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by

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ON OPTIMAL CONTROL OF A BROWNIAN MOTION*

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ON OPTIMAL CONTROL OF A BROWNIAN MOTION

by

Yu-Chung Liao

Abstract

This report discusses

~~Consider~~ a controlled diffusion process which evolves as a reflected Brownian motion under each control action. A switching cost is incurred when the control action is switched. The control problem turns out to be a sequential decision problem, i.e., to find a sequence of optimal stopping times to switch control. The dynamic programming equation for a discounted cost criterion is a quasi-variational inequality. By allowing the discount factors tend to zero, we show a new Q.V.I. has a solution that serves as a potential function to give direction to attain the optimality for a long-run average cost criterion.

Key words: diffusion, switching cost, quasi-variational inequality, potential function, long-run average cost.

1. Introduction

Optimal control of reflected Brownian motion arises naturally from input-output systems. Faddy [4] models a dam by a Brownian motion with two reflecting barriers. Puterman [9] uses diffusion processes to model production and inventory processes. In both cases they assume the existence of a stationary optimal strategy and start from there. In Rath [10] a Bang-Bang style strategy is proved to be optimal among stationary strategies by using a random walk to approximate Brownian motion. Chernoff and Petkau [2] prove that the optimal conditions are satisfied by certain strategies. All those papers discuss the case of linear holding costs and two control actions.

Here, we consider a controlled diffusion process which evolves as a reflected Brownian motion. A switching cost is incurred when the control action is switched. Since the instants of switches are crucial, the optimal control problem turns out to be a sequential decision problem. We can write the dynamic programming equation for a discounted cost criterion by the principle of dynamic programming in Fleming-Rishel [5]. It is a quasi-variational inequality which can be solved by the penalty method in Bensoussan-Lions [1]. By allowing the discount factors tend to zero, a new quasi-variational inequality arises as the dynamic programming equation for a long-run average cost criterion. We solve it to prove the existence of a stationary optimal strategy.

2. Model

Let $(\Omega, \mathcal{F}, P)$ be a probability space on which a standard Brownian motion W_t is defined. $\{\mathcal{F}_t\}_{t=0}^{\infty}$ is the increasing family of complete σ -fields generated

by W_t . Let S be the set of F_t -stopping times. Let $A = \{1, 2, \dots, M\}$ be the set of control actions. Under control action i , the controlled process evolves as the reflected Brownian motion

$$(1) \quad Rf(x+d_i t+a_i W_t)$$

where Rf is a function on $C[0, \infty)$ defined as

$$Rf(w)(t) = w(t) - \inf\{0; w(s), s \leq t\}$$

for all $w \in C[0, \infty)$. The operating and holding cost is $f(x, i)$ per unit time if the state of the process is x and action i is used. When switching from action i to j a switching cost $C(i, j)$ is incurred. Since infinitely many switches in a finite time interval will make the total cost blow up, we can, without loss of generality, define a strategy $u = \{s(n), u(n)\}_{n=1}^{\infty}$ to be admissible if

- i) $s(n) \in S$ for all n ,
- ii) $0 \leq s(n) < s(n+1)$ for all n
- iii) $s(n) \rightarrow \infty$ as $n \rightarrow \infty$ w.p.1.,
- iv) $u(n): \Omega \rightarrow A$ is $F_{s(n)}$ -measurable and $u(n) \neq u(n+1)$ for all n .

Then given initial state (x, i) and admissible strategy u the control process is

$$u(t) = \begin{cases} i & t < s(1), \\ u(n) & s(n) \leq t < s(n+1) \end{cases}$$

and the controlled process is

$$Rf(x + \int_0^t d u(s) ds + \int_0^t a u(s) dW_s).$$

We assume the following conditions throughout this paper.

- (2) $a_i \neq 0 \quad i \in A,$
- (3) $d_i < 0 \quad i \in A,$
- (4) $f(\cdot, i): R^+ \rightarrow R^+$ is bounded measurable and nondecreasing $i \in A,$
- (5) $C(i, j) > 0$ and $C(i, j) + C(j, l) > C(i, l) \quad i \neq j \text{ and } j \neq l.$

Here, (3) is a stability condition. See Kushner [7] and [8].

3. Preliminary Results.

To use variational inequality techniques for solving sequential decision problems has been studied extensively in [1]. For completeness we briefly discuss some results in a form which is suited for use in the next section.

Let G be an open subset of R^+ , $\mu \geq 0$, $p \geq 1$ and $D = \frac{d}{dx}$. We denote the space of all functions f on G such that

$$\sum_{k=0}^n \|e^{-\mu x} D^k f(x)\|_{L^p(G)} < \infty$$

by $W^{n,p,\mu}(G)$ and $W^{n,p}(G)$ if $\mu = 0$. Since the generalized Itô's formula in [1] holds for the diffusion processes with reflecting boundary in Stroock-Varadhan [13], the following theorem is proved by the penalty method in [1].

We state it without proof.

Let I_Q be the indicator function of the set Q , $\|\cdot\|_G$ be the sup-norm in $L^\infty(G)$ and

$$L^i = \frac{-1}{2} a_i^2 D^2 - d_i D \quad i \in A.$$

Theorem 1. Let $G = [0, B]$, $r > 0$, $f \in L^\infty(G)$, $v \in \mathbb{R}$ and h satisfy

- (a) $h \in W^{2,\infty}(G)$,
- (6) (b) $Dh(0) = 0$,
- (c) $v \leq h(B)$.

Then the variational inequality

- (a) $Z \in W^{2,p}(G) \quad 1 < p < \infty$,
- (b) $L^i Z + rZ - f \leq 0 \quad \text{a.e. on } G$,
- (c) $Z - h \leq 0$,
- (7) (d) $(b) \times (c) = 0$,
- (e) $DZ(0) = 0$
- (f) $Z(B) = v$

has a unique solution for any fixed $i \in A$. In addition we have

$$(8) \quad \|L^i Z\|_G \leq |v| + 2 \|f\|_G - d_i \|Dh\|_G + \frac{1}{2} a_i^2 \|(D^2 h)^+\|_G$$

and

$$(9) \quad Z(x) = \inf_{s \in S} E_x \left\{ \int_0^{s/\tau} e^{-rt} f(x(t)) dt + e^{-rs} I_{\{s < \tau\}} h(x(s)) + e^{-r\tau} I_{\{s \geq \tau\}} v \right\}.$$

Here, $x(t)$ is the process in (1), and $\tau = \inf\{t: x(t) = B\}$.

Corollary 1. Given $h \in W^{1,\infty}(G)$ and satisfy

(10) there is a constant c and a sequence h_n such
that h_n satisfies (6), $D^2 h_n < c$ for all n
and $h_n \rightarrow h$ as $n \rightarrow \infty$ in $W^{1,\infty}(G)$.

Then Theorem 1 holds. In this case, $\|(D^2 h)^+\|_G$ in (8) is replaced by
 $C' = \text{Max}\{0, c\}$.

Proof. Let Z_n be the solution of (7) with respect to h_n . By (8) there is
a subsequence of n , still denoted by n , such that $Z_n \rightarrow Z$ weakly in
 $W^{2,p}(G)$ and strongly in $W^{1,p}(G)$. Hence Z satisfies (7), (8) and (9).

We have the same conclusions as in Corollary 1 when the Neumann boundary
condition (7e) is replaced by the Dirichlet boundary condition.

Corollary 2. Let $G = [B_1, B_2]$ and $B_1 > 0$. Then Corollary 1 is true if (7e)
and (7f) are replaced by

$$(e') \quad Z(B_i) = v_i \quad \text{and} \quad v_i \leq h(B_i) \quad i = 1, 2.$$

In case $r|v_i| \leq \|f\|_G$ for $i = 1, 2$ then $|v|$ in (8) can be removed,
i.e.

$$(11) \quad \|L^1 Z\| \leq 2\|f\|_G - d_1 \|DH\|_G + \frac{1}{2} a_1^2 C'.$$

Another result from Robin [11] is

Theorem 2. Given $f \in L^{\infty}(R^+)$, $r > 0$, h bounded continuous on R^+ . Let $x(t)$

be the process in (1) and

$$(12) \quad Z(x) = \inf_{s \in S} E_x \left\{ \int_0^s e^{-rt} f(x(t)) dt + e^{-rs} h(x(s)) \right\}.$$

Then Z is bounded continuous on R^+ , $Z \leq h$ and

$$Z(x) = E_x \int_0^\tau e^{-rt} f(x(t)) dt + e^{-r\tau} h(x(\tau))$$

where

$$\tau = \inf\{t: Z(x(t)) = h(x(t))\}.$$

The next lemma gives useful estimates.

Lemma 1. If both f and h in Theorem 2 are nondecreasing and non-negative then so is Z .

Proof. Given $x > y$ then

$$Rf(x+d_i t+a_i W_t) \geq Rf(y+d_i t+a_i W_t) \quad \text{w.p.1.}$$

By (12), it is clear that $Z(x) \geq Z(y) \geq 0$.

Following the assumptions of Lemma 1 we have

$$0 \leq Z(x) - Z(y) \leq E_x \left\{ \int_0^\tau e^{-rt} f(x(t)) dt + e^{-r\tau} Z(y) \right\} - Z(y)$$

where

$$\tau = \inf\{t: x(t) = y\}.$$

By Karlin-Taylor [6],

$$E_x \tau = -\frac{1}{d_i}(x-y).$$

Hence

$$0 \leq Z(x) - Z(y) \leq -\frac{1}{d_i}(x-y) \cdot \|f\|_{R^+}$$

and

$$(13) \quad 0 \leq DZ(x) \leq -\frac{1}{d_i} \|f\|_{R^+}.$$

Let

$$V_0^T(x, i) = E_x \int_0^\infty e^{-rt} f(x(t), i) dt$$

and

$$V_m^T(x, i) = \inf_{s \in S} E_x \left\{ \int_0^s e^{-rt} f(x(t), i) dt + e^{-rs} M_i V_{m-1}^T(x(s)) \right\} \quad m \geq 1$$

where

$$M_i V_m^T(x) = \min_{j \neq i} C(i, j) + V_m^T(x, j) \quad m \geq 1.$$

By induction and Lemma 1, $V_m^T(x, i)$ is non-decreasing and non-negative for all i and m . Hence

$$(14) \quad 0 \leq DV_m^T(x, i) \leq -\frac{1}{d_i} \|f(x, i)\|_{R^+}.$$

Now $M_i V_m^T(x)$ is satisfied by (10) when $V_m^T(x, j) \in W^{2,\infty}(R^+)$ for all $j \in A$.

By induction, we can choose boundary condition $v = V_m^r(B)$ and use Corollary 1 to show locally, hence globally, that

- (a) $V_m^r(x, i) \in W^{2, \infty}(R^+)$ $i \in A$ and $m \geq 0$,
- (b) $V_m^r(x, i) - M_i V_{m-1}^r(x) \leq 0$ $i \in A$ and $m \geq 1$,
- (15) (c) $L^i V_m^r(x, i) + r V_m^r(x, i) - f(x, i) \leq 0$ a.e. on R^+ $i \in A$ and $m \geq 0$,
- (d) $(b) \times (c) = 0$,
- (e) $DV_m^r(0, i) = 0$ $i \in A$.

Here (a) comes from (11) and (14) because the upper bound in (11) is actually independent of G . Let

$$V^r(x, i) = \inf_{u \in U} E_{x, i}^u \left\{ \int_0^\infty e^{-rt} f(x(t), u(t)) dt + \sum_{n=1}^\infty e^{-rs(n)} C(u(n-1), u(n)) \right\}$$

where U is the set of all admissible strategies and $C(u(0), u(1)) = C(i, u(1))$.

Theorem 3. $V^r(x, i)$ is the unique solution of the following quasi-variational inequality

- (a) $V^r(x, i) \in W^{2, \infty}(R^+)$ $i \in A$,
- (b) $L^i V^r(x, i) + r V^r(x, i) - f(x, i) \leq 0$ a.e. $i \in A$
- (16) (c) $V^r(x, i) - M_i V^r(x) \leq 0$ $i \in A$,
- (d) $(b) \times (c) = 0$,
- (e) $DV(0, i) = 0$ $i \in A$.

Also $V^r(x,i)$ is non-decreasing for all i and there is a constant K independent of r such that

$$(17) \quad \|DV^r(x,i)\|_{R^+} \leq K$$

and

$$(18) \quad \|D^2V^r(x,i)\|_{R^+} \leq K \quad \text{for all } i \in A.$$

Proof. The same as in Evans-Freidman [3]: $V_m^r(x,i)$ is the optimal cost function to control the process with no more than m switches,

$$(19) \quad \|rV_m^r(x,i)\|_{R^+} \leq \|f(x,i)\|_{R^+} \quad i \in A$$

and

$$(20) \quad V_m^r(x,i) \rightarrow V^r(x,i) \quad \text{as } m \rightarrow \infty$$

uniformly on R^+ . Now all we need is to estimate $D^2V_m^r(y,i)$ for any $y \in R^+$, $i \in A$ and $m \geq 2$. If

$$V_m^r(y,i) < M_i V_{m-1}^r(y),$$

then there is a neighborhood G of y such that

$$(21) \quad L^i V_m^r(x,i) + rV_m^r(x,i) - f(x,i) = 0 \quad \text{a.e. on } G.$$

By (13) and (19), we have

$$(22) \quad \|D^2V_m^r(x,i)\|_G \leq 6a_i^{-2} \|f(x,i)\|_{R^+}.$$

In case

$$V_m^r(y, i) = M_i V_{m-1}^r(y),$$

there is a set $A' \subset A$ such that $i \notin A'$,

$$V_m^r(y, i) = C(i, j) + V_{m-1}^r(y, j) \quad j \in A'$$

and

$$V_m^r(y, i) < C(i, j) + V_{m-1}^r(y, j) \quad j \notin A' \text{ and } i \neq j.$$

By (5), we have

$$V_{m-1}^r(y, j) < M_j V_{m-2}^r(y) \quad j \in A'.$$

So there is a neighborhood G of y on which $V_{m-1}^r(x, j)$ satisfies (21), hence (22), for all $j \in A'$. Thus,

$$(23) \quad V_m^r(x, i) = \inf_{s \in S} E_x \left\{ \int_0^{s \wedge \tau} e^{-rt} f(x(t)) dt + e^{-rs} I_{\{\tau > s\}} h(x(s)) + e^{-r\tau} I_{\{\tau \leq s\}} V_m^r(x(\tau), i) \right\} \text{ on } G$$

where

$$h(x) = \min_{j \in A'} C(i, j) + V_{m-1}^r(x, j) \text{ on } G$$

and

$$\tau = \inf\{t: x(t) \in \partial G \text{ and } x(t) > 0\}.$$

By Corollary 2, (11) and (22), we have

$$(1.24) \quad \|L^i V_m^r(x, i)\|_G \leq 6 \|f(x, i)\|_{R^+}.$$

From (14), (22) and (24) there is a constant K independent of r and m such that

$$\|DV_m^r(x, i)\|_{R^+} \leq K$$

and

$$\|D^2 V_m^r(x, i)\|_{R^+} \leq K.$$

So the theorem is proved by allowing $m \rightarrow \infty$ in (15).

4. Minimum Average Cost Problem.

The total cost to control the process by strategy u up to time T with initial state (x, i) is

$$J(u, x, i, T) = E_{x, i}^u \int_0^T f(x(t), u(t)) dt + \sum_{n=1}^{\infty} I_{\{s(n) \leq T\}} C(u(n-1), u(n)).$$

The long run average cost is

$$(25) \quad \theta(u, x, i) = \liminf_{T \rightarrow \infty} \frac{J(u, x, i, T)}{T}.$$

The related dynamic programming equation to minimize $\theta(u, x, i)$ is solved as Theorem 4. Theorem 5 is a verification theorem that shows that there is a stationary optimal strategy such that the minimum average cost can be attained as a real limit in (25).

By (19) and (20), there are $j \in A$, $\theta \in \mathbb{R}$ and a subsequence of r , still denoted by r , such that

$$V^r(0,j) \leq V^r(0,i) \quad i \in A$$

and

$$rV^r(0,j) \rightarrow \theta \quad \text{as } r \rightarrow 0.$$

By Theorem 3, we have

$$\begin{aligned} 0 &\leq \bar{V}^r(x,i) \equiv V^r(x,i) - V^r(0,j) \\ &\leq C(i,j) + V^r(x,j) - V^r(0,j) \\ &\leq C(i,j) + Kx. \end{aligned}$$

Since $\bar{V}^r(x,i)$ has the same derivatives as $V^r(x,i)$ has, we have

- (a) $\bar{V}^r(x,i) \in W^{2,\infty}(\mathbb{R}^+)$ $i \in A$,
- (b) $L^i \bar{V}^r(x,i) + rV^r(x,i) - f(x,i) \leq 0$ a.e. on $\mathbb{R}^+$ $i \in A$,
- (26) (c) $\bar{V}^r(x,i) - M_i \bar{V}^r(x) \leq 0$ $i \in A$,
- (d) $(b) \times (c) = 0$,
- (e) $D\bar{V}^r(0,i) = 0$ $i \in A$.

By (17), there is a function $V(x,i)$ and a subsequence of r , still denoted by r , such that

$$\bar{V}^r(x,i) \rightarrow V(x,i)$$

and

$$rV^r(x,i) \rightarrow \theta$$

uniformly on compact subsets of $\mathbb{R}^+$ as $r \rightarrow 0$ for all i .

Theorem 4. $V(x,i)$ satisfies

- (a) $V(x,i) \in W^{2,p,\mu}(R^+)$ $p > 1, \mu > 0,$
- (b) $L^i V(x,i) + \theta - f(x,i) \leq 0$ a.e. on R^+ $i \in A,$
- (c) $V(x,i) - M_i V(x) \leq 0,$
- (d) $(b) \times (c) = 0,$
- (e) $DV(0,i) = 0$ $i \in A$

and

$$(27) \quad 0 \leq V(x,i) \leq C(i,j) + Kx.$$

Proof. Let $r \rightarrow 0$ in (26).

Let g be a twice continuous differentiable function on R^+ such that

$$\begin{aligned} g(x) &\geq 0 \text{ on } R^+, \\ Dg(0) &= 0, \\ g(x) &= e^{\alpha x} \quad x \geq B \end{aligned}$$

and

$$g(x) + |Dg(x)| + |D^2g(x)| < K' \quad x \leq B$$

for some constants B and K' .

Lemma 1.2. For any u, x and i , $E_{x,i}^u x(T)$ is a bounded function of T .

Proof. By (3), there is an $\alpha > 0$ and $\beta < 0$ such that

$$\frac{1}{2} \alpha^2 + d_i \alpha < \beta \quad i \in A.$$

Let $u = \{s(n), u(n)\}_{n=1}^\infty$ with $s(m) = \infty$ for some $m > 0$. By Stroock-Varadhan [21],

$$\begin{aligned}
 h(T) &\equiv E_{x,i}^u g(x(T)) \\
 &= g(x) + E_{x,i}^u \left\{ \sum_{n=1}^m \int_{T \wedge s(n-1)}^{T \wedge s(n)} -L^{u(t)} g(x(t)) dt \right\} \\
 &= g(x) + E_{x,i}^u \int_0^T -L^{u(t)} g(x(t)) dt
 \end{aligned}$$

is finite for all T . So

$$\begin{aligned}
 dh(T) &= E_{x,i}^u - L^{u(T)} g(x(T)) \\
 &= E_{x,i}^u \left\{ I_{\{x(T) > B\}} \left(\frac{1}{2} a_{u(T)}^2 \alpha^2 + d_{u(T)} \alpha \right) e^{\alpha x(T)} \right. \\
 &\quad \left. + I_{\{x(T) \leq B\}} - L^{u(T)} g(x(t)) \right\}.
 \end{aligned}$$

Hence $Dh(T) < 0$ if

$$h(T) > K'' > K' - \frac{1}{\beta} K' \left(\frac{1}{2} a_i^2 - d_i \right) \quad i \in A$$

for some constant K'' . Thus

$$h(T) \leq K'' + g(x) \quad T \geq 0.$$

For any admissible strategy u , we have

$$\begin{aligned}
 E_{x,i}^u g(x(T)) &= \lim_{m \rightarrow \infty} E_{x,i}^u I_{\{s(m) \leq T\}} g(x(T)) \\
 &\leq K'' + g(x).
 \end{aligned}$$

This proves the lemma.

Let $u^* = \{s(n), u(n)\}_{n=1}^{\infty}$ be defined by

$$s(1) = \inf\{t \geq 0: V(x(t), i) = M_i V(x(t))\},$$

$$u(1) = \min\{j \in A: V(x(s(1)), i) = C(i, j) + V(x(s(1)), j)\}$$

and

$$s(n) = \inf\{t > s(n-1): V(x(t), u(n-1)) = M_{u(n-1)} V(x(t))\},$$

$$u(n) = \min\{j \in A: V(x(s(n)), u(n-1)) = C(u(n-1), j) + V(x(s(n)), j)\}$$

for $n > 1$.

Theorem 5. $\theta(u^*, x, i) = \theta \leq \theta(u, x, i)$ for any admissible strategy u .

Proof. By Theorem 4,

$$\begin{aligned} & E_{x,i}^{u^*} \{V(x(T), u(T)) - V(x, i)\} \\ (27) \quad &= E_{x,i}^{u^*} \sum_{n=1}^{\infty} \{V(x(T \wedge s(n)), u(T \wedge s(n))) - V(x(T \wedge s(n-1)), u(T \wedge s(n-1)))\} \\ &= E_{x,i}^{u^*} \sum_{n=1}^{\infty} \left\{ \int_{T \wedge s(n-1)}^{T \wedge s(n)} L^{u(n-1)} V(x(t), u(n-1)) dt - I_{\{s(n) \leq T\}} C(u(n-1), u(n)) \right\}, \end{aligned}$$

where $u(T \wedge s(0)) = i$. Hence

$$(28) \quad \theta = \frac{J(u^*, x, i, T)}{T} - \frac{\{V(x, i) - E_{x,i}^{u^*} V(x(T), u(T))\}}{T}$$

and then

$$(29) \quad \theta = \theta(u^*, x, i)$$

by Lemma 2. To prove $\theta \leq \theta(u, x, i)$ for any u , we simply have inequality at (27), (28) and (29).

Remark. u^* is a stationary strategy.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Consider a controlled diffusion process which evolves as a reflected Brownian motion under each control action. A switching cost is incurred when the control action is switched. The control problem turns out to be a sequential decision problem, i.e., to find a sequence of optimal stopping times to switch control. The dynamic programming equation for a discounted cost criterion is a quasi- variational inequality. By allowing the discount factors tend to zero, the author shows a new W.V.I. has a solution that serves as a potential function to give direction to attain the optimality for a long-run average cost criterion.		

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