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AN APPLICATION OF INVARIANCE
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FOR NT-33 AIRCRAFT WITH
VARIABLE COEFFICIENTS AND DELAYS*

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
A method is presented for analyzing Pilot-induced oscillations (PIO) for the NT-33 closed-loop pilot model when retardations and coefficients are not constant. The variation of retardations and coefficients results from the effect of wind shear and the neuro-muscular dynamics of the pilot reported in available data. Nonlinearities in the model are also considered. The method is based on the use of a new description of such systems in terms of convolution equations. Spectral factorization is applied to the entire functions of exponential order. The result is a criterion for the PIO-system with (CONT.)

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ITEM #20, CONTINUED: variable coefficients and variable delays. The criterion assumes continuity and boundedness of the coefficients and delays. A Lyapunov functional is constructed which gives a criterion on the roots of a certain 'quasi-polynomial', i.e., a polynomial in a variable and the exponential of that variable. The largest domain of attraction is obtained from the Invariance Principle.

ABSTRACT

A method is presented for analyzing Pilot-induced oscillations
(PIO) for the NT-33 closed-loop pilot model when retardations and
coefficients are not constant. The variation of retardations and
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certain "quasi-polynomial," i.e., a polynomial in a variable and the
exponential of that variable. The largest domain of attraction is
obtained from the Invariance Principle.

I. INTRODUCTION

The Air Force Flight Dynamics Laboratory (AFWAL/FI) has been conducting research on the effects of control system dynamics on longitudinal flying qualities during fighter approach and landing. A sharp degradation in flying qualities takes place during this critical phase of the landing task. Severe pilot-induced oscillations during the flare have been reported. The objective in the program has been to investigate pilot-induced oscillations (PIO) of the NT-33 aircraft due to significant control system lags, to effects of wind shear and to pilot delays. Advance digital control schemes add much greater flexibility and logic capabilities when compared to analog systems. However, this increase in complexity of future aircraft flight control systems may also add larger control system lags. It has been observed that large control system lags, high pilot gains, pilot-lag due to neuromuscular dynamics and aerodynamic transport lag are all possible causes of pilot-induced oscillation problems. These phenomena all require careful theoretical analysis.

It should be stressed that the use of digital control system is now a reality and its effects on flying qualities of these fighter aircrafts need careful analysis. The variable stability NT-33 is capable of producing a wide range of aircraft and control system characteristics. The main reason for selecting the NT-33 aircraft was to test the flying qualities of simulated YF-12 and YF-17 aircraft. The simulation of the YF-17 with the NT-33 aircraft has encountered serious PIO difficulties in flare whereas no such problems have been reported for YF-17 [1]. Some detailed studies of PIO during the NT-33 aircraft simulation can be found in earlier works of USAF/Calspan [2]. Calspan diagnosed the PIO-problem as excessive control lags. They modified the

simulated control system dynamics to reduce the lag contribution to
longitudinal dynamics and found that it reduced the problem. The effects
of significant control dynamics on fighter approach and landing longitudinal
flying qualities were also investigated in flight using the USAF/Calspan
NT-33 aircraft [3]. Pilot-induced oscillations occurred during the
landing task. The flight tests reported in [3] provide a data base for
the development of suitable flying qualities requirements which are
applicable to aircraft with significant control system dynamics.

The properties of solutions of linear differential equations of
the retarded type with constant coefficients and constant time-delays
for the pilot model has been considered by several authors. However, the
formulation has not been considered when the coefficients and retardations
in the closed-loop pilot model are variable. Such formulation may now
be justified when the effect of wind shear and the neuromuscular system
dynamics are included. This extension of the analysis is suggested by
the recent measurements that have been cited. A generalized closed-
loop nonlinear pilot model for NT-33 aircraft, with variable retardations
and coefficients is considered herein. The theoretical analysis is
developed in the time domain to analyze the pilot-induced oscillations
problem in the most general format.

II. OBJECTIVES

The structure of the research is as follows. First, a formulation
of the closed-loop NT-33 pilot model is introduced. The NT-33 air-frame
dynamic equations, linearized about the trim conditions and representing
the manual flare and landing of the aircraft, have been used. The pilot
dynamics are assumed to have variable gain and variable retardation,
possibly due to wind shear and the neuromuscular effects. It is
assumed that the pilot forms the closed-loop, thus changing the overall
characteristics of the system.

After having introduced the required formulation of the closed-loop NT-33A pilot model, certain theorems dealing with the spectral factorization of entire functions of the exponential order were used to generate Lyapunov functionals. The reference source for this material is the English translation of the book by Levine [4], which gives a comprehensive treatment of the properties of the zeros of the entire functions and related topics. The spectral factorization theorems play a central role in constructing the Lyapunov functionals. Spectral factorization is emphasized for the role that these equations play in generating Lyapunov functionals for a class of system that represents PIO-systems, rather than on the mathematical proofs. Convolution equations involving distributions which satisfy assumptions made by Hale and Meyer [5] for the functional equations of the delay type are used to describe the dynamical systems in analyzing the PIO system.

III. THE PILOT MODEL

Figure 1 represents a nonlinear pilot model. The NT-33 airframe dynamic equations linearized about the trim condition representing the manual flare and landing of the aircraft have been used. These equations are the same as those given in USAF/CALSPAN [2] and Smith [3], except that only the longitudinal transfer functions have been derived. The longitudinal equations representing the open-loop aircraft dynamics about the trim conditions during the flare and landing maneuvers are represented as

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & -g\cos\theta_o & -w_o \\ Z_u & Z_w & -g\sin\theta_o & u_o \\ M_u & M_w & 0 & M_q \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_{ES}} \\ Z_{\delta_{ES}} \\ M_{\delta_{ES}} \\ 0 \end{bmatrix} \delta_{ES} \qquad (1)$$

where symbols represent as follows: u, w, q, θ are perturbation velocity from trim along x-body axis, perturbation velocity from trim along z-body axis, body axis pitch rate and pitch attitude respectively. δ_{ES} is pitch stick deflection at grip. Also, notationally,

$$X_u = \frac{1}{m}\frac{\partial X}{\partial u} \;,\quad X_w = \frac{1}{m}\frac{\partial X}{\partial w} \;,\quad X_{\delta_{ES}} = \frac{1}{m}\frac{\partial X}{\partial \delta_{ES}}$$

$$Z_u = \frac{1}{m}\frac{\partial Z}{\partial u} \;,\quad Z_w = \frac{1}{m}\frac{\partial Z}{\partial w} \;,\quad Z_{\delta_{ES}} = \frac{1}{m}\frac{\partial Z}{\partial \delta_{ES}}$$

$$M_u = \frac{1}{I_y}\frac{\partial M}{\partial u} \;,\quad M_w = \frac{1}{I_y}\frac{\partial M}{\partial w} \;,\quad M_{\delta_{ES}} = \frac{1}{m}\frac{\partial M}{\partial \delta_{ES}}$$

X_u, X_w, Z_u, Z_w are body axis dimensional x-force derivative and z-force derivatives respectively. M_u, I_x, I_y are aircraft mass, moment of inertia about body x-axis and body y-axis respectively, u_o, w_o, q_o, and θ_o are trim values.

These equations imply that the reference axis are body axis and the wings are always level. For small angles, $u_o \simeq V_T$, the trim true airspeed, and $\alpha_o \simeq W_o/V_T$. The variables u, w (α), θ and δ_{ES} are all incremental values from the reference trim conditions.

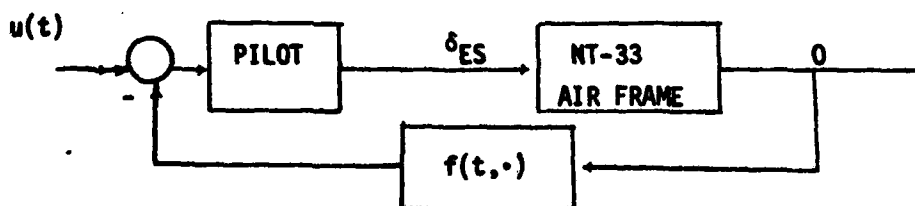
Fig. 1

The transfer function for longitudinal stick input (δ_{ES}) and pitch attitude control output (θ) can be derived from the equations of motion (1) as

$$\frac{\theta}{\delta_{ES}} = \frac{M_{\delta_{ES}} (s + \frac{1}{\tau_{\theta_1}})(s + \frac{1}{\tau_{\theta_2}})}{(s^2 + 2\zeta_{sp} w_{sp} s + w_{sp}^2)(s^2 + 2\zeta_{ph} w_{ph} s + w_{ph}^2)} = \frac{\rho(s)}{\sigma(s)} \quad (2)$$

where $\tau_{\theta 1,2}$ represents airframe lead time constants ζ_{sp}, w_{sp}, and ζ_{ph}, w_{ph} are short period damping ratio, short period undamped natural frequency and phugoid damping ratio, phugoid undamped natural frequency respectively. $\rho(s)$ and $\sigma(s)$ are polynomials in s. The polynomials $\rho(s)$ and $\sigma(s)$ are such that the degree of $\sigma(s)$ is assumed to exceed to that of $\rho(s)$. An inspection of the data base of ref. [3, 10-12] suggests that a reasonable model for pilot dynamics in pitch tracking would be of the form

$$L(s) = \sum_{i=0}^{m} a_i (t) s^i e^{-s\tau(t)} + \sum_{i=0}^{n} b_i (t) s^i \; ; n > m \quad (3)$$

where $a_i (t)$, and $b_i (t)$ are bounded coefficients and $\tau(t)$ is a bounded time-delay, the time-delay $\tau(t)$ is unknown and can be assumed due to neuromuscular effect of the pilot. A small transmission lag may also be present. The pilot dynamics is assumed to have variable gains $a_i (t)$, $b_i (t)$ possibly due to wind shear and the neuromuscular effects. It is assumed that the pilot forms the closed-loop, thus changing the overall characteristics of the system.

A closed-loop analysis can be performed by considering the pilot to be controlling to some desired attitude which minimizes the pitch attitude error e. The non-linearities of the artificial feel system are included in the model, as well as the nonlinearities in the stability augmentation systems.

This completes our formulation of the closed-loop nonlinear model with the pilot in the loop. The dynamics of the NT-33 airframe, the pilot and the nonlinearities of the artificial feel system as well as the nonlinearities of the stability augumentation system have all been defined. In the next section, we give some notations, theoretical backgrounds and our method of analysis.

IV APPLICATION OF INVARIANCE PRINCIPLE

The 'invariance principle' of J. P. LaSalle played an important role in the theory of abstract dynamical systems. Very few practical applications [7,13] of 'invariance principle' have been attempted because of its complexity. Nevertheless, it is also particularly useful in solving practical problems. By applying the invariance principle, we are able to derive stability criteria for the dynamical systems which are optimal in a certain sense. Thus, the stability results can be improved considerable when the study of the system is based on LaSalle's invariance principle [6].

In this section, we first introduce our notations. This is followed by some lemmas on spectral factorization and its application to the invariance principle. Levin [4] has already given comprehensive treatment of the properties of zeros of the entire functions. The spectral factorization of the entire function plays an important role in studying the properties of solutions as we shall see in our analysis of the pilot-induced oscillation problem.

The following notation from Hale [8] will be used in this paper: E^n is complex Euclidean n-space, and for $x \in E^n$, $|x|$ denotes any vector norm. For a given $\tau(t) \geq \bar{\tau} > 0$, C denotes the linear space of continuous functions mapping the internal $[-\bar{\tau}, 0]$ into E^n and for $\phi \in C$, $||\phi||_0 = \sup | \phi(\theta)|$, $-\bar{\tau} \leq \theta \leq 0$. Of course C with this norm is a Banach space. For $H > 0$, C_H denotes the set of ϕ in C for which $|| \phi || < H$, for any continuous function $x(s)$ whose domain is $-\bar{\tau} \leq s \leq \alpha$, $\alpha \geq 0$, and whose range is in E^n, and for any t, $0 \leq t \leq \alpha$ the symbol x_t will denote $x_t (\theta) = x (t + \theta)$, $-\bar{\tau} \leq \theta \leq 0$; that is x_t is in C_H, and is that segment of the function $x(s)$ defined by letting s range in the interval $t - \bar{\tau} \leq s \leq t$.

Let $G(t,\phi)$ be a function defined on $(0,\infty) \times C_H$ into E^n and let $\dot{x}(t)$ denote the right hand derivative of $x(s)$ at $s = t$. The system

$$\dot{x}(t) = G(t, x_t), t \geq 0$$

is called a functional - differential equation (FDE).

<u>Definition</u> - Let $t_0 \geq 0$, and let ϕ be any given function in C_H. A function $x(t_0, \phi)$ (t) is said to be solution of FDE with initial function ϕ at $t = t_0$ if there exists a number $A \geq 0$ such that

(a) for each t, $t_0 \leq t \leq t_0 + A$, x_t (t_0, ϕ) is defined in C_H.

(b) x_{t_0} $(t_0, \phi) = \phi$

(c) $x(t_0, \phi)$ (t) satisfied the functional differential equation (FDE) for $t_0 \leq t \leq t_0 + A$.

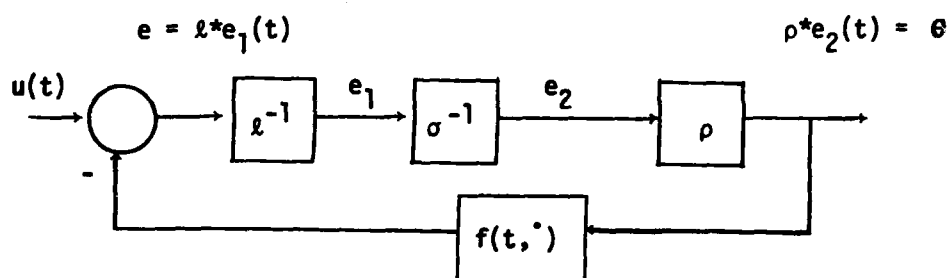To analyze the PIO-problem the original model in fig. 1 is redrawn as

Fig. 2

Note that in the block diagram

$\ell * e_1$ (t)

$$= (\sum_{i=o}^{m} a_i(t) \; \delta_{T(t)}^{i} \; + \; \sum_{i=o}^{n} b_i(t) \; \delta^{i} \;) * e_i(t) \tag{4}$$

Notationally, we have

$$\delta^m * e(t) = e^m (t)$$

and

$$\delta^m_{\tau(t)} * e(t) = e^m(t-\tau(t))$$

where * denotes the convolution operator and δ^m denotes the m-th derivative of Dirac delta function. Throughout we are assuming that the distribution functions are measureable and have compact support on $[0,\tau]$.

Next, we define the convolution equations.

$$e_1(t) = \sigma * e_2(t)$$

$$e(t) = \ell * e_1(t) = \ell * \sigma * e_2(t) \qquad (5)$$

The Laplace transform of the distribution $e_1(t)$ can be represented by a function

$$< e_1, e^{-st} > = \int_{-\infty}^{\infty} e_1(t) \, e^{-st} \, dt$$

It is assumed, however, that the distributions are of the finite order. In other words the distributions throughout have compact support. For details of such distribution functions, we refer to Schwartz [9]. Our objective here is to provide some background material rather than vigorous derivations. Next, we state a Theorem which establishes the properties of solutions $e(t)$. This result will be used later in analysis of the closed-loop pilot model.

<u>Theorem 1.</u> The solutions of the equation

$$e(t) = \ell * \sigma * e_2(t) = 0$$

are exponentially stable provided the transcendental polynomial $\ell(s)\sigma(s)$ satisfy the following conditions:

(i) $\mathrm{Re} \, [\ell(s) \, \sigma(s) \, |_{\tau(t)} = 0] \leq 0$

(ii) $\ell(w) \, \sigma(w) \neq 0$

Proof of Theorem 1. It is sufficient to show that the real parts of the roots of the transcendental polynomial

$$P(s;\tau(t)) = \ell(s) \cdot \sigma(s)$$

$$= \left(\sum_{i=0}^{m} a_i(t) \, s^i \, e^{-\tau(t)s} + \sum_{i=0}^{n} b_i(t)s^i \right)$$

$$\cdot (s^2 + 2 \, \zeta_{sp} \, w_{sp} \, s + w_{sp}^2)(s^2 + 2 \, \zeta_{ph} \, w_{ph} \, s + w_{ph}^2) = 0 \qquad (6)$$

are all negative for $\tau(t) \geq 0$. It is obvious that $\mathrm{Re} \, \sigma(s) < 0$ provided

$0 \leq \zeta_{sp} \leq w_{sp}$ and $0 \leq \zeta_{ph} \leq w_{ph}$. To show that the real part of the transcendental polynomial $\ell(s;\tau(t))$ to be negative, we expand in the form

$$\ell(s;\tau(t)) = s^n + [a_{n-1}(t) e^{-\tau(t)s} + b_{n-1}(t)] s^{n-1}$$

$$+ [a_{n-2}(t) e^{-\tau(t)s} + b_{n-2}(t)] s^{n-2} + \dots$$

$$+ [a_0(t) e^{-\tau(t)s} + b_0(t)] \qquad (7)$$

we have assumed that $m = n-1$. If m is order less than n-1, we can set the coefficients a_{n-1} etc. zero.

$$\ell(s;\tau(t)) = s^n + p_{n-1}(t) s^{n-1} + p_{n-2}(t) s^{n-2} + \dots + p_0(t) \qquad (8)$$

where

$$p_{n-1}(t) = a_{n-1}(t) e^{-\tau(t)s} + b_{n-1}(t)$$

$$p_{n-2}(t) = a_{n-2}(t) e^{-\tau(t)s} + b_{n-2}(t)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$p_0(t) = a_0(t) e^{-\tau(t)s} + b_0(t)$$

Since, $|\exp[-\tau(t)s]| \leq 1$ for all $\tau(t) \geq 0$, hence, when $\tau(t) \geq 0$ and $s \geq 0$, the coefficients $p_i(t)$, $i=1,\dots,n$ are bounded. Let $\bar{p}$ denote a constant such that $\bar{p} = \max |p_i(t)|$ and let $D = \max[1, (n+1)\bar{p}] > 0$. We will now show that $1 \leq i \leq n$ under the assumptions of Theorem 1, all roots of $\ell(s;\tau(t))$ lie in the left half plane. To prove this, we consider two cases: (i) when $|s| \geq D$ and (ii) when $|s| < D$. Now suppose $|s| \geq D$, then

$$|\ell(s;\tau(t))| = |s^n + p_{n-1}(t) s^{n-1} + \dots + p_0(t)|$$

$$\geq |s|^n [1 - \frac{|p_{n-1}(t)|}{|s|} - \dots - \frac{|p_0(t)|}{|s|^n}]$$

$$\geq |s|^n [1 - \frac{n\bar{p}}{(n+1)\bar{p}}] > 0 \qquad (9)$$

The last inequality follows from the fact that $|s| \geq D = (n+1)\bar{p} \geq 1$. Thus, in the domain $|s| \geq D$ and $\text{Re}(s) > 0$ the polynomial $\ell(s;\tau(t))$ possesses no

root for any bounded $\tau(t) \geq 0$. Now suppose that $|s| < D$. From condition (i) of Theorem 1 roots of $\ell(s)\sigma(s)$ are all in the semi-plane Re $(s) < 0$ for $\tau(t) = 0$. Now when $\tau(t) \neq 0$, the only possibility for the characteristic roots to fall within Re $(s) > 0$ is that for $\tau(t) \neq 0$ the variable s runs along the imaginary axis on the s-plane from $-D$ to D. But the condition (ii) does not allow the roots to run along the imaginary axis of s-plane and therefore under our assumptions the characteristic roots must remain within the semi-plane Re $(s) < 0$. This completes the proof.

For the nonlinear model in fig. 2, the description of the system is obtained as

$$\ell * \sigma * e_2 (t) + f (t, p*e_2 (t)) = u(t); \; t \geq 0 \tag{10}$$

To analyze stability of the functional equation (10), we shall construct a Lyapunov functional. The spectral factorization of the entire function plays an important role in the construction of this Lyapunov functional as will be seen later. We depend heavily upon the works of Levin [4] who has shown that spectral factorization may be applied to entire functions of exponential type. We state these results for our convenience.

<u>Lemma 1</u>. In order that an entire function $F(s)$ of exponential type may be of class A it is necessary and sufficient that for some fixed $\lambda > 0$ and for every $R > \lambda$ the following inequality be valid:

$$\lambda \int^R \frac{\ln F (w) F (-w)|}{w^2} \; dw < M_{f,\lambda}$$

where $M_{f,\lambda}$ is a constant.

<u>Lemma 2</u>. For an entire function $F(s)$ of exponential type to have the representation

$$F(w) = \emptyset (w) \, \emptyset (-w)$$

where $\emptyset (w)$ is an entire function of type $\frac{T}{2}$ ($| \tau(t) | \leq T$) with zeros in the half-plane Re(s) ≥ 0, if, and only if $F(s)$ belongs to class A and $F(w) \geq 0$.

We will now introduce our main results to establish asymptotic stability of the nonlinear system (10). Stability of the largest invariant set is then derived from the application of the 'invariance principle'.

5. MAIN RESULT

Consider a Lyapunov functional

$$V (\cdot) = e'_2 q e_2$$

$$= \int_{t(o)}^{t(e_2)} [2 (\ell * \sigma * e_2 (t)) (\rho * e_2 (t))$$

$$- (\phi * e_2 (t))^2] dt \qquad (11)$$

where we have assumed that $\ell(s) \sigma(s)$ and $\rho(s)$ have no common zero. Let the assumptions of lemmas 1 and 2 hold, then

$$F(s) \stackrel{\text{def.}}{=} \ell(s) \sigma(s) \rho(-s) + \ell(-s) \sigma(-s) \rho(s) \qquad (12)$$

has spectral factorization such that

$$F(s) = \phi(s) \phi(-s). \qquad (13)$$

Let $\phi(s)$ be the transform of a distribution of order $\leq$ n and support in $[0,T]$. If the assumption of Theorem 1 holds, then $F(s)|_{s=jw} > 0$. We now show that $V(\cdot)$ is positive. The state $e_2 (t) = e_2(t_0, \psi_0)(t)$ is such that at time $t_1 = t(0)$, $(e_2(t))_t = 0$. And similarly at $t_2 = t(e_2)$, $(e_2)_t = e_2$. Therefore $t(0) = \infty$, and $t(e_2) = 0$. Hence the functional (11) can be written as

$$V(\cdot) = e'_2 (t) q e_2 (t)$$

$$= -\int_{o}^{\infty} [2 (\ell * \sigma * e_2 (t)) (\rho * e_2 (t))$$

$$- (\phi * e_2 (t))^2] dt \qquad (14)$$

For examining the invariance properties of the nonlinear system (15) with $u(t) = 0$,

$$\ell * \sigma * e_2(t) + f(t, \rho * e_2(t)) = 0 \qquad (15)$$

we compare its solution with the linear system

$$\ell * \sigma * \bar{e}_2(t) = 0. \qquad (16)$$

Next, we show that $V(\cdot) > 0$ along the solution of eqn. (16). Obviously

$$V(\cdot) = \int_0^\infty (\phi * e_2(t))^2 \, dt > 0 \qquad (17)$$

provided $\phi * e_2(t) \neq 0$. We of course assume that $\phi * e_2(t)$ is defined. Now, computing $D^+ V_t(\bar{e}_2)$ along the trajectories of eqn. (16) yields:

$$D^+ V_t(\bar{e}_2) = \limsup_{h \to 0^+} \frac{1}{h} [V_{t+h}(\bar{e}_2) - V_t(\bar{e}_2)]$$

$$= \limsup_{h \to 0^+} \frac{1}{h} [\int_{t+h}^\infty (\phi * \bar{e}_2(t))^2 \, dt$$

$$- \int_t^\infty (\phi * \bar{e}_2(t))^2 \, dt]$$

$$= -[\phi * \tilde{e}_2(t)]^2. \qquad (18)$$

Thus the solution of eqn. (16) is decaying and any two solutions $\bar{e}_{2t}(t_0, \psi_0)(t)$, $\bar{e}_{2t}(t_0, \eta_0)$ of (16) satisfy the estimate,

$$|| \bar{e}_{2t}(t_0, \psi_0) - \bar{e}_{2t}(t_0, \eta_0) ||_0 \leq \Lambda(t, \tau) || \psi_0 - \eta_0 ||_0 e^{-\alpha(t-t_0)} \qquad (19)$$

where notationally $||\psi||_0 = \max_{-\tau \leq s \leq 0} ||\psi(s)||$. Now consider functional (17) and compute along the trajectories of (15)

$$D^+ V_t(e_2)_{(15)} = \limsup_{h \to 0^+} \frac{1}{h}[V_{t+h}(e_2) - V_t(e_2)]$$

$$= \limsup_{h \to 0^+} \frac{1}{h}\{[V_{t+h}(e_2(t)) - V_{t+h}(\bar{e}_2(t))]$$

$$+ [V_{t+h}(\bar{e}_2(t)) - V_t(e_2(t))]\}$$

$$\leq \limsup_{h \to 0^+} \frac{1}{h}||[V_{t+h}(e_2(t)) - V_{t+h}(\bar{e}_2(t))]|| + D^+V_t(\bar{e}_2)_{(16)}$$

$$\leq -[\phi * e_2(t)]^2 + C||f(t,\rho * e_2(t))||$$

$$\leq -[\phi * e_2(t)]^2 + Cg(t,||e_2(t)||_0)$$

$$\leq -W(t,e_2(t_0,\psi_0)(t)) \tag{20}$$

We have assumed that the nonlinear system admits unique solution and the nonlinearity is such the $||f(t,\rho * e_2(t))|| \cdot \leq g(t,||e_{2t}(t)||_0)$ where $g(t,u)$ is nondecreasing in u for $t \epsilon R^+$ and W is some non negative function. It is of course, possible to obtain the function W ($\cdot$) if the distributions involved are restricted further. However, a few examples will be presented to illustrate the application of the method and indicate the type of functionals that are obtained. To examine the invariance properties of the nonlinear system, we first note that the solution $e_2(t_0, \psi_0)(t)$ is bounded for all $t \geq t_0$ because of the existence of a Lypunov functional. If the limit point Ω of $e_2(t_0, \psi_0)(t)$ does not exist on the boundary G, G is an open set in C, then by lemma 4.8 in [6], $e_2(t_0, \psi_0)(t)$ is pre-compact. Now define the largest invariant set $\Omega \subset E = \{e_2(t_0,\psi_0)(t) : W(t,e_2(t_0,\psi_0)(t) = 0\}$ then

$$e_2(t_0,\psi_0)(t) \to E \cap V^{-1}(C)$$

for some C and all $t \geq 0$.

For our assertion, we are making use of the Theorem 4.7, P.78 in [6]. Thus, these results are combined into the following theorem.

Theorem 2. If there exists $\phi(s)$ whose distribution is of order $\leq \eta$ and compact support in $[0,T]$ such that $F(s)$ defined in eqn. 12 has the spectral factorization such that

    i) $F(s) = \phi(s) \, \phi(-s)$;

    ii) Conditions of Theorem 1, and lemmas 1 and 2 hold;

    iii) $||f(t, \rho * e_2(t))|| \leq g(t, ||e_{2t}(t)||_0$

        where $g(t,u)$ is non decreasing in u for $t \in R^+$.

Then there exists a Lyapunov functional $V_t(\cdot)$ on G, for system (10) with $u(t) = 0$. Furthermore if

    iv) $\Omega \subset E \overset{\text{def.}}{=} \{e_2(t_0, \psi_0)(t) : W(t, e_2(t_0, \psi_0)(t) \doteq 0 \}$

        where W is some non-negative function then

$$e_2(t_0, \psi_0)(t) \rightarrow E \cap V^{-1}(c)$$

        for some c and $t \geq 0$.

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