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INJECTION OF A HIGH CURRENT BEAM INTO A MODIFIED BETATRON ACCEL--ETC(U)
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Table I — Parameters for the computer runs of Figures 3 and 4

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A method is proposed for injecting and trapping a high current, high energy, non-neutral electron beam into a modified betatron accelerator. The injection is substantially simplified as a result of the balancing of two effects that are prominent in high current beams.		

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CONTENTS

INJECTION OF A HIGH CURRENT BEAM INTO A MODIFIED BETATRON ACCELERATOR

I. Introduction

Recently, there is increasing interest in the development of compact accelerators that are capable of generating ultra high current beams. One of the most promising is the modified betatron accelerator.[1-4] This device consists of a conventional betatron[5] magnetic field configuration as well as a toroidal magnetic field. It has been shown both analytically and numerically that the toroidal field substantially improves the stability[2,6] of the high current beams. However, the injection and extraction of the electron beam is substantially more involved as a result of the toroidal field.

In this paper, we report on an injection scheme that is conceptually simple and rather easily realizable. Although the proposed scheme has some similarities with previous injection techniques of relativistic beams into toroidal geometries,[7-9] several of its key features are different.

The proposed injection scheme is closely related to two effects that are very important in high current beams. The first effect is associated with the reduction of the kinetic energy of the injected beam (inductive effect) and the second is associated with the additional force that appears on the geometric center of the beam as a result of the finite radius of curvature of the circulating electron beam (toroidal effect). Either of these effects could drastically change the major radius of the electron ring and thus drive the injected beam to the wall of the vacuum chamber.

II. Balancing of Inductive and Toroidal Effects

Consider a non-neutral electron beam emitted from a diode that is located inside the torus, as shown in Fig. 1. During injection, the beam kinetic energy is reduced in order to provide the necessary energy to build-up the electromagnetic fields inside the torus. The reduction of the beam's kinetic energy may be computed from the conservation of energy

$$N(\gamma_o - 1)\, m_o c^2 = N \langle \gamma - 1 \rangle\, m_o c^2 + \frac{1}{2c}\int \vec{J}\cdot\vec{A}\, dV + \frac{1}{2}\int \rho\phi\, dV, \qquad (1)$$

where N is the total number of electrons in the beam, $(\gamma_o - 1)\, m_o c^2$ is the

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kinetic energy of electrons at the anode and $\langle\gamma - 1\rangle m_o c^2$ is the average kinetic energy of the electrons after equliibrium has been established. The last two terms in Eq. (1) represent the magnetic and electric field energies respectively. For highly relativistic, large aspect ratio rings, it is shown later on that the two potentials are about equal and thus the field energy terms are also about equal. Thus, for J constant, Eq. (1) gives

$$\gamma_o - \langle\gamma\rangle \approx \frac{-|e|\int A_\theta{}^s dV}{V m_o c^2} ,$$

where $A_\theta{}^s$ is the self magnetic vector potential and V is the volume occupied by the beam. Assuming that in the equilibrium state all the electrons have the same canonical angular momentum, this equation becomes for a large aspect ratio (R_b/r_b) ring

$$\gamma_o - \gamma(R_b) \approx \frac{-|e|}{mc^2} A_\theta{}^s(R_b), \qquad (2)$$

where R_b is the equilibrium radius and r_b is the minor radius of the ring.

The *equilibrium position of the beam is determined from the force balance* equation

$$-\frac{v_\theta^2}{R_b} = -\frac{|e|}{\gamma m_o}\left(E_r + \frac{v_\theta}{c} B_z\right), \qquad (3)$$

where E_r is the radial electric field and B_z is the total axial magnetic field at R_b. Since $E_r = -\frac{\partial\phi}{\partial r}$ and $B_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)$, where ϕ is the electrostatic potential and A_θ is the total magnetic vector potential and since $1/\gamma^2 \ll 1$, i.e., $v_\theta \approx c$, Eq. (3) becomes

$$\frac{1}{R_b} = \frac{|e|}{\gamma m_o c^2}\left(-\frac{\partial\phi}{\partial r} + \frac{\partial A_\theta{}^s}{\partial r} + \frac{A_\theta{}^s}{r} + B_z^{ext}\right)_{r = R_b}, \qquad (4)$$

where B_z^{ext} is the external field at $r = R_b$.

In the coordinate system shown in Fig. 1, the two potentials ϕ and $A_\theta{}^s$

satisfy the equations

$$\frac{\partial^2\phi}{\partial\rho^2} + \frac{1}{(\rho+R_b)}\frac{\partial\phi}{\partial\rho} + \frac{\partial^2\phi}{\partial z^2} = 4\pi|e|n_o \ , \tag{5}$$

and

$$\frac{\partial^2 A_\theta^s}{\partial\rho^2} + \frac{1}{(\rho+R_b)}\frac{\partial A_\theta^s}{\partial\rho} - \frac{A_\theta^s}{(\rho+R_b)^2} + \frac{\partial^2 A_\theta^s}{\partial z^2} = 4\pi|e|n_o\frac{v_\theta}{c} \ , \tag{6}$$

where $\rho = r-R_b$. For $R_b/a >> 1$, the third term in Eq. (6) can be omitted and since $v_\theta \approx c$ the two potentials satisfy the same equation and the same boundary conditions. Thus Eq. (4) becomes

$$\frac{1}{R_b} = \frac{|e|}{\gamma m_o c^2}\left[\frac{A_\theta^s}{R_b} + B_z^{ext}(R_b)\right]. \tag{7}$$

Substituting Eq. (2) into Eq. (7), it is obtained

$$R_b = c\gamma_o \Big/ \left(\frac{1}{V}\int \frac{|e|B_z^{ext}}{m_o c}\, dV\right) \simeq \frac{m_o c^2\gamma_o}{|e|B_z^{ext}} \ . \tag{8}$$

Therefore, when $\gamma^2 >> 1$ and $a/R_b << 1$ inductive and toroidal effects balance each other, even when the beam is injected off-axis. As a consequence, the injection process is considerably simplified and the local external magnetic field required to confine the ultra high current electron beam in its equilibrium position is given by the simple relation of Eq. (8).

The balancing may be computed explicitly when the electron ring is located near the minor axis of the torus. When this condition is satisfied, the self magnetic potential is given by

$$\frac{rA_\theta^s}{(IR_b/c)} = 1 + 2\ \ln\frac{a}{r_b} - \frac{\xi^2}{r_b^2} - \frac{3}{4}\frac{\xi^3}{R_b r_b^2}\cos\phi + C_1\frac{\xi}{r_b}\sin\phi + C_2\frac{\xi}{R_b}\cos\phi, \quad \xi < r_b, \tag{9}$$

where C_1 and C_2 are constants, ξ is the radial distance from the center of the electron ring and ϕ is the polar angle.

When the axis of the electron ring coincides with the minor axis of the torus, A_θ^s becomes

$$A_\theta^s\,(r_o,t) = \frac{2I(t)}{c}\left\{\frac{1}{2} + \ell n\,\frac{a}{r_b}\right\}. \tag{10}$$

The change in γ on the axis of the ring may be computed from Eqs. (2) and (10) and is

$$\Delta\gamma = (\gamma_a - \gamma_o) = -2\nu\left\{\frac{1}{2} + \ell n\,\frac{a}{r_b}\right\}, \tag{11}$$

where ν is the Budker parameter. The average value of γ can be obtianed by integrating Eq. (9) over the cross-section of the beam and is

$$\langle\gamma\rangle - \gamma_o = -2\nu\left\{\frac{1}{4} + \ell n\,\frac{a}{r_b}\right\}.$$

Because of the toroidal effect neither the self electric nor the self magnetic field are equal to zero at the axis of the electron ring even when the axis of the ring lies along the minor axis of the torus. For a constant current density ring the fields at the center of the ring (ξ=o) are[2,6]

$$E_r = -\pi|e|n_o\,\frac{r_b^2}{r_o}\,\ell n\,\frac{a}{r_b}, \tag{12}$$

and

$$B_z = -\pi|e|n_o\left(\frac{v}{c}\right)\frac{r_b^2}{r_o}\left(1 + \ell n\,\frac{a}{r_b}\right). \tag{13}$$

Substituting Eqs. (11), (12) and (13) into the radial force balance equation, it is easy to show that the equilibrium radius of the beam remains constant, although the kinetic energy of the beam is substatially reduced.

The above treatment is based on an asymmetric injection, i.e., when the canonical angular momentum P_θ of the equilibrium state is not the same with that of the diode. The same results are obtained for a symmetric "injection". Although it is not practical, the symmetric "injection" can be easily analyzed and is realized when, for example, the current of an intially very weak ring incrases rapidly with time. The main advantage of the symmetric "injection" is that it can be simulated with two dimensional codes.

The balancing of inductive and toroidal effects both on-axis and off-axis was verified in several computer simulation runs. Excellent agreement was found between theory and simulation, provided the assumptions of the theoretical model were satisfied. Typical result from the computer simulation when the beam is injected on axis are shown in Fig. 2. The electron ring remained at the center of the torus, although its current increased from zero to 10 kA. The betatron field $\left(B_{oz}^{ext}\right)$ used in the simulation was the single particle magnetic field corresponding to the diode energy.

If the assumptions of the theoretical model are not adequately satisfied, the lowest order correction to the local, single particle magnetic field is

$$\delta B_z = 0.17 \times 10^4 \left(\frac{\Delta\gamma}{r_o^2} + \frac{2\nu}{\gamma_a^2 a^2} \right) \Delta r \text{ Gauss},$$

where displacement Δr, the major radius r_o and the minor radius a of the torus are measured in centimeters.

III. Injection

As a result of the balancing of inductive and toroidal effects, the center of the beam would remain stationary if the betatron field at the point of injection is equal to the equilibrium field defined in Eq. (8). Thus, after a revolution around the major axis of the torus, the beam will come back to hit the injector. However, if the value of the betatron field is a few percent different than its equilibrium value, the center of the beam will drift away from the injector as the beam propagates along the torus. The distance Δs the beam will drift in one revolution is given by

$$\Delta s = 4\pi r_o \left(\frac{B_{oz}}{B_{o\theta}}\right) \left(\frac{c}{\Omega_{oz}}\right) \frac{\nu}{\gamma_a^2 a^2} \frac{\Delta}{(1-\Delta^2/a^2)},$$

where r_o is the major radius, B_{oz} and $B_{o\theta}$ are the value of betatron and toroidal fields at r_o, $\Omega_{oz} = eB_{oz}/m_o c$, a is the minor radius of the torus and Δ is the displacement of the center of the beam from the center of the torus. If the betatron magnetic field remains constant in time or both the flux and the local magnetic field vary in sychronism, the beam would drift and come back to the injector after a bounce period[2] (or approximately after 10 revolutions around the major axis). This is shown in the computer simulation results of Fig. 3. The values of the various parameters are listed in Table I. Shown in the figure is the projection of the center of the rotating beam in the r-z plane as a function of time. It is apparent that after about 250 nsec (one bounce period) the beam would return and strike the injector. However, when the local magnetic field is reduced slightly during the bounce period the beam drifts away from the injector. This is shown in the computer simulation results of Fig.4. The various parameters have the same values as those in Fig. 3, except that the value of the betatron field during a bounce period is reduced by two gauss, i.e., from 146 to 144 Gauss. If the fields were held constant after a bounce period, the center of the beam would continue rotating around a fixed equilibrium position. However, during the acceleration both the field and the flux through the orbit increase at the same rate, the equilibrium position moves closer to the center of the minor cross section of the torus and the beam rotates around it with a progressively smaller radius.

In addition to the circular cross-section vacuum chamber, the proposed injection scheme was tested in vacuum chambers with elliptical and rectangular cross-sections. Results from the elliptical cross-section chamber is shown in Fig. 5. The values of the various parameters are shown in Table II. During the first 200 nsec the betatron field was reduced 148.5 to 146.5 Gauss. For $t > 200$ nsec the betatron field was kept constant at 146.5 Gauss. The time difference between two adjacent dots on the trajectory curve is 20 nsec.

This paper reports on the injection and trapping of an ultra-high current electron beam into a modified betatron accelerator. For computational convenience the various parameters in the simulation were such that the induced fields dominate the external fields. This parameter regime should be avoided in an actual device because during the acceleration it is possible that the low frequency of rotation (bounce) will change sign and thus the beam will become unstable. However, the main features of the proposed injection are not sensitive to the relative magnitude of the fields but rather to the magnitude of their difference. Both analysis and simulation are based on the cold beam approximation. Presently, work is in progress with finite emittance beams. Finally, it has been assumed that a hard vacuum is continuously maintained inside the confining chamber and thus the plasma formation and its effect on the beam[10] was neglected.

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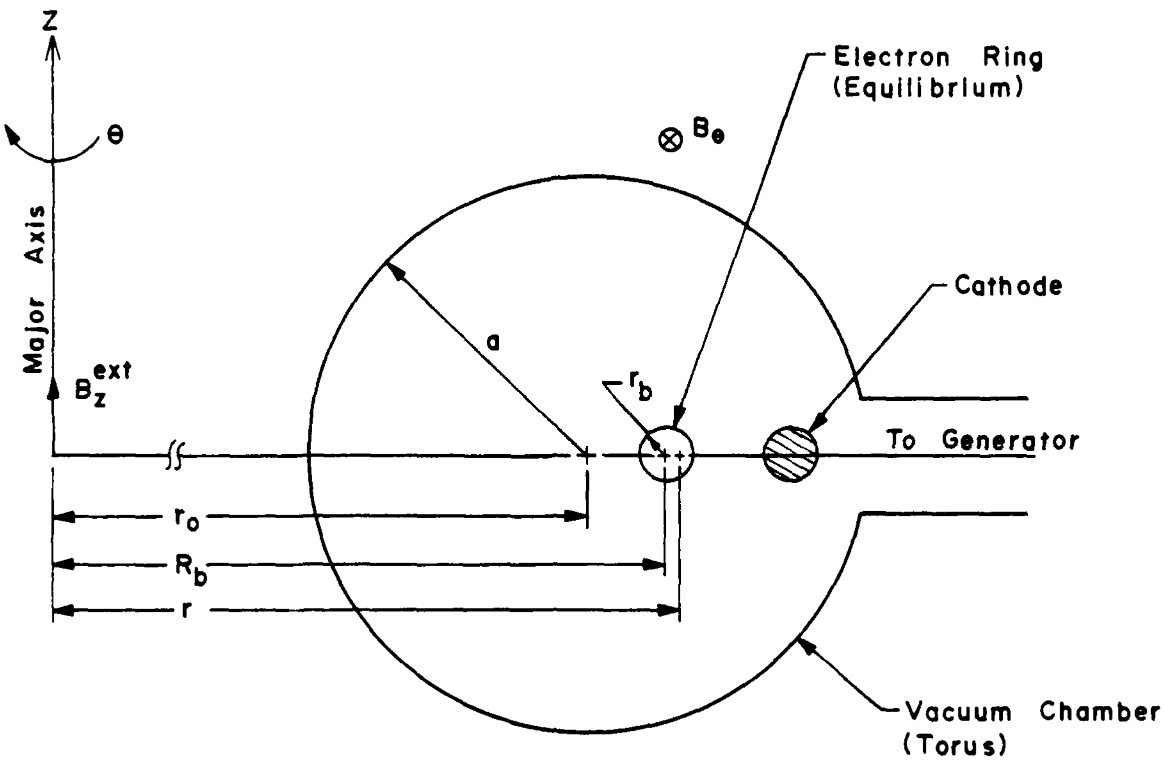

Fig. 1 — System of coordinates used in the analysis

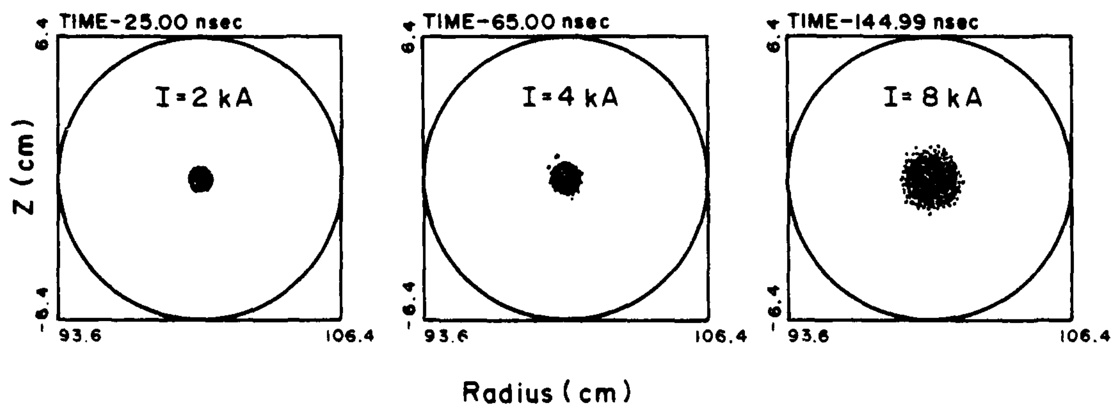

Fig. 2 — Snap-shots of the minor cross-section of the electron ring injected along the minor axis of the torus

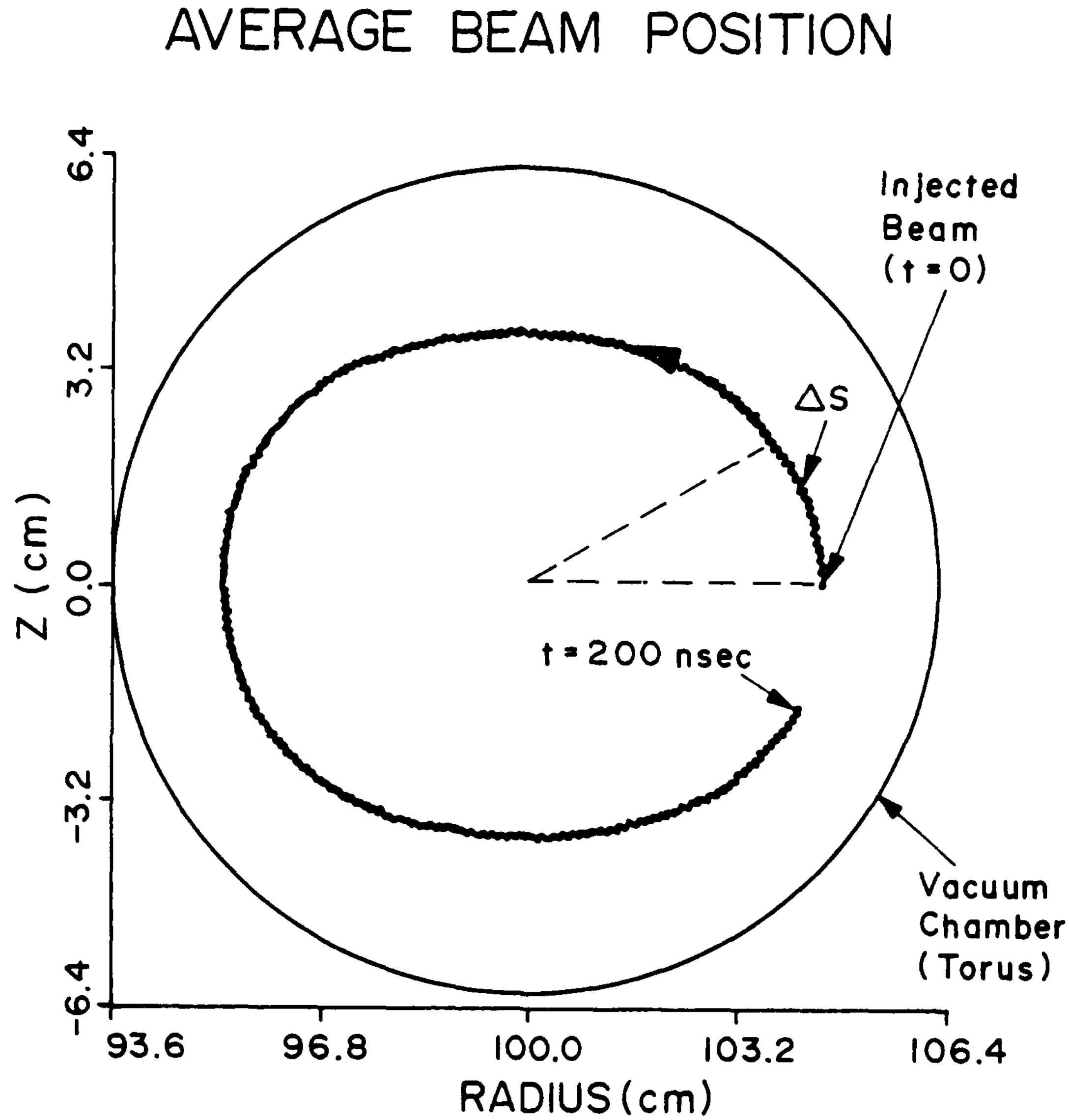

Fig. 3 — Center of the 10 kA electron beam as a function of time. The various parameters are listed in Table I. The bounce period, i.e., the time required by the beam to rotate once around the equilibrium position is ~250 nsec.

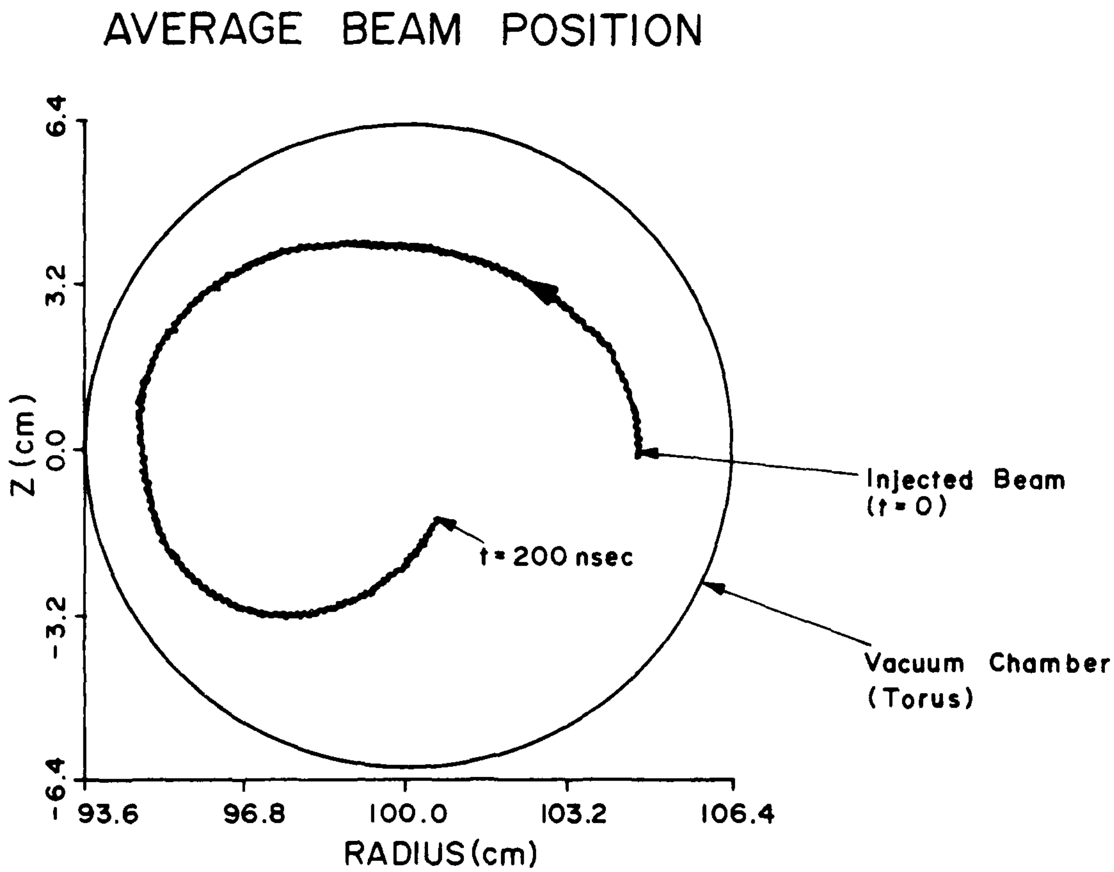

Fig. 4 — Center of the 10 kA electron beam as a function of time. During the bounce period the local betatron field was reduced by two Gauss.

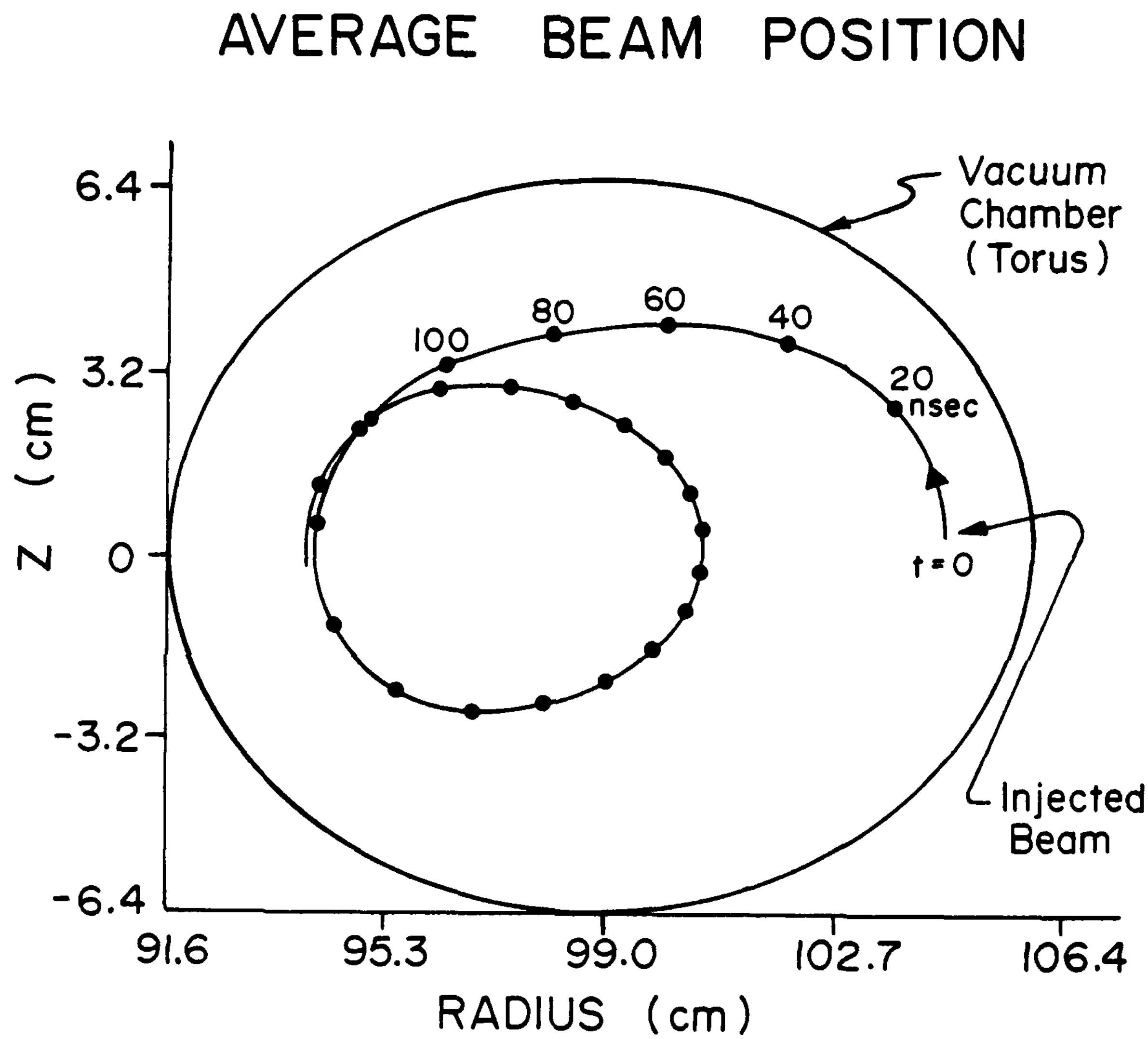

Fig. 5 — Trajectory of the beam's center in an elliptical cross-section torus. The values of the various parameters are listed in Table II.

Table I — Parameters for the computer runs of Figures 3 and 4

Beam Current	$I = 10$ kA
Beam Energy at the Diode	$E_o = 3.84$ MeV ($\gamma_o = 8.5$)
Beam Energy after Injection	$E = 3.05$ MeV ($\gamma = 6.97$)
Injection Radius	$R_b = 104.5$ cm
Initial Beam Minor Radius	$r_{bi} = 1.0$ cm
Final Beam Minor Radius	$r_{bf} = 1.2$ cm
Major Radius	$r_o = 100$ cm
Torus Minor Radius	$a = 6.4$ cm
Toroidal Magnetic Field	$B_{o\theta} = 1415$ G
Equilibrium Betatron Field	$B_{oz}^{eq}(r_o,o) = 152$ G
Betatron Magnetic Field	$B_{oz}(r_o,o) = 146$ G

Table II — Parameters for the computer run of Figure 5

Elliptical cross-section vacuum chamber

Beam Current	$I = 10$ kA
Beam Energy at the Diode	$E_o = 3.84$ MeV
Beam Energy after Injection	$E = 3.05$ MeV
Injection Radius	$R_b = 104.5$ cm
Initial Beam Minor Radius	$r_{bi} = 1$ cm
Final Beam Minor Radius	$r_{bf} = 1.2$ cm
Major Radius	$r_o = 99$ cm
Torus Minor Axes	$a = 7.4$ cm
	$b = 6.4$ cm
Toroidal Magnetic Field	$B_{o\theta} = 1415$ Gauss
Equilibrium Betatron Field	$B_{oz}^{eq}(r_o,o) = 154$
Betatron Magnetic Field	$B_{oz}(r_o,o) = 148.5$

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