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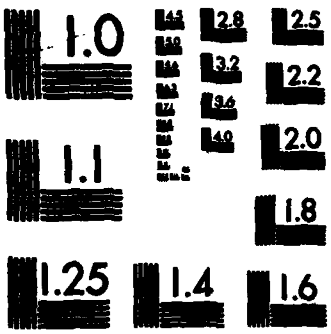

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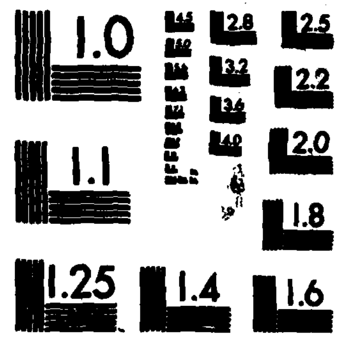

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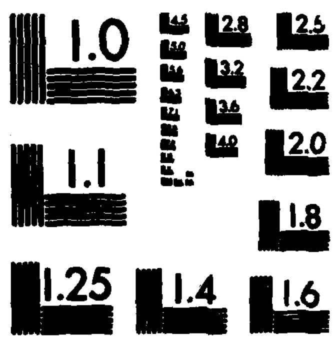

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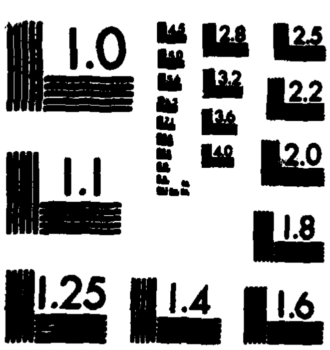

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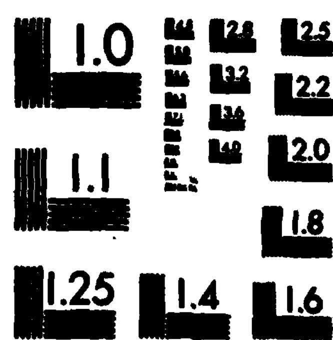

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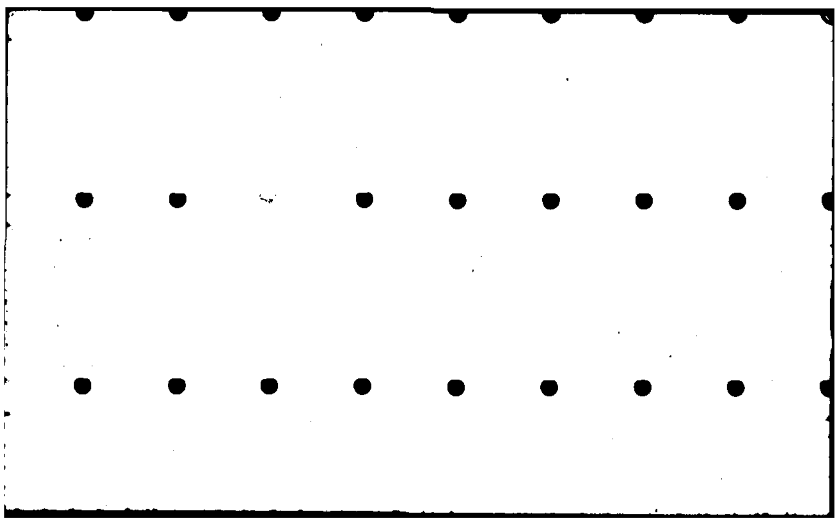

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A GLOBALLY STABLE ADAPTIVE CONTROLLER

FOR MULTIVARIABLE SYSTEMS

R. P. Singh and K. S. Narendra

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A Globally Stable Adaptive Controller

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Introduction

Soon after the problem of adaptively controlling a single-input single-output (SISO) system in a stable fashion was resolved in 1979 [1-3], interest shifted to related theoretical problems. One of the principal questions currently being investigated is the possibility of extending these results to multivariable systems [4-10].

The adaptive control problem can be broadly divided into two parts - an algebraic part dealing with a specific parametrization of the plant and an analytic part dealing with the adaptive laws and the resulting problems of convergence. For stable adaptive control of SISO plants, certain assumptions regarding the plant transfer function have to be made. In particular it is assumed that the designer has the knowledge of

(i) the relative degree n^* of the plant transfer function,

(ii) the sign of the high frequency gain k_p,

and (iii) an upper bound n on the order of the plant transfer function,

and that

(iv) the zeros of the plant transfer function lie in the open left half plane.

Of these, conditions (i) and (ii) are quite restrictive. As might be expected, the corresponding conditions for the multivariable systems are considerably more stringent. The principal aims of this paper are:

i) to elucidate these conditions and to discuss the considerations which arise in the design of globally stable adaptive controllers for mxm multi-input multi-output (MIMO) systems,

and ii) to examine the nature of the prior information needed for a complete solution of the adaptive control problem for 2x2 systems.

In section IVa it is shown that the knowledge of the relative degree of an SISO system generalizes to the knowledge of the Hermite normal form of the plant transfer matrix. The Hermite form plays a central role in the choice of a reference model in the multivariable case. In section Vb the definiteness of a gain matrix associated with the plant transfer matrix is shown to correspond to the multivariable version of condition (ii) and is needed to generate stable adaptive laws. Hence, the feasibility of using adaptive control for the MIMO case in practical situations hinges strongly on the availability of prior information needed to satisfy the above conditions.

Early attempts to extend SISO results to the MIMO case were made by Monopoli and Hsing [4] for continuous time systems and by Borison [5] and Goodwin et al. [6] for discrete time systems. All of them tacitly assumed that the plant transfer matrix can be diagonalized. More recently, Monopoli and Subbarao [10] have considered a special class of such 2x2 systems for practical applications. In [7] Elliott and Wolovich introduced the concept of the interactor [15] and later Goodwin and Long used this concept to generalize the results in [8]. Independently of this work, recently, Morse [9] discussed the importance of the Hermite normal form of a transfer matrix defined over the principal ideal domain (PID) of proper rational functions [11] in the context of general MIMO adaptive control. In [9] it is stated that the Hermite form and the interactor contain equivalent information.

As mentioned earlier, the Hermite form (or equivalently the interactor) of the plant transfer matrix and the sign definiteness of the high frequency gain matrix have to be known apriori before a stable adaptive controller can be designed. In general, as discussed in section IVa, the Hermite form has a triangular structure. In the approach used here, it is assumed that the relative degree of each element of the plant transfer matrix is known. This information is adequate to determine whether the Hermite form is diagonal or triangular, and whether, in the latter case, it can be made generically diagonal using a known prefilter. When the Hermite form is triangular, its off-diagonal elements, in general, depend upon the unknown plant parameters and hence can not be specified apriori. Therefore, a sufficient condition for adaptive control to be practically feasible is that the Hermite form be diagonal. Even when this condition is satisfied, the high frequency gain matrix K_p can be either diagonal or non-diagonal. In the former case only the sign of each diagonal element needs to be known for generating stable adaptive laws. When K_p is not diagonal, the additional prior information regarding the definiteness of its symmetric part must be available for the adaptive control of the multivariable plant. These different cases are illustrated by considering 2x2 systems in detail. It is shown there that all stably invertible 2x2 plants can be generically adaptively controlled subject to the definiteness of the gain matrix.

Section II states the problem of multivariable adaptive control in a general setting. Section III contains four important lemmas from the adaptive control and multivariable literature. These are used extensively in the subsequent two sections in setting up the reference model and realizing the controller in the feedback form and in proving the stability of the overall system for different types of 2x2 plants in section V.

II. Statement of the Problem

An mxm multi-input multi-output linear time invariant plant P is completely represented by the m-input m-output vector pairs $\{u(\cdot), y_p(\cdot)\}$. It is assumed that P can be modeled by a rational transfer matrix

$$W_p(s) = Z_p(s)R_p^{-1}(s) \tag{1}$$

with $\{Z_p(s)$ and $R_p(s)\}$ right coprime polynomial matrices, both of dimension mxm and $R_p(s)$ column-proper (i.e., the constant matrix $[R_p]_h$ formed by the coefficients of the highest powers in each column of $R_p(s)$ is nonsingular). Further, $W_p(s)$ is of full rank and is strictly proper. The zeros of the plant transfer matrix, given by the roots of the polynomial det $[Z_p(s)]$, lie in the open left half plane, while the plant poles may be unstable. The parameters of $W_p(s)$ are assumed to be unknown.

A reference model represents the behavior expected from the plant when the latter is augmented with a suitable differentiator free controller (a cascade controller in combination with linear state feedback). The model is linear time-invariant and has a piecewise continuous and uniformly bounded reference input vector $r(\cdot)$ and output vector $y_m(\cdot)$. The transfer matrix, denoted by $W_m(s)$, is strictly proper and stable.

The error between the plant and the model outputs is defined as

$$e_1(t) \triangleq y_p(t) - y_m(t). \tag{2}$$

The adaptive control problem is to determine a suitable control vector $u(\cdot)$ such that

$$\lim_{t\to\infty} \|e_1(t)\| = \lim_{t\to\infty} \|y_p(t) - y_m(t)\| = 0 \tag{3}$$

As in the scalar case, the solution to the above problem can be divided into two parts - an algebraic part and an analytic part. The algebraic part is concerned with the equivalence class of reference models which can be used as well as the structure of the adaptive controller whose parameters are to be adjusted. The existence of a solution then corresponds to the existence of a constant controller parameter matrix such that condition (3) is satisfied for any arbitrary input $r(\cdot)$. The structure of the controller also determines the uniqueness (or nonuniqueness) of the solution.

Once the existence of a solution is established, the analytic part deals with adaptive schemes for updating the unknown control parameter matrix, so that error $e_1(t)$ evolves asymptotically to zero.

III. Mathematical Preliminaries

The algebraic and analytic aspects of the adaptive control problem are discussed in the following two sections and use well known results in linear multivariable theory and stability theory of dynamical systems. These results are presented here as four principal lemmas and their relevance to the multivariable adaptive control problem is briefly discussed. Lemma 1 (Bezout Identity for polynomial matrices) is proved here following the proof given in [1] for scalar polynomials. Proofs of lemmas 2 and 3 can be found in standard texts on multivariable systems [13,14]. A brief outline of the proof of lemma 4, which is the multivariable version of the Lin-Narendra error model [17], is presented here for easy reference.

a) Bezout Identity

Lemma 1: Let $Q(s)$ and $T(s)$ be $m \times m$ right coprime polynomial matrices with each column degree of $T(s)$ strictly less than the corresponding column degree d_j of $Q(s)$, with $Q(s)$ column proper (i.e., $T(s)Q^{-1}(s)$ is a strictly proper transfer matrix). Then $m \times m$ polynomial matrices $P(s)$ and $R(s)$, each having highest degree $(\nu-1)$, exist such that $P(s)Q(s) + R(s)T(s)$ can be made equal to any arbitrary mxm polynomial matrix $M(s)$ with each column degree less than or equal to $(d_j + \nu-1)$, where ν is the observability index of the minimal transfer matrix $T(s)Q^{-1}(s)$.

Proof: Since $Q(s)$ and $T(s)$ are right coprime polynomial matrices, there exist polynomial matrices $A(s)$ and $B(s)$ [12] such that

$$A(s)Q(s) + B(s)T(s) = I.$$

Let $M(s)$ be an mxm arbitrary polynomial matrix with column degree $\leq (d_j + \nu-1)$. Then

$$M(s)A(s)Q(s) + M(s)B(s)T(s) = M(s). \tag{4}$$

The right coprime factorization $T(s)Q^{-1}(s)$ can also be expressed by a left coprime factorization $E^{-1}(s)F(s)$ with $E(s)$ row proper and each row degree of $F(s)$ strictly less than the corresponding row degree of $E(s)$. The highest degree of $E(s)$ is ν, the observability index of the minimal transfer matrix [14]. Hence,

$$T(s)Q^{-1}(s) = E^{-1}(s)F(s) \tag{5}$$

(4) and (5) can be represented as a composite matrix equation

$$\begin{bmatrix} M(s)A(s) & M(s)B(s) \\ F(s) & -E(s) \end{bmatrix} \begin{bmatrix} Q(s) \\ T(s) \end{bmatrix} = \begin{bmatrix} M(s) \\ 0 \end{bmatrix}.$$

By elementary column operations, the above matrix equation can be reduced to

$$\begin{bmatrix} P(s) & R(s) \\ F(s) & -E(s) \end{bmatrix} \begin{bmatrix} Q(s) \\ T(s) \end{bmatrix} = \begin{bmatrix} M(s) \\ 0 \end{bmatrix}$$

such that every column degree of $R(s)$ is strictly less than the corresponding column degree of $E(s)$. Since the highest degree of $E(s)$ is ν, the highest degree of $R(s)$ can be at most $(\nu-1)$. Further, since $Q(s)$ is column proper and each column of $M(s)$ has degree less than or equal to $(d_j + \nu - 1)$, the polynomial matrix $P(s)$ can have degree at most $(\nu-1)$.

This lemma is used to establish the existence of a controller structure so that the transfer matrix of the plant together with the controller is identical to that of the model.

b) Decoupling by State Feedback

Let $G(s)$ be a nonsingular mxm strictly proper transfer matrix. Let d_i denote the minimum relative degree in the ith row of $G(s)$, i.e., $d_i \triangleq$ min (degree difference in s of the denominator and the numerator of each entry of the ith row of $G(s)$) - 1. Let (1xm) constant row vector E_i be defined as

$$E_i \triangleq \lim_{s\to\infty} s^{d_i+1} G(s).$$

It is known [13] that d_i and E_i are invariant under linear state feedback.

Lemma 2: Let $G(s)$ and E_i, $i = 1,\ldots,m$ be defined as above. $G(s)$ can be decoupled by linear state feedback if and only if the constant matrix

$$E = \begin{bmatrix} E_1 \\ E_2 \\ \cdot \\ \cdot \\ \cdot \\ E_m \end{bmatrix} \qquad (6)$$

is nonsingular.

The entries in the matrix E are the high frequency scalar gains associated with scalar transfer functions of minimum relative degree in each row of the transfer matrix. This lemma is used in section Va to specify the model from a knowledge of the relative degree of each entry in the plant transfer matrix.

c) Number of Inputs and Outputs

Lemma 3: A transfer matrix $G(s)$ is output function controllable if and only if it has linearly independent rows over the field of rational functions, i.e., the rank of $G(s)$ is equal to the number of outputs.

In model reference adaptive control, for the plant output to follow the model output asymptotically, the plant transfer matrix must be output function controllable. If such a transfer matrix has more columns than rows, then the number of inputs in excess of the number of outputs of the plant can be set to an arbitrary value, and in particular, to zero. This pertains to the columns of zeros in the Hermite normal form [cf. 11] of a rectangular transfer matrix. Alternatively, this is equivalent to selecting inputs corresponding to linearly independent columns of the transfer matrix, which in turn is reflected in the nonzero columns of its Hermite form. Hence, in general, it is sufficient to consider square transfer matrices with the same number of inputs and outputs.

d) Multivariable Error Model Prototype 3

Lemma 4

Given a stable m-input, m-output n-dimensional minimal triple (C,A,B), two symmetric positive definite matrices Γ and Γ_1 and $\omega(t): [0,\infty) \to R^p$ whose elements are piecewise continuous functions, the equilibrium state of the set of differential equations

$$\dot{e}(t) = Ae(t) + B\nu(t)$$

$$e_1(t) = Ce(t)$$

$$\nu(t) = \Phi(t)\omega(t) - \omega^T(t)\Gamma_1\omega(t)e_1(t) \tag{7}$$

$$\dot{\Phi}(t) = -\Gamma e_1(t)\omega^T(t) \tag{8}$$

is stable if the transfer matrix $T(s) = C(sI-A)^{-1}B$ is strictly positive real (SPR).

Proof:

The proof follows directly by using a Lyapunov function candidate

$$V(e,\Phi) \triangleq e^T(t)Pe(t) + \text{trace}\,[\Phi^T(t)\Gamma^{-1}\Phi(t)] \text{ with } P = P^T > 0 \qquad (9)$$

and the matrix version of the Kalman-Yacubovich Lemma [18]. This yields

$$\dot{V}(e,\Phi) = -e^T(t)[GG^T + \varepsilon N]e(t) - 2e_1^T(t)e_1(t)\omega^T(t)\Gamma_1\omega(t) \leq 0 \qquad (10)$$

for some G and $N = N^T > 0$ matrices and $\varepsilon > 0$.

It follows that the error model is uniformly stable and $e(t)$ (hence $e_1(t)$) and $\Phi(t)$ are uniformly bounded for all finite initial conditions. Further, from (8) and (10) it follows that $e(\cdot)$ and $\dot{\Phi}(\cdot)\varepsilon L^2$. If in addition it is assumed that $\omega(t)$ and $\dot{\omega}(t)$ are uniformly bounded, it can be concluded that $\lim_{t\to\infty} \varepsilon(t) = 0$ and $\lim_{t\to\infty} \dot{\Phi}(t) = 0$. However, very little can be said about the convergence of $\Phi(\cdot)$ to a constant matrix.

Remark: If $T(s) = 1$ in the above lemma, the third error model (eqns. 7,8) degenerates into the first error model. The equations describing such a model and the corresponding adaptive equations may be expressed as follows

$$\Phi(t)\omega(t) = e_1(t) \qquad (11)$$

$$\dot{\Phi}(t) = -\frac{\Gamma e_1(t)\omega^T(t)}{1+\omega^T(t)\Gamma_1\omega(t)} \qquad \begin{array}{l}\Gamma = \Gamma^T > 0\\ \Gamma_1 = \Gamma_1^T > 0.\end{array} \qquad (12)$$

Using similar arguments, it can be shown that when $\omega(t)$ and $\dot{\omega}(t)$ are uniformly bounded, $\lim_{t\to\infty} e_1(t) = 0$ and $\lim_{t\to\infty} \dot{\Phi}(t) = 0$.

IV. Structure of the Adaptive System: General Case

a) Hermite Normal Form

In a model reference adaptive system, the output of the plant is required to follow the output of a reference model. The plant together with the controller, whose parameters are adjusted, asymptotically approaches a linear time-invariant system. The transfer matrix of the latter should therefore be identical to that of the reference model for perfect model following, if the class of inputs is sufficiently general. An important question that has to be resolved in the initial stages of design is the choice of the model transfer matrix, i.e., the class of rational transfer matrices from which the model transfer matrix should be selected. It is in this context that the Hermite normal form is found to be important. The principal idea here is that the set of all stable reference models can be generated by the stable Hermite form of the plant transfer matrix.

The following concepts from linear systems theory are found to be relevant for a discussion of the Hermite form.

The set of all proper rational transfer function $\mathbb{R}_p(s)$ is a principal ideal domain (PID). A matrix with elements in $\mathbb{R}_p(s)$ is invertible if and only if its determinant is a unit in $\mathbb{R}_p(s)$. A unimodular matrix is an invertible proper transfer matrix whose inverse is also a proper transfer matrix. If the 'relative degree' (degree of the denominator minus the degree of the numerator) of each rational transfer function is taken as the 'degree' of each element, a division rule can be established, making this a Euclidean domain [16].

Two transfer matrices $T_1(s)$ and $T_2(s)$ over $\mathbb{R}_p(s)$ are said to be dynamically equivalent if and only if there exists a unimodular matrix $C(s)$ over $\mathbb{R}_p(s)$ such that $T_1(s) = T_2(s)C(s)$. The Hermite normal form for nonsingular matrices over PID [12], obtained by performing elementary column operations on the matrix, represents the canonical form within each equivalence class. This together with the rank of the transfer matrix represents the complete set of invariants within each class [11].

The Hermite form of an mxm matrix T(s) over $\mathbb{R}_p(s)$ is a lower triangular mxm rational matrix of the form

$$H(s) = \begin{bmatrix} \frac{1}{\pi^{n_1}} & & & \\ h_{21} & \frac{1}{\pi^{n_2}} & & \\ \vdots & & \ddots & \\ h_{m1} & & & \frac{1}{\pi^{n_{12}}} \end{bmatrix} \tag{13}$$

where $h_{ij}(s) = \{\frac{\delta(s)}{\pi^{n_{ij}}}, n_{ij} < n_i\}$, $\frac{\delta(s)}{\pi^{n_{ij}}}$ is proper and n_i and n_{ij} are positive integers. $\pi(s)$ is any monic polynomial of degree 1. The choice of $\pi(s)$ is immaterial, but once it is chosen the Hermite form H(s) is unique. If $H_1(s)$ is π_1-Hermite form and $H_2(s)$ is π_2-Hermite form, then either can be obtained from the other by elementary column operations. In other words, $H_1(s)$ and $H_2(s)$ are dynamically equivalent. However, if the roots of $\pi(s)$ lie in the open left half plane, H(s) corresponds to a stable Hermite form.

From the foregoing discussion it follows that every plant transfer matrix $W_p(s)$ generates a class $\mathcal{C}$ of dynamically equivalent models. The set of all stable reference models which the plant transfer matrix can follow, denoted by $\mathcal{W}$, is a proper subset of $\mathcal{C}$. Hence, by the preceding considerations, a stable Hermite form of the plant transfer matrix itself can be chosen as a reference model. The entire class $\mathcal{W}$ of the stable reference models can then be generated by postmultiplying the Hermite form by a known and fixed dynamic controller. The importance of the Hermite form $H(s) \varepsilon \mathcal{W}$ lies in the fact that it can be determined a priori directly from reasonable information about the plant transfer matrix $W_p(s)$. In algebraic terms, the Hermite form represents a basis for the free module over $\mathbb{R}_p(s)$ spanned by the columns of the plant transfer matrix. Dynamic equivalence then implies that the free modules spanned by the columns of the plant and model transfer matrices are the same.

For SISO systems the Hermite form is simply $\frac{1}{\pi^{n^*}}$, where π is as defined before and n^* is the relative degree of the transfer function. The unimodular form is just a unit in $\mathbb{R}_p(s)$. The invariance of the relative degree and the realization of the controller in the feedback and the feedforward form are well known in adaptive control literature. In that sense deg[det $H(s)$] represents the multivariable analog of the relative degree (deg[det $R_p(s)$] minus deg[det $Z_p(s)$]) It is shown in the next subsection how the multivariable controller can be realized.

b) Controller Structure

From the discussion in section IVa it follows that the reference model transfer matrix $W_m(s)$ is dynamically equivalent to the plant transfer matrix $W_p(s)$, i.e.,

$$W_p(s)Q(s) = W_m(s) \tag{14}$$

for some unimodular matrix $Q(s)$. If $H(s)$ is the Hermite form in the equivalence class, then

$$W_p(s)Q_p(s) = H(s) = W_m(s)Q_m(s) \tag{15}$$

where $Q_p(s)$ and $Q_m(s)$ are unimodular matrices. From the above two equations it readily follows that

$$W_p(s)Q_p(s)Q_m^{-1}(s) = W_p(s)Q(s) = W_m(s). \tag{16}$$

The basic structure of the adaptive system is shown in Fig. 1. The feedforward controller $Q_m^{-1}(s)$ is fixed and known. The controller $Q_p(s)$ can be realized as shown below.

Let $Q_p(s)$ be factorized as $R_p(s)P_F^{-1}(s)K_0 = R_p(s)(K_0^{-1}P_F(s))^{-1}$ such that

$$\begin{aligned} \partial_{cj}[R_p(s)] &= \partial_{cj}[P_F(s)] \\ [R_p]_h &= [P_F]_h , \end{aligned} \tag{17}$$

and $$\lim_{s\to\infty} Q_p(s) = \lim_{s\to\infty} R_p(s)P_F^{-1}(s)K_0 = K_0. \tag{18}$$

$\partial_{cj}[\cdot]$ denotes the column degree of a polynomial matrix and $[\cdot]_h$ represents the constant matrix formed by taking the coefficients of the highest powers in each column of a matrix. From eqn. (15)

$$[H^{-1}(s)W_p(s)]Q_p(s) = I . \tag{19}$$

Since $Q_p(s)$ is unimodular, $[H^{-1}(s)W_p(s)]$ is also unimodular. This implies

$$\lim_{s\to\infty} [H^{-1}(s)W_p(s)] = \lim_{s\to\infty} [(H^{-1}(s)Z_p(s))(R_p^{-1}(s))] = K_p \tag{20}$$

where K_p is a constant nonsingular matrix, and

$$\partial_{cj}[H^{-1}(s)Z_p(s)] = \partial_{cj}[R_p(s)].$$

K_p is the high frequency gain matrix of the transfer matrix. In view of (17) and (19), let

$$K_0^{-1}P_F(s) = H^{-1}(s)Z_p(s) \tag{21}$$

such that

$$\partial_{cj}[H^{-1}(s)Z_p(s)] = \partial_{cj}[R_p(s)] = \partial_{cj}[P_F(s)] = d_j.$$

From (19)

$$\lim_{s\to\infty} (H^{-1}(s)Z_p(s)R_p^{-1}(s))Q_p(s) = \lim_{s\to\infty} (H^{-1}(s)Z_p(s))R_p^{-1}(s)\cdot\lim_{s\to\infty} Q_p(s)$$
$$= K_p\cdot K_0 = I$$

Hence, $K_0 = K_p^{-1}$.

Then (21) becomes

$$P_F(s) = K_p^{-1}H^{-1}(s)Z_p(s).$$

The overall transfer matrix is given by

$$[Z_p(s)R_p^{-1}(s)][R_p(s)P_F^{-1}(s)K_0] = Z_p(s)[Z_p^{-1}H(s)K_p]K_0 = H(s) \quad (22)$$

The controller $Q_p(s) = R_p(s)P_F^{-1}(s)K_0$ consists of gain matrix K_0 in the forward path and two auxiliary signal generators F_1 and F_2 in the feedback path. F_1 contains a system with transfer matrix $W_1(s) = N^{-1}(s)C(s)$ and F_2 contains a system with transfer matrix $W_2(s) = N^{-1}(s)D(s) + D_0$. For constant values of parameters, the overall transfer matrix from $v(t)$ to $y_p(t)$ is $W(s)$, where

$$W(s) = Z_p(s)R_p^{-1}(s)R_p(s)[(N(s) + C(s))R_p(s) + (D(s) + D_0N(s))Z_p(s)]^{-1}$$

$$\cdot N(s)\cdot K_0 \ . \quad (23)$$

By Lemma 1, given a polynomial matrix $P_F(s)$ of column degree d_j and an arbitrary polynomial matrix $N(s)$ with $\partial_{rj}[N(s)] = \nu-1$ such that $\partial_{cj}[N(s)\cdot P_F(s)] = d_j + \nu-1$, polynomial matrices $C(s), D(s)$ and a constant matrix D_0 of appropriate degrees can be determined from the following Bezout identity.

$$[N(s) + C(s)]R_p(s) + [D(s) + D_0N(s)]Z_p(s) = N(s)P_F(s).$$

ν is the observability index of the plant transfer matrix which is assumed to be known.*

The arbitrary mxm polynomial matrix $N(s)$ is chosen such that it is row proper and it commutes with both $C(s)$ and $D(s)$. Hence, $N(s)$ is chosen as diagonal $[n(s)]$ where $n(s)$ is an arbitrary monic Hurwitz polynomial of degree $(\nu-1)$.

F_1 generates a set of auxiliary signals

$$\omega_i^T(t) = \frac{1}{n(s)}[s^{\nu-2}u^T(t),\ldots,s^0u^T(t)] \qquad i = 2,\ldots,\nu \quad (24)$$

and contains $(\nu-1)$ mxm parameter matrices $C_i (i = 1,\ldots,\nu-1)$. F_2 generates a set

* An upper bound on ν is sufficient, but then the above equation may not have a unique solution.

of auxiliary signals

$$\omega_i^T(t) = \frac{1}{n(s)}[s^{\nu-1}y_p^T(t),\ldots,s^0 y_p(t)] \qquad i = \nu+1,\ldots,2\nu \tag{25}$$

and contains ν $m\times m$ parameter matrices $D_i(i = 0,1,\ldots,\nu-1)$. Together with $\omega_1(t) = v(t)$, output of the feed forward controller, and K_0 these constitute 2ν m-vector signals and $2\nu(m\times m)$ matrices of adjustable parameters of the controller denoted by the elements of a parameter matrix

$$\theta(t) \triangleq [K_0(t);C_1(t),\ldots,C_{\nu-1}(t);D_0(t),D_1(t),\ldots,D_{\nu-1}(t)] \tag{26}$$

The control input $u(t)$ to the plant is given by $\theta(t)\omega(t) - e_1(t)\omega^T(t)\Gamma_1\omega(t)$ where $\Gamma_1 = \Gamma_1^T > 0$. For $\theta(t) = \theta^*$, the desired feedback controller parameter matrix, the plant together with the feedback controller yields its Hermite form. The parameter error matrix $\phi(t)$ is defined as

$$\phi(t) \triangleq \theta(t) - \theta^*$$

and this gives rise to the error transfer matrix.

c) <u>Error Equation</u>

The error transfer matrix representing the error equation between the plant and its Hermite form is given by

$$W_e(s) = H(s)K_p \tag{27}$$

Hence, the error model can be represented as shown in Fig. 2 with $\phi(t)\omega(t) - e_1(t)\omega^T(t)\Gamma_1\omega(t)$ as the input to a system with transfer matrix $W_e(s)$. It is worth noting that the signals $\omega_i(t)(i = 1,\ldots,2\nu)$ are derived from $v(t)$, output vector of a known and fixed feedforward compensator $Q_m^{-1}(s)$, $u(t)$, the plant control input vector and $y_p(t)$, the output vector of the plant.

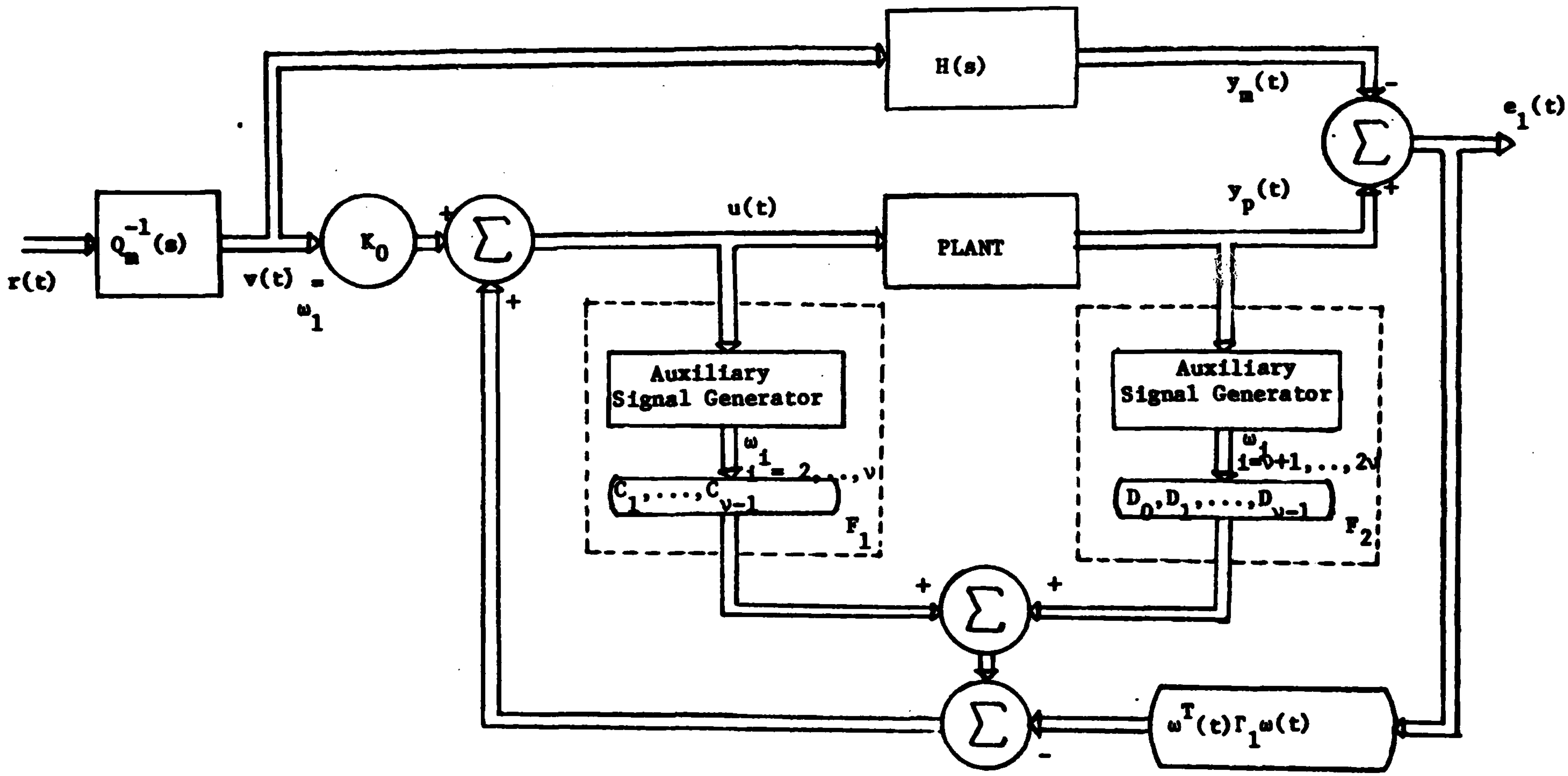

Fig. 1. Basic Structure of the Adaptive System

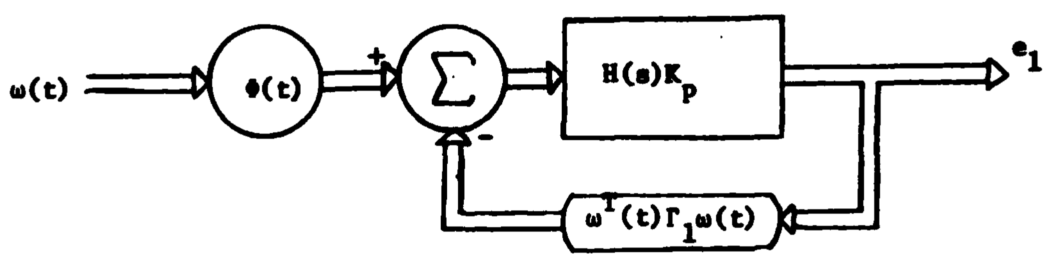

Fig. 2. The Error Model

V. Adaptive Control Problem: 2x2 Systems

a. In this subsection the concepts discussed in section IV are applied to the adaptive control of 2x2 systems. It is assumed that the relative degree of each of the four scalar transfer functions in the plant transfer matrix is known and that adequate information regarding the matrix E (cf. eqn. (6)) is available so that the nature of the Hermite form can be concluded using lemma 2. The different situations that can arise are discussed and it is shown through examples that adaptive control is, in general, feasible only when the Hermite form is diagonal. In other words, the plant transfer matrix is decoupled using the controller structure described in the previous section. In such cases, the exponents n_j along the diagonal in H(s) represent the minimum relative degree in the corresponding rows of the plant transfer matrix. In the discrete case a diagonal transfer matrix implies that each output is affected by an independent input (or inputs) with minimum delay. The nonzero entries in the high frequency gain matrix K_p in (eqn. 20) are the scalar gains associated with the transfer functions of minimum relative degree in each row (cf. lemma 2).

Throughout this section the term 'controller' will refer to the feedback controller whose parameters are to be adjusted and the term 'model' will be used interchangeably with the Hermite form of the plant transfer matrix.

Example 1 Let $W_p(s) = \begin{bmatrix} \dfrac{k_1}{s+\alpha_1} & \dfrac{k_2}{s+\alpha_2} \\ \dfrac{k_3}{s+\alpha_3} & \dfrac{k_4}{s+\alpha_4} \end{bmatrix}$

represent the plant transfer matrix in which k_i and α_i ($i = 1,2,3,4$) are unknown parameters. It is further known that the zeros of the plant transfer matrix lie in the open left half of the complex plane. In the following three cases, the prior information that is assumed regarding the unknown parameters is successively greater.

<u>Case (i)</u> Let $k_1k_4 \neq k_2k_3$

In this case the model (Hermite form) H(s) is diagonal and the high frequency gain matrix K_p is the same as E in eqn. 6 and

$$H(s) = \begin{bmatrix} \frac{1}{s+a} & 0 \\ 0 & \frac{1}{s+a} \end{bmatrix} \quad a > 0; \; K_p = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} .$$

<u>Case (ii)</u> Let $k_1k_4 = k_2k_3$ and $\alpha_1 + \alpha_4 \neq \alpha_2 + \alpha_3$

The singularity of the matrix E for $W_p(s)$ implies that the model will be triangular. The Hermite form H(s) and the corresponding high frequency gain matrix K_p are respectively:

$$H(s) = \begin{bmatrix} \frac{1}{s+a} & 0 \\ \frac{k_3/k_1}{s+a} & \frac{1}{(s+a)^2} \end{bmatrix} \quad a > 0; \; K_p = \begin{bmatrix} k_1 & k_2 \\ k_3(\alpha_1-\alpha_3) & k_4(\alpha_2-\alpha_4) \end{bmatrix}$$

It is clear that considerable information regarding the plant parameters will be needed to set up the model as well as to generate the adaptive laws.

<u>Case (iii)</u> Even greater knowledge of the plant parameters will be needed for adaptive control when $k_1k_4 = k_2k_3$ and $\alpha_1 + \alpha_4 = \alpha_2 + \alpha_3$ resulting in

$$H(s) = \begin{bmatrix} \frac{1}{s+a} & 0 \\ \frac{\frac{k_3}{k_1}[s+a+(\alpha_1-\alpha_3)]}{(s+a)^2} & \frac{1}{(s+a)^3} \end{bmatrix} a > 0; \; K_p = \begin{bmatrix} k_1 & k_2 \\ k_3(\alpha_1-\alpha_3)\cdot(a-\alpha_3) & k_4(\alpha_2-\alpha_4)\cdot(a-\alpha_4) \end{bmatrix}$$

Hence, even in the simple case described in this example, adaptive control may be practically feasible (in terms of the prior information needed) only in case (i) where H(s) is diagonal and K_p has a simple structure.

Example 2 In this case the transfer matrix

$$W_p(s) = \begin{bmatrix} \frac{1}{s+\alpha_1} & \frac{1}{(s+\alpha_2)^2} \\ \frac{k_3}{s+\alpha_3} & \frac{1}{(s+\alpha_4)^2} \end{bmatrix}$$

and has transfer functions of minimum relative degree in both rows in the first column. Once again the matrix E is singular so that the model H(s) is triangular and the high frequency gain matrix K_p is no longer the same as E.

$$H(s) = \begin{bmatrix} \frac{1}{s+a} & 0 \\ \frac{k_3}{(s+a)} & \frac{1}{(s+a)^2} \end{bmatrix} \quad a > 0 \; ; \; K_p = \begin{bmatrix} 1 & 0 \\ k_3(\alpha_1-\alpha_3) & (1-k_3) \end{bmatrix}$$

$$k_3 \neq 1$$

For the same reasons as before, considerable prior information regarding the unknown plant parameters will be needed in this case also for the implementation of an adaptive controller.

The above two simple examples illustrate the two different ways in which the Hermite form of a plant transfer matrix may become triangular. By completely classifying (2x2) transfer matrices in terms of their Hermite forms we can establish the prior information needed to control them adaptively.

Let the transfer matrix of a 2x2 plant be represented by

$$W_p(s) = \begin{bmatrix} k_1 \frac{\alpha_1}{\beta_1} & k_2 \frac{\alpha_2}{\beta_2} \\ k_3 \frac{\alpha_3}{\beta_3} & k_4 \frac{\alpha_4}{\beta_4} \end{bmatrix}$$

where k_i $(i = 1,2,3,4)$ are the high frequency scalar gains and n_i^* $(i = 1,2,3,4)$ are the relative degrees of the four transfer functions. Using lemma 2, and knowledge of n_1^*, the models and the gain matrices corresponding to four different classes of all 2x2 plants are delineated in Table 1. The number of elements in each class is shown in column 1 (for example $n_1^* < n_2^*$, $n_3^* > n_4^*$ and $n_1^* > n_2^*$, $n_3^* < n_4^*$ are the two elements of class I). The structure of only one typical element in each class is given in column 2 and every element of the class can be reduced to the same form by relabling the rows and/or columns of $W_p(s)$.

In class I both the model H(s) and the gain matrix K_p are diagonal. Adaptive control is possible in this case if the sign of the elements of K_p are known. In class II, which contains four elements, H(s) is diagonal but K_p is triangular.

I	II	III	IV
	Condition on the rel. deg.	Structure of Model, H(s)	Structure of gain matrix K_p
I 2 elements	$n_1^* < n_2^*$ $n_3^* > n_4^*$	$\begin{bmatrix} \frac{1}{(s+a)^{n_1^*}} & 0 \\ 0 & \frac{1}{(s+a)^{n_4^*}} \end{bmatrix}$	$\begin{bmatrix} k_1 & 0 \\ 0 & k_4 \end{bmatrix}$
II 4 elements	$n_1^* < n_2^*$ $n_3^* = n_4^*$	$\begin{bmatrix} \frac{1}{(s+a)^{n_1^*}} & 0 \\ 0 & \frac{1}{(s+a)^{n_3^*=n_4^*}} \end{bmatrix}$	$\begin{bmatrix} k_1 & 0 \\ k_3 & k_4 \end{bmatrix}$
III 1 element	$n_1^* = n_2^*$ $n_2^* = n_4^*$	$\begin{bmatrix} \frac{1}{(s+a)^{n_1^*=n_2^*}} & 0 \\ 0 & \frac{1}{(s+a)^{n_3^*=n_4^*}} \end{bmatrix}$	$\begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}$ $k_1k_4 \neq k_2k_3$
IV 2 elements	$n_1^* < n_2^*$ $n_3^* < n_4^*$	Non diagonal	Not obvious

Table I

A sufficient condition for adaptive control is that K_p be sign definite (shown in the next subsection) and hence considerably more prior information is needed in this case. In class III, which has one element, the minimum relative degree in each row occurs in both columns. If $k_1k_4 \neq k_2k_3$ and the matrix K_p is sign definite, the plant can be adaptively controlled.

In class IV the transfer functions with minimum relative degree (in each row) occurs in the same column so that H(s) is triangular in structure. The corresponding gain matrix K_p also depends in general upon the unknown plant parameters in a complex way. Hence, for this class of transfer matrices, adequate prior information is generally not available to enable adaptive controllers to be designed directly. However, as shown in example 3, using dynamic compensation, the Hermite form of the modified transfer matrix may be made diagonal. The following simple example illustrates how this can be achieved.

Example 3: In example 2 it was shown that when

$$W_p(s) = \begin{bmatrix} \frac{1}{(s+\alpha_1)} & \frac{1}{(s+\alpha_2)^2} \\ \frac{k_3}{(s+\alpha_3)} & \frac{1}{(s+\alpha_4)^2} \end{bmatrix}$$

the resulting H(s) would be triangular and contain the unknown plant parameters. If such a plant is augmented by a matrix $D(s) = \begin{bmatrix} \frac{1}{s+\beta} & 0 \\ 0 & 1 \end{bmatrix}$, $\beta > 0$, then the modified plant transfer matrix

$$W_p^1(s) = W_p(s)D(s) = \begin{bmatrix} \frac{1}{(s+\alpha_1)(s+\beta)} & \frac{1}{(s+\alpha_2)^2} \\ \frac{k_3}{(s+\alpha_3)(s+\beta)} & \frac{1}{(s+\alpha_4)^2} \end{bmatrix}$$

has a diagonal Hermite form $H^1(s)$ where

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has a diagonal Hermite form $H^1(s)$ where

$$H^1(s) = \begin{bmatrix} \frac{1}{(s+a)^2} & 0 \\ 0 & \frac{1}{(s+a)^2} \end{bmatrix} \qquad a > 0$$

The corresponding high frequency gain matrix K_p is $\begin{bmatrix} 1 & 1 \\ k_3 & 1 \end{bmatrix}$ which is the same as the matrix E for $W_p^1(s)$.

Using the above approach we now show that all transfer matrices $W_p(s)$ which correspond to class IV can be reduced to one of the previous classes using dynamic prefilters. The three distinct cases which can arise are treated separately below and the minimal order diagonal compensators are specified in each case.

1) $n_1^* + n_4^* < n_2^* + n_3^*$

Using a dynamic precompensator with transfer matrix D(s) where

$$D(s) = \begin{bmatrix} \frac{1}{(s+\alpha)^{(n_4^*-n_3^*)}} & 0 \\ 0 & 1 \end{bmatrix} \quad ; \alpha > 0$$

the Hermite form H(s) and the gain matrix K_p of the modified system can be obtained as

$$H^1(s) = \begin{bmatrix} \frac{1}{\pi^{(n_1^*+n_4^*-n_3^*)}} & 0 \\ 0 & \frac{1}{\pi^{n_4^*}} \end{bmatrix} , \quad K_p^1 = \begin{bmatrix} k_1 & 0 \\ k_3 & k_4 \end{bmatrix}$$

(ii) $n_1^* + n_4^* = n_2^* + n_3^*$ and $k_1k_4 \neq k_2k_3$

If $u_1(t)$ is filtered by $\frac{1}{(s+\alpha)^{(n_4^*-n_3^*)}}$ and $n_2(t)$ by 1, the new Hermite form $H^1(s)$ and gain matrix K_p^1 are given by the following.

$$H^1(s) = \begin{bmatrix} \frac{1}{\pi^{n_2^*-n_1^*}} & 0 \\ 0 & \frac{1}{\pi^{n_3^*-n_4^*}} \end{bmatrix} , \quad K_p^1 = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}$$

(iii) $n_1^* + n_4^* > n_2^* + n_3^*$

Filter $u_1(t)$ by $\dfrac{1}{(s+\alpha)^{n_2^*-n_1^*}}$ and $u_2(t)$ by 1 to obtain

$$H^1(s) = \begin{bmatrix} \dfrac{1}{\pi^{n_2^*}} & 0 \\ 0 & \dfrac{1}{\pi^{n_3^*+n_2^*-n_1^*}} \end{bmatrix}, \quad K_p^1 = \begin{bmatrix} k_1 & k_2 \\ k_3 & 0 \end{bmatrix}.$$

<u>Remark</u> In [19], using a geometric approach, the generic decoupling of a plant represented by the triple (C,A,B) using only state feedback is discussed. However, since the mapping from the parameter space of (C,A,B) to the parameter space of T(s), the transfer matrix, is not bicontinuous, properties true in (C,A,B) space may not be true in the parameter space of T(s).

Since the construction of the Hermite form or the interactor is done in the parameter space of the transfer matrix, its generic decoupling can not be concluded on the above basis. The problem arises because the matrix E(Eqn. 6) is not a continuous function of its arguments, namely the minimum relative degree in each row of the transfer matrix.

Hence, the point made in [8] that the diagonality of the interactor matrix (equivalently the Hermite normal form) is a generic property does not apply here. In fact, as shown above, almost one third of all 2x2 transfer matrices have non-diagonal Hermite forms.

b) <u>Adjustment Laws</u>:

In this subsection adaptive laws for the adjustment of the controller parameters are developed for all 2x2 systems categorized in table 1. Since, by introducing known prefilters, transfer matrices which belong to class IV can be reduced to one of classes I-III, only the latter are considered here. The analysis is limited to error models which arise in the various cases. Only a brief description of the corresponding controller structures is given since they are merely multivariable extensions of those used in SISO systems discussed extensively in [1] and [2] and modifications of the basic structure described in the previous section. The proof of stability of the overall control loop for SISO systems in [2] can also be directly extended to these multivariable systems and hence is not discussed here.

The complexity of the controller used in the various cases depends upon the amount of prior information available regarding the gain matrix K_p as well as the Hermite form $H_p(s)$. The simplest cases occur when K_p is known and $H_p(s)$ is strictly positive real. In the following, the various cases are arranged in increasing order of generality.

(i) $\underline{K_p \text{ Known}}$: If K_p is known, considerable simplification is achieved by including a fixed gain matrix K_p^{-1} in the control loop. The modified plant transfer matrix then has a high frequency gain matrix which is unity (i.e. $K_0 \equiv I$). The adaptive controller contains $2x(4\nu-2)$ parameters which are the elements of a parameter error matrix $\bar{\Phi}(t)$ and the corresponding $(4\nu-2)$ dimensional vector signal is denoted by $\bar{\omega}(t)$.

a) H(s) SPR (strictly positive real)*:

When H(s) is SPR the parameter error matrix is updated according to the law

$$\dot{\bar{\Phi}}(t) = \dot{\bar{\theta}}(t) = -\Gamma e_1(t)\omega(t) \qquad \Gamma = \Gamma^T > 0$$

By lemma 4 the output error $e_1(t)$ and $\bar{\Phi}(t)$ are bounded. Since the output of the model is bounded, this ensures (as in the SISO case) that the state of the plant and hence $\bar{\omega}(t)$ is bounded. Hence $e_1(t) \to 0$ and $\dot{\Phi}(t) \to 0$ as $t \to \infty$. It is worth pointing out that for this case the feedback term $e_1\bar{\omega}^T(t)\Gamma\bar{\omega}(t)$ is not needed to prove global stability of the adaptive loop.

* A diagonal matrix of rational functions is strictly positive real if and only if each diagonal element is strictly positive real.

b) H(s) not SPR

If H(s) is not SPR, auxiliary signals have to be generated and added to the model output to avoid differentiation.* The controller of Fig. 1 is modified as shown in Fig. 3. The simple error model of Fig. 2. changes to the augmented error model shown in Fig. 4.

Let $\theta_i^T(t)$ and $\phi_i^T(t)$ denote the ith row in $\Theta(t)$ and $\phi(t)$ matrices respectively. Let $h_{11}(s)$ and $h_{22}(s)$ be the two non-zero diagonal elements of H(s). In constructing the auxiliary signals the matrices $\Theta(t)$ and H(s) have been expanded, so that cancelations for $\Theta(t) = \Theta^*$, the true parameter matrix, can occur in the auxiliary loop of the error model in Fig. 4.

From Fig. 4, the error equations can be derived as

$$\varepsilon(t) = \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{1+\bar{\xi}_1^T(t)\Gamma_1\bar{\xi}_1(t)} \cdot \bar{\phi}_1^T(t)\bar{\xi}_1(t) \\ \dfrac{1}{1+\bar{\xi}_2^T(t)\Gamma_2\bar{\xi}_2(t)} \cdot \bar{\phi}_2^T(t)\bar{\xi}_2(t) \end{bmatrix}$$

where $\bar{\xi}_1(t) = h_{11}(s)\bar{\omega}(t)$ and $\bar{\xi}_2(t) = h_{22}(s)\bar{\omega}(t)$.

The adaptive laws are given by

$$\dot{\bar{\theta}}_1(t) = \dot{\bar{\phi}}_1(t) = -\Gamma_1\varepsilon_1(t)\bar{\xi}_1(t) \qquad \Gamma_1 = \Gamma_1^T > 0 \quad \text{and}$$

$$\dot{\bar{\theta}}_2(t) = \dot{\bar{\phi}}_2(t) = -\Gamma_2\varepsilon_2(t)\bar{\xi}_2(t) \qquad \Gamma_2 = \Gamma_2^T > 0.$$

By the remark following lemma 4, the error system is uniformly stable and $\varepsilon(t)$ and $\Phi(t)$ are uniformly bounded regardless of $\omega(t)$.

---

* Similar to the operator L(·) in [1] for SISO systems, a diagonal matrix of operators S(·) exists which makes H(s)·S(s) SPR. For simplicity of analysis $S^{-1}(s) = H(s)$ is chosen.

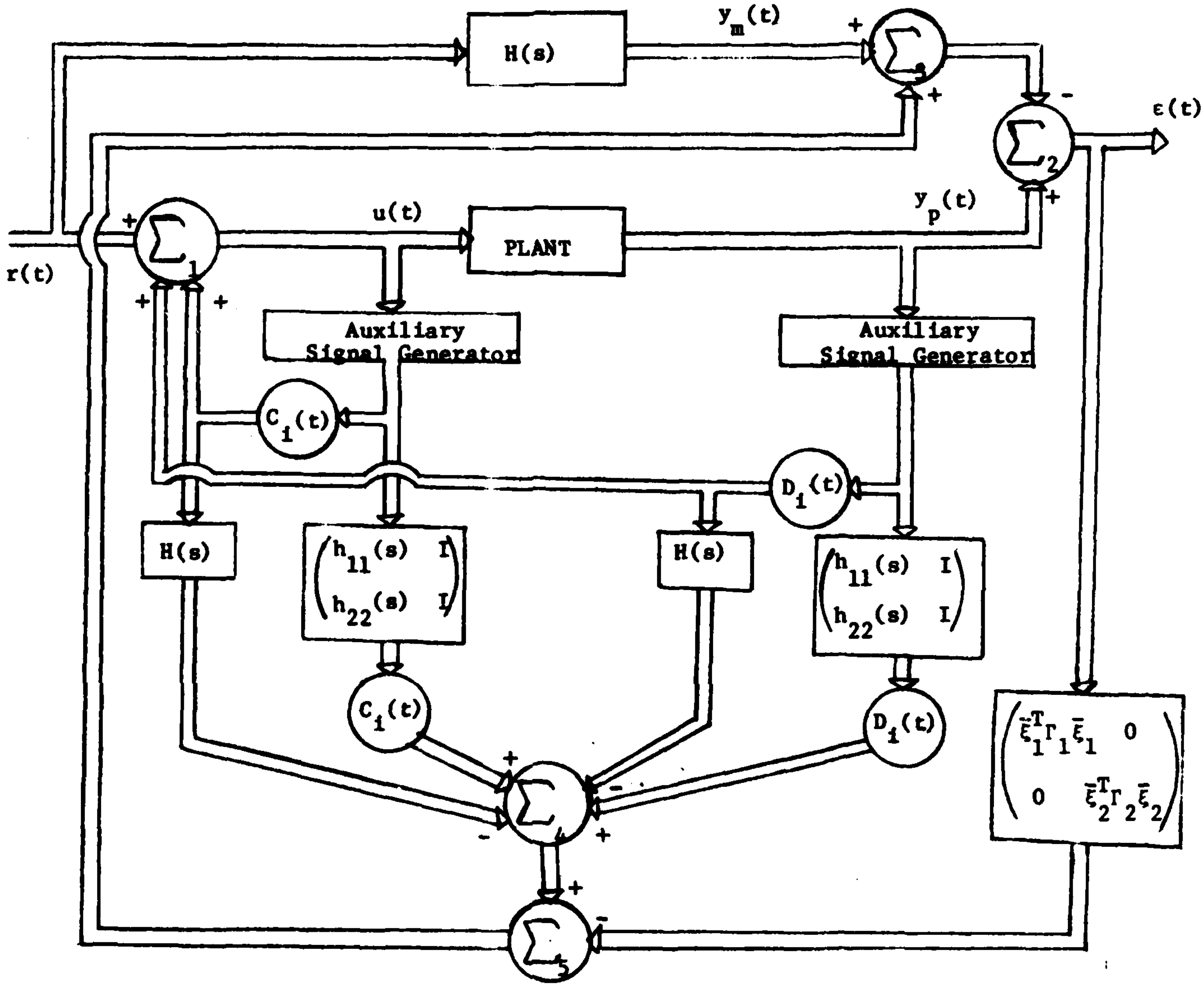
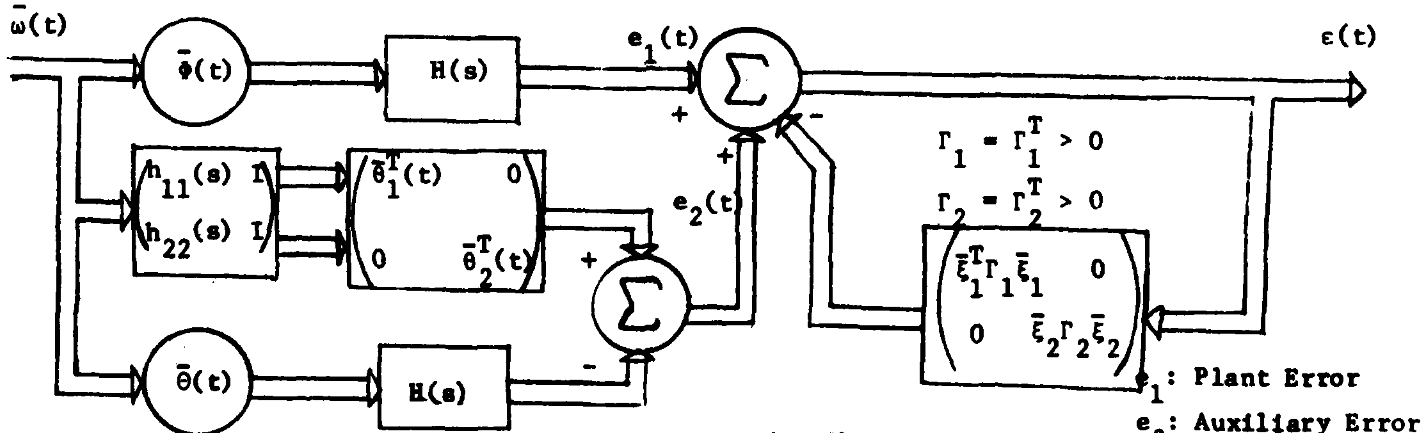


Fig. 3. Adaptive Controller for K_p Known and H(s) Not SPR.



I: Identity matrix of $(4\nu-2)$ dimension

Fig. 4. Augmented Error Model [K_p Known, H(s) not SPR]

(ii) Unknown K_p

When K_p is unknown the controller structure is considerably more involved. More prior information about the plant transfer matrix is needed to obtain a stable adaptive controller.

a) H(s) SPR

The controller structure remains the same as in Fig. 1. The error model also remains the same as in Fig. 2. The parameter error matrix $\Phi(t)$ is updated according to the law

$$\dot{\Phi}(t) = \dot{\Theta}(t) = -\Gamma e_1(t)\omega^T(t)$$

The boundedness of $e_1(t)$ and $\Phi(t)$ can be proved by lemma 4. By the same argument as in the previous case, $e_1(t)$ and $\dot{\Phi}(t) \to 0$ as $t \to \infty$. However, now the input $\nu(t)$ in eqn. 7 is $K_p[\Phi(t)\omega(t) - \omega^T(t)\Gamma_1\omega(t)e_1(t)]$. It can be easily shown that K_p must be positive definite to prove that $V > 0$ and $\dot{V} \leq 0$ in lemma 4.

For the transfer matrices in class I, the knowledge of the sign of each entry in K_p is enough to meet this sign definiteness condition.

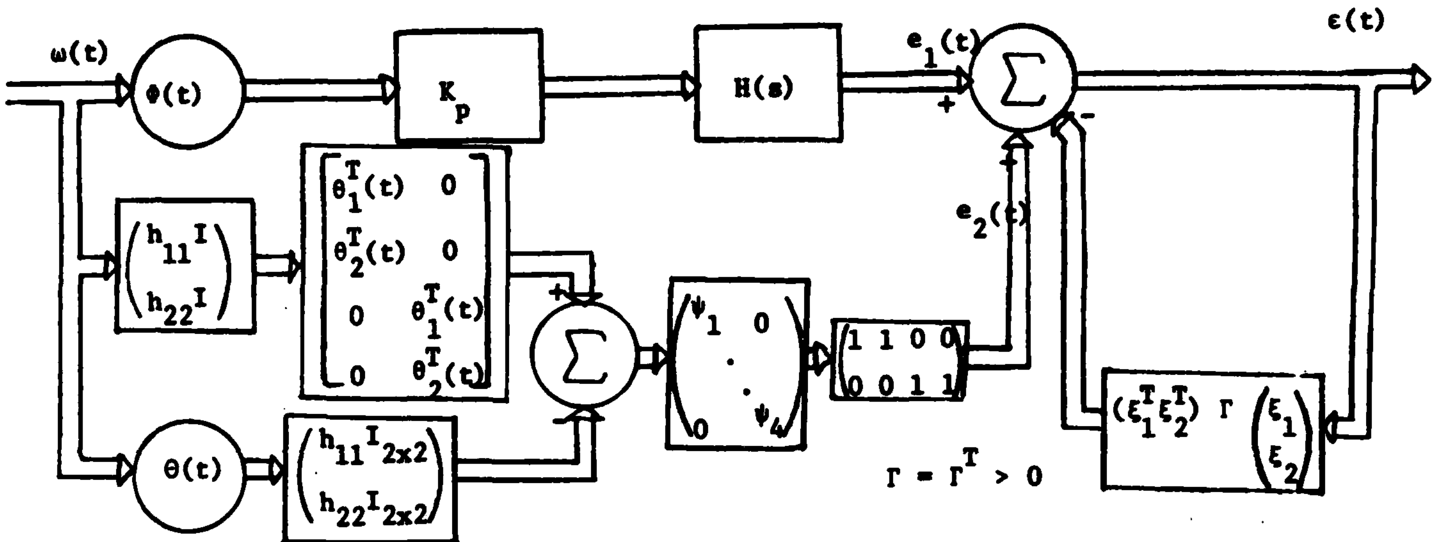
b) H(s) not SPR

Since H(s) is not SPR, auxiliary signals have to be generated to augment the model output. To account for the unknown gain matrix, an additional gain parameter matrix $\Psi(t)$ is introduced in series with the auxiliary signals. The controller structure in fig. 3 is modified slightly to include $\Psi(t)$ between the summing junctions 4 and 5. The error equation changes somewhat and the error model is shown in fig. 5 for class III (K_p nonsingular) transfer matrices. Other cases in classes I and II can also be suitably specialized. $\Psi(t)$ is also adjusted along with $\Theta(t)$ such that as $t \to \infty, \Psi(t) \to K^*$ (K^* is a diagonal matrix formed by the elements of K_p arranged along the diagonal) and $\Theta(t) \to \Theta^*$.

The augmented error vector from fig. 5 for this case can be computed to be

$$\varepsilon(t) = \begin{bmatrix} \varepsilon_1(t) \\ \\ \varepsilon_2(t) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{1+x}\left[(k_1\phi_1^T(t)+k_2\phi_2^T(t))\,\xi_1(t)+k_1 g_1(t)\zeta_1(t)+k_2 g_2(t)\zeta_2(t)\right] \\ \\ \dfrac{1}{1+x}\left[(k_3\phi_1^T(t)+k_4\phi_2^T(t))\,\xi_2(t)+k_3 g_3(t)\zeta_3(t)+k_4 g_4(t)\zeta_4(t)\right] \end{bmatrix}$$

where $\xi_1(t) = h_{11}(s)\omega(t)$ and $\xi_2(t) = h_{22}(s)\omega(t)$ and $x = (\xi_1^T(t)\,\xi_2^T(t))\Gamma \begin{pmatrix} \xi_1(t) \\ \xi_2(t) \end{pmatrix}$.



e_1: Plant Error

e_2: Auxiliary Error

ε: Augmented Error

I: Identity matrix of 4ν dimension.

Fig. 5. Augmented Error Model [unknown K_p and H(s) not SPR]

$$\text{and} \quad \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \zeta_3(t) \\ \zeta_4(t) \end{bmatrix} = \left[\begin{bmatrix} \phi_1^T(t) & 0 \\ \phi_2^T(t) & 0 \\ 0 & \phi_1^T(t) \\ 0 & \phi_2^T(t) \end{bmatrix} \begin{bmatrix} h_{11}(s)I \\ h_{22}(s)I \end{bmatrix} - \begin{bmatrix} h_{11}(s)I_{2x2} \\ h_{22}(s)I_{2x2} \end{bmatrix} \Phi(t) \right] \cdot \omega(t)$$

and $G(t) = \mathrm{diag}(g_i(t)) = (K^*)^{-1}\Psi(t) - I$.

The corresponding adaptive laws are

$$\begin{bmatrix} \dot{\theta}_1^T(t) \\ \dot{\theta}_2^T(t) \end{bmatrix} = \begin{bmatrix} \dot{\phi}_1^T(t) \\ \dot{\phi}_2^T(t) \end{bmatrix} = -\Gamma \begin{bmatrix} \epsilon_1(t)\xi_1^T(t) \\ \epsilon_2(t)\xi_2^T(t) \end{bmatrix}$$

and
$$\dot{g}_1(t) = -\gamma_1\epsilon_1(t)\zeta_1(t), \quad \dot{g}_2(t) = -\gamma_2\epsilon_1(t)\zeta_2(t)$$
$$\dot{g}_3(t) = -\gamma_3\epsilon_2\zeta_3(t), \quad \dot{g}_4(t) = -\gamma_4\epsilon_2(t)\zeta_4(t),$$
$$\gamma_i > 0, \ i = 1,2,3,4.$$

Using the Lyapunov function approach it can be shown that the error system is stable and $\epsilon(t)$ and $\Phi(t)$ are bounded. But a sufficient condition for this is that K_p be such that there exists a matrix Γ such that the symmetric part of $(K_p\Gamma)$ be positive definite.

As pointed out in [2] for SISO systems, the main stability question arises when H(s) is not SPR. In this case auxiliary signals are used which cannot be assumed to be bounded. So even if $\epsilon(t)$ is bounded, boundedness of the plant output $y_p(t)$ and hence all the relevant signals can not be concluded. The nature of the stability problem is the same whether K_p is unknown or known. The fact that $\epsilon(\cdot)$ and $\dot{\Phi}(\cdot) \epsilon L^2$ can be concluded through the existence of a Lyapunov function as was done in lemma 1. Boundedness of $\omega(t)(\bar{\omega}(t))$ and $\dot{\omega}(t)(\dot{\bar{\omega}}(t))$ can be shown using the same arguments as in [2] making it possible to conclude the stability of the adaptive control loop in the large.

Remark 1

A similar condition on the gain matrix has been obtained in [8] for the discrete case and in [9] for the continuous case. Elliott et.al. in [7] have made somewhat more restrictive assumptions regarding the structure of K_p to generate stable adaptive laws.

VI. Conclusions

Hermite normal forms of nonsingular transfer matrices play a central role in determining the class of model transfer matrices which the plant can follow. However, due to inherent complexity in specifying the Hermite form in general, it has been argued that adaptive control is practically feasible only for those plants which have diagonal Hermite forms, i.e., which can be decoupled by using state feedback only. The sign definiteness of the high frequency gain matrix K_p has been found to be sufficient to generate stable adaptive laws.

For 2x2 systems, the knowledge of relative degree of each scalar transfer function has been used in determining the Hermite form. A globally stable adaptive controller has been developed and it has been shown that all 2x2 stably invertible systems can be generically adaptively controlled subject to the definiteness condition on the gain matrix.

References

1. K. S. Narendra and Lena S. Valavani, "Stable Adaptive Controller Design - Direct Control," IEEE TAC Vol. 23, No. 4, Aug. 1978.

2. K. S. Narendra, Yuan-Hao Lin and Lena S. Valavani, "Stable Adaptive Controller Design, Part II: Proof of Stability," IEEE TAC Vol. 25, No. 3, June 1980.

3. A. S. Morse, "Global Stability of Parameter-Adaptive Control Systems," IEEE TAC Vol. 25, Nov. 3, June 1980.

4. R. V. Monopoli and C. C. Hsing, "Parameter Adaptive Control of Multivariable Systems," Int. J. Control, Vol. 22, No. 3, pp. 313-327, 1975.

5. U. Borison, "Self Tuning Regulators for a Class of Multivariable Systems," Automatica, Vol. 15, No. 2, pp. 209-215, March 1979.

6. G. C. Goodwin, P. J. Ramadge and P. E. Caines, "Discrete Time Multivariable Adaptive Control," IEEE TAC, Vol. 25, No. 3, June 1980.

7. H. Elliott and W. A. Wolovich, "Parameter Adaptive Control of Linear Multivariable Systems," Proc. of 13th Asilomar Conference on Circuits, Systems and Computers, Nov. 1979.

8. G. C. Goodwin and R. S. Long, "Generalization of Results on Multivariable Adaptive Control," Proc. 19th CDC, Dec. 1980.

9. A. S. Morse, "Parametrizations for Multivariable Adaptive Control," Proc. of Yale Workshop on Adaptive Control, May 1981.

10. R. V. Monopoli and V. N. Subbarao, "Design of a Multivariable Model Following Adaptive Control System," to be presented at the 20th CDC, Dec. 1981.

11. A. S. Morse, "System Invariants under Feedback and Cascade Control," Mathematical System Theory, Vol. 131. Springer Verlag: New York.

12. C. C. MacDuffee, The Theory of Matrices. New York: Chelsea, 1956.

13. C. T. Chen, Introduction to Linear System Theory. New York: Holt, Rinehart and Winston, Inc., 1970.

14. W. A. Wolovich, Linear Multivariable Systems. New York: Springer, 1974.

15. W. A. Wolovich and P. L. Falb, "Invariants and Canonical Forms Under Dynamic Compensation," SIAM Journal on Control, Vol. 14, No. 6, Nov. 1976.

16. N. T. Hung and B.D.O. Anderson, "Triangularization Technique for the Design of Multivariable Control Systems," IEEE TAC Vol. 24, No. 3, June 1979.

17. Y.-H. Lin and K. S. Narendra, "A New Error Model for Adaptive Systems," IEEE TAC Vol. 25, No. 3, June 1980.

18. K. S. Narendra and J. H. Taylor, Frequency Domain Criteria for Absolute Stability. New York: Academic, 1973.

19. M. E. Warren and S. K. Mitter, "Generic Solvability of Morgan's Problem," IEEE TAC Vol. 20, No. 2, April 1975.

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