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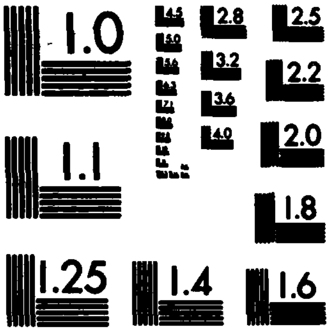

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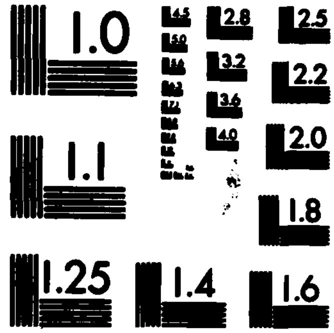

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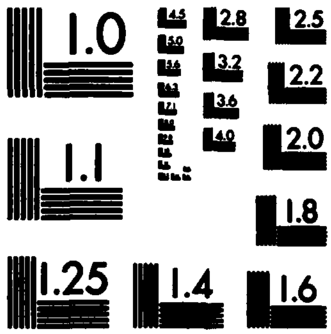

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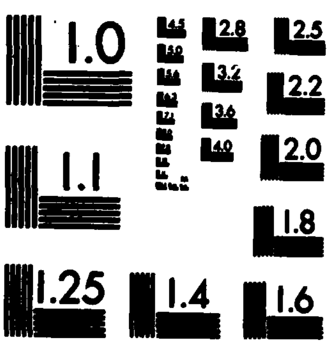

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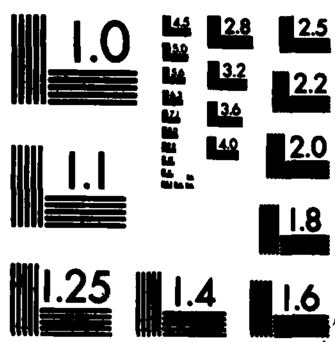

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Report Number 5

DETECTION IN A NON-GAUSSIAN ENVIRONMENT

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INFORMATION SCIENCES AND SYSTEMS LABORATORY

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by

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ABSTRACT

Techniques for the detection of a weak signal in non-Gaussian, ill-defined noise are considered. Statistical characterizations used are moments, tail measures related to quantiles, and a measure related to the score function. For multivariate densities, the characterization is by means of a nonlinear transformation. Initial results seem to indicate that assuming a particular family of probability densities does not necessarily result in a significant degradation in performance when the observations actually come from a density outside the assumed family. More important to performance are accurate estimates of the moments, tail measures, or other parameters which are used to specify the detector.

INTRODUCTION

In most engineering studies, there is usually a tradeoff between model complexity and analytical tractability: the more complicated (and realistic) the model is assumed to be, the more difficult the subsequent analysis becomes. This balance is especially delicate in the area of non-Gaussian signal processing.

If departures from Gaussian statistics are investigated, it is usually assumed that the sequence of observations is independent and, often, identically distributed. With these basic assumptions, analytical results are often available. When non-Gaussian dependencies are taken into account, results are often only obtainable by means of Monte Carlo simulation since multivariate distributions are usually not available in analytical form.

It is within this framework that we summarize some of our preliminary results for detection in a non-Gaussian environment. We first consider the detection of a signal in nearly Gaussian skewed noise. Surprisingly, a small degree of skewness can lead to a significant degradation in performance of the linear detector as measured by false-alarm rate. A detector which exhibits the desired robustness is introduced. Some of the potential difficulties in using adaptive procedures are illustrated in the context of under-ice ambient noise data.

A general adaptive procedure is then outlined which uses the skewness-kurtosis plane to measure departures from Gaussian statistics. Overlays, which specify the probability density family of the observations, are introduced and used to determine the form of the nonlinear processor.

Non-Gaussian statistics are then characterized by the derivative of the logarithm of the probability density. This expression, $f'(x)/f(x)$, is estimated using the observations and is then used to form an optimum detector. It is shown by example that, under reasonable conditions, assuming a particular family of probabilty densities does not significantly degrade detector performance when the observations actually come from a density outside the family. This general conclusion is also arrived at using another approach, in which quantiles are used to measure the tail behavior of heavy-tailed probability densities.

In the last section, a class of multivariate non-Gaussian probability densities is defined. For this class, the locally optimum detector is seen to separate into a zero-memory nonlinear part and a part with memory. The results of a simulation to evaluate a number of detectors are presented. It is observed that simplified versions of the optimal processor also lead to improved performance when compared to conventional detectors.

DETECTION IN SKEWED NOISE

In robust and general nonparametric studies (Refs. 1,2), a symmetry assumption on the underlying noise density is usually made. (Another frequent nonparametric assumption is that the noise density is unimodal or that the median is 1/2.) In this section, we report on an investigation which assumes nonzero skewness and which studies the sensitivity of the linear and sign detectors to this lack of symmetry. Surprisingly, a small amount of skewness can lead to marked deterioration in system performance for the linear detector as measured by false-alarm rate. A modified detector is also introduced which exhibits a desired robustness to skewness in the observations.

Consider the detection problem

$$H_0: X_i = N_i \ , \ i=1,2,\ldots k$$
$$H_1: X_i = \Theta + N_i$$

where the constant signal $\Theta>0$. The noise is independent,identically distributed with first three moments

$$E(X_i)=0,\ E(X_i^2)=\sigma^2,\ E(X_i^3)=\mu_3>0$$

The skewness is defined as

$$\zeta = \mu_3/\sigma^3$$

and is assumed to be small.

If skewness were zero and the noise Gaussian, the optimum receiver is the sample mean or linear detector

$$T_1(x) = \frac{1}{k}\sum_{i=1}^{k} X_i$$

With a small amount of skewness and nearly Gaussian noise, the test statistic T can be represented by the first few terms of the Cornish-Fisher expansion (Refs.3,4). Under hypothesis H_0

$$T_2(x) = \frac{\sigma}{\sqrt{k}} Z + \frac{\mu_3}{6\sigma^2 k}(Z^2-1) + 0(k^{-3/2})$$

where Z is the standard normal variable. (Under hypothesis H_1 the above expression includes a shift by the signal.)

Utilizing the first two terms of this expansion, one can generate the probability density function for T_2, along with analytic expressions for the false-alarm rate and detectability. Details can be found in [3],[4].

Figure 1 compares the Gaussian density to the density of T_2 with skewness ζ=0.5. Figure 2 gives the normalized false-alarm rate as a function of skewness. The number of observations for the test is 100. The constant α_0 is the false-alarm rate under the strict Gaussian assumption (ζ=0) and provides the threshold setting for T_2. A slight departure from symmetry is hardly discernible (Fig.1). Yet, for ζ=0.5, there is an increase in false-alarm rate of over 80% for a nominal $\alpha_0=10^{-5}$. With smaller α_0, the degradation is even more severe (Ref. 4).

Clearly, the linear detector can be modified so as to account for skewness. One approach is to use a nonparametric test such as the sign detector, which keeps the false alarm rate relatively constant. (See Ref. 3 for details.) A second approach is to directly modify the linear detector. Here, a natural way to proceed would be to "subtract off" that part of the observation due to skewness. The following test statistic is the simplest version of this approach:

$$T_3(x) = T_1(x) - \frac{\zeta}{6\sigma}(T_1(x))^2$$

where T_1 is the sample mean detector given above. Analytic expressions can again be developed for false alarm-rate and power (Ref.4). Here, we choose to present the results of a Monte Carlo simulation which also serves to verify the accuracy of the analytic approximations. The results for false-alarm rates in Fig. 3 indicates that the modified sample mean detector has the desired robustness.

Implicit in the improved performance is the ability to measure the skewness accurately. This is clearly illustrated with a simple experiment performed with under-ice ambient noise. Fram II data (Ref.5 and also discussed elsewhere in these Proceedings) was used to compute empirically the false-alarm rate for the linear detector and the modified version discussed above. Required estimates of the variance and skewness were obtained by straightforward sample moment methods over (assumed stationary) blocks of data. Figure 4 summarizes the results of our first experiment. Performance of both detectors is essentially the same, in sharp contrast to the results obtained from computer-generated data discussed in the above paragraph. The primary reason for this is, we suspect, related to the nature of the nonstationarity of the data (See Ref. 5, page 8 and Fig. 5 below) and the need to use more accurate estimates of skewness with better tracking properties. This aspect of data-adaptive estimation and detection is an area of current research activity.

To conclude this section, we will outline one possible adaptive system which uses sample moments. Computed skewness and kurtosis are shown versus time for a representative Fram II data set in Figs. 5,6. (See Ref. 7 for further details.) Kurtosis is defined as the normalized fourth central moment:

$$\beta_2 = E(X-m)^4/\sigma^4$$

Figure 7 presents the same skewness (β_1) versus kurtosis (β_2), with time an implicit parameter. Observe the cluster of points around $\beta_1=0, \beta_2=3$, which are the values for a Gaussian density. The overlaid lines are different regions of the Johnson family of densities. (The Johnson family can be defined as a nonlinear transformation of Gaussian variates. The symbol S_u, for example, represents the sinh transformation, while S_l is a logarithmic transformation, leading to a lognormal random variable. (See Ref. 6, section 2.2 for further details.)

Each point in the skewness-kurtosis plane defines a unique member of the Johnson family. This one-to-one relationship can be utilized in a data-adaptive detector in the following manner. A region around the Gaussian point ($\beta_1=0, \beta_2=3$) can be defined. For points lying in this region, it is assumed that the observations are governed by Gaussian statistics and the optimum processor is the linear detector. When the computed moments fall outside the region, one declares that the observations are non-Gaussian and another detector is switched in to process the data. The point in the $\beta_1-\beta_2$ plane would determine the appropriate density which then specifies the likelihood processor (locally optimum detector in the weak signal case.) In practice, the $\beta_1-\beta_2$ plane would be quantized into, say, rectangular regions. Then, either due to nonstationary statistics or because of sampling variances of the moment or other estimators, the point wanders around in the region. When it leaves one region, another likelihood (nonlinear) processor can be be switched in. Clearly, there are two key steps. The first is to obtain good tracking estimates. The second is to specify which family of probability densities to overlay on the $\beta_1-\beta_2$ plane, e.g., the Johnson, Pearson, or mixture model. This point will be discussed in more detail in the next section.

DENSITY FAMILIES AND ADAPTIVE DETECTION

Rather than focus on a particular moment measure such as skewness or kurtosis to charcterize non-Gaussian statistics, one can attempt to generalize, but still parameterize, the problem in the following manner.

The score function of the density f(x) is related to f'(x)/f(x) and plays an important role in estimation theory [8]. In addition, $-f'(x)/f(x)$ determines the form of the processor for the locally most powerful test or the detection of a weak signal in noise. (This expression is also related to the test which maximizes efficacy in the small signal case.)

These observations lead us to focus on using f'/f as a characterization of non-Gaussian statistics. Observe that for the Gaussian density f'/f is linear, so departures from Gaussian statistics can be conveniently measured by departures from linearity. A straightforward way to proceed is to specify f'/f as a ratio of two polynomials, Specializing to first-order numerator and second-order denominator polynomials gives the classical Pearson family of probability densities:

$$\frac{f'(x)}{f(x)} = \frac{a+x}{b_0+b_1x+b_2x^2}$$

which includes the Gaussian, gamma, and beta among its members.

The procedure would now be to use the observations to estimate the above coefficients. This determines the particular density of the Pearson family and the corresponding detector. The two main approaches for estimating the coefficients are maximum likelihood estimates of the paramters directly, and indirect estimates using sample moments.

Of immediate interest is the question: what happens when there is a mismatch, i.e., when we use data generated by a non-Pearson density to fit a Pearson-type detector as described above? We proceed, using the non-Pearson mixture model ([7]):

$$f(x) = (1-\varepsilon)\frac{1}{\sqrt{2\pi}\sigma}e^{-x^2/2\sigma^2} + \frac{\varepsilon\alpha}{2}e^{-\alpha|x|}$$

The ratio of the component variances is defined to be

$$\gamma = (\alpha\sigma)^2/2$$

This mixture density has been used to model heavy-tailed impulsive noise. (See

Ref. 9 and the references cited therein.)

For the results that follow, the first four moments of the mixture model are used to obtain analytically the parameters a, b_0, b_1, b_2. This then determines the processor, in this case the locally optimum detector. Figure 8 shows the curve $g(x)=-f'(x)/f(x)$ which is the zero-memory nonlinear part of the locally optimum detector. This is for a symmetric density ($a=b_1=0$), with kurtosis shown as a parameter. Figures 9a-9d present a comparison of ARE, as a function of the mixing constant ε, with γ as a parameter. (The ARE, asymptotic relative efficiency, is the ratio of efficacies. Efficacies, in turn, are incremental signal-to-noise ratio measures in the small signal case.) $ARE(l_0, ld)$ is the ratio of the locally optimum to the linear detector, while $ARE(\hat{g}, ld)$ compares the Pearson fit as discussed above to the linear detector. It is seen that for small ε and γ, the Pearson-fit detector compares quite favorably to the locally optimum, even though $\hat{g}(x)$ was determined from a non-Pearson density. (Additional details and further examples can be found in Ref. 7.)

Based on these preliminary computions, it would appear that, for nearly Gaussian noise (a mixing parameter of $\varepsilon < 0.1$), the assumption of operating within a Pearson family does not significantly degrade detector performance when the true density turns out to be a Gaussian mixture with a Laplace contaminant.

One of the difficulties with moment estimators is the potentially large sampling variance for the higher-order moments. Oftentimes, it is more appropriate to use quantile measures to characterize the underlying statistics. We now outline a study using these measures. (See Refs. 10 and 11 for details.)

Let F(x) be the cumulative distribution function for the noise. Let p_1 represent a quantile on the tail of the density, e.g., $p_1=.999$, and let $p_2<p_1$ be a lower quantile. Then, two other measures which can be used to characterize the

density (and its tails) are:

$$\tau = F^{-1}(p_1)/F^{-1}(p_2)$$

and

$$\rho = \frac{d}{dp_1}\ln F^{-1}(p_1)$$

Figure 10 illustrates the one-to-one relationship between these measures (and kurtosis) and the Johnson S_u family as one of the two parameters δ of this density varies. The other parameter is determined from the variance (measured or given) and δ. (See [10] or [11], Chapt. 4.)

Using these measures, one can parallel the adaptive procedure outlined above. That is, the data is used to estimate δ. This, in combination with an estimate of σ, uniquely determines the Johnson probability density and, hence, the nonlinear detector. This has been simulated when the noise does not actually come from a Johnson density but, rather, from other heavy-tailed densities (Gaussian-Gaussian mixtures, Lapacian noise). The results of a preliminary computer simulation are encouraging: the detector based on Johnson statistics performs close to the optimum detector (which is based on exact knowledge of the noise statistics) and always better than the linear detector ([11]). Again we see that detector performance does not appear to be critically dependent on specifying the correct family of densities. More important, apparently, are accurate estimates of the parameters which characterize the family and the corresponding detector

DETECTION IN MULTIVARIATE NOISE

The previous discussion assumed the observations were independent; in order to account for the dependencies, multivariate probability densities need to be considered. In this section, we study nonlinear transformations of a known multivariate density. This type of noise, called transformation noise in Refs. 7

and 12, is a multivariate generalization of the Johnson family discussed earlier. Any output marginal can be obtained by prescribing a zero-memory nonlinear transformation. It is difficult, however, to obtain analytically the output dependencies. Figure 11 is a schematic of the transformation noise generation. The output multivariate density is denoted by $f(\mathbf{n})$ and the input density by $\varphi(\mathbf{u})$. With $\mathbf{u}$ multivariate Gaussian, there is a further decomposition, since one can easily define the linear transformation $\mathbf{u}=L\mathbf{z}$, where $\mathbf{z}$ is a vector of independent Gaussian variates, and L defines the covariance structure.

For the case of a weak signal in noise, the appropriate receiver is the locally optimum detector. It is shown in Fig.12, where it is assumed that the nonlinearity g is twice differentiable. The symbol ● denotes element-by-element vector multiplication and ⊙ is the vector dot product. Observe that the detector consists of a zero-memory nonlinear part (g,g', etc...) and the locally optimal nonlinearity with memory, $\nabla\varphi/\varphi$, for a signal in noise with a density φ. With $\mathbf{u}$ multivariate Gaussian, $\nabla\varphi/\varphi$ reduces to the usual linear matched filter, $R^{-1}\mathbf{z}$.

Figure 13 summarizes the results of a Monte Carlo simulation to evaluate ARE. (See [7],[12] for details.) The output marginal was specified as Laplacian:

$$f_1(n) = \frac{\alpha}{2} exp(-\alpha|n|)$$

which was generated by a suitable transformation of multivariate Gaussian noise. The Gaussian vector was assumed to be m-dependent, with the correlation function taken as triangular:

$$\rho_i = 1 - |i/m|, \ |i| \le m$$
$$\rho_i = 0, \ |i| > m$$

The nonlinearities required in the detector are determined from $f_1(n)$, and the signal was taken as a constant.

Four detectors were simulated. The first was the locally optimum detector, and the second assumed independent noise. The third was a simplification of the first; the indicated vector multiplication was removed. The fourth detector was the linear matched filter. As indicated above, values for the ARE were computed via simulation; consequently, it is difficult to make general statements. Nevertheless, some observations seem appropriate.

It is clear that for small correlation time, $m \leq 3$ or 4, an independence assumption does not significantly degrade detector performance. Second, with some sort of nonlinear processing, i.e., taking into account non-Gaussian statistics, a reasonable improvement can usually be obtained: for any of the nonlinear detectors compared to the linear one, the ARE for this simulation does not fall below 2. Note that detectors 2 and 3 give similar ARE values. This is interesting since one detector assumes independent noise, while the other simplfies the optimal detector, but keeps the dependency assumption. Clearly, one would like to learn what are the essential common features of the various detectors that lead to improved performance. Then, only these features need be incorporated into a practical receiver.

SUMMARY

Techniques for detection of weak signals in non-Gaussian noise have been considered. The importance of both learning and robust procedures was illustrated by means of an example where a modest deviation from a Gaussian noise assumption (in skewness) led to a substantial increase in the false alarm rate for a linear detector. In contrast, a modified (non-linear) detector was robust and maintained a relatively constant false alarm rate for a wide range of skewness. These analytical results were verified by computer simulation.

Under-ice ambient noise was used in an experiment to illustrate some of the practical difficulties with adaptive procedures; in this case, because of the non-stationarity of the noise, there was a need to incorporate into the adaptive detector parameter estimates with enhanced tracking properties.

The skewness-kurtosis plane was presented as a convenient graphical display to place in evidence the time-varying nature of the non-Gaussian statistics. The notion of overlays was also introduced to define density families and the corresponding likelihood processors. Densities were also characterized in terms of a function related to the score function and tail measures using quantiles. Our preliminary results seem to indicate that detector performance does not appear to be critically dependent on specifying the correct family of densities. More important to performance are accurate estimates of the moments, tail measures, or other parameters which are used to specify the detector.

A particular class of multivariate non-Gaussian densities was defined and the canonical form of the locally optimum detector derived. The results of a simulation to evaluate ARE for the optimum detector and simplified versions were presented. These results indicate that some sort of nonlinear processing which takes into account deviations from Gaussian statistics leads to a large portion of the improved performance.

ACKNOWLEGEMENT

Much of the work covered in this paper is an explication of the doctoral dissertations of Y.F. Huang, A.B. Martinez, E.J. Modugno, and P.F. Swaszek done under the supervision of J.B. Thomas. It is always a pleasure to interact with and acknowledge bright, serious-minded graduate students as they begin their professional careers.

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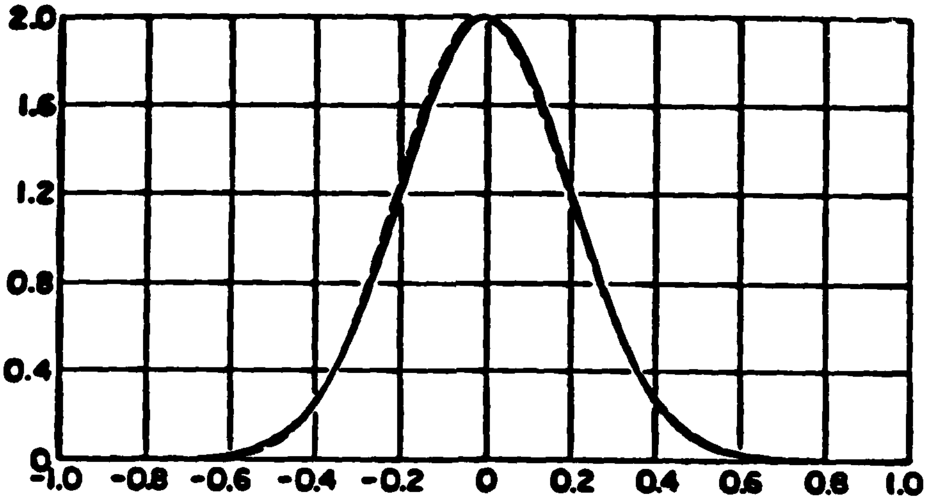

Fig. 1 Probability Density Functions
—— Gaussian, ---- Skewed, ξ_k=0.5

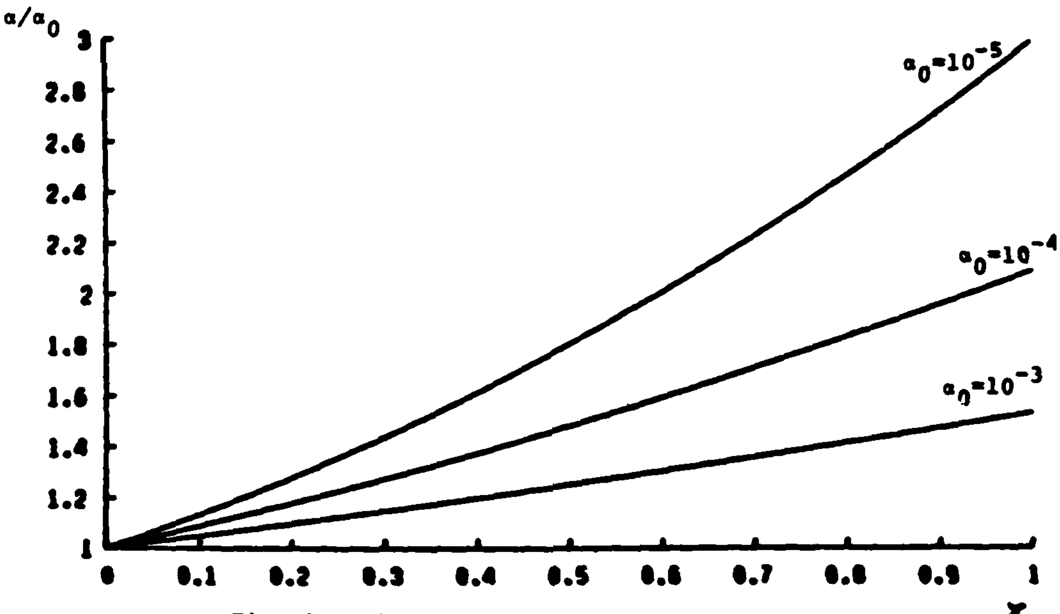

Fig. 2 False Alarm (Normalized) versus Skewness
(Number of observation for a decision =100)

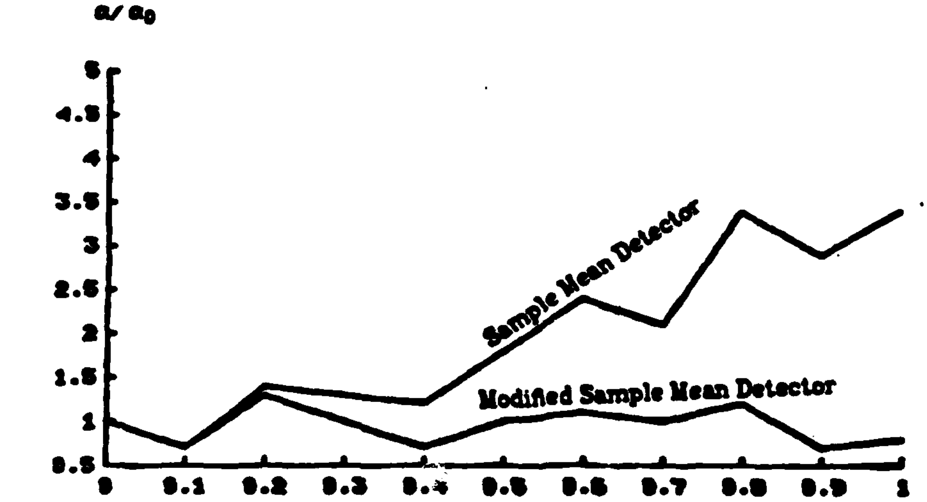

Fig. 3 False Alarm Rate via Simulation

$\alpha_0=10^{-4}$, sample size = 100, 10^4 trials per point

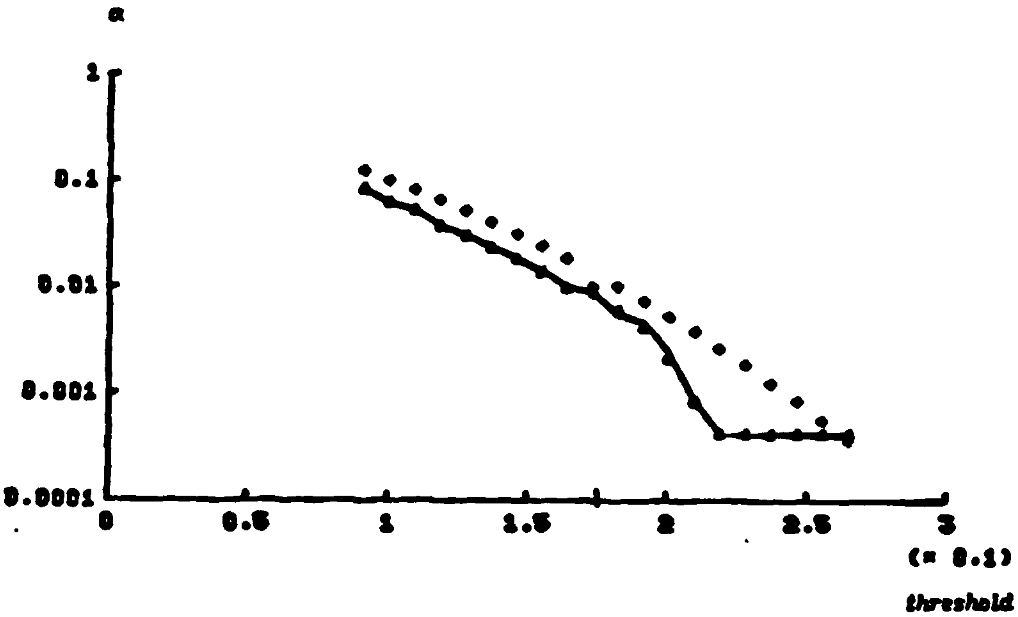

Fig. 4 False Alarm Rate vs. Threshold

++++ Sample mean detector in Gaussian noise
—— Sample mean detector (under-ice noise)
ΔΔΔΔ Modified sample mean detector (under-ice noise)

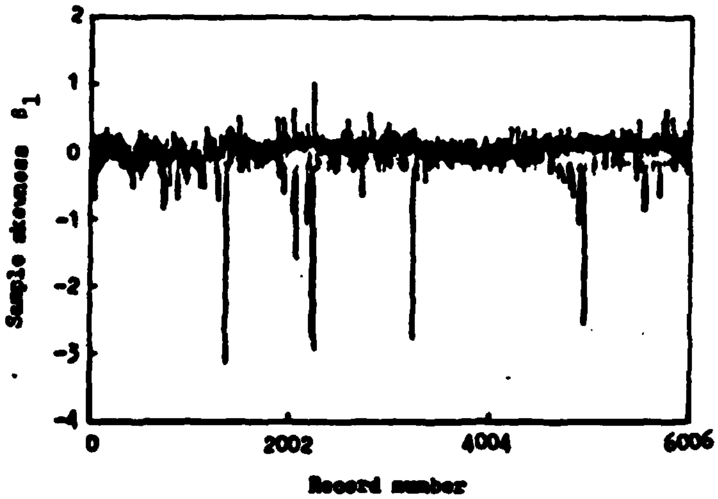

Fig. 5 Sample skewness versus record number.

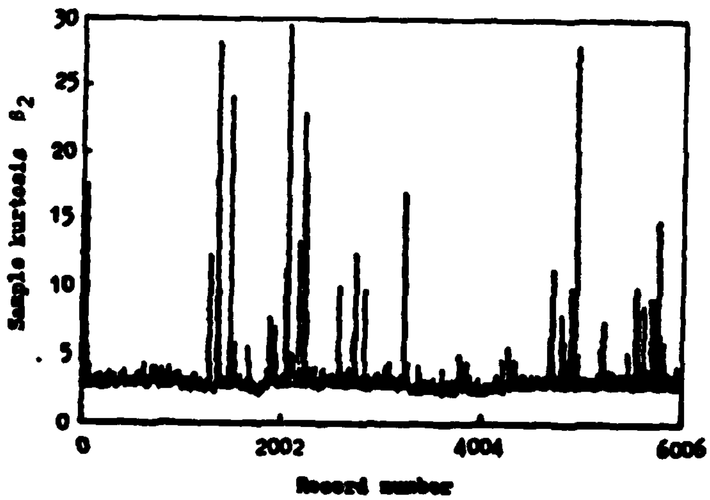

Fig 6 Sample kurtosis versus record number.

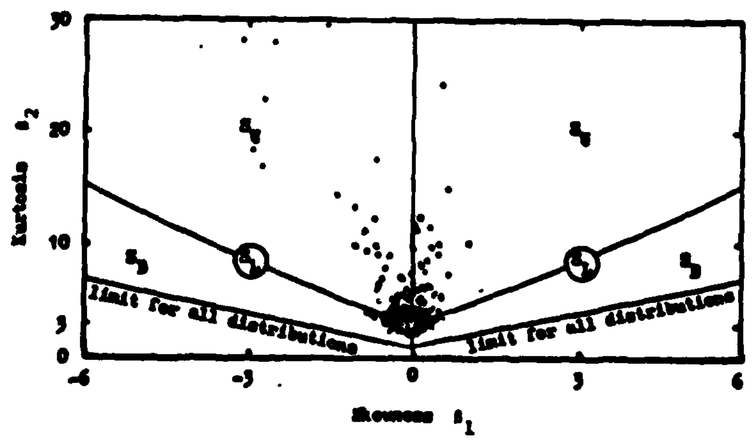

Fig. 7 Scatter-plot of (β_1, β_2) for Johnson family

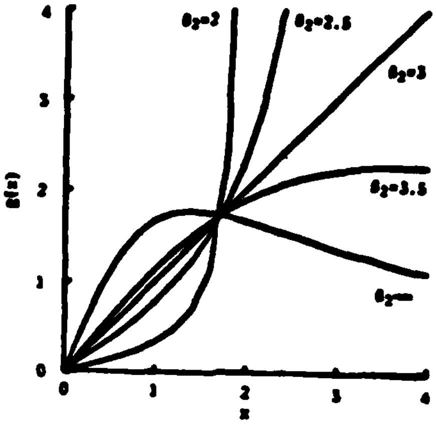

Fig. 8 Pearson Locally Optimal ZNL's

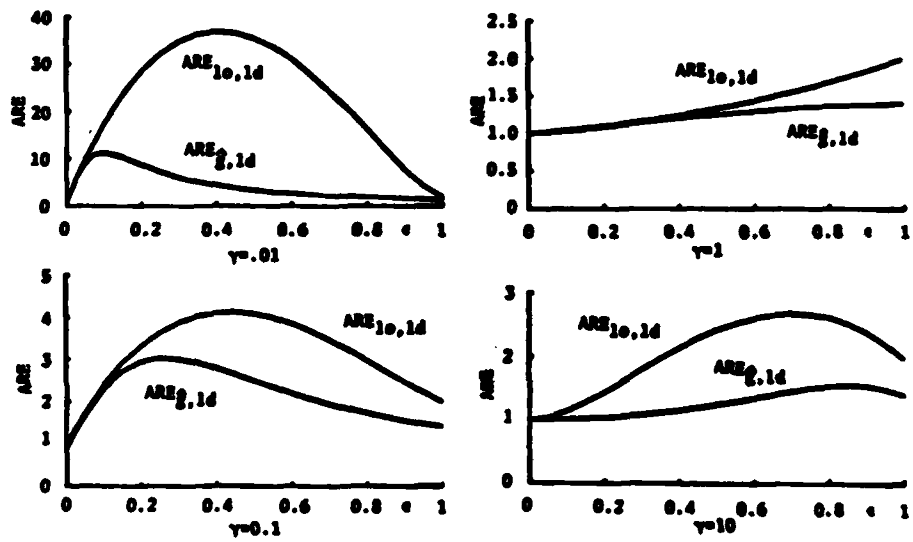

Fig. 9 - $ARE_{lo,ld}$ and $ARE_{g,ld}$ for Gauss-Laplace mixture

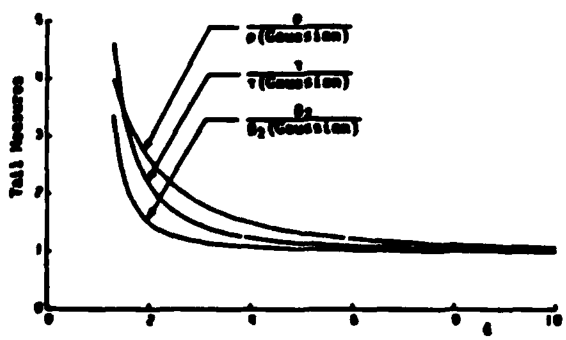

Fig. 10 - Johnson's S_u System: β_2, τ, ρ vs. δ

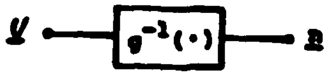

Fig. 11 Transformation Noise

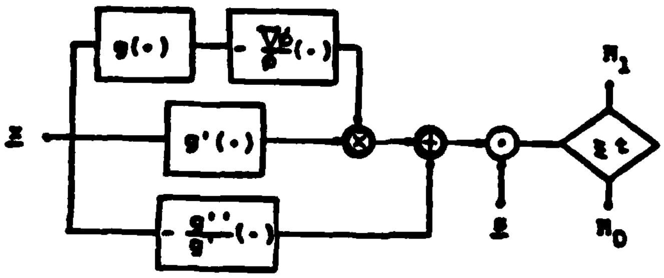

Fig. 12 Locally Optimum Detector

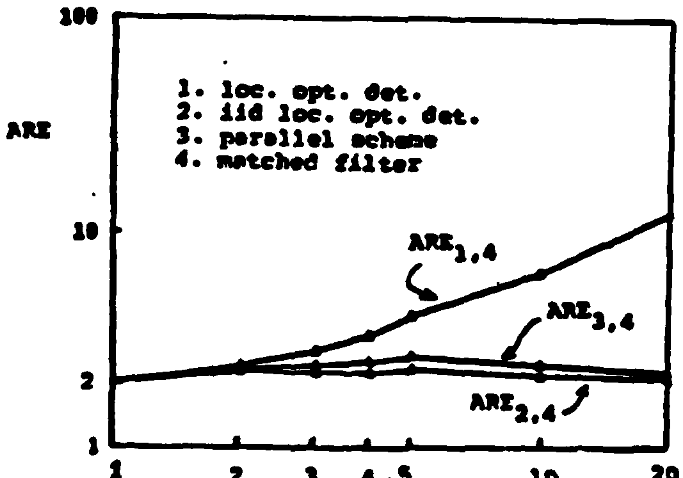

Fig. 13 ARE Comparison

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