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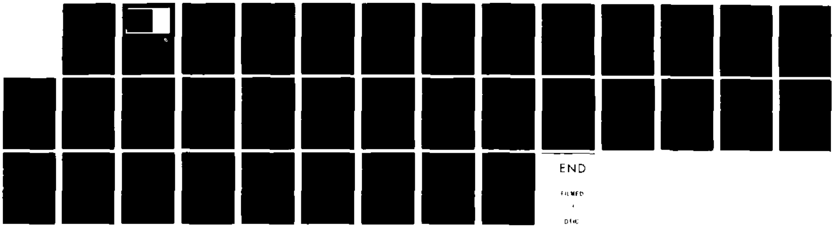

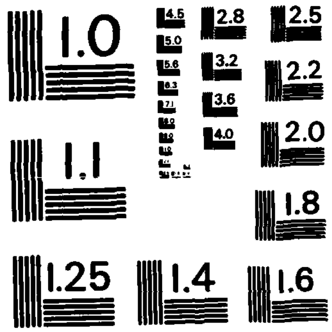
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BOUNDARY CONDITIONS FOR SUPPRESSING RAPIDLY MOVING COMPONENTS IN HYPERBOLIC SYSTEMS II: AN EXAMPLE

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ABSTRACT

This paper is a continuation of [2]. Here we apply the ideas of that paper to the linearized shallow water equations, or, equivalently, the linearized two-dimensional isentropic Euler equations of gas dynamics.

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Key Words: hyperbolic systems, absorbing boundary conditions, multiple time scales, reflection of singularities.

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SIGNIFICANCE AND EXPLANATION

When one computes a numerical approximation to the solution of a partial differential equation, it is sometimes necessary to restrict the computation to a domain which is only a portion of the domain on which the problem is naturally defined. This is done when one is interested in the behavior of the solution only on that part and when a computation over the entire domain would be prohibitively expensive.

In such cases a portion of the boundary of the computational domain represents merely the edge of the computation and corresponds to nothing physical. The task of finding suitable boundary conditions to impose on this part of the boundary can present some analytical and numerical difficulties. This work is concerned with one such difficulty which arises in limited-area meteorological computations. It is also related to the problem of finding conditions which prevent the nonphysical reflection of waves at an artificial boundary.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

BOUNDARY CONDITIONS FOR SUPPRESSING RAPIDLY MOVING COMPONENTS IN HYPERBOLIC SYSTEMS II: AN EXAMPLE

Robert L. Higdon

1. Introduction

This paper continues the discussion of hyperbolic systems begun in [2]. The goal of this work is to find boundary conditions which prevent rapidly moving high-frequency waves from entering the spatial domain on which the system is defined. In the present paper we assemble the ideas of [2] and apply them in detail to an example of physical interest. This is done partly for the sake of the example and partly for the sake of developing the concepts of [2] into a definite method.

We will consider the system

$$(1.1)\quad \frac{\partial}{\partial t}\begin{pmatrix} u \\ v \\ p \end{pmatrix} = \begin{pmatrix} \alpha & & -c \\ & \alpha & \\ -c & & \alpha \end{pmatrix} \frac{\partial}{\partial x}\begin{pmatrix} u \\ v \\ p \end{pmatrix} + \begin{pmatrix} \beta & & \\ & \beta & -c \\ & -c & \beta \end{pmatrix} \frac{\partial}{\partial y}\begin{pmatrix} u \\ v \\ p \end{pmatrix} + (\gamma_{ij})\begin{pmatrix} u \\ v \\ p \end{pmatrix} .$$

The unknowns u, v, p are functions of $x, y,$ and t. For most of this paper the spatial domain we consider will be the half-space $x > 0$. We assume $\alpha \neq 0$ and $|\alpha|, |\beta| << c$. The latter assumption guarantees the presence of propagation speeds having substantially different magnitudes.

The system (1.1) is a symmetric version of the linearized shallow water equations. In this notation $(-\alpha,-\beta)$ is the velocity of the flow about which the system has been linearized, and c is the speed of gravity waves relative to the underlying flow.

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Up to multiplicative factors, the dependent variables u and v are the perturbations in the components of velocity. The variable p is the perturbation in the geopotential. The coefficients γ_{ij} in the undifferentiated term are due partly to Coriolis effects and partly to the process of linearization.

The linearized two-dimensional isentropic Euler equations of gas dynamics can also be written in the form (1.1). In this case c is the speed of sound waves, and p is essentially the perturbation in the density.

In order to find the desired boundary conditions, we first transform the system to an approximate diagonal form in such a way that each of the new dependent variables can be identified as a slow, incoming fast, or outgoing fast portion of the solution. We then set the incoming fast part equal to zero at the boundary, at least approximately. The calculations presented here follow the outline given in Section 5.4 of [2].

Pseudo-differential operators are used extensively throughout this work. The use of these operators is analogous to the formal manipulations of Fourier transforms which are used to state the outline in [2]. The operators are introduced in order to treat systems with variable coefficients and to settle a technical point mentioned in Section 3.2 of [2]. An informal discussion of the operators is given in Section 4.1 of [2], and in the Appendix of the present paper we define them precisely and state some of their basic properties. In this work they are regarded as a formalism for studying the high-frequency asymptotic behavior of the solution.

In Section 2 we uncouple the system (1.1), and in Sections 3 and 4 we derive boundary conditions from the results of the uncoupling. Numerical tests of the conditions are discussed in Section 5. An analytical justification of the boundary conditions is outlined in Section 4.2 of [2].

2. Uncoupling the system

We first diagonalize the coefficient matrix of the normal derivative $\frac{\partial}{\partial x}$ in (1.1). To do this we use the matrix

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 1 \\ \sqrt{2} & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \quad . \tag{2.1}$$

The columns of this matrix are normalized eigenvectors of the coefficient matrix in question. When we multiply (1.1) on the left by the inverse of (2.1) and make the appropriate change of dependent variable, the result is the system

$$w_t = \begin{pmatrix} \alpha & & \\ & \alpha-c & \\ & & \alpha+c \end{pmatrix} w_x + Dw_y + Ew \quad , \tag{2.2}$$

where

$$w = \begin{pmatrix} v \\ \frac{1}{\sqrt{2}}(u+p) \\ \frac{1}{\sqrt{2}}(u-p) \end{pmatrix} \quad . \tag{2.3}$$

D and E can be written explicitly, but we will wait until we solve for w_x.

The system (2.2) can be used to make a preliminary identification of the slow, incoming fast, and outgoing fast portions of the solution. Since $|\alpha| << c$, we can say roughly that the first component v is a slow component and that $u+p$ and $u-p$ are incoming and outgoing fast components, respectively. In order to suppress the incoming fast part of the solution one could therefore require $u+p = 0$ at the boundary $x = 0$. If $\alpha < 0$, then one would also prescribe a value for v in order to have a well-posed problem. The trouble with this approach is that it ignores the effect of the terms Dw_y and Ew. Our identification of the various parts of the solution is therefore not very accurate. The purpose of the uncoupling process is to produce a more accurate identification and thereby make it possible to find boundary conditions which are more effective at suppressing the incoming fast part.

2.1. Uncoupling the leading order part of the system

It is necessary to solve for w_x in (2.2). When this is done the result is

$$w_x = Aw_t + Bw_y + C_w \quad , \tag{2.4}$$

where

$$A = \begin{pmatrix} \frac{1}{\alpha} & & \\ & \frac{1}{\alpha-c} & \\ & & \frac{1}{\alpha+c} \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{-\beta}{\alpha} & \frac{c}{\alpha\sqrt{2}} & \frac{-c}{\alpha\sqrt{2}} \\ \frac{c}{\sqrt{2}\,(\alpha-c)} & \frac{-\beta}{\alpha-c} & 0 \\ \frac{-c}{\sqrt{2}\,(\alpha+c)} & 0 & \frac{-\beta}{\alpha+c} \end{pmatrix} \tag{2.5}$$

$$C = -\begin{pmatrix} \frac{\gamma_{22}}{\alpha} & \frac{\gamma_{21}+\gamma_{23}}{\alpha\sqrt{2}} & \frac{\gamma_{21}-\gamma_{23}}{\alpha\sqrt{2}} \\ \frac{\gamma_{12}+\gamma_{32}}{\sqrt{2}\,(\alpha-c)} & \frac{\gamma_{11}+\gamma_{13}+\gamma_{31}+\gamma_{33}}{2(\alpha-c)} & \frac{\gamma_{11}-\gamma_{13}+\gamma_{31}-\gamma_{33}}{2(\alpha-c)} \\ \frac{\gamma_{12}-\gamma_{32}}{\sqrt{2}\,(\alpha+c)} & \frac{\gamma_{11}+\gamma_{13}-\gamma_{31}-\gamma_{33}}{2(\alpha+c)} & \frac{\gamma_{11}-\gamma_{13}-\gamma_{31}+\gamma_{33}}{2(\alpha+c)} \end{pmatrix} .$$

In analogy with the outline in Section 5.4 of [2], we write the system (2.4) in the form

$$w_x = Hw + Cw + Ew \quad , \tag{2.6}$$

where H is the operator whose symbol is given by

$$\sigma_H = i\xi A + i\omega\phi B \quad , \tag{2.7}$$

and E is the operator with symbol $i\omega(1-\phi)B$. ϕ is the smooth cutoff function which is used to restrict attention to the conical set Γ in the (ω,ξ) space in which fast waves can be found. As noted in the outline, the term Ew can be regarded as an insignificant error term.

In order to uncouple the leading order part of (2.6), we need to find a symbol q such that $q\sigma_H q^{-1}$ is closer to diagonal form than σ_H, at least for small $\frac{\omega}{\xi}$. To do this we will use the perturbation method given in the Appendix in [2]. That is, we will find a matrix M such that

$$\begin{aligned}(I + \tfrac{\omega}{\xi}\phi M)(A + \tfrac{\omega}{\xi}\phi B)(I + \tfrac{\omega}{\xi}\phi M)^{-1} \\ = \text{diagonal matrix} + (o(\tfrac{\omega}{\xi}\phi)^2) \quad .\end{aligned} \tag{2.8}$$

We will then let $q = I + \frac{\omega}{\xi}\phi M$ and apply to (2.6) the pseudo-differential operator which is defined by the symbol q.

Satisfying the condition (2.8) amounts to setting the off-diagonal elements of $MA - AM + B$ equal to zero. A calculation shows that M can be taken to be

$$M = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -(\alpha-c) & -(\alpha+c) \\ \alpha & 0 & 0 \\ \alpha & 0 & 0 \end{pmatrix} \tag{2.9}$$

and that $(I + \varepsilon M)(A + \varepsilon B)(I + \varepsilon M)^{-1}$ is then equal to

$$\begin{pmatrix} \frac{1}{\alpha} & & \\ & \frac{1}{\alpha-c} & \\ & & \frac{1}{\alpha+c} \end{pmatrix} + \varepsilon \begin{pmatrix} -\frac{\beta}{\alpha} & & \\ & -\frac{\beta}{\alpha-c} & \\ & & -\frac{\beta}{\alpha+c} \end{pmatrix} + o(\varepsilon^2) \quad . \tag{2.10}$$

The off-diagonal elements in (2.9) are determined uniquely by the condition (2.8), but the diagonal elements may be chosen arbitrarily. For convenience we have set these equal to zero.

Now define the symbol q by

$$q = I + \frac{\omega}{\xi}\phi M \quad , \tag{2.11}$$

where M is given in (2.9), and let Q be the pseudo-differential operator with symbol q. When the operator Q is applied to the system (2.6), the result is

$$(Qw)_x = (QHQ^{-1})Qw + (QCQ^{-1} + Q_xQ^{-1})Qw + QEw \quad . \tag{2.12}$$

Here Q_x denotes the operator with symbol q_x, and Q^{-1} is a parametrix, or approximate inverse, of the operator Q. Parametrices are defined in the Appendix.

The composition law for pseudo-differential operators implies that the leading symbol of QHQ^{-1} is equal to $q\sigma_H q^{-1}$. According to (2.7) and (2.11) this is given by

$$q\sigma_H q^{-1} = (I + \frac{\omega}{\xi}\phi M)(i\xi A + i\omega\phi B)(I + \frac{\omega}{\xi}\phi M)^{-1} \quad .$$

If we factor out $i\xi$ and identify $\frac{\omega}{\xi}\phi$ with ε in (2.10), we can conclude that $q\sigma_H q^{-1}$ is equal to $\sigma_G + (\frac{\omega^2}{\xi}\phi^2)$, where

$$\sigma_G = i\xi\begin{pmatrix} \frac{1}{\alpha} & & \\ & \frac{1}{\alpha-c} & \\ & & \frac{1}{\alpha+c} \end{pmatrix} + i\omega\phi\begin{pmatrix} \frac{-\beta}{\alpha} & & \\ & \frac{-\beta}{\alpha-c} & \\ & & \frac{-\beta}{\alpha+c} \end{pmatrix} \quad . \tag{2.13}$$

Let G denote the operator whose symbol is σ_G. The system (2.12) can then be written in the form

$$\frac{\partial w_0}{\partial x} = Gw_0 + Zw_0 + o(\frac{\omega^2}{\xi}\phi^2)w + o(\omega(1-\phi))w \quad , \tag{2.14}$$

where $w_0 = Qw$.

The operator Z is associated with the zero-order terms appearing explicitly in (2.12) and with the terms of order zero or less which arise in the expansion of the symbol of QHQ^{-1}. In a moment we will discuss this further. The term $o(\frac{\omega^2}{\xi}\phi^2)w$ denotes the effect of an operator whose symbol is dominated by $\frac{\omega^2}{\xi}\phi^2$ and which is a result of the error in uncoupling the leading order part of the system (2.6). The term $o(\omega(1-\phi))w$ represents the effect of the operator E which appears in (2.12). This is an insignificant error term.

2.2. Uncoupling the term of order zero

The system is now partially uncoupled near normal incidence, since the symbol of G is diagonal. We next need to reduce the coupling caused by the zero-order operator Z, which is given by

$$Z = QCQ^{-1} + Q_xQ^{-1} + (\text{terms of order zero or less arising from the expansion of } QHQ^{-1}) \quad . \tag{2.15}$$

The immediate task is to identify explicitly the leading symbol of this operator.

A short calculation shows that the first two terms in (2.15) are given by

$$QCQ^{-1} + Q_xQ^{-1} = C + \text{order } (-1) + o(\frac{\omega}{\xi}\phi) \quad .$$

We will regard terms of order $\frac{\omega}{\xi}\phi$ as error terms, since $\frac{\omega}{\xi}\phi$ is no larger than the term $\frac{\omega^2}{\xi}\phi^2$ which has already appeared in (2.14). The terms of order zero or less in the expansion of QHQ^{-1} can be calculated using the composition law for pseudo-differential operators. The calculation is long and tedious and will not be given here. Details can be found in [1]. The terms are given by

$$\phi M \frac{\partial A}{\partial y} + o(\frac{\omega}{\xi}\phi) + \text{order } (-1) \quad .$$

Terms involving derivatives of ϕ should also appear here, but these have been omitted since they are nonzero only near the edge of Γ. The symbol of the operator Z is then given by

$$C + \phi MA_y + o(\frac{\omega}{\xi}\phi) + \text{order } (-1) \quad .$$

The system (2.14) can therefore be written in the form

$$\frac{\partial w_0}{\partial x} = Gw_0 + Z_0 w_0 + o(\frac{\omega^2}{\xi}\phi) + o(\frac{\omega}{\xi}\phi)w + o(\omega(1-\phi))w + (\text{order } -1)w \quad , \tag{2.16}$$

where Z_0 is the operator whose symbol is $\phi(C + MA_y)$. The matrix C has been split into the sum $\phi C + (1-\phi)C$ in order to give a neater form to certain formulas which will appear later.

We now use the method outlined in Section 5.4 of [2] to uncouple the term of order zero. That is, we apply an operator of the form $I + K$ to (2.16), where K has order -1, and then make the corresponding change of dependent variable. We then choose K so that the zero-order operator in the transformed system has a diagonal leading symbol. A short calculation shows that this operator is $KG - GK + Z_0$. The calculation requires the expansion $(I+K)^{-1} \sim I-K+\ldots$, which is given in the Appendix. It follows that we need to choose K so that

$$\sigma_K\sigma_G - \sigma_G\sigma_K + \phi(C + MA_y) = \text{diagonal matrix} \quad . \tag{2.17}$$

Here σ_K and σ_G are the symbols of K and G, respectively, and the third term is the symbol of Z_0. σ_G is given in (2.13), A and C are given in (2.5), and M is given in (2.9).

A method for solving (2.17) is given in the Appendix in [2]. After a certain amount of labor one can obtain

$$\sigma_K = \frac{\phi}{i\xi(1 - \frac{\omega}{\xi}\phi\beta)c}\begin{pmatrix} 0 & k_{12} & k_{13} \\ k_{21} & 0 & k_{23} \\ k_{31} & k_{32} & 0 \end{pmatrix} \quad , \tag{2.18}$$

where

$$k_{12} = \frac{1}{\sqrt{2}} [(\gamma_{21}+\gamma_{23})(\alpha-c) - \alpha(\alpha_y-c_y)]$$

$$k_{13} = \frac{1}{\sqrt{2}} [(-\gamma_{21}+\gamma_{23})(\alpha+c) + \alpha(\alpha_y+c_y)]$$

$$k_{21} = \frac{1}{\sqrt{2}} [-\alpha(\gamma_{12}+\gamma_{32}) - \alpha_y(\alpha-c)]$$

$$k_{31} = \frac{1}{\sqrt{2}} [\alpha(\gamma_{12}-\gamma_{32}) + \alpha_y(\alpha+c)]$$

$$k_{23} = \frac{1}{4} (-\gamma_{11}+\gamma_{13}-\gamma_{31}+\gamma_{33})(\alpha+c)$$

$$k_{32} = \frac{1}{4} (\gamma_{11}+\gamma_{13}-\gamma_{31}-\gamma_{33})(\alpha-c) \quad .$$

Equation (2.17) does not impose any conditions on the diagonal elements of K. For convenience we have set these equal to zero.

The operator $I + K$ transforms the system (2.16) into the form

$$\frac{\partial w_1}{\partial x} = Gw_1 + (\text{diagonal operator of order zero})w_1 + (\text{order } (-1))w$$

(2.19)

$$+ o(\frac{\omega^2}{\xi} \phi)w + o(\frac{\omega}{\xi} \phi)w + o(\omega(1-\phi))w \quad ,$$

where $w_1 = (I+K)w_0 = (I+K)Qw$. The symbol of K is given in (2.18), the symbol of Q is given in (2.11), and the components of w are given in (2.3). This represents all of the uncoupling which we will do for this system.

3. Derivation of boundary conditions for the fast part of the solution.

It is now time to use the results of the uncoupling process to derive boundary conditions for the system (1.1). It is necessary to identify the incoming fast component for the partially uncoupled system (2.19) and then find conditions which suppress this component at the boundary $x = 0$.

The symbol of the operator G which appears in (2.19) is given in (2.13) and is equal to

$$\sigma_G = i\xi \begin{pmatrix} \frac{1}{\alpha} & & \\ & \frac{1}{\alpha-c} & \\ & & \frac{1}{\alpha+c} \end{pmatrix} + i\omega\phi \begin{pmatrix} \frac{-\beta}{\alpha} & & \\ & \frac{-\beta}{\alpha-c} & \\ & & \frac{-\beta}{\alpha+c} \end{pmatrix} .$$

Since $|\alpha \pm c| >> |\alpha|$, the second and third components of w_1 in (2.19) are the rapidly moving portions of the solution. The second is the incoming component, since $\alpha - c < 0$. We need to use the identity $w_1 = (I+K)Qw$ to find an explicit formula for this component and then use this formula to find suitable boundary conditions for (1.1).

The dependent variable w_1 is given by

$$\begin{aligned} w_1 &= (I+K)Qw \\ &= "(I + \sigma_k)" \circ "(I + \frac{\omega}{\xi}\phi M)"w \quad . \end{aligned} \tag{3.1}$$

Here we have used the expression (2.11) for the symbol of Q. The quote marks denote pseudo-differential operators having the stated symbols, and the small circle denotes composition of operators. (3.1) can be written

$$w_1 = "(\phi + \sigma_k)" \circ "(\phi + \frac{\omega}{\xi}\phi M)"w + o(1-\phi) \quad . \tag{3.2}$$

In order to produce cleaner formulas later on we have used the cut-off function ϕ to restrict the solution to the set Γ in the (ω,ξ) space in which the fast waves can be found.

According to the composition law, the symbol of the composition in (3.2) is

(3.3)
$$(\phi + \sigma_K)(\phi + \frac{\omega}{\xi}\phi M) + \frac{1}{i}\frac{\partial}{\partial \xi}(\phi + \sigma_k)\frac{\partial}{\partial t}(\phi + \frac{\omega}{\xi}\phi M)$$
$$+ \frac{1}{i}\frac{\partial}{\partial \omega}(\phi + \sigma_K)\frac{\partial}{\partial y}(\phi + \frac{\omega}{\xi}\phi M) + \text{order } (-2) \quad .$$

The derivatives of $\phi + \sigma_K$ with respect to ω and ξ are of order -2, since σ_K has order -1 and we are ignoring derivatives of ϕ. (3.3) is therefore

$$(\phi + \sigma_K)(\phi + \frac{\omega}{\xi}\phi M) + \text{order } (-2)$$

(3.4)
$$= \phi^2 + \phi\sigma_K + \frac{\omega}{\xi}\phi^2 M + \frac{\omega}{\xi}\phi\sigma_K M + \text{order } (-2)$$

$$= \phi^2 + \phi\sigma_K + \frac{\omega}{\xi}\phi^2 M + o(\frac{\omega}{\xi^2}\phi^2) + \text{order } (-2) \quad .$$

To obtain the last line we used (2.18) to conclude $\sigma_K = o(\frac{1}{\xi}\phi)$. (3.2) and (3.4) now imply

(3.5) $$w_1 = "(\phi^2 + \phi\sigma_K + \frac{\omega}{\xi}\phi^2 M)"w + o(\frac{\omega}{\xi^2}\phi^2) + o(1-\phi) + \text{order } (-2) \quad .$$

The error terms in (3.5) can be ignored in the system (2.19), since their only contributions in that equation are error terms having the same order as terms which are already present. In particular, $G \circ o(\frac{\omega}{\xi^2}\phi^2) = (o(\xi) + o(\omega)) \circ o(\frac{\omega}{\xi^2}\phi^2) = o(\frac{\omega}{\xi}\phi^2) + o(\frac{\omega^2}{\xi^2}\phi^2)$. The system (2.19) can therefore be written

(3.6)
$$\frac{\partial z}{\partial x} = Gz + (\text{diagonal of order zero})z + (\text{order } (-1))w$$
$$+ o(\frac{\omega^2}{\xi}\phi)w + o(\frac{\omega}{\xi}\phi)w + o(\omega(1-\phi))w \quad ,$$

where

(3.7) $$z = "(\phi^2 + \phi\sigma_K + \frac{\omega}{\xi}\phi^2 M)"w$$

σ_K is given in (2.18) and M is given in (2.9).

The second component of z is the incoming fast component which we need to suppress. According to (3.7) and the expressions for σ_K and M, this component is

$$z_2(x,y,t) =$$

$$(3.8) \quad \int e^{i(\omega y+\xi t)}\phi^2 \left\{\left[(\frac{\omega}{\xi})\frac{\alpha}{\sqrt{2}} - (\frac{1}{\alpha\sqrt{2}})\frac{\alpha(\gamma_{12}+\gamma_{32}) + \alpha_y(\alpha-c)}{i\xi(1-\frac{\omega}{\xi}\phi\beta)}\right]\hat{w}^{(1)} + \hat{w}^{(2)} - (\frac{1}{4c})\frac{(\alpha+c)(\gamma_{11}-\gamma_{13}+\gamma_{31}-\gamma_{33})}{i\xi(1-\frac{\omega}{\xi}\phi\beta)}\hat{w}^{(3)}\right\} d\omega d\xi \quad .$$

Here $w^{(1)}$, $w^{(2)}$, and $w^{(3)}$ denote the components of w. These are given explicitly in (2.3). We note that (3.8) is a perturbation of $w^{(2)}$.

This expression can be simplified somewhat. The factor $(1 - \frac{\omega}{\xi}\phi\beta)^{-1}$ can be approximated by $1 + o(\frac{\omega}{\xi}\phi)$. When this is multiplied by $(i\xi)^{-1}$ the result is $(i\xi)^{-1} + o(\frac{\omega}{\xi^2}\phi)$. For reasons stated earlier, terms of order $\frac{\omega}{\xi^2}\phi$ can be omitted without affecting the order of the error terms in the system (3.6). We can therefore replace (3.8) with a new fast quantity

$$(3.9) \quad \int e^{i(\omega y+\xi t)}\phi^2 \left\{\left[(\frac{\omega}{\xi})\frac{\alpha}{\sqrt{2}} - \frac{1}{i\xi}(\frac{1}{\alpha\sqrt{2}})(\alpha(\gamma_{12}+\gamma_{32}) + \alpha_y(\alpha-c))\right]\hat{w}^{(1)} + \hat{w}^{(2)} - \frac{1}{i\xi}(\frac{1}{4c})(\alpha+c)(\gamma_{11}-\gamma_{13}+\gamma_{31}-\gamma_{33})\hat{w}^{(3)}\right\} d\omega d\xi \quad .$$

We need to find a condition which suppresses (3.9) at the boundary $x = 0$. If the coefficients are independent of y and t, then the bracketed quantity in the integrand is the Fourier transform of (3.9), give or take a factor ϕ^2, and we can accomplish what we want by setting this quantity equal to zero at the boundary. If we do this, clear denominators, and then invert the Fourier transform, we obtain

$$\frac{\partial w^{(2)}}{\partial t} + \frac{\alpha}{\sqrt{2}}\frac{\partial w^{(1)}}{\partial y} - \frac{1}{c\sqrt{2}}[\alpha(\gamma_{12}+\gamma_{32}) + \alpha_y(\alpha-c)]w^{(1)}$$

$$- (\frac{\alpha+c}{4c})(\gamma_{11} - \gamma_{13} + \gamma_{31} - \gamma_{33})w^{(3)} = 0 \quad \text{for} \quad x = 0 \quad . \tag{3.10}$$

In the next section we will say more about the validity of this boundary condition.

The condition (3.10) is written in terms of the components of the vector w which appears in the system (2.2). We now use the definition (2.3) of w to write the condition in terms of the variables u, v, and p in the original system (1.1). The result is

$$\frac{\partial}{\partial t}(u+p) + \alpha\frac{\partial v}{\partial y} - \frac{1}{c}[\alpha(\gamma_{12} + \gamma_{32}) + \alpha_y(\alpha-c)]v$$

$$- (\frac{\alpha+c}{4c})(\gamma_{11} - \gamma_{13} + \gamma_{31} - \gamma_{33})(u-p) = 0 \tag{3.11}$$

$$\text{for} \quad x = 0 \quad .$$

4. Discussion of the boundary conditions

In this section we discuss the boundary condition (3.10) and generate some additional conditions which will be needed for the test problem presented in Section 5.

If the coefficients in the system depend on y or t, the derivation of (3.10) from (3.9) is not valid. However, we can show that (3.10) is still useful in this case. Suppose that this condition holds, and write it in the simpler form

$$\frac{\partial w^{(2)}}{\partial t} + \frac{\alpha}{\sqrt{2}} \frac{\partial w^{(1)}}{\partial y} + F_1 w^{(1)} + F_3 w^{(3)} = 0 \quad . \tag{4.1}$$

If we apply the operator having the symbol $\frac{\phi^2}{i\xi}$ to (4.1), the result is

$$\begin{aligned} 0 &= \frac{"\phi^2"}{i\xi} \circ \left["i\xi" w^{(2)} + "(i\omega \frac{\alpha}{\sqrt{2}} + F_1)" w^{(1)} + F_3 w^{(3)} \right] \\ &= "\phi^2" w^{(2)} + "\phi^2 \left[(\frac{\omega}{\xi}) \frac{\alpha}{\sqrt{2}} + \frac{F_1}{i\xi} + o(\frac{\omega}{\xi^2}) + o(\frac{1}{\xi^2}) \right]" w^{(1)} \\ &\quad + "\phi^2 \left[\frac{F_3}{i\xi} + o(\frac{1}{\xi^2}) \right]" w^{(3)} \\ &= \int e^{i(\omega y + \xi t)} \phi^2 \left[\left\{ (\frac{\omega}{\xi}) \frac{\alpha}{\sqrt{2}} + \frac{F_1}{i\xi} \right\} \hat{w}^{(1)} + \hat{w}^{(2)} + \frac{F_3}{i\xi} \hat{w}^{(3)} \right] d\omega d\xi \\ &\quad + o(\frac{\omega}{\xi} \phi^2) w + (\text{order } (-2)) w \quad . \end{aligned} \tag{4.2}$$

According to (3.9) and the definitions of F_1 and F_3 implied by (4.1), the integral in the last line is the approximation to the incoming fast part of the solution. The entire last line is equal to zero, so this fast part must be equal to

$$o(\frac{\omega}{\xi^2} \phi^2) w + \text{order } (-2) w$$

at $x = 0$. The incoming fast part is therefore small compared to w for large frequencies and for angles away from tangential incidence.

It may be worthwhile to compare the boundary condition (3.11), or (3.10), with the condition

$$w^{(2)} = u+p = 0 \tag{4.3}$$

which was mentioned early in Section 2. This condition was derived from the system (2.2), in which the coefficient of the x-derivative is a diagonal matrix. The newer condition (3.11) is based on the incoming fast variable (3.8) which was obtained from a more extensive uncoupling of the system. An inspection of (3.8) shows that this variable can be considered a perturbation of $w^{(2)} = u+p$, so in some sense (3.11) is a refinement of (4.3). One obvious difference between the two is the presence of derivatives in (3.11). This is a result of the need to clear denominators in the Fourier transform of the incoming fast part. The other difference is the presence of terms which do not involve $u + p$. The term $\alpha \frac{\partial v}{\partial y}$ is a result of uncoupling the leading symbol, and the other terms in (3.11) are the result of reducing the coupling caused by terms of order zero. The term $\alpha_y(\alpha-c)$ corresponds to the part of the zero-order coupling which resulted from the prior uncoupling of the leading symbol. If we had not carried out the lower-order uncoupling, then the boundary condition would have been $\frac{\partial}{\partial t}(u+p) + \alpha \frac{\partial v}{\partial y} = 0$.

Up to now we have discussed boundary conditions only for the incoming fast part of the solution. If the boundary $x = 0$ is an inflow boundary, i.e., if $\alpha < 0$ in (2.2), then it is also necessary to specify a condition for the slow part. One possibility is to use the system (2.2) to obtain the condition

$$w^{(1)} = v = \text{given function, for } x = 0 \quad . \tag{4.4}$$

Another possibility is to try to base a boundary condition on the more extensively uncoupled system (3.6). We could presumably prescribe a value for the Fourier transform of the slow component in (3.6), clear denominators, and then apply an inverse transform to obtain an inhomogeneous boundary condition analogous to (3.11).

The first approach suggested here is acceptable, but the second one is not. The use of the cutoff function ϕ means that we have uncoupled the system only on the wedge Γ in the (ω,ξ) space which corresponds to rapidly moving waves. This is clearly no restriction when we are seeking boundary conditions which suppress the incoming fast part of the solution. But in the present case it is a major restriction, since the slow part of the solution is associated with the entire (ω,ξ) space. The partially uncoupled system (3.6) cannot give a full description of the slow part, so there is no point in trying to use this system to find an improvement of the condition (4.4).

We therefore prescribe the conditions

$$\begin{aligned} &\frac{\partial}{\partial t}(u+p) + \alpha\frac{\partial v}{\partial y} - \frac{1}{c}[\alpha(\gamma_{12}+\gamma_{32}) + \alpha_y(\alpha-c)]v \\ &\qquad - \left(\frac{\alpha+c}{4c}\right)(\gamma_{11}-\gamma_{13}+\gamma_{31}-\gamma_{33})(u-p) = 0 \qquad (4.5) \\ &v = \text{given function, for } x = 0\ , \end{aligned}$$

if $x = 0$ defines an inflow boundary. In the case of an outflow boundary only the first condition should be used.

Up to this point the discussion has been limited to problems defined on the half-plane $x > 0$. In the next section we present the results of some numerical computations for the system (1.1) defined on the unit square $0 < x < 1$, $0 < y < 1$. For these computations we need boundary conditions analogous to (4.5) for the boundary segments $y = 0$, $x = 1$, and $y = 1$.

These conditions can be obtained through suitable orthogonal transformations of the independent space variables x and y, together with the corresponding transformations of the velocity components u and v. The necessary calculations are outlined in [1]. The results are:

Boundary segment $y = 0$:

$$\frac{\partial}{\partial t}(v+p) + \beta \frac{\partial u}{\partial x} - \frac{1}{c}[\beta(\gamma_{21}+\gamma_{31}) + \beta_x(\beta-c)]u$$

$$- \left(\frac{\beta+c}{4c}\right)(\gamma_{22}-\gamma_{23}+\gamma_{32}-\gamma_{33})(v-p) = 0$$

u = given function.

Boundary segment $x = 1$:

$$\frac{\partial}{\partial t}(-u+p) - \alpha \frac{\partial v}{\partial y} - \frac{1}{c}[-\alpha(-\gamma_{12}+\gamma_{32}) - \alpha_y(-\alpha-c)]v$$

(4.6) $$- \left(\frac{-\alpha+c}{4c}\right)(\gamma_{11}+\gamma_{13}-\gamma_{31}-\gamma_{33})(-u-p) = 0$$

v = given function .

Boundary segment $y = 1$:

$$\frac{\partial}{\partial t}(-v+p) - \beta \frac{\partial u}{\partial x} - \frac{1}{c}[-\beta(-\gamma_{21}+\gamma_{31}) - \beta_x(-\beta-c)]u$$

$$- \left(\frac{-\beta+c}{4c}\right)(\gamma_{22}+\gamma_{23}-\gamma_{32}-\gamma_{33})(-v-p) = 0$$

u = given function .

For each segment the first condition is the one which is intended to suppress the incoming fast part of the solution. The second condition prescribes a value for the slow variable, and it should be imposed only when the segment is an inflow boundary.

5. Numerical Computations

In this section we present the results of some numerical computations involving the boundary conditions which have just been derived. We consider the system

$$(5.1) \quad \frac{\partial}{\partial t}\begin{pmatrix} u \\ v \\ p \end{pmatrix} = \begin{pmatrix} -1 & & -3 \\ & -1 & \\ -3 & & -1 \end{pmatrix} \frac{\partial}{\partial x}\begin{pmatrix} u \\ v \\ p \end{pmatrix} + \begin{pmatrix} 0 & & \\ & 0 & -3 \\ & -3 & 0 \end{pmatrix} \frac{\partial}{\partial y}\begin{pmatrix} u \\ v \\ p \end{pmatrix} + (\gamma_{ij}) \begin{pmatrix} u \\ v \\ p \end{pmatrix}$$

on the unit square $0 < x < 1, 0 < y < 1$. This system is a special case of the system (1.1). Two different choices for the matrix (γ_{ij}) will be considered.

We wish to compare three types of boundary conditions for this system. The first of these is obtained by diagonalizing the coefficient of the normal derivative and then defining boundary conditions in terms of the dependent variables in the new system. These variables will be referred to as "characteristic variables". This was discussed early in Section 2, immediately after equation (2.3). For the four sides of the unit square the incoming fast characteristic variables are the quantities in (4.5) and (4.6) which are differentiated with respect to time. The second set of conditions is obtained by uncoupling the leading symbol in the manner described earlier, but then doing nothing about the zero-order coupling in the system. These conditions can be obtained by deleting the zero-order terms in the derivative conditions appearing in (4.5) and (4.6). The third set of boundary conditions is obtained by also uncoupling the zero-order terms in the system of differential equations. These are the conditions (4.5) and (4.6).

We present two separate tests of these conditions, one to demonstrate the effect of uncoupling the leading symbol, and the other to demonstrate the effect of uncoupling both the leading symbol and the zero-order term. In the first case we let $\gamma_{ij} = 0$ for all i,j and use the first two sets of boundary conditions. In the second case we use all three sets of conditions, and we let (γ_{ij}) be the matrix

$$\text{(5.2)} \qquad \begin{pmatrix} 0 & 10 & 0 \\ -10 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

In the computations we set the solution equal to zero when $t = 0$. At the boundary $x = 0$ we set v (see (4.5)) equal to a pulse consisting of half a sine wave in t multiplied by half a sine wave in the tangential variable y. We use homogeneous conditions on the other boundaries. The nonzero part of the solution is due entirely to the nonzero data at the boundary $x = 0$, so it is possible to study the influence of these data by examining the size of the solution in various parts of the (x,y) plane at various times.

In the computation the system is approximated by the leap frog difference scheme. The derivative boundary conditions in (4.5) and (4.6) are approximated by centered differences in the time and tangent variables. The outgoing fast characteristic variables are extrapolated at the boundary using the given differential equation. For this we use centered differences in the time and tangent variables, and we approximate the normal derivative with a forward difference which uses a time average at the back point. At an outflow boundary the slow characteristic variable is extrapolated in the same manner.

The boundaries $y = 0$ and $y = 1$ are characteristic for the system (5.1). At these boundaries we integrate the slow characteristic variable in the boundary using a centered difference approximation. This is an experiment to see if the incoming fast modes can be activated at a characteristic boundary. In our earlier discussion we always assumed that the boundary was noncharacteristic.

The surface pictured in Figure 5.2 and 5.3 are graphs of $(u^2 + v^2 + p^2)^{1/2}$ as functions of x and y for fixed t. The configuration is shown in Figure 5.1.

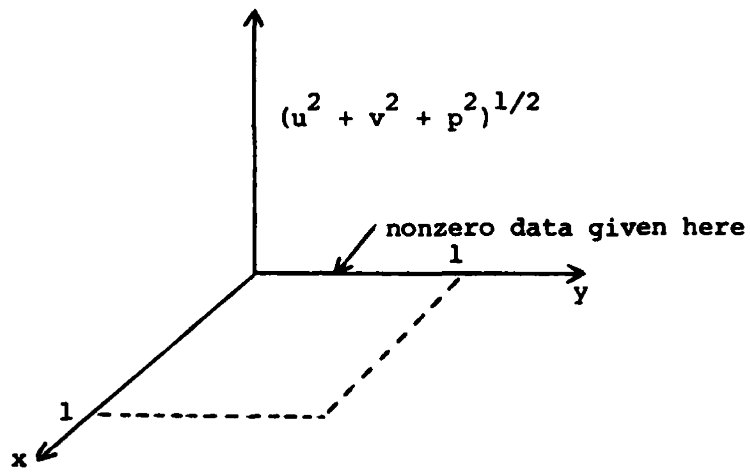


Figure 5.1

We show solutions at times $t = .125$, $.25$, and $.375$. The fast mode entering through the boundary $x = 0$ has normal velocity 4 since $\alpha = -1$ and $c = -3$. Pulses entering on this mode should therefore be visible near the nearest boundary $(x = 1)$ in the graphs for $t = .25$.

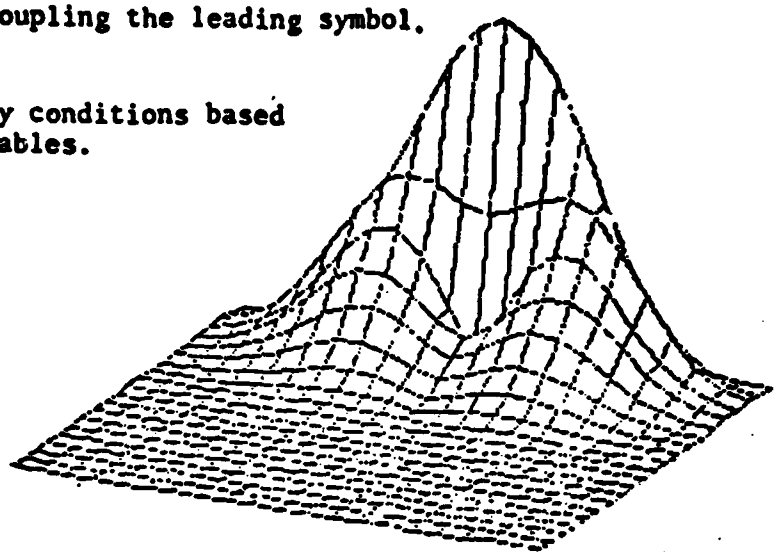
In Figure 5.2 we show the effect of uncoupling the leading symbol. In this case $\gamma_{ij} = 0$ for all i,j. Figure 5.2(a) shows the solution corresponding to the boundary conditions defined in terms of characteristic variables. The solution in Figure 5.2(b) corresponds to the more refined boundary conditions. The second set of conditions is clearly more effective at suppressing the incoming fast part of the solution.

In Figure 5.3 we show the effect of uncoupling both the leading symbol and the term of order zero. In this case the matrix (γ_{ij}) is given by (5.2). The simplest boundary conditions are used in part (a). In part (b) we use the boundary conditions obtained by uncoupling the leading symbol only. The boundary conditions for part (c) are obtained by uncoupling both the leading symbol and the term of order zero. The third set of conditions is clearly the most effective.

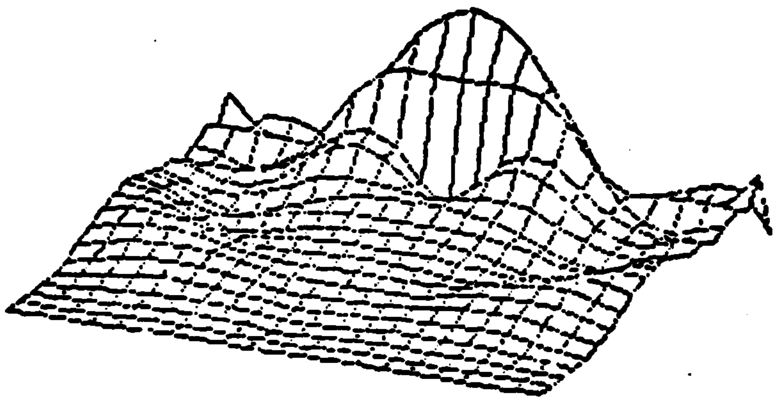
Figure 5.2. Effect of uncoupling the leading symbol.

(a) Solution using boundary conditions based on characteristic variables.

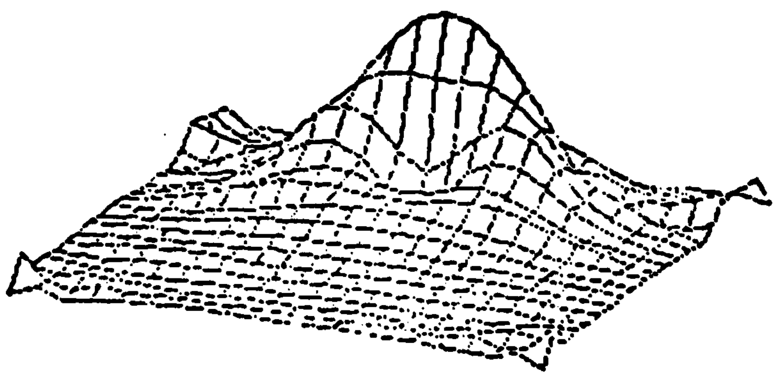
t = .125



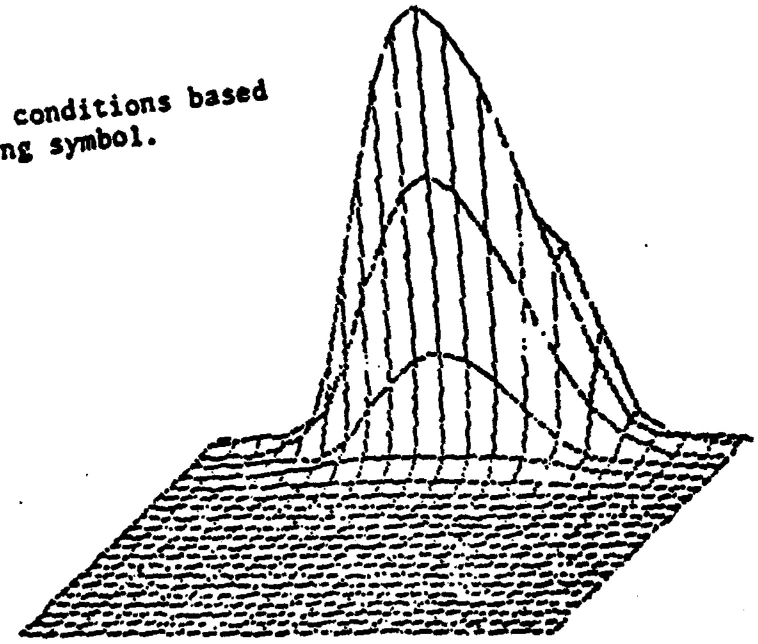
t = .25



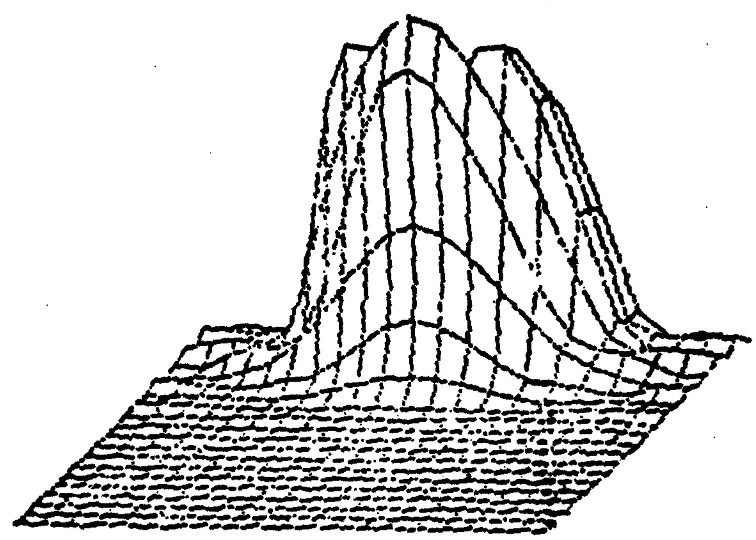
t = .375



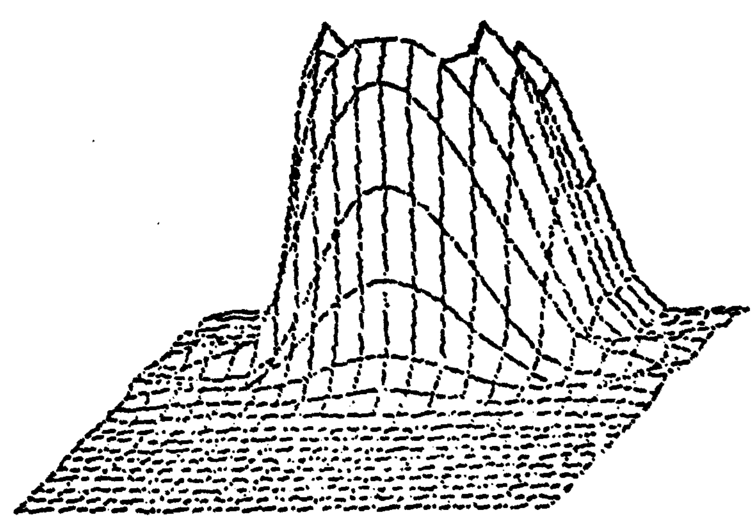
(b) Solution using boundary conditions based on uncoupling the leading symbol.



t = .125



t = .25

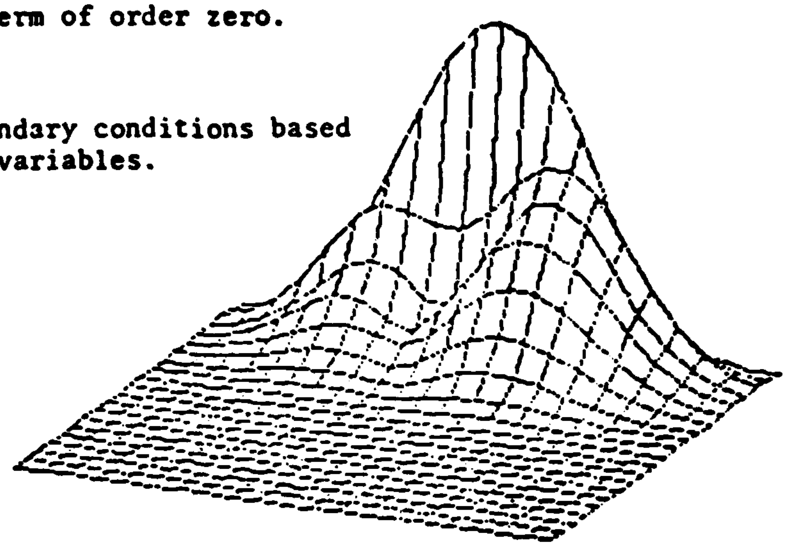


t = .375

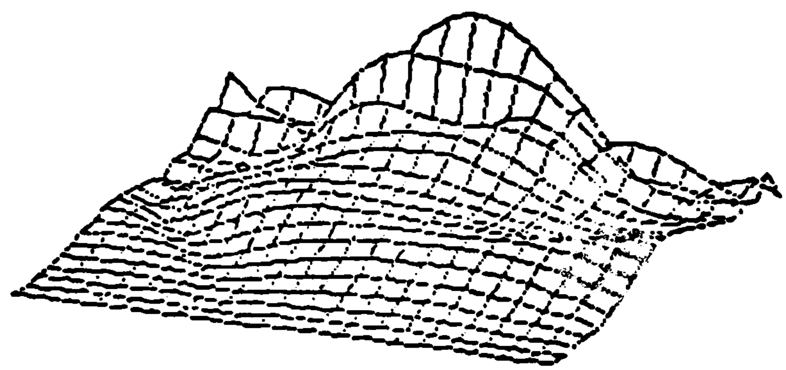
Figure 5.3. **Effect of uncoupling both the leading symbol and the term of order zero.**

(a) Solution using boundary conditions based on characteristic variables.

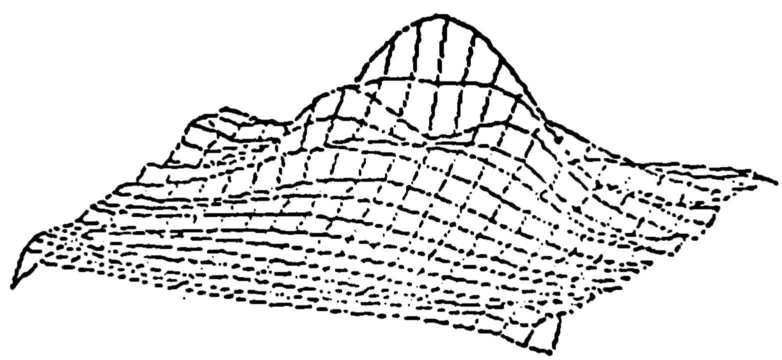
t = .125



t = .25

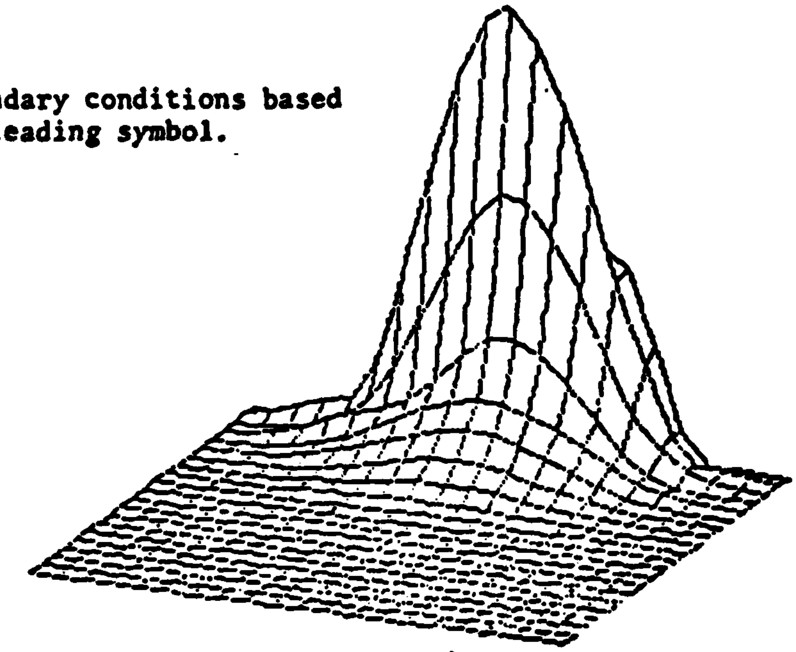


t = .375

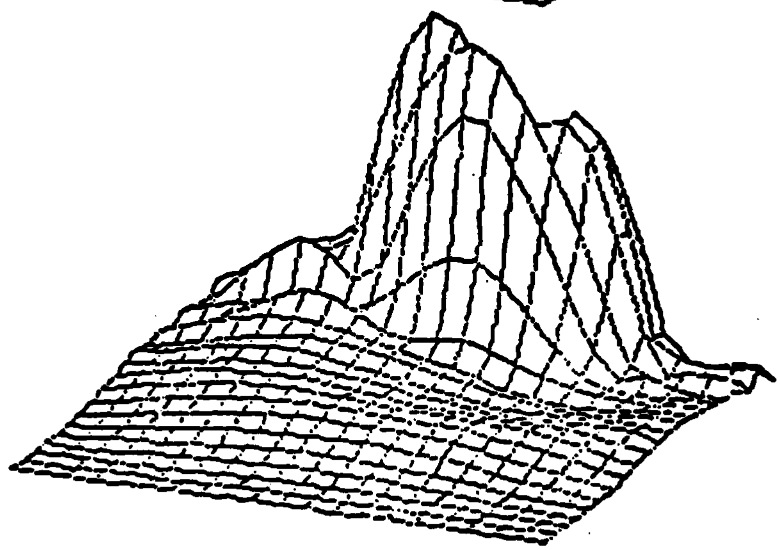


(b) Solution using boundary conditions based on uncoupling the leading symbol.

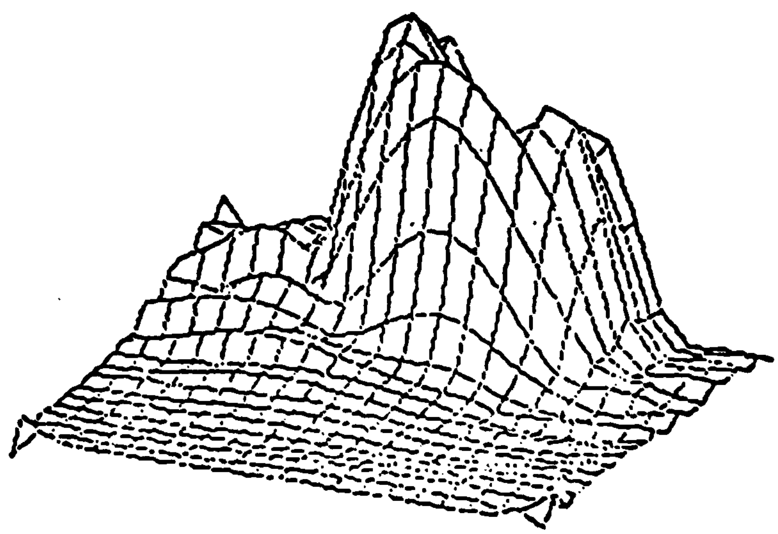
t = .125



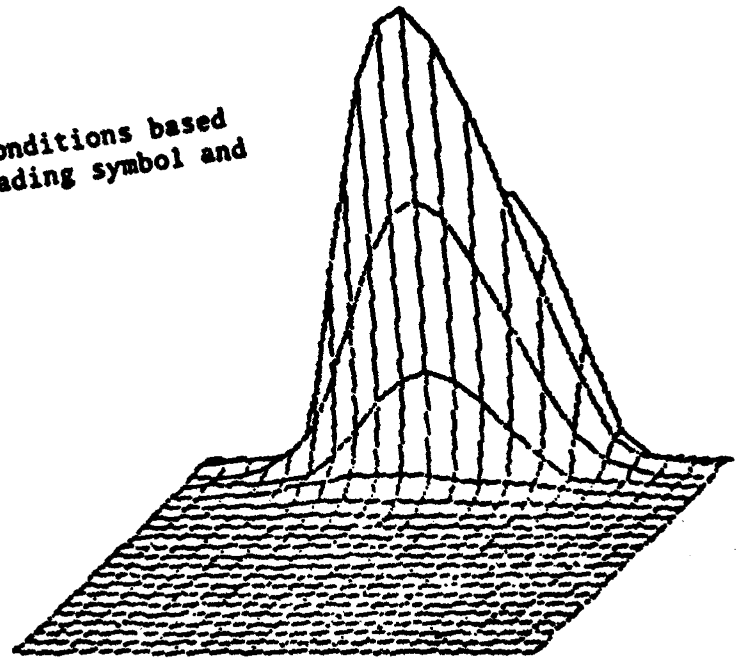
t = .25



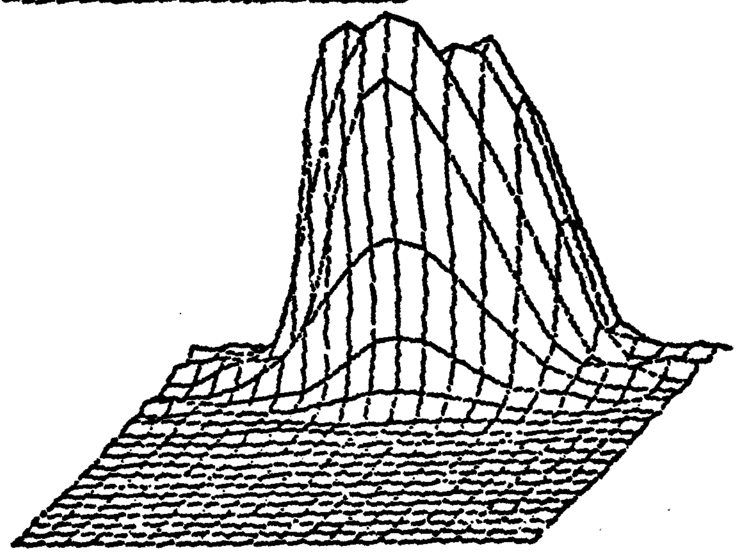
t = .375



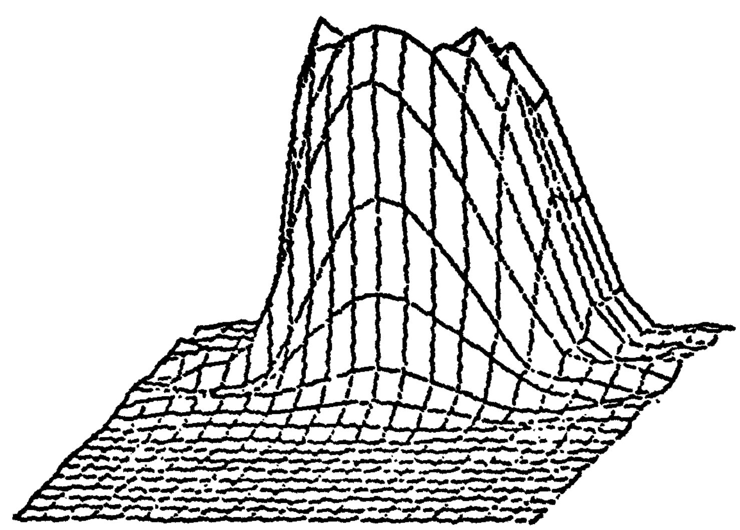
(c) Solution using boundary conditions based on uncoupling both the leading symbol and the term of order zero.



t = .125



t = .25



t = .375

APPENDIX

PROPERTIES OF PSEUDO-DIFFERENTIAL OPERATORS

In this appendix we define pseudo-differential operators and state without proof of some of their basic properties. More extensive treatments can be found in Nirenberg [3], Taylor [4], and Treves [5].

We must first establish some notation. Partial derivatives in $\mathbb{R}^n$ will be denoted by ∂^α, where $\alpha = (\alpha_1, \ldots, \alpha_n)$, and

$$\partial^\alpha = \partial_1^{\alpha_1} \cdot \ldots \cdot \partial_n^{\alpha_n} = \left(\frac{\partial}{\partial x_1}\right)^{\alpha_1} \cdot \ldots \cdot \left(\frac{\partial}{\partial x_n}\right)^{\alpha_n} .$$

The α_j are nonnegative integers. Differential operators can then be written in the form

$$P = \Sigma\, a_\alpha \partial^\alpha .$$

The coefficients a_α are functions on $\mathbb{R}^n$, and the sum is taken over finitely many multi-indices α. We will allow the possibility that P may act on vector-valued functions. In that case the a_α may be either scalars or matrices.

The Fourier transform on $\mathbb{R}^n$ will be denoted by

$$\hat{u}(\xi) = \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} e^{-ix\cdot\xi} u(x)\,dx ,$$

where $x \cdot \xi = \Sigma_{j=1}^n x_j \xi_j$. The inverse Fourier transform is then given by

$$\int_{\mathbb{R}^n} e^{ix\cdot\xi}\, \hat{u}(\xi)\,d\xi .$$

Differential operators can be represented in terms of the Fourier transform. For suitable functions u, we have

$$\begin{aligned}(Pu)(x) &= \Sigma\, a_\alpha(x)\partial^\alpha u(x) \\ &= \Sigma\, a_\alpha(x)\partial_x^\alpha \int e^{ix\cdot\xi}\, \hat{u}(\xi)\,d\xi \\ &= \int e^{ix\cdot\xi}\, [\Sigma\, a_\alpha(x)(i\xi)^\alpha]\hat{u}(\xi)\,d\xi .\end{aligned}$$

Here $(i\xi)^\alpha$ denotes the product $(i\xi_1)^{\alpha_1} \cdot \ldots \cdot (i\xi_n)^{\alpha_n}$. The equation can be written in the form

(A.1) $$(Pu)(x) = \int e^{ix\cdot\xi} p(x,\xi)\hat{u}(\xi)d\xi \quad ,$$

where $p(x,\xi) = \Sigma\, a_\alpha(x)(i\xi)^\alpha$. The function p is sometimes called the symbol of the operator P.

Pseudo-differential operators are obtained by allowing a larger class of symbols to be used in (A.1). Every differential operator is a pseudo-differential operator, but not vice versa. One fairly general symbol class is the class $S^m_{\rho,\delta}$, $0 \leq \delta < \rho \leq 1$, which was introduced by Hörmander. This is defined to be the set of all C^∞ functions p which satisfy estimates of the form

(A.2) $$|\partial^\alpha_\xi \partial^\beta_x p(x,\xi)| \leq C_{K,\alpha,\beta}(1 + |\xi|)^{m-\rho|\alpha|+\delta|\beta|} \quad ; \quad x \in K,\ \xi \in R^n$$

for all α, β and for every compact subset K of R^n. The constant is allowed to depend on α, β, and K. The symbol of a differential operator of order m having smooth coefficients clearly belongs to the class $S^m_{1,0}$. For the operators considered in this paper we always have $\rho = 1$ and $\delta = 0$. In general, the number m appearing in (A.2) is called the order of the operator P whose symbol is p. The order need not be positive, and it need not be an integer.

If $u \in C^\infty_0$, then $Pu \in C^\infty$. It is possible to extend P so that Pu is defined for any distribution u having compact support. In this case Pu is a distribution.

If an operator P has negative order, then Pu is smoother than u, since $\hat{u}(\xi)$ is multiplied by a symbol which tends to zero as $|\xi| \to \infty$. P is said to have order $-\infty$ if $P \in S^m$ for all $m < 0$.

A useful concept is that of an asymptotic expansion of a symbol. Suppose that $\{m_j\}_{j=0}^{\infty}$ is a sequence of real numbers such that $m_j > m_{j+1}$ for all j and $m_j \to -\infty$ as $j \to \infty$. Let $\{p_j\}$ be a sequence of symbols such that $p_j \in S_{\rho,\delta}^{m_j}$ for each j. A symbol p is said to be an asymptotic sum of the p_j, written

$$p \sim \sum_{j=0}^{\infty} p_j \quad ,$$

provided $p - \Sigma_{j=0}^{k} p_j \in S_{\rho,\delta}^{m_{k+1}}$ for all k. That is, the error in each partial sum must have the same order as the first term omitted from the partial sum. This concept is analogous to the usual concept of asymptotic expansion. In fact, if a function $p(\xi)$ of one variable has an asymptotic expansion

$$p(\xi) \sim \sum_{j=-m}^{\infty} \frac{a_j}{\xi^j} \quad \text{as} \quad \xi \to \infty$$

in the usual sense, then this expansion is also asymptotic in the sense described above.

Pseudo-differential operators can be composed. Let P and Q be operators with symbols $p(x,\xi)$ and $q(x,\xi)$, respectively, and suppose that q has compact support in x. The composition $P(Qu)$ is then well-defined and is given by a pseudo differential operator whose symbol has the asymptotic expansion

(A.3) $$\sigma_{PQ} \sim \sum_{|\alpha|\geq 0} \frac{1}{i^{|\alpha|}\alpha!} \partial_{\xi}^{\alpha} p(x,\xi) \partial_{x}^{\alpha} q(x,\xi) \quad .$$

The sum is taken over all multi-indices $\alpha = (\alpha_1,\ldots,\alpha_n)$ having nonnegative components. The order of α is given by $|\alpha| = \Sigma\alpha_j$, and the factorial $\alpha!$ denotes the product $\alpha_1!\alpha_2! \cdot\ldots\cdot \alpha_n!$.

It follows from (A.2) that when a symbol is differentiated with respect to ξ, the result is a symbol of lower order. This implies that the leading order term in (A.3) corresponds to $|\alpha| = 0$ and is equal to $p(x,\xi)q(x,\xi)$.

The symbol of the product of two operators is therefore equal to the product of their symbols, up to certain terms of lower order.

The makes sense when we consider the special case of differential operators. The composition of two operators $a(x)\partial^{\alpha}$ and $b(x)\partial^{\beta}$ is equal to $a\partial^{\alpha}(b\partial^{\beta}) = ab\partial^{\alpha+\beta}$ + lower order terms involving derivatives of b. In this case the composition law can be derived using Leibniz' rule. For general pseudo-differential operators the derivation is much more complicated.

In this paper we use the concept of parametrix, or approximate inverse, of a pseudo-differential operator. A parametrix of an operator Q is an operator Q^{-1} such that $QQ^{-1} - I$ and $Q^{-1}Q - I$ are operators of order $-\infty$. At high frequencies Q^{-1} is very nearly a true inverse of Q. Q^{-1} is uniquely determined up to order $-\infty$.

Let q be the symbol of the operator Q. If the function q is a nonzero scalar or a nonsingular matrix, then it can be shown that Q^{-1} exists and that its leading symbol is q^{-1}. The proof consists of writing a formal expansion for Q^{-1} and solving for the terms one by one.

An operator of the form $I + K$, where K has negative order, has a parametrix which is given by the expansion $(I+K) \sim I - K+K^2 - \dots$.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper is a continuation of [2]. Here we apply the ideas of that paper to the linearized shallow water equations, or, equivalently, the linearized two-dimensional isentropic Euler equations of gas dynamics.

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