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VIRGINIA POLYTECHNIC INST AND STATE UNIV BLACKSBURG --ETC F/G 20/40
THREE-DIMENSIONAL DISTURBANCES IN HIGH SPEED BOUNDARY LAYER FLO--ETC(11)
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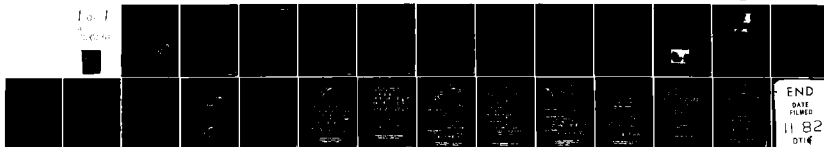
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report summarizes developments on a new theory which was devised to identify and describe the cause of the pronounced spanwise-periodic disturbances observed in reattaching separated flows. This theory was subsequently extended to include surface mass transfer effects and the coupling of the boundary layer disturbances and adjacent surface material thermal and ablative response of the body surface. It was shown for the first time that a similar three-dimensional phenomenon can occur near the separation line of a high speed boundary layer. In parallel, the theoretical study of reattachment-		

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>stagnation region 3-D vortices which was refined using a systematic inner-outer match asymptotic analysis approach appropriate to finite-sized bodies is described. A very careful detailed formulation of the associated boundary value problem for the case of a cylinder and its detailed numerical treatment (which also required great effort and care) has nearly been completed. Finally, a new turbulent viscous-disturbance sublayer theory which was developed as the inner deck of a generalized non-asymptotic triple-deck theory of two-dimensional turbulent shock interaction problems in both transonic and supersonic flows is discussed. This theory provides a much improved theoretical account of the skin friction, upstream influence and displacement thickness growth in a wide variety of such problems.



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~~Abstract~~

*Three-Dimensional Disturbances in
High Speed Boundary Layer Flows*

FINAL SCIENTIFIC REPORT

AFOSR CONTRACT F49620-76-C-0013

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MATTHEW J. KERPER

Chief, Technical Information Division

1. Introduction

The objective of this research is the basic theoretical investigation of three-dimensional pressure, skin friction and heat transfer disturbances in both laminar and turbulent boundary layer flows including viscous-inviscid interaction effects, separation and reattachment. A sound understanding of these phenomena is required in modern aerodynamic design analyses of high speed flight vehicles.

The aforementioned need has led to a systematic basic research effort treating in parallel the twin problems of (1) viscous-inviscid interaction with flow separation and/or attachment and (2) three-dimensional effects (including those due to streamwise vortices) in high speed laminar and turbulent boundary layer flows along two-dimensional and axi-symmetric bodies, with emphasis on pressure distribution, skin friction and heat transfer effects. Such a study was initiated October 1, 1971, and has continued under AFOSR support. To date, it has been very successful both in solving a number of the target flow problems and in developing new ideas and tools for attacking new aspects of (1) and (2). This report summarizes these research results.

2. Summary of Technical Accomplishments

A new theory was devised¹ which identified and described the cause of the pronounced spanwise-periodic disturbances observed² in reattaching separated flows (see Figure 1). This theory was subsequently extended³ to include surface mass transfer effects and the coupling of the boundary layer disturbances and adjacent surface material thermal and ablative response of the body surface (Figure 2). Furthermore, it was shown⁴ for the first time that a similar three-dimensional phenomenon can occur near the separation line of a high speed boundary layer (Figure 3). During the author's Fulbright-von Humboldt Research Fellowship at the DFVLR-AVA in Göttingen, West Germany,

an improved theory of two-dimensional separation and reattachment regions of either laminar or turbulent flow was also developed^{5,6} to determine the stream-line angle and flow curvature therein, both of which are very important to the 3-D disturbance study (Figure 4). These separation-region theories were further subjected to verifying experimental comparisons with data (see Figure 5) supplied by Professor Bogdonoff's group at Princeton¹¹, with whom active collaboration has been established. In parallel, the theoretical study of reattachment-stagnation region 3-D vortices was refined using a systematic inner-outer matched asymptotic analysis approach appropriate to finite-sized bodies. A very careful detailed formulation of the associated boundary value problem for the case of a cylinder and its detailed numerical treatment (which also required great effort and care) was nearly finished; the goal was to predict a unique spanwise wavelength for these disturbances as experimentally observed. Informal collaboration on this subject was established with Professors Mark Morkovin of the Illinois Institute of Technology (Figure 6) and W. Sadeh of Colorado State University.

Turning to work on viscous-inviscid interactions, several fruitful developments occurred. First, a new turbulent viscous-disturbance sublayer theory was developed as the inner deck of a generalized non-asymptotic triple-deck theory¹² of two-dimensional turbulent shock interaction problems in both transonic and supersonic flows (Figure 7). This theory provides a much improved theoretical account of the skin friction, upstream influence and displacement thickness growth in a wide variety of such problems. In particular, it provides the heretofore-missing link between Lighthill's theory for lower Reynolds numbers and modern asymptotic theory pertaining to the infinite Reynolds number limit (Figure 8). Second, somewhat as a fortuitous side development, some fundamental studies^{13,14} were carried out on how vectored suction or injection

through the surface influences laminar boundary layer viscous-inviscid flow properties; one of the major results was the discovery of some new branches of the family of self-similar solutions to the laminar boundary layer equations. Third, the author spent three months (6/16-9/18) as a visiting researcher at the Princeton University Gas Dynamics Laboratory for the purpose of developing a basic analytical framework for certain Princeton experimental studies of three-dimensional shock-boundary layer interactions. Princeton's experimental approach offers two important advantages to the theoretician: (a) the physical configurations are simple geometries offering the best opportunity for fundamental diagnosis; (b) the measurements involve a systematic variation of the interaction strength over a wide range including the weaker non-separating regime. A detailed study of various theses and papers describing their results (plus many of the references therein) indicated that the most promising problem for initial attack was the sharp vertical fin study by Oskam¹⁵ (Figure 8). A mathematically well-posed analytical model of this 3-D turbulent interaction field was formulated for fin deflection angles below that causing separation, and a solution approach then devised using operational methods. This theory constitutes a three-dimensional generalization of the author's aforementioned non-asymptotic turbulent triple-deck theory for 2-D interactions.¹² In Appendix A we show some of the essential steps involved in setting up the solution approach to this vertical fin problem, based on illustrations from a lecture presented by the author at Princeton in September 1980. Although considerable detailed work is still required to complete the solution, the sought-after foundation was established; the ultimate results are expected to shed valuable light on (among other things) upstream and lateral influence properties, skin friction line geometry and incipient separation, and deviations from cylindrically-symmetric behavior.

3. Summary of Publications and Presented Papers on Behalf of This Contract

Overall this research effort produced nine major papers (Refs. 1, 3, 4, 5, 6, 7, 12, 13, 14) of which six have been published so far in the open literature (Refs. 1, 3, 5, 6, 13, 14) and seven were presented at major technical meetings (Refs. 1, 3, 6, 7, 12, 13, 14).

In addition, seminars on selected aspects of the research were presented at various times throughout the contract period at the University of Colorado, Johns Hopkins University Applied Physics Laboratory, University of Arizona, Princeton University (as a Baetjer Lecturer), University of Delaware, State University of New York at Buffalo, the von Karman Institute (Brussels), University of Trondheim (Norway), Université de Poitiers (France), and the University of Toronto (at the 1980 IUTAM Meeting).

4. Educational Impact of the Research

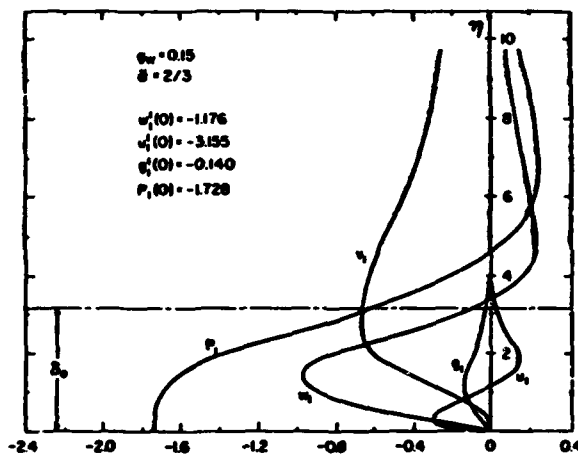
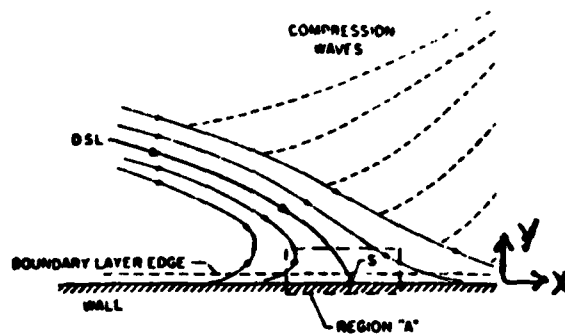
Various phases of this work have formed the graduate training of three Ph.D. students (R. C. Swanson and T. F. Swean of VPI&SU plus M. Namtu of Romania) and two M.S.-level students (K. M. Cho, subsequent Ph.D. candidate at Cornell and V. Sonnad, subsequent Ph.D. candidate at M.I.T.).

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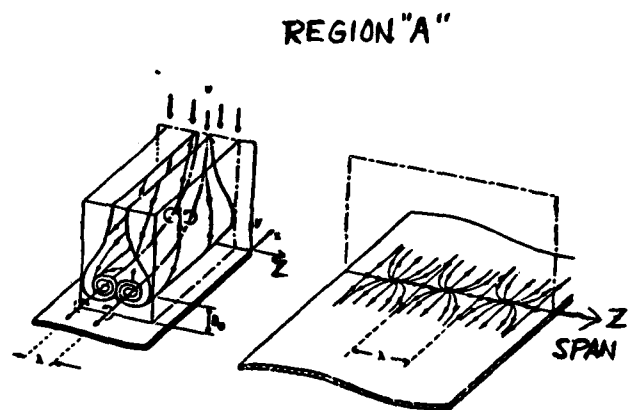
1. Inger, G. R., "Three-Dimensional Disturbances in Reattaching Separated Flows," Proc. AGARD Symposium on Flow Separation, CP-168, pp. 18-1 to 18-12, Göttingen, May 1975.
2. Ginoux, J., "The Existence of Three-Dimensional Perturbations in the Reattachment of a Two-Dimensional Supersonic Boundary Layer After Separation," AGARD Report 272, April 1960.
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Eigenvalue Disturbance Profiles Across Boundary Layer



Vortex-Instability Disturbance Flow (Schematic)

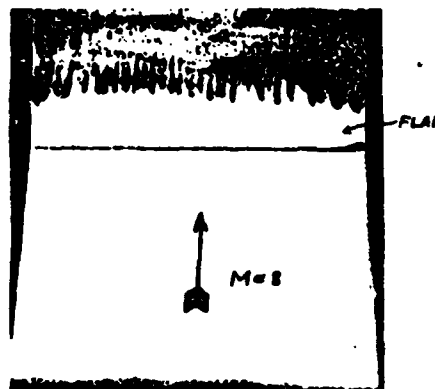


Experimental Oil Flow Visualization of Reattachment Flow Pattern

Figure 1. Spanwise-Periodic Disturbance-Vortices in Reattaching Shear Flows (Ref. 1)



Typical surface thermal response (subliming film)⁴



Spanwise scorching pattern in hypersonic reattachment region (steel flap)¹⁰

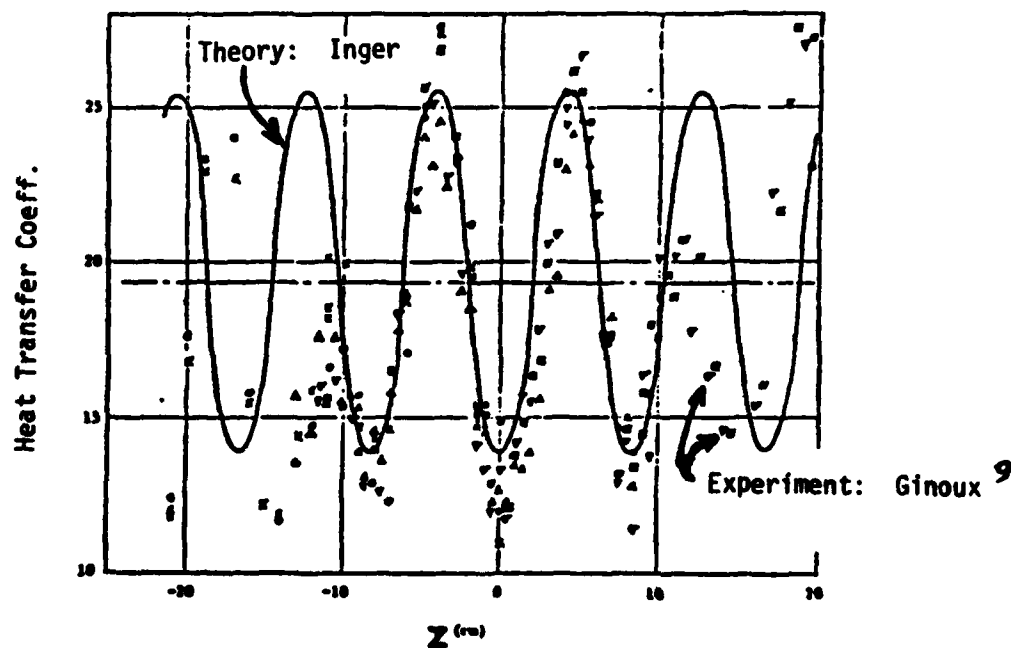


Figure 2. Heat Transfer and Surface-Material Response Aspects of the Spanwise-Disturbance Vortices in Reattaching Flows

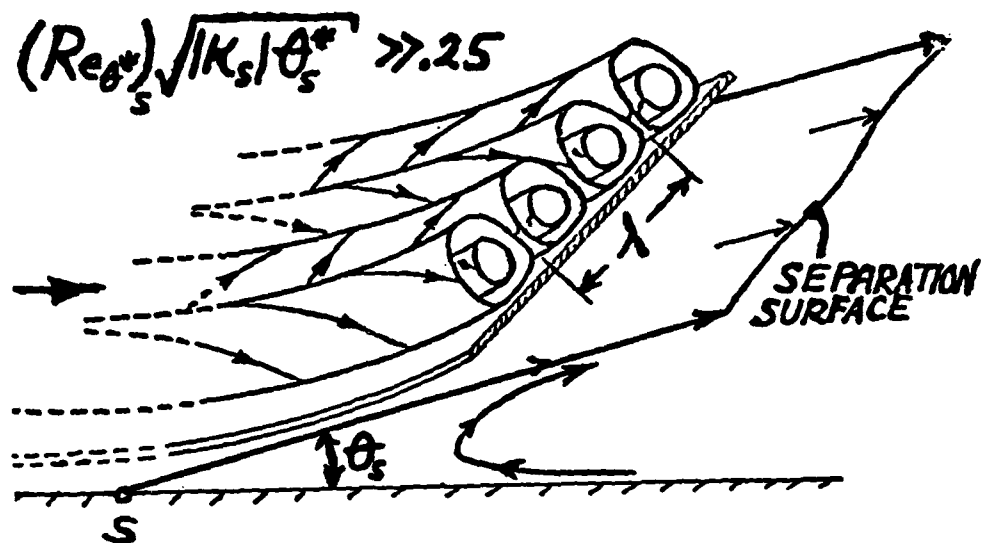
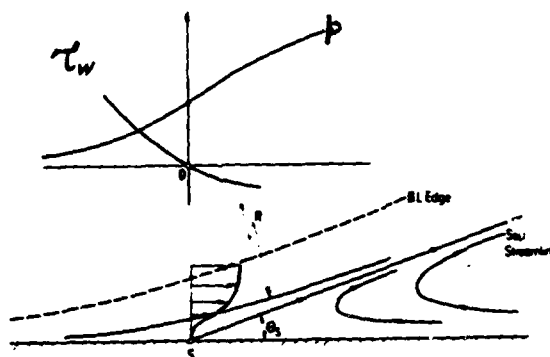


Figure 3. Streamwise Vortex-Disturbances in Separating Flows (Ref. 4)



$$\tan \theta_s \approx -3 (\partial \tau_w / \partial x)_s / (dp / dx)_s$$

$$R \approx - \frac{3 (\partial \tau_w / \partial x)_s}{(\partial^2 \tau_w / \partial x^2)_s} \cdot \cot \theta_s \approx \frac{(dp / dx)_s}{(\partial^2 \tau_w / \partial x^2)_s}$$

Figure 4. Separation-Neighborhood Analysis for Two-Dimensional Laminar or Turbulent Flows (Ref. 6,7)

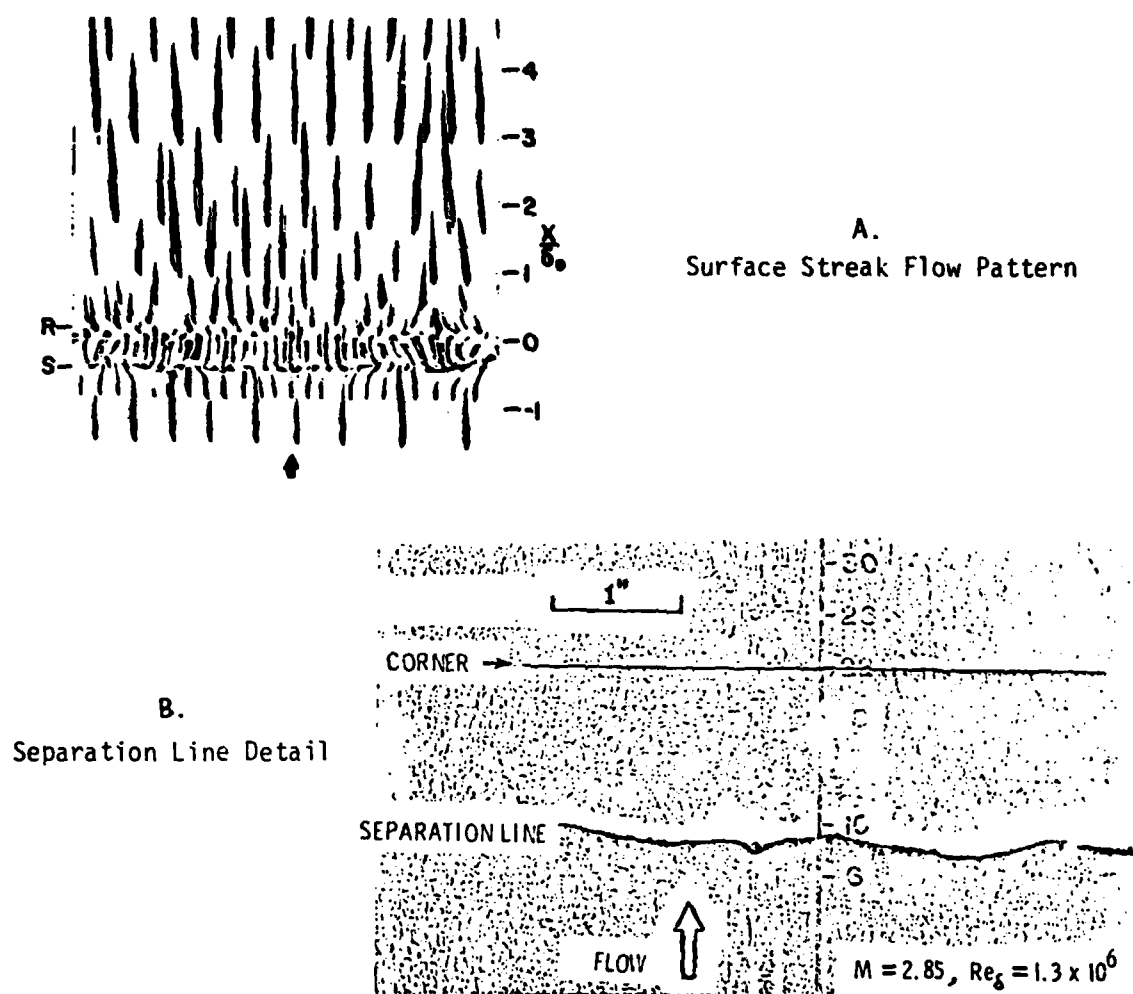


Figure 5. Experimental Confirmation of Theoretically-Predicted Separation-Region Vortices (Ref. 11)

NASA Contractor Report 3231

On the Question of Instabilities Upstream of Cylindrical Bodies

Mark V. Morkovin
Illinois Institute of Technology
Chicago, Illinois

" The rather unconventional views presented here in a searching spirit were first written up in May 1976. Eighteen copies of that report were circulated in the United States, Europe, and Japan to elicit critique and suggestions and to provide an opportunity for private prepublication rebuttals for those whose earlier work was questioned in the text. The present report incorporates, then, not only the information and views from publications since 1976 but also the advice and comments of a number of discussors, in particular Prof. G. Inger, E. Roshotko, and I. Tani. Figures based on the reviewed publications are explicitly credited. The original NASA study under Grant NSG-1120 was guided, with understanding, by Ivan Beckwith. The 1979 revision was supported under AFOSR Contract F49620-77-C-013 concerned with instabilities and transition to turbulence. "

Figure 6.

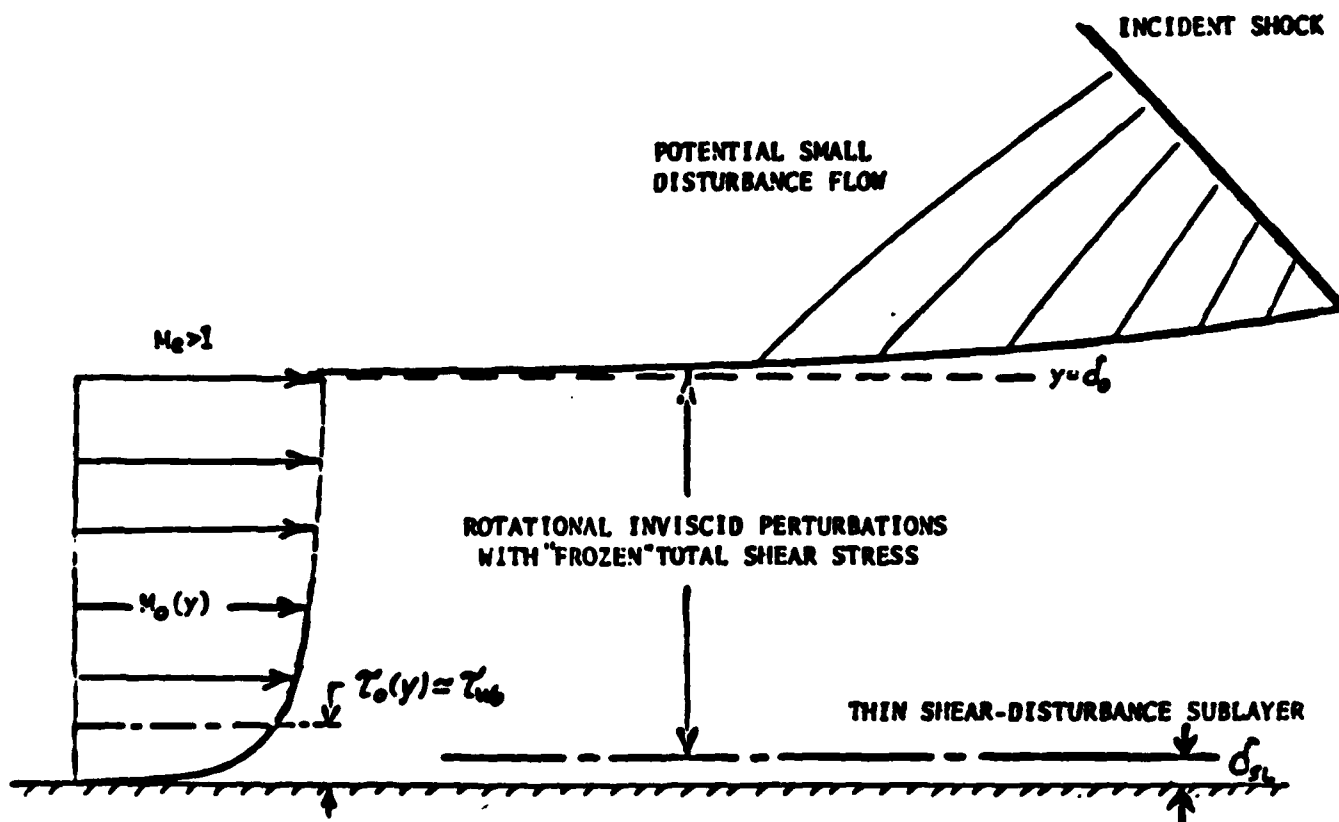


Figure 7. Non-Asymptotic Triple-Deck Model of Shock-Turbulent Boundary Layer Interactions (Ref. 12)

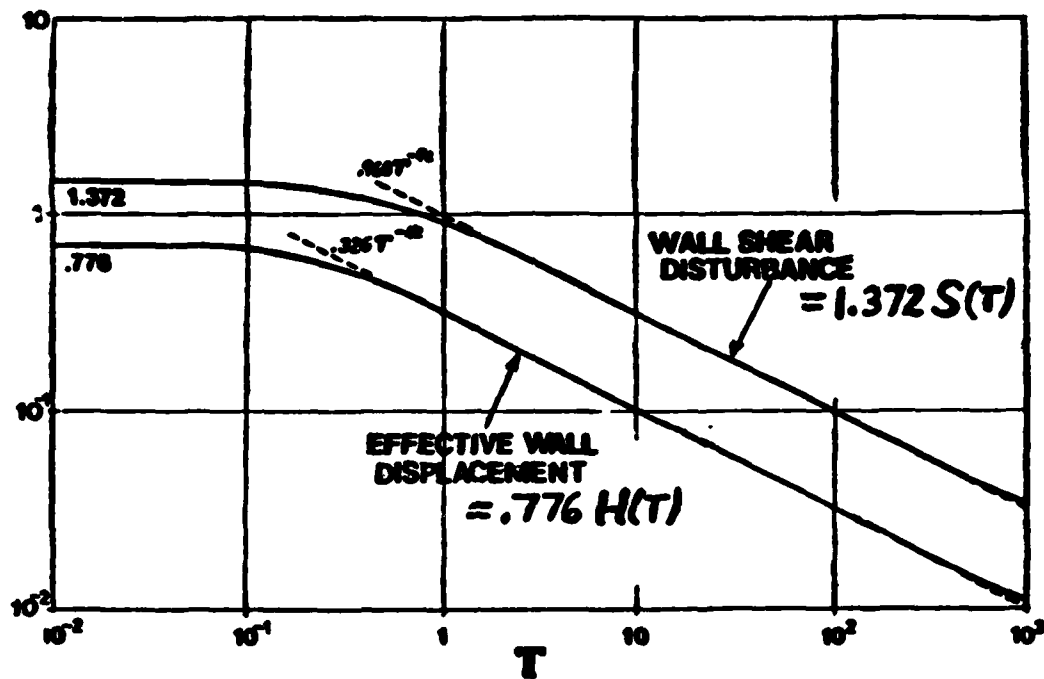


Figure 8. Effect of Interactive Turbulence Parameter T on Inner Deck Properties (from Ref. 12)

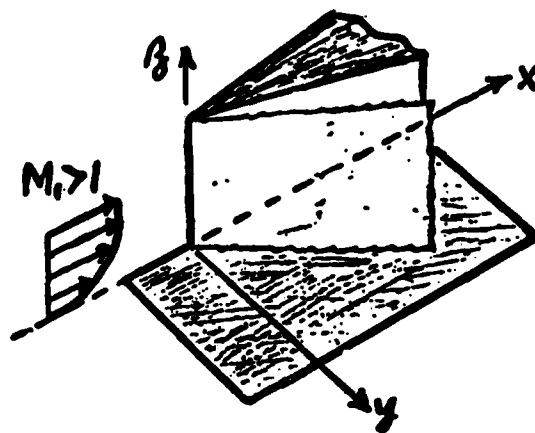
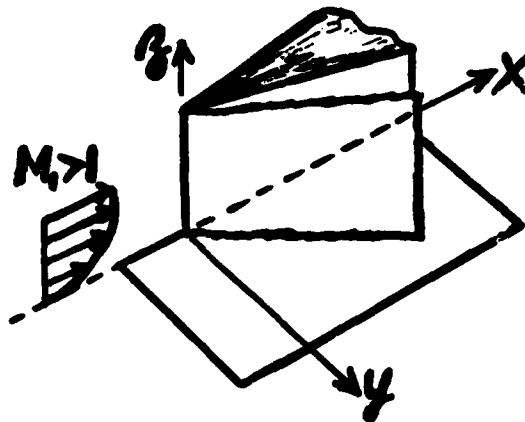


Figure 9. The Slender Sharp Vertical Fin-Boundary Layer Interaction Problem (Schematic)

Appendix A

Details of the 3-D Triple-Deck Analysis of the Vertical Fin Shock-Boundary Layer Interaction Problem



$$\frac{\partial \rho'}{\partial x} U_0 + \rho_0 \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} \right) + w' \frac{d\rho_0}{dz} = 0 \quad \text{CONTINUITY}$$

$$U_0 \frac{\partial u'}{\partial x} + w' \frac{dU_0}{dz} + \frac{1}{\rho_0} \frac{\partial p'}{\partial x} \approx 0 \quad \text{STREAMWISE MOMENTUM}$$

$$U_0 \frac{\partial w'}{\partial x} + \frac{1}{\rho_0} \frac{\partial p'}{\partial z} \approx 0 \quad \text{VERTICAL MOMENTUM}$$

$$U_0 \frac{\partial v'}{\partial x} + \frac{1}{\rho_0} \frac{\partial p'}{\partial y} \approx 0 \quad \text{LATERAL MOMENTUM}$$

$$U_0 \frac{\partial p'}{\partial x} \approx a_0^2 \left(U_0 \frac{\partial \rho'}{\partial x} + w' \frac{d\rho_0}{dz} \right) \quad \text{ENERGY}$$

- Notes :
- ADIABATIC
 - FROZEN TOTAL SHEAR STRESS
 - "PARALLEL" UNDISTURBED FLOW ($V_0 \approx \frac{\partial U_0}{\partial x} = 0$)

**GOVERNING 3-D DISTURBANCE
EQUATIONS FOR INVISCID OUTER
AND MIDDLE DECK REGIONS**

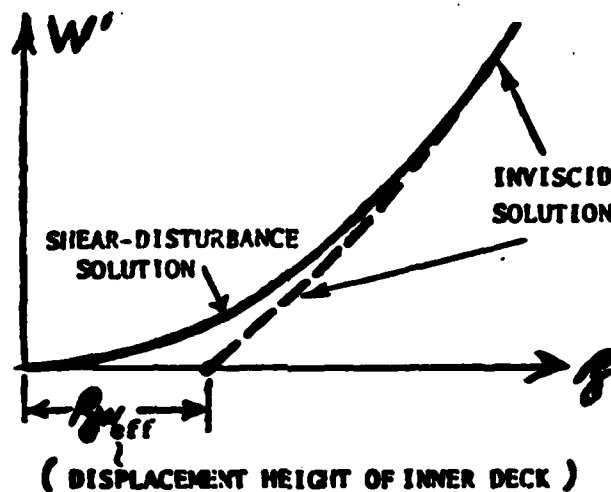
- INCOMPRESSIBLE DISTURBANCE MOTION
- VERY THIN LAYER, $\partial p' / \partial g = 0$ & $\rho_0 \approx \rho_{ow}$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial g} = 0$$

$$U_0(g) \frac{\partial u'}{\partial x} + \frac{dU_0}{dg} w' + \frac{1}{\rho_0} \frac{\partial p'}{\partial x} = \nu_{ow} \left(\frac{\partial^2 u'}{\partial g^2} + \frac{\partial^2 u'}{\partial y^2} + \frac{\partial^2 u'}{\partial x^2} \right)$$

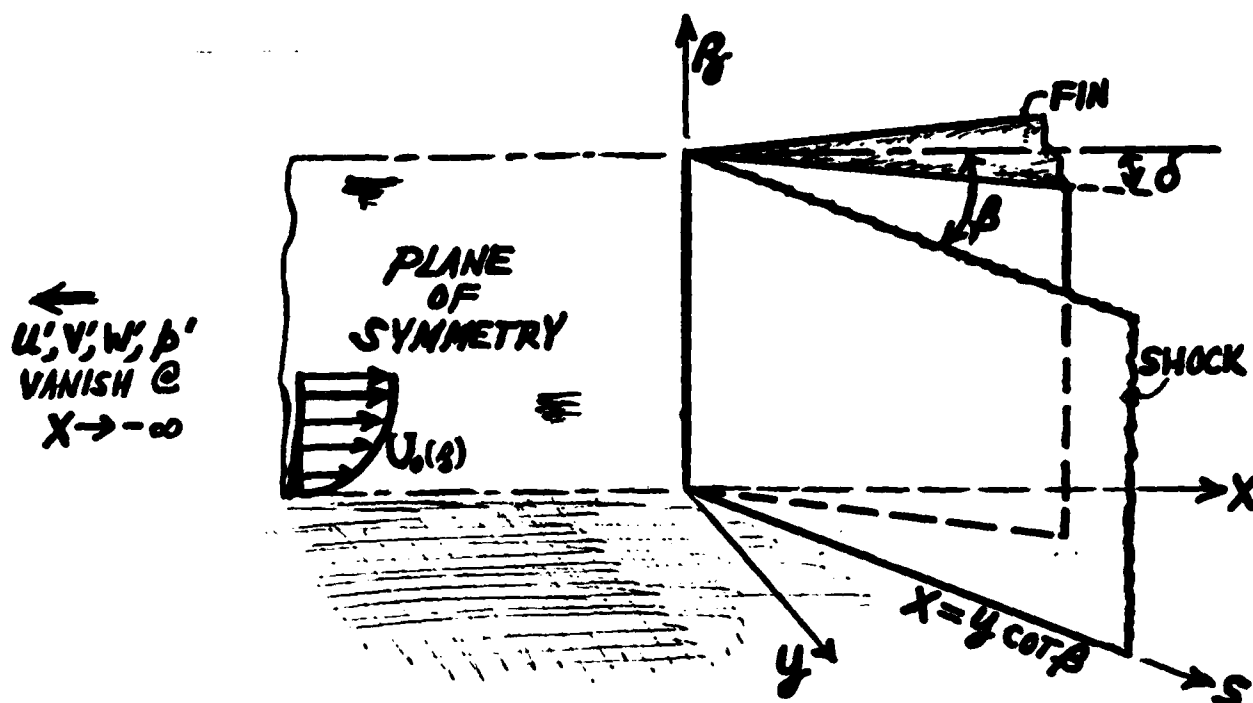
$$U_0(g) \frac{\partial v'}{\partial x} + \frac{1}{\rho_{ow}} \frac{\partial p'}{\partial y} = \nu_{ow} \left(\frac{\partial^2 v'}{\partial g^2} + \frac{\partial^2 v'}{\partial y^2} + \frac{\partial^2 v'}{\partial x^2} \right)$$

$$\frac{\partial}{\partial x} \left(U_0 \frac{\partial^2 w'}{\partial g^2} - \frac{d^2 U_0}{dg^2} w' \right) = \nu_{ow} \frac{\partial^2}{\partial g^2} \left(\frac{\partial^2 w'}{\partial g^2} + \frac{\partial^2 w'}{\partial y^2} + \frac{\partial^2 w'}{\partial x^2} \right)$$



GENERAL 3-D VISCOUS DISTURBANCE PROBLEM for INNER DECK

TRY
IN 1



INNER LATERAL ($y=0$)

$x < 0$ (PLANE OF SYMMETRY):

$$\partial p' / \partial y = 0$$

$$v' = 0$$

$$w' \approx 0 \quad (\text{NEGLIGIBLE UPWASH FOR SHARP FIN})$$

$x \geq 0$ (FIN FACE): $v'(x, 0, z) \approx U_0(z) \delta$

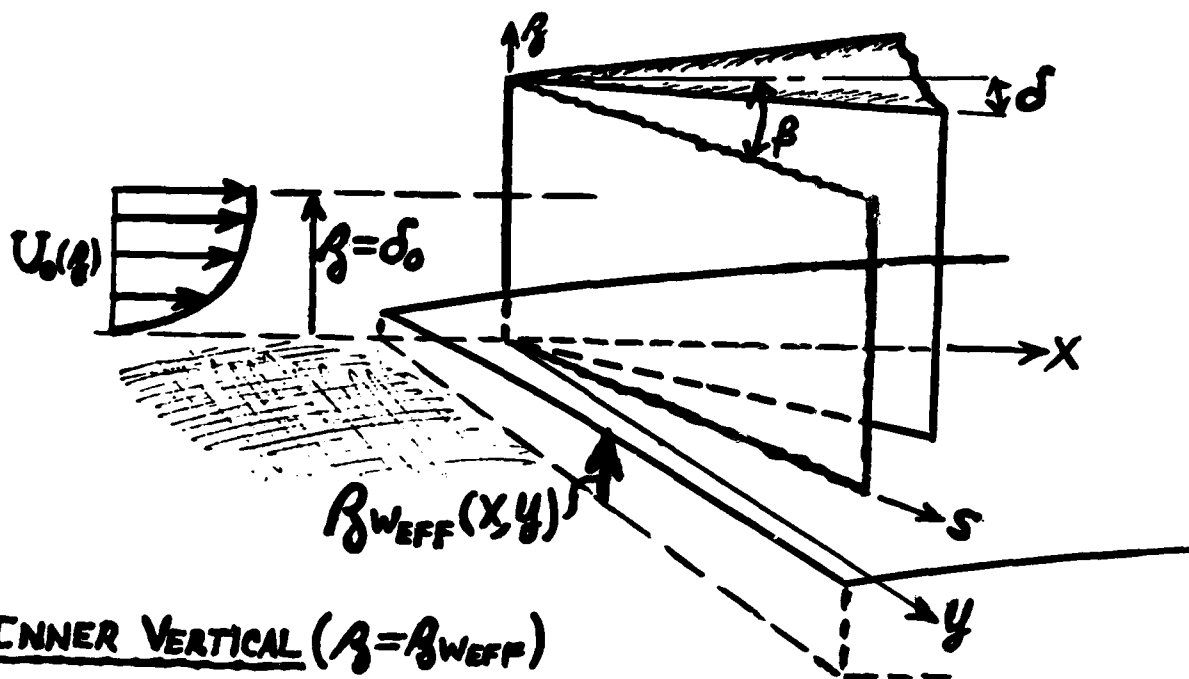
$$[\partial p' / \partial y = -\rho_0(z) U_0(z) \frac{\partial v'}{\partial x}]$$

OUTER LATERAL ($y \rightarrow \infty$)

DISTURBANCE FIELD BECOMES CYL.-SYMM., $\frac{\partial}{\partial z} \rightarrow 0$:

$$\frac{\partial p'}{\partial y}(x, \infty, z) + \cot \beta \cdot \frac{\partial p'}{\partial x}(x, \infty, z) = 0$$

**INVISCID BOUNDARY CONDITIONS
FOR VERTICAL FIN PROBLEM**



INNER VERTICAL ($\beta = \beta_{\text{WEFF}}$)

$$W' = \frac{\partial p'}{\partial \beta} = 0$$

MIDDLE-OUTER DECK MATCHING ($\beta = \delta_0$)

p' and W' (hence $\frac{\partial p'}{\partial \beta}$) continuous

FAR FIELD VERTICAL ($\beta \rightarrow \infty$)

$$p' = 0, \quad X < Y \cot \beta$$

$$= \gamma p_1 M_1^2 \delta / \sqrt{M_1^2 - 1}, \quad X \geq Y \cot \beta$$

(UNIFORM
PRESSURE DISTURB.
DUE TO FIN δ)

(NOTE : ENFORCING THIS CONDITION AUTOMATICALLY
INSURES SATISFACTION OF
 $\partial p' / \partial s \rightarrow 0$ AT $Y \rightarrow \infty$)

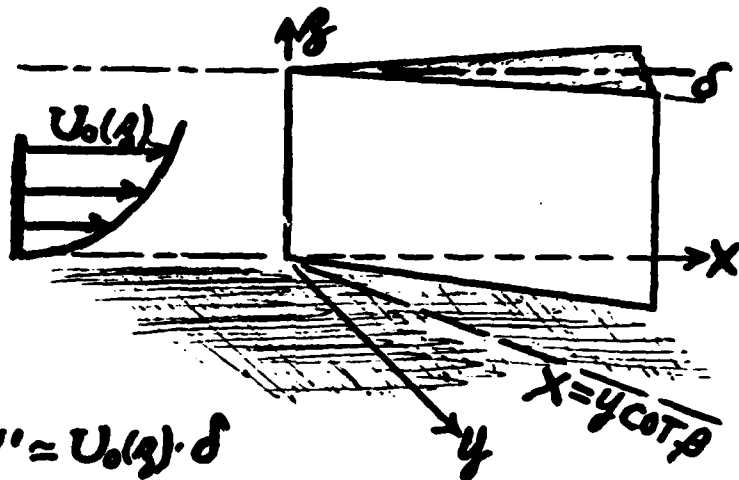
**INVISCID BOUNDARY CONDITIONS
(CONTINUED)**

INNER LATERAL ($y=0$)

$X < 0$ (PLANE OF SYMM.):

$$V' = \frac{\partial V'}{\partial y} = 0$$

$$W' = 0$$



$X \geq 0$ (FIN FACE): $V' = U_0(y) \cdot \delta$

$W' = 0$ (NO SLIP)

$U' = -U_0(y)$ (NO SLIP)

OUTER LATERAL ($y \rightarrow \infty$)

$\partial(\cdot)/\partial s \rightarrow 0$ { AUTOMATIC SINCE DRIVING $p'(x, y)$ SOLUTION HAS THIS PROPERTY

INNER VERTICAL ON FLOOR ($\beta = 0$)

$W' = 0$ (IMPERMEABLE & FLAT)

$V' = U' = \frac{\partial W'}{\partial \beta} = 0$ (NO SLIP & FLAT)

$$\mu \left[\frac{\partial^3 W'}{\partial \beta^3} + \frac{\partial^3 W'}{\partial y^2 \partial \beta} \right] = - \left(\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2} \right) \left\{ \begin{array}{l} \text{"COMPATIBILITY"} \\ \text{EQN} \end{array} \right.$$

OUTER VERTICAL ($\beta \gg \delta_{SL}$)

$U', V', W' \rightarrow U'_{inv}, V'_{inv}, W'_{inv}$ { INVISCID FORM OF X, Y MOM. EQNS }

**VISCOUS DISTURBANCE FLOW
BOUNDARY CONDITIONS IN
INNER DECK**

STEP 1 : FOURIER TRANSFORM W.R.T. X

$$\tilde{P}(K, y, \beta) \equiv \int_{-\infty}^{\infty} e^{-iKX} p'(X, y, \beta) dX$$

YIELDS

$$K^2(M_1^2 - 1)\tilde{P} + \frac{\partial^2 \tilde{P}}{\partial y^2} + \frac{\partial^2 \tilde{P}}{\partial \beta^2} = 0$$

SUBJECT TO

$$\tilde{P}(K, y, \infty) = M_1^2 \delta e^{-iKy \cot \beta / iK \sqrt{M_1^2 - 1}}$$

$$\frac{\partial \tilde{P}}{\partial y}(K, 0, \beta) = -M_1^2 \delta$$

STEP 2 : FOURIER COSINE TRANSFORM W.R.T. y

$$\tilde{\tilde{P}}(K, m, \beta) \equiv \int_0^{\infty} \tilde{P}(K, y, \beta) \cdot \cos my dy$$

YIELDS

$$(K^2 \cot^2 \beta - m^2) \tilde{\tilde{P}} + \frac{\partial^2 \tilde{\tilde{P}}}{\partial \beta^2} = 0$$

SUBJECT TO

$$\tilde{\tilde{P}}(K, m, \infty) = M_1^2 \delta / [(iK)^2 \cot^2 \beta + m^2]$$

whose solution is

$$\tilde{\tilde{P}}(K, m, \beta) = \frac{M_1^2 \delta}{(iK)^2 \cot^2 \beta + m^2} + \tilde{\tilde{P}}(K, m, \beta_0) e^{-\lambda(\beta - \beta_0)}$$

$$\text{with } \lambda \equiv i \sqrt{K^2 \cot^2 \beta - m^2}$$

$$\frac{\partial \tilde{\tilde{P}}}{\partial \beta} + \lambda \tilde{\tilde{P}} = \frac{-i \delta M_1^2}{\sqrt{K^2 \cot^2 \beta - m^2}}$$

**SOLUTION METHOD for OUTER
DECK PRESSURE FIELD**

Attack : FOURIER X-TRANSFORM FOLLOWED BY
FOURIER COSINE TRANSFORM W.R.T. y

THIS YIELDS THE O.D.E. SPLIT BOUNDARY
VALUE PROBLEM

$$\{[M_0^2(\beta)-1]K^2-m^2\}\tilde{\bar{P}} + \frac{d^2\tilde{\bar{P}}}{d\beta^2} - \frac{2}{M_0} \frac{dM_0}{d\beta} \frac{d\tilde{\bar{P}}}{d\beta} =$$

$$= -\underbrace{\delta M_0^2(\beta)}_{\substack{\text{SIDE DEFLECTION} \\ \text{EFFECT OF FIN}}}$$

$$\frac{d\tilde{\bar{P}}}{d\beta}(K, m, \beta_{WEFF}) = 0$$

$$\frac{d\tilde{\bar{P}}}{d\beta}(K, m, d_0) + i\sqrt{K^2 \cot^2 \beta - m^2} \cdot \tilde{\bar{P}}(K, m, d_0) = \underbrace{\frac{-i\delta M_1^2}{\sqrt{K^2 \cot^2 \beta - m^2}}}_{\text{EFFECT OF OVERLYING FIN - INDUCED SHOCK}}$$

SOLUTION CAN BE OBTAINED
BY STANDARD METHODS

MIDDLE DECK SOLUTION

Attack: FOURIER X-TRANSFORM FOLLOWED BY FOURIER
SINE TRANSFORM

$$\tilde{V}_s(k, m, \beta) \equiv \int_0^{\infty} \tilde{V}(k, y, \beta) \sin my \, dy$$

THIS YIELDS THE VISCOUS SIDEWASH PROBLEM

$$\frac{d^2 \tilde{V}_s}{d\beta^2} - \left[\frac{i k U_0(\beta)}{\nu_{ow}} + \underset{\substack{\uparrow \\ \text{"BOUNDARY REGION" TERMS}}}{m^2} \right] \tilde{V}_s = -m \left[\frac{\tilde{P}_w}{\mu} + \frac{\delta U_0(\beta)}{i k} \right]$$

WHERE

OVERLYING
PRESSURE
SPECTRUM
FROM BOTTOM OF MIDDLE DECK

$$\tilde{P}_w = \tilde{P}(k, m, \beta_{\text{EFF}}) \quad \langle \text{COSINE TRANSFORM} \rangle$$

$$\tilde{V}_s(k, m, 0) = 0, \quad \tilde{V}_s(k, m, \beta_{\infty}) = \text{INVISCID SOLUTION}$$

THE COMPANION UP-WASH PROBLEM IS :

$$i k \left[U_0(\beta) \frac{d^2 \tilde{W}_s}{d\beta^2} - \frac{d^2 U_0}{d\beta^2} \tilde{W}_s \right] = \nu_{ow} \frac{d^2}{d\beta^2} \left[\frac{d^2 \tilde{W}_s}{d\beta^2} - m^2 \tilde{W}_s \right]$$

subject to $\tilde{W}_s(0) = \frac{d\tilde{W}_s}{d\beta}(0) = 0$

$$\mu_{ow} \frac{d^3 \tilde{W}_s}{d\beta^3}(0) = -\left(\frac{k^2 + m^2}{m}\right) \frac{d\tilde{P}_w}{dm} - \frac{m}{2\pi} \int_0^{\infty} \tilde{P}_w \, dm$$

SOLUTION OF VISCOUS DISTURBANCE
FLOW IN THE INNER DECK

DATE
FILMED

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