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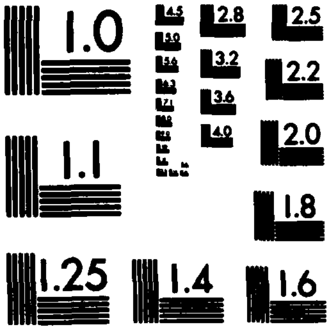

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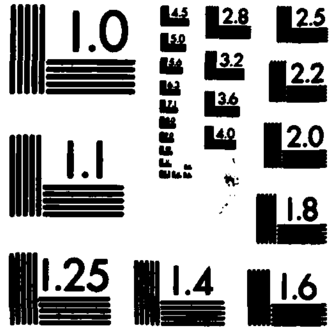

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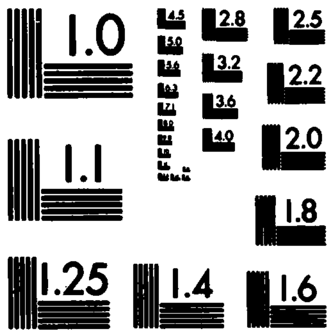

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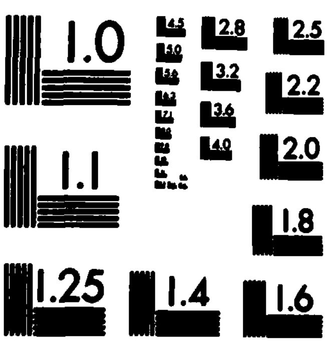

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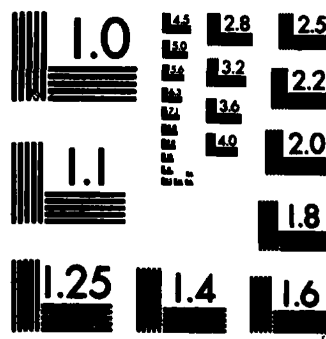

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Chernick (1980) showed that when the parameter ρ in an EARMA (1,1) sequence is equal to one, the distribution of the maximum term converges to a distribution of the general form given by Galambos (1978). This distribution is not one of the three extreme-value types. In this report the asymptotic joint distribution of the maximum and minimum is obtained using the same conditioning argument as in Chernick (1980). The maximum and minimum are asymptotically independent and the minimum behaves as if the sequence were a collection of —

19. KEY WORDS (Continued)

20. ABSTRACT (Continued)

independent and identically distributed random variables even though the maximum does not behave as such. The asymptotic distribution for the range and midrange are also obtained.

CONTENTS

1. INTRODUCTION

The EARMA (1,1) process was introduced by Jacobs and Lewis (Ref. 1). When the parameter $\rho = 1$ the sequence is given by

$$X_n = \beta\varepsilon_n + U_n A_o \qquad \text{for } n > 1,$$

where ε_n's are i.i.d. with density $f(x) = \lambda e^{-\lambda x}$, $x > 0$, $\lambda > 0$, $0 < \beta < 1$ and $\{U_n\}$ is an i.i.d. Bernoulli sequence with $P[U_n = 1] = 1 - \beta$. A_o has density $f(x)$ and is independent of the ε_n's.

Chernick (Ref. 2) showed that for $M_n = \max(X_1, X_2, \ldots, X_n)$

$$\lim_{n\to\infty} P\left[M_n < \frac{\beta(x + \ln(n))}{\lambda}\right] = \int_0^\infty \lambda e^{-\lambda a} \exp(-e^{-x}(\beta + (1-\beta)e^{\lambda a}\beta))\, da$$

This asymptotic distribution denoted by $G_\beta(x)$ is not an extreme-value type but is of the general form given by Galambos (Ref. 3) p. 144. The sequence $\{X_n\}$ is exchangeable.

The conditioning argument given in Chernick (Ref. 2) can be used to obtain the joint asymptotic distribution for the maximum and minimum. This result will be given in the next section. The maximum and minimum are asymptotically independent and the minimum behaves asymptotically similar to the minimum of an i.i.d. sequence of exponential random variables.

Lemma 2.2.1, p. 58, Galambos (Ref. 3) can be applied to show that the range and midrange behave similar to the maximum asymptotically.

2. ASYMPTOTIC RESULTS

When $\beta = 1$ the sequence is i.i.d., and when $\beta = 0$, $X_n = A_o$ for each n. So we will not consider these cases.

<u>Theorem 2.1.</u> Let $\{X_n\}_{n=1}^{\infty}$ be an EARMA (1,1) process with $\rho = 1$ and $1 > \beta > 0$.

Let

$$M_n = \max (X_1, X_2, \ldots, X_n) \text{ and}$$

$$W_n = \min (X_1, X_2, \ldots, X_n).$$

Then

$$\lim_{n\to\infty} P[W_n > \frac{y}{\lambda n}, M_n < \beta\frac{(x + \ln(n))}{\lambda}] = e^{-y} G_{\beta}(x).$$

Proof: Let K_n = number of U_i which are 0. We consider

$$P[W_n > \frac{y}{\lambda n}, M_n < \beta\frac{(x + \ln(n))}{\lambda}] =$$

$$P[\frac{y}{\lambda n} < X_i < \beta\frac{(x + \ln(n))}{\lambda} \text{ for each } i].$$

We note that K_n has a binomial distribution with parameters n and $1 - \beta$. Also given $A_o = a$, $X_i = \beta\epsilon_i$ if $U_i = 0$ and $X_i = \beta\epsilon_i + a$ if $U_i = 1$. Let $u_n = \beta\frac{(x + \ln(n))}{\lambda}$ and $v_n = \frac{y}{\lambda n}$. Conditioning on K_n and A_o as in Chernick (Ref. 2) we have

$$P[v_n < X_i < u_n \text{ for each } i] =$$

$$\int_0^\infty \sum_{k=0}^{n} \binom{n}{k} \beta^k(1 - \beta)^{n-k} P^k\left[\frac{v_n}{\beta} < \epsilon_1 < \frac{u_n}{\beta}\right] P^{n-k}\left[\frac{v_n}{\beta} - \frac{a}{\beta} < \epsilon_1 < \frac{a_n}{\beta} - \frac{a}{\beta}\right]\lambda e^{-\lambda a}\, da$$

For $0 < a < v_n$ $\quad P\left[\frac{v_n}{\beta} - \frac{a}{\beta} < \varepsilon_1 < \frac{u_n}{\beta} - \frac{a}{\beta}\right] = e^{\lambda a/\beta}\left(e^{\frac{-\lambda v_n}{\beta}} - e^{\frac{-\lambda u_n}{\beta}}\right)$,

for $v_n < a < u_n$ $\quad P\left[\frac{v_n}{\beta} - \frac{a}{\beta} < \varepsilon_1 < \frac{u_n}{\beta} - \frac{a}{\beta}\right] = 1 - e^{\lambda a/\beta} e^{\frac{-\lambda u_n}{\beta}}$,

and for $u_n < a$ $\quad P\left[\frac{v_n}{\beta} - \frac{a}{\beta} < \varepsilon_1 < \frac{u_n}{\beta} - \frac{a}{\beta}\right] = 0.$

Equation (1) simplifies to

$$P[v_n < W_n, M_n < u_n] =$$

$$\beta^n \left(e^{-y/(\beta n)} - \frac{e^{-x}}{n}\right)^n \frac{e^{-\beta x}}{n^\beta} + \int_0^{v_n} \left(\beta\left(e^{-y/(\beta n)} - \frac{e^{-x}}{n}\right)\right.$$

$$\left. + (1-\beta)e^{\lambda a/\beta}\left(e^{-y/(\beta n)} - \frac{e^{-x}}{n}\right)\right)^n \lambda e^{-\lambda a}\, da$$

$$+ \int_{v_n}^{u_n} \left(\beta\left(e^{-y/(\beta n)} - \frac{e^{-x}}{n}\right) + (1-\beta)\left(1 - e^{-x}\frac{e^{\lambda a/\beta}}{n}\right)\right)^n \lambda e^{-\lambda a}\, da.$$

The first term in Eq. (2) clearly tends to zero as $n \to \infty$. The second term is bounded by

$$\int_0^{v_n} \lambda e^{-\lambda a}\, da = 1 - e^{-\lambda v_n} = \lambda v_n + o\left(\frac{1}{n}\right)$$

and hence the second term tends to zero also. Now

$$\lim_{n\to\infty} \int_{v_n}^{u_n} \left(\beta\left(e^{-y/(\beta n)} - \frac{e^{-x}}{n}\right) + (1-\beta)\left(1-\frac{e^{-x}e^{\lambda a/\beta}}{n}\right)\right)^n \lambda e^{-\lambda a}\, da$$

$$= \int_0^\infty \lim_{n\to\infty} \left(1 - \frac{y}{n} - \frac{e^{-x}}{n}\left(\beta + (1-\beta)e^{\lambda a/\beta}\right)\right)^n \lambda e^{-\lambda a}\, da$$

$$= \int_0^\infty e^{-y} \exp\left(-e^{-x}(\beta + (1-\beta)e^{\lambda a/\beta}\right)\lambda e^{-\lambda a}\, da$$

$= e^{-y}\, G_\beta(x)$. This completes the proof.

If we let $x\to\infty$ and then let $n\to\infty$ we see that $\lim_{n\to\infty} P\left[W_n > \frac{y}{\lambda n}\right] = e^{-y}$ and because $\lim_{n\to\infty} P\left[M_n < \frac{\beta(x + \ln(n))}{\lambda}\right] = G_\beta(x)$, W_n and M_n are asymptotically independent. If $\hat{W}_n$ represents the minimum of n i.i.d. exponential random variables with density $f(x) = \lambda e^{-\lambda x}$, then $P\left[\hat{W}_n > \frac{y}{\lambda n}\right] = e^{-y}$ for each n. So asymptotically, W_n behaves similar to $\hat{W}_n$. On the other hand, if $\hat{M}_n$ is the maximum of n i.i.d. exponential variables $P\left[\hat{M}_n < \frac{x + \ln n}{\lambda}\right] \to \exp(-e^{-x})$ as $n\to\infty$. So M_n does not behave at all similar to $\hat{M}_n$, and in fact the norming constants are different.

Let $R_n = M_n - W_n$ and $T_n = (M_n + W_n)/2$. R_n is called the range and T_n is the midrange.

<u>Theorem 2.2.</u> For the EARMA (1,1) process with $\rho = 1$ and $1 > \beta > 0$ $\lim_{n\to\infty} P\left[R_n < \beta\frac{(x + \ln(n))}{\lambda}\right] = G_\beta(x)$, (3) and $\lim_{n\to\infty} P\left[T_n < \frac{\beta}{2}\frac{(x + \ln(n))}{\lambda}\right] = G_\beta(x)$. (4)

Proof. Because $R_n = M_n - W_n$ and $2\ T_n = M_n + W_n$ and for every $\delta > 0$

$$\lim_{n\to\infty} P(W_n > \delta) = 0$$

direct application of Lemma 2.2.1, Galambos (Ref. 3) yields Eqs. (3) and (4).

REFERENCES

1. P. A. Jacobs and P. A. W. Lewis, "A Mixed Autoregressive Moving Average Exponential Sequence and Point Process EARMA (1,1)," Advances in Applied Probability 9, pp. 87-104, (1977).

2. M. R. Chernick, "A Limit Theorem for the Maximum Term in a Particular EARMA (11) Sequence," Journal of Applied Probability 17, pp. 869-873, (1980).

3. J. Galambos, The Asymptotic Theory of Extreme Order Statistics, John Wiley and Sons, (1978).

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