

AD-A118 679

NAVAL SURFACE WEAPONS CENTER SILVER SPRING MD
ULTRASONIC PULSE PROPAGATION IN CONSTANT-GROUP-VELOCITY MEDIA. (U)

F/G 20/2

JUN 82 P L EDWARDS

UNCLASSIFIED

NSWC/TR-82-296

SBI-AD-F500 058

NL

1-1

4 02 PM



END
DATE
FILMED
9 82
DTIC

AD A118679

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC TR 82-296	2. GOVT ACCESSION NO. AD-A118 679	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ULTRASONIC PULSE PROPAGATION IN CONSTANT-GROUP-VELOCITY MEDIA	5. TYPE OF REPORT & PERIOD COVERED Interim March 1982-July 1982	
	6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s) Palmer L. Edwards	8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center (Code R34) White Oak Silver Spring, MD 20910	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61152N; ZR00001; ZR01301; R01AA190	
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE 1 June 1982	
	13. NUMBER OF PAGES 30	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified	
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Ultrasonic Velocities Nondestructive Testing Signal Distortion Composite Materials Dispersion Constant-Group-Velocity Media		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Results are presented of calculations made of distortion experienced by ultrasonic pulses in transmission through dispersive constant-group-velocity media, and the effects that it may have on velocity measurements. Three types of pulses were considered; a pulsed sine wave of constant amplitude, a pulsed sine wave with amplitude varying as sine-squared, and a rectangular pulse. It is shown that the individual waves in the pulsed sine waves move with the phase velocity of a continuous wave, and the envelope moves with the group		

DD FORM 1473
1 JAN 73

EDITION OF 1 NOV 68 IS OBSOLETE
S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

BLOCK 20 (cont'd)

velocity. It was also found that a pulsed sine wave in going a certain distance in a constant-group-velocity medium repeats itself, and midway between these distances is the negative of the values at the repetition distances. Because of this repetition phenomenon it is seen that the pulses do not spread as is normally the case in dispersive media. The envelopes of the propagated waves were also calculated, and in agreement with Brillouin's results (Wave Propagation and Group Velocity, Academic, New York, 1960) it was found that if the envelope contains negligible frequency components above the carrier frequency, then the envelope is not distorted in passing through a dispersive constant-group-velocity medium.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

FOREWORD

A theoretical study has been made of distortion that a pulsed ultrasonic signal undergoes in a dispersive medium in which the wave velocity varies with frequency in such a manner that the group velocity is constant, and the effect that this has on measurement of velocity in the medium. Funding for this work was provided by the Independent Research Program of the Naval Surface Weapons Center.

The author wishes to express his appreciation to Charles H. Dyer and Robert D. Corsaro for helpful discussions and comments.

J. R. Dixon
 J. R. DIXON
 By direction



Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

CONTENTS

	<u>Page</u>
INTRODUCTION.....	5
THEORY.....	5
EXAMPLES.....	11
CONCLUSION.....	22
APPENDIX A--FOURIER COEFFICIENTS.....	27

ILLUSTRATIONS

<u>Figure</u>		<u>Page</u>
1	THE THREE TYPES OF PULSED ULTRASONIC SIGNALS CONSIDERED. THE SINE WAVE PERIOD IS t_0 , THE PULSE LENGTH IS T_p , AND THE PULSE REPETITION PERIOD IS T_1	7
2	PHASE VELOCITY AS A FUNCTION OF FREQUENCY FOR A CONSTANT-GROUP-VELOCITY MEDIUM.....	12
3	FREQUENCY SPECTRUM FOR PULSED SINE WAVE WITH CONSTANT AMPLITUDE, WITH FIVE WAVES PER PULSE, AND WITH A PULSE REPETITION PERIOD FOUR TIMES THE PULSE DURATION.....	14
4	CONSTANT-AMPLITUDE PULSE PROPAGATED THROUGH A CONSTANT-GROUP-VELOCITY MEDIUM AS A FUNCTION OF TIME FOR SEVERAL DISTANCES X	15
5	CONSTANT-AMPLITUDE PULSED SINE WAVE AND MODULATION ENVELOPES AT THREE DISTANCES.....	17
6	THE FREQUENCY SPECTRUM FOR PULSED SINE WAVE WITH SINE-SQUARED AMPLITUDE. THERE ARE FIVE WAVES PER PULSE, AND THE PULSE REPETITION PERIOD IS FOUR TIMES THE PULSE DURATION.....	19
7	SINE-SQUARED-AMPLITUDE PULSE, WITH ENVELOPE, PROPAGATED THROUGH A CONSTANT-GROUP-VELOCITY MEDIUM AS A FUNCTION OF TIME FOR SEVERAL DISTANCES X . MODULATION ENVELOPES ARE SHOWN FOR ALL DISTANCES.....	20
8	THE FREQUENCY SPECTRUM FOR A RECTANGULAR PULSE WITH REPETITION PERIOD FOUR TIMES THE PULSE DURATION.....	21
9	RECTANGULAR PULSE PROPAGATED THROUGH A CONSTANT GROUP VELOCITY MEDIUM. AS IN THE PREVIOUS EXAMPLES, THE TIMES AND DISTANCES ARE NORMALIZED TO THE TWENTIETH HARMONIC OF THE FOURIER SERIES, AND THE GROUP VELOCITY IS 1.2 TIMES THE PHASE VELOCITY OF THE TWENTIETH HARMONIC.....	23

INTRODUCTION

Pulsed ultrasonic measurements have been employed extensively in recent years in measuring elastic moduli of materials under varying conditions. In order to do this, the phase velocity must be determined, and because of a distortion that pulsed signals undergo in transmission through dispersive and attenuative materials, there is often some question as to just what velocity a given measurement determines. A number of procedures have been developed to assure that the proper phase velocities are being measured, and have been reviewed by Papadakis¹ and by Sachse and Pao.²

In this paper the distortion is examined that ultrasonic pulses undergo in transmission through lossless dispersive media in which the phase velocity varies with frequency in such a manner that the group velocity is constant. Brillouin³ has shown that under certain limited conditions the envelope of a pulsed signal will be transmitted through such media with no distortion, and it will be shown below that also at certain distances the pulsed signals themselves are reproduced without distortion, and thus do not, as is usually the case in dispersive media, spread out in time.

THEORY

NORMALIZED FOURIER SERIES. In order to study the transmission of a pulsed ultrasonic wave, the pulsed pressure signal is expressed in terms of a Fourier series, the transmission of the various terms considered separately, and the terms then recombined at positions and times of interest. The Fourier series for the propagating pulse is

$$F(x,t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos n\omega_1(t - \frac{x}{v_n}) + \sum_{n=1}^{\infty} B_n \sin n\omega_1(t - \frac{x}{v_n}) \quad (1)$$

¹Papadakis, E. P., "Ultrasonic Velocity and Attenuation: Measurement Methods with Scientific and Industrial Applications," in Physical Acoustics, edited by W. P. Mason and R. N. Thurston (Academic Press, New York, 1976), Vol. XII, Chap. 5, pp 277-374.

²Sachse, W. and Pao, Y-H, "On the Determination of Phase and Group Velocities of Dispersive Waves in Solids," J. Appl. Phys., 49, 1978, pp 4320-4327.

³Brillouin, L., Wave Propagation and Group Velocity, (Academic Press, New York, 1960), pp 7-10.

where A_n and B_n are the amplitudes of the cosine and sine terms, ω_0 is the fundamental angular frequency of the series, x is the distance in the direction of propagation, t is the time, and v_n is the phase velocity of the n th harmonic.

Three types of pulsed ultrasonic signals, as shown in Figure 1, will be considered. The first consists of a pulsed sine wave of constant amplitude, the second is a pulsed sine wave with the amplitude varying as the sine squared, and the third is a rectangular pulse.

It is convenient to normalize the distance, time, and velocity in the Fourier series to that of the pulsed sine wave, the carrier wave. The normalized values are

$$X = x/\lambda_0 , \quad (2)$$

$$T = t/t_0 , \quad (3)$$

$$V_n = v_n/v_0 , \quad (4)$$

where λ_0 , t_0 and v_0 are, respectively, the wavelength, period, and phase velocity of a continuous sine wave of the same frequency as the carrier wave. The pulse duration T_p is given by

$$T_p = Jt_0 , \quad (5)$$

where J is the number of waves per pulse, and the repetition period T_0 is M times the pulse duration, so that

$$T_1 = MT_p = MJt_0 \quad (6)$$

where M and J are limited to integer values for this study. The angular frequency ω_0 of the carrier wave is the same as that of the MJ th harmonic of the series, thus,

$$\omega_0 = MJ\omega_1 . \quad (7)$$

The Fourier series for the propagating pulses becomes

$$F(X,T) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos \frac{2\pi n}{MJ} \left(T - \frac{X}{V_n} \right) + \sum_{n=1}^{\infty} B_n \sin \frac{2\pi n}{MJ} \left(T - \frac{X}{V_n} \right) . \quad (8)$$

In general, this is an expression for the Fourier series normalized to the quantities of the MJ th harmonic of the series, which in this case corresponds to the carrier wave. The same normalization will be used for the pulsed sine wave with the sine-squared amplitude, and for the rectangular pulse.

NORMALIZED PHASE VELOCITY IN CONSTANT-GROUP-VELOCITY MEDIA. The group velocity v_g is given by the expression

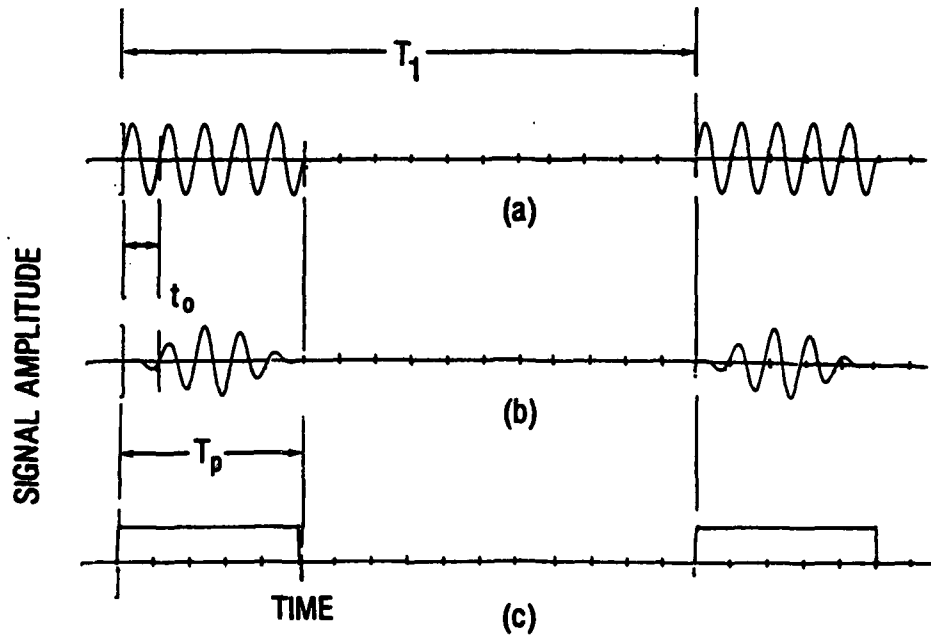


FIGURE 1. THE THREE TYPES OF PULSED ULTRASONIC SIGNALS CONSIDERED. THE SINE WAVE PERIOD IS t_0 THE PULSE LENGTH IS T_p AND THE PULSE REPETITION PERIOD IS T_1

$$v_g = \frac{\partial \omega}{\partial k}, \quad (9)$$

where k is the propagation constant,

$$k = 2\pi/\lambda = \omega/v. \quad (10)$$

For a constant group velocity, from Eq. (9)

$$kv_g = \omega + b, \quad (11)$$

where b is a constant of integration.

Using Eqs. (10) and (11), the phase velocity v may be written

$$v = \frac{\omega v_g}{\omega + b} \quad (12)$$

and the velocity for the n th harmonic of the Fourier series is

$$v_n = \frac{n\omega_1 v_g}{n\omega_1 + b} = \frac{nv_g}{n + H}, \quad (13)$$

with $H = b/\omega_1$, and ω_1 is the fundamental angular frequency of the Fourier series.

The velocity V_n , Eq. (4), normalized to the velocity of the carrier wave v_0 , with

$$v_0 = v_{MJ} = \frac{MJv_g}{MJ + H} \quad (14)$$

becomes

$$V_n = \frac{v_n}{v_0} = \frac{n(MJ + H)}{MJ(n + H)}. \quad (15)$$

Let the group velocity be Z times the velocity v_0 of the carrier wave, then from Eq. (14)

$$v_g = Zv_0 = \frac{ZMJv_g}{MJ + H} \quad (16)$$

from which

$$H = MJ(Z-1) \quad (17)$$

and Eq. (15) may be written

$$v_n = \frac{nZ}{n+H} \quad (18)$$

The Fourier series for the pulses propagating in a constant velocity medium becomes

$$F(X,T) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos \frac{2\pi n}{MJ} \left(1 - \frac{(n+H)X}{nZ}\right) + \sum_{n=1}^{\infty} B_n \sin \frac{2\pi n}{MJ} \left(1 - \frac{(n+H)X}{nZ}\right) \quad (19)$$

If the signal is to be propagated with no distortion, then at some normalized distance X_r and time T_r from X and T , the signal would repeat, or

$$F(X + X_r, T + T_r) = F(X, T) \quad (20)$$

and for this relationship to hold requires that

$$\frac{2\pi n}{MJ} \left(T - \frac{(n+H)X}{nZ}\right) = \frac{2\pi n}{MJ} \left(T + T_r - \frac{(n+H)(X + X_r)}{nZ}\right) + 2\pi I(n), \quad (21)$$

where $I(n)$ must have integral values. This simplifies to

$$MJZI(n) + (T_r A - X_r)n - HX_r = 0. \quad (22)$$

There are several cases that may be considered here:

1. $I(n) = 0$

In this case, for Eq. (22) to hold for all n , if there is to be no distortion for a finite X , then H must be zero, which with Eq. (17) requires that Z be unity, and thus from Eqs. (16) and (18) we see that the phase velocities are the same for all terms of the series, and are equal to the group velocity.

There are no limitations on X_r and T_r except that $T_r = X_r$. Thus the signal propagates with no distortion at all distances.

2. $I(n) = n$

In this case for Eq. (22) to hold for all n these relations must be satisfied:

$$X_r = MJZ/H = Z/(Z-1), \quad (23)$$

and

$$T_r = X_r/Z . \quad (24)$$

If the value of $I(n)$ were taken as An , where A is an integer, then X_r and T_r would be increased by the factor A .

3. $I(n) = n^2$

Eq. (19) then becomes

$$MJZn^2 + (T_r Z - X_r) n - HX_r = 0 , \quad (25)$$

and there are no meaningful values of X_r and T_r to satisfy this equation for all values of n . The same is true if higher powers of n are considered.

From the above it is seen that for a non-dispersive medium, in which the group velocity is equal to the constant phase velocity, the signal is repeated for all values of X_r , and at a time T_r , which is the time that is required for the signal to travel the distance X_r at the common group and phase velocity. For the dispersive media with constant group velocity but not constant phase velocity, it was found that the signal repeats itself at a normalized distance X_r given by Eq. (23), and at a normalized time T_r later. From Eqs. (2), (3), (4), and (24) it can be seen that the signal moves the distance X_r at the group velocity;

$$T_r = X_r/Z ,$$

$$\frac{t_r}{t_0} = \frac{X_r}{\lambda_0} \frac{v_0}{v_g} .$$

and since for the carrier wave

$$\lambda_0 = v_0 t_0 ,$$

then

$$X_r = v_g t_r . \quad (26)$$

If, in Eq. (21) $I(n)$ equals an integer plus one-half, rather than an integer as considered above, it can be shown that

$$F(X,T) - A_0/2 = - (F(X + X_r/2, T + T_r/2) - A_0/2) \quad (27)$$

for the case of a dispersive constant-group-velocity medium. From this it is seen that if $A_0 = 0$, which holds for the pulsed sine waves, then the negative of $F(X,T)$ will occur at $X + X_r/2$ and $T + T_r/2$.

Although a medium with a velocity that varies with frequency is called a dispersive medium, it is seen from the above that in such a medium with a constant velocity the signals repeat at certain distances and therefore do not spread out as is the usual case. Also, it is seen from Eqs. (20) and (27) that knowledge of the signal over a range of half a repetitive distance and one period provides information for all distances and times.

MODULATION ENVELOPE. At $x = 0$ the equation for the constant-amplitude and the sine-squared-amplitude pulsed sine wave may be written in the form

$$C_1(0,t) = C(0,t) \sin \omega_0 t \quad (28)$$

where $C(0,t)$ is the modulation envelope at the origin, and ω_0 is the angular frequency of the carrier wave. For the constant-amplitude pulse of Figure 1, $C(0,t)$ would be a rectangular pulse of the amplitude and duration of the pulse. Brillouin³ has shown that if the maximum frequency component ω_m of the modulation envelope, $C(0,t)$, is less than ω_0 , then the propagated pulse at a distance x is given by

$$C_1(x,t) = C\left(t - \frac{x}{v_g}\right) \sin \omega_0 \left(t - \frac{x}{v_0}\right), \quad (29)$$

from which it is seen that the envelope propagates undistorted at the group velocity v_g and the carrier wave propagates at its own phase velocity v_0 .

EXAMPLES

As indicated above, the propagation of the three types of pulses in Figure 1 through dispersive constant-group-velocity media are considered. In these examples the pulse repetition period is four times the pulse duration, and for the pulsed sine waves there are five complete waves in each pulse, i.e., $M = 4$ and $J = 5$. The pulsed sine wave is therefore of the same frequency as the twentieth harmonic of the Fourier series, and the times, distances, and velocities, are normalized to t_0 , λ_0 and v_0 for this twentieth harmonic. The group velocity in the medium is taken as $1.2v_0$, thus $Z = 1.2$. The phase velocity as a function of frequency is given for this case in Figure 2, where the frequency is normalized relative to the fundamental frequency of the series, and the velocity is normalized to that of the pulsed, or carrier, wave, i.e., to the twentieth harmonic of the series.

PULSED SINE WAVE -- CONSTANT AMPLITUDE. The equation for the pulsed sine wave is

$$\begin{aligned} F(x,t) &= A \sin MJ\omega_1 t & 0 \leq t \leq T_p \\ &= 0 & T_p \leq t \leq T_1 \end{aligned} \quad (30)$$

and the cycle repeats with the period T_1 . The equations for the Fourier coefficients A_n and B_n are given in Appendix A.

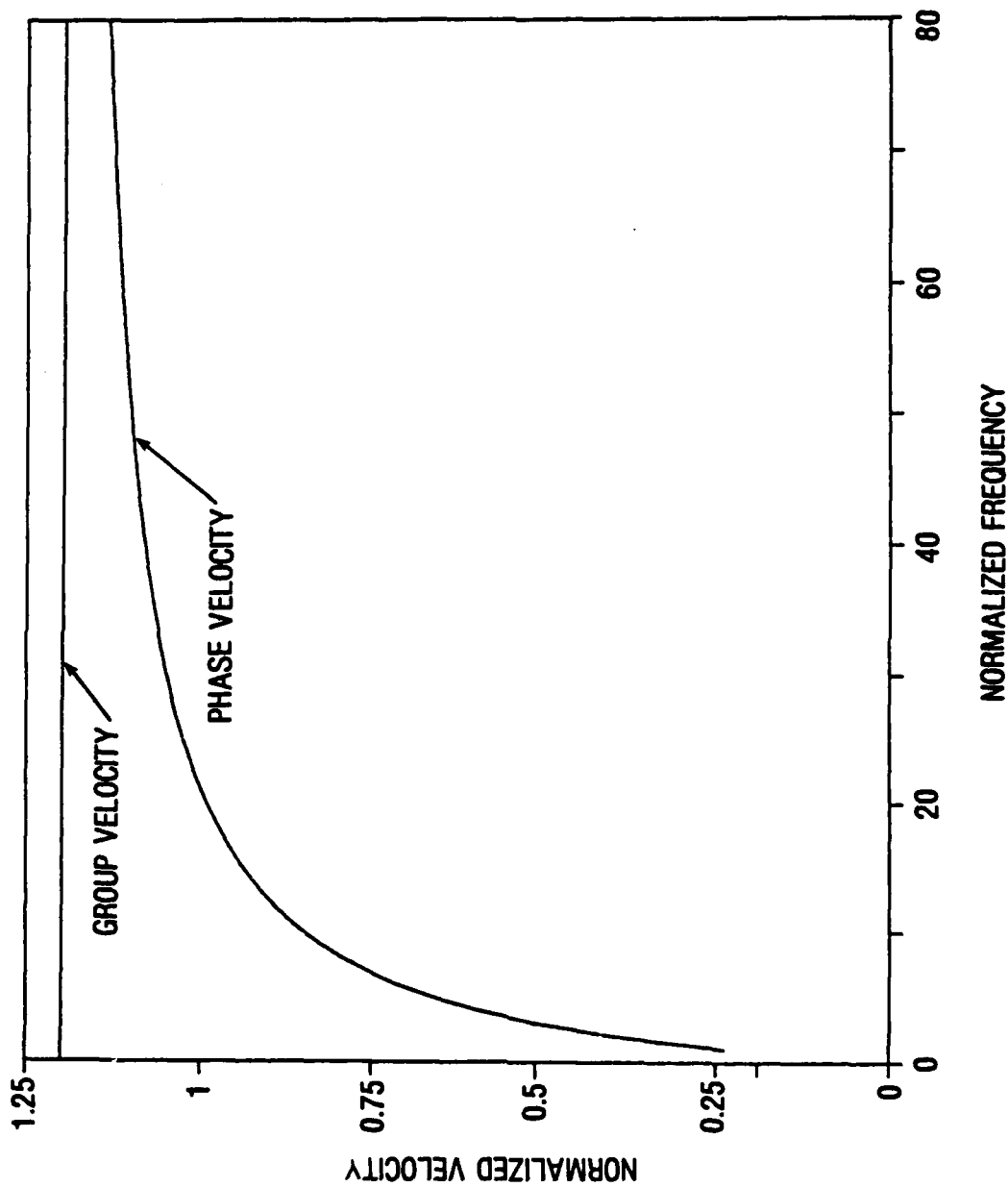


FIGURE 2. PHASE VELOCITY AS A FUNCTION OF FREQUENCY FOR A CONSTANT-GROUP-VELOCITY MEDIUM

The frequency spectrum for the pulsed sine wave with $J = 5$ and $M = 4$ is given in Figure 3. The peak amplitude is at the carrier-wave frequency, the twentieth harmonic, and decreases rather slowly for both increasing and decreasing frequencies.

The propagated pulse is shown in Figure 4 as a function of time, measured in periods of the pulsed sine wave, for the propagation of distances of integer values of the distance X from 0 to 7. The value of X gives the number of wavelengths λ_0 that the pulse has traveled.

At $X = 0$ the waves are identified as A, B, C, D, and E. As X increases by unity, the time at which a particular wave arrives increases by almost exactly one period, t_0 . This can be seen, for example, by noting that the peak of wave A is at $X = 0$ at about $T = 0.25$ (as indicated by the short, vertical line perpendicular to the time axis); at $X = 1$ it is at about 1.25, and so forth for higher values of X . From this it is seen that the peak of the wave is moving with the phase velocity of the wave, the twentieth harmonic, and in fact it is seen that all of the waves in the interior of the pulse move with the velocity of a continuous wave of the carrier frequency.

The group velocity is 1.2 times the phase velocity v_0 of the pulsed wave, and so a point moving at the group velocity which was at $T = 0$ at $X = 0$, would arrive at $X = 1$ at $T = 0.83$ periods, at $X = 2$ at 1.67 periods, etc. This is indicated in Figure 4 by circular dots at $T = 0$, at 0.83, etc. A point which was initially at $T = 5$ periods at $X = 0$ and moving at the group velocity would at the various values of X be 5 periods after the above mentioned point that started at $X = 0$ at $T = 0$. These points are also indicated by dots, which are located at $T = 5, 5.63, \text{etc.}$ From Figure 4 it can be seen that the pulsed sine wave remains essentially between these two dots, which means that the pulse spreads very little, and as a whole moves with the group velocity.

It is also seen from Figure 4 that the pulsed sine wave undergoes little distortion in propagating through the constant-group-velocity medium. That distortion which does occur is primarily at beginning and the end of the pulse. Note also, that since the waves move with the phase velocity v_0 and the pulse as a whole moves at the higher group velocity, namely $1.2v_0$, a new cycle of the wave joins the group each time X increases by six; this cycle in Figure 4 is designated F. The addition of the complete cycle F and the loss of cycle E leads to the duplication at the position $X = 6$, and beginning at the time $T = 5$, of the pulse which at $X = 0$ began at $T = 0$. These values agree with Eqs. (23) and (24) for distances and times at which the signal is duplicated. The negative of the signal at $X = 0$ is seen to occur at $X = 3$, as is indicated by Eq. (27). Similar duplications hold for any X value, for example the signal at $X = 1$ is duplicated at $X = 7$, and its negative occurs at $X = 4$. For the particular example considered here, the negative of the signal is reproduced every time X increases by three, and thus knowledge of the signal over a range of three in X provides information for all distances.

The modulation envelope of the pulse at $X = 0$, $C(0,T)$ from Eq. (21), is a rectangular pulse with a frequency spectrum decreasing relatively slowly so that Brillouin's requirement that $\omega_m < \omega_0$ is not met, and thus distortion of the envelope as the signal propagates is to be expected. At distances, however, at which the signal is equal to the signal at $X = 0$, or to its negative, the

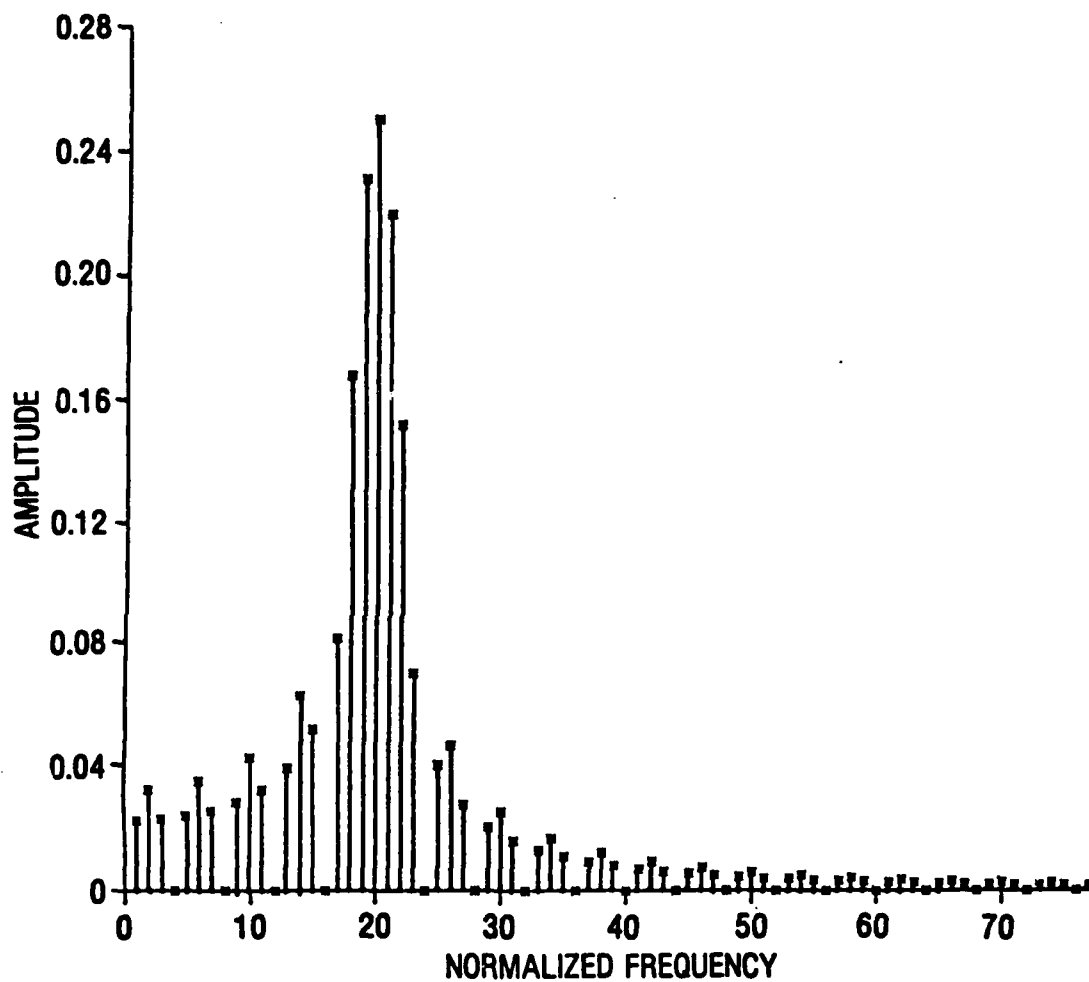


FIGURE 3. FREQUENCY SPECTRUM FOR PULSED SINE WAVE WITH CONSTANT AMPLITUDE, WITH FIVE WAVES PER PULSE, AND WITH A PULSE REPETITION PERIOD FOUR TIMES THE PULSE DURATION

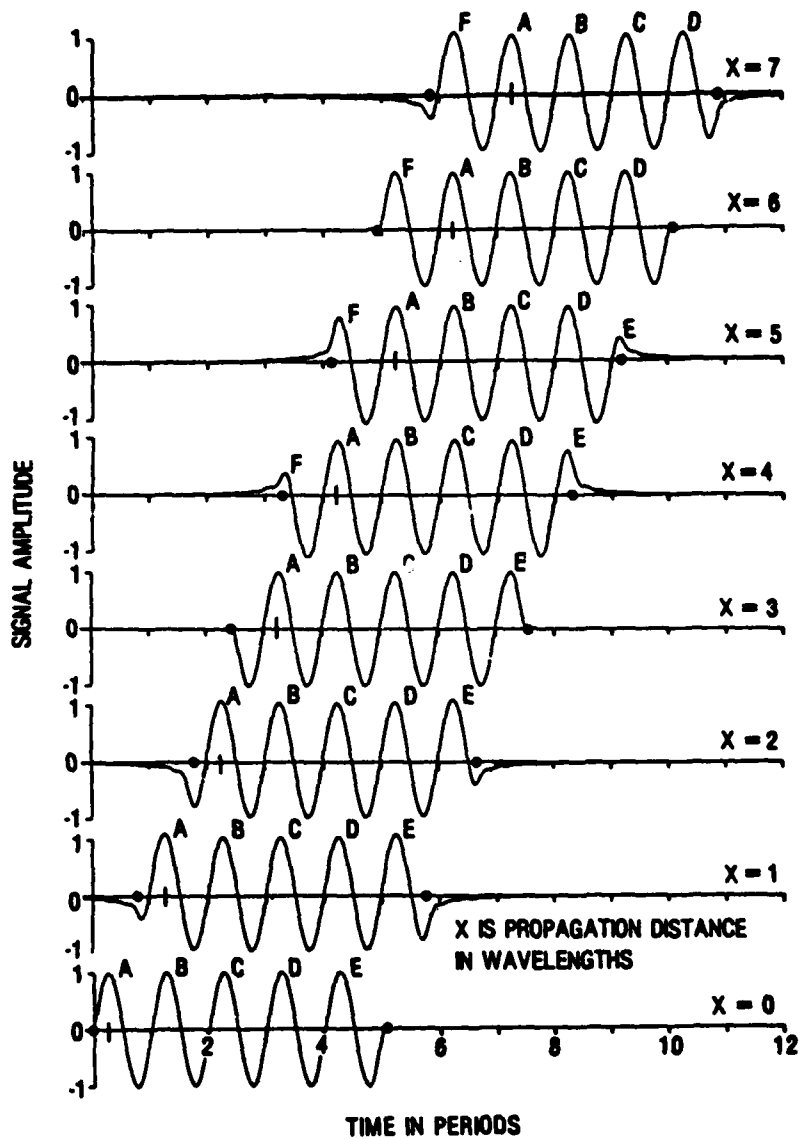


FIGURE 4. CONSTANT-AMPLITUDE PULSE PROPAGATED THROUGH A CONSTANT-GROUP-VELOCITY MEDIUM AS A FUNCTION OF TIME FOR SEVERAL DISTANCES X

modulation envelope will be undistorted. For the pulses considered here this will occur for X equal to multiples of three. This is shown in Figure 5, in which the pulses and modulation envelopes are shown for $X = 0, 1, \text{ and } 3$. The envelopes were determined by dividing the propagated pulse by $\sin \omega_0(t - \frac{X}{V_0})$, Eq. (29).

The modulation envelope used for the constant-amplitude pulsed sine wave was a rectangular pulse for which theoretically ω_m would be infinite. As a check of Brillouin's theory, the rectangular-pulse modulation envelope was approximated in one case by the terms for the first 20 harmonics of the Fourier series for the rectangular pulse and in the second case by the terms for the first 21 harmonics. In both cases $J = 5$ and $M = 4$, so the Brillouin criterion for no envelope distortion was met for the first case and not for the second. Calculations similar to those in Figure 5 showed, in agreement with the theory, no distortion for the twenty-term approximation and distortion similar to that in Figure 5 for $X = 1$ for the twenty-one-term approximation.

The experimental determination of ultrasonic velocities by comparing the transit times of pulses over known distances can often be confusing because of pulse distortion. This is true for the cases considered here as can be seen from Figures 4 and 5, even though zero attenuation has been assumed to simplify the situation. Let two receiving transducers be separated by a distance Δx , and the measured transit time be Δt , then the measured velocity is

$$v = \frac{\Delta x}{\Delta t} = \left(\frac{\Delta x/\lambda_0}{\Delta t/t_0}\right)v_0 = \left(\frac{\Delta X}{\Delta T}\right)v_0 \quad (31)$$

Suppose that in an experiment the receiving transducers are at $X = 0$ and 6, Figure 4, and the transit time is taken as the time between the first peak at each distance. With $\Delta X = 6$ and $\Delta T = 5$, the measured velocity would be $1.2v_0$, or the group velocity. If the transit time were taken as the time between the first peak at $X = 0$ and the second peak at $X = 6$ (the A peaks in both cases), then the measured velocity would be the phase velocity v_0 . Since the phase velocity is often the velocity of interest, being able to identify the A peak in the second pulse is of great importance, and procedures for doing this have been described by McSkimin^b and Papadakas.¹ It is of interest to note that if the second transducer is at an X greater than 30, the A peak is no longer in the propagating pulse. If the transit time were taken to be the time between the first peaks on the $X = 0$ and $X = 5$ pulses, the experimental velocity would be about $1.27v_0$, neither the phase velocity nor the group velocity.

The envelope is sometimes used in velocity measurements. If the transit times for the middles of the envelopes at $X = 0$ and 3 in Figure 5 were used, the group velocity would be obtained. If a full-wave rectifier were used to determine the envelope for these positions, the same result would be obtained. However, if a half wave rectifier were used, the phase velocity would be obtained, since at both positions the C peak (Figure 5) would be at the center of the rectified signal. This would not, however, be the case for greater distances, as the

^bMcSkimin, H. J., "Pulse Superposition Method for Measuring Ultrasonic Wave Velocities in Solids," *J. Acoust. Soc. Am.*, 33, 1961, pp 12-16.

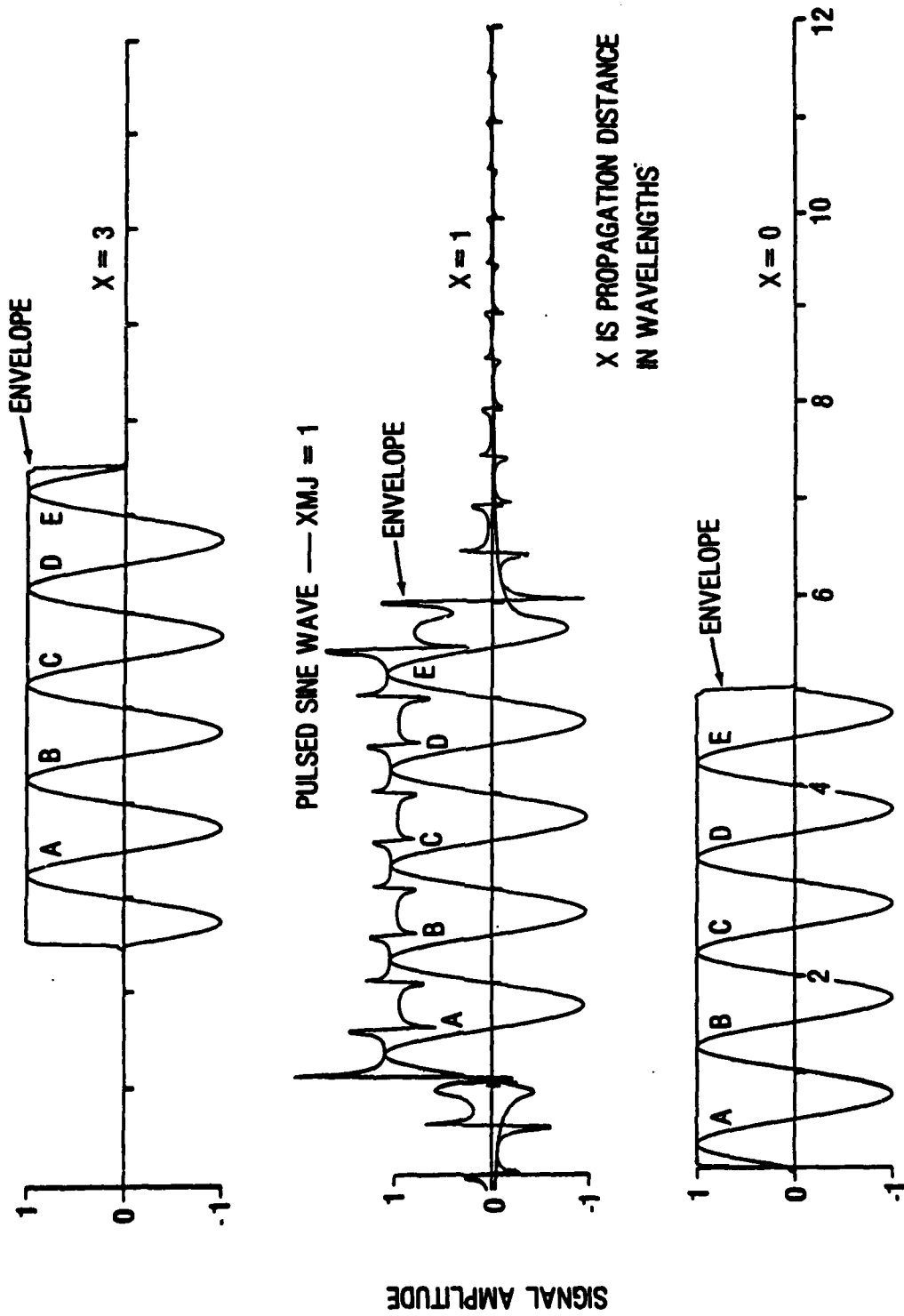


FIGURE 5. CONSTANT-AMPLITUDE PULSED SINE WAVE AND MODULATION ENVELOPES AT THREE DISTANCES

distance is increased the determined velocity would approach the group velocity.

PULSED SINE WAVE -- SINE-SQUARED AMPLITUDE. The equation for the pulsed sine wave with the sine-squared amplitude at $X = 0$ is, for $0 \leq t \leq T_p$,

$$F(0,t) = A \sin^2(M\pi t/T_1) \sin(2\pi M J t/T_1) . \quad (32)$$

The frequency spectrum is given in Figure 6 for $M = 4$ and $J = 5$. The peak amplitude is at the carrier frequency, the twentieth harmonic of the Fourier series, and although there are theoretically an infinite number of frequencies in the spectrum, the amplitudes drop off very fast and become negligible as zero frequency and the 40th harmonic are approached. Because of this rapid drop in amplitude, this spectrum essentially satisfies Brillouin's requirement that the maximum angular frequency ω_m in the spectrum of the modulation envelope be less than the carrier frequency ω_0 , and thus little distortion of the envelope is to be expected as the pulse is propagated.

The propagated pulse is shown in Figure 7 for the same values of X as is shown for the constant-amplitude pulse in Figure 4. The times and distances are normalized to the period and wavelength of the pulsed wave, and the group velocity is 1.2 times its phase velocity. The waves at $X = 0$ are identified as A, B, C, D, and E as in Figure 4, and are followed as the pulse progresses to larger X values. The envelopes for all values of X are also shown in the figure, and are seen to be essentially undistorted, which is to be expected as indicated above. A square dot is placed on the X axis underneath the peak of the envelope for all the curves. From the positions of the dot as X increases it is seen that the peak of the envelope moves with the group velocity $1.2v_0$. Circular dots are also placed at $T = 0$ and 5 at $X = 0$, and move with the group velocity as in Figure 4.

As was the case in Figure 4, this pulse also changes shape as it progresses through the medium, but because of the small amplitudes the distortions at the beginning and end are much less evident than in Figure 4. The changes observed are primarily due to waves moving at the phase velocity v_0 and the envelope moving at $1.2v_0$, the group velocity. As was the case before, it is seen that the pulse at $X = 0$ is duplicated five periods later at $X = 6$, and its negative is duplicated 2.5 periods later at $X = 3$.

The experimental determination of velocities by comparing the pulse transit times poses problems similar to those discussed earlier relative to the constant-amplitude pulsed sine wave and Figure 4.

RECTANGULAR PULSE. The equation for the rectangular pulse is

$$\begin{aligned} F(x,t) &= A & 0 \leq t \leq T_p , \\ &= 0 & T_p < t < T_1 , \end{aligned}$$

and the cycle repeats with the period T_1 . The coefficients for the Fourier series are given in the Appendix.

The frequency spectrum is shown in Figure 8, and it is seen that the maximum

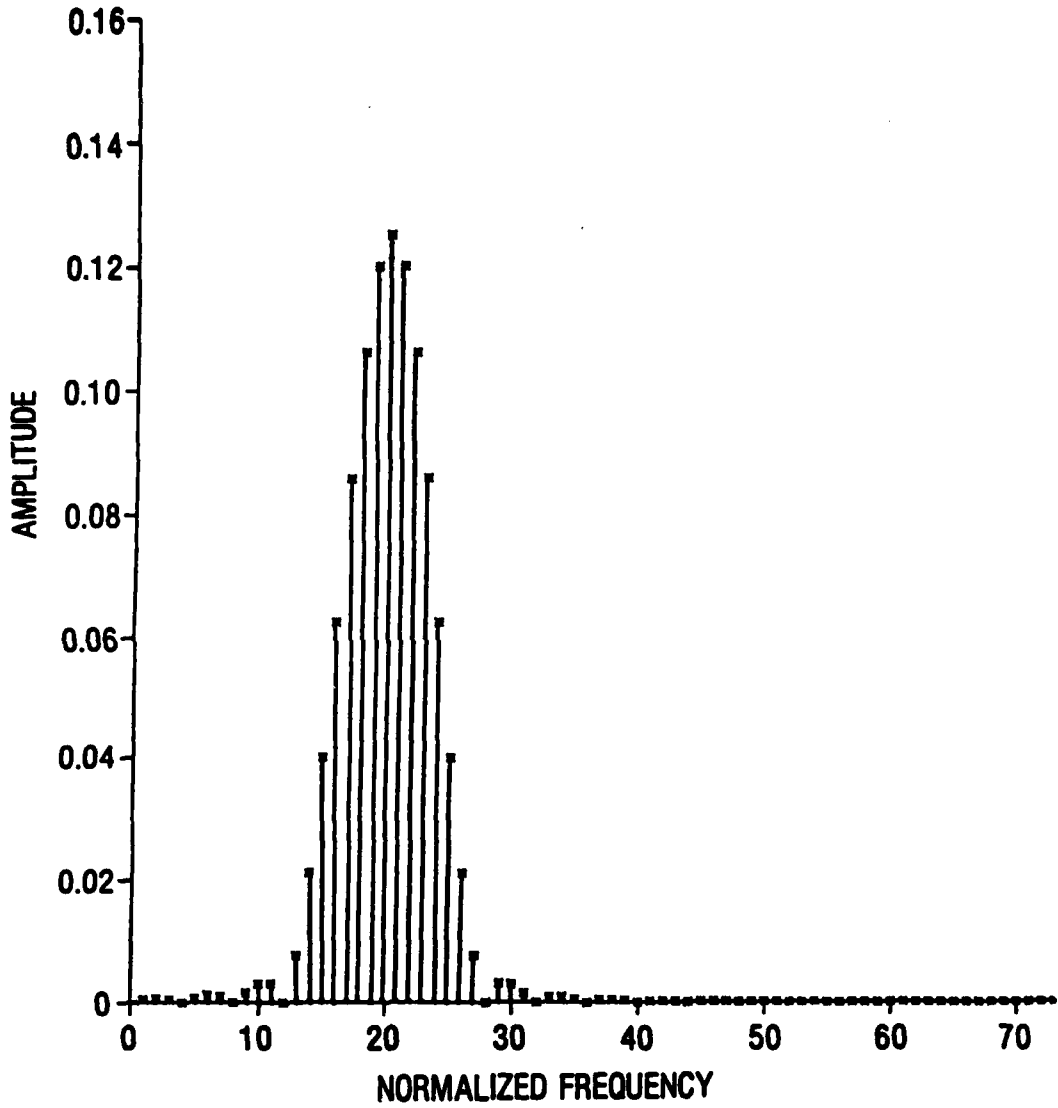


FIGURE 6. THE FREQUENCY SPECTRUM FOR PULSED SINE WAVE WITH SINE-SQUARED AMPLITUDE. THERE ARE FIVE WAVES PER PULSE, AND THE PULSE REPETITION PERIOD IS FOUR TIMES THE PULSE DURATION

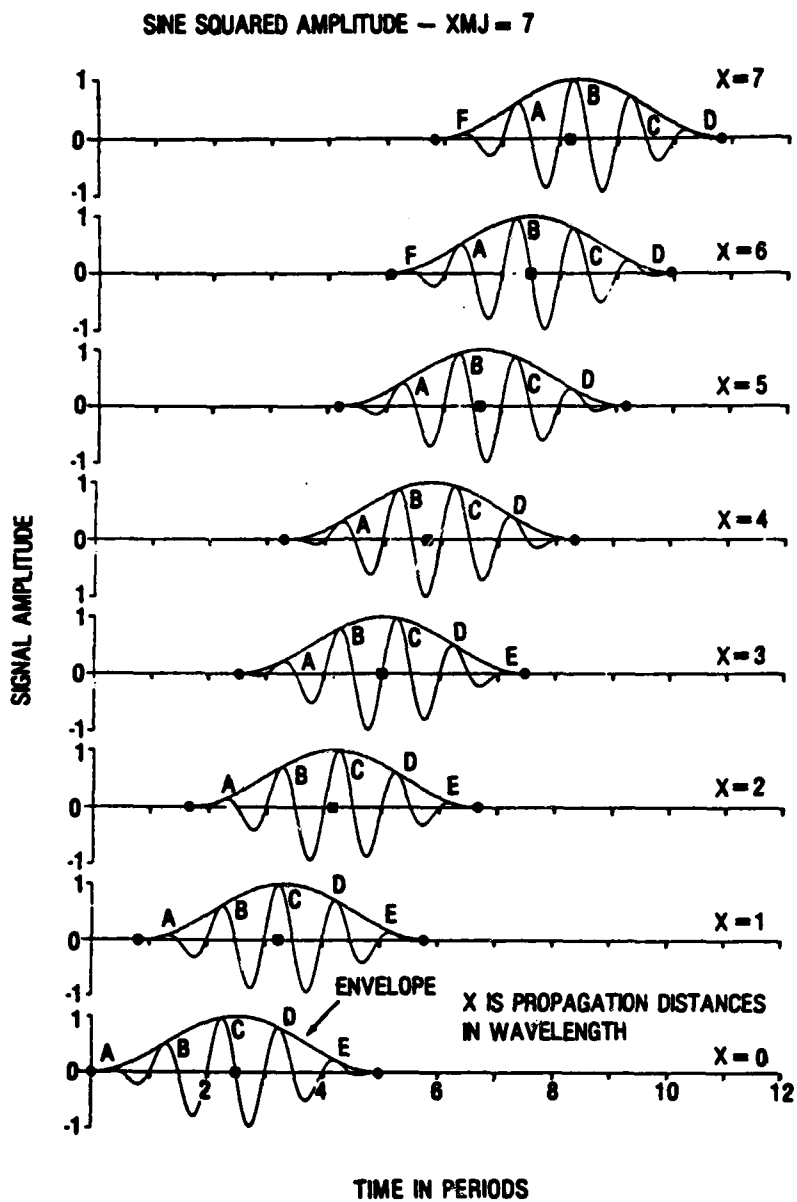


FIGURE 7. SINE-SQUARED-AMPLITUDE PULSE, WITH ENVELOPE, PROPAGATED THROUGH A CONSTANT-GROUP-VELOCITY MEDIUM AS A FUNCTION OF TIME FOR SEVERAL DISTANCES X. MODULATION ENVELOPES ARE SHOWN FOR ALL DISTANCES

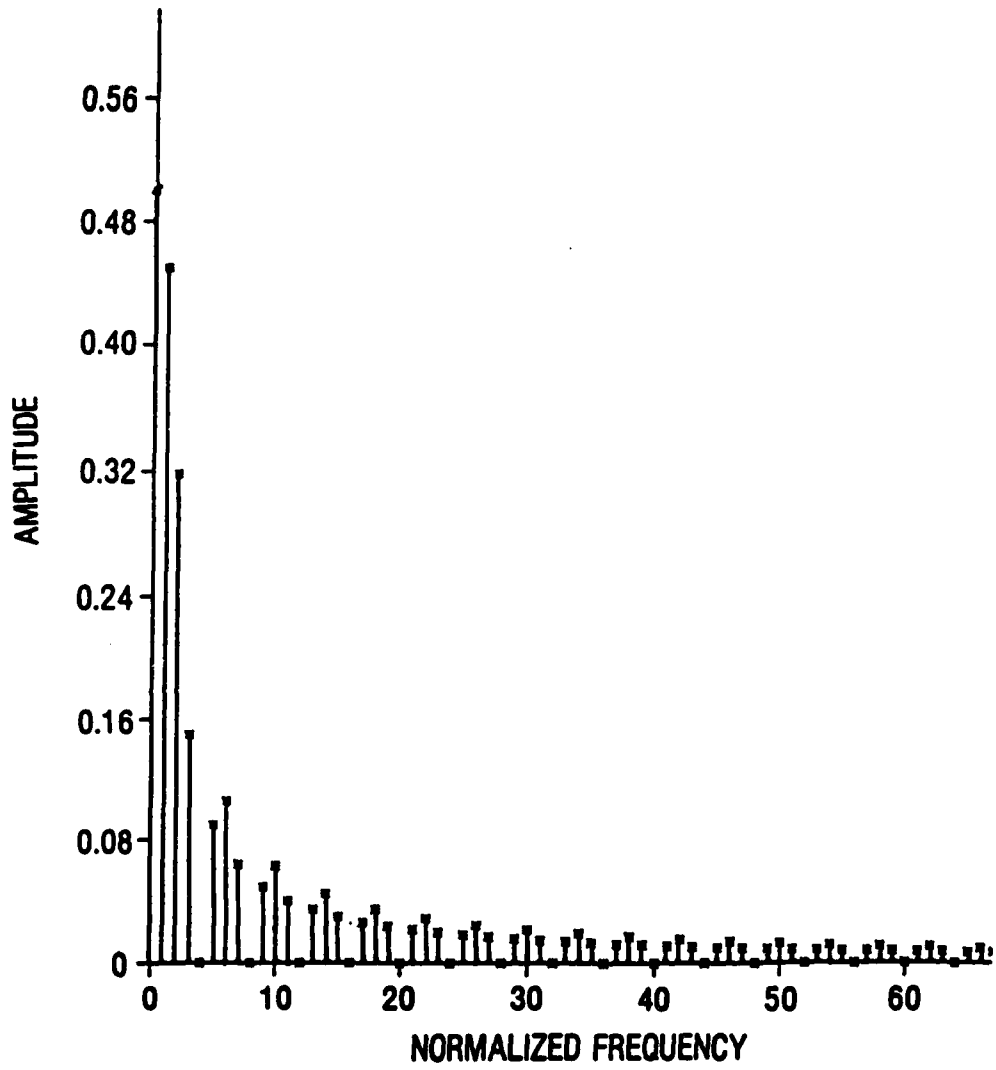


FIGURE 8. THE FREQUENCY SPECTRUM FOR A RECTANGULAR PULSE WITH REPETITION PERIOD FOUR TIMES THE PULSE DURATION

amplitude is at zero frequency, and the amplitude decreases rather slowly with increased frequency.

The propagated pulse is shown for $X = 0$ to 7 in Figure 9. There is much distortion of the pulse as it propagates, which is not surprising considering that its frequency spectrum has its maximum values in the frequency region in which the greatest velocity changes occur. The signal is seen to repeat with each increase of six in X , and an increase of three in X is seen to reverse the signal, except for the constant term $A_0/2$, as indicated by Eq. (27) and observed for the other two cases.

Two dots are placed on the $X = 0$ plot at the beginning and end of the rectangular pulse, and they are positioned for the other values of X as moving with the group velocity, and from this it is seen that the pulse moves at the group velocity.

CONCLUSION

A study has been made of the transmission and distortion of ultrasonic pulses in lossless dispersive media with constant group velocity. It was found that a pulsed-sine-wave signal moves through the media at the group velocity undergoing some distortion, but after traveling a certain distance -- the repetition distance X_r -- the signal is repeated, and at positions midway between repetition positions the signal is the negative of that at the repetition positions. Also, although the pulsed-sine-wave signal moves with the group velocity, the individual cycles within the pulse move with the phase velocity of the pulsed, or carrier, wave as if they are cycles within a continuous wave. It was also found that the envelope of the pulsed sine wave was distorted except at positions where the signal at the origin, or its negative, was repeated, and except for those cases, as indicated by Brillouin, in which the maximum angular frequency components of the envelope were less than the carrier frequency ω_0 .

The rectangular-pulse signal underwent great distortion in propagating through the constant-group-velocity media, but it repeated itself in traveling the repetition distance X_r , and inverted midway between repetition distances as indicated by Eq. (27). The signal, though badly distorted for most positions, moved at the group velocity.

It was seen that great care must be taken in velocity measurements using pulsed sine waves. Measurements of transit times using the initial peaks of the signal at two positions may give the phase velocity if the two positions are close enough together, but it may give other values, and it approaches the group velocity at large distances. Measurements of transit times of the middle of the envelope will at large distances give the group velocity, but as indicated above, other values may be obtained. Velocity measurements in lossy and non-constant-group-velocity media could be expected to be more complicated and to require greater care for lossless constant-group-velocity situation.

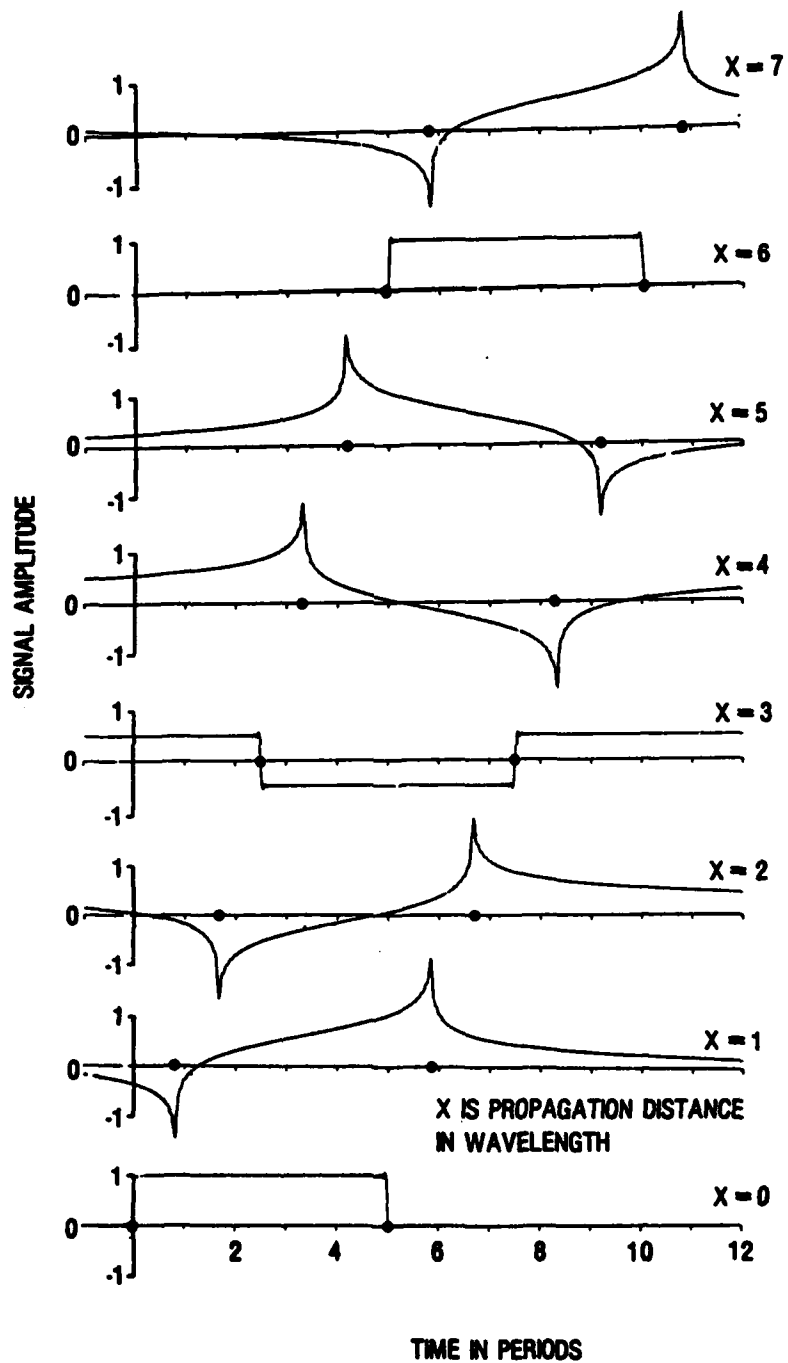


FIGURE 9. RECTANGULAR PULSE PROPAGATED THROUGH A CONSTANT GROUP VELOCITY MEDIUM. AS IN THE PREVIOUS EXAMPLES, THE TIMES AND DISTANCES ARE NORMALIZED TO THE TWENTIETH HARMONIC OF THE FOURIER SERIES, AND THE GROUP VELOCITY IS 1.2 TIMES THE PHASE VELOCITY OF THE TWENTIETH HARMONIC

BIBLIOGRAPHY

Brillouin, L., Wave Propagation and Group Velocity, (Academic Press, New York, 1960), pp 7-10.

McSkimin, H. J., "Pulse Superposition Method for Measuring Ultrasonic Wave Velocities in Solids," J. Acoust. Soc. Am., 33, 1961, pp 12-16.

Papadakis, E. P., "Ultrasonic Velocity and Attenuation: Measurement Methods with Scientific and Industrial Applications," in Physical Acoustics, edited by W. P. Mason and R. N. Thurston (Academic Press, New York, 1976), Vol. XII, Chap. 5, pp 277-374.

Sachse, W. and Pao, Y-H, "On the Determination of Phases and Group Velocities of Dispersive Waves in Solids," J. Appl. Phys., 49, 1978, pp 4320-4327.

APPENDIX A
FOURIER COEFFICIENTS

For pulses starting at t equal zero and of duration T_p with a repetition period T_1 , the equations for the Fourier coefficients A_n and B_n (see Equation (1)) are

$$A_n = \frac{2}{T_1} \int_0^{T_p} P_F(0,t) \cos n\omega_1 t \, dt ,$$

and

$$B_n = \frac{2}{T_1} \int_0^{T_p} P_F(0,t) \sin n\omega_1 t \, dt .$$

The amplitudes C_n of the Fourier spectrum are determined from the relation

$$C_n = (A_n^2 + B_n^2)^{1/2}$$

The Fourier coefficients for the three cases considered are given below.

- a. Pulsed Sine Wave, Constant Amplitude

For $n \neq MJ$,

$$A_n = - \frac{AMJ}{\pi} \left[\frac{\cos(2n\pi/M) - 1}{(MJ)^2 - n^2} \right] ,$$

$$B_n = - \frac{AMJ}{\pi} \left[\frac{\sin(2n\pi/M)}{(MJ)^2 - n^2} \right] ,$$

and for $n = MJ$

$$A_n = 0 ,$$

and

$$B_n = A/M .$$

b. Pulsed Sine Wave, Sine-Squared Amplitude

$$A_n = A_{1n} + A_{2n} + A_{3n}$$

$$B_n = B_{1n} + B_{2n} + B_{3n}$$

$$A_{1n} = \frac{-AMJ}{2\pi} \left[\frac{\cos(2\pi n/M) - 1}{(MJ)^2 - n^2} \right] \quad n \neq MJ$$

$$A_{1n} = 0 \quad n = MJ$$

$$A_{2n} = \frac{A(MJ+M)}{4\pi} \left[\frac{\cos(2\pi n/M) - 1}{(MJ+M)^2 - n^2} \right] \quad n \neq MJ+M$$

$$A_{2n} = 0 \quad n = MJ+M$$

$$A_{3n} = \frac{A(MJ-M)}{4\pi} \left[\frac{\cos(2\pi n/M) - 1}{(MJ-M)^2 - n^2} \right] \quad n \neq MJ-M$$

$$A_{3n} = 0 \quad n = MJ-M$$

$$B_{1n} = \frac{-AMJ}{2\pi} \left[\frac{\sin(2\pi n/M)}{(MJ)^2 - n^2} \right] \quad n \neq MJ$$

$$B_{1n} = \frac{A}{2M} \quad n = MJ$$

$$B_{2n} = \frac{A(MJ+M)}{4\pi} \left[\frac{\sin(2\pi n/M)}{(MJ+M)^2 - n^2} \right] \quad n \neq MJ+M$$

$$B_{2n} = \frac{-A}{4M} \quad n = MJ+M$$

$$B_{3n} = \frac{A(MJ-M)}{4\pi} \left[\frac{\sin(2\pi n/M)}{(MJ-M)^2 - n^2} \right] \quad n \neq MJ-M$$

$$B_{3n} = \frac{-A}{4M} \quad n = MJ-M$$

c. Rectangular Pulse

$$A_n = \frac{A}{n\pi} \left[\sin(2\pi n/M) \right]$$

$$B_n = \frac{A}{n\pi} \left[1 - \cos(2\pi n/M) \right]$$

DISTRIBUTION

	<u>Copies</u>		<u>Copies</u>
Deputy Under Secretary of Defense for Research and Engineering The Pentagon Washington, DC 20301	1	Commander Naval Sea Systems Command Attn: M. Kinna, Code 62R4 H. Vanderveldt, Code 05R15 Washington, DC 20362	2 1
Office of Deputy Under Secretary of Defense for Research and Engineering Attn: J. Persh Staff Specialist for Materials and Structures Room 3D1089, The Pentagon Washington, DC 20301	1	Commander Naval Air Systems Command Attn: R. Schmidt, Code 320A Washington, DC 20361	1
Defense Advanced Research Projects Agency Attn: E. VanReuth L. Jacobsen R. E. Green, Jr. 1400 Wilson Blvd. Arlington, VA 22209	1 1 1	Department of the Army Attn: A. Levitt J. Greenspan AMMRC Watertown, MA 02172	1 1
Naval Material Command Attn: J. Kelly, MAT-0725 W. Holden, MAT 064 Department of the Navy Washington, DC 20360	1 1	U.S. Army Research Office Associate Director, Metallurgy and Materials Sci. Div. Attn: J. C. Hurt P. O. Box 1221 Triangle Park, NC 27709	1
Office of Naval Research Attn: B. MacDonald, Code 471 Y. Rajapakse, Code 432 Washington, DC 20375	2 1	Commanding Officer Air Force Materials Laboratory Attn: M. Duhl, Code MB T. Ronald, Code LLS Wright-Patterson Air Force Base Dayton, OH 45433	1 1
Naval Research Laboratory Attn: S. C. Sanday, Code 6370 I. Wolock, Code 6307 H. Chaskelis, Code 8431 Washington, DC 20375	1 1 1	Air Force Materials Laboratory Attn: R. M. Neff, Nonmetallic Materials Division D. R. Beeler, MB WPAFB, OH 45433	1 1

DISTRIBUTION (Cont.)

	<u>Copies</u>		<u>Copies</u>
NASA Lewis Research Center Attn: R. A. Signorelli 21000 Brookpark Road Cleveland, OH 44135	1	United Technologies Research Center Attn: K. M. Prewo East Hartford, CT 06108	1
Defense Technical Information Center Cameron Station Alexandria, VA 22314	12	Vought Corporation Advanced Technology Center Attn: R. C. Van Siclen C. M. Standard P. O. Box 226144 Dallas, TX 75266	1 1
National Bureau of Standards Attn: G. Blessing Sound Bldg: A147 Washington, DC 20234	1	Measurements Technology Inc. Attn: M. F. Duggan 3029 Greer Road Palo Alto, CA 94303	1
NETCO Attn: L. Davis 2225 E. 28th Street Long Beach, CA 90806	1	The Applied Research Laboratory Attn: R. Reed P. O. Box 30 State College, PA 16801	1
Aerospace Corporation Attn: L. Rubin H. Katzman G. Hawkins P. O. Box 92957 Los Angeles, CA 90009	1 1 1	International Harvester Company Attn: M. Amateau 7 South 500 County Line Road Hinsdale, IL 60521	1
Exxon Enterprises Attn: Jack L. Cook Materials Division P. O. Drawer H Greer, SC 29651	1	General Electric Space Center Attn: C. Zweben M4018 P. O. Box 8555 Philadelphia, PA 19101	1
DWA Composite Specialties, Inc. Attn: J. Dolowy 21133 Superior Street Chatsworth, CA 91311	1	Metal Matrix Composites Information Analysis Center Kaman Tempo 816 State Street P. O. Drawer QQ Santa Barbara, CA 93102	2
Material Concepts, Inc. Attn: S. Paprocki 666 North Hague Avenue Columbus, OH 43204	1	The University of Texas Attn: H. L. Marcus Tay 167 Austin, TX 78712	1
Material Sciences Corporation Attn: B. W. Rosen Blue Bell Office Campus Merion Towle House Blue Bell, PA 19422	1	Library of Congress Attn: Gift and Exchange Division Washington, DC 20540	4

DATE
FILME
— 8