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On the Optimal Location of Vibration Supports

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ABSTRACT

The problem of optimal positioning of vibration supports to raise the fundamental natural frequency of a system is studied. It is established that for the optimal locations criterion the corresponding lowest antiresonant frequency is a maximum. A numerical example illustrates this criterion.

1. Introduction

Intermediate supports are often introduced in engineering structures to increase the resonant frequencies of the system as well as to support weights. These supports, when realized by actual structural components, are elastic supports. Thus, the problem of designing vibration supports to raise the fundamental frequency involves both finding the location and the required stiffness of the supports.

In an earlier paper, Bezler and Curreri [1] studied the design of vibration supports for piping systems. They used the transfer matrix method to study a spring supported cantilever beam and a spring supported "L" bend. They found the optimum spring location, i.e., the most effective location to put a spring to increase the fundamental frequency, from numerical experimentation. They concluded that a "near optimal" position for a flexible spring is at a node of the second mode. For a rigid support this would be the optimal location.

In the present paper, a criterion for selecting the optimal spring locations will be derived. This criterion can also be used to compare the relative effectiveness of sets of proposed support locations.

2. Problem Formulation

For a multiple-degree-of-freedom, undamped system with a spring introduced at dof J, the frequency equation is

$$\frac{1}{k} + R_{JJ}(\omega) = 0 \tag{1}$$

where $R_{JJ}(\omega)$ is the receptance of dof J. Equation (1) can be derived using the receptance method [2]. Alternatively, it can be found by considering the addition of a spring to a system as a local modification [3,4,5]. The receptance $R_{JJ}(\omega)$ can be expressed in modal summation form as

$$R_{JJ}(\omega) = \sum_{\ell=1}^{n} \frac{\rho_{J\ell}^2}{G_\ell(\omega_\ell^2-\omega^2)} \tag{2}$$

where ω_ℓ is the natural frequency of the ℓth mode of the unsupported system. $\{\rho_\ell\}$ is the corresponding eigenvector, $\rho_{J\ell}$ is the Jth component of $\{\rho_\ell\}$, and $G_\ell = \{\rho_\ell\}^T[m]\{\rho_\ell\}$ is the generalized mass of the ℓth mode. Thus, for any given spring rate k, Eq. (1) along with Eq. (2) can be used to solve for the new frequencies ω. The natural frequencies of the supported system increase as the spring rate increases. In the limit, as k approaches infinity, i.e., as the support becomes ideally rigid, the frequency equation becomes

$$R_{JJ}(\omega) = 0 \tag{3}$$

Denote the lowest ω that satisfies Eq. (3) as $a^{(J)}$. Then $a^{(J)}$ is the lowest antiresonant frequency of dof J. That is, $a^{(J)}$ is the highest fundamental frequency achievable when the support at dof J becomes rigid. It follows from the eigenvalue separation property [6], that $a^{(J)} \leq \omega_2$, where ω_2 is the second natural frequency of the unsupported system. Thus, by choosing dof J for a rigid support as a node in the second mode of the unsupported system, we have $a^{(J)} = \omega_2$, which is the maximum obtainable fundamental frequency. This result has been known for some time [1].

Now consider the case of introducing s springs at dof $J_1, J_2 \ldots J_s$. Following the procedure of Ref. 5, the frequency equation of the supported system is given by

$$\det([I] + [\hat{R}][\Delta K]) = 0 \tag{4}$$

where [I] is an sxs identity matrix

[$\hat{R}$] is the receptance matrix associated with the dof $J_1, J_2, \ldots, J_s$, i.e.,

$$\hat{R}_{ij} = R_{J_i J_j} \tag{5}$$

$$[\hat{\Delta K}] = \begin{bmatrix} \Delta k_1 & & & \\ & \Delta k_2 & & \\ & & \ddots & \\ & & & \Delta k_s \end{bmatrix} = \text{an sxs diagonal matrix}$$

Δk_j is the spring rate of the support at dof j.

In the limiting case when all $\Delta k_j \rightarrow \infty$, Eq. (4) becomes

$$\det[R] = 0 \tag{6}$$

Let $A^{(J_s)}$ be the lowest root of Eq. (6). Then the optimal support locations will be where $a^{(J_s)}$ is a maximum. We are now in a position to establish a criterion for optimal support locations.

3. Maximum Antiresonant Frequency Criterion

> For given sets of support locations, the best set of locations is where the corresponding lowest antiresonant frequency is a maximum.

We will call this criterion the **Maximum Antiresonant Frequency Criterion (MAFC)**. To find the antiresonant frequency, one can either solve an eigenvalue problem of order (n-s) or solve the nonlinear Eq. (6).

4. Numerical Example

To illustrate the basic contention of the MAFC criterion, consider the simply supported beam of Fig. 1. The fundamental frequency of this beam is 15.71 Hz. It is desired to introduce two intermediate supports to increase the fundamental natural frequency to above 25 Hz. For this example it is practical to restrict the support locations to two possible sets of positions, say A (x_1 = .1L, x_2 = 0.5L) and B (x_1 = 0.34L, x_2 = 0.67L).

For this case with two supports, we have

$$[\hat{\Delta K}] = \begin{bmatrix} \Delta K_1 & 0 \\ 0 & \Delta K_2 \end{bmatrix} \tag{6}$$

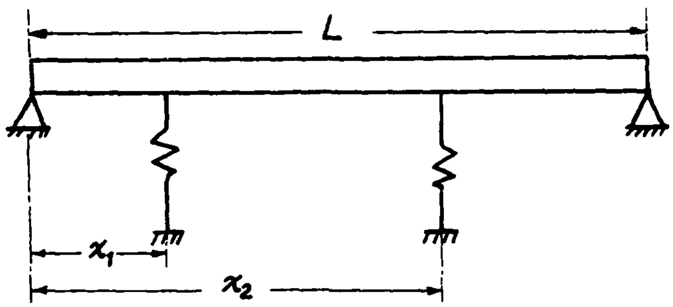
and

$$[\hat{R}] = \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} \\ \hat{R}_{21} & \hat{R}_{22} \end{bmatrix} \tag{7}$$

It is convenient to calculate the elements R_{ij} with a modal summation. Thus,

$$\hat{R}_{ij} = R(x_1, x_2)$$

$$= \sum_{\ell=1}^{n} \frac{\rho_\ell(x_1)\rho_\ell(x_2)}{G_\ell(\omega_\ell^2 - \omega^2)} \tag{8}$$



$L = 2.54\text{m}\ (100\ \text{in.})$

$E = 69\ \text{GPa}\ (10^7\ \text{psi})$

$\rho = 8748.73\ \frac{\text{kg}}{\text{m}^3}\ (0.01\ \frac{\text{lb-sec}^2}{\text{in}^2})$

$I = 4.1623\text{x}10^{-6}\ \text{m}^4\ (10\ \text{in.}^4)$

Figure 1   A simply supported beam

where, for a simply supported beam,

$$\rho_\ell(x_1) = \sin \frac{\ell \pi x}{L}$$

$$G_\ell = 1/2\ \rho L$$

$$\omega_\ell = \frac{(\ell\pi^2)}{L^2}\sqrt{\frac{EI}{\rho}}$$

In the numerical calculation n = 20 is used, or, in other words, 20 modes are used to evaluate the receptances in Eq. (8). The frequency determinant of Eq. (6) gives

$f_1^{(A)}$ = fundamental natural freqency for rigid supports at $x_1 = 0.1L$, $x_2 = 0.5L$
= 70.9 Hz

$f_1^{(B)}$ = fundamental natural frequency for rigid supports at $x_1 = 0.34L$, $x_2 = 0.67L$
= 180.1 Hz

Since $f_1^{(B)} > f_1^{(A)}$, we conclude that the location pair B is more effective than location pair A in raising the fundamental frequency of the system.

To check the above proposition, we will compute the fundamental frequencies of the spring supported beam for the special case of equal spring rates. The resuls are summarized in Table 1. Alternatively, we can compute the required (equal) spring rates for both springs for given fundamental frequencies. The results are summarized in Table 2. We observe that to raise the fundamental frequency above 25 Hz, springs with rates of about 1.23×10^6 N/m (7000 lb/in) are needed at location $x_1 = 0.1L$ and $x_2 = 0.5L$, while less stiff springs with rates of 0.88×10^6 N/m (5000 lb/in) are needed if they are located at $x_1 = 0.34L$, $x_2 = 0.67L$.

Table 1

Natural Frequencies for Simply Supported Beam with Two Equal Intermediate Springs

Spring Stiffness N/m	(lb/in.)	Fundamental Natural Frequency of the Supported System (Hz): x_1 = 0.1L, x_2 = 50	x_1 = 34, x_2 = 0.67L
17513	(100)	15.88	15.95
87565	(500)	16.67	16.88
175130	(1000)	17.38	17.98
350268	(2000)	18.90	20.00
525390	(3000)	20.30	21.81
700520	(4000)	21.61	23.48
875650	(5000)	22.82	25.05
1751300	(10,000)	28.07	31.72
	∞	76.9	180.1

Table 2

Required Spring Rate to Achieve Prescribed Natural Frequency

Fundamental Natural Frequency (Hz)	Required Spring Stiffness (lb/in) for Springs at x_1 = 0.1L, x_2 = 0.5L: N/m	(lb/in)	x_2 = 0.34L, x_2 = 0.67L: N/m	(lb/in)
16	29238	(166.95)	21220.5	121.17
17	133608	(762.91)	968962.7	553.09
18	244632	(1396.86)	177135	1011.45
19	362386	(2069.24)	262047	1496.3
20	486935	(2780.42)	351610	2007.71
25	1214360	(6934.07)	869482	4964.78
30	2125640	(12137.50)	1505170	8594.61

5. Conclusion

In summary, a simple criterion has been derived that will allow a designer to choose the optimal locations for placing vibration supports. This will narrow the design problem to that of determining the required stiffness to achieve a desired fundamental natural frequency.

Acknowledgment

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