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MODIFIED BETATRON STABILITY CALCULATIONS.(U)  
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MODIFIED BETATRON STABILITY CALCULATIONS

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August 1982

Prepared for: Office of Naval Research  
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Under Contract: N00014-81-C-0647

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. <b>AD-A119584</b>	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle)  <b>MODIFIED BETATRON STABILITY CALCULATIONS</b>		5. TYPE OF REPORT & PERIOD COVERED  <b>Interim</b>
		6. PERFORMING ORG. REPORT NUMBER <b>AMRC-N-207</b>
7. AUTHOR(s) <b>Brendan B. Godfrey Thomas P. Hughes</b>		8. CONTRACT OR GRANT NUMBER(s)  <b>N00014-81-C-0647</b>
9. PERFORMING ORGANIZATION NAME AND ADDRESS <b>MISSION RESEARCH CORPORATION 1720 Randolph Road, S.E. Albuquerque, New Mexico 87106</b>		10. PROGRAM ELEMENT PROJECT TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS <b>Office of Naval Research 800 North Quincy Street Arlington, Virginia 22217</b>		12. REPORT DATE <b>August 1982</b>
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES <b>14</b>
		15. SECURITY CLASS (of this report)  <b>Unclassified</b>
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  <b>Approved for Public Release: Distribution Unlimited</b>		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  <b>Modified Betatron Resistive Wall Instability Negative Mass Instability</b>		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  <b>Cold beam linear growth rates of the negative mass and resistive wall instabilities in a high current modified betatron have been determined numerically. The resistive wall instability is much slower than the negative mass instability even for large wall resistivities. The two instabilities can be suppressed by a 10% electron energy spread provided <math>I(kA) &lt; 2[B_0(kg)]^{1/3} \cdot (\gamma/10)^{5/2}</math>.</b>		

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## MODIFIED BETATRON STABILITY CALCULATIONS

The Naval Research Laboratory presently is engaged in the point design of a high current modified betatron<sup>1,2</sup> in preparation for building such a device. This note summarizes linear growth rate computations of the negative mass and resistive wall instabilities performed in support of the design effort. Table 1 lists the base parameters assumed. In the following we first consider instability growth in a cold beam and then estimate reduction of the growth rates caused by a spread in beam particle energy.

Cold beam results were obtained by solving numerically the dispersion relation derived in Ref. 3. The dispersion relation predicts two distinct regimes of negative mass instability, one lying above and the other below a transition energy

$$\gamma_{tr} = (4 \nu r_0^2 / a^2)^{1/3} . \quad (1)$$

For the parameters of Table 1 with  $I = 10$  kA,  $\gamma_{tr} \approx 6.2$ . Figure 1 shows the growth rates versus beam energy above the transition point for the first three toroidal mode numbers. Wall conductivity is infinite, and  $I = 10$  kA,  $B_0 = 5$  kg. Note that the peak growth scales as  $\lambda^{1/2}$ . We also ran infinite conductivity cases for  $I = 10$  kA,  $B_0 = 1.46$  kg and  $I = 2$  kA,  $B_0 = 1.42$  kg. Results for  $\lambda = 1$ , plotted in Figure 2, indicate a  $(\gamma I / B^2)^{1/3}$  scaling of the growth rate for  $\gamma$ 's below the peaks of the growth rate curves, in agreement with Ref. 3. The peak growth rate, on the other hand, scales roughly as  $(I/B)^{1/2}$ .

Behavior of the growth rate curves in the vicinity of the transition energy is illustrated in Figure 3 for  $\lambda = 1$  and in Figure 4 for  $\lambda = 2$ ;  $I = 10$  kA,  $B_0 = 5$  kg. The curves go to zero with infinite slope at  $\gamma_{tr}$ .



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TABLE 1. NOMINAL MODIFIED BETATRON PARAMETERS

Major Radius	$r_0 = 100 \text{ cm}$	
Minor Radius	$a = 10 \text{ cm}$	
Beam Radius	$r_b = 1 \text{ cm}$	
Guide Field	$B_\theta \leq 5 \text{ kg}$	$(\omega_\theta \leq 2.933 \text{ cm}^{-1})$
Vertical Field	$B_z$	$(\omega_z = \beta\gamma/r_0)$
Field Index	$n = 1/2$	
Beam Current	$I \leq 10 \text{ kA}$	$(\nu < 0.588)$
Beam Energy	$U = 2-50 \text{ MeV}$	$(\gamma = 5-100)$
Resistivity	$250, 0 \text{ } \mu\Omega\text{-cm}$	$(\sigma = 1.5 \cdot 10^6, \infty \text{ cm}^{-1})$

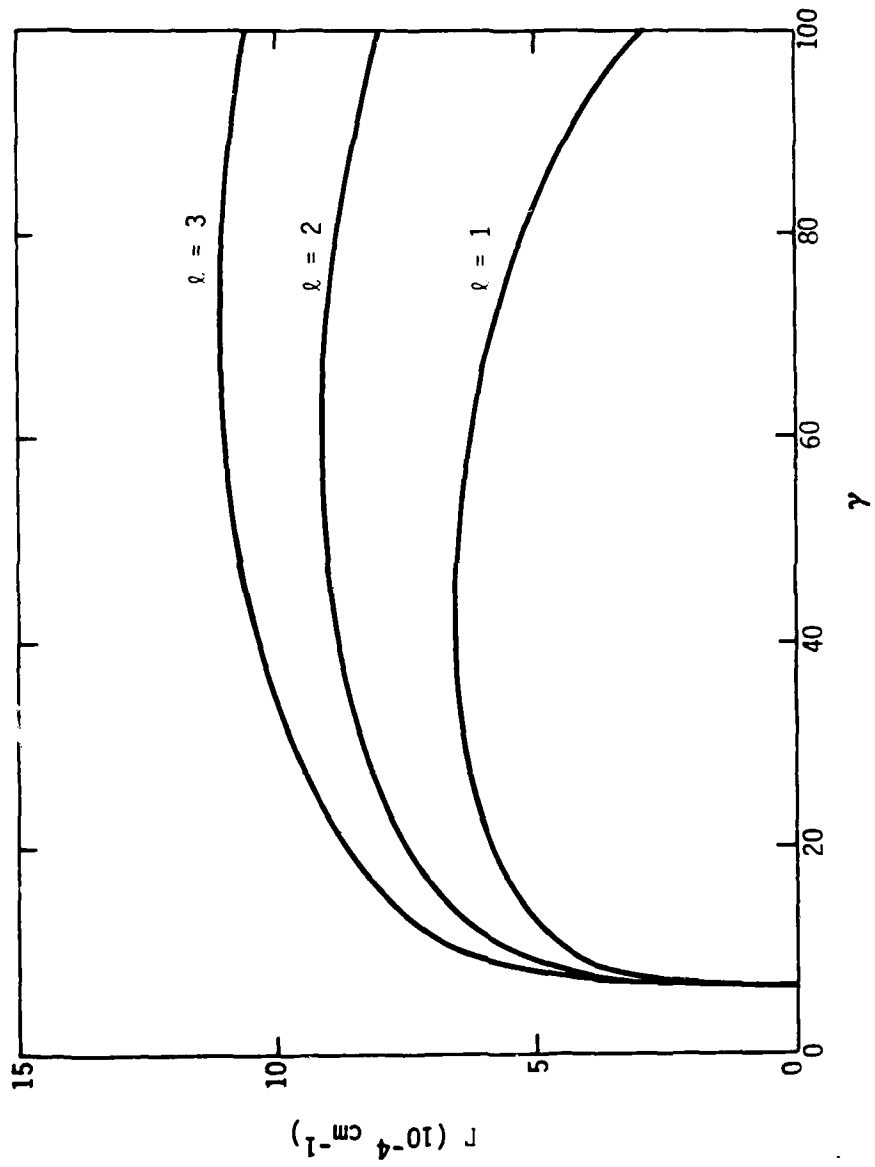


Figure 1.  $L = 1-3$  negative mass instability growth rates versus beam energy above the transition point for  $I = 10 \text{ kA}$ ,  $B_{\theta} = 5 \text{ kg}$ .

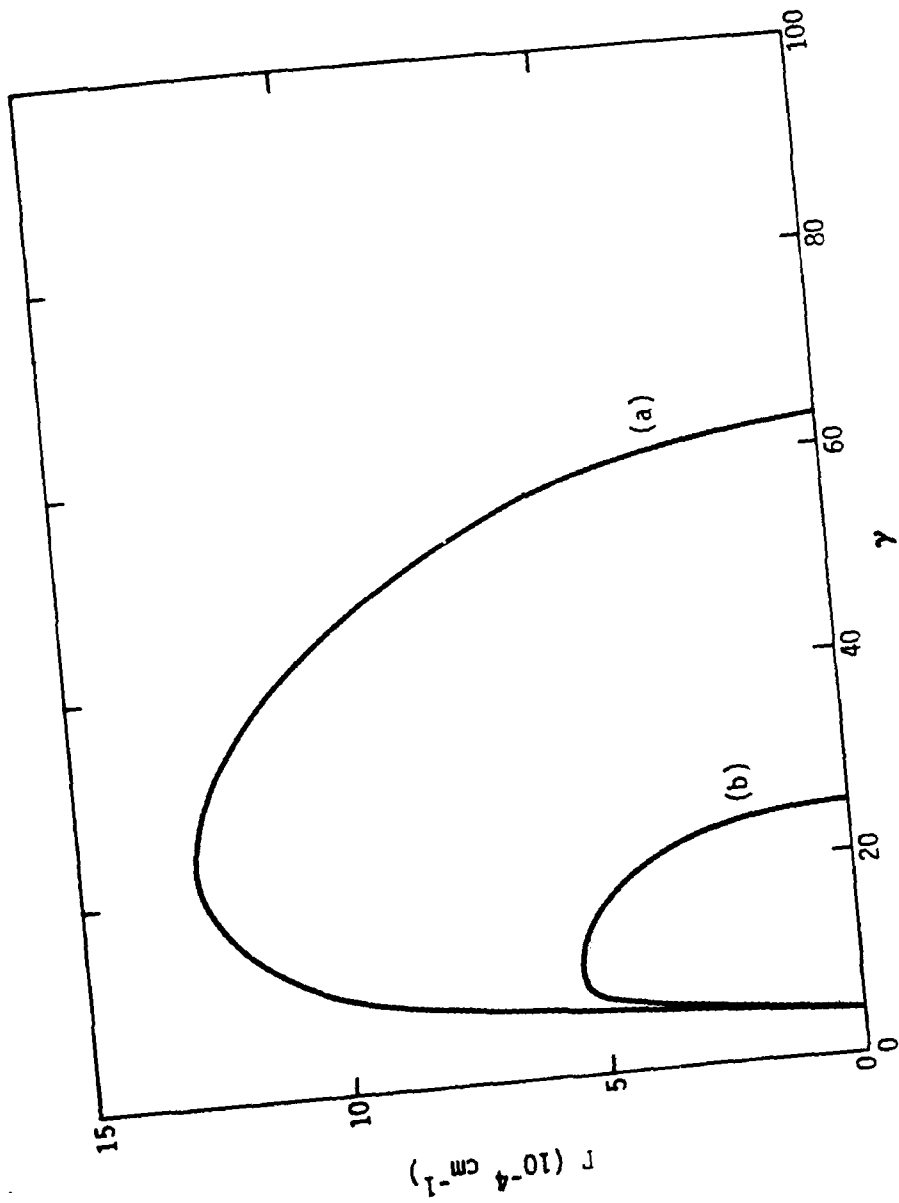


Figure 2.  $L = 1$  negative mass instability growth rates versus beam energy above the transition point for (a)  $I = 10 \text{ kA}$ ,  $B_0 = 1.42 \text{ kG}$ , and (b)  $I = 2 \text{ kA}$ ,  $B_0 = 1.42 \text{ kG}$ .

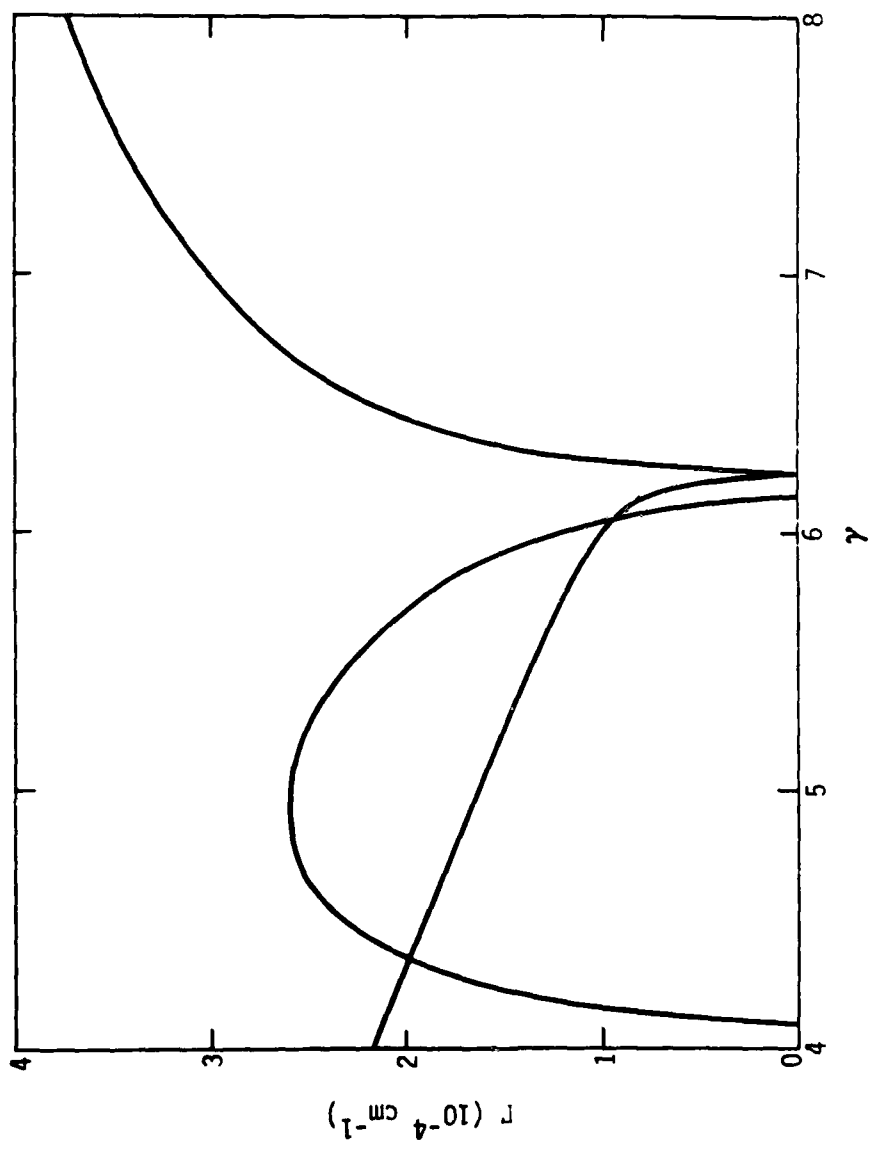


Figure 3.  $L = 1$  negative mass instability growth rates versus beam energy in the vicinity of the transition point for  $I = 10$  kA,  $B_0 = 5$  kg.

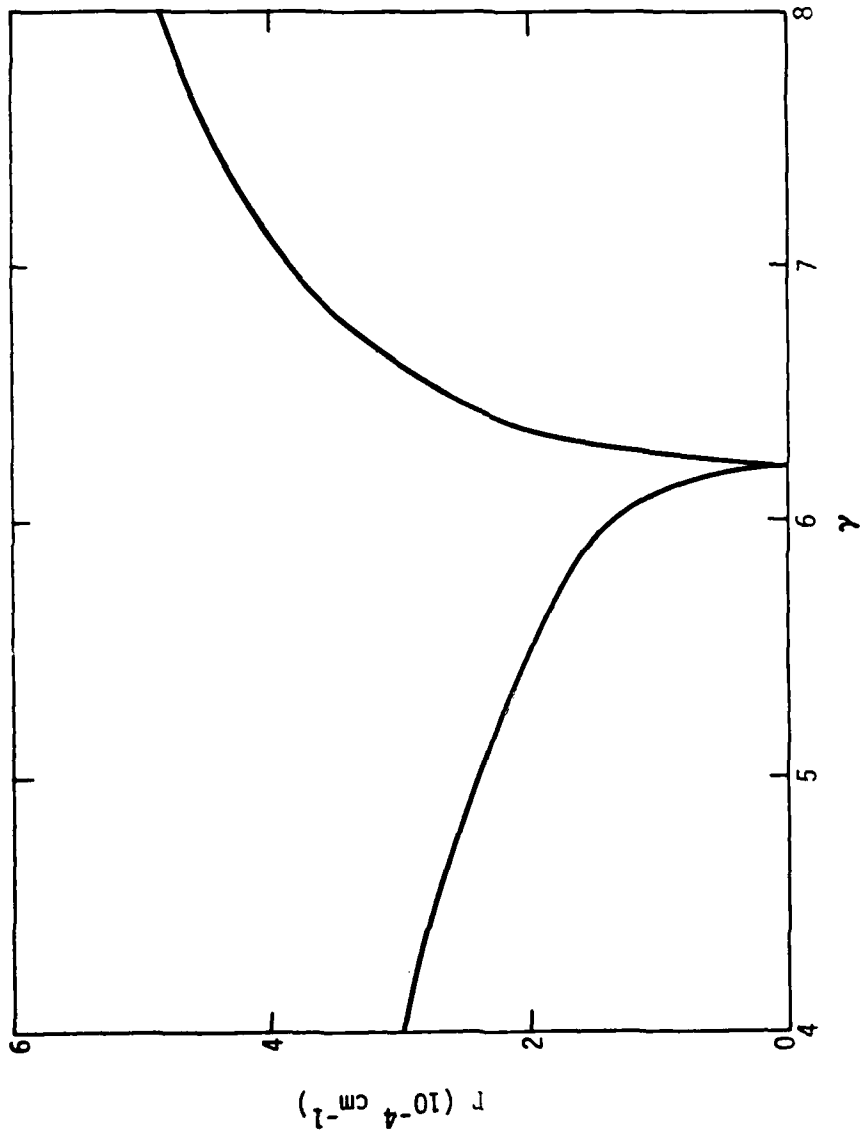


Figure 4.  $L = 2$  negative mass instability growth rates versus beam energy in the vicinity of the transition point for  $I = 10 \text{ kA}$ ,  $B_{\theta} = 5 \text{ kg}$ .

Below  $\gamma_{tr}$  one or two unstable modes occur, depending on the parameters chosen. The growth rate dependence on  $\lambda$  in this region typically is weak, while the scaling with current and toroidal magnetic field crudely follows the expressions valid above  $\gamma_{tr}$ .

Introducing a 250  $\mu\Omega$ -cm wall resistivity changes the negative mass growth curves of Figures 1 through 4 by only a few percent except in the immediate vicinity of  $\gamma_{tr}$ , where the otherwise occurring sharp dip in the growth rate curve is partially filled in. Additionally, resistivity drives unstable a previously stable negative mass mode but with a growth rate down from those of the original modes by an order of magnitude. Compare Figure 5 to Figure 3.

A transverse cyclotron wave instability also is triggered by wall resistivity, when  $\omega = \lambda/R - \omega_{\theta}/\gamma > 0$ . Figure 6 depicts growth rates for  $\lambda = 4, 8$  at  $I = 10$  kA,  $B_z = 5$  kg. As noted in Ref. 4, this instability is well described by models developed for beams in straight drift tubes. In particular, the peak growth rate formula

$$\Gamma = \frac{1}{2} \sigma^{-1/3} \left( \frac{\omega_p^2 r_b^2}{\omega_{\theta}^2 a^3} \right)^{2/3} \quad (2)$$

reproduces data in Figure 6 to within 5%. (The conductivity  $\sigma$  is measured in  $\text{cm}^{-1}$  and contains a factor  $4\pi$ .) The transverse resistive wall instability is seen to be somewhat slower than the negative mass instability even for the rather high resistivity assumed. Moreover, the growth curves are much narrower in  $\gamma$ . We conclude that wall resistivity is not a significant factor in modified betatron stability.

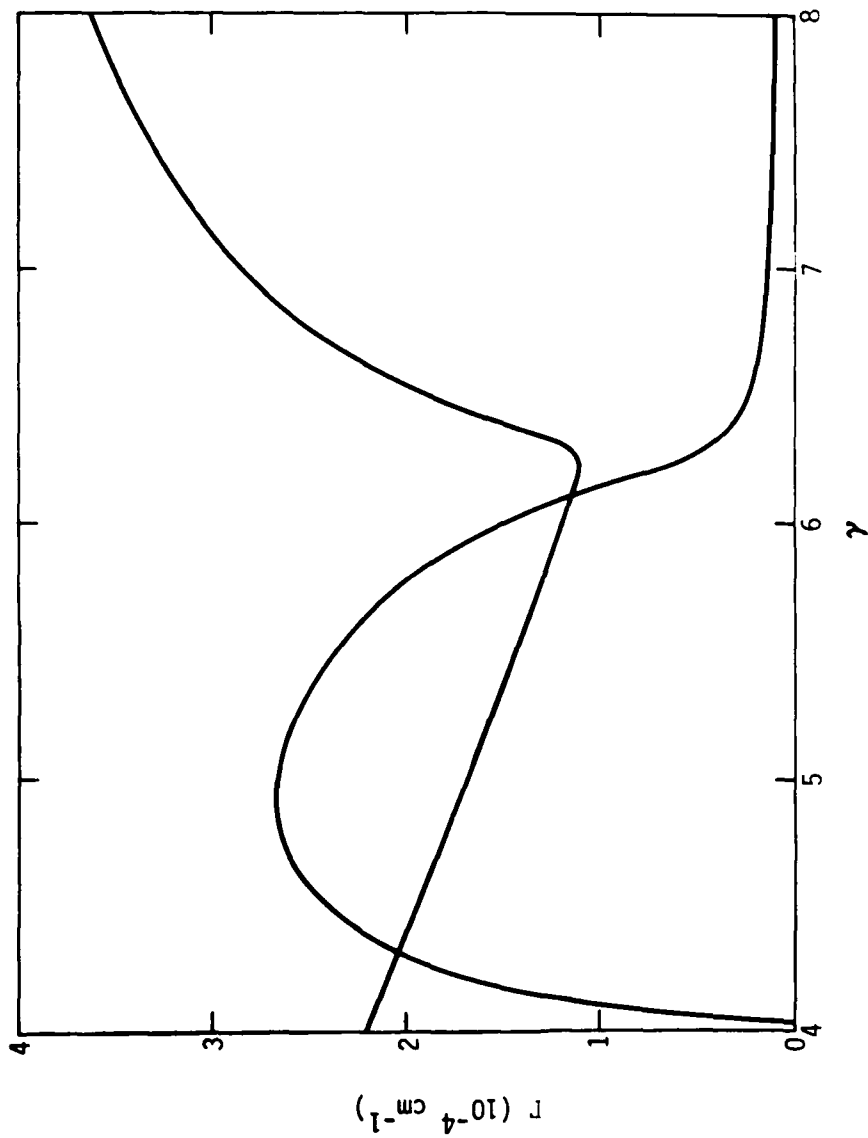


Figure 5.  $L = 1$  negative mass instability growth rates, as modified by 250  $\mu\Omega$ -cm wall resistivity, versus beam energy in the vicinity of the transition point for  $I = 10$  kA,  $B_{\theta} = 5$  kg.

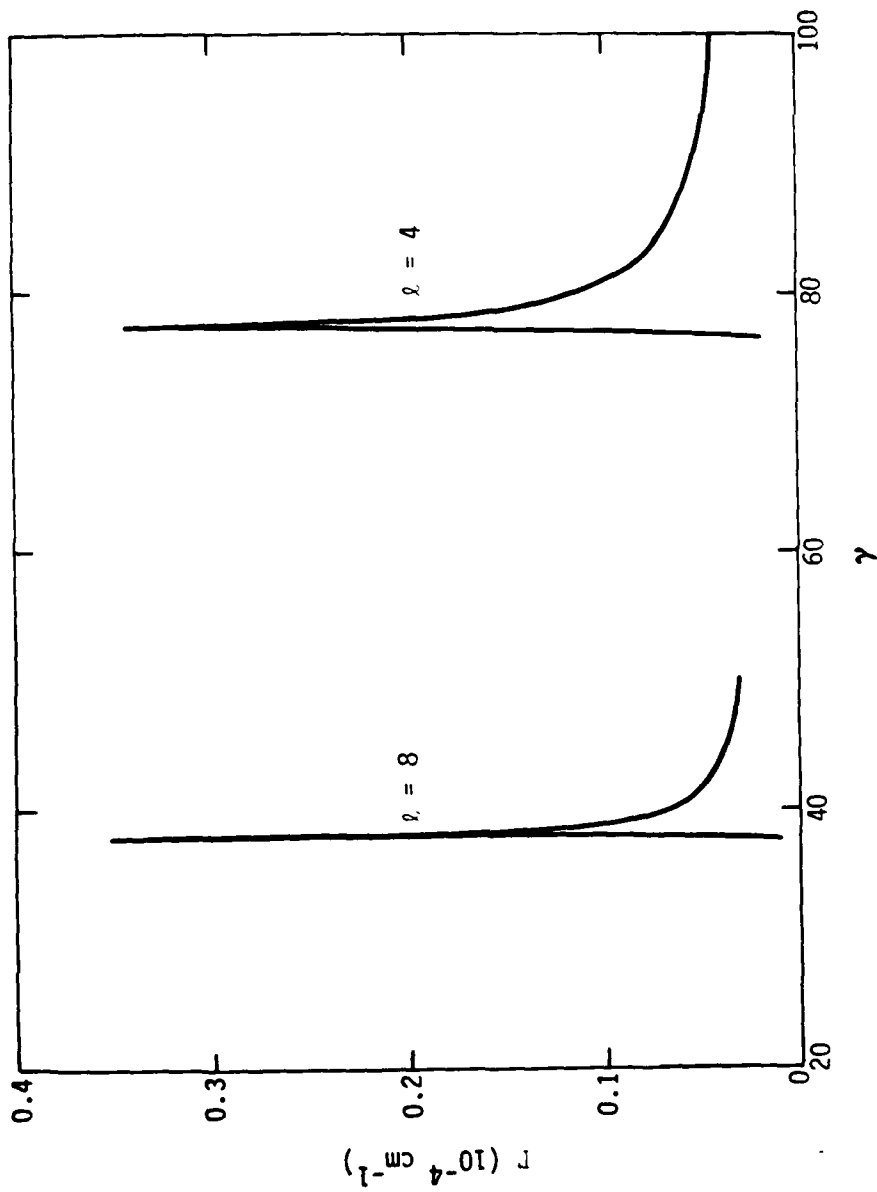


Figure 6.  $L = 4, 8$  transverse resistive wall growth rates, due to  $250 \mu\Omega\text{-cm}$  wall resistivity, ve  $I = 10 \text{ kA}$ ,  $B_{\theta} = 5 \text{ kg}$ .

Table 2 summarizes peak growth rates for the various cases discussed above. In view of the long acceleration times required ( $\sim 10^7$  cm), thermal effects must be involved to ameliorate the instabilities.

Reference 5 estimates the (Lorentzian) electron energy spread necessary to suppress an instability of growth rate  $\Gamma$  as

$$\Delta\gamma \geq \Gamma/\ell r_0 |k| \quad (3)$$

with

$$k \equiv [\gamma^{-2} + (n_s + n - 1)^{-1}]/\gamma r_0^2 \quad (4)$$

and

$$n_s = n_0/2\gamma\omega_z^2 \approx 2\nu r_0^2/r_b^2 \gamma^3 . \quad (5)$$

For parameters of interest here, the second term in Eq. (4) dominates, giving

$$\Delta\gamma/\gamma \geq (\Gamma r_0/\ell) |n_s - 1/2| . \quad (6)$$

Since  $\Gamma$  increases with  $\ell$  as  $\ell^{1/2}$  or slower, the greatest energy spread is required for  $\ell = 1$ . Furthermore, the  $\gamma^{-3}$  dependence of  $n_s$  and the weak  $\gamma$  dependence of  $\Gamma$  dictate evaluating  $\Delta\gamma$  for  $\gamma \sim 5-10$ , the worst case.

From Figure 1 and Table 1, evaluating  $\Delta\gamma$  at  $\gamma = 10$  yields  $\Delta\gamma/\gamma \approx 0.5$ , which is unacceptably large. The corresponding spread for  $\gamma = 5$  is, of course, several times worse. To obtain a more realistic energy spread, say

TABLE 2. INSTABILITY PEAK GROWTH RATES

<u>CASE</u>	<u><math>\Gamma</math> (<math>10^{-4}</math> cm<math>^{-1}</math>)</u>
10 kA, 5 kg, $\sigma = \infty$	
$\ell = 1$ negative mass	6.57
$\ell = 2$ negative mass	9.13
$\ell = 3$ negative mass	11.1
10 kA, 1.42 kg, $\sigma = \infty$	
$\ell = 1$ negative mass	12.7
2 kA, 1.42 kg, $\sigma = \infty$	
$\ell = 1$ negative mass	5.41
10 kA, 5 kg, $\sigma = 1.5 \cdot 10^6$ cm $^{-1}$	
$\ell = 4$ resistive wall	0.35
$\ell = 8$ resistive wall	0.36

TABLE 3. COMPUTED MAXIMUM CURRENT FOR WHICH 10% ENERGY SPREAD STABILIZES INSTABILITIES

<u><math>B_z</math> (kg)</u>	<u><math>\gamma</math></u>	<u><math>I</math> (kA)</u>
5.0	10	3.2
1.42	10	2.0
5.0	5	0.6
1.42	5	0.4

0.1, we recall that  $n_s$  scales as  $I/\gamma^3$  and  $\Gamma$  as  $(\gamma I/B_\theta^2)^{1/3}$ . Combining these dependencies yields a semi-empirical upper bound on the beam current for which a 10% energy spread damps out the negative mass and resistive wall instabilities,

$$I(\text{kA}) < 2 [B_\theta(\text{kg})]^{1/3} (\gamma/10)^{5/2} . \quad (7)$$

The exponent of  $B_\theta$  in (7) was adjusted from 1/2 to 1/3 to agree better with the numerical data of Table 3.

In summary, we have determined the cold beam linear growth rates of the negative mass and resistive wall instabilities for a modified betatron with dimensions and guide fields listed in Table 1. The resistive wall instability was found to be relatively insignificant. The negative mass instability, on the other hand, probably would disrupt the beam during its long acceleration time unless suppressed by electron thermal spread. A model based on a Lorentzian distribution of particle energies predicts that a 10% spread in energy is sufficient to eliminate the negative mass instability provided the beam current satisfies Eq. (7). Since this inequality is quite restrictive for small  $\gamma$ , every effort should be made to have the beam injection energy as high as possible. Increasing the toroidal guide field also helps. Not considered here are transverse betatron oscillations, which also should have a stabilizing effect.

These calculations were suggested by Phillip Sprangle of the Naval Research Laboratory and were supported by the Office of Naval Research.

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