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The assessment of uncertainties in component costs, a method of combining these uncertainties for determining the total cost uncertainty, and a method of presentation for risk analysis results are discussed in this report. An extension of the method of statistical risk analysis which uses the Weibul distribution and the method of moments is developed for incorporating covariance between component costs.

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FINAL REPORT

APPLIED RISK ANALYSIS WITH DEPENDENCE AMONG COST COMPONENTS

November 1981

Prepared by George H. Worm

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ABSTRACT

The assessment of uncertainties in component costs, a method of combining these uncertainties for determining the total cost uncertainty, and a method of presentation for risk analysis results are discussed in this paper. An extension of the method of statistical risk analysis which uses the Weibul distribution and the method of moments is developed for incorporating covariance between component costs. A computer program is given for implementing the results.

I. Introduction

When estimates and objectives are being established for contract negotiations, numerous sources of information are available and must be sensitized by the persons responsible for preparing a pre-negotiation briefing. Within ASD these persons are assigned to the Directorate of Pricing and are referred to as price analysts. The price analyst has the responsibility of determining and negotiating a fair and reasonable price for a contract.

The information available to the price analyst is generally point estimates of components which directly affect the total cost (e.g., material cost, overheads, labor hours, etc.). These point estimates for components are generally derived by engineering, cost, or price analysis. By combining these point estimates for components, one can derive a point estimate for the total cost of the contract.

Seldom are the component costs known with certainty unless there are firm purchase orders, negotiated overhead rates, fixed wage rates, etc. The components which are not fixed at the time of negotiations are the components for which a point estimate is not sufficient unless one has a crystal ball. The components not fixed will be referred to as uncertain components.[1] Risk analysis is a procedure for taking information about the uncertain components and reflecting how much uncertainty exists in the total cost. An uncertain component such as the manufacturing labor hours is often referred to as random. This is not to say that management cannot control labor hours, but simply implies that under good management the specific amount of manufacturing labor hours required cannot be determined specifically; therefore, there is still some uncertainty or randomness.

Risk analysis is basically a three-phased procedure. First, the contract must be analyzed to determine where uncertainties exist and to

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[1]Technically many authors differentiate between risk and uncertainty, but the two terms will be used interchangeably in this paper.

determine the magnitude of the uncertainty for each component. Second, the component uncertainties must be combined to reflect the uncertainty in the total cost. Third, the resulting total cost uncertainty must be used and presented in a way that aids in the decision-making and understanding of the contract under consideration.

The second phase of the risk analysis is the link between component uncertainty and total cost uncertainty. This phase is often considered to be the risk and analysis; however, the first and third phases are the most important in implementation. The second phase has been approached many different ways making different assumptions and using different methodologies [1, 2, 3, 6, 7, 10]. The methodology used in this paper is given in Appendix I. The first part of the appendix is based on results presented in [6]. The last part is original work which appears here for the first time. Section II contains the assumptions made, the information concerning components required, and the results provided by the second phase of the risk analysis. Section III contains an explanation of how the resulting total cost uncertainty can be used and how it can be presented in order to provide the decision-maker with valuable information. Section IV contains a description of how the price analyst can assess the uncertainty in the components. Section V provides an example of the use of a computer program designed to perform the calculations to combine the component uncertainties into the total cost uncertainty. In the last section, several recommendations are made concerning improving the usefulness of risk analysis.

II. Statistical Risk Analysis

In order to discuss the component uncertainty and total cost uncertainty, we use probability distributions. The area under a probability distribution represents a probability of an uncertain quantity having a value between two points. For instance, in Figure 1, the probability (p(c)) of the uncertain component cost being between $1,000 and $2,000 is given by the amount of area shaded.

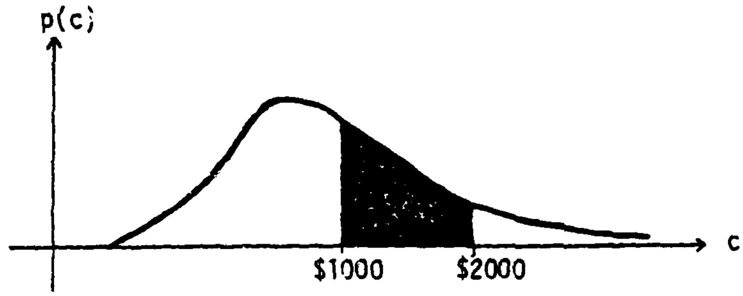

Figure 1

In order to describe component uncertainty and total cost uncertainty, the Weibul distribution has been chosen. The term "Weibul distribution" is used to describe characteristics of the probability distributions. For instance, the Weibul distribution can have many different shapes. (See Figure 2). There is a lower limit on component costs but no upper limit. By no upper limit we mean that the curve is asymptotic to the axis. The probability of extremely large over-runs is, however, approximately zero. The skewness of the Weibul distribution is a desirable characteristic for describing cost.

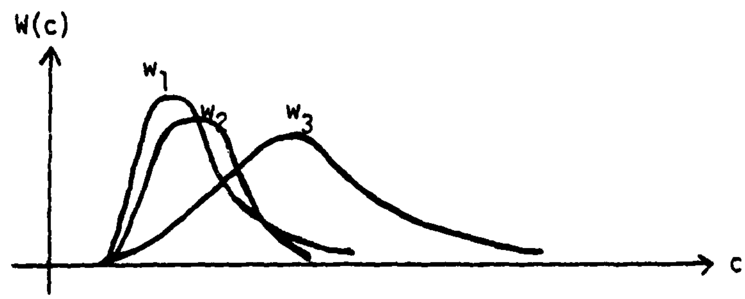

Figure 2

The price analyst must supply the information necessary to choose the appropriate Weibul distribution. The information required is three estimates of the component. First, the price analyst must estimate the component's most likely value. This is the value which has the highest probability of occurring. Second, the analyst must estimate for the component a value for which there is only a one percent chance of being less than that value. In other words, under the best circumstances, what would the component cost actually be? The third estimate is the high value for the component, one which has a one percent chance of being exceeded. In other words, under the worst circumstances what would the component cost actually be? The three points are denoted ML, L, and H respectively. The three points and a resulting Weibul distribution are shown in Figure 3.

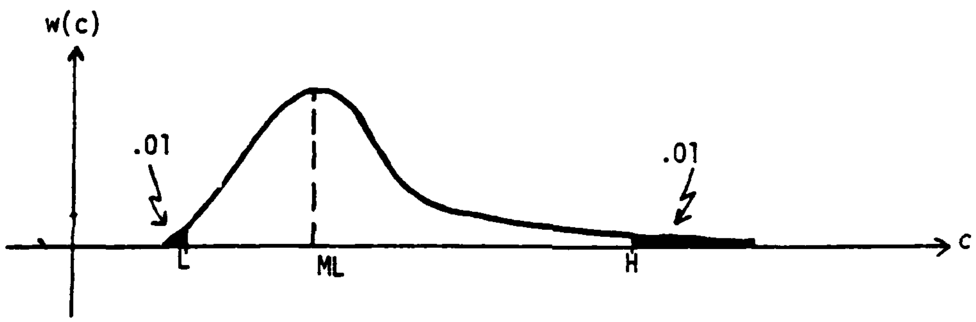

Figure 3

Naturally, a different L, ML, and H would give a different shape to the distribution for the component cost.

The components for which the analyst must supply the L, ML, and H are basically the same as those used in the DD Form 633. They are:

Material Cost
Material Overhead (not based on Material Cost)
Material Overhead Rate
Interdivisional Transfer Cost
Engineering Hours
Engineering Wage Rate
Engineering Overhead (not based on Engineering Cost)
Engineering Overhead Rate

Manufacturing Hours
Manufacturing Wage Rate
Manufacturing Overhead (not based on Manufacturing Cost)
Manufacturing Overhead Rate
Other Cost
General and Administrative (G&A) Expenses
Other Cost with no G&A Expenses

If a component is known with certainty, then L = ML = H. Section 4 contains information which should be useful to the price analyst in determining the L, ML, and H for each component.

By putting the three estimates for each component into the computer model described later, the distribution for the total cost uncertainty is determined. That is, the probability of exceeding different values is printed. These probability statements are made using a Weibul distribution for the total cost. The computer program takes the information supplied by the analyst for the components and using the methodology described in Appendix I, determines the best Weibul distribution for the total cost uncertainty.

The preceding estimates are used when the analyst is asked to assess each component independently. But quite often when one assesses the uncertainty for different components (e.g., the amount of material and the amount of manufacturing labor), there is a common underlying reason for this uncertainty. This is called a covariance between the components. The model being used allows the component uncertainty to be broken into two parts. The first part of the uncertainty for a component is independent of events which affect other components. The second part of the uncertainty in each component is dependent on a common set of events. For instance, part of the uncertainty concerning the amount of material and manufacturing labor hours may be dependent on the amount of rework. Therefore, material cost and labor hours would not be independent. To provide this information to the model requires the price analyst to assess the portion of the uncertainty in each component between the Low and High which is attributable to the same influences.

If one of the components is independent, then the portion of the uncertainty attributable to common factors should be zero. If the effects are in different directions (one positive and the other negative) this can be reflected by using a negative proportion. Again assistance in assessing the amount of covariance is presented in Section IV.

The price analyst is asked to assess the amount of dependence between the set of components listed below:

Material Cost
Material Overhead (not based on Material Cost)
Interdivisional Transfer Cost
Engineering Hours
Engineering Overhead (not based on Engineering Cost)
Manufacturing Hours
Manufacturing Overhead (not based on Manufacturing Cost)
Other Cost
Other Cost with no G&A Expenses

It should be observed that the covariance between the components is a function of one set of common factors or events. This single set of factors is assumed to cause the specific proportion of variation.

In summary, the assumptions made in the statistical risk analysis are:

1. Component uncertainty can be represented using a Weibul distribution. This assumption is based on what most analysts feel is a reasonable expression for the behavior of cost uncertainties.

2. Total cost uncertainty can be represented using a Weibul distribution. This assumption is based on the same reasoning as above plus the fact that the Weibul chosen to represent the total cost will have the correct first three moments (i.e., mean, variance, and skewness).

3. The dependence between components' uncertainties is related to a common set of influences or factors.

Naturally, the results of the risk analysis are only as good as the information supplied by the analyst. It is the opinion of the author that these assumptions are intuitively reasonable and represent the current state of the art.

III. Results, Uses, and Presentation

As with any mathematical or statistical technique, the purpose of risk analysis is to supply information to a decision maker. The forms of this information and the interpretation are critical in risk analysis. Its use must be considered as only one input into development of a negotiation objective. The overall business strategy considers many factors. The risk analysis should be performed in an impartial, objective manner without regard to gaming, political climate, etc.. Analysts should prepare and/or present analyses based on their best judgment of the uncertainties involved in the contract.

Quite often the amount of uncertainty in the total cost is considered unacceptable by a decision maker. This would indicate that more analysis is needed on some of the components. The specific component or components causing the large amount of uncertainty in the total cost can be determined through sensitivity analysis. An unacceptable total cost uncertainty may be the result of viewing the results with a knowledge of the political climate. Care should be taken not to "fix" the risk analysis results. This is one piece of information for decision making and not the decision.

When the information concerning the amount of uncertainty is presented, the primary drivers should identified. Effectively, one says how much uncertainty exists and why. Actions may be desired on the reasons for uncertainty. Ways of presenting the total cost uncertainty will now be discussed.

At the time of negotiations the future actual cost, since unknown, must be estimated. Risk analysis does not estimate the cost but rather estimates its distribution. A distribution is a pictorial representation of the probabilities of different final actual costs. The probability density function, Figure 4, is one way of presenting the probabilities of different final actual costs. This figure relates area to probability.

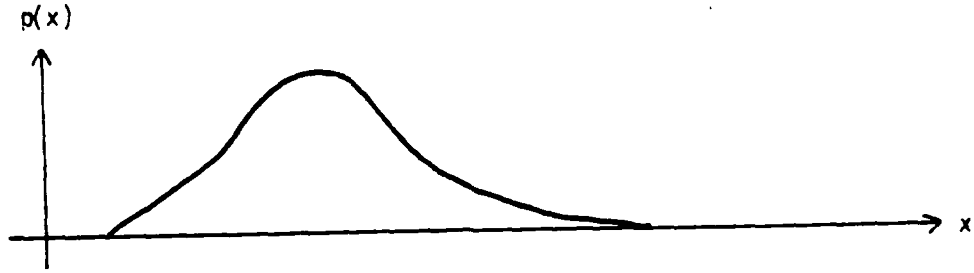

Figure 4

An easier way of presenting this same information is to use the cumulative distribution shown in Figure 5. From this figure one can determine the probability of exceeding a given total cost by reading the Y axis. For example, from Figure 5 we can see that the probability of exceeding $3,000 is .3. This would mean that there is a .7 probability of being less than $3,000.

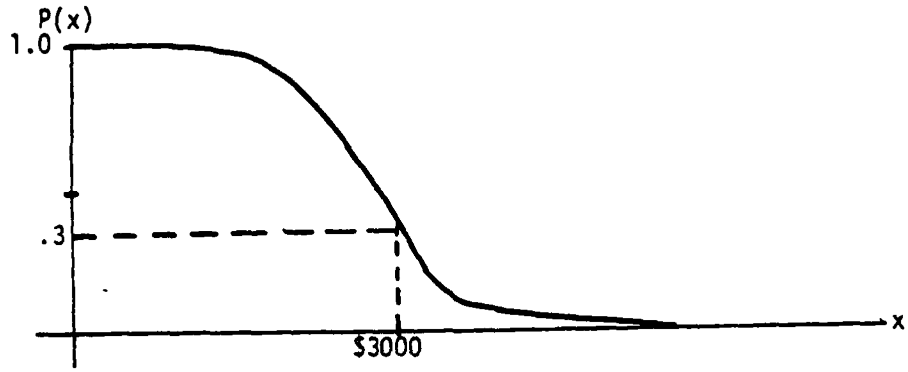

Figure 5

Usually from a risk analysis, two other pieces of information are available. These are the mean and the mode. The mean reflects the average total cost if the contract was executed many times. Of course, the contract is executed only once and, therefore, the mean may not be the best point estimate. The mode, on the other hand, is a good point estimate since it represents the most probable total cost. Contrary to intuition, the most probable total cost is <u>not</u> the sum of the most probable component costs. This can best be illustrated by using a simple example. Suppose we have two loaded dice, where the "one" is twice as likely to occur as the 2, 3, 4, 5, or 6. The analogy is that the dice represent the uncertain components, each having a most likely value of 1 since each is loaded. Intuition may tell us that the most likely total of the two dice would be 2. This is not the case however. The probability of a 2 with these two dice is 4/49, whereas, the probability of a 7 with these two dice is 8/49. The total of 7 is twice as likely. The same is true for the most likely total cost. That is, the resulting total cost by summing the most likely values for each component is not the most likely total cost.

The presentation of the risk analysis results to convey the most information might best be accomplished using a display similar to the one in Figure 6. This figure gives the distribution of total cost, the important drivers, and the negotiation positions.

The rest of this section is devoted to discussing the use of risk analysis in structuring contract types. These results should not be considered rules, but rather suggestions which may be helpful. If the total cost uncertainty[2] is small (e.g., 4 percent), then a Firm Fixed Price (FFP) type contract would be appropriate. The price would be the most likely total cost plus profit as determined by the Weighted Guidelines (WG).

Although incentive contracts were designed to provide incentives to the contractor to keep the cost down, they can also be structured to distribute the risk in a contract. For instance, if the uncertainty in total costs is between 4 and 16% and the share ratio is 40/60 (contractor/government), then the contractor is only accepting 40% of the risk or between 1.6 and 6.4% reduction in profit. One might thus consider structuring a Fixed Price Incentive Firm (FPIF) contract by using the following rules of thumb:

Target Cost = Most Likely Total Cost

Target Profit = WG (including risk)

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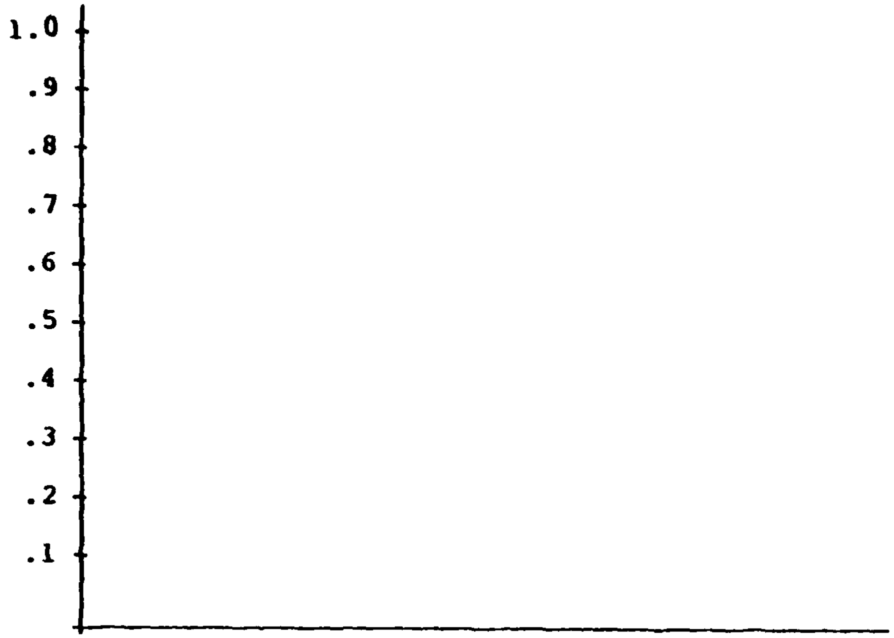
[2]When a single number is used to represent the uncertainty, it is usually the percent that the 99 percentile total cost exceeds the most likely total cost.

Contract # ____________________

Date ____________________

Analyst ____________________

PROBABILITY OF EXCEEDING TOTAL COST



TOTAL COST ($)

Uncertainty Primary Drivers

1.

2.

3.

4.

5.

Most Likely Total Cost = ________

Results of Risk Analysis

Figure 6

Ceiling Price = 99 percentile Total Cost Plus WG (excluding risk)

$$\text{Share} = \frac{\text{WG (including risk)} - \text{WG (excluding risk)}}{\text{99 percentile} - \text{Most Likely Cost}} \times 100$$

See the example in Section V for structuring an incentive contract.

Of course, these are only guidelines and may be totally unacceptable for a particular contract. For instance, if the decision has been made to use a FFP type contract or specific share ratios or ceilings, then these guidelines need not be applied. If a contractor has the same uncertainties as the government (say 16%), then the contractor will most likely not consider a FFP Contract with a price equal to the most likely cost plus WG profit. The contractor would naturally try to negotiate a price which would cover a majority of the risk. It is important to note that the uncertainty in total cost might be totally different from the contractor's point of view as opposed to the government's point of view.

In summary, the total cost uncertainty is a result of the uncertainty assessed at the component level. The accurate representation of the total cost uncertainty and the reasons for this uncertainty constitute one input into the decision making. This information reflects the uncertainties involved in a contract and should be only one part of sound business decisions. Its use in decisions on contract type, again is only one of many factors to be considered. It can, however, show the effect of different contract types when uncertainties exist.

IV. Assessing Uncertainty

The assessment of uncertainty is by far the most difficult task in a risk analysis. It is important to have the proper perspective of the problem when one is attempting to develop the estimates for the low, most likely, high and the dependence for the components of the total cost. This perspective includes:

1. Estimates are for the actual future cost and not the cost which can be negotiated. Actual future cost should be estimated when determining a fair and reasonable price.

2. Risk is associated with uncontrollable factors without regard to controllable factors such as gaming, political climate, etc.

3. Risk analysis is only one input into development of a negotiation objective.

4. Results of the risk analysis will reflect the inputs; nothing more, nothing less.

It is sometimes difficult for a decision maker on one side of a contract to relate to a risk analysis which is useful for determining a fair and reasonable price (the one mentioned above). Therefore, another analysis viewing the risk from the Air Force point of view might be useful. This analysis would not attempt to estimate the future actual total cost, but rather attempt to estimate the price which the contractor would accept. The two analyses differ considerably because of the point of view. There are really two questions. First, what is the distribution of actual total cost? Second, what is the distribution of the resulting negotiated cost at the end of negotiations? In the strictest sense, the second question is not generally addressed by a risk analysis, but can be answered by estimating for each component the cost acceptable to the contractor. Even though the bottom line is the result of the negotiations, acceptable levels of each component can be estimated. The discussion in this section will address the estimation of component costs when the distribution of actual total cost is desired.

Table 1 contains a list of drivers for the uncertainty involved in a contract. This list is presented here as a thought provoker and is not meant to be all inclusive.

The analyst must use all of the resources available in order to estimate the uncertainty in each component. These resources include:

- historical records
- experience
- support groups (engineering, etc.)
- contractor's track record
- similar contracts

The resulting estimates may be either subjective or a combination of subjective and standardized factors.

Subjective estimates are difficult to make and to evaluate; therefore, there is often differing opinions of the correct values. When subjective estimates are used, try both a conservative and a liberal set of estimates. This requires running the model twice. Many times you will find that the results are approximately the same. A risk analysis model is very useful in assessing the "What If" questions. Don't be afraid to try different inputs to determine the effect on the total cost distribution.

Table 1

Cost Uncertainty Drivers[3]

Material Elements

Extent of firm POs
Extent of established vendors
Projected inflation
Reliability of estimated allowance factors
Design maturity
Critical items

Labor Elements

Design maturity/deficiencies
Reliability of estimating methodology (L/C/ratios/estimates/LOE)
Impact of schedule slippages
Relatable historical experience
Employment population
Production capacity

Labor and Overhead Rates

Status of union agreements
Status of FPRA
Projected inflation
Period of performance
Variance of direct cost elements
Variance of plant volume

Other Costs

Reliability in estimating methodology
Impact from direct cost variances
Reliability of factors

[3]Source - Myron Bailey, ASD Pricing

Assessing the dependence between component costs involves listing the common cost drivers inherent to all cost elements, e.g., design maturity, manufacturing methods, etc.; then estimating the proportion of uncertainty between the low and high which is due to these cost drivers. For instance, if the low material = 3000 and the high material = 7000, and the common cost drivers cause a total fluctuation in materials of 1000, then the dependence for material is 25% (i.e., 1000/(7000-3000)). Each cost dependence is assessed using the same set of drivers. See the list in Section II for the cost elements for which dependence is allowed.

A preliminary set of standardized factors have been developed at ASD Pricing. These factors are listed in Table 2.

This list is a first draft and should be modified and added to as needed. Once an acceptable set of factors is developed, it will only be necessary for the analyst to supply point estimates and identify the characteristics of the contract as in Table 2. The computer could then apply the L% and the H% to the component estimates and perform the risk analysis. The decision maker would then know specifically the drivers, the amount of uncertainty introduced by each, and the resulting distribution of total cost.

The advantages of using standardized factors are:

1. Uncertainty is based on objective characteristics of the contract.

2. Uncertainty becomes less dependent on the analyst performing the analysis.

3. The amount of uncertainty due to characteristics of the contract can incorporate different view points of analysts and management, and

4. Uncertainty assessment is consistent between contracts.

An effort should be made to modify and revise the standardized factors to encompass additional reasons for uncertainty.

The next section demonstrates the use of a computer program useful for determining the total cost distribution from three point estimates of the components' costs.

Table 2

Draft of Standardized Factors for Risk Analysis

1. Material: This cost element is broken into several distinct sub-elements due to their peculiar nature.

L%	H%	Score
		a. Raw Material:
-5	+7	1 - 90% firm purchase orders (POs), established sources - 2% or less critical material (titanium, chromium, etc.)
-5	+9	2 - 80% firm POs; 5% critical material
-5	+12	3 - 70% firm POs; 10% critical material
-8	+20	4 - 60% firm POs; 15% or more critical material
		b. Purchased Parts (Supplier designed items):
-5	+7	1 - 90% POs
-5	+9	2 - 75% POs; 25% current quotes
-5	+12	3 - 50% POs; 25% current quotes, 25% history
-8	+20	4 - Less than 50% POs greater than 25% history
		c. Subcontracts (Prime designed items):
-5	+7	1 - 90% POs and current quotes
-5	+9	2 - 75% POs and quotes; 25% engineering estimates
-5	+12	3 - Less than 75% POs and quotes; more than 25% engineering estimates
-10	+25	4 - Inhouse engineering estimates + 50%
		d. Special Material Factors (Scrap, rework, freight, receiving inspections, attribution):
-5	+5	1 - Historical factors well supported.
-5	+9	2 - Design in minor state of flux
-6	+12	3 - Design not set; factors not reliable
-9	+18	4 - New program with little relevant history

2. Engineering Labor:

L%	H%	Score
-5	+5	1 - Firm design sustaining type effort
-[illegible]	+9	2 - 50/50 mix of changes and sustaining effort
-8	+15	3 - Design set but many changes; 1st production phase
-10	+25	4 - New program; design not determined; little historical basis to estimates

3. Manufacturing Labor:

L%	H%	Score
-2	+4	1 - Firm standards, well documented learning curves
-5	+7	2 - 80% firm standards less reliable variance data due to limited history
-5	+10	3 - 1st production limited standards & variance data extensive tooling effort
-7	+15	4 - Model shop operation or RSED. Tool design not yet determined.

Table 2
(continued)

L%	H%	Score
4.		**Labor and Overhead Rates:**
		a. Status of Union Agreement:
-2	+5	1 - Firm 2-3 yrs before renegotiation
-5	10	2 - No agreement through 1 yr before expiration
		b. Status of FPRA:
-3	+7	1 - FPRA negotiated
-2	+8	2 - Recommended rates within 5% of proposed
-5	+5	3 - Recommended rates at more than 5%
		c. Projected Inflation:
-2	+2	1 - Contract includes EPA clause
-2	-15	2 - No EPA - inflation less than/equal to field rec.
-2	+10	3 - No EPA - inflation more than field rec.
		d. Period of Performance:
-5	+5	1 - 1 year
-2	+10	2 - 2-3 years
-0	+15	3 - 4-5 years
-0	+20	4 - More than 5 years
5.		**Considerations That Are Equally Inherent to All Cost Elements:**
		a. Design:
-2	+5	1 - Firm design, mature program
-5	+7	2 - 2nd/3rd production lot changes predictable
-5	+10	3 - 1st production/pre-production
-8	+15	4 - Design/development phase
		b. Manufacturing Methods:
-2	+5	1 - Standard defined, hard tooling
-5	+7	2 - 50% standards, production rate building
-5	+10	3 - First production hard tooling being developed
-8	+15	4 - Model shop, FSED

V. Example Use of Computer Program

Before presenting an example of the usage of this program, several input peculiarities need to be explained. A brief explanation is given below.

1. Overhead rates and dependency are entered as whole numbers (e.g., 50% is entered as 50).

2. The distance between the low and the most likely value should be less than 65% of the distance between the low and high. (i.e., $(ML-L)/(H/L) \leq .65$)

3. If the computer responds with a question mark, additional input is needed.

4. If labor is entered as cost rather than hours, the L, ML, and H for wage rate should be 1, 1, 1.

5. Independence of component costs is given when D% is 0.

6. Conversion to 1,000 or 1,000,000 of dollars and hours may be desirable.

7. When dependence is considered, D% for material must be other than zero.

The user should always check the reasonableness of the results. If a problem is perceived, the inputs should be checked.

Blank forms for organizing the inputs and displaying the results are provided in Appendix II. The input form has been completed for a specific contract and is shown on the next page, Form I. Example 1 shows the execution of the risk analysis program illustrating the input and the output. All of the user supplied input is underlined.

The results of this analysis indicate that the most likely total cost is $19,708 and that there is only a 1% chance of the total cost exceeding $21,011. The results of the computer run have been transferred to the form for organizing the results of the risk analysis (see Form II).

Contract # ______________________
Date ______________________
Analyst ______________________

INPUTS FOR RISK ANALYSIS

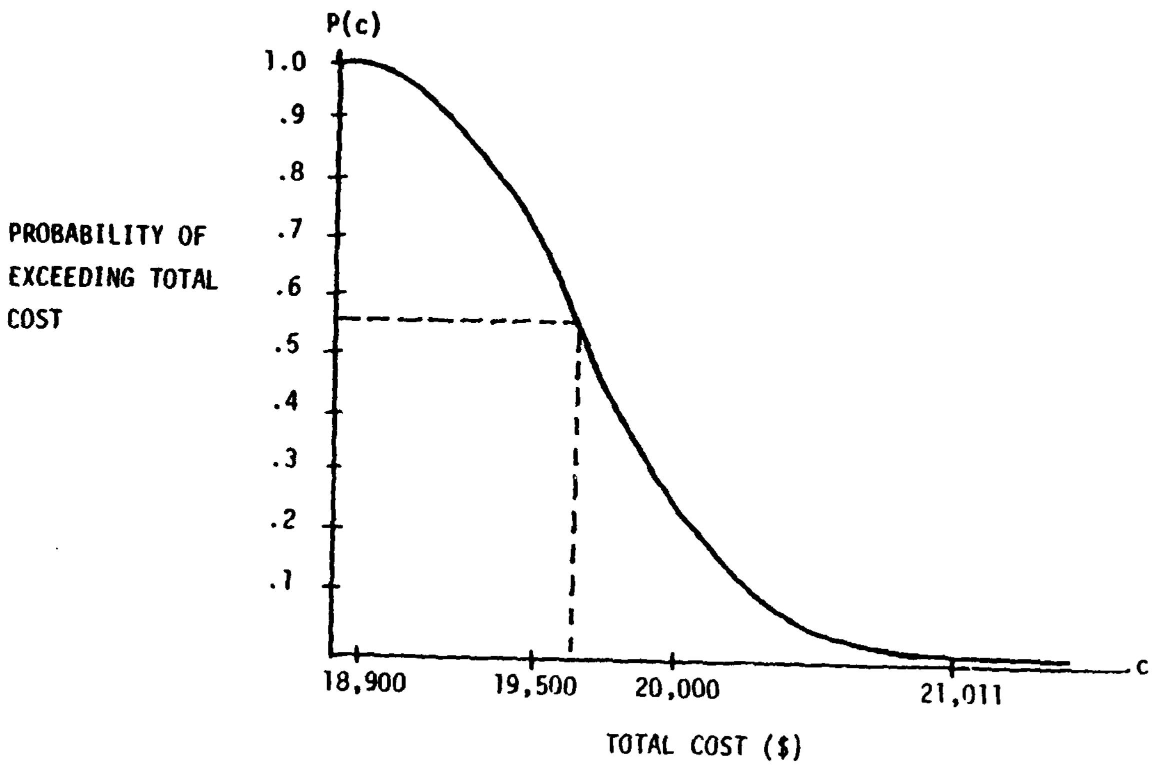
Elements	Minimum	Most Likely	Maximum	Dependence	Comment
Material Cost	8,400	8,900	10,000	0	
Independent O.H.	--	--	--	--	
Rate for Mat. O.H.	5	5.2	6	xxx	
Interdivisional Transfer	1,700	1,850	2,200	0	
Engineering Labor Direct	35	45	65	0	
Wage rate	11.57	11.57	11.57	xxx	
Independent O.H.	--	--	--	--	
Rate for Eng. Lab. O.H.	102	102	102	xxx	
MFG Labor Direct	200	205	215	0	
Wage rate	11.04	11.5	12	xxx	
Independent O.H.	--	--	--	--	
Rate for MFG Lab. O.H.	150	150	150	xxx	
Other Cost with G & A	400	450	500	--	
G & A Rate applied to Subtotal	4.92	4.92	4.92	xxx	
Other cost with no G & A	--	--	--	--	

FORM I

Contract # ____________

Date ____________

Analyst ____________



Most Likely Total Cost = $19,708

Results of Risk Analysis

Uncertainty Primary Drivers

1.
2.
3.
4.
5.

FORM II

In this example the 99 percentile is 6% larger than the most likely total cost. This example would be a case in which either an FFP or an FPIF type contract might be used. The price would be the target cost plus the weighted guidelines profit for an FFP contract, where the target cost is set by analyzing the results of the risk analysis. There is nc way to state a hard and fast rule for determining the target cost; however, the following list of possibilities should be considered.

1. Target Cost = $19,708
   Target set at most likely total cost.

2. Target Cost = $19,774
   Target set so there is a 50% chance of over and under run.

3. Target Cost = $21,011
   Target set where the government absorbs 99% of the cost risk.

4. Target Cost = $18,900
   Target set where the contractor absorbs 99% of the cost risk.

The setting of a specific target would consider factors not included in the risk analysis but can be evaluated by the total cost distribution. For instance, if a target cost of $20,000 is chosen, there is approximately a 33% chance that the total cost will exceed $20,000.

For an FPIF contract, the procedure given in Section III might be used. For example, suppose that the weighted guideline was (including risk) 16% or (excluding risk) 12%, then the following would structure an incentive contract so that the most probable profit for the contractor would be 16% and there would be only a 1% chance of his having less than 12% profit. This is achieved by setting

Target Cost = $19,708
WG (excluding risk) = 19,708 · .12 = $2,365
WG (including risk) = 19,708 · .16 = $3,153
Point of total assumption = $21,011

Target Profit = $3,153

Ceiling Price = 21,011 + 2,365 = $23,376 (119% of target)

Share = (3,153 - 2,365)/(21,011 - 19,708) = .60 or 60%

Note that quite often this procedure will come up with unacceptable results. Always apply sound business judgment when analyzing the results of these calculations.

Example 1 - Example Run of Risk Analysis Program

```
OLD RISKAN
RUN-100

THIS PROGRAM WAS WRITTEN TO PERFORM THE NECESSARY
CALCULATIONS FOR A RISK ANALYSIS BY GEORGE WORM, 1981
THE LINES REQUIRING THREE INPUTS END WITH L, ML, H
THE LINES WHICH ALLOW FOR DEPENDENCE END WITH L,ML,H,D%
IF THERE IS NO DEPENDENCE ENTER ZERO FOR D
MATERIAL COST L, ML, H, D%?8400 8900 10000 0

MATERIAL OVERHEAD INDEPENDENT L, ML, H, D%?0 0 0 0

MATERIAL OVERHEAD RATE% L, ML, H?5 5.2 6

INTERDIV TRSFRS L, ML, H, D%?1700 1850 2200 0

DIRECT ENGRG LABOR (HOURS OR COST) L,ML,H,D%?35 45 65 0

ENGRG WAGE RATE (ENTER ONES IF LABOR IS COST AND NOT HOURS L,ML,H)
 ?11.57 11.57 11.57

ENGRG OVERHEAD INDEPENDENT L, ML, H, D%?0 0 0 0

ENGRG OVERHEAD RATE% L,ML,H?102 102 102

DIRECT MFG LABOR (HOURS OR COST) L, ML, H, D%?200 205 215 0

MGT WAGE RATE (ENTER ONES IF LABOR IN COST AND NOT HOURS L,ML,H
 ?11.04 11.5 12 0

MFG OVERHEAD INDEPENDENT L, ML, H, D%?0 0 0 0

MFG OVERHEAD RATE% L,ML,H?150 150 150

OTHER COSTS L, ML, H, D%?400 450 500 0

G AND A EXPENSE (PERCENT OF SUBTOTAL)% L,ML,H?4.92 4.92 4.92

OTHER COST WITH NO G&A L,ML,H,D%?0 0 0 0
```

Example 1 - (Continued)

ESTIMATES FOR RISK ANALYSIS

ELEMENTS		MINIMUM	MOST LIKELY	MAXIMUM	DEPENDANCE
MATERIAL	COST	8400.00	8900.00	10000.00	0.00
MAT OVERHEAD	INDEPENDENT	0.00	0.00	0.00	0.00
RATE FOR MATERIAL		5.00	5.20	6.00	
INTERDIV TRSFRS	COST	1700.00	1850.00	2200.00	0.00
DIRECT ENGRG LABOR	HOURS	35.00	45.00	65.00	0.00
WAGE RATE		11.57	11.57	11.57	
ENGRG OVERHEAD	INDEPENDENT	0.00	0.00	0.00	0.00
RATE FOR ENGRG		102.00	102.00	102.00	
DIRECT MFG LABOR	HOURS	200.00	205.00	215.00	0.00
WAGE RATE		11.04	11.50	12.00	
MFG OVERHEAD	INDEPENDENT	0.00	0.00	0.00	0.00
RATE FOR MFG		150.00	150.00	150.00	
OTHER COST	COST	400.00	450.00	500.00	0.00
G&A EXPENSE		4.92	4.92	4.92	
OTHER COST WITH NO G&A		0.00	0.00	0.00	0.00

PROBABILITY OF EXCEDING	TOTAL COST
0.0100	21011.5647
0.0500	20641.7915
0.1000	20445.4526
0.1500	20313.8118
0.2000	20209.9187
0.2500	20121.4702
0.3000	20042.7068
0.3500	19970.3938
0.4000	19902.4751
0.4500	19837.5090
0.5000	19774.3882
0.5500	19712.1807
0.6000	19650.0215
0.6500	19587.0203
0.7000	19522.1555
0.7500	19454.1206
0.8000	19381.0381
0.8500	19299.8350
0.9000	19204.5029
0.9500	19079.0403
0.9900	18900.4202

THE MOST LIKELY TOTAL COST IS 19708.52

VI. Recommendations

Risk analysis can provide valuable information to decision makers if it is developed carefully and presented in a meaningful manner. Several suggestions are made here to facilitate its usefulness.

1. Uncertainty information should be requested from engineering and other support groups.

2. Several computer runs would be useful to answer the "what if" questions and to analyze the contract from different points of view.

3. Continued development of standardized factors would help to make the risk analyses consistent and, therefore, more meaningful.

4. Good business judgment should be used when using the results of the risk analysis.

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APPENDIX I

This technical appendix provides the theory necessary to perform a statistical risk analysis when subcomponents[1] and total cost are assumed to have a Weibul distribution. The method used converts information concerning subcomponents to moments of their corresponding distributions and combines these moments to determine the moments of the total cost. By fitting a Weibul distribution to the moments for total cost probability statements can be made concerning the total cost.

The appendix is divided into four sections. The first section gives a procedure for converting the user supplied information listed below to moments of the subcomponent distribution:

1. L - Low Estimate (Probability of the actual being lower is δ_1)
2. ML - Most Likely
3. H - High estimate (Probability of the actual being higher is δ_2)

The second section contains methods of combining the moments of the subcomponents to obtain the moments of the total cost. The third section contains the procedure for converting from moments of the total cost to probabilities about the total cost. In the last section a method of incorporating covariance between subcomponents is developed. All but the last section of the appendix is a restatement of the work of McNichols [6].

Calculating Moments from L, ML and H

In the risk analysis discussed here the user will supply an estimate of L, ML and H for specific values of δ_1 and δ_2. The Weibul distribution is illustrated in Figure 1 showing the relationship between L, ML, H, δ_1 and δ_2.

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[1] In this appendix the terms subcomponent and component are used interchangeably

Weibul Distribution

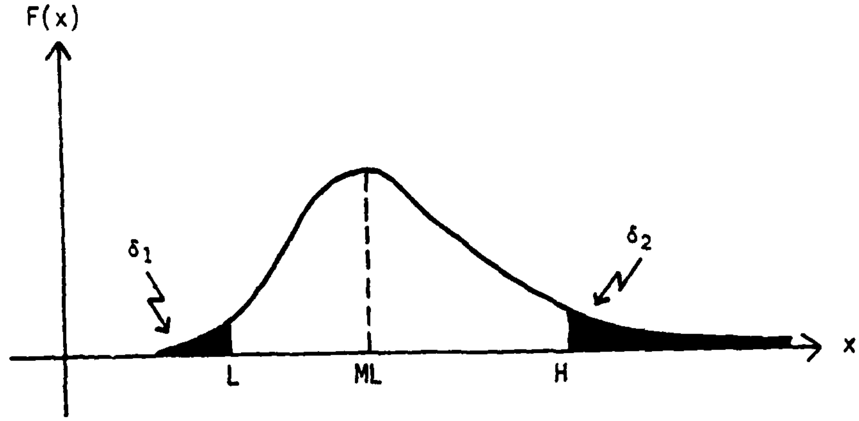


Figure 1

The Weibul distribution with parameters ξ, α and c has a functional form given by $F(x) = c/\alpha\ ((x-\xi)/\alpha)^{c-1}\ \exp[-((x-\xi)/\alpha)^c]$ for $x>\xi$. If c, ξ and α are known then the moments are

$$\mu = E(x) = \alpha\Gamma(1/c+1) + \xi$$
$$\mu_2 = E((x-\mu)^2) = \alpha^2[\Gamma(2/c+1) - \Gamma^2(1/c+1)] \quad \text{and} \qquad \text{(A.1)}$$
$$\mu_3 = E((x-\mu)^3) = \alpha^3[\Gamma(3/c+1)-3\Gamma(1/c+1)\Gamma(2/c+1)+2\Gamma^3(1/c+1)]$$

$$\text{Mode} = \alpha[(c-1)/c]^{(1/c)} + \xi$$

The notation $\Gamma(\mu)$ is used to denote the Gamma Function defined as

$$\Gamma(\mu) = \int_0^\infty y^{\mu-1}\exp[-y]dy$$

Table 1 gives $\Gamma(1/c+1)$, $\Gamma(2/c+1)$, and $\Gamma(3/c+1)$ for several values of c.

The problem then is to determine c, ξ and α from L, ML, H, δ_1 and δ_2. These are found by using the cumulative distribution given by

$$F(x) = 1-\exp[-((x-\xi)/\alpha)^c]$$

evaluated at the points L and H, and using the Mode (ML). These relationships are presented below.

$$ML = \alpha[(c-1)/c]^{1/c} + \xi$$
$$F(L) = 1-\exp[-(L-\xi)/\alpha)^c = \delta_1$$
or $$L = \alpha[-\ln(1-\delta_1)]^{1/c} + \xi$$
$$F(H) = 1-\exp[-(H-\xi)/\alpha)^c] = 1-\delta_2$$
or $$H = \alpha[-\ln(\delta_2)]^{1/c} + \xi$$

TABLE I

C	Γ(1/C+1)	Γ(2/C+1)	Γ(3/C+1)
1.010	.99585	1.96394	5.78098
1.020	.99187	1.92951	5.57543
1.030	.98803	1.89662	5.38215
1.040	.98435	1.86518	5.20043
1.050	.98080	1.83506	5.02920
1.060	.97738	1.80626	4.86768
1.070	.97409	1.77864	4.71529
1.080	.97092	1.75218	4.57126
1.090	.96786	1.72781	4.43503
1.100	.96490	1.70245	4.30606
1.110	.96208	1.67904	4.18387
1.120	.95935	1.65657	4.06794
1.130	.95671	1.63494	3.95796
1.140	.95416	1.61416	3.85337
1.150	.95170	1.59416	3.75398
1.160	.94934	1.57490	3.65940
1.170	.94706	1.55633	3.56932
1.180	.94485	1.53846	3.48345
1.190	.94271	1.52121	3.40154
1.200	.94066	1.50459	3.32336
1.210	.93868	1.48854	3.24871
1.220	.93676	1.47303	3.17733
1.230	.93491	1.45807	3.10908
1.240	.93312	1.44361	3.04378
1.250	.93138	1.42963	2.98119
1.260	.92972	1.41612	2.92129
1.270	.92811	1.40305	2.86381
1.280	.92655	1.39039	2.80882
1.290	.92504	1.37814	2.75578
1.300	.92358	1.36629	2.70493
1.310	.92217	1.35480	2.65607
1.320	.92081	1.34367	2.60913
1.330	.91949	1.33288	2.56397
1.340	.91823	1.32242	2.52047
1.350	.91700	1.31227	2.47863
1.360	.91581	1.30241	2.43831
1.370	.91464	1.29285	2.39939
1.380	.91354	1.28357	2.36196
1.390	.91246	1.27457	2.32578
1.400	.91143	1.26584	2.29091
1.410	.91042	1.25734	2.25721
1.420	.90946	1.24907	2.22466
1.430	.90851	1.24104	2.19322
1.440	.90761	1.23323	2.16284
1.450	.90672	1.22565	2.13341
1.460	.90588	1.21825	2.10500

TABLE I
(Continued)

C	Γ(1/C+1)	Γ(2/C+1)	Γ(3/C+1)
1.470	.90505	1.21106	2.07746
1.480	.90426	1.20408	2.05084
1.490	.90349	1.19734	2.02504
1.500	.90276	1.19068	2.00000
1.510	.90204	1.18420	1.97579
1.520	.90134	1.17791	1.95230
1.530	.90068	1.17178	1.92951
1.540	.90002	1.16581	1.90742
1.550	.89940	1.15999	1.88599
1.560	.89879	1.15433	1.86518
1.570	.89821	1.14880	1.84491
1.580	.89765	1.14341	1.82533
1.590	.89710	1.13815	1.80626
1.600	.89658	1.13300	1.78773
1.700	.89225	1.08797	1.62793
1.800	.88930	1.05219	1.50459
1.900	.88737	1.02342	1.40735
2.000	.88623	1.00000	1.32934
2.100	.88570	.98080	1.26584
2.200	.88563	.96490	1.21345
2.300	.88593	.95170	1.16978
2.400	.88649	.94066	1.13300
2.500	.88726	.93138	1.10180
2.600	.88822	.92358	1.07515
2.700	.88928	.91700	1.05219
2.800	.89046	.91143	1.03234
2.900	.89173	.90672	1.01509
3.000	.89301	.90276	1.00000
3.100	.89431	.89940	.98679
3.200	.89566	.89658	.97518
3.300	.89702	.89422	.96490
3.400	.89839	.89225	.95585
3.500	.89976	.89062	.94781

Note that

$$\frac{ML-L}{H-L} = \frac{[(c-1)/c]^{1/c} - [-\ln(1-\delta_1)]^{1/c}}{[-\ln(\delta_2)]^{1/c} - [-\ln(1-\delta_1)]^{1/c}}$$

which is always between 0 and 1 and is a function of c. Furthermore, the left hand side can be calculated from estimates. To solve for c, a table has been constructed (see Table II) for determining c given (ML-L)/(H-L). The difference between H and L can be used to solve for α and yields

$$\alpha = \frac{H-L}{[-\ln(\delta_2)]^{1/c} - [-\ln(1-\delta_1)]^{1/c}} \qquad (A.2)$$

And from the equation for ML, we have

$$\xi = ML-\alpha[(c-1)/c]^{1/c} \qquad (A.3)$$

In summary, the steps for finding the moments of the subcomponents are

Step 1: Estimate L, ML and H for a given δ_1 and δ_2

Step 2: Find c from Table II using (ML-L)/(H-L)

Step 3: Calculate α and ξ using (A.2) and (A.3)

Step 4: Calculate μ, μ_2 and μ_3 using (A.1)

Calculating Moments of Products and Sums

The total cost is the product and sum of subcomponents. The moments of the total cost can be determined by using what are called additive and multiplicative moments. The only assumption needed is that the subcomponents are independent. This assumption can be relaxed using the results in section four.

When two subcomponents are added the moments of their total are the sum of the moments for each subcomponent. That is

$$\mu(X_1+X_2) = \mu(X_1) + \mu(X_2)$$
$$\mu_2(X_1+X_2) = \mu_2(X_1) + \mu_2(X_2) \quad \text{and}$$
$$\mu_3(X_1+X_2) = \mu_3(X_1) + \mu_3(X_2),$$

where $\mu_j(X)$ denotes the jth moment of X about the mean. Note that this is only true for the first three moments and for variables which are independent.

TABLE II

(ML-L)/(H-L)	C
.01	1.05014
.02	1.08514
.03	1.11767
.04	1.14869
.05	1.17873
.06	1.20814
.07	1.23717
.08	1.26598
.09	1.29472
.10	1.32349
.11	1.35239
.12	1.38150
.13	1.41089
.14	1.44063
.15	1.47079
.16	1.50142
.17	1.53258
.18	1.56434
.19	1.59675
.20	1.62986
.21	1.66375
.22	1.69847
.23	1.73409
.24	1.77068
.25	1.80831
.26	1.84706
.27	1.88701
.28	1.92824
.29	1.97086
.30	2.01497
.31	2.06065
.32	2.10806
.33	2.15730
.34	2.20852
.35	2.26188
.36	2.31754
.37	2.37569
.38	2.43653
.39	2.50029
.40	2.56723
.41	2.63763
.42	2.71179
.43	2.79008
.44	2.87290
.45	2.96069
.46	3.05397
.47	3.15332
.48	3.25941

TABLE II
(Continued)

(ML-L)/(H-L)	C
.49	3.37303
.50	3.49505
.51	3.626
.52	3.769
.53	3.923
.54	4.091
.55	4.275
.56	4.473
.57	4.702
.58	4.95
.59	5.23
.60	5.544
.61	5.901
.62	6.312
.63	6.79
.64	7.352
.65	8.025

When two subcomponents are multiplied the moments of the product are not the product of the moments of the subcomponents. However, if we define M_1, M_2 and M_3 as the moments about the origin, then

$$\begin{aligned} M_1(X_1X_2) &= M_1(X_1)\ M_1(X_2) \\ M_2(X_1X_2) &= M_2(X_1)\ M_2(X_2) \text{ and} \\ M_3(X_1X_2) &= M_3(X_1)\ M_3(X_2) \end{aligned} \qquad (A.4)$$

These moments about the origin are related to the additive moments by the relationships given below

$$\begin{aligned} M_1 &= \mu_1 \\ M_2 &= \mu_2 + \mu_1^2 \\ M_3 &= \mu_3 + 3\mu_1\mu_2 + \mu_1^3 \end{aligned} \qquad (A.5)$$

or

$$\begin{aligned} \mu_1 &= M_1 \\ \mu_2 &= M_2 - M_1^2 \\ \mu_3 &= M_3 - 3M_1M_2 + 2M_1^3 \end{aligned} \qquad (A.6)$$

Therefore, if two subcomponents are multiplied, the moments about the mean of the product $\mu_i(X_1X_2)$ can be found by

Step 1: Determine $\mu_i(X_1)$ and $\mu_i(X_2)$, $i = 1, 2, 3$

Step 2: Find $M_i(X_1)$ and $M_i(X_2)$, $i = 1, 2, 3$ using (A.5)

Step 3: Find $M_i(X_1X_2)$, $i = 1, 2, 3$ using (A.4) and

Step 4: Find $\mu_i(X_1X_2)$, $i = 1, 2, 3$ using (A.6).

Note that we start with $\mu_i(X_1)$ and $\mu_i(X_2)$ and end up with $\mu_i(X_1X_2)$.

To illustrate the application of the method of moments for determining the moments of the total cost a simple example will be used. Consider the total cost equation

$$\text{Total cost} = X_1+X_2X_3+X_4X_5X_6,$$

where $X_1 \ldots, X_6$ are six subcomponents with estimates of L, ML, and H. First we find $\mu_i(X_j)$ $i=1,2,3$ and $j=1,2,\ldots,6$.

Second, we use (A.5), (A.4) and (A.6) to find $\mu_i(X_2X_3)$, $\mu_i(X_4X_5)$ and $\mu_i(X_4X_5X_6)$ and finally

$\mu_i(\text{Total cost}) = \mu_i(X) + \mu_i(X_2X_3) + \mu_i(X_4X_5X_6)$.

Note that $\mu_i(X_4X_5)$ is not needed if one calculates $M_i(X_4X_5X_6)$ as $M_i(X_4)\ M_i(X_5)\ M_i(X_6)$.

The moments of the total cost can thus be obtained by applying the procedure outlined in this section. Next we will show how to estimate a Weibul distribution having the moments of the total cost and then how probabilities can be calculated.

Calculating Probabilities About Total Cost

If the first three moments of the total cost are known, and we wish to make probability statements concerning the total cost, it is necessary to assume a distribution for the total cost. Since the total cost distribution is generally positively skewed, the Weibul will again be used. First, a set of parameters μ, c and ξ which give a Weibul distribution with approximately the same moments are determined.

From (A.1) we can see that

$$G(c) = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{[\Gamma(3/c+1)-3\Gamma(1/c+1)\Gamma(2/c+1)+2\Gamma^3(1/c+1)]^2}{[\Gamma(2/c+1)-\Gamma^2(1/c+1)]^3}$$

is a function of only c and the left hand side can be calculated for the total cost moments. Table III gives the G(c) for different values of c. Using this table and the calculated value of $(\mu_3)^2/(\mu_2)^3$ for the total cost, an appropriate value of c can be determined. Then using the equation for μ_2 and μ_1 in (A.1) and the calculated values of μ_2 and μ_1, α and ξ are

$$\alpha = [\mu_2/(\Gamma(2/c+1)-\Gamma^2(1/c+1))]^{1/2}$$

$$\xi = \mu_1-\alpha\Gamma(1/c+1)$$

Now that c, α and ξ can be determined from μ_1, μ_2 and μ_3, the following can be calculated

Mode of Total Cost = $\alpha[(c-1)/c]^{1/c}+\xi$

Probability (Total Cost > X) = $\exp[-((X-\xi)/\alpha)^c]$

Total cost for which there is a δ probability of exceeding is

$$\alpha[-\ln(\delta)]^{1/c}+\xi$$

Being able to make these probability statements would be the completion of the risk analysis. It is important to note the assumptions made in order to use the analysis in the first three sections. They are

Assumption 1: Subcomponents have a Weibul distribution

Assumption 2: Total cost has a Weibul distribution

Assumption 3: Subcomponents are independent

The purpose of the next section is to relax assumption 3 by structuring the dependence of the subcomponents and to illustrate how this can be incorporated in the above analysis.

TABLE III

u_3^2/u_2^3	C
.0	3.60
.1	2.60026
.2	2.31480
.3	2.13207
.4	1.98141
.5	1.89207
.6	1.80521
.7	1.73158
.8	1.66832
.9	1.61292
1.0	1.56380
1.1	1.51994
1.2	1.48046
1.3	1.44440
1.4	1.41128
1.5	1.38120
1.6	1.35301
1.7	1.32690
1.8	1.30265
1.9	1.28026
2.0	1.25884
2.1	1.23894
2.2	1.22015
2.3	1.20237
2.4	1.18563
2.5	1.16975
2.6	1.15458
2.7	1.14014
2.8	1.12652
2.9	1.11338
3.0	1.10074
3.1	1.08884
3.2	1.07742
3.3	1.06639
3.4	1.05580
3.5	1.04562
3.6	1.03579
3.7	1.02637
3.8	1.01730
3.9	1.00849
4.0	1.00000

Covariance Between Subcomponents

Historically, risk analysis has assumed that the subcomponents of total cost were independent. This assumption was made because of two basic reasons. First, the user of the models could not estimate the covariance because of the lack of historical data and (or) the lack of a way of capturing the covariance from subjective estimates. Covariance is a relative figure which has no intuitive interpretation of its magnitude. A covariance of say 50 is meaningless unless it is compared with another covariance or variance.

The second reason covariance (lack of independence) has been assumed away is because of the difficulty of incorporating it into the risk analysis. To incorporate it into most risk analysis procedures would require an assumption about the multivariate distribution. Even then complications arise as to how to combine these distributions in order to obtain the distribution of the total cost.

The purpose of this section is to propose a means of structuring the covariance between subcomponents and a means of posing questions to the user in such a way that the covariance takes on an intuitive meaning. Finally, a means of incorporating this covariance information into the total cost distribution is given. Conceptually, the user is asked to estimate for each subcomponent a proportion of the total variation due to a commonality between the subcomponents. See the example below for a discussion of how this can be done. This information is used to break the variance of the subcomponent into two parts. Each subcomponent is considered to be the sum of two independent random variables, one of the two being a part of another subcomponent. Putting these two random variables into the cost model in place of the subcomponent makes the total cost a function of independent random variables. The methods discussed in sections two and three would then be applicable.

The structure to be placed on the subcomponents is that each subcomponent X_i is of the form

$$X_i = I_i + \alpha_i D,$$

where I_i is a random variable which represents the independent variation of subcomponent X_i and where D is a random variable which is common to other subcomponents. $\alpha_i D$ will be referred to as the dependent portion of subcomponent X_i.

The user is asked to supply the proportion of the total variation in each X_i which is being caused by the exogenous events or factors constituting the commonality between the subcomponents. In other words, part of the variation in each X_i is thought to have some common reason for occurring. For instance, labor cost and material cost may both be high because of excessive rework. Here excessive rework would be the commonality. In this example, suppose that the low and high for labor cost and material cost were ($5,000, $10,000) and ($25,000, $50,000) respectively. Now if the labor cost were to fluctuate by $2,000 because of excessively high or low rework and material cost were to fluctuate by $5,000 because of rework, the user would say that 40% (i.e., 2,000/(10,000-5,000)) of labor cost and 20% (i.e., 5,000/(50,000-25,000)) of material cost were dependent on rework. The rest of the variation in labor and material cost would be considered independent.

The commonality between subcomponents will, in general, consist of more than one factor and their effect on subcomponents should be estimated cumulatively.

The proportion of the total variation of subcomponent X_i due to a common dependence will be denoted by P_i. Since P_i represents the proportion of the total variation represented by $\alpha_i D$, the standard deviation of $\alpha_i D$ should be approximately $P_i\sqrt{\mu_2(X_i)}$. The variance of X_i is the sum of the variance of I_i and $\alpha_i D$; therefore, the variance of I_i is given as

$$\mu_2(I_i) = \mu_2(X_i) - \mu_2(\alpha_i D)$$

or

$$\mu_2(I_i) = (1-P_i^2)\mu_2(X_i) \tag{A.7}$$

By taking $\alpha_1 = 1$

$$\mu_2(D) = P_1^2\mu_2(X_1) \tag{A.8}$$

and for $i = 2, \ldots, n$

$$\mu_2(\alpha_i D) = \alpha_i^2\mu_2(D) = P_i^2\mu_2(X_i)$$

or

$$\alpha_i = \frac{P_i}{P_1}\sqrt{\frac{\mu_2(X_i)}{\mu_2(X_1)}} \tag{A.9}$$

The results can now be summarized for finding $\mu_2(D)$, $\mu_2(I_i)$, α_i from $\mu_2(X_i)$ and P_i.

Step 1: Use section 1 to estimate $\mu_2(X_i)$ and estimate P_i.

Step 2: Calculate $\mu_2(I_i)$ using (A.7).

Step 3: Calculate $\mu_2(D)$ using (A.8).

Step 4: $\alpha_1 = 1$ and calculate α_i using (A.9).

For the above analysis it is necessary to assume that $P_1 \neq 0$, $\mu_1(D) = 0$, $\mu_3(D) = 0$, $\mu_2(X_1) \neq 0$. In the event $P_1 = 0$ or $\mu_2(X_1) = 0$ it is only necessary to redefine X_1 to be a subcomponent for which $P_1 \neq 0$ and $\mu_2(X_1) \neq 0$.

Returning to our example using labor cost (X_1) and material cost (X_2), suppose the variances were

$$\mu_2(X_1) = 1{,}000{,}000$$
$$\mu_2(X_2) = 25{,}000{,}000$$

then for

$$P_1 = .4 \text{ and } P_2 = .2$$
$$\mu_2(I_1) = (1-.16)\ 1{,}000{,}000$$
$$\mu_2(I_2) = (1-.04)25{,}000{,}000$$
$$\mu_2(D) = .16(1{,}000{,}000)$$
$$\alpha_1 = 1$$
$$\alpha_2 = .2/.4\ \sqrt{25{,}000{,}000/1{,}000{,}000}$$

In a cost model with X_1 and X_2 we would use instead $I_1 + D$ and $I_2 + \alpha_2 D$. The three random variables I_1, I_2 and D would be independent and have moments $\mu_1(I_1) = \mu_1(X_1)$, $\mu_3(I_1) = \mu_3(X_1)$, $\mu_1(I_2) = \mu_1(X_2)$, $\mu_3(I_2) = \mu_3(X_2)$, $\mu_1(D) = 0$ $\mu_3(D) = 0$ and $(\mu_2(I_1), \mu_2(I_2), \mu_2(D))$ as calculated above.

Now that the moments of I_i and D, and α_i can be determined, we now look at how they can be combined into the moments of the total cost.

The cost model used here is

$$\{X_1(1+X_3)+X_2+X_4+X_5X_6(1+X_8)+X_7+X_9X_{10}(1+X_{12})+X_{11}+X_{13}\}\ (1+X_{14})+X_{15}$$

where

X_1 = Material Cost,

X_2 = Material Overhead (not based on Material Cost),

X_3 = Material Overhead Rate,

X_4 = Interdivisional Transfer Cost,

X_5 = Engineering Hours,

X_6 = Engineering Wage Rate,

X_7 = Engineering Overhead (not based on Engineering Cost),

X_8 = Engineering Overhead Rate,

X_9 = Manufacturing Hours,
X_{10} = Manufacturing Wage Rate,
X_{11} = Manufacturing Overhead (not based on Manufacturing Cost),
X_{12} = Manufacturing Overhead Rate,
X_{13} = Other Cost,
X_{14} = General and Administrative Expenses, and
X_{15} = Other Cost with no G&A Expenses

In this model X_1, X_2, X_4, X_5, X_7, X_9, X_{11}, X_{13} and X_{15} will be allowed to have covariance. The resulting model when X_i is replaced by $I_i + \alpha_i D$ is

$$[\{I_1(1+X_3)+I_2+I_4+I_5X_6(1+X_8)+I_7+I_9X_{10}(1+X_{12})+I_{11}+I_{13}\}(1+X_{14})+I_{15}]+$$
$$[\{\alpha_1(1+X_3)+\alpha_2+\alpha_4+\alpha_5X_6(1+X_8)+\alpha_7+\alpha_9X_{10}(1+X_{12})+\alpha_{11}+\alpha_{13}\}(1+X_{14})+\alpha_{15}]D$$

For notational purposes this will be written as

$$A_1+A_2D.$$

Observe that using additive and multiplicative moments the first three moments of A_1 and A_2 can be determined. But A_1 and A_2 are not independent. The first two additive moments of A_1+A_2D are given by

$$\mu_1(A_1+A_2D) = \mu_1(A_1)$$
$$\mu_2(A_1+A_2D) = \mu_2(A_1)+\mu_2(A_2)\mu_2(D)+\mu_1{}^2(A_2)\mu_2(D) \qquad \text{(A.10)}$$

Therefore, the first two moments of the total cost can be determined. The third moment of the total cost is

$$\mu_3(A_1+A_2D) = \mu_3(A_1) -3 \quad \mu_1(A_1)\mu_1(A_2^2)\mu_2(D)+3 \quad \mu_1(A_1A_2^2)\mu_2(D) \qquad \text{(A.11)}$$

where all of the terms are known except for $\mu_1(A_1A_2^2)$ which is

$$\mu_1(A_1A_2^2) = \mu_1(c_1c_2^2) \; \mu_1((1+X_{14})^3)+ \mu_1(c_1)\mu_1(1+X_{14})\alpha_{15}^2$$
$$+2\alpha_{15}\mu_1(c_1c_2)\mu_1((1+X_{14})^2)+\mu_1(c_2^2)\mu_1((1+X_{14})^2)(\mu_1 I_{15})$$
$$+\alpha_{15}^2\mu_1(I_{15})+2\alpha_{15}\mu_1(c_2)\mu_1(1+X_{14})\mu_1(I_{15}) \qquad \text{(A.12)}$$

where

$$c_1 = (I_2+I_4+I_7+I_{11}+I_{13})+I_1(1+X_3)+I_5X_6(1+X_8)+I_9X_{10}(1+X_{12})$$

$$c_2 = (\alpha_2+\alpha_4+\alpha_7+\alpha_{11}+\alpha_{13})+\alpha_1(1+X_3)+\alpha_5X_6(1+X_8)+\alpha_9X_{10}(1+X_{12})$$

The terms $\mu_1(c_1c_2)$ and $\mu_1(c_1c_2^2)$ are given by

$$\mu_1(c_1c_2) = \sum_{ij} a_i b_j \mu_1(y_i y_j)$$
$$\mu_1(c_1c_2^2) = \sum_{ijk} a_i b_j b_k \mu_1(y_i y_j y_k)$$

where

$$a_1 = \mu_1(I_2+I_4+I_7+I_{11}+I_{13}),$$
$$a_2 = \mu_1(I_1),$$
$$a_3 = \mu_1(I_5),$$
$$a_4 = \mu_1(I_9),$$
$$b_1 = \alpha_2+\alpha_4+\alpha_7+\alpha_{11}+\alpha_{13},$$
$$b_2 = \alpha_1,$$
$$b_3 = \alpha_5,$$
$$b_4 = \alpha_9,$$
$$y_1 = 1,$$
$$y_2 = (1+X_3),$$
$$y_3 = X_6(1+X_8), \text{ and}$$
$$y_4 = X_{10}(1+X_{12}),$$

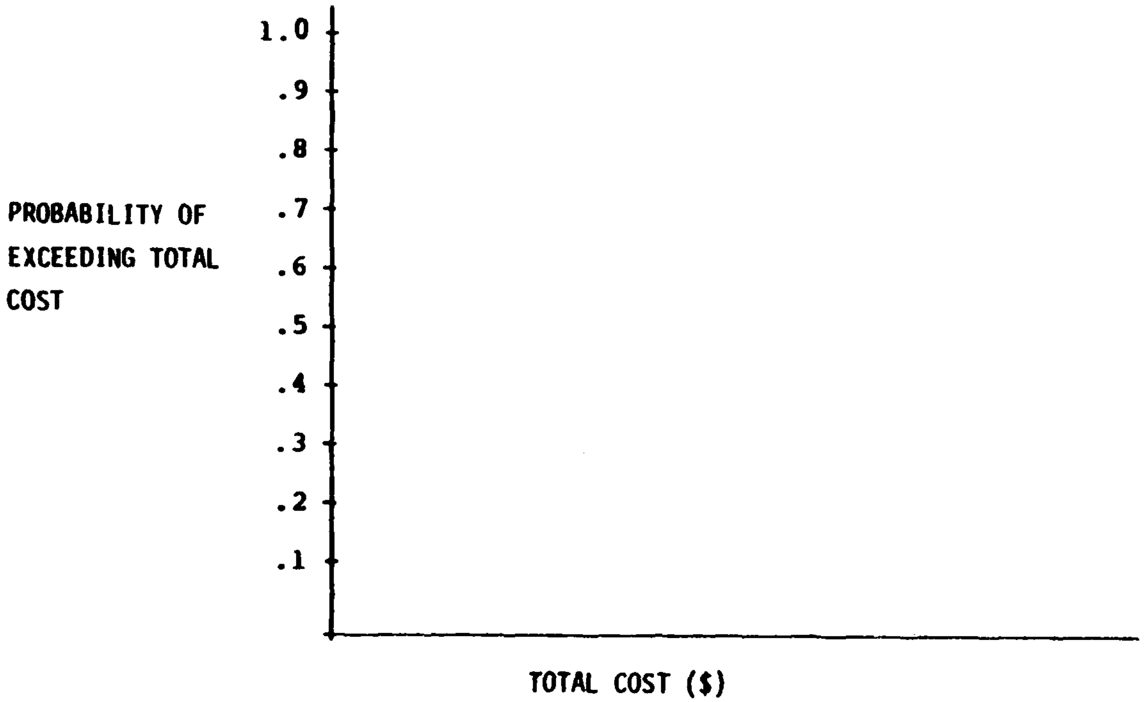
The term $\mu_1(y_i y_j y_k)$ must be evaluated differently when $i=j$, $i=k$, $j=k$ and $i=j=k$.

In summary the first three moments of total cost can be found by using equations (A.10) through (A.13) and the procedures given in the first three sections of this appendix.

APPENDIX II

Forms for Organizing Inputs and
Displaying Outputs of Risk Analysis

Contract # ______________________
Date ______________________
Analyst ______________________



Uncertainty Primary Drivers

1.

2.

3.

4.

5.

Most Likely Total Cost = ________

Results of Risk Analysis

Contract # ____________________
Date ____________________
Analyst ____________________

INPUTS FOR RISK ANALYSIS

Elements	Minimum	Most Likely	Maximum	Dependence	Comment
Material Cost					
Independent O.H.					
Rate for Mat. O.H.				xxx	
Interdivisional Transfer					
Engineering Labor Direct					
Wage rate				xxx	
Independent O.H.					
Rate for Eng. Lab. O.H.				xxx	
MFG Labor Direct					
Wage rate				xxx	
Independent O.H.					
Rate for MFG Lab. O.H.				xxx	
Other Cost with G & A					
G & A Rate applied to Subtotal				xxx	
Other cost with no G & A					

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