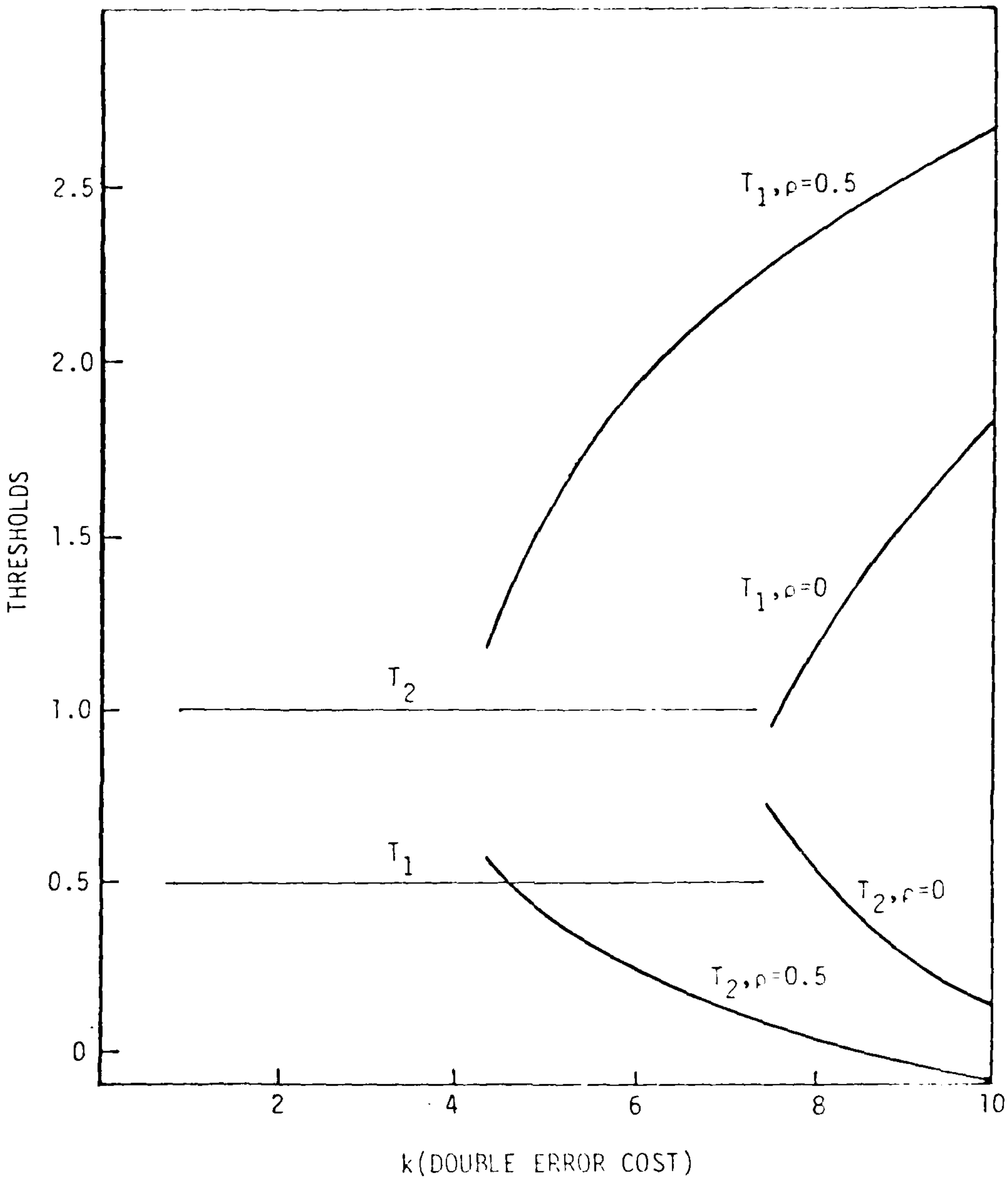


Figure 4-2. Thresholds as a Function of k for DLRT and LO Laws with $E_1=1$, $E_2=4$ and $\rho=0$ or 0.5.

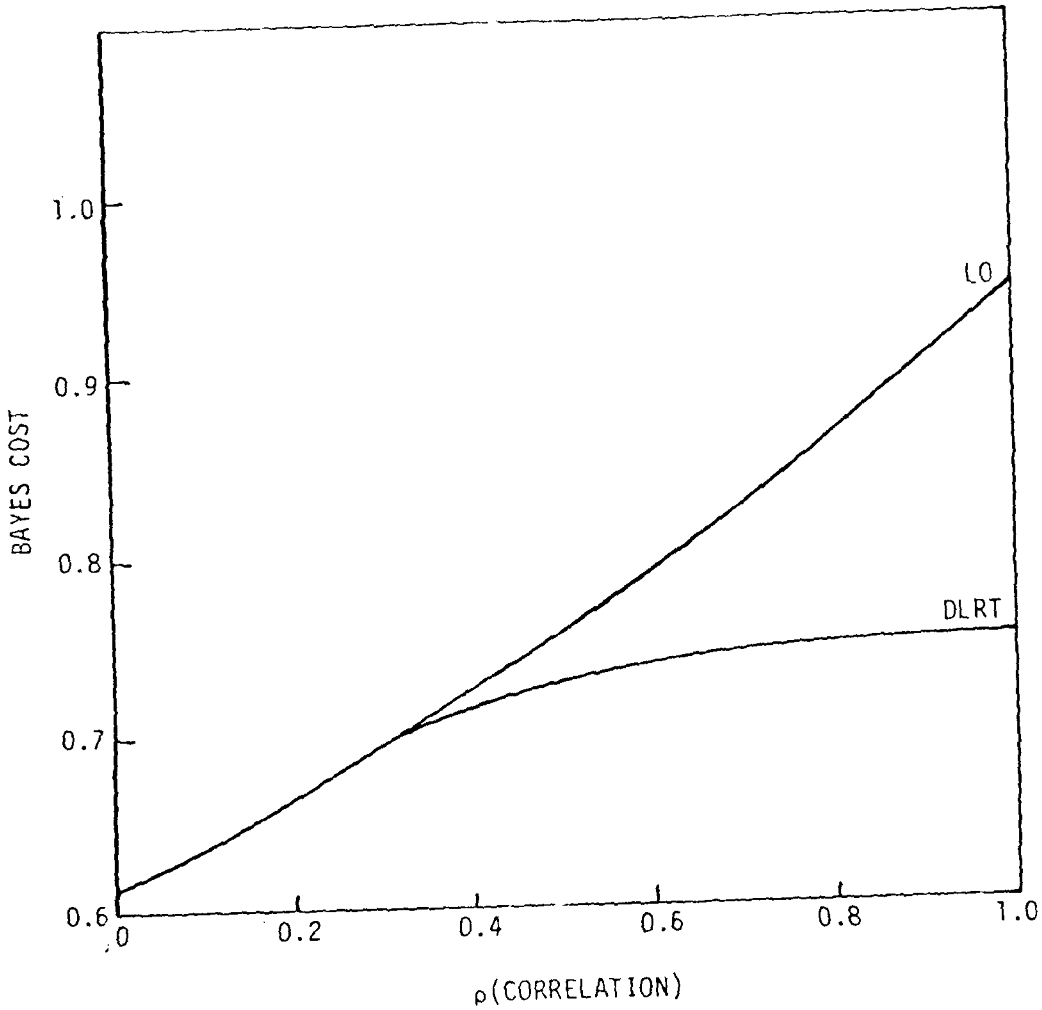


Figure 4-3. Bayes Cost as a Function of $\rho$ for DLRT and LO Laws with $E_1=1$, $E_2=4$ and $k=5$.

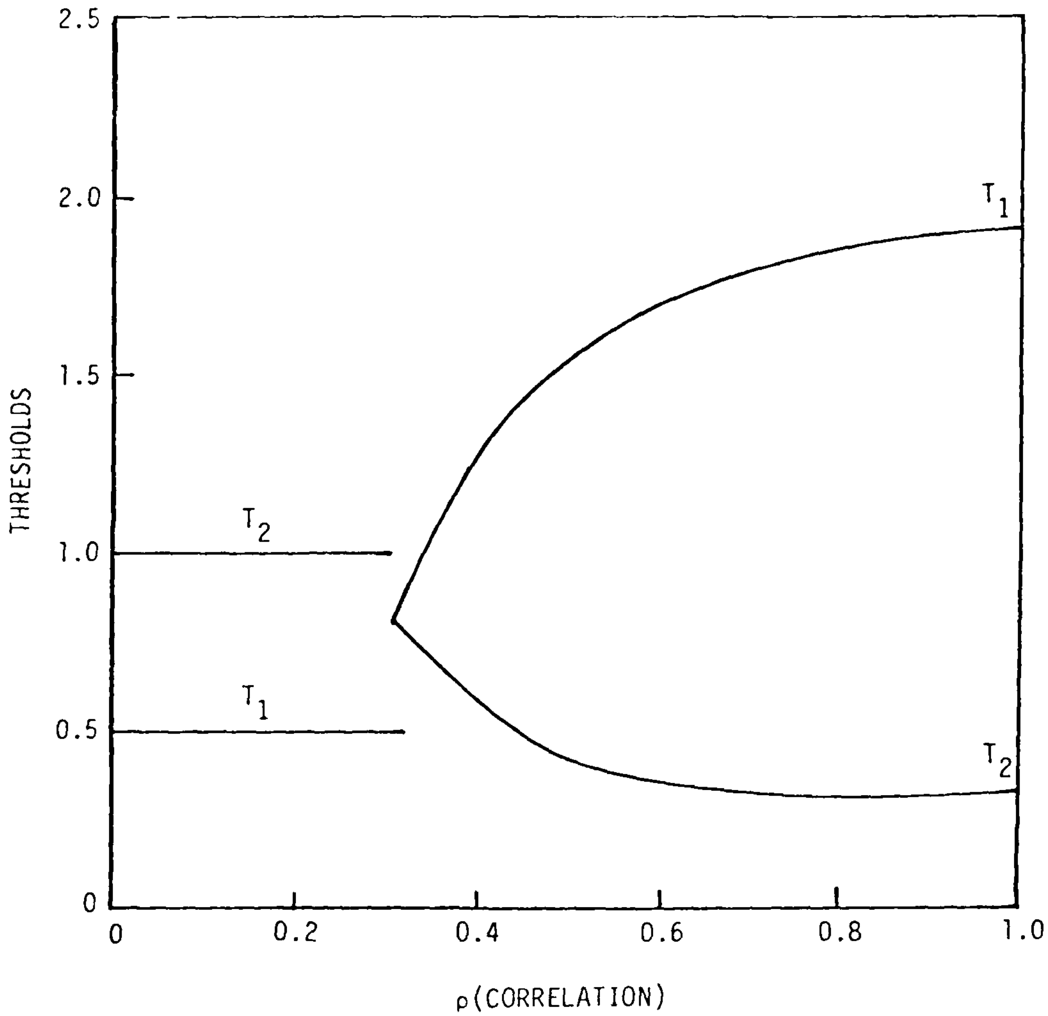


Figure 4-4. Thresholds as a Function of ρ for DLRT and LO Laws with $E_1$=1, $E_2$=4 and k=5.

to determine how much is lost by using the decentralized likelihood ratio test as a substitute. We discuss this issue in more detail in section 5.

## 4.2 FUSION CENTER DESIGN

In this subsection we assume that each sensor sends its local decision $u_i$ to a fusion center where a global decision u is made. We assume that the fusion center law is defined as $u = 1$ if and only if $u_1 = u_2 = 1$. We display the performance of the fusion center via receiver operating characteristic (ROC) curves which plot probability of detection versus probability of false alarm. In addition to plotting the performance of the optimal DLRT we also plot the performance of the optimal centralized detection law. This allows us to determine how the performance of the detection system is degraded by requiring that only local decisions rather than sensor data be transmitted to the fusion center.

The ROC curve can be obtained by varying the ratio of the cost of a false alarm to the cost of a missed detection. If we let the false alarm cost be unity and the missed detection cost be $\alpha$ then the necessary conditions for optimality become

$$T_1 = \frac{a_1}{2} - \frac{1}{a_1} \ln \left\{ \alpha \frac{1-\mathrm{erf}\left[\left(T_1 - a_1 - \rho(T_2 - a_2)\right) / \sqrt{1-\rho^2}\right]}{1-\mathrm{erf}\left[(T_1 - \rho T_2) / \sqrt{1-\rho^2}\right]} \right\} \tag{4-14}$$

$$T_2 = \frac{a_2}{2} - \frac{1}{a_2} \ln \left\{ \alpha \frac{1-\mathrm{erf}\left[\left(T_2 - a_2 - \rho(T_1 - a_1)\right) / \sqrt{1-\rho^2}\right]}{1-\mathrm{erf}\left[(T_2 - \rho T_1) / \sqrt{1-\rho^2}\right]} \right\}. \tag{4-15}$$

Figures 4-5 through 4-8 are plots of the performance of the optimal

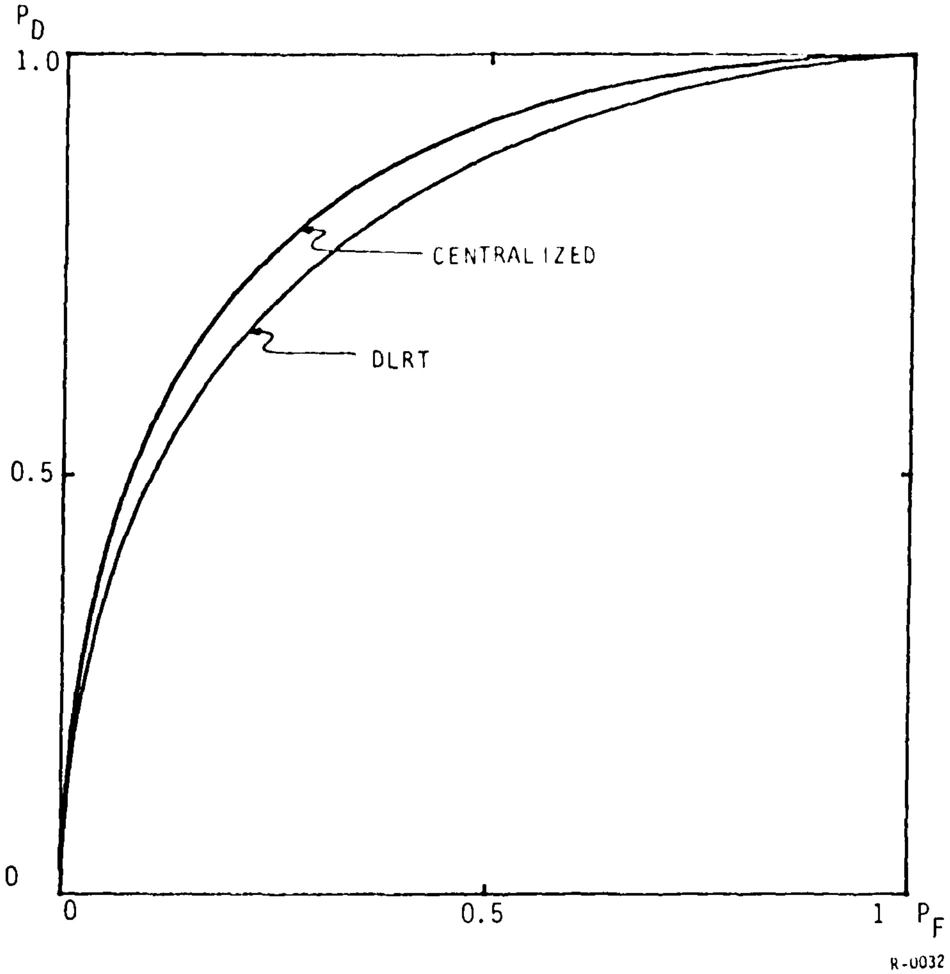


Figure 4-5. ROC Curves for Centralized and DLRT Laws with $E_1=1$, $E_2=1$ and $\rho=0.0$.

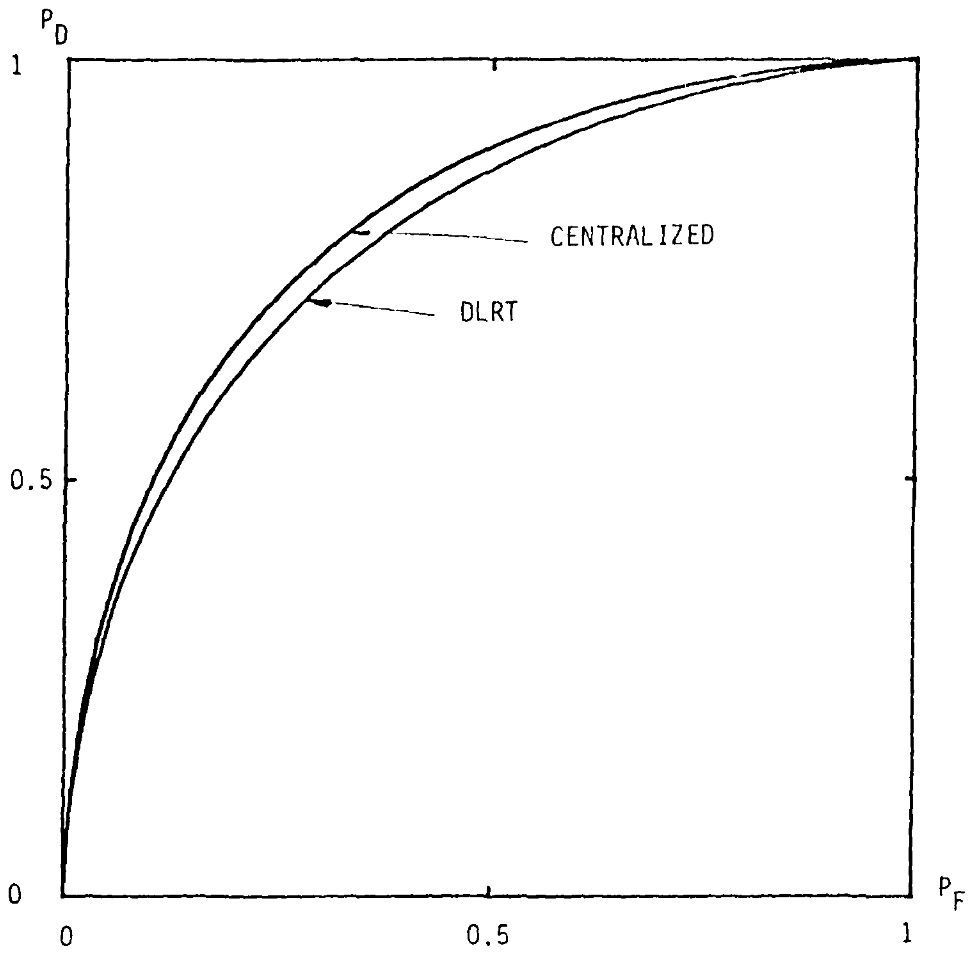


Figure 4-6. ROC Curves for Centralized and DLRT Laws with $E_1=1$, $E_2=1$ and $\rho=0.25$.

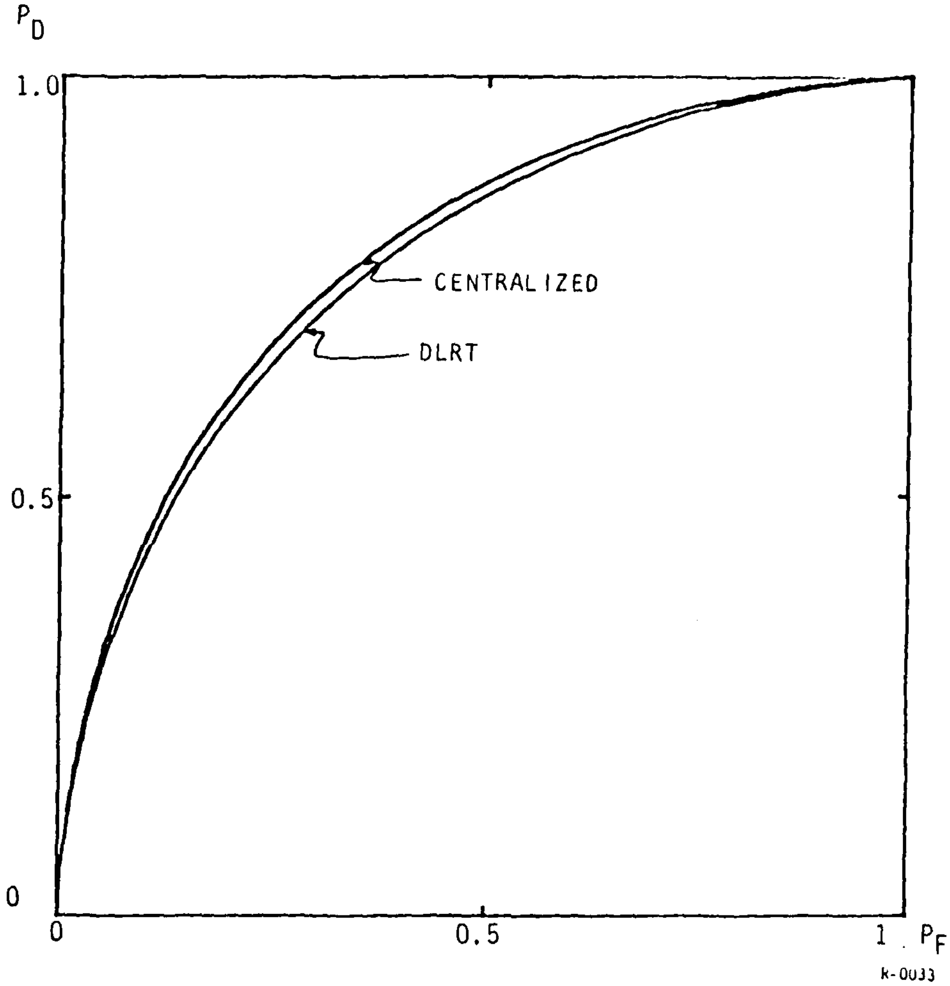


Figure 4-7. ROC Curves for Centralized and DLRT Laws with $E_1$=1, $E_2$=1 and $\rho$=0.5.

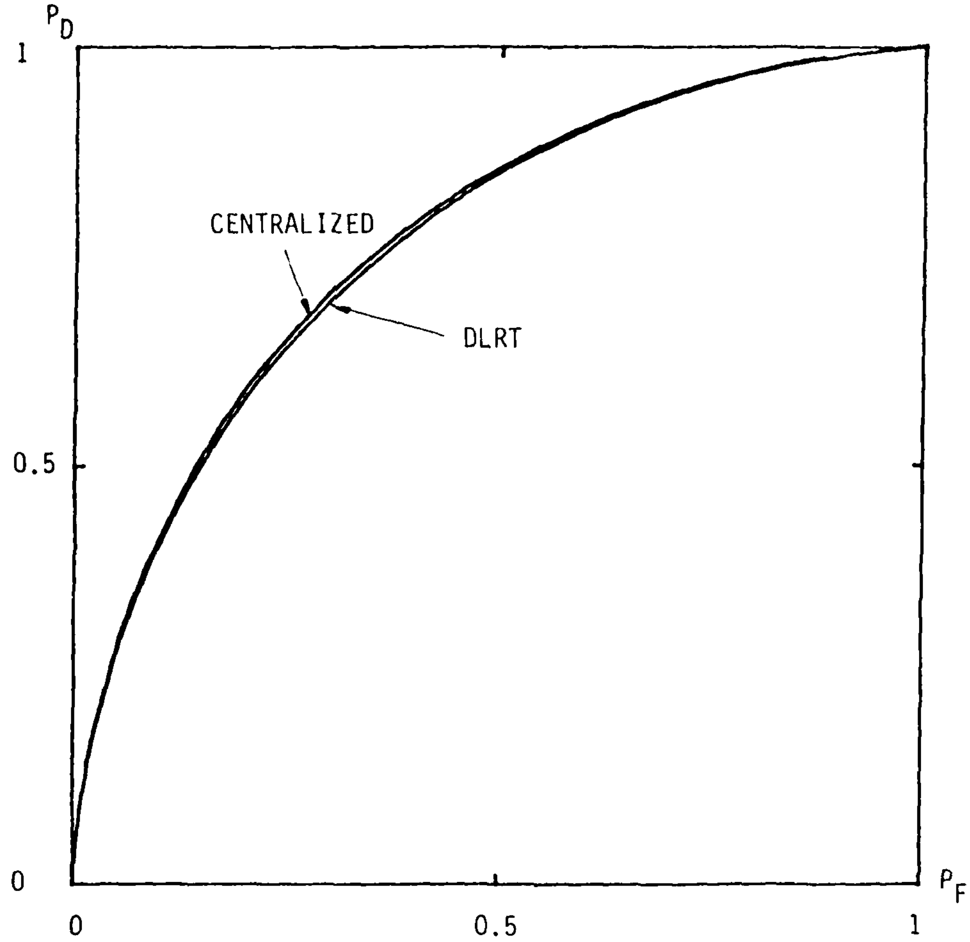


Figure 4-8. ROC Curves for Centralized and DLRT Laws with $E_1=1$, $E_2=1$ and $\rho=0.75$.

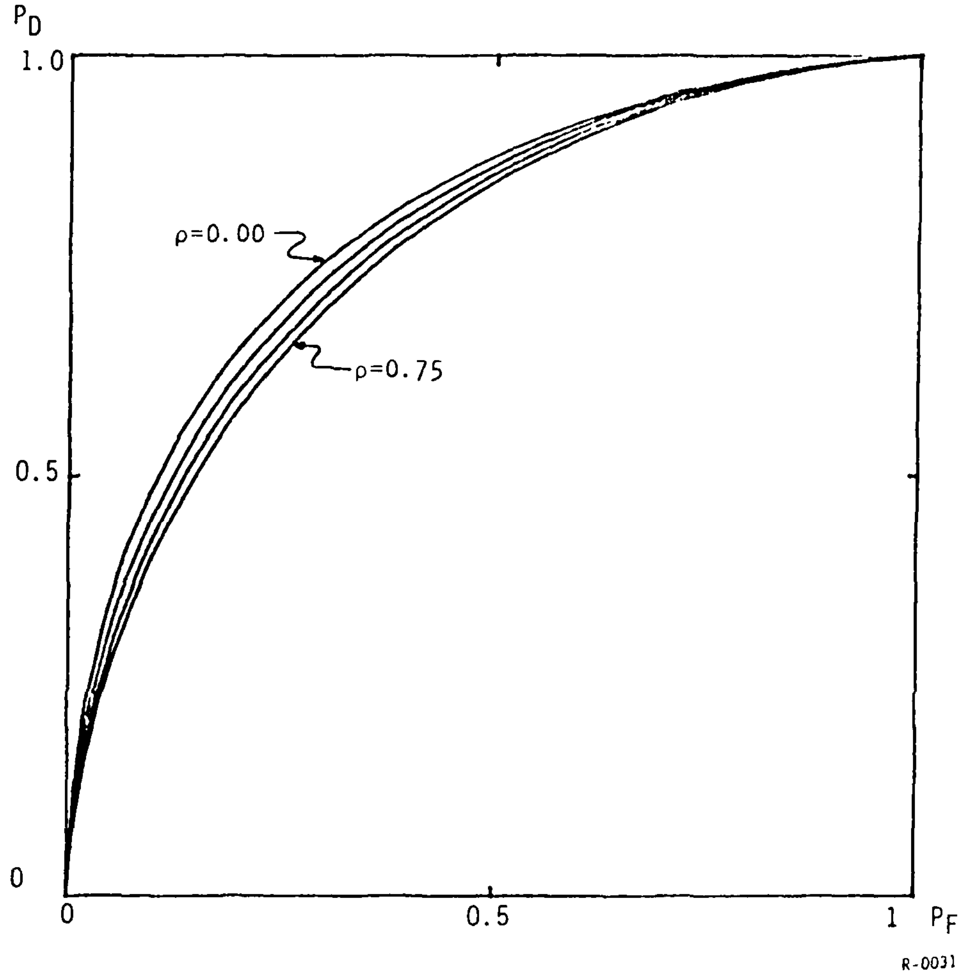


Figure 4-9. ROC Curves for DLRT Laws with $E_1$=1, $E_2$=1 and ρ=0.0, 0.25, 0.50 and 0.75.

centralized test and that of the DLRT for the case where the sensors are identical, $a_1 = a_2 = 1$. We see that as $\rho$ increases the centralized and DLRT results become more and more similar. Figure 4-9 shows that as $\rho$ increases the performance degrades.

This occurs for the centralized problem because as $\rho$ increases the information available for decision making effectively decreases from 2 independent observations with $\rho = 0$ to 1 with $\rho = 1$. As $\rho$ increases in the DLRT case, each sensor has a better and better indication of what the other observation was, and thus the centralized solution can be more closely approximated by the decentralized solution.

Note however that the performance of the DLRT while more closely approaching that of the centralized test degrades as $\rho \to 1$. This can be understood by noting that if p is the probability of a local decision being wrong then the probability of a double error is approximately $p^2$ when $\rho = 0$ but is p when $\rho = 1$. Since the fusion center always makes an incorrect decision when both local decisions are wrong, the performance degrades as $\rho \to 1$.

The phenomenon of the DLRT and centralized results growing closer together as $\rho$ increases is not universal. Figures 4-10 through 4-14 illustrate the behavior of these two decision laws for the case of asymmetric sensors, $a_1 = 1$ and $a_2 = 2$. In Figure 4-10 we see that for the DLRT as $\rho \to 1$ the performance degrades. The reason is exactly as in the case of $a_1 = a_2 = 1$. For the centralized case however the performance becomes perfect as $\rho \to 1$. This occurs (Figs. 4-11 through 4-14) because by differencing the sensor observations one has

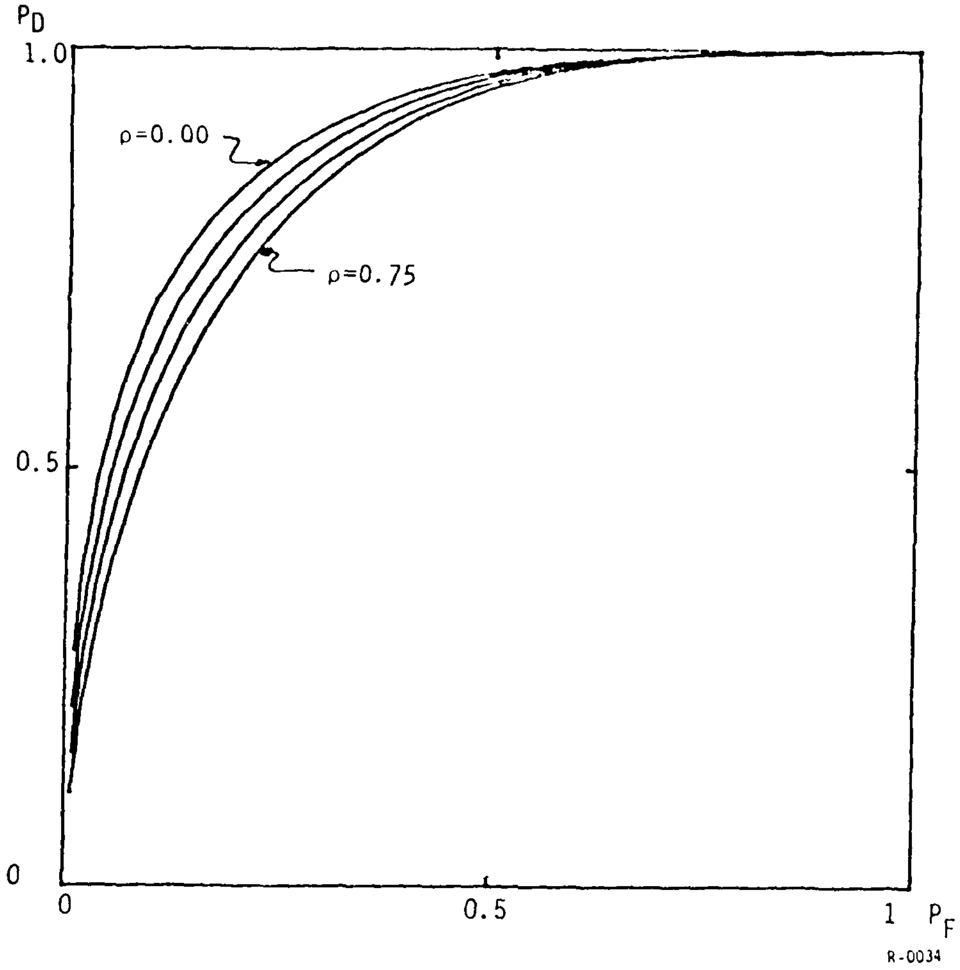


Figure 4-10. ROC Curves for DLRT Laws with $E_1=1$, $E_2=4$ and $\rho=0.0$, 0.25, 0.50 and 0.75.

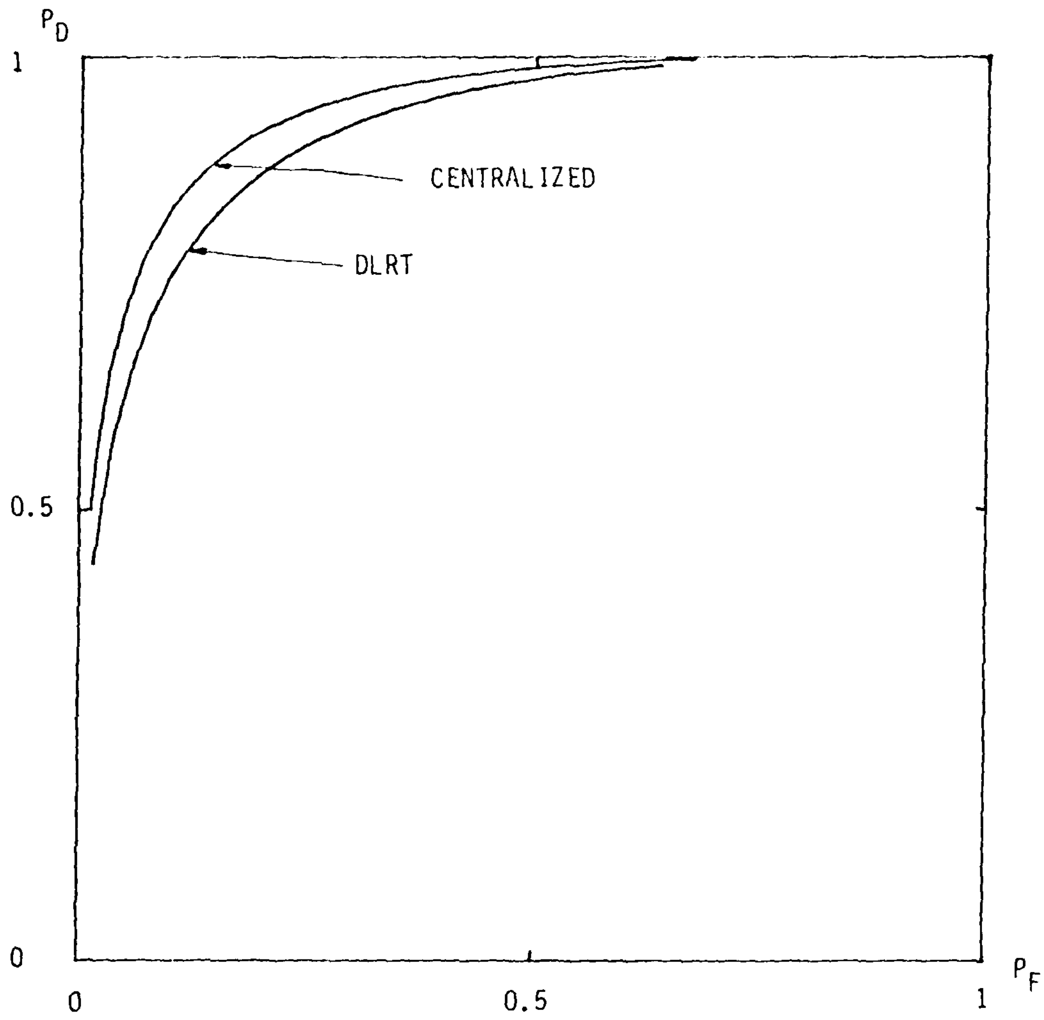


Figure 4-11. ROC Curves for Centralized and DLRT Laws with $E_1=1$, $E_2=4$ and $\rho=0.0$.

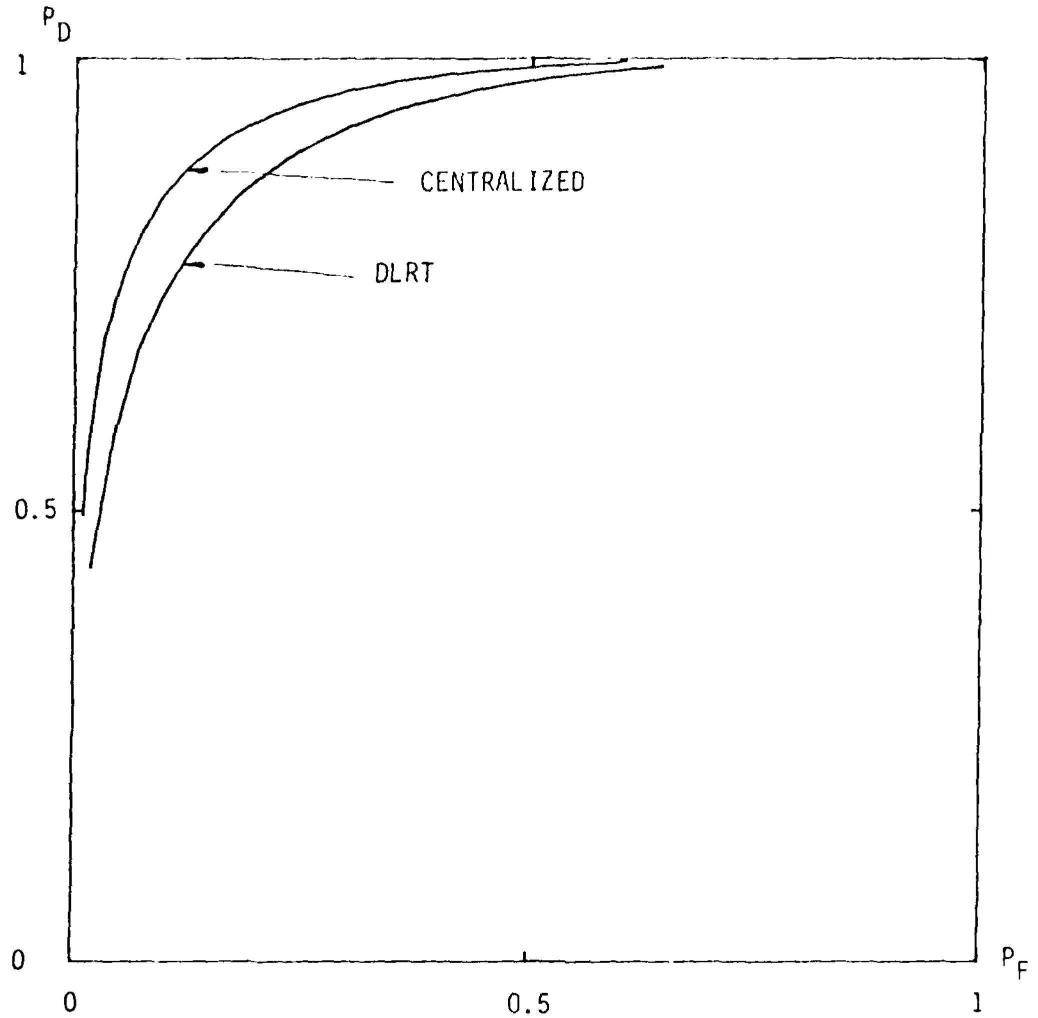


Figure 4-12. ROC Curves for Centralized and DLRT Laws with $E_1=1$, $E_2=4$ and $\rho=0.25$.

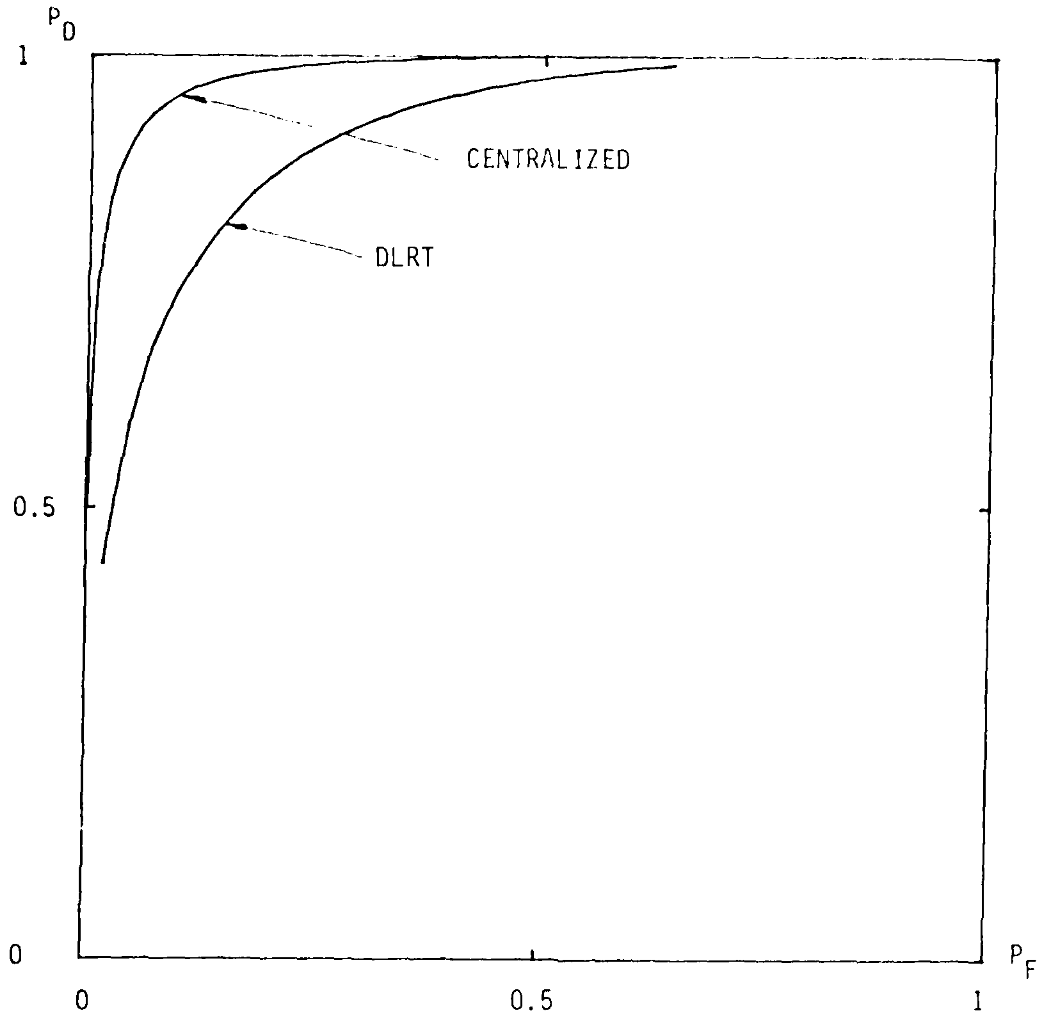


Figure 4-13. ROC Curves for Centralized and DLRT Laws with $E_1=1$, $E_2=4$ and $\rho=0.5$.

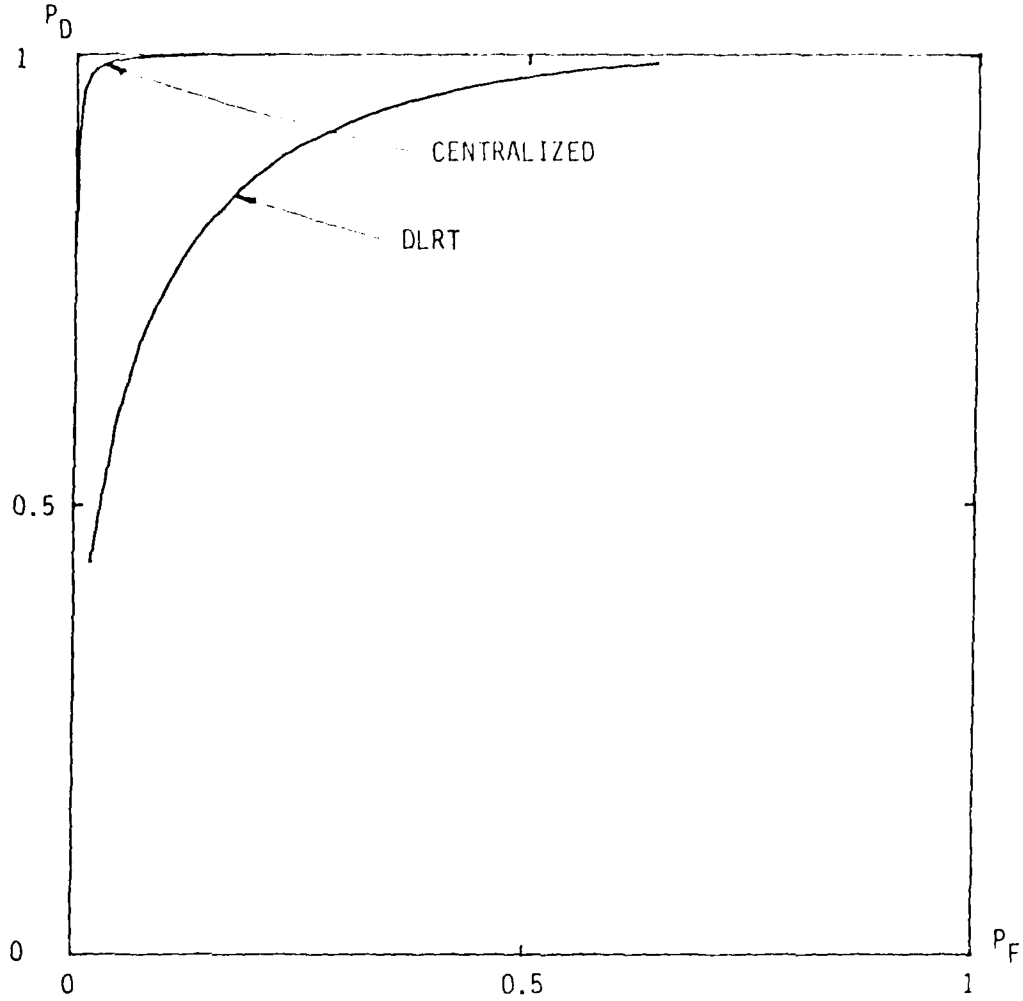


Figure 4-14. ROC Curves for Centralized and DLRT Laws with $E_1=1$, $E_2=4$ and $\rho=0.75$.

$$H^1: \quad \Delta y(t) = y_1(t)-y_2(t) = (\sqrt{2}-1)\sin(2\pi t)+n_1(t)-n_2(t) \tag{4-16a}$$

$$H^0: \quad \Delta y(t) = y_1(t)-y_2(t) = n_1(t)-n_2(t) \ . \tag{4-16b}$$

As $\rho \to 1$, $\Delta y \to (\sqrt{2}-1)\sin(2\pi t)$ if $H^1$ is true and $\Delta y = 0$ if $H^0$ is true, thus perfect detection is possible.

These graphs illustrate that the price involved in using a DLRT law is strongly dependent on the problem being considered. Similarly, the benefit in reduction of communications requirements is highly problem dependent. It is thus the case that detailed analysis is required to determine whether a DLRT or a centralized detection law should be implemented in a given situation. An interesting questions remains however: is the poor performance of the DLRT in this second problem due to the decentralization constraint or due to the fact that the DLRT need not be the optimal decentralized law? We briefly address this question in the next section.

## SECTION 5

## ANALYSIS

For the case considered in the previous section, the structure of the optimal distributed decision law is known. More generally, if $s_1(t) = s_2(t)$ and the noise corrupting the observations is white Gaussian noise, the optimal distributed decision law structure is known. Consider,

$$\left.\begin{aligned} H^1:\quad y_i(t) &= \sqrt{E_i}\,s(t)+n_i(t) \qquad (5\text{-}1a)\\ H^0:\quad y_i(t) &= n_i(t) \qquad (5\text{-}1b)\end{aligned}\right\} \quad 0\le t\le T$$

where $n_i(t)$ is zero-mean unit spectral height white Gaussian noise with

$$E\{n_1(t)n_2(t)\} = \rho\delta(t-\tau). \tag{5-2}$$

As usual we expand the signal into K-L coefficients where we now choose the orthonormal functions $\phi_i^k(t)$ so that

$$\phi_1^k(t) = \phi_2^k(t) \triangleq \phi^k(t)\ , \quad 0\le t\le T \tag{5-3}$$

and

$$\phi^1(t) = s(t)\ , \quad 0\le t\le T\ . \tag{5-4}$$

We have

$$y_i(t) = \underset{K\to\infty}{\text{l.i.m.}} \sum_{k=1}^{K} y_i^k \phi^k(t) \tag{5-5}$$

and

$$y_i^1 = \begin{cases} \int_0^T s(t)[\sqrt{E_i}s(t)+n_i(t)]dt = \sqrt{E_i}+w_i^1 & : H^1 \quad (5\text{-}6a) \\ \int_0^T s(t)n_i(t)dt = w_i^1 & : H^0, \end{cases}$$

$$y_i^k = \begin{cases} \int_0^T \phi^k(t)[\sqrt{E_i}s(t)+n_i(t)]dt = w_i^k & : H^1 \quad (5\text{-}7a) \\ \int_0^T \phi^k(t)n_i(t)dt = w_i^k & : H^0, \quad (5\text{-}7b) \end{cases}$$

Note that the $y_i^k$ for $k \geq 2$ have the same statistics under both hypotheses and thus any optimal decision law will use only the $y_i^1$'s. This implies that for problems with observations given by 5-1 the optimal decision laws are functions of a scalar "observation" $y_i^1$.

For this problem <u>any</u> decision law can be specified by defining the regions of the real line in which $y_i^1$ must lie for a decision of $H^1$ to be made. These regions can be specified by their endpoints, and thus the decision law can be characterized by a set of endpoints or thresholds. This is not the cause for arbitrary vector or waveform observations since, for these problems, a decision law must specify regions in a higher dimensional space.

The DLRT for both problems in the previous section tests a scalar against a threshold. If $y_i^1$ is greater than the threshold, then $H^1$ is declared, otherwise $H^0$ is declared. The DLRT is thus optimal if the optimal decentralized test is such that there is only one region in which $H^1$ is declared. This need not, however, be the case in general.

Consider Figure 5-1 wherein the DLRT law for a Bayes problem with $\rho = 1$ and large k are illustrated. The laws are given by a tick mark on a line where the threshold $T_i$ lies. Note that, since $\rho = 1$, there are 3 regions of interest. In A, both local decisions are $u_i = 0$, in B, $u_1 = 0$ and $u_2 = 1$ and in C both $u_i = 1$.

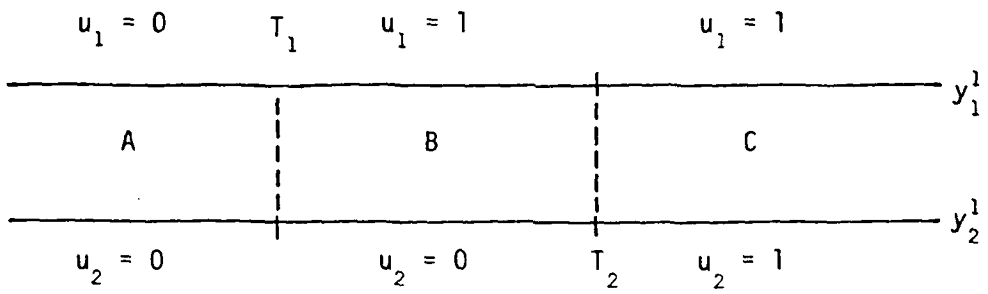


Figure 5-1. Decision Laws for Bayes Problem with $\rho$=1.

A decision law with exactly identical performance is illustrated in Figure 5-2. Here in A both $u_i = 0$, in C both $u_i = 1$ and in B one $u_i = 0$ and the other $u_i = 1$. This shows that the optimal decentralized law may have multiple thresholds.

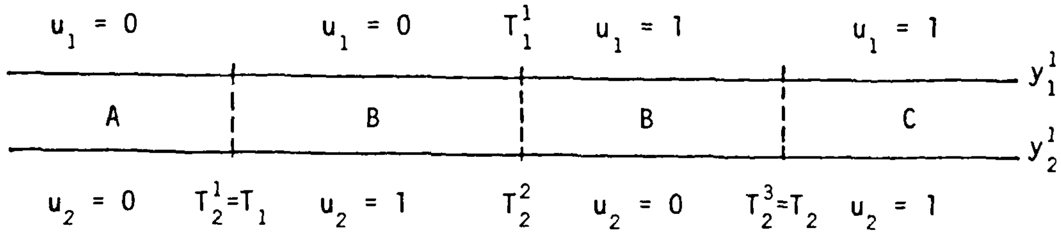


Figure 5-2. An Equivalent Decision Law.

We have derived necessary conditions for the optimality of local decision laws using up to 3 thresholds. Only for problems with $|\rho| > .9$ have we found local decision laws which satisfy the necessary conditions and have more than one threshold. For all of these cases the performance of the multiple threshold law was inferior to that of the optimal DLRT.

While this does not prove that the DLRT is optimal for this example it does indicate that the poor performance of the DLRT for the case where $a_1 = 1$ and $a_2 = 2$ is probably due to the requirement of a decentralized rule rather than the restriction of our attention to the particular case of DLRT laws.

## SECTION 6

## SUMMARY AND EXTENSIONS

In this paper we have extended the results of Lauer and Sandell [1] to the case of correlated observation noise. As in [2] this leads to a pair of functional equations which must be solved to determine the optimal decentralized decision rule. Since these equations are not readily solved, we introduce a suboptimal law, the decentralized likelihood ratio test.

Several examples were considered, contrasting the DLRT to naive decentralized laws and to optimal centralized laws. These examples indicate the advantages of using the optimal DLRT over a naive "locally optimal" rule and also indicated that the DLRT may perform almost as well or much worse than a centralized law depending on the given situation. This indicates that judgement must be exercised when deciding when a decentralized decision rule should be used.

Finally, we presented arguments to the effect that, while the DLRT may not be optimal even for problems which reduce to scalar tests, the limitations indicated by the examples of section 4 are grounded in the requirement of decentralization rather than being due to consideration of suboptimal decentralized decision laws.

In ongoing research we are considering the problem of detecting an unknown signal in noise using distributed sensors. The approach we take is similar to the one herein — we consider suboptimal laws based on testing the local log likelihood ratio against globally optimized thresholds.

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# APPENDIX C

## DISTRIBUTED DETECTION OF AN UNKNOWN SIGNAL IN NOISE

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# DISTRIBUTED DETECTION OF AN UNKNOWN SIGNAL IN NOISE

By

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May 1982

## ABSTRACT

In this paper, we extend the results of Lauer and Sandell [1] for distributed detection of a known signal in correlated noise to the case in which the signal is a random process. Optimal decision laws cannot be obtained so a class of suboptimal laws based on local likelihood ratio tests is considered. The first and second moments of the local likelihood ratios are computed and used in a numerical example to illustrate the effect of using distributed detection laws with unknown signals.

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## SECTION 1

## INTRODUCTION

In this paper, we extend the methodology developed by Lauer and Sandell [1] for distributed detection of known signals in the presence of correlated noise to the case of unknown signals in white noise. As in [1] the optimal detection policy requires the solution of coupled functional equations and thus is not practical. We therefore consider a suboptimal detection policy based on assuming that the structure of the local detection laws is a likelihood ratio test (LRT) and then jointly optimizing the thresholds of the local decision laws.

Because the likelihood ratio (LR) is not a linear function of the observations, it is difficult to determine the p.d.f. of the LR for the purpose of threshold optimization. We therefore compute the first two moments of the LR and use these to determine the local thresholds and to evaluate the performance of the distributed detection strategy.

The structure of this paper is as follows: In Section 2, we formulate the problem of distributed detection of an unknown signal in noise as well as the suboptimal policy we will analyze. In Section 3, we determine the mean and covariance of the LR under each hypothesis. In Section 4, we consider a scalar example which illustrates some of the tradeoffs to be considered when implementing a distributed detection policy. Finally, in Section 5, we summarize the results and discuss extensions we are investigating.

## SECTION 2

## PROBLEM FORMULATION

Let us consider the problem of detecting a sample function from a Gaussian random process in the presence of additive white Gaussian noise based on measurements by two sensors. As in [1]-[3] we assume that it is not desirable to transmit the received waveforms to a central location for optimal processing. Thus a local decision $u_i$ must be made at each sensor and we assume that the decisions are to be made so as to minimize the expected value of a global cost which depends upon the $u_i$ and the true hypothesis.

For this problem we assume the $i^{th}$ sensor's observation under the two hypotheses is given by:

$$H^1: \quad dy_i(t) = s_i(t)dt + dv_i(t) \quad 0<t<T \tag{2-1a}$$

$$H^0: \quad dy_i(t) = dv_i(t) \quad 0<t<T \tag{2-1b}$$

where $dv_i(t)$ are independent zero-mean Brownian motion processes with $E\{v_i(t)v_i(s)\} = \min(t,s)$. The signals $s_i(t)$ are zero-mean and have known covariances given by

$$E\{s_i(t)\ s_j(\tau)\} \triangleq K^s_{ij}(t,\tau) \quad 0<t,\tau<T \ . \tag{2-2}$$

We assume that, based on $y_i(t)$, $u_i$ is chosen so that

$$u_i = \begin{cases} 0 & \text{indicates } H^0 \text{ selected} \\ 1 & \text{indicates } H^1 \text{ selected} \end{cases} \tag{2-3}$$

and that the $u_i$ are to be chosen so as to minimize a global cost $E\{J(u_1,u_2,H)\}$, where $J(u_1,u_2,H)$ is the cost associated with local decision $u_1$ and $u_2$ when H is true.

From arguments similar to those in [2]-[5] the optimal distributed policy has the following structure:*

$$\ell_i^*(y_i) \underset{0}{\overset{1}{\gtrless}} f_i(y_i) \tag{2-4}$$

where the $f_i(\cdot)$ are the solution to two coupled functional equations and $\ell_i^*(y_i)$ is the log likelihood ratio. From [5] we have that

$$\ell_i^*(y_i) = \int_0^T \hat{s}_i(t)\, dy_i(t) - \frac{1}{2}\int_0^T \hat{s}_i^2(t)dt \tag{2-5}$$

where the first integral is an Itô integral and

$$\hat{s}_i(t) = E\{s_i(t)|y_i(\tau),\ 0\leq\tau\leq t,\ H^1\} \quad . \tag{2-6}$$

*The notation $\ell \underset{0}{\overset{1}{\gtrless}}$ indicates that u=1 if $\ell>t$, u=0 if $\ell<t$ and u=0 or 1 if $\ell=t$.

The equations for $f_i(\cdot)$ are not readily solved so we consider instead the suboptimal policy given by

$$\ell_i = \ell_i(y_i) \underset{0}{\overset{1}{\gtrless}} t_i \tag{2-7}$$

where we define

$$\ell_i(y_i) \triangleq \int_0^t r_i(t)\, dy_i(t) - \frac{1}{2} r_i^2(t)dt \tag{2-8}$$

and $r_i(t)$ is some causal linear function of $y_i(t)$:

$$r_i(t) = \int_0^t h_i(t,\tau)\, dy_i(\tau) \tag{2-9}$$

The definition of $\ell_i$ reduces to Eq. 2-5 when $r_i(t)=\hat{s}_i(t)$ but allows us also to consider signal estimates which are more readily computed than $\hat{s}_i(t)$.

Equation 2-8 yields $\ell_i$ as a quadratic form on the observations $y_i(t)$ and thus $\ell_i$ is not a Gaussian random variable. For the problems considered in Section 4 accurate results can be obtained by assuming $\ell_i$ is approximately Gaussian* and thus the $\ell_i$ can be characterized by computing the first two moments of $\ell_i$ and the cross correlation between $\ell_1$ and $\ell_2$.

The global cost associated with thresholds $t_1$ and $t_2$ is given by

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*For the cases we consider in Section 4, the Gaussian assumption leads to results which agree for the centralized case with those in [4].

$$J = \sum_{j=0}^{1} \left\{ \int_{t_1}^{\infty} \int_{t_2}^{\infty} p(\ell_1,\ell_2|H^j)\, d\ell_2 d\ell_1 J(1,1,H^j) \right.$$

$$+ \int_{-\infty}^{t_1} \int_{t_2}^{\infty} p(\ell_1,\ell_2|H^j) d\ell_2 d\ell_1 J(0,1,H^j)$$

$$+ \int_{t_1}^{\infty} \int_{-\infty}^{t_2} p(\ell_1,\ell_2|H^j) d\ell_2 d\ell_1 J(1,0,H^j)$$

$$\left. + \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} p(\ell_1,\ell_2|H^j) d\ell_2 d\ell_1 J(0,0,H^j) \right\} p^j \tag{2-10}$$

This can be rewritten as

$$J = \Delta J_0 + \Delta J_1 \int_{-\infty}^{t_1} p(\ell_1|H^1) d\ell_1 + \Delta J_2 \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} p(\ell_1,\ell_2|H^1) d\ell_2 d\ell_1$$

$$- \Delta J_3 \int_{-\infty}^{t_1} p(\ell_1|H^0) d\ell_1 - \Delta J_4 \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} p(\ell_1,\ell_2|H^0) d\ell_2 d\ell_1$$

$$+ \Delta J_5 \int_{-\infty}^{t_2} p(\ell_2|H^1) d\ell_2 - \Delta J_6 \int_{-\infty}^{t_2} p(\ell_2|H^0) d\ell_2 \tag{2-11}$$

where

$$\Delta J_0 = J(1,1,H^0)p^0 - J(1,1,H^1)p^1 \tag{2-12a}$$

$$\Delta J_1 = [J(0,1,H^1) - J(1,1,H^1)]p^1 \tag{2-12b}$$

$$\Delta J_2 = [J(0,0,H^1) + J(1,1,H^1) - J(1,0,H^1) - J(0,1,H^1)]p^1 \tag{2-12c}$$

$$\Delta J_3 = [J(1,1,H^0)-J(0,1,H^0)]p^0 \tag{2-12d}$$

$$\Delta J_4 = [J(1,0,H^0)+J(0,1,H^0)-J(0,0,H^0)-J(1,1,H^0)]p^0 \tag{2-12e}$$

$$\Delta J_5 = [J(1,0,H^1)-J(1,1,H^1)]p^1 \tag{2-12f}$$

and

$$\Delta J_6 = [J(1,1,H^0)-J(1,0,H^0)]p^0 \ . \tag{2-12g}$$

Differentiating Eq. 2-11 with respect to $t_1$ and setting it to zero yields one half of the necessary condition for optimality of the thresholds:

$$\Delta J_1 \ p(t_1|H^1)+\Delta J_2 \int_{-\infty}^{t_1} p(t_1,\ell_2|H^1)d\ell_2=\Delta J_3 \ p(t_1|H^0)+\Delta J_4 \int_{-\infty}^{t_2} p(t_1,\ell_2|H^0)d\ell_2 \tag{2-13}$$

Differentiating Eq. 2-11 with respect to $t_2$ and setting it to zero yields

$$\Delta J_5 \ p(t_2|H^1)+\Delta J_2 \int_{-\infty}^{t_1} p(\ell_1,t_2|H^1)d\ell_1=\Delta J_6 \ p(t_2|H^0)+\Delta J_4 \int_{-\infty}^{t_1} P(\ell_1,t_2|H^0)d\ell_1 \tag{2-14}$$

Under the Gaussian assumption we have that*

$$P(\ell_1,\ell_2|H^0) \cong \frac{1}{\sqrt{2\pi v_1^0}} e^{-\frac{1}{2v_1^0}(\ell_1-m_1^0)^2} \frac{1}{\sqrt{2\pi v_2^0}} e^{-\frac{1}{2v_2^0}(\ell_2-m_2^0)^2} \tag{2-15}$$

*Since the observations are independent under $H^0$, $\ell_1$ and $\ell_2$ are uncorrelated.

and

$$p(\ell_1,\ell_2|H^1) \cong \frac{1}{2\pi\sqrt{v_1^1 v_2^1(1-\rho^2)}}\, e^{-\frac{1}{2v_1^1 v_2^1(1-\rho^2)}\Gamma} \tag{2-16a}$$

where

$$\Gamma = \left((\ell_1-m_1^1)^2 v_2^1 - 2\rho\sqrt{v_1^1 v_2^1}\,(\ell_1-m_1^1)(\ell_2-m_2^1)+(\ell_2-m_2^1)^2 v_1^1\right) \tag{2-16b}$$

$$m_i^j \;\underline{\Delta}\; E\{\ell_i|H^j\} \tag{2-17}$$

$$m_i^j \;\underline{\Delta}\; E\{(\ell_i-m_i^j)^2|H^j\} \tag{2-18}$$

$$j=0,1 \quad i=1,2$$

$$\rho = c^1/\sqrt{v_1^1 v_2^1} \tag{2-19}$$

and

$$c^1 \;\underline{\Delta}\; E\{(\ell_1-m_1^1)(\ell_2-m_2^1)|H^1\} \tag{2-20}$$

Using Eq. 2-15 through 2-20 we can rewrite Eqs. 2-13 and 2-14 as

$$\frac{(t_1-m_1^1)^2}{2v_1^1} - \frac{(t_1-m_1^0)^2}{2v_1^0} = \ell n\left\{\sqrt{\frac{v_1^0}{v_1^1}}\;\frac{\Delta J_1+\Delta J_2\ \mathrm{erf}\left[\dfrac{\left(t_2-m_2^1-\rho(v_2^1/v_1^1)^{1/2}(t_1-m_1^1)\right)}{\sqrt{v_2^1(1-\rho)}}\right]}{\Delta J_3+\Delta J_4\ \mathrm{erf}\left[\left(t_2-m_2^0\right)/\sqrt{v_2^0}\right]}\right\} \tag{2-21}$$

$$\frac{(t_2-m_2^1)^2}{2v_2^1} - \frac{(t_2-m_2^0)^2}{2v_2^0} = \ln\left\{\sqrt{\frac{v_2^0}{v_2^1}}\;\frac{\Delta J_5+\Delta J_2 \operatorname{erf}\left[\dfrac{\left(t_1-m_1^1-\rho(v_1^1/v_2^1)^{1/2}(t_2-m_2^1)\right)}{\sqrt{v_1^1(1-\rho^2)}}\right]}{\Delta J_6+\Delta J_4 \operatorname{erf}\left[(t_1-m_1^0)/\sqrt{v_1^0}\right]}\right\}$$

(2-22)

These equations are necessary conditions for optimality and thus if multiple solutions exist the corresponding costs must be computed to determine the optimal thresholds. To solve these equations, however, we need $m_i^j$, $v_i^j$ and $\rho$. These are computed in the next section.

## SECTION 3

## MOMENT COMPUTATION

Recall that

$$\ell_i \triangleq \int_0^T r_i(t)dy_i(t) - \frac{1}{2}\int_0^T r_i^2(t)dt \tag{3-1}$$

where

$$H^1: \quad dy_i(t) = s_i(t)dt + dv_i(t) \tag{3-2}$$

$$H^0: \quad dy_i(t) = \qquad dv_i(t) \quad , \tag{3-3}$$

and $r_i(t)$ is a linear causal function of $y_i(\tau)$.

We denote by $E_j\{\cdot\}$ , $j=0,1$ the expectation operator assuming the model of $dy_i(t)$ given by $H^0$ and $H^1$ respectively.

We have

$$m_i^0 \triangleq E_0\{\ell_i\} = \int_0^T E_0\{r_i(t)dv_i(t)\} - \frac{1}{2}\int_0^T E_0\{r_i^2(t)\}dt$$

$$= -\frac{1}{2}\int_0^T E_0\{r_i^2(t)\}dt \tag{3-4}$$

$$m_i^1 \triangleq E_1\{\ell_i\} = \int_0^T E_1\{r_i(t)s_i(t)dt + r_i(t)dv_i(t)\} - \frac{1}{2}\int_0^T E_1\{r_i^2(t)\}dt$$

$$= \int_0^T E_1\{r_i(t)s_i(t)\}dt - \frac{1}{2}\int_0^T E_1\{r_i^2(t)\}dt \quad . \tag{3-5}$$

The second moment under $H^0$ is given by

$$v_i^0 \triangleq E_0\{(\ell_i - m_i^0)^2\}$$

$$= E_0\left\{\left[\int_0^T r_i(t)dy_i(t) - \frac{1}{2}\int_0^T r_i^2(t)dt + \frac{1}{2}\int_0^T E_0\{r_i^2(t)\}dt\right]^2\right\}$$

$$= \int_0^T E_0\{r_i^2(t)\}dt + \frac{1}{4}\int_0^T\int_0^T E_0\{r_i^2(t)r_i^2(\tau)\} - E_0\{r_i^2(t)\}\, E_0\{r_i^2(\tau)\}dtd\tau \tag{3-6}$$

Using Gaussian moment factoring [6] we have that

$$E_0\{r_i^2(t)r_i^2(\tau)\} = E_0\{r_i^2(t)\}\, E_0\{r_i^2(\tau)\} + 2E\{r_i(t)r_i(\tau)\}^2 \tag{3-7}$$

So $v_i^0$ is given by

$$v_i^0 = \int_0^T E_0\{r_i^2(\tau)\}dt + \frac{1}{2}\int_0^T\int_0^T [E_0\{r_i(t)r_i(\tau)\}]^2 dtd\tau \tag{3-8}$$

The second moment under $H^1$ is given by (again using moment factoring)

$$v_i^1 \triangleq E_1\left\{\left[\int_0^T r_i(t)s_i(t)-E_1\{r_i(t)s_i(t)\} - \frac{1}{2}\, r_i^2(t) + \frac{1}{2}\, E_1\{r_i^2(t)\}dt\right]^2\right\}$$

$$+ \int_0^T E_1\{r_i^2(t)\}dt$$

$$= \int_0^T E_1\{r_i^2(t)\}dt + \int_0^T\int_0^T E_1\{r_i(t)r_i(\tau)\} \cdot$$

$$[E_1\{s_i(t)s_i(\tau) - E_1\{s_i(t)r_i(\tau)\} - E_1\{r_i(t)s_i(\tau)\} + \frac{1}{2}\, E_1\{r_i(t)r_i(\tau)\}]$$

$$+ E_1\{r_i(t)s_i(\tau)\}\; E_1\{s_i(t)r_i(\tau)\}dtd\tau$$

(3-9)

Finally we have, using similar arguments,

$$c^1 \triangleq E_1\{(\ell_1-m_1^1)(\ell_2-m_2^1)\}$$

$$= \int_0^T\int_0^T E_1\{r_1(t)r_2(\tau)\}[E_1\{s_1(t)s_2(\tau)\} - E_1\{r_1(t)s_2(\tau)\} - E_1\{s_1(t)r_2(\tau)\}$$

$$+ \frac{1}{2}\, E_1\{r_1(t)r_2(\tau)\}] + E_1\{r_1(t)s_2(\tau)\}\; E_1\{s_1(t)r_2(\tau)\}dtd\tau$$

(3-10)

These quantities can be computed once $h_i(t,\tau)$ is specified in Eq. 2-9. We shall do so for a simple example in the next section.

## SECTION 4

## EXAMPLE

In this section we consider an example in which the unknown signal is a sample path from a stationary Gauss-Markov process and in which the filter gains $K_i(t)$ of the Kalman filter that can be used to compute $\hat{s}_i(t)$ are chosen as the steady state values of the optimal gains. The observations processes of concern under the two hypotheses are

$$H^0: \quad dy_i(t) = dv_i(t) \tag{4-1}$$

$$H^1: \quad dy_i(t) = c_i x(t)dt + dv_i(t) \tag{4-2}$$

with

$$dx(t) = -ax(t)dt + \sqrt{2a}\, dw(t) \tag{4-3}$$

where $v_i(t)$ and $w(t)$ are independent zero-mean Brownian motion processes with $E\{v_i(t)v_i(s)\} = \min(t,s)$ and $E\{w(t)w(s)\} = \min(t,s)$.

The $r_i(t)$ are given by

$$r_i(t) = c_i \hat{x}_i(t) \tag{4-4}$$

where

$$d\hat{x}_i(t) = -a\hat{x}_i(t)dt + k_i\left(dy_i(t) - c_i\hat{x}_i(t)dt\right) \tag{4-5}$$

The signal is assumed to be in steady state so

$$K^s_{ii}(t,\tau) = c_i^2\, e^{-a|t-\tau|} \tag{4-6}$$

$$K^s_{12}(t,\tau) = c_1 c_2\, e^{-a|t-\tau|} \tag{4-7}$$

The filter gains $k_i$ are the steady state Kalman gains:

$$k_i = -a\left[1 - \sqrt{1+c_i^2}\,\right]/c_i \tag{4-8}$$

We shall also assume that the observation time T is long in comparison to the inverse bandwidth of the signal so that the signals $r_i(t)$ are in steady state for most of the interval $[0,T]$. This implies that, to a good approximation, we can assume that the $r_i(t)$ are stationary. Thus, for $t>\tau$

$$E_1\{x(t)x(\tau)\} = e^{-a(t-\tau)} \tag{4-9}$$

$$E_1\{x(t)\hat{x}_i(\tau)\} = A_i\, e^{-a(t-\tau)} \tag{4-10}$$

$$E_1\{\hat{x}_i(t)x(\tau)\} = A_i\, e^{-\alpha_i(t-\tau)} + e^{-a(t-\tau)} - e^{-\alpha_i(t-\tau)} \tag{4-11}$$

$$E_1\{\hat{x}_i(t)\hat{x}_i(\tau)\} = A_i\, e^{-a(t-\tau)} \tag{4-12}$$

$$E_1\{\hat{x}_i(t)\hat{x}_2(\tau)\} = B\, e^{-\alpha_1(t-\tau)} + A_2\left(e^{-a(t-\tau)} - e^{-\alpha_1(t-\tau)}\right) \quad , \tag{4-13}$$

$$E_0\{\hat{x}_1(t)\hat{x}_2(\tau)\} = \frac{a\ A_i}{\alpha_i}\, e^{-\alpha_i(t-\tau)} \quad , \tag{4-14}$$

where

$$A_i = \frac{a}{2c_i^2}\left(-1 + \sqrt{1+2c_i^2/a}\,\right)^2 \quad , \tag{4-15}$$

$$\alpha_i = a\sqrt{1+2c_i^2/a} \quad , \tag{4-16}$$

and

$$B = \frac{A_1(\alpha_2-a)+A_2(\alpha_1-a)}{\alpha_1+\alpha_2} \quad , \tag{4-17}$$

We can thus rewrite Eqs. 3-4 through 3-10 as:

$$m_i^0 = -\frac{c_i^2 A_i aT}{\alpha_i} \quad , \tag{4-18}$$

$$m_i^1 = \frac{c_i^2 A_i T}{2} \quad , \tag{4-19}$$

$$v_i^0 = \frac{c_i^2 a A_i T}{\alpha_i} + \frac{c_i^4 a^2 A_i^2}{2\alpha_i^3}\left[T - \frac{1}{2\alpha_i}\left(1-e^{-2\alpha_i T}\right)\right] \quad , \tag{4-20}$$

$$v_i^1 = c_i^2 A_i T + \frac{c_i^4 (2A_i - A_i^2)}{2a} \left[ T - \frac{1}{2a} \left( 1 - e^{-2aT} \right) \right] , \tag{4-21}$$

$$c^1 = c_1^2 c_2^2 \left\{ \frac{D_1}{2\alpha_1} \left[ T - \frac{1}{2\alpha_1} \left( 1 - e^{-2\alpha_1 T} \right) \right] + \frac{D_2}{2\alpha_2} \left[ T - \frac{1}{2\alpha_2} \left( 1 - e^{-2\alpha_2 T} \right) \right] \right.$$

$$\left. + \frac{E}{2a} \left[ T - \frac{1}{2a} \left( 1 - e^{-2aT} \right) \right] \right\} , \tag{4-22}$$

where

$$D_1 = -BA_1 + B + \frac{1}{2} B^2 + A_1 A_2 - A_2 - BA_2 + \frac{1}{2} A_2^2 , \tag{4-23}$$

$$D_2 = -BA_2 + B + \frac{1}{2} B^2 + A_1 A_2 - A_1 - BA_1 + \frac{1}{2} A_1^2 , \tag{4-24}$$

and

$$E = A_1 + A_2 - \frac{1}{2} A_1^2 - \frac{1}{2} A_2^2 . \tag{4-25}$$

In the figures that follow we have selected $c_1 = c_2$ so that the sensors are identical. In the cases we examine the Gaussian approximation is accurate except for the tails of the distributions. This implies that we cannot get accurate results for cases in which $P_F$ is constrained to be small (<.01 for example). An approximation which is valid in these cases has been developed and will be reported on in [8].

Figure 4-1 plots the probability of miss, $P_M$, as a function of signal energy, E, and time-bandwidth product, $\Delta = aT/2$, when the false alarm probability, $P_F$, is fixed equal to 0.1. Three curves are given on this plot. That labeled "two sensors (centralized)" corresponds to the case in which both $y_1(t)$ and $y_2(t)$ are available at the fusion center and are used in making an optimal decision. That labeled "one sensor" corresponds to assuming only $y_1(t)$ is available for decisionmaking. Finally, that labeled "two sensors (distributed)" corresponds to the performance obtained by the optimal DLRT law. In all of these cases the same signal energy per sensor is assumed.

The curve for "two sensors (centralized)" is horizontal indicating $P_M$ does not vary with $\Delta$. This was achieved by adjusting the energy per sensor E as a function of $\Delta$. Note that, as $\Delta$ increases from 10 to $10^5$, the energy required to maintain a constant $P_M$ increases from 10 to 606.6.

This can be explained as follows. For a fixed observation time T, increases in $\Delta$ correspond to increases in signal bandwidth a. As the signal bandwidth increases so too does the Kalman filter bandwidths associated with the computation of $\hat{s}_i(t)$. Since the noise is assumed to be white, the noise energy in $\hat{s}_i(t)$ is proportional to the Kalman filter bandwidth and thus, as $\Delta$ increases, the signal energy-to-noise energy <u>in the signal bandwidth</u> decreases. The signal-to-noise ratio needs to be constant to obtain a constant $P_M$ and thus the energy required increases with $\Delta$.

For a fixed bandwidth, increases in $\Delta$ correspond to increases in the observation time T. As T increases the noise energy <u>in the signal bandwidth</u> increases and thus, for fixed signal energy, the signal-to-noise ratio decreases. Thus as $\Delta$ increases, the energy per sensor must be increased if constant $P_M$ is to be obtained.

If $\Delta$ is decreased below 10 then other effects enter and eventually dominate the computation of energy required to attain a given $P_M$. For fixed T as a $\Delta$ decreases the observations $y_i(t)$ becomes more strongly correlated (i.e., $y_i(0)$ and $y_i(\tau)$ are more strongly correlated). Thus for $\Delta$ small enough, less information is available for decisionmaking and the energy required to attain a fixed $P_M$ increases. This effect occurs, however, in a region of $\Delta$ and E in which the Gaussian assumption on $\ell_i$ fails and thus is not illustrated in Fig. 4-1.

The curve labled "one sensor", in this scalar problem, can be generated by the same computations as for "two sensors (centralized)" except that the energy available for decisionmaking is reduced by half. Note that $P_M$ increases with bandwidth, despite the increase in energy, indicating an increased sensitivity of $P_M$ with respect to energy as bandwidth increases.

The optimal DLRT performance, labeled "two sensors (distributed)," degrades with increasing $\Delta$. The best DLRT performance thus occurs, with respect to the centralized two sensor case, for small $\Delta$. The one sensor case is also best for small $\Delta$ and thus the best DLRT performance, with respect to the one sensor case, occurs for large $\Delta$.

The requirement of using a distributed law effectively reduces the energy available for decisionmaking. We have determined the energy required by a single sensor to perform as well as the DLRT law and plotted the result in Fig. 4-2. The curve labled "two sensors (distributed)" is simply a plot of the energy per sensor versus $\Delta$ as given by the two horizontal axes of Fig. 4-1. The curve labeled "one sensor" is the energy required by a single sensor to perform as well as the optimal DLRT law. The top curve plots the ratio of the lower two curves.

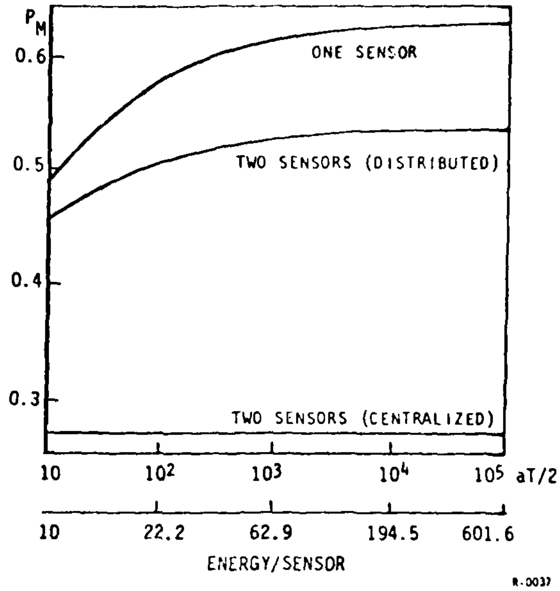


Figure 4-1. $P_M$ as a Function of Signal Energy E and Time-bandwidth Product $\Delta$ for Different Combinations of Sensors.

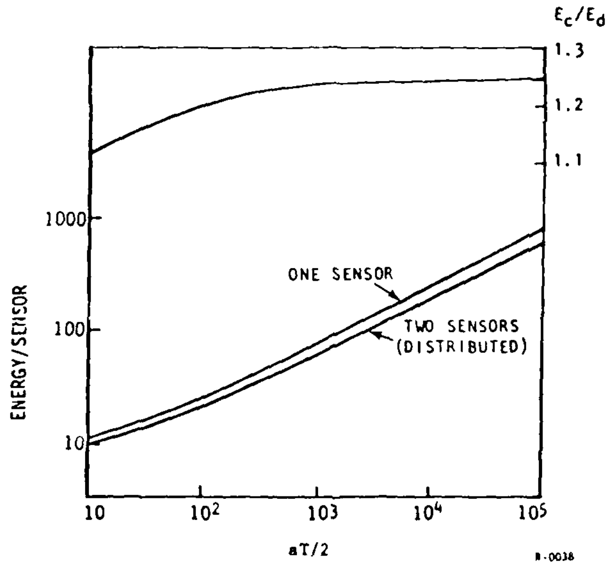


Figure 4-2. Comparison of Energy Required for Equivalent Performance of DLRT Law and Single Sensor Law.

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We see that the "energy loss" entailed by using DLRT laws is not fixed: at $\Delta=10$ two DLRT sensor are equivalent to about 1.12 centralized sensors whereas at $\Delta=10^5$ two DLRT sensors are equivalent to about 1.25 centralized sensors. In part, the effective energy loss is not fixed since the correlation $\rho$ between $\ell_1$ and $\ell_2$ decreases with increasing $\Delta$. This implies that for smaller $\Delta$ less information is available to the fusion center and poorer performance results.

## SECTION 5

## CONCLUSIONS

We have introduced a class of suboptimal distributed decision laws for the problem of detecting an unknown signal in noise which we denoted DLRT laws. Using a Gaussian approximation we were able to determine the performance of these laws for a simple scalar example. For the problem considered an optimal DLRT law with two sensors was roughly equivalent to 1.25 centralized sensors, thus indicating a substantial penalty exists for using distributed detection laws and the potentially high payoff for allowing sensors to communicate.

We are currently investigating the case in which the sensors are not the same. For the problem considered in Section 4, the optimal DLRT thresholds are identical to those which would be used if each likelihood ratio test were optimized individually (a locally optimal DLRT). This occurs because the sensors are identical and is not the case when the sensors are different. The optimal DLRT thus performs better than the locally optimal DLRT when the sensors are different. It is interesting to characterize the difference in performance between these two types of laws since the difference is effectively free. That is, the improved performance of the optimal DLRT does not require more communication or different hardware - only the thresholds are different.

We are also developing approximations to $P_M$ and $P_F$ which are better than those resulting from the Gaussian assumption of Section 2. These bounds are based on those developed in [4] and [7]; however significant modifications are required to handle distributed decisionmaking.

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