AD-A121 407 STRENGTH OF MECHANICALLY FASTENED COMPOSITE JOINTS(U) 1/2
MICHIGAN UNIV ANN ARBOR DEPT OF MECHANICAL ENGINEERING
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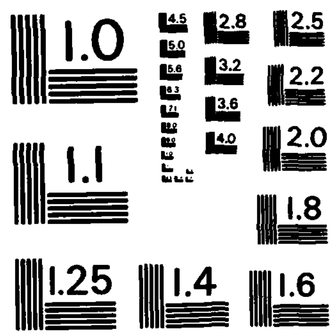

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STRENGTH OF MECHANICALLY FASTENED COMPOSITE JOINTS

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July, 1982

Final Report for Period June 1981-May 1982

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FOR THE COMMANDER

F. D. Cherry

F. D. CHERRY, Chief
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFWAL-TR-82-4095	2. GOVT ACCESSION NO. AD-A121407	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) STRENGTH OF MECHANICALLY FASTENED COMPOSITE JOINTS		5. TYPE OF REPORT & PERIOD COVERED June, 1981-May, 1982
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Fu-kuo Chang Richard A. Scott George S. Springer		8. CONTRACT OR GRANT NUMBER(s) F33615-81-C-5050
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mechanical Engineering and Applied Mechanics, The University of Michigan Ann Arbor, Michigan 48109		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS FY1457-81-02013
11. CONTROLLING OFFICE NAME AND ADDRESS Materials Laboratory (AFWAL/MLBM) Air Force Wright Aeronautical Laboratories Wright-Patterson, AFB, OH 45433		12. REPORT DATE July 1982
		13. NUMBER OF PAGES 97
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Composite Materials
Failure Hypothesis
Joints
Bolted Joints

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A method is presented for predicting the failure strength and failure mode of mechanically fastened joints made of fiber reinforced composite laminates. The method includes two steps. First, the stress distribution in the laminate is calculated by the use of a finite element method. Second, the failure load and the failure mode are predicted by means of a proposed failure hypothesis together with Yamada's failure criterion. A computer code was developed which can be used to calculate the maximum load and the mode of failure of joints involving laminates with different ply orientations,

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) different material properties, and different geometries. Results generated by the present method were compared to data and to existing analytical and numerical solutions. The results of the present method were found to agree well with those reported previously. Parametric studies were also performed to evaluate the effects of joint geometry and ply orientation on the failure strength and on the failure mode.

FOREWORD

This report was prepared by Fu-kuo Chang, Richard A. Scott, and George S. Springer, Department of Mechanical Engineering and Applied Mechanics, The University of Michigan for the Mechanics and Surface Interactions Branch (AFWAL/MLBM), Nonmetallic Materials Division, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio. The work was performed under Contract Number F 33615-81-C-5050, Project number FY1457-81-02013.

This report covers work accomplished during the period June, 1981-May 1982.

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A	Total surface area of the laminate
A_L	Stress prescribed area
A_F	Stress free area
A_R	Displacement prescribed area (fixed boundary)
A_{Lg}	Surface area of an element g on which surface tractions is applied
B	Bearing stress
D	Diameter of the hole
E	Edge distance
E_{ijkl}	Elastic moduli
E_{mn}	Reduced laminate moduli
e	Failure indicator ($e<1$ non-failure, $e\geq 1$ failure)
e_o	Maximum value of e on the characteristic curve
$F_{i\beta}$	Assembled load vector
H	Thickness of the laminate
h^p	Thickness of the p-th ply
$k^g_{i\beta k\alpha}$	Stiffness matrix of the g-th element
$\bar{K}_{i\beta k\alpha}$	Assembled stiffness matrix
L	Plate length
M	Number of elements
N	Number of plies in the laminate
N_α	Shape function
n_j	Unit vector normal to the surface
P	Applied Load
P_{max}	Failure (maximum) load
$\bar{Q}^p_{ij}$	Transformed reduced stiffness matrix of the p-th ply

$q_{i\alpha}$	Nodal displacement
r	Radial distance
r_c	Radial distance to the characteristic curve
R_{ot}	Characteristic length for tension
R_{oc}	Characteristic length for compression
S	Ply shear strength
S_c	Shear strength of cross-ply laminate
s	Total area of the two-dimensional laminate
s_g	Area of element g (2-dimensions)
SCF	Stress concentration factor
T_i	Surface traction components
u_i	Displacement
$\overline{u}_i$	Arbitrary displacement functions
V_o	Total volume of the laminate
v_g	Volume of element g
W	Width of the plate
X	Ply tensile strength
x	Coordinate along the fiber direction in each ply
x_1	Coordinate perpendicular to the loading direction in the laminate plane
x_2	Coordinate opposed to the loading direction and perpendicular to the x_1-axis
x_3	Coordinate perpendicular to the x_1 and x_2 axes
y	Coordinate perpendicular to the fiber direction in each ply
Γ_L	Boundary curve of the hole on which the surface traction is applied
Γ_{Lg}	Boundary curve of the element g on which the surface traction is applied
$\Delta\theta$	Ply continuity

ε_{ij}	Strain components in the x_1-x_2 coordinate system
η	Angle measured counterclockwise from the x_1-axis
θ_f	Angle at which failure occurs
σ_{ij}	Stress components in the x_1-x_2 coordinate system
$\sigma_x, \sigma_y, \sigma_{xy}$	Stress components in the x-y coordinate system
ϕ	Maximum angular range of the ply orientation

SECTION I

INTRODUCTION

Among the major advantages of laminated composite structures over conventional metal structures are their comparatively high strength to weight and stiffness to weight ratios. As a result, fiber reinforced composite materials have been gaining wide application in aircraft and spacecraft construction. These applications require joining composites either to composites or to metals. Most commonly, joints are formed using mechanical fasteners. Therefore, suitable methods must be found to determine the failure strengths of mechanically fastened joints. A knowledge of the failure strength would help in selecting the appropriate size joint in a given application.

Owing to the significance of the problem, several investigators have developed analytical procedures for calculating the strength of bolted joints in composite materials. Among the recent studies are those of Waszczak and Cruse [1], Agarwal [2], and Garbo and Ogonowski [3]. As will be discussed in Section VII, the previous methods provide conservative results and underestimate the failure strength, often by as much as fifty percent.

The major objective of this investigation was, therefore, to develop a method which a) predicts the failure strength and failure mode of mechanically fastened composite joints with better accuracy than the existing analytical methods and b) can be used readily in the design of mechanically fastened composite joints. In the present method first the stress distribution around the hole is calculated by the use of a finite element method. Second, the failure load and the failure mode are predicted by means of a proposed new failure hypothesis together with Yamada's [4] failure criterion. On the basis of this analysis a computer code was developed which can be applied to joints involving laminates with different ply orientations, different material properties, and different configurations, including different hole sizes, hole positions, and joint thicknesses. Because of the accuracy of the method and the flexibility of the computer code, the code can be applied to the analysis and the design of mechanically fastened composite joints.

SECTION II

PROBLEM STATEMENT

Consider a plate (length L, width W, thickness H) made of N fiber reinforced unidirectional plies. The ply orientation is arbitrary, but must be symmetric with respect to the $x_3=0$ plane (symmetric laminate, Figure 1). Perfect bonding between each ply is assumed.

A hole of diameter D is located along the centerline of the plate ($x_1=0$) at a distance E from one end of the plate. A rigid pin (diameter D), supported outside the laminate, is inserted into the hole (Figure 1). A uniform tensile load P is applied to the plate, as shown in Figure 1. The load is parallel to the plate (in-plane loading) and is symmetric with respect to the centerline. Hence the load cannot create bending moments about either the x_1, x_2 or x_3 axes. Moreover, for symmetric laminates, in plane and bending effects are uncoupled. It is desired to find

1) the stresses and strains in each ply,
2) the maximum (failure) load (P_{max}) that can be applied before the joint fails, and
3) the mode of failure.

Point 2 refers to the fact that, according to experimental evidence, mechanically fastened joints under tensile loads generally fail in three basic modes referred to as tension mode, shearout mode, and bearing mode. The type of damage resulting from each of these modes is illustrated in Figure 2. The objective, listed in point 3 above, is to determine which of these modes will most be responsible for the failure.

The calculation proceeds in three steps. For a given geometry and

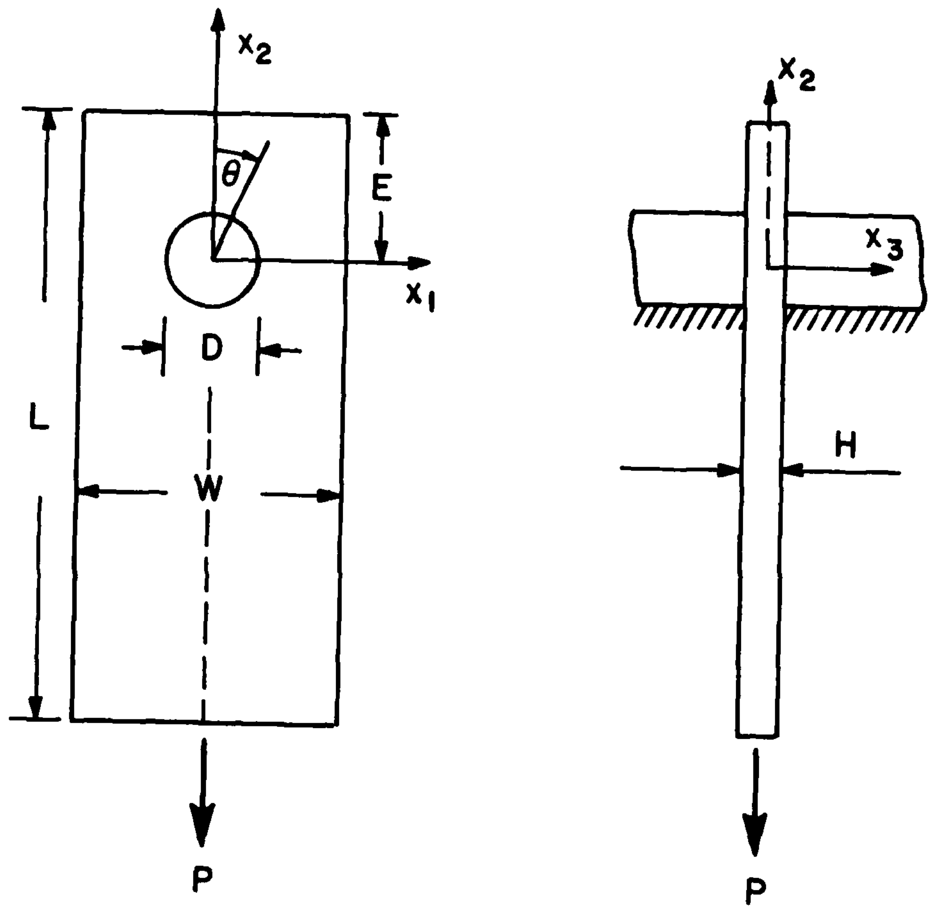

Figure 1. Geometry of the problem

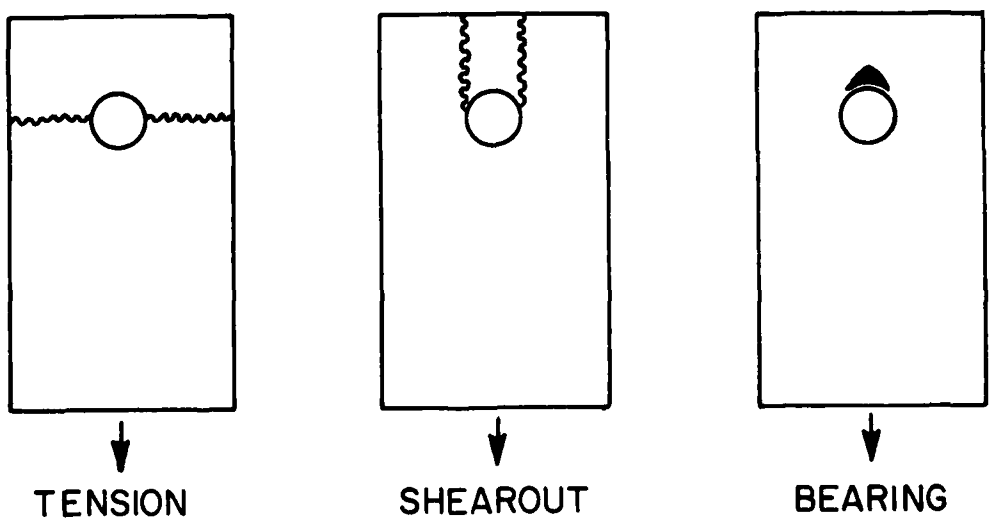

Figure 2. Illustration of the three basic failure modes

load

1) the stress distribution around the hole is calculated,

2) the maximum (failure) load is predicted , and

3) the mode of failure is determined.

The details of these steps are presented in Sections III-V.

SECTION III

STRESS ANALYSIS-GOVERNING EQUATIONS

The stresses in the laminate are calculated on the basis of anisotropic theory of elasticity and classical lamination plate theory. Accordingly, in the analysis planes are taken to remain planes, the strain across the thickness is taken to be constant [$\varepsilon_{ij}=f(x_1,x_2)$] and only plane stresses are considered ($\sigma_{13}=\sigma_{23}=\sigma_{33}=0$). Under these conditions, in the absence of body forces, the condition of force equilibrium can be expressed as [5]

$$\frac{\partial\sigma_{11}}{\partial x_1} + \frac{\partial\sigma_{12}}{\partial x_2} = 0$$
$$\frac{\partial\sigma_{21}}{\partial x_1} + \frac{\partial\sigma_{22}}{\partial x_2} = 0 \tag{1}$$

In index notation eq. (1) becomes

$$\sigma_{ij,j} = 0 \tag{2}$$

σ_{ij} is the stress in the plane normal to the x_i axis and is in the x_j direction. The subscripts i and j may have the values 1 or 2. Consider now an elastic laminate of volume V_o containing a loaded hole as shown in Figure 3. Stresses are applied over the suface area A_L. The surface area A_R is rigidly fixed (no displacement), while the surface area A_F is free of applied stress. The total surface area is

$$A = A_L + A_R + A_F \tag{3}$$

Let us denote by $\bar{u}_i$ any arbitrary displacement inside the body. $\bar{u}_i$ is a test function. The only requirement is that $\bar{u}_i$ be continuous, differentiable and be zero on A_R. By multiplying eq. (2) by $\bar{u}_i$ and by taking the volume integral of the

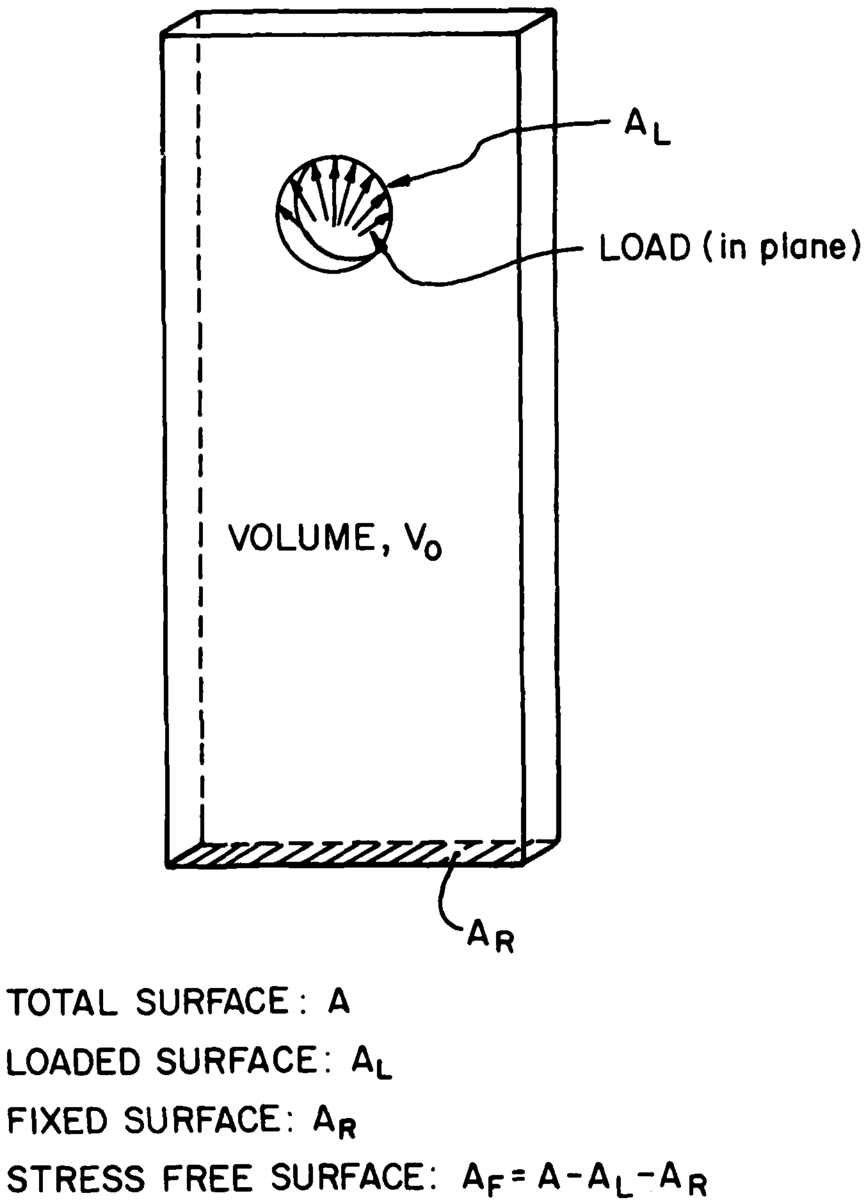

Figure 3. Configuration of an elastic laminate with a loaded hole

resulting expression we obtain

$$\iiint_{V_o} \sigma_{ij,j}\, \bar{u}_i \, dV = 0 \tag{4}$$

By employing the identity

$$\sigma_{ij,j}\, \bar{u}_i = (\sigma_{ij}\bar{u}_i)_{,j} - \sigma_{ij}\, \bar{u}_{i,j} \tag{5}$$

and by utilizing Gauss' theorem, eq. (4) may be written as

$$\iint_{A} \sigma_{ij}\, n_j\, \bar{u}_i \, dA - \iiint_{V_o} \sigma_{ij}\, \bar{u}_{i,j} \, dV = 0 \tag{6}$$

where n_j is the unit vector normal to the surface. On the free surface A_F the stresses are zero, while on the surface A_R the displacement is zero. These conditions give

$$\iint_{A_F} \sigma_{ij}\, n_j\, \bar{u}_i \, dA = 0 \tag{7}$$

$$\iint_{A_R} \sigma_{ij}\, n_j\, \bar{u}_i \, dA = 0 \tag{8}$$

The force per unit area (called surface traction) at each point of the surface area A_L is [5]

$$T_i = \sigma_{ij} n_j \tag{9}$$

Equations (6) - (9) yield

$$\iint_{A_L} T_i\, \bar{u}_i \, dA = \iiint_{V_o} \sigma_{ij}\, \bar{u}_{i,j} \, dV \tag{10}$$

The stress is related to the displacement through the stress-strain relationship, which for an elastic body is

$$\sigma_{ij} = E_{ijk\ell}\, \varepsilon_{k\ell} \tag{11}$$

The subscripts k and ℓ may take on the values of 1 or 2. The strains are related to the displacements u_i by the expression

$$\varepsilon_{k\ell} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_\ell} + \frac{\partial u_\ell}{\partial x_k}\right) \tag{12}$$

By combining eqs. (10) - (12) we obtain

$$\iiint\limits_{V_o} E_{ijk\ell}\, \bar{u}_{i,j}\, u_{k,\ell}\, dV = \iint\limits_{A_L} T_i\, \bar{u}_i\, dA \tag{13}$$

$E_{ijk\ell}$ are the moduli of elasticity of the laminate. Because of the laminate symmetry, the following simplification may be made

$$E_{ijk\ell} \equiv E_{mn} \tag{14}$$

The subscripts i,j,k, and ℓ are related to m and n as follows

$$\begin{array}{llll} i=j=1 & \rightarrow m=1 & k=\ell=1 & \rightarrow n=1 \\ i=j=2 & \rightarrow m=2 & k=\ell=2 & \rightarrow n=2 \\ i\neq j & \rightarrow m=3 & k\neq\ell & \rightarrow n=3 \end{array} \tag{15}$$

The reduced laminate modulus E_{mm} is given by

$$E_{mn} = \sum_{p=1}^{N} \frac{h^P}{H}\, \bar{Q}^P_{ij} \tag{16}$$

where h^P is the thickness of the p-th ply. $\bar{Q}^P_{ij}$ is the transformed reduced stiffness matrix for the p-th ply [6] (Appendix A).

SECTION IV

STRESS ANALYSIS-FINITE ELEMENT METHOD

In order to perform the calculations, the problem shown in Figure 1 was simulated by the geometry given in Figure 4. Because of symmetry the stresses were calculated only in one half of the body. Along the symmetry axis, displacement is allowed only in the x_2 direction. Along the lower edge of the plate, displacement is allowed only in the x_1 direction. The intersection of the symmetry axis and the lower edge is considered to be rigidly fixed.

The surface of the hole is subjected to a surface traction T_i. The parameter T_i is related to the applied load. The spatial distribution of T_i depends on the magnitude of the applied load, on the material properties, and on the geometry in a complex manner. It is extremely difficult, if not impossible, to determine the exact distribution of T_i inside the hole. To overcome this difficulty a cosine normal load distribution was assumed. With this approximation T_i becomes

$$T_i = -\frac{4P}{\pi D} n_i \cos\theta \tag{17}$$

The angle θ is in the x_1-x_2 plane and is measured clockwise from the x_2 axis (Figure 1). For isotropic materials the cosine normal load distribution (eq. 17) was found to represent closely the actual load distribution [7]. Calculations performed by previous investigators also showed that for composite materials the stress distribution inside the body is insensitive to the assumed load distribution [1,3,8]. Therefore, eq. (17) should suffice for the purpose of the present analysis which is to determine the overall strength of the joint.

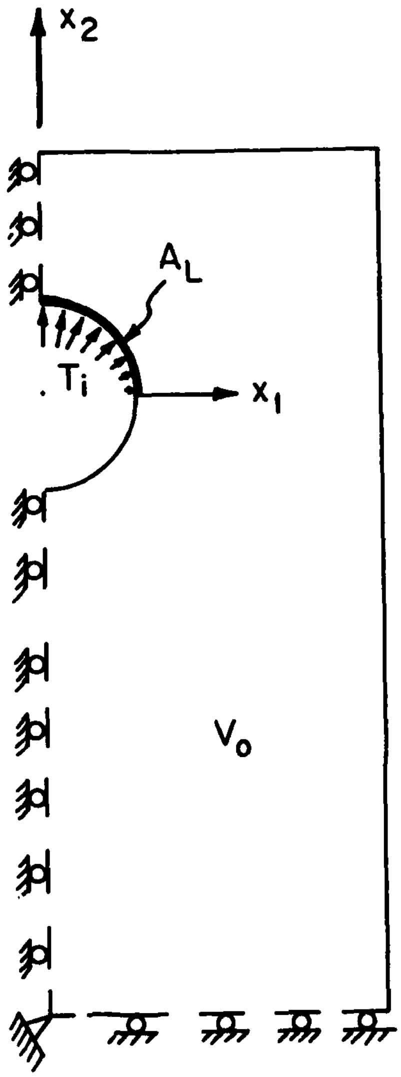

Figure 4. Configuration of a joint approximated in the finite element method

Equations (13) and (17) give

$$\iiint_{V_o} E_{ijk\ell}\, \bar{u}_{i,j}\, u_{k,\ell}\, dV = \iint_{A_L} -\frac{4P}{\pi D} n_i\, u_i \cos\theta\, dA \tag{18}$$

We recall that $\bar{u}_i$ are functions that can be selected arbitrarily. The unknowns in eq. (18) are the displacements $\bar{u}_i$. Once $\bar{u}_i$ are known the stress at every point can be calculated from eqs (11) and (12). The method of solution of eq. (18) is described below.

Solutions to eq.(18) were obtained by a finite element method (FEM). As a first step in the solution procedure the volume V_o is subdivided into M subdomains of volume V_g

$$V_o = \sum_{g=1}^{M} V_g \tag{19}$$

Equation (18) may now be written as

$$\sum_{g=1}^{M} \iiint_{V_g} E_{ijk\ell}\, \bar{u}_{i,j}\, u_{k,\ell}\, dV = \sum_{g=1}^{M} \iint_{A_{Lg}} -\frac{4P}{\pi D} n_i\, \bar{u}_i \cos\theta\, dA \tag{20}$$

A_{Lg} is the surface of an element on the inside of the hole where the surface traction is applied ($0 \leq \theta \leq \pi/2$). At any surface where load is not applied, A_{Lg} is zero. Advantage is taken now of the assumption that the strains ($\varepsilon_{11}, \varepsilon_{22}$ and ε_{12}) are independent of the thickness, i.e. the strains are independent of x_3 and depend only on x_1 and x_2. Thus, the three dimensional grid consisting of M volume elements may be replaced by a two dimensional grid consisting of M surface elements of area s_g (Figure 5)

$$s = \sum_{g=1}^{M} s_g \tag{21}$$

Equation (20) thus becomes

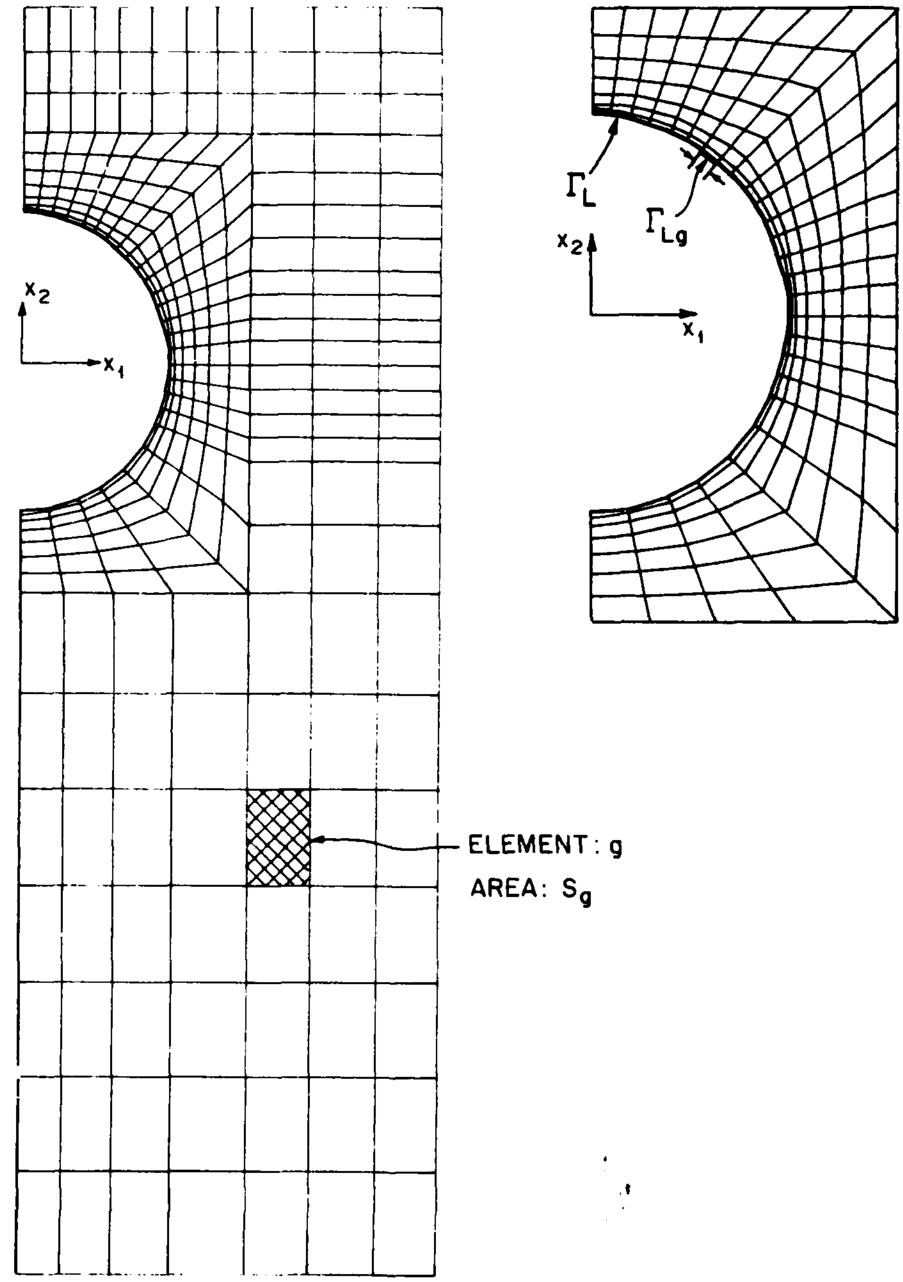

Figure 5. Grid used in the finite element method. Right hand figure is an enlarged view of the grid around the hole

$$\sum_{g=1}^{M} \iint_{S_g} E_{ijk\ell}\, \bar{u}_{i,j}\, u_{k,\ell}\, ds = \sum_{g=1}^{M} \int_{\Gamma_{Lg}} - \frac{4P}{\pi D}\, n_i\, \bar{u}_i \cos\theta\, d\Gamma \tag{22}$$

Γ_L is a line along the boundary of the hole where the surface traction is applied ($0 \le \theta \le \pi/2$, Figure 5). Each segment of this line (denoted by Γ_{Lg}) coincides with the boundary of an element g adjacent to the hole. For those elements which do not lie along this line Γ_{Lg} equals zero. Isoparametric 4-node elements were used in this investigation. The grid was generated using a grid generator [9]. The grid sizes were unequal. Smaller grids were used in the vicinity of the hole to obtain a better resolution of the stresses. Utilizing the symmetry about the x_2 axis, a grid (consisting of 306 elements) was placed on one half of the laminate, as illustrated in Figure 5.

The displacement in each element can be expressed in terms of the displacements of the four nodal points [10]

$$\begin{aligned} u_i &= N_\alpha\, q_{i\alpha} \\ \bar{u}_i &= N_\alpha\, \bar{q}_{i\alpha} \end{aligned} \tag{23}$$

The subscript α designates the nodal points (α=1,2,3, or 4). N_α is the shape function described in detail in Appendix B. $q_{i\alpha}$ is the displacement at the nodal point α in the i direction.

We define a stiffness matrix for the g-th element as

$$K^g_{i\beta k\alpha} \equiv \iint_{S_g} E_{ijk\ell}\, N_{\alpha,\ell}\, N_{\ell,j}\, ds \tag{24}$$

$K^g_{i\beta k\alpha}$ is an eight by eight matrix. The subscript β may take on the values 1,2,3, and 4. The nodal displacements $q_{k\alpha}$ and $\bar{q}_{i\beta}$ are independent of the surface and line integrations. Accordingly, eqs. (22) - (24) yield

$$\sum_{g=1}^{M} K^{g}_{i\beta k\alpha}\, q_{k\alpha}\, \bar{q}_{i\beta} = \sum_{g=1}^{M} \bar{q}_{i\beta} \int_{\Gamma_{Lg}} - \frac{4P}{\pi D}\, n_i\, N_\beta \cos\theta \, d\Gamma \tag{25}$$

The nodal displacements $\bar{q}_{i\beta}$ are arbitrary functions and hence eq. (25) can be written

$$\bar{K}_{i\beta k\alpha}\, q_{k\alpha} = \bar{F}_{i\beta} \tag{26}$$

where the global stiffness matrix, $\bar{K}_{i\beta k\alpha}$ and the load vector $\bar{F}_{i\beta}$ are given by

$$\bar{K}_{i\beta k\alpha} \equiv \sum_{g=1}^{M} K^{g}_{i\beta k\alpha} \tag{27}$$

$$\bar{F}_{i\beta} \equiv \sum_{g=1}^{M} \int_{\Gamma_{Lg}} - \frac{4P}{\pi D}\, n_i\, N_\beta \cos\theta \, d\Gamma \tag{28}$$

The elements of $\bar{K}_{i\beta k\alpha}$ and the components of the vector $\bar{F}_{i\beta}$ are known. Hence, $q_{k\alpha}$ can be obtained from eq. (26) using the Gaussian elimination method [12]. Once $q_{k\alpha}$ are known the displacements u_i are calculated from eq. (23). A computer code was developed for performing the calculations and for generating solutions (section VI).

SECTION V

PREDICTION OF FAILURE

In order to determine the load at which a joint fails and the mode of failure, the conditions for failure must be established. In this investigation the joint is taken to have failed when the combined stresses have exceeded a prescribed limit in any of the plies along an approximately chosen curve (denoted as the characteristic curve). The combined stress limit is evaluated using the failure criterion proposed by Yamada [4]. The coordinates of the characteristic curve are established by extending Whitney and Nuismer's failure hypothesis [13] (developed for open, unloaded holes) to loaded holes.

1) Failure Criterion

Numerous criteria for failure have been proposed in the past [14,15,16,17]. Although the concepts underlying the different failure criteria may be different, the results of the various criteria are generally quite similar. In this investigation Yamada's failure criterion was adopted [4]. This criterion is based on the assumption that just prior to failure of the laminate every ply has failed due to cracks along the fibers. The validity of this assumption is supported by tests performed during this investigation with 64 ply graphite-epoxy (AS/3501-6) laminates. Yamada's criterion states that failure occurs when the following condition is met in any one of the plies

$$\left(\frac{\sigma_x}{X}\right)^2 + \left(\frac{\sigma_{xy}}{S_c}\right)^2 = e^2 \begin{cases} e < 1 & \text{no failure} \\ e \geqq 1 & \text{failure} \end{cases} \tag{29}$$

σ_x and σ_{xy} are the longitudinal and shear stresses in a ply, respectively (x and y being the coordinates parallel and normal to the fibers in the ply). X is the longitudinal tensile strength of the ply. S_c is the shear strength of a symmetric, cross ply laminate which has the same number of plies as the laminate under consideration. As indicated in eq. (29) failure occurs when e is equal to or greater than unity.

2) Failure Hypothesis-Characteristic Curve

The hypothesis is proposed here that failure occurs when in any one of the plies the combined stresses satisfy an appropriately chosen failure criterion at any point on a characteristic curve. The characteristic curve (Figure 6) is specified by the expression

$$r_c(\theta) = D/2 + R_{ot} + (R_{oc}-R_{ot}) \cos \theta \qquad (30)$$

The angle θ, measured clockwise from the x_2 axis, may range in value from $-\pi/2$ to $\pi/2$. R_{ot} and R_{oc} are the characteristic lengths for tension and compression [13,18]. These parameters can be determined experimentally by measuring the tensile and compressive strengths of notched laminates. R_{ot} and R_{oc} depend only on the material. Therefore, the coordinates of the characteristic curve also depend only on the material, and are independent of the geometry and the stress distribution.

In this investigation the characteristic curve is used together with the Yamada failure criterion. Accordingly (see eq. 29), failure occurs when the parameter e is equal to or is greater than unity at any point on the characteristic curve

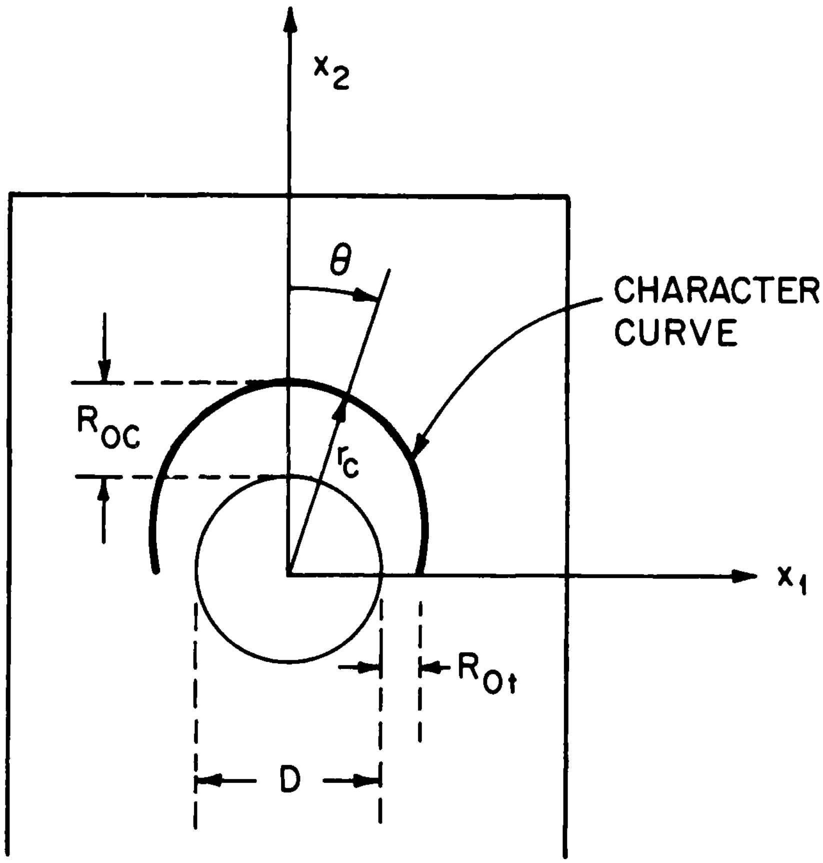

Figure 6. Description of the characteristic curve

$$
\left.\begin{array}{ll}\text{No failure} & e < 1 \\ \text{Failure} & e \geq 1\end{array}\right\} \text{ at } r = r_c \qquad (31)
$$

It is emphasized that the above failure hypothesis is used here in conjunction with the Yamada failure criterion (eq. 29). However, the hypothesis is general, and is not restricted to Yamada's criterion. The characteristic curve proposed here may be used with any other failure criterion.

3) Solution Procedure

Whether or not a joint fails under a given condition is determined as follows. For a given load

a) the stresses ($\sigma_1, \sigma_2, \sigma_{12}$) are calculated in each ply using eqs. (11), (14) and (16), and the FEM described in Sections II, and III,

b) the longitudinal and shear stresses (σ_x, σ_{xy}) are evaluated in each ply employing the transformation

$$
\begin{aligned}
\sigma_x &= \sigma_1 \cos^2\eta + \sigma_2 \sin^2\eta + 2\sigma_{12} \sin\eta \cos\eta \\
\sigma_{xy} &= -\sigma_1 \sin\eta \cos\eta + \sigma_2 \sin\eta \cos\eta + \sigma_{12}(\cos^2\eta - \sin^2\eta)
\end{aligned} \qquad (32)
$$

where η is the angle measured counter clockwise from the x_1-axis to the x-axis of each ply.

c) the parameter e is calculated (eq. 29) along the characteristic curve

d) if e equals or exceeds the value of unity ($e \geq 1$) in any ply along the characteristic curve, the joint is taken to have failed.

The procedure outlined above is used to predict whether or not failure occurs under a given load. Due to the assumption of a cosine normal load distribution around the hole (eq. 17), the calculated stresses are linearly proportional to the applied load P. This fact together with Yamada's failure criterion (eq. 29) gives

$P \sim e$

This relationship is utilized to determine the maximum load (P_{max}) which can be imposed on the joint. For a given load P, values of e are calculated on the characteristic curve as discussed above (points a-d). The highest value of e (e_o) is then determined, and the maximum load is calculated by the expression

$$P_{max} = \frac{P}{e_o} \tag{34}$$

The calculation procedure described in the foregoing also provides the location (angle θ_f) at which e first reaches the value of unity (e=1) on the characteristic curve. (Figure 7). A knowledge of θ_f provides an estimate of the mode of failure. When θ_f is small ($\theta_f \approx 0°$) failure is by the bearing mode. When $\theta_f \approx 45°$ failure is due to shearout. When $\theta_f \approx 90°$ failure is caused by tension. In summary

$$\begin{array}{ll} -15° < \theta_f < 15° & \text{bearing mode} \\ 30° < \theta_f < 60° & \text{shearout mode} \\ 75° < \theta_f < 90° & \text{tension mode} \end{array} \tag{35}$$

At intermediate values of θ_f failure may be caused by a combination of these modes.

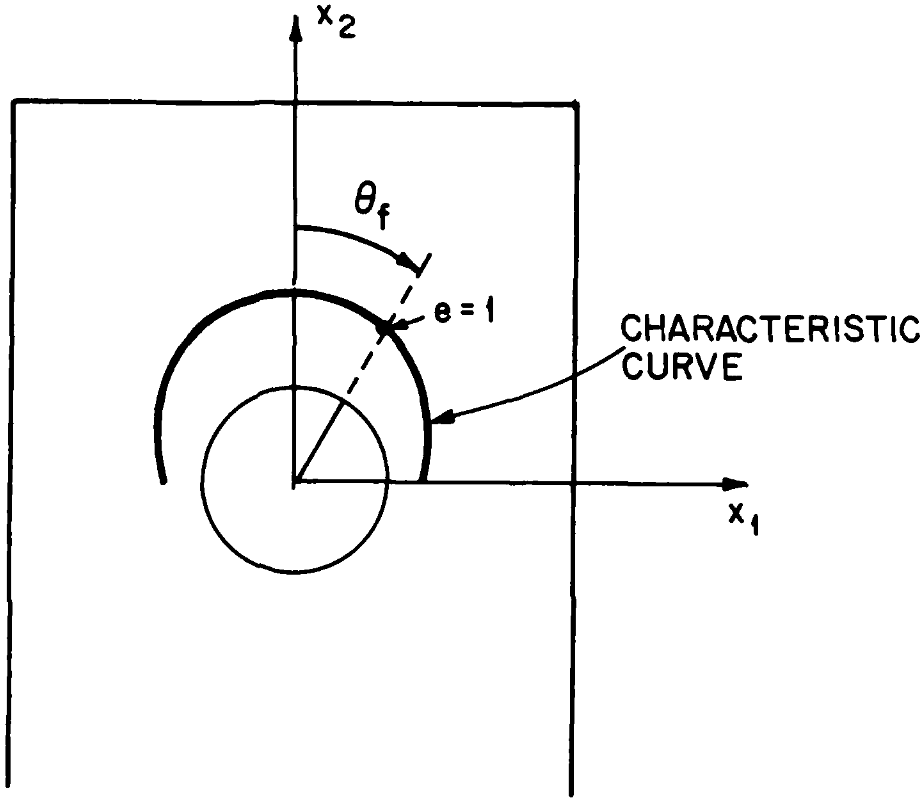

Figure 7. Location of failure (e=1) along the characteristic curve

SECTION VI

NUMERICAL SOLUTION

A computer code (designated as BOLT) was developed which is suitabe for generating solutions to the problem formulated in Sections II-V. The required input parameters and the output provided by the code are summarized in Table 1.

A Fortran listing of the code and a sample input-output are included in Appendix C.

Table 1. Input parameters required by the computer code and the output provided by the code.

Input Parameters

1) Ply properties

    a) Young's modulus, E_1

    b) Shear modulus, G_{12}

    c) Poisson's ratio, ν_{12}

2) Ply orientations

3) Geometry

    a) hole diameter, D

    b) thickness, H

    c) width, W

    d) length, L

    e) edge distances, E

4) Characteristic lengths, R_{ot} and R_{oc}

5) Longitudinal tensile strength of each ply, X

6) Shear strength of cross ply laminate S_c

Output Parameters

1) Failure load

2) The failure mode

3) Stresses in the laminate

SECTION VII

RESULTS AND DISCUSSIONS

Results were generated in order to assess the validity and accuracy of the method and the computer code and to compare the results of the present method with the results of other existing methods of solutions. In addition, parametric studies were performed to evaluate the major characteristics of bolted joints.

1) Isotropic and Orthotropic Plates

Stress distributions were calculated in isotropic plates containing both unloaded (open) and loaded holes and in orthotropic plates containing unloaded holes. These problems were selected because analytical solutions are available for comparisons with the results of the present method.

An analytical solution for the stress distribution in an infinite ($W \to \infty$) isotropic plate containing an unloaded hole was given by Timoshenko [19]. The stress distribution in such a plate was also calculated by the present method. The parameters used in the numerical calculations are given in Figure 9. A large width ($W/D=14$) was used in the calculation to approximate an infinite plate. The results of the present method and the analytical solution of Timoshenko are compared in Figure 8. There is excellent agreement between the stresses calculated by the two methods.

The stresses in isotropic plates containing loaded holes were also calculated. Plates of infinite and finite widths were considered. Calculations were performed for the parameters given in Figure 9 and Table 2. From the calculated stresses, the stress concentration factor was determined. The stress concentration factor is defined as

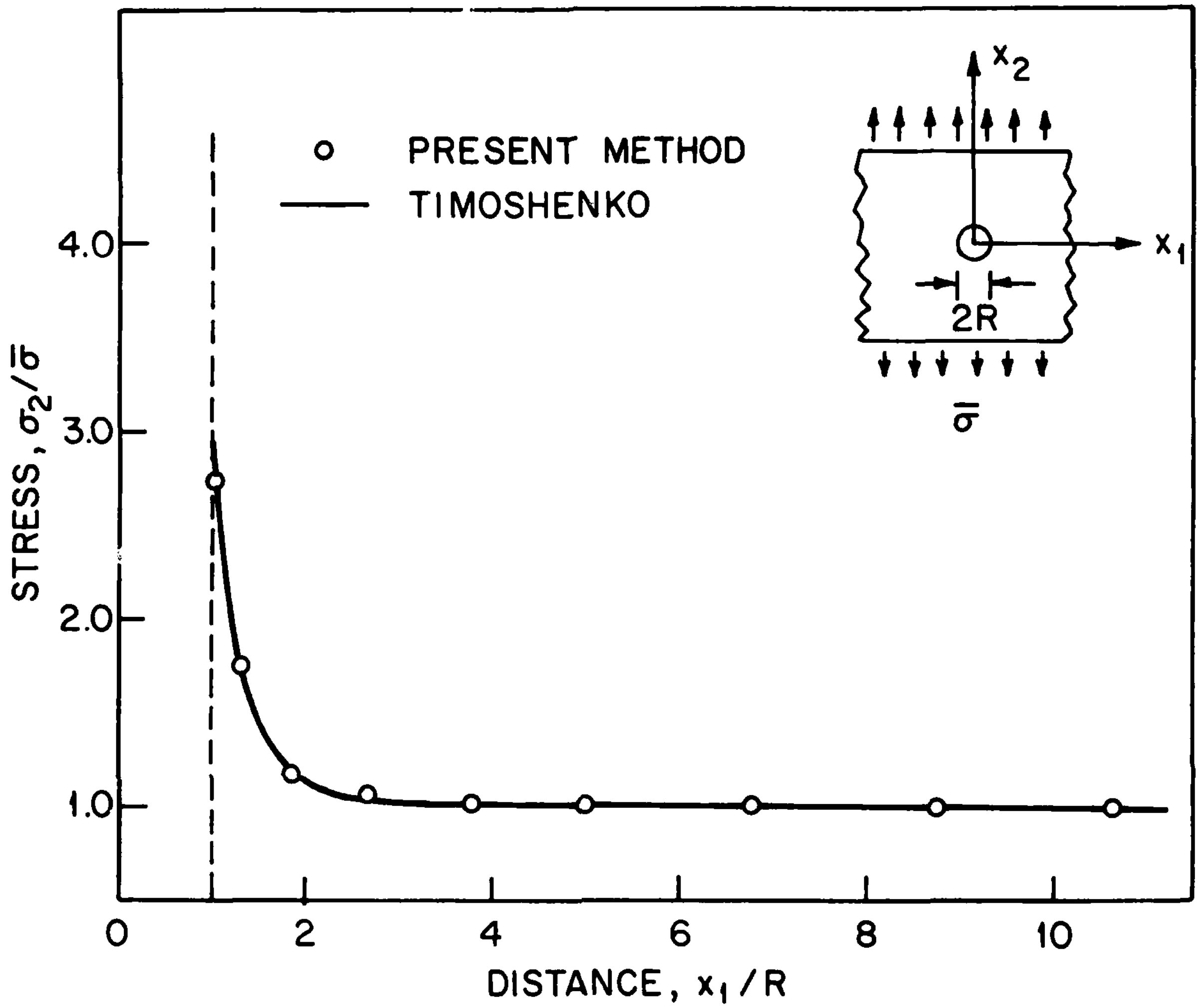

Figure 8. The stress σ_2 along the x_1-axis in an isotropic infinite plate containing a c'rcular hole. Comparison of the present results with the theoretical results given by Timoshenko [19]. Parameters used in the numerical calculations: $\bar{\sigma}$=1.64 MPa, D=2R=7.62 mm, W/D=14, E/D=14, L/D=28

$$SCF \equiv \frac{(\sigma_2)_{max}}{B} \tag{36}$$

where $(\sigma_2)_{max}$ is the maximum stress in the plate (perpendicular to the x_1-x_3 plane, Figure 1) and B is the bearing stress

$$B \equiv \frac{P}{(H)(D)} \tag{37}$$

The stress concentration factors obtained by the present method were compared to those reported by previous investigators (Table 2). The maximum difference in the stress concentration factors given by the different methods is about 20 percent. The stress concentration factor given by the present method differs from the values given by previous investigators at most by 15 percent.

The stresses in an isotropic plate of finite width containing a loaded hole are shown in Figure 9. The stresses calculated by the present method are in excellent agreement with De Jong's approximate solution [8].

The stress distribution in an orthotropic plate of finite width containing an open (unloaded) hole was also calculated. The calculations were performed for a plate with the symmetric laminate lay up of $[0/90]_S$. An analytical solution for this problem was provided previously by Nuismer and Whitney [18], who modified Lekhnitskii's earlier solution [20] for an infinite plate. The results given in Figure 10 show excellent agreement between the stresses calculated by the present method and by the analytical solution.

The aforementioned comparisons indicate that the present method predicts the stress distribution around loaded and unloaded holes with high accuracy.

Table 2. Stress Concentration Factor (SCF) Around a Pin Loaded Hole Contained in an Isotropic Plate of Infinite Width. Comparison of Present Result with Those Obtained by Previous Investigators.

Investigators	SCF
Present Result*	0.985
Hong [22]	0.955
De Jong [8]	1.058
Eshwar et al [23]	0.922
Bickley [7]	0.81

*Present results was calculated for the case:

$D = 7.62$ mm, $W/D = 8$, $E/D = 4$, $L/D = 14$.

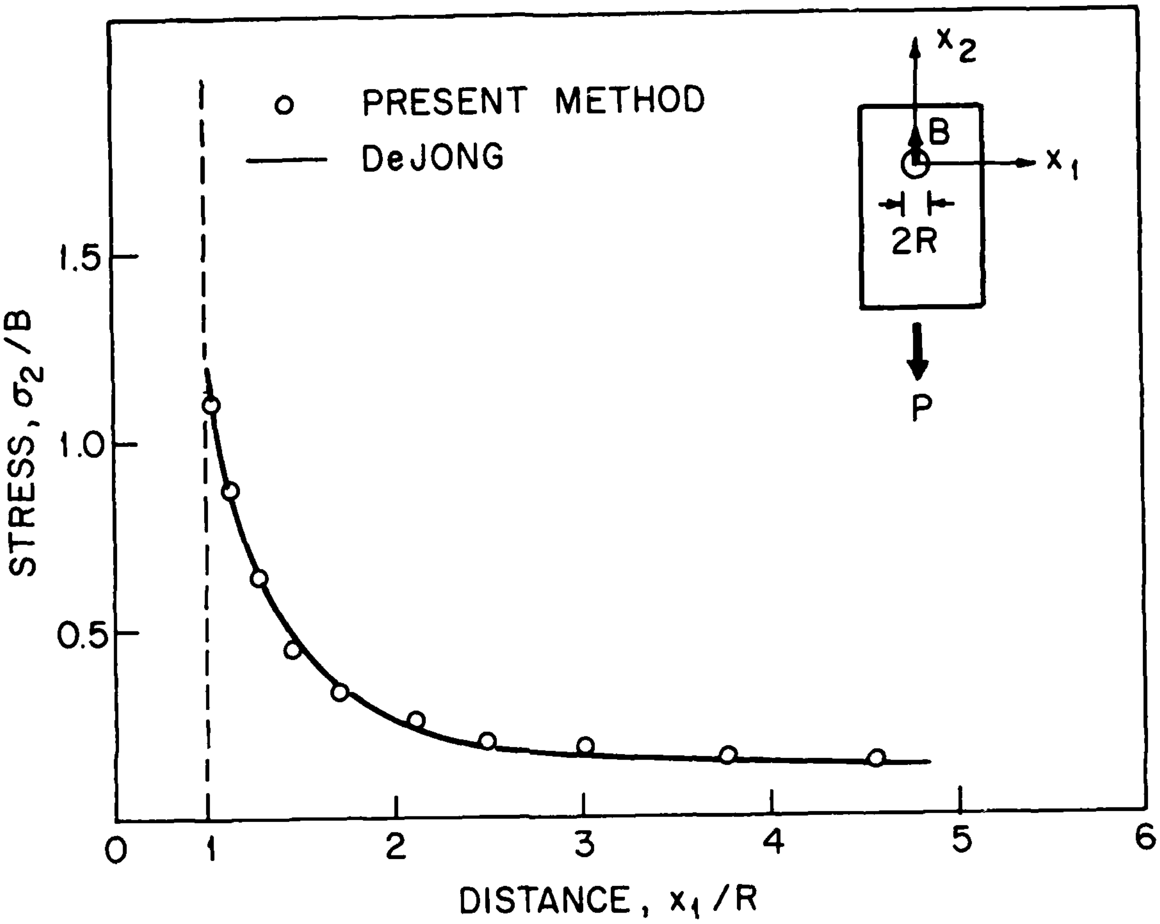

Figure 9. The stress σ_2 along the x_1-axis in an isotropic plate of finite width containing a loaded hole. Comparison of the present results with the theoretical results given by De Jong [8]. Parameters used in the numerical calculations: D=7.62 mm, W/D=5.0, E/D=4.0, L/D=14.0

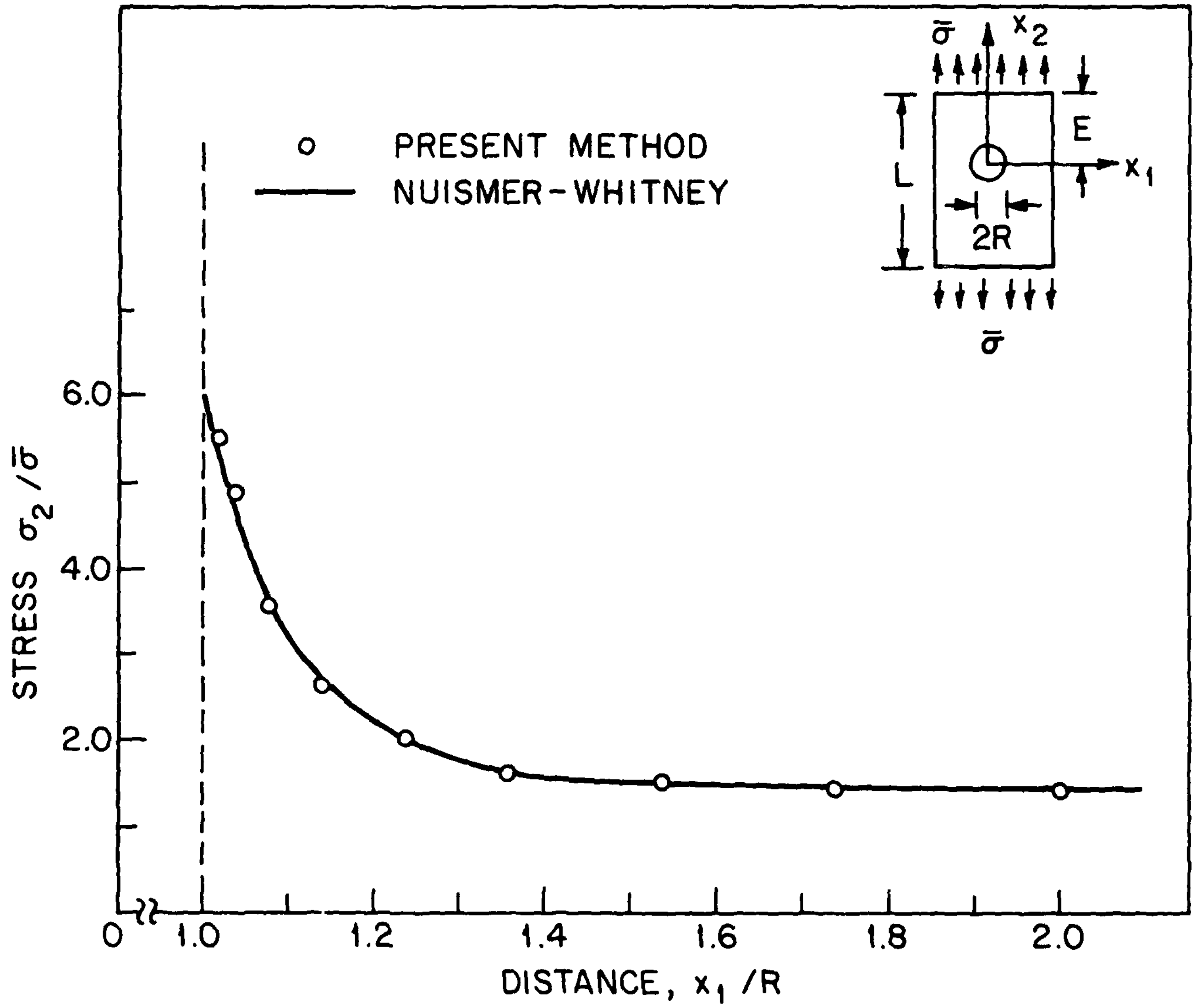

Figure 10. The stress σ_2 along the x_1-axis in an orthotropic finite plate $[0/90]_s$ containing a circular hole. Comparison of the present results with the theoretical results obtained by Nuismer and Whitney [18]. Parameters used in the numerical calculations: Material: Graphite/Epoxy T300/5208, E_1=149.8 GPa, E_2=11.2 GPa, G_{12}=5.39 GPa, ν_{12}=0.29, $\bar{\sigma}$=2.3 MPa, D=24.5 mm, W/D=3.0, E/D=4.0, L/D=14.0

2) <u>Failure Strength and Failure Mode</u>

Failure strengths of mechanically fastened composite joints calculated by the present method were compared to available data and to failure strengths predicted by other numerical methods.

Failure strengths of joints in graphite-epoxy laminates were measured by Van Siclen [21] and by Garbo and Oganowski [3]. The test conditions are summarized in Table 3. The failure strengths of the joints under these conditions were also calculated by the present method. The material properties used in the calculations are summarized in Table 4. Comparisons between the experimental and predicted failure strengths are given in Table 5. In these tables comparisons between the data and the failure strengths predicted by Agarwal [2] and by Garbo and Oganowski [3] are also included.

As can be seen, of the three analytical methods included in Table 5 the present one predicts the failure most accurately. In most cases, the failure load given by the present method agrees with the data within about 10%. On the other hand, the failure loads given by the Agarwal and by the Garbo and Oganowski methods are in error by as much as 50 percent.

One point is of interest here. When there is a high fraction of zero degree plies in the laminate (parallel to the body direction) the laminate fails in the shearout mode. In this case the calculated results are sensitive to the value of the laminate shear strength S_c. For example, when the S_c value obtained with cross ply laminates is used to calculate the failure load of $[0/\pm45/90]_s$ laminates containing 70 percent of 0° plies the calculated and

Table 3. Summary of Test Conditions

Investigator	Material	Case No.	D(mm)	W/D	E/D	L/D	H(mm)	Lay up	Volume Fraction %0°	%45°	%90°
Van Siclen [21]	T300/sp286	1	4.76	8.025	2.983	14.68	1.067	$[0/\pm45/90]_s$	25	50	25
		2	4.76	8.025	2.083	14.68	1.067	$[0/\pm45/90]_s$	25	50	25
		3	4.76	5.336	2.083	14.68	2.235	$[0/\pm45/90]_s$	25	50	25
		4	4.76	5.336	2.983	14.68	1.397	$[0_2/\pm45/90]_s$	40	40	20
		5	4.76	5.336	2.983	14.68	1.067	$[0/\pm45/90]_s$	25	50	25
		6	4.76	8.025	4.013	14.68	1.067	$[0/\pm45/90]_s$	25	50	25
		7	4.76	5.336	2.983	14.68	1.118	$[\pm45]_{2s}$	0	100	0
		8	4.76	5.336	2.983	14.68	1.118	$[0/90]_{2s}$	50	0	50
		9	4.76	5.336	2.983	14.68	1.067	$[0_2/\pm45]_s$	50	50	0
Garbo and Ogonowski [3]	AS/3501-6	10	6.35	6	3	14.68	5.283	$[0/\pm45/90]_s$	30	60	10
		11	6.35	6	3	14.68	5.283	$[0/\pm45/90]_s$	50	40	10
		12	6.35	6	3	14.68	5.283	$[0/\pm45/90]_s$	70	20	10

Table 4. Material Properties Used in the Calculations

Ply Properties	T300/SP286	AS/3501-6
Longitudinal Modulus E_1 Gpa(10 ksi)	130 (18.7)[21]	130(18.85)[3]
Transverse Modulus E_2 Gpa(10 ksi)	8.274(1.2)[21]	13.1(1.9)[3]
Shear Modulus G_{12} Gpa(10 ksi)	5.033(0.73)[21]	5.86(0.85)[3]
Poisson Ratio ν_{12}	0.30[21]	0.30[3]
Tensile Strength X Gpa(10 ksi)	1.23(0.178)[21]	1.58(0.23)[3]
Shear Strength S Gpa(10 ksi)	0.05(0.0073)[21]	0.12(0.017)[3]

Laminate Properties	T300/SP286	AS/3501-6
Cross-ply Laminate Shear Strength S_c Gpa(10 ksi)	0.125 [a](0.018)	0.204 [b](0.03) 0.12[c](0.017)
Characteristic Length R_{ot} mm(in)	1.092 [d](0.043)	0.584 (0.023)[3]
Characteristic Length R_{oc} mm(in)	3.048 [e](0.12)	1.727[e](0.068)

(a) Tests with Glass/Epoxy showed S_c to be about 2.5 times higher than the ply shear strength S [4]. The 2.5 multiplier was used to obtain S_c of Graphite/Epoxy T300/SP286 from the ply shear strength S given in [21].

(b) The value of S_c for AS/3501-6 was taken to be 1.7 times the ply shear strength S.

(c) This value was used for laminates containing more than 70% (by volume) of 0 degree plies.

(d) For T300/SP286 the value was chosen from [18] for T300/5208.

(e) The values of R_{oc} for T300/SP286 and AS/3501-6 were evaluated by the method given in [18] together with the data in [24,25].

Table 5. Comparisons Between the Experimental (P) and Calculated (P_c) Failure Loads. Case Numbers Correspond to Test Conditions Given in Table 3.

Material	Case No.	Lay Up	Percent Difference $(1-P/P_c)x100$	
			Present Results	Agarwal(1980)
T300/SP286	1	$[0/\pm45/90]_s$	3.7	8.7
	2	$[0/\pm45/90]_s$	0.01	0.01
	3	$[0/\pm45/90]_{2s}$	1.81	7.78
	4	$[0_2/\pm45/90]_s$	0.01	0.01
	5	$[0/\pm45/90]_s$	8.66	1.16
	6	$[0/\pm45/90]_s$	11.18	1.2
	7	$[\pm45]_{2s}$	11.1	49.43
	8	$[0/90]_{2s}$	12.3	49.85
	9	$[0_2/\pm45]_s$	7.64	20.0
			Present Results	Garbo and Ogonowski(1981)
AS/3501-6	10	$[0/\pm45/90]_s$	0.5	45
	11	$[0/\pm45/90]_s$	3.8	27
	12	$[0/\pm45/90]_s$	15.0	14

measured values differ by 35 percent. On the other hand, if the shear stress of the individual ply is used in the calculations the difference between the calculated failure load and the data is only 15 percent. This indicates that the value of S_c must be selected carefully.

Failure loads calculated by Waszczak and Cruse [1] are compared to data in Table 6. Waszczak and Cruse's method also yields failure loads which are in error by as much as 50 percent. The present method was not applied to Waszczak and Cruse's data because the material properties needed for the calculation were unavailable.

It is interesting to note that the accuracies of all four methods (present, Agarwal, Garbo and Ogonowski, and Waszczak and Cruse) depend on the arrangements of the plies in the laminate. In general, the the analytical predictions are most accurate for quasi-isotropic laminates, and are least accurate for angle ply and cross ply laminates. However, even for angle ply and cross ply laminates the present method yields results within about 10 percent accuracy, in contrast with the results of other existing methods of solutions, which may be in error by as much as 50 percent.

The failure modes predicted by the present method were also compared to failure modes observed experimentally. These comparisons, given in Table 7, show that the present method predicts well the mode of failure.

The aforementioned comparisons between the results of the present method and the data show that the method predicts with good accuracy both the load at which the joint fails and the mode of failure. The good agreements between the predictions of the model and the data

Table 6. Comparisons Between the Experimental Failure Loads and the Values Predicted by Waszczak and Cruse [1]

Material	Lay Up	Volume Fraction % 45	0	90	W/D	E/D	Percent Difference (1-[P/P$_c$)x100
Graphite/Epoxy	[±45]	100	0	0	4	3	53
Boron/Epoxy	$[(\pm45/\bar{0})_s/\overline{90}]_s$	72.3	18.2	9.1	4	2	24
	$[\pm45_5/90_6]$	62.5	0	37.5	4	3	7
	$[\pm45/(0_6/90)_s]_s$	13.3	80.0	6.7	6	6	42
	$[0_6/\pm45_5]$	62.5	37.5	0	7.5	2.5	2
	$[0_6/\pm45]$	62.5	37.5	0	7.35	4	35

Table 7. Comparisons of Predicted Failure Modes with those Observed Experimentally
T-Tension Mode S-Shearout Mode B-Bearing Mode

Material	Case No.	Lay Up	Observed Failure Mode	Predicted Failure Mode Present	Predicted Failure Mode Agarwal (1980)
T300/SP286	1	$[0/\pm45/90]_s$	S/B	S	S
	2	$[0/\pm45/90]_s$	S	S	S
	3	$[0/\pm45/90]_{2s}$	T/S	T	T
	4	$[0_2/\pm45/90]_s$	S	S	S
	5	$[0/\pm45/90]_s$	T	T	T
	6	$[0/\ 45/90]_s$	B	B	B
	8	$[0/90]_{2s}$	S	S	S
	9	$[0_2/\pm45]_s$	B/S	S	S
AS/3501-6				Present	Garbo & Ogonowski (1981)
	10	$[0/\pm45/90]_s$	B/S	B	-
	11	$[0/\pm45/90]_s$	B/S	S	-
	12	$[0/\pm45/90]_s$	B/S	S	-

(illustrated in Tables 5-7) create further confidence in the model.

3) Effects of Geometry and Ply Orientations

Parametric studies were performed to evaluate the effects of joint geometry and ply orientation on the failure strength and on the failure mode.

The effects of joint width on joint failure is illustrated in Figure 11. In this, and in subsequent figures, the failure load is normalized with respect to the ultimate tensile load of the laminate (without the hole) in the direction of the applied load. As is shown by the results in Figure 11, in general, the maximum load the joint can carry decreases as the hole size decreases, when the width to hole diameter ratio is greater than about 3. As the hole diameter approaches the width ($W/D \rightarrow 1$) the strength reduces to zero ($P \rightarrow 0$). Here failure loads were not calculated for W/D less then three because at such low W/D ratios the assumption of the cosine load distribution (eq. 17) is inaccurate [8].

The effect of edge distance E (Figure 1) on the failure load is shown in Figure 12. For the lay-ups $[0/\pm 45/90]_s$ and $[0/90]_{2s}$ increasing edge distance results in higher failure loads, as long as E/D is less than about 4. For higher edge ratios ($E/D>4$) an increase in edge distance does not seem to influence significantly the failure load. For the lay-up $[0_2/\pm 45]_s$, the failure load does not vary significantly with E/D.

The effects of ply orientation on the failure load are given in Figure 13. This figure illustrates the effects of two parameters 1) the maximum ply angle ϕ in the laminate and 2) the change in

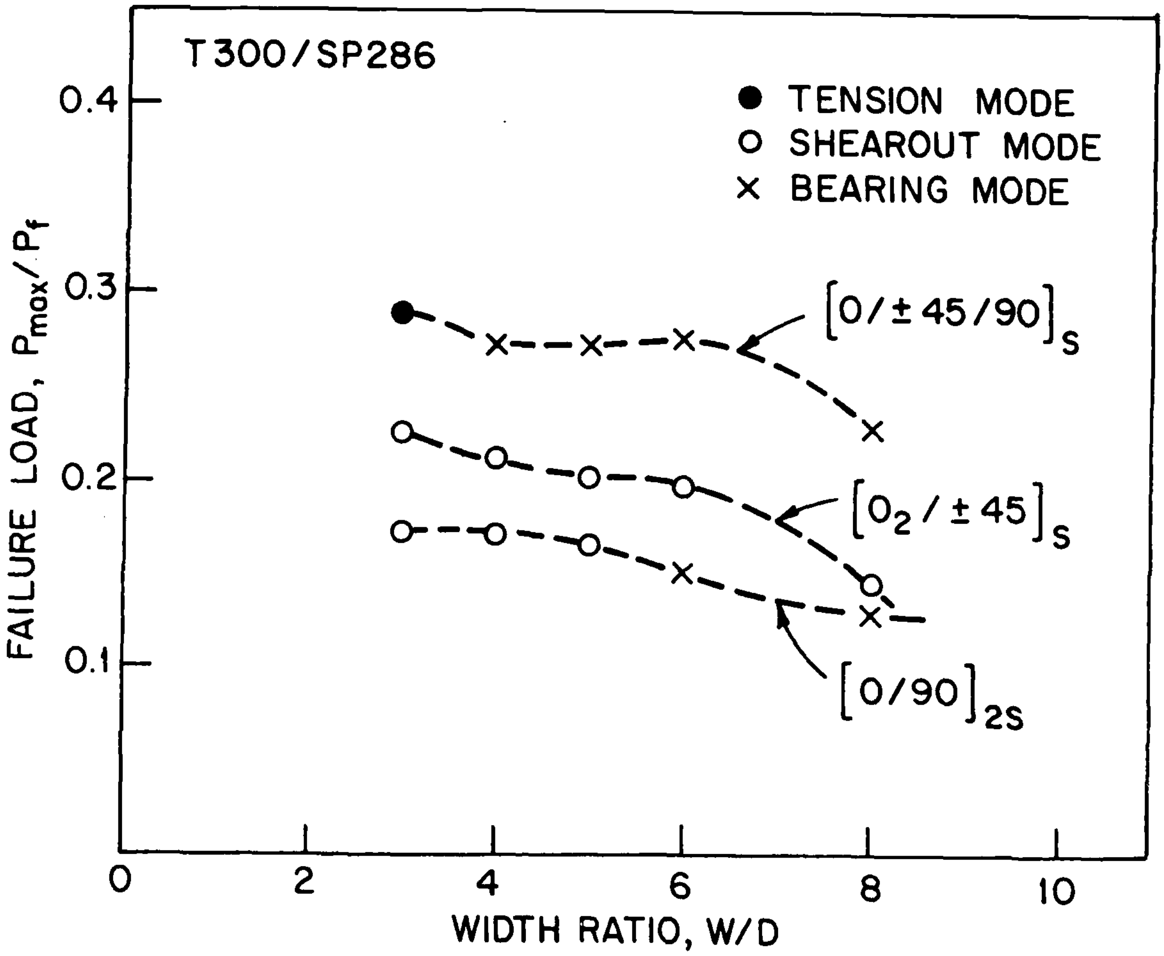

Figure 11. The effects of width ratio on the failure load of laminates with different ply orientations. P_f is the tensile failure load of laminates without holes. Parameters used in the numerical calculations: Material: Graphite/Epoxy T300/SP286, W=38 mm, E=50.8 mm, L=203.2 mm, H=1.067 mm for $[0/\pm45/90]_s$ and $[0_2/\pm45]_s$, and H=1.18 mm for $[0/90]_{2s}$

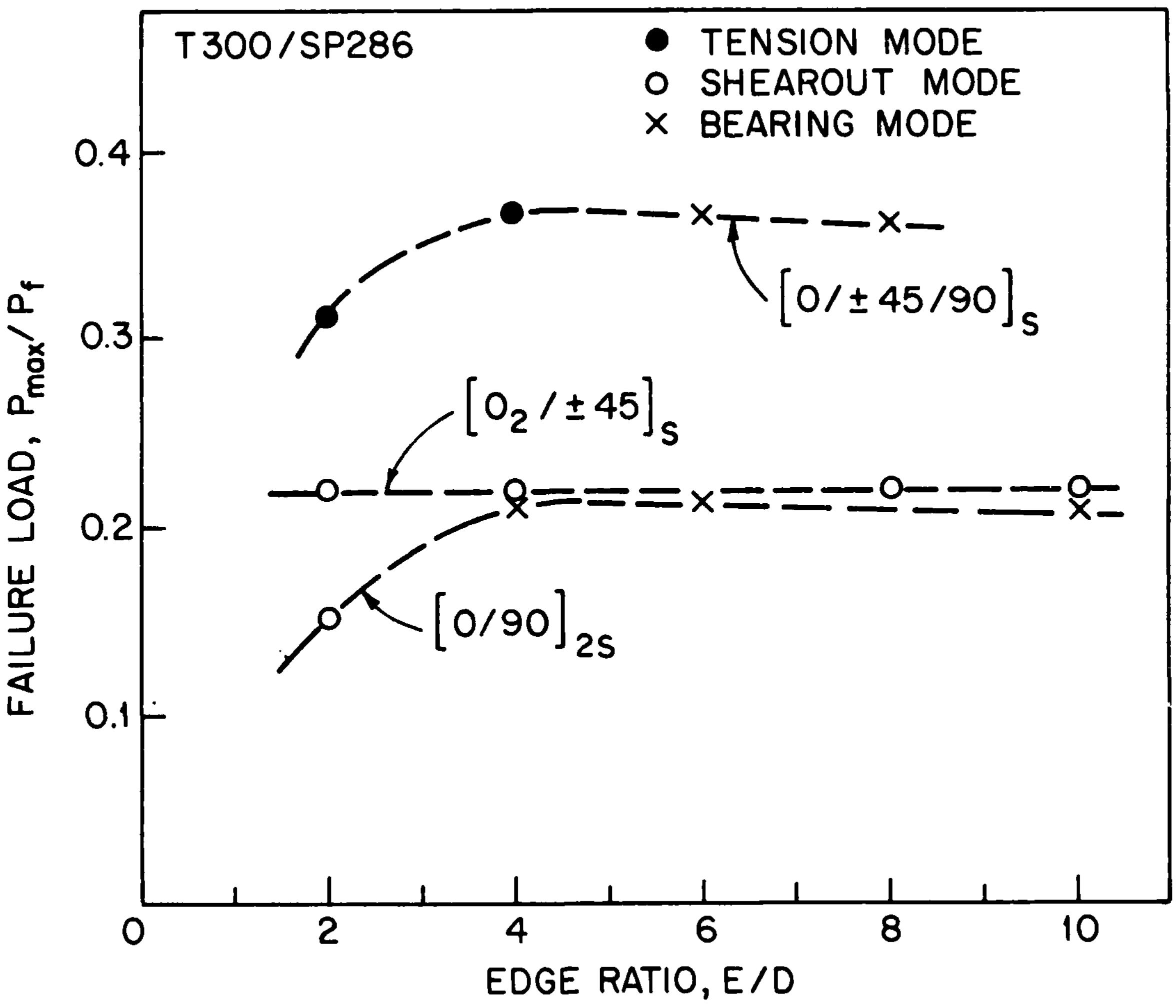

Figure 12. The effects of edge ratio on the failure load of laminates with different ply orientations. Parameters used in the numerical calculations: Material: Graphite/Epoxy T300/SP286, D=5.08 mm, W/D=5, L/D=14

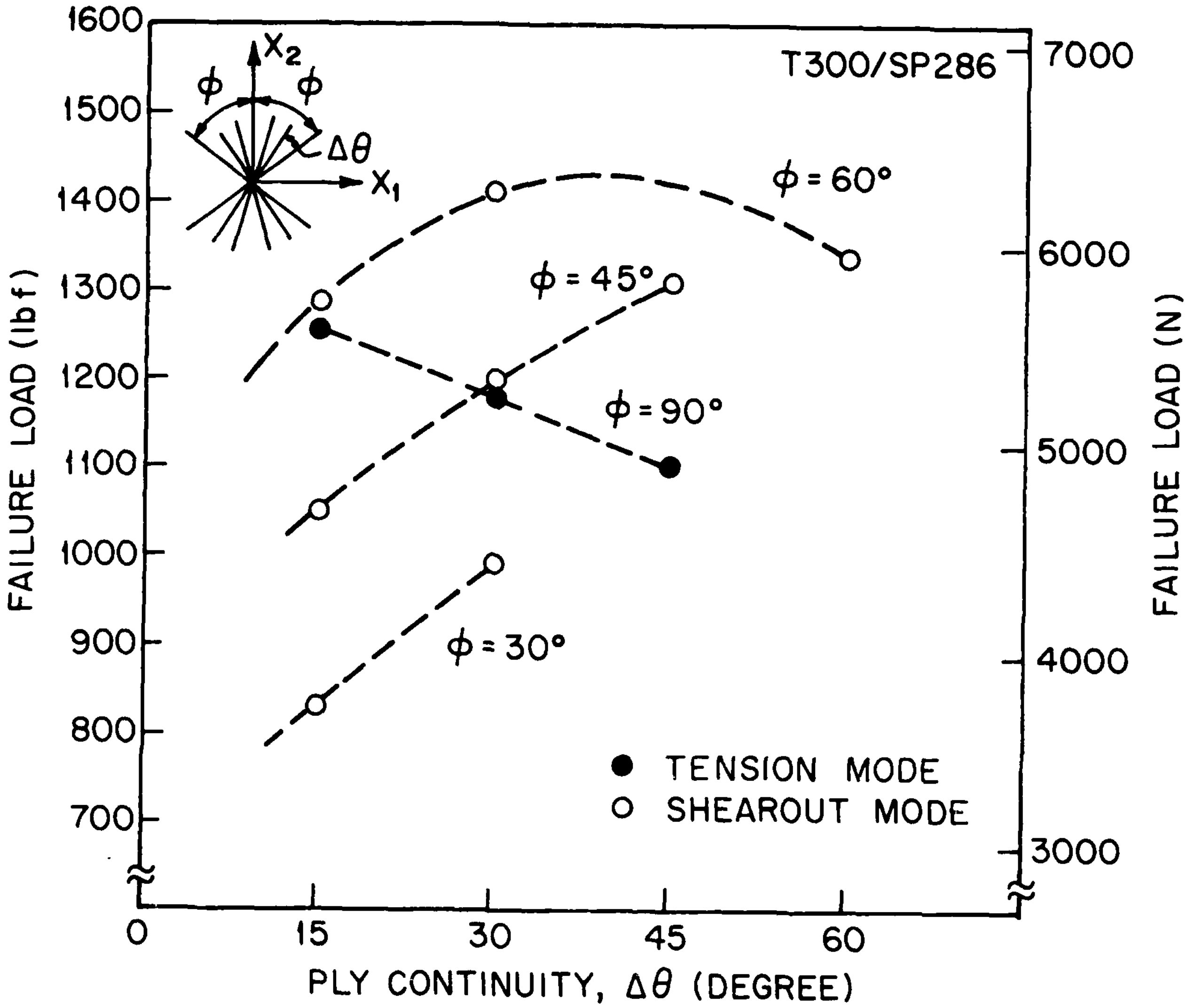

Figure 13. The effects of maximum ply angle ϕ and ply continuity $\Delta\theta$ on the failure load of mechanically fastened joints. Parameters used in the numerical calculations: Material: Graphite/Epoxy T300/SP286, D=4.76 mm, W/D=5.336, E/D=2.983, L/D=14.68, H=1.397 mm

orientation between two adjacent plies $\Delta\theta$. The latter parameter is referred to here as "ply continuity". The results in Figure 13 show that the failure load increases both with increasing ϕ and with increasing $\Delta\theta$, as long as failure is by shearout mode. On the other hand, the failure load decreases with increasing ϕ and with increasing $\Delta\theta$ when the failure is by tension mode. These results indicate that care must be exercised in designing bolted joints. If there are no other design constraints, the range of ply orientation ϕ and the ply continuity $\Delta\theta$ should be determined with the use of the computer code such that the joint can withstand the highest load.

SECTION VIII

CONCLUDING REMARKS

The model and computer code developed in this investigation can be used in the design of mechanically fastened joints involving fiber reinforced laminates. The computer code can be used to determine

a) the optimum geometry of a joint for a given load,

b) whether or not the joint will fail under a given load,

c) the failure load,

d) the mode of failure, and

e) the ply in which failure first occurs.

The good accuracy of the method suggests that it might be worth it to extend the method to joints consisting of two or more fasteners.

The results of parametric studies performed with the present computer code show that the material properties, joint geometry, and ply orientation, all effect significantly the strength of mechanically fastened joints.

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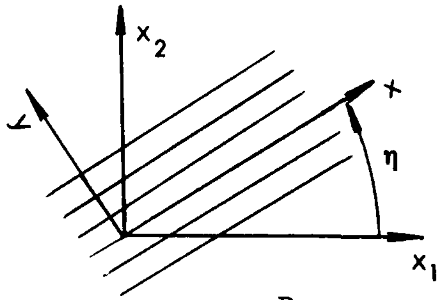
Appendix A - The Transformed Reduced Stiffness Matrix Q_{ij}



The components of the matrix $\bar{Q}^P_{ij}$ appearing in Eq. (16) are

$$
\begin{aligned}
\bar{Q}^P_{11} &= Q^P_{11}\cos^4\eta+2(Q^P_{12}+2Q^P_{66})\sin^2\eta\cos^2\eta+Q^P_{22}\sin^4\eta \\
\bar{Q}^P_{12} &= (Q^P_{11}+Q^P_{22}-4Q^P_{66})\sin^2\eta\cos^2\eta+Q^P_{12}(\sin^4\eta+\cos^4\eta) \\
\bar{Q}^P_{22} &= Q^P_{11}\sin^4\eta+2(Q^P_{12}+2Q^P_{66})\sin^2\eta\cos^2\eta+Q^P_{22}\cos^4\eta \\
\bar{Q}^P_{13} &= (Q^P_{11}-Q^P_{12}-2Q^P_{33})\sin\eta\cos^3\eta+(Q^P_{12}-Q^P_{22}+2Q^P_{33})\sin^3\eta\cos\eta \\
\bar{Q}^P_{23} &= (Q^P_{11}-Q^P_{12}-2Q^P_{33})\sin^3\eta\cos\eta+(Q^P_{12}-Q^P_{22}+2Q^P_{33})\sin\eta\cos^3\eta \\
\bar{Q}^P_{33} &= (Q^P_{11}+Q^P_{22}-2Q^P_{12}-2Q^P_{33})\sin^2\eta\cos^2\eta+Q^P_{33}(\sin^4\eta+\cos^4\eta)
\end{aligned}
$$

in which

$$Q^P_{11} = \frac{E^P_1}{1-\nu^P_{12}\nu^P_{21}}$$

$$Q^P_{12} \quad \frac{\nu^P_{12}\,E^P_2}{1-\nu^P_{12}\nu^P_{21}} = \frac{\nu^P_{21}\,E^P_1}{1-\nu^P_{12}\nu^P_{21}}$$

$$Q^P_{22} = \frac{E^P_2}{1-\nu^P_{12}\nu^P_{21}}$$

$$Q^P_{33} = G^P_{12}$$

The superscript p denotes the material properties of the p-th ply and the angle η is measured from the x_1-axis to the x-axis. E_1^P, E_2^P and G_{12}^P are the longitudinal, transverse and shear moduli of the p-th ply, respectively. ν_{12}^P and ν_{21}^P are Poisson's rations of p-th ply and satisfy the relation

$$\frac{\nu_{12}^P}{E_1^P} = \frac{\nu_{21}^P}{E_2^P}$$

Appendix B - Shape Function Used in the Finite Element Code.

In the isoparametric element, the geometry and the displacement of the element are described in terms of the shape function N_α by a transformation from a Master Element in the r-s coordinate system to the element in the x_1-x_2 coordinate system (Figure 14)

$$X_i = N_\alpha(r,s)\,\bar{x}_{i\alpha} \qquad i = 1,2$$

$$u_i = N_\alpha(r,s)\,q_{i\alpha} \qquad \alpha = 1,2,3 \text{ or } 4$$

$$N_\alpha(r,s) = 1/4(1+r\,r_\alpha)(1+ss_\alpha) \qquad -1 \leq r,s \leq 1$$

where $x_{i\alpha}$ is the coordinate of node α in the i-direction and $q_{i\alpha}$ is the displacement of node α in the i-direction, and r_α and s_α are the coordinates of node α referred to th Master Element

Note the property

$$N_\alpha(r_\beta, s_\beta) = \begin{cases} 1, & \text{if } \alpha=\beta \\ 0, & \text{if } \alpha\neq\beta \end{cases}$$

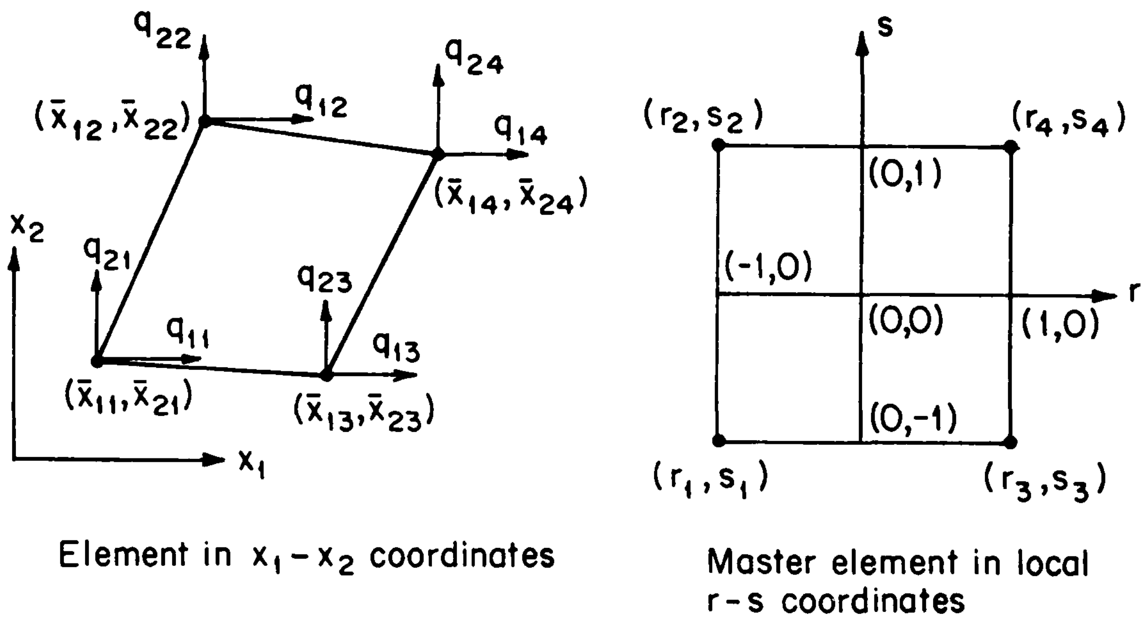


Figure 14. Geometry of an element used in the finite element calculations. Left: Element in the x_1-x_2 coordinate system. Right: Element (master element) in the local (r-s) coordinate system. $x_{i\alpha}$ is the coordinate of node α in the i direction, $q_{i\alpha}$ is the displacement of node α in the i direction and (r_α, s_α) are the coordinates of node α in the r-s coordinate system, i=1,2, α=1,2,3 or 4

Appendix C - Listing of the Computer Code "BOLT", and a Sample of Input and Output.

```
C
C *******************************************************************
C
C
C                          <  BOLT  >
C
C
C                   THIS PROGRAN IS DEVELOPED FOR PREDICTING THE FAILURE
C              LOAD AND FAILURE MODE OF SINGLE FASTENER COMPOSITE JOINTS AS
C              WELL AS LAMINATES CONTAINING A OPEN HOLE (WITHOUT LOAD) .  A
C              DISPLACEMENT MODLE OF FINITE ELEMENT METHOD IS USED TO SOLVE
C              THE STRESSES  AROUND  THE BOUNDARY OF THE HOLE  AND THEN THE
C              FAILURE HYPOTHESIS-CHARACTERISTIC CURVE TOGETHER WITH YAMADA
C              FAILURE CRITERION  IS USED TO  PREDICT THE  FAILURE LOAD AND
C              FAILURE MODE.
C
C
C
C                         -------   FU-KUO CHANG    ----------
C                          APPLIED MECHANICS AT UNIVERSITY OF MICHIGAN
C                           ANN ARBOR, MICHIGAN, 48109
C
C
C
C
C *******************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
     1RAN
      COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
      COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
      COMMON / AFORE/ PF,DP,F(706)
      COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
      COMMON /ALEM/ NODXY,NELX,IJK(4,400)
      COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
     1              ,SI(4),RI(4),P(20),Q(20)
      COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
     1)
      COMMON /AMAX/ SK(706,90),R1(706)
C
C
      DIMENSION DIS(2,400)
      EQUIVALENCE (DIS(1),R1(1))
C
C
      CALL PGAUSS
      CALL INPUT
      CALL MATRL
      CALL FORMK
      CALL BOUND
C
   10 DO 5 I=1,NEQ
      F(I)=F(I)*PF
    5 CONTINUE
      CALL SOLVE
```

```
               C      DO 15 I=1,353                                                        52.000
               C      WRITE(6,400) I,(DIS(J,I),J=1,2)                                      53.000
               C 400  FORMAT(/,5X,'POINT=',I3,5X,'X-DIS',E15.8,5X,'Y-DIS',E15.8/)          54.000
               C  15  CONTINUE                                                             55.000
  0022                CALL STRESS(DIS)                                                     56.000
  0023                CALL CFAIL(KT,MT,FAL)                                                57.000
  0024                IF(FAL .GT. 1.) GO TO 100                                            58.000
               C                                                                           59.000
  0025                TN=DSQRT(1.0/FAL)                                                    60.000
  0026                PF=TN*PF*HI*4.0                                                      61.000
               C                                                                           62.000
  0027            50 CALL OUTPUT(KT,MT,FAL)                                                63.000
  0028                GO TO 11                                                             64.000
  0029           100 WRITE(6,111)                                                          65.000
  0030           111 FORMAT (/,5X,' ****** THE APPLIED LOAD IS TOO HIGH   ****',/)         66.000
  0031            11 STOP                                                                  67.000
  0032                END                                                                  68.000
   *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NUDECK,LOAD,NOMAP
   *OPTIONS IN EFFECT*  NAME = MAIN    , LINECNT =       57
   *STATISTICS*    SOURCE STATEMENTS =       32,PROGRAM SIZE =       798
   *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
MICHIGAN TERMINAL SYSTEM FORTRAN G(21.8)              PGAUSS        06-01-82        16:14:20        PAGE P001

 0001                SUBROUTINE PGAUSS
 0002                IMPLICIT REAL*8 (A-H,O-Z)
 0003                COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
                    1               ,SI(4),RI(4),P(20),Q(20)
 0004                COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
 0005                LINT=2
 0006                RG(1)=0.577350269187
 0007                RG(2)=-RG(1)
 0008                SG(1)=RG(1)
 0009                SG(2)=RG(2)
           C
 0010                WG(1)=1.
 0011                WG(2)=1.
           C
 0012                RI(1)=-1.0
 0013                SI(1)=-1.0
 0014                RI(2)=-1.0
 0015                SI(2)= 1.0
 0016                RI(3)= 1.0
 0017                SI(3)=-1.0
 0018                RI(4)=1.0
 0019                SI(4)= 1.0
           C
 0020                RETURN
 0021                END
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = PGAUSS  , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       21,PROGRAM SIZE =      424
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
 0001               SUBROUTINE INPUT
              C
              C
              C
              C *********************************************************************
              C
              C
              C        -----  INPUT INSTRUCTION
              C
              C        1) INDICATOR
              C
              C            1.1) TEST :  A INDICATOR TO SPECIFY THE PROBLEM
              C
              C                  TEST=0  :  OPEN HOLE (WITHOUT LOAD)
              C
              C                  TEST=1  :  LOADED HOLE (JOINT)
              C
              C            1.2) NRAN  :  A INDICATOR TO SPECIFY THE PLY ORIENTATION
              C
              C                  NRAN=0  :  MAXIMUM   PLY ANGLE (ANG) AND  PLY
              C                             CONTINUITY (DANG) AS INPUT PARAMETERS
              C
              C                  NRAN=1  :  INDIVIDAL PLY ANGLE (ANT(I)) AS INPUT
              C                             PARAMETERS
              C
              C        2)  MATERIAL PROPERTIES
              C
              C              2.1) E1   :  PLY LONGITUDINAL TENSIL MODULOUS
              C
              C              2.2) V12 :  POISSON'S RATIO FOR TRANSVERSE STRAIN IN
              C                          THE Y-DIRECTION WHEN STRESSED IN THE
              C                          X-DIRECTION
              C
              C              2.3) G12 :  PLY SHEAR MODULUS
              C
              C              2.4) XX   :  PLY LONGITUDINAL TENSILE STRENGTH
              C
              C              2.5) SS   :  LAMINATE SHEAR STRENGTH OF CROSS-PLY
              C
              C              2.6) RT   :  CHARACTERISTIC LENGTH FOR TENSION
              C
              C              2.7) RC   :  CHARACTERISTIC LENGTH FOR COMPRESSION
              C
              C        3)  PLY ORIENTATION
              C
              C             3.1) ANG   :  MAXIMUM PLY ANGLE  (NRAN=0)
              C
              C             3.2) DANG  :  PLY CONTINUITY  (NRAN=0)
              C
              C             3.3) ANT(I)  :  INDIVIDUAL PLY ANGLE  (NRAN=1)
              C
              C        4)  GEOMETRY
              C
              C             4.1) D   :  DIAMETER
              C
```

```
C             4.2) WD  : WIDTH RATIO W/D
C
C             4.3) ED  :  EDGE RATIO E/D
C
C             4.4) LD  :  LENGTH RATIO L/D
C
C             4.5) NLY  :  A HALF NUMBER OF TOTAL PLIES IN THE
C                          SYMMETRIC LAMINATE
C
C             4.6) HI  :  ONE HALF OF LAMINATE THICKNESS
C
C             4.7) THICK(I)  :  THICKNESS OF I-TH PLY  (NRAN=1)
C
C
C
C      * NOTE  :
C                 IF  NRAN=0. NLY. THICK(I)'AND ANT(I) DO NOT
C                 HAVE TO BE INPUT.
C
C                 IF NRAN=1. INPUT ANG=0.0 AND DANG=1.0.
C
C
C
C
C ***********************************************************************
C ********************
C
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
     1RAN
      COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
      COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
      COMMON / AFORE/ PF,DP,F(706)
      COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
      COMMON /ALEM/ NODXY,NELX,IJK(4,400)
      COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
     1             ,SI(4),RI(4),P(20),Q(20)
      READ (5,1) ITEST,NRAN
    1 FORMAT (I5)
      READ (5,10) ANG,DANG
      READ (5,10) E1,V12,G12
      READ (5,10) D,WD,ED,ALD
      READ (5,10) XX,SS,RT,RC
      READ (5,10) HI
C
C
      IF(NRAN .EQ. 0) GO TO 6
      READ (5,1) NLY
      READ (5,10) (ANT(I),THICK(I),I=1,NLY)
   10 FORMAT (4F15.5)
C
    6 CONTINUE
      NP=4
```

```
0023             NX=353
0024             NELX=306
0025             NEQ=706
0026             NBAND=82
0027             E2=100
0028             PF=500
           C
           C
           C
0029             W=WD*D
0030             E=ED*D
0031             AL=ALD*D
0032             TP=4.0*PF*HI
           C
0033             CALL AMESH
           C
           C  ****  INPUT FIXED BOUNDARY POINTS  *****
           C
0034             NBC(1)=353
0035             NBC(2)=352
0036             NBC(3)=351
0037             NBC(4)=350
0038             NBC(5)=349
0039             NBC(6)=348
0040             NBC(7)=347
0041             NBC(8)=346
0042             NBC(9)=345
0043             NBC(10)=344
0044             NBC(11)=343
0045             NBC(12)=342
0046             NBC(13)=341
0047             NBC(14)=340
0048             NBC(15)=339
0049             NBC(16)=1
0050             NBC(17)=2
0051             NBC(18)=3
0052             NBC(19)=4
0053             NBC(20)=5
0054             NBC(21)=6
0055             NBC(22)=7
0056             NBC(23)=8
0057             NBC(24)=9
0058             NBC(25)=10
0059             NBC(26)=11
0060             IF(ITEST .EQ. 0) GO TO 40
0061             NBC(27)=338
0062             NBC(28)=323
0063             NBC(29)=308
0064             NBC(30)=290
0065             NBC(31)=291
0066             NBC(32)=292
0067             NBC(33)=293
           C
0068             IF(ITEST .EQ. 1) GO TO 50
           C
```

```
0069        40 NBC(27)=22
0070           NBC(28)=33
0071           NBC(29)=44
0072           NBC(30)=55
0073           NBC(31)=66
0074           NBC(32)=77
0075           NBC(33)=88
0076           NBC(34)=99
0077           NBC(35)=103
0078           NBC(36)=107
0079           NBC(37)=111
         C
         C
0080        50 NFIX(1)=11
0081           IF(ITEST .EQ. 0) NFIX(1)=10
0082           NFIX(2)=10
0083           NFIX(3)=10
0084           NFIX(4)=10
0085           NFIX(5)=10
0086           NFIX(6)=10
0087           NFIX(7)=10
0088           NFIX(8)=10
0089           NFIX(9)=10
0090           NFIX(10)=10
0091           NFIX(11)=10
0092           NFIX(12)=10
0093           NFIX(13)=10
0094           NFIX(14)=10
0095           NFIX(15)=10
0096           NFIX(16)=10
         C
         C     IF(ITEST .EQ. 1) NFIX(16)=11
         C
0097           NFIX(17)=10
0098           NFIX(18)=10
0099           NFIX(19)=10
0100           NFIX(20)=10
0101           NFIX(21)=10
0102           NFIX(22)=10
0103           NFIX(23)=10
0104           NFIX(24)=10
0105           NFIX(25)=10
0106           NFIX(26)=11
         C
0107           IF(ITEST .EQ. 1) NFIX(26)=10
         C
         C
0108           NFIX(27)=1
0109           NFIX(28)=1
0110           NFIX(29)=1
0111           NFIX(30)=1
0112           NFIX(31)=1
0113           NFIX(32)=1
0114           NFIX(33)=1
0115           IF(ITEST .EQ. 1) GO TO 55
```

```
      NFIX(34)=1
      NFIX(35)=1
      NFIX(36)=1
      NFIX(37)=1
C
C  *****  INPUT  NODAL UNIT FORCES  ************
C
   55 FT=X(262)/(W/2.0)
      FT3=X(316)/(W/2.0)
      FT2=FT-FT3
      FT1=1-FT
C
      DO 100 I=1,706
      F(I)=0.0
  100 CONTINUE
      IF(ITEST .EQ. 1) GO TO 155
C
      F(586)=-FT1/6.0
      F(584)=-FT1/3.0
      F(582)=-FT1/3.0
      F(580)=-(FT1/6.0)-(FT2/4.0)
      F(616)=-FT2/2.0
      F(646)=-(FT2/4.0)-(FT3/4.0)
      F(676)=-FT3/2.
      F(706)=-FT3/4.0
      IF(ITEST .EQ. 0) GO TO 150
C
  155 PI=3.141596535
      F(1)=2.0*(4.0/(D*PI))*0.0
      F(2)=0.062499988
      F(23)=0.012193140
      F(24)=0.12379906
      F(45)=0.023917705
      F(46)=0.12024245
      F(67)=0.034723127
      F(68)=0.11446683
      F(89)=0.044194158
      F(90)=0.10669416
      F(111)=0.051966834
      F(112)=0.097223130
      F(133)=0.057742454
      F(134)=0.086417711
      F(155)=0.061299064
      F(156)=0.074693147
      F(177)=0.062499988
      F(178)=0.062500007
      F(223)=0.061299072
      F(224)=0.050306867
      F(245)=0.057742469
      F(246)=0.038582300
      F(267)=0.051966855
      F(268)=0.027776877
      F(289)=0.044194186
      F(290)=0.018305843
      F(311)=0.034723159
```

```
 0165              F(312)=0.010533165                                                      365.000
 0166              F(333)=0.023917741                                                      366.000
 0167              F(334)=0.0047575414                                                     367.000
 0168              F(355)=0.012193179                                                      368.000
 0169              F(356)=0.0012009269                                                     369.000
 0170              F(377)=0.0020453255                                                     370.000
 0171              F(378)=2.0*0.00                                                         371.000
            C                                                                              372.000
            C                                                                              373.000
 0172           150 CONTINUE                                                               374.000
            C                                                                              375.000
 0173              RETURN                                                                  376.000
 0174              END                                                                     377.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = INPUT    , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =      174,PROGRAM SIZE =      2894
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001            SUBROUTINE AMESH                                                   378.000
0002            IMPLICIT REAL*8(A-H,O-Z)                                           379.000
0003            COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N 380.000
               1RAN                                                                380.000
0004            COMMON /ADIM/ D,WD,ED,ALD,W,E,AL                                   381.000
0005            COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND                382.000
0006            COMMON /ALEM/ NODXY,NELX,IJK(4,400)                                383.000
0007            COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)                     384.000
               1          ,SI(4),RI(4),P(20),Q(20)                                 385.000
0008            DIMENSION BX(33,10),BY(33,10)                                      386.000
0009            DIMENSION NES(30),NS(30),K12(30),K43(30),K4(30),MX(30),MY(30),KY(3 387.000
               10),KSC(30)                                                         387.000
0010            DIMENSION SH(8)                                                    388.000
          C                                                                        389.000
          C      ******  READ INPUT DATA FOR THE GLOBAL MODEL*****                 390.000
          C                                                                        391.000
          C                                                                        392.000
          C                                                                        393.000
0011            TH=0.09817477                                                      394.000
0012            Z=W/3.6                                                            395.000
0013            ZN=Z-(D/2.0)                                                       396.000
          C                                                                        397.000
          C                                                                        398.000
0014            BX(1,1)=0.0                                                        399.000
0015            BY(1,1)=D/2.                                                       400.000
0016            TH2=TH*2                                                           401.000
0017            TH4=TH*4                                                           402.000
0018            TH6=TH*6                                                           403.000
0019            TH8=TH*8                                                           404.000
0020            TH10=TH*10                                                         405.000
0021            TH12=TH*12                                                         406.000
0022            TH14=TH*14                                                         407.000
0023            BX(1,5)=D/2.*DSIN(TH2)                                             408.000
0024            BY(1,5)=D/2.*DCOS(TH2)                                             409.000
0025            BX(1,2)=D/2.*DSIN(TH4)                                             410.000
0026            BY(1,2)=D/2.*DCOS(TH4)                                             411.000
0027            BX(1,6)=FTH(TH4)*DSIN(TH4)                                         412.000
0028            BY(1,6)=FTH(TH4)*DCOS(TH4)                                         413.000
0029            BX(1,3)=(D/2.+ZN)/DCOS(TH4)*DSIN(TH4)                              414.000
0030            BY(1,3)=(D/2.+ZN)                                                  415.000
0031            BX(1,7)=(D/2.+ZN)/DCOS(TH2)*DSIN(TH2)                              416.000
0032            BY(1,7)=(D/2.+ZN)                                                  417.000
0033            BX(1,4)=0.0                                                        418.000
0034            BY(1,4)=(D/2.+ZN)                                                  419.000
0035            BX(1,8)=0.0                                                        420.000
0036            BY(1,8)=D/2.+ZN/2.0                                                421.000
          C                                                                        422.000
          C                                                                        423.000
0037            BX(2,1)=0.0                                                        424.000
0038            BY(2,1)=(D/2.)+ZN                                                  425.000
0039            BX(2,2)=BX(1,3)                                                    426.000
0040            BY(2,2)=BY(1,3)                                                    427.000
0041            BX(2,3)=BX(2,2)                                                    428.000
0042            BY(2,3)=E                                                          429.000
0043            BX(2,4)=0.0                                                        430.000
```

```
0044        BY(2,4)=E                                   431.000
         C                                              432.000
         C                                              433.000
0045        BX(3,1)=BX(1,2)                             434.000
0046        BY(3,1)=BY(1,2)                             435.000
0047        BX(3,5)=D/2.*DSIN(TH6)                      436.000
0048        BY(3,5)=D/2.*DCOS(TH6)                      437.000
0049        BX(3,2)=D/2.*DSIN(TH8)                      438.000
0050        BY(3,2)=D/2.*DCOS(TH8)                      439.000
0051        BX(3,6)=FTH(TH8)*DSIN(TH8)                  440.000
0052        BY(3,6)=FTH(TH8)*DCOS(TH8)                  441.000
0053        BX(3,3)=((D/2.+ZN)/DCOS(TH8))*DSIN(TH8)     442.000
0054        BY(3,3)=(D/2.)+ZN                           443.000
0055        BX(3,7)=(D/2.+ZN)/DCOS(TH6)*DSIN(TH6)       444.000
0056        BY(3,7)=(D/2.)+ZN                           445.000
0057        BX(3,4)=BX(1,3)                             446.000
0058        BY(3,4)=BY(1,3)                             447.000
0059        BX(3,8)=BX(1,6)                             448.000
0060        BY(3,8)=BY(1,6)                             449.000
         C                                              450.000
         C                                              451.000
0061        BX(4,1)=BX(1,3)                             452.000
0062        BY(4,1)=BY(1,3)                             453.000
0063        BX(4,2)=BX(3,3)                             454.000
0064        BY(4,2)=BY(3,3)                             455.000
0065        BX(4,3)=BX(4,2)                             456.000
0066        BY(4,3)=E                                   457.000
0067        BX(4,4)=BX(2,3)                             458.000
0068        BY(4,4)=BY(2,3)                             459.000
         C                                              460.000
         C                                              461.000
0069        BX(5,1)=BX(3,3)                             462.000
0070        BY(5,1)=BY(3,3)                             463.000
0071        BX(5,2)=W/2.0                               464.000
0072        BY(5,2)=BY(3,3)                             465.000
0073        BX(5,3)=W/2.0                               466.000
0074        BY(5,3)=BY(2,4)                             467.000
0075        BX(5,4)=BX(4,3)                             468.000
0076        BY(5,4)=BY(4,3)                             469.000
         C                                              470.000
         C                                              471.000
0077        BX(6,1)=BX(3,2)                             472.000
0078        BY(6,1)=BY(3,2)                             473.000
0079        BX(6,5)=D/2.*DSIN(TH10)                     474.000
0080        BY(6,5)=D/2.*DCOS(TH10)                     475.000
0081        BX(6,2)=D/2.*DSIN(TH12)                     476.000
0082        BY(6,2)=D/2.*DCOS(TH12)                     477.000
0083        BX(6,6)=FTH(TH4)*DSIN(TH12)                 478.000
0084        BY(6,6)=FTH(TH4)*DCOS(TH12)                 479.000
0085        BX(6,3)=(D/2.)+ZN                           480.000
0086        BY(6,3)=BX(6,3)/DCOS(TH4)*DSIN(TH4)         481.000
0087        BX(6,7)=(D/2.)+ZN                           482.000
0088        BY(6,7)=BX(6,7)/DCOS(TH6)*DSIN(TH6)         483.000
0089        BX(6,4)=BX(3,3)                             484.000
0090        BY(6,4)=BY(3,3)                             485.000
```

```
0091          BX(6,8)=BX(3,6)                                         486.000
0092          BY(6,8)=BY(3,6)                                         487.000
        C                                                             488.000
        C                                                             489.000
0093          BX(7,1)=D/2.+ZN                                         490.000
0094          BY(7,1)=((BY(6,4)-BY(6,3))/4.)*3.+BY(6,3)               491.000
0095          BX(7,2)=BX(6,3)                                         492.000
0096          BY(7,2)=BY(6,3)                                         493.000
0097          BX(7,3)=W/2.0                                           494.000
0098          BY(7,3)=BY(6,3)                                         495.000
0099          BX(7,4)=W/2.0                                           496.000
0100          BY(7,4)=BY(7,1)                                         497.000
        C                                                             498.000
        C                                                             499.000
0101          BX(8,1)=BX(6,2)                                         500.000
0102          BY(8,1)=BY(6,2)                                         501.000
0103          BX(8,5)=D/2.*DSIN(TH14)                                 502.000
0104          BY(8,5)=D/2.*DCOS(TH14)                                 503.000
0105          BX(8,2)=D/2.                                            504.000
0106           BY(8,2)=0.0                                            505.000
0107          BX(8,6)=D/2.+ZN/2.0                                     506.000
0108          BY(8,6)=0.0                                             507.000
0109          BX(8,3)=(D/2.)+ZN                                       508.000
0110          BY(8,3)=0.0                                             509.000
0111          BX(8,7)=(D/2.)+ZN                                       510.000
0112          BY(8,7)=BX(8,7)/DCOS(TH2)*DSIN(TH2)                     511.000
0113          BX(8,4)=BX(6,3)                                         512.000
0114          BY(8,4)=BY(6,3)                                         513.000
0115          BX(8,8)=BX(6,6)                                         514.000
0116          BY(8,8)=BY(6,6)                                         515.000
        C                                                             516.000
        C                                                             517.000
0117          BX(9,1)=BX(8,4)                                         518.000
0118          BY(9,1)=BY(8,4)                                         519.000
0119          BX(9,2)=BX(8,3)                                         520.000
0120          BY(9,2)=BY(8,3)                                         521.000
0121          BX(9,3)=W/2.0                                           522.000
0122          BY(9,3)=0.0                                             523.000
0123          BX(9,4)=W/2.0                                           524.000
0124          BY(9,4)=BY(8,4)                                         525.000
        C                                                             526.000
        C                                                             527.000
0125          BX(10,1)=D/2.                                           528.000
0126          BY(10,1)=0.0                                            529.000
0127          BX(10,5)=BX(8,5)                                        530.000
0128          BY(10,5)=-1*BY(8,5)                                     531.000
0129          BX(10,2)=BX(8,1)                                        532.000
0130          BY(10,2)=-BY(8,1)                                       533.000
0131          BX(10,6)=BX(8,8)                                        534.000
0132          BY(10,6)=-BY(8,8)                                       535.000
0133          BX(10,3)=BX(8,4)                                        536.000
0134          BY(10,3)=-BY(8,4)                                       537.000
0135          BX(10,7)=BX(8,7)                                        538.000
0136          BY(10,7)=-BY(8,7)                                       539.000
0137          BX(10,4)=BX(8,3)                                        540.000
```

```
0138          BY(10,4)=-BY(8,3)
0139          BX(10,8)=BX(8,6)
0140          BY(10,8)=-BY(8,6)
        C
        C
0141          BX(11,1)=BX(9,2)
0142          BY(11,1)=-BY(9,2)
0143          BX(11,2)=BX(9,1)
0144          BY(11,2)=-BY(9,1)
0145          BX(11,3)=BX(9,4)
0146          BY(11,3)=-BY(9,4)
0147          BX(11,4)=BX(9,3)
0148          BY(11,4)=-BY(9,3)
        C
        C
0149          BX(12,1)=BX(6,2)
0150          BY(12,1)=-BY(6,2)
0151          BX(12,5)=BX(6,5)
0152          BY(12,5)=-BY(6,5)
0153          BX(12,2)=BX(6,1)
0154          BY(12,2)=-BY(6,1)
0155          BX(12,6)=BX(6,8)
0156          BY(12,6)=-BY(6,8)
0157          BX(12,3)=BX(6,4)
0158          BY(12,3)=-BY(6,4)
0159          BX(12,7)=BX(6,7)
0160          BY(12,7)=-BY(6,7)
0161          BX(12,4)=BX(6,3)
0162          BY(12,4)=-BY(6,3)
0163          BX(12,8)=BX(6,6)
0164          BY(12,8)=-BY(6,6)
        C
        C
0165          BX(13,1)=BX(7,2)
0166          BY(13,1)=-BY(7,2)
0167          BX(13,2)=BX(6,4)
0168          BY(13,2)=-BY(6,4)
0169          BX(13,3)=W/2.0
0170          BY(13,3)=-(D/2.+ZN)
0171          BX(13,4)=BX(7,3)
0172          BY(13,4)=-BY(7,3)
        C
        C
0173          BX(14,1)=BX(13,2)
0174          BY(14,1)=BY(13,2)
0175          BX(14,2)=BX(13,2)
0176          BY(14,2)=-(AL-E)
0177          BX(14,3)=W/2.0
0178          BY(14,3)=BY(14,2)
0179          BX(14,4)=W/2.0
0180          BY(14,4)=BY(13,3)
        C
        C
0181          BX(15,1)=BX(3,2)
0182          BY(15,1)=-BY(3,2)
```

```
0183          BX(15,5)=BX(3,5)                                   596.000
0184          BY(15,5)=-BY(3,5)                                  597.000
0185          BX(15,2)=BX(3,1)                                   598.000
0186          BY(15,2)=-BY(3,1)                                  599.000
0187          BX(15,6)=BX(3,8)                                   600.000
0188          BY(15,6)=-BY(3,8)                                  601.000
0189          BX(15,3)=BX(3,4)                                   602.000
0190          BY(15,3)=-BY(3,4)                                  603.000
0191          BX(15,7)=BX(3,7)                                   604.000
0192          BY(15,7)=-BY(3,7)                                  605.000
0193          BX(15,4)=BX(3,3)                                   606.000
0194          BY(15,4)=-BY(3,3)                                  607.000
0195          BX(15,8)=BX(3,6)                                   608.000
0196          BY(15,8)=-BY(3,6)                                  609.000
        C                                                        610.000
        C                                                        611.000
0197          BX(16,1)=(BX(15,4)-BX(15,3))/2.+BX(15,3)           612.000
0198          BY(16,1)=-(D/2.+ZN)                                613.000
0199          BX(16,2)=BX(15,3)                                  614.000
0200          BY(16,2)=-(D/2.+ZN)                                615.000
0201          BX(16,3)=BX(15,3)                                  616.000
0202          BY(16,3)=-(AL-E)                                   617.000
0203          BX(16,4)=BX(16,1)                                  618.000
0204          BY(16,4)=-(AL-E)                                   619.000
        C                                                        620.000
        C                                                        621.000
0205          BX(17,1)=BX(1,2)                                   622.000
0206          BY(17,1)=-BY(1,2)                                  623.000
0207          BX(17,5)=BX(1,5)                                   624.000
0208          BY(17,5)=-BY(1,5)                                  625.000
0209          BX(17,2)=BX(1,1)                                   626.000
0210          BY(17,2)=-BY(1,1)                                  627.000
0211          BX(17,6)=BX(1,8)                                   628.000
0212          BY(17,6)=-BY(1,8)                                  629.000
0213          BX(17,3)=BX(1,4)                                   630.000
0214          BY(17,3)=-BY(1,4)                                  631.000
0215          BX(17,7)=BX(1,7)                                   632.000
0216          BY(17,7)=-BY(1,7)                                  633.000
0217          BX(17,4)=BX(1,3)                                   634.000
0218          BY(17,4)=-BY(1,3)                                  635.000
0219          BX(17,8)=BX(1,6)                                   636.000
0220          BY(17,8)=-BY(1,6)                                  637.000
        C                                                        638.000
        C                                                        639.000
0221          BX(18,1)=BX(17,4)                                  640.000
0222          BY(18,1)=BY(17,4)                                  641.000
0223          BX(18,2)=0.0                                       642.000
0224          BY(18,2)=BY(15,4)                                  643.000
0225          BX(18,3)=0.0                                       644.000
0226          BY(18,3)=-(AL-E)                                   645.000
0227          BX(18,4)=BX(17,4)                                  646.000
0228          BY(18,4)=-(AL-E)                                   647.000
        C                                                        648.000
0229          NBLOCK=18                                          649.000
0230          NES(1)=0                                           650.000
```

```
0231            NES(2)=28
0232            NES(3)=40
0233            NES(4)=68
0234            NES(5)=80
0235            NES(6)=89
0236            NES(7)=117
0237            NES(8)=126
0238            NES(9)=154
0239            NES(10)=166
0240            NES(11)=194
0241            NES(12)=206
0242            NES(13)=220
0243            NES(14)=226
0244            NES(15)=247
0245            NES(16)=261
0246            NES(17)=268
0247            NES(18)=282
          C
          C
0248            NS(1)=0
0249            NS(2)=7
0250            NS(3)=44
0251            NS(4)=51
0252            NS(5)=95
0253            NS(6)=88
0254            NS(7)=118
0255            NS(8)=144
0256            NS(9)=151
0257            NS(10)=188
0258            NS(11)=195
0259            NS(12)=232
0260            NS(13)=239
0261            NS(14)=261
0262            NS(15)=254
0263            NS(16)=300
0264            NS(17)=308
0265            NS(18)=315
          C
          C
0266            DO 11 I=1,18
0267         11 K12(I)=0
          C
0268            K12(2)=7
0269            K12(4)=7
0270            K12(7)=7
0271            K12(9)=7
0272            K12(11)=7
0273            K12(13)=7
0274            K12(16)=7
0275            K12(18)=7
          C
          C
0276            DO 12 I=1,18
0277         12 K43(I)=0
          C
```

```
0278              K43(1)=3                                    706.000
0279              K43(3)=3                                    707.000
0280              K43(6)=3                                    708.000
0281              K43(8)=3                                    709.000
0282              K43(10)=3                                   710.000
0283              K43(12)=3                                   711.000
0284              K43(15)=7                                   712.000
0285              K43(17)=7                                   713.000
            C                                                 714.000
            C                                                 715.000
0286              K2=0                                        716.000
            C                                                 717.000
            C                                                 718.000
0287              DO 13 I=1,18                                719.000
0288           13 K4(I)=0                                     720.000
            C                                                 721.000
0289              K4(6)=12                                    722.000
0290              K4(15)=24                                   723.000
            C                                                 724.000
            C                                                 725.000
0291              DO 14 I=1,18                                726.000
0292           14 MX(I)=4                                     727.000
            C                                                 728.000
0293              MX(5)=3                                     729.000
0294              MX(7)=3                                     730.000
0295              MX(12)=2                                    731.000
0296              MX(13)=2                                    732.000
0297              MX(14)=7                                    733.000
0298              MX(15)=2                                    734.000
0299              MX(16)=1                                    735.000
0300              MX(17)=2                                    736.000
0301              MX(18)=2                                    737.000
            C                                                 738.000
            C                                                 739.000
0302              DO 15 I=1,18                                740.000
0303           15 MY(I)=7                                     741.000
            C                                                 742.000
            C                                                 743.000
0304              MY(2)=3                                     744.000
0305              MY(4)=3                                     745.000
0306              MY(5)=3                                     746.000
0307              MY(7)=3                                     747.000
0308              MY(9)=3                                     748.000
0309              MY(11)=3                                    749.000
0310              MY(13)=3                                    750.000
0311              MY(14)=3                                    751.000
            C                                                 752.000
            C                                                 753.000
0312              KX=1                                        754.000
            C                                                 755.000
            C                                                 756.000
0313              DO 16 I=1,18                                757.000
0314           16 KY(I)=2                                     758.000
0315              KY(2)=1                                     759.000
0316              KY(4)=1                                     760.000
```

```
0317          KY(5)=1
0318          KY(7)=1
0319          KY(9)=1
0320          KY(11)=1
0321          KY(13)=1
0322          KY(14)=1
0323          KY(16)=1
0324          KY(18)=1
      C
      C
0325          DO 17 I=1,18
0326       17 KSC(I)=4
      C
0327          KSC(1)=8
0328          KSC(3)=8
0329          KSC(6)=8
0330          KSC(8)=8
0331          KSC(10)=8
0332          KSC(12)=8
0333          KSC(15)=8
0334          KSC(17)=8
      C
      C
      C
0335          DO 300 I=1,NBLOCK
      C
      C
      C
      C     *********COORDINATE  ***********
      C
0336          MXX=MX(I)
0337          MYY=MY(I)
0338          MXX1=MXX+1
0339          MYY1=MYY+1
0340          DO 100 IX=1,MXX1
0341          IF(IX.EQ.1) NSI=NS(I)
0342          IF(IX.GT.1.AND.IX.LT.MXX1) NSI=NS(I)+K4(I)+(IX-1)*(MYY1+K12(I)+K43
             1(I))
0343          IF(IX.EQ.MXX1) NSI=NS(I)+K4(I)+K2+(IX-1)*(MYY1+K12(I)+K43(I))
0344          XLI=FUN1(IX,MXX,KX)
0345          DO 100 IY=1,MYY1
0346          KY1=KY(I)
0347          YLI=FUN1(IY,MYY,KY1)
0348          CONTINUE
0349          IF(KSC(I).EQ.4) CALL SEREND(SH,XLI,YLI,4)
0350          IF(KSC(I).EQ.8) CALL SEREND(SH,XLI,YLI,8)
0351          XI=0.
0352          YI=0.
0353          KSC1=KSC(I)
0354          DO 102 K=1,KSC1
0355          SHK=SH(K)
0356          XI=XI+BX(I,K)*SHK
0357      102 YI=YI+BY(I,K)*SHK
0358          NIXY=NSI+IY
0359          X(NIXY)=XI
```

```
      Y(NIXY)=YI
  100 CONTINUE
C
C
C    ******   ELEMENT  ********
C
C      ***** 4-NODE ELEMENT  *******
      DO 202 IX=1,MXX
      NSI=NS(I)+(IX-1)*(MYY1+K12(I)+K43(I))
      DO 202 IY=1,MYY
      NN=NSI+IY
      NEL=NES(I)+(IX-1)*MY(I)+IY
      IJK(1,NEL)=NN+K4(I)
      IF(IX.EQ.1) IJK(1,NEL)=NN
      IJK(3,NEL)=NN+K4(I)+(MYY1+K12(I)+K43(I))
      IJK(4,NEL)=IJK(3,NEL)+1
      IJK(2,NEL)=IJK(1,NEL)+1
      IF(IX.NE.MXX) GO TO 202
      IJK(2,NEL)=IJK(2,NEL)+K2
      IJK(3,NEL)=IJK(3,NEL)+K2
  202 CONTINUE
C
  206 CONTINUE
C
  300 CONTINUE
      IJK(1,297)=96
      IJK(2,297)=100
      IJK(3,297)=119
      IJK(4,297)=120
      IJK(1,298)=100
      IJK(2,298)=104
      IJK(3,298)=120
      IJK(4,298)=121
      IJK(1,299)=104
      IJK(2,299)=108
      IJK(3,299)=121
      IJK(4,299)=122
      IJK(1,300)=262
      IJK(2,300)=266
      IJK(3,300)=301
      IJK(4,300)=302
      IJK(1,301)=266
      IJK(2,301)=270
      IJK(3,301)=302
      IJK(4,301)=303
      IJK(1,302)=270
      IJK(2,302)=274
      IJK(3,302)=303
      IJK(4,302)=304
      IJK(1,303)=274
      IJK(2,303)=278
      IJK(3,303)=304
      IJK(4,303)=305
      IJK(1,304)=278
      IJK(2,304)=282
```

```
 0408             IJK(3,304)=305                                              870.000
 0409             IJK(4,304)=306                                              871.000
 0410             IJK(1,305)=282                                              872.000
 0411             IJK(2,305)=286                                              873.000
 0412             IJK(3,305)=306                                              874.000
 0413             IJK(4,305)=307                                              875.000
 0414             IJK(1,306)=286                                              876.000
 0415             IJK(2,306)=290                                              877.000
 0416             IJK(3,306)=307                                              878.000
 0417             IJK(4,306)=308                                              879.000
            C     WRITE (6,900)                                               880.000
            C 900 FORMAT(//5X,'<COORDINATE>',/)                               881.000
            C     WRITE(6,902) (I,X(I),Y(I),I=1,NX)                           882.000
            C 902 FORMAT(10X,I5,2X,2F10.3,5X,I5,2X,2F10.3)                    883.000
            C     WRITE(6,904)                                                884.000
            C 904 FORMAT(//5X,'<ELEMENT>',/)                                  885.000
            C     DO 906 NEL=1,NELX                                           886.000
            C 906 WRITE(6,908) NEL,(IJK(I,NEL),I=1,NP)                        887.000
            C 908 FORMAT(10X,I5,5X,9I5)                                       888.000
            C     WRITE(6,909)(P(I),Q(I),LEB(I),I=1,18)                       889.000
            C 909 FORMAT(//,5X,'P=',E15.6,5X,'Q=',E15.6,5X,'LEB=',I3/)        890.000
 0418             RETURN                                                      891.000
 0419             END                                                         892.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = AMESH    , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       419,PROGRAM SIZE =     13950
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
 0001                SUBROUTINE SEREND(SH,XL,YL,KKK)
           C
           C
 0002                IMPLICIT REAL*8(A-H,O-Z)
 0003                DIMENSION SH(8)
           C
 0004                K4=KKK/4
 0005                GO TO (100,200),K4
           C          ***** 4-NODE ELEMENT  ******
 0006            100 CONTINUE
 0007                SH(1)=0.25*(1-XL)*(1.-YL)
 0008                SH(2)=0.25*(1.+XL)*(1.-YL)
 0009                SH(3)=0.25*(1.+XL)*(1.+YL)
 0010                SH(4)=0.25*(1.-XL)*(1.+YL)
 0011                RETURN
           C
           C       ***8-NODE ELEMENT ****
 0012            200 CONTINUE
 0013                SH(1)=-0.25*(XL+YL+1.)*(XL-1.)*(YL-1.)
 0014                SH(2)=-0.25*(XL-YL-1.)*(XL+1.)*(YL-1.)
 0015                SH(3)= 0.25*(XL+YL-1.)*(XL+1.)*(YL+1.)
 0016                SH(4)= 0.25*(XL-YL+1.)*(XL-1.)*(YL+1.)
 0017                SH(5)= 0.5*(XL*XL-1.)*(YL-1.)
 0018                SH(6)=-0.5*(XL+1.)*(YL*YL-1.)
 0019                SH(7)=-0.5*(XL*XL-1.)*(YL+1.)
 0020                SH(8)= 0.5*(XL-1.)*(YL*YL-1.)
 0021                RETURN
 0022                END
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = SEREND   , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       22,PROGRAM SIZE =      868
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
MICHIGAN TERMINAL SYSTEM FORTRAN G(21.8)            FUN1           06-01-82        16:14:59        PAGE P001

  0001                FUNCTION FUN1(I,M,K)                                                921.000
             C                                                                            922.000
             C                                                                            923.000
  0002                IMPLICIT REAL*8(A-H,O-Z)                                            924.000
  0003                RI=I                                                                925.000
  0004                RM=M                                                                926.000
  0005                GO TO (100,102,104),K                                               927.000
  0006            100 FUN1=-1.+2.*(RI-1.)/RM                                              928.000
  0007                RETURN                                                              929.000
  0008            102 FUN1=-1.+2.*(RI-1.)*RI/(RM*(RM+1.))                                 930.000
  0009                RETURN                                                              931.000
  0010            104 FUN1=-1.+2.*(RI-1.)*(2.*RM-RI+2.)/(RM*(RM+1.))                      932.000
  0011                RETURN                                                              933.000
  0012                END                                                                 934.000
   *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
   *OPTIONS IN EFFECT*  NAME = FUN1    , LINECNT =       57
   *STATISTICS*    SOURCE STATEMENTS =        12,PROGRAM SIZE =      634
   *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001          FUNCTION FTH(TH)                                           935.000
         C                                                               936.000
         C                                                               937.000
0002          IMPLICIT REAL*8(A-H,O-Z)                                   938.000
0003          COMMON /ADIM/ D,WD,ED,ALD,W,E,AL                           939.000
0004          Z=W/3.6                                                    940.000
0005          ZN=Z-(D/2.0)                                               941.000
0006          FTH=((((D/2.)+ZN)/DCOS(TH))-(D/2.))/2.+(D/2.)              942.000
0007          RETURN                                                     943.000
0008          END                                                        944.000
 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = FTH     , LINECNT =       57
 *STATISTICS*    SOURCE STATEMENTS =        8,PROGRAM SIZE =      442
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE   POINT(LEB)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
     1RAN
      COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
      COMMON /ALEM/ NODXY,NELX,IJK(4,400)
      COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
     1               ,SI(4),RI(4),P(20),Q(20)
      DIMENSION LEB(20)
C
C
      RLC=RC+(D/2.0)
      RL=RT+(D/2.0)
      CA=0.09817477
      PI=3.141596535
      P(1)=(RL+(RC-RT)*DCOS(CA/6.0))*DSIN(CA/6.0)
      Q(1)=(RL+(RC-RT)*DCOS(CA/6.0))*DCOS(CA/6.0)
      P(2)=(RL+(RC-RT)*DCOS(CA/2.0))*DSIN(CA/2.)
      Q(2)=(RL+(RC-RT)*DCOS(CA/2.0))*DCOS(CA/2.)
      P(3)=(RL+(RC-RT)*DCOS(3*CA/2.0))*DSIN(3*CA/2.)
      Q(3)=(RL+(RC-RT)*DCOS(3*CA/2.0))*DCOS(3*CA/2.)
      P(4)=(RL+(RC-RT)*DCOS(5*CA/2.0))*DSIN(5*CA/2.)
      Q(4)=(RL+(RC-RT)*DCOS(5*CA/2.0))*DCOS(5*CA/2.)
      P(5)=(RL+(RC-RT)*DCOS(7*CA/2.0))*DSIN(7*CA/2.)
      Q(5)=(RL+(RC-RT)*DCOS(7*CA/2.0))*DCOS(7*CA/2.)
      P(6)=(RL+(RC-RT)*DCOS(9*CA/2.0))*DSIN(9*CA/2.)
      Q(6)=(RL+(RC-RT)*DCOS(9*CA/2.0))*DCOS(9*CA/2.)
      P(7)=(RL+(RC-RT)*DCOS(11*CA/2.0))*DSIN(11*CA/2.)
      Q(7)=(RL+(RC-RT)*DCOS(11*CA/2.0))*DCOS(11*CA/2.)
      P(8)=(RL+(RC-RT)*DCOS(13*CA/2.0))*DSIN(13*CA/2.)
      Q(8)=(RL+(RC-RT)*DCOS(13*CA/2.0))*DCOS(13*CA/2.)
      P(9)=(RL+(RC-RT)*DCOS(15*CA/2.0))*DSIN(15*CA/2.)
      Q(9)=(RL+(RC-RT)*DCOS(15*CA/2.0))*DCOS(15*CA/2.)
      P(10)=(RL+(RC-RT)*DCOS(17*CA/2.0))*DSIN(17*CA/2.)
      Q(10)=(RL+(RC-RT)*DCOS(17*CA/2.0))*DCOS(17*CA/2.)
      P(11)=(RL+(RC-RT)*DCOS(19*CA/2.0))*DSIN(19*CA/2.)
      Q(11)=(RL+(RC-RT)*DCOS(19*CA/2.0))*DCOS(19*CA/2.)
      P(12)=(RL+(RC-RT)*DCOS(21*CA/2.0))*DSIN(21*CA/2.)
      Q(12)=(RL+(RC-RT)*DCOS(21*CA/2.0))*DCOS(21*CA/2.)
      P(13)=(RL+(RC-RT)*DCOS(23*CA/2.0))*DSIN(23*CA/2.)
      Q(13)=(RL+(RC-RT)*DCOS(23*CA/2.0))*DCOS(23*CA/2.)
      P(14)=(RL+(RC-RT)*DCOS(25*CA/2.0))*DSIN(25*CA/2.)
      Q(14)=(RL+(RC-RT)*DCOS(25*CA/2.0))*DCOS(25*CA/2.)
      P(15)=(RL+(RC-RT)*DCOS(27*CA/2.0))*DSIN(27*CA/2.)
      Q(15)=(RL+(RC-RT)*DCOS(27*CA/2.0))*DCOS(27*CA/2.)
      P(16)=(RL+(RC-RT)*DCOS(29*CA/2.0))*DSIN(29*CA/2.)
      Q(16)=(RL+(RC-RT)*DCOS(29*CA/2.0))*DCOS(29*CA/2.)
      P(17)=(RL+(RC-RT)*DCOS(31*CA/2.0))*DSIN(31*CA/2.)
      Q(17)=(RL+(RC-RT)*DCOS(31*CA/2.0))*DCOS(31*CA/2.)
      P(18)=(RL+(RC-RT)*DSIN(CA/6.0))*DCOS(CA/6.0)
      Q(18)=(RL+(RC-RT)*DSIN(CA/6.0))*DSIN(CA/6.0)
      I=0
    5 I=I+1
      IF(I .GT. 8) GO TO 110
      RLX=RLC-Y(I)
```

```
      IF(RLX .GT. 0.0`    GO TO 5
      LEB(1)=I-1
      LEB(2)=I-1
    6 K=1
      LI=0
   50 K=K+1
      IF(K .GT. 16) GO TO 100
      KP=7
      IF(K .EQ. 5) KP=19
      IF(K .EQ. 9) KP=28
      IF(K .EQ. 13) KP=16
C
      M=LEB(K)+KP+LI
      LI=0
      II=IJK(1,M)
      KK=IJK(3,M)
      K1=K+1
   40 IF(X(II) .EQ. X(KK)) TT=P(K1)-X(KK)
      IF(X(II) .EQ. X(KK)) GO TO 45
      TT=(-(P(K1)-X(II))*(Y(II)-Y(KK))/(X(II)-X(KK)))-Y(II)+Q(K1)
   45 IF(TT .GT. 0.0) GO TO 10
      LEB(K+1)=M-1
      II=IJK(1,M-1)
      KK=IJK(3,M-1)
      M=LEB(K+1)
      LI=LI+1
      GO TO 40
   10 LEB(K1)=M
      GO TO 50
  100 LEB(K+1)=M
  110 RETURN
      END
*OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = POINT   , LINECNT =       57
*STATISTICS*    SOURCE STATEMENTS =       82,PROGRAM SIZE =     5002
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE  MATRL
0002          IMPLICIT REAL*8(A-H,O-Z)
0003          COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
             1RAN
0004          COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
0005          COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
             1)
        C
        C
0006          IF(NRAN .EQ. 0) NLY=2.0*ANG/DANG+1
0007          V21=E2*V12/E1
0008          DIV=1-V12*V21
0009          Q22=E2/DIV
0010          Q11=E1/DIV
0011          Q12=V12*E2/DIV
0012          Q66=G12
        C
        C   *** COMPUTE INVARIANT PROPERTIES
0013          U1=(3.*Q11+3.*Q22+2.*Q12+4.*Q66)/8.
0014          U2=(Q11-Q22)/2.
0015          U3=(Q11+Q22-2.*Q12-4.*Q66)/8.0
0016          U4=(Q11+Q22+6.*Q12-4.*Q66)/8.0
0017          U5=(Q11+Q22-2.*Q12+4.*Q66)/8.0
        C
        C  ***  COMPUTRE TRANSFORMED REDUCED STIFFNESS PER LAYER *******
        C
0018          DO 100 I=1,NLY
0019          L=I-1
0020          THTA=90.0-(ANG-DANG*L)
0021          IF(NRAN .EQ. 1) THTA=90.0-ANT(I)
0022          IF(THTA .EQ. 180.0) THTA=0.0
0023          DEG=THTA*3.1415926535/180.0
0024          QA(I,1,1)=U1+U2*DCOS(2.*DEG)+U3*DCOS(4.*DEG)
0025          QA(I,1,2)=U4-U3*DCOS(4.*DEG)
0026          QA(I,2,2)=U1-U2*DCOS(2.*DEG)+U3*DCOS(4.*DEG)
0027          QA(I,1,3)=+0.5*U2*DSIN(2.*DEG)+U3*DSIN(4.*DEG)
0028          QA(I,2,3)=+0.5*U2*DSIN(2.*DEG)-U3*DSIN(4.*DEG)
0029          QA(I,3,3)=U5-U3*DCOS(4.*DEG)
0030          QA(I,2,1)=QA(I,1,2)
0031          QA(I,3,1)=QA(I,1,3)
0032          QA(I,3,2)=QA(I,2,3)
        C
        C   ***  COMPUTE ROTATION TRANSFORMATION PER PLY  *****
        C
0033          T(I,1,1)=DCOS(DEG)**2
0034          T(I,1,2)=DSIN(DEG)**2
0035          T(I,1,3)=2*DSIN(DEG)*DCOS(DEG)
0036          T(I,2,1)=T(I,1,2)
0037          T(I,2,2)=T(I,1,1)
0038          T(I,2,3)=-T(I,1,3)
0039          T(I,3,1)=T(I,2,3)/2 O
0040          T(I,3,2)=T(I,1,3)/2.0
0041          T(I,3,3)=T(I,1,1)-T(I,1,2)
        C
        C
```

```
0042          100 CONTINUE
             C
             C    *** COMPUTE LAMINATE PROPERTIES  *****
             C
0043              DO 200 M=1,3
0044              DO 200 N=1,3
0045              C(M,N)=0.0
0046              DO 150 I=1,NLY
0047              AZ=THICK(I)
0048              IF(NRAN .EQ. 0) AZ=1.0
0049          150 C(M,N)=C(M,N)+QA(I,M,N)*AZ
0050              IF(NRAN .EQ. 0) GO TO 10
0051              C(M,N)=C(M,N)/HI
0052              GO TO 200
0053           10 C(M,N)=C(M,N)/NLY
0054          200 CONTINUE
0055              RETURN
0056              END
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = MATRL    , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       56,PROGRAM SIZE =      2044
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE STIFF(MM,A)
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
      COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
      COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
     1              ,SI(4),RI(4),P(20),Q(20)
      COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
     1)
C
      DIMENSION A(8,8)
      DO 10 I=1,8
      DO 10 J=1,8
      A(I,J)=0.0
   10 CONTINUE
C
      DO 100 N=1,LINT
      R=RG(N)
      DO 100 M=1,LINT
      S=SG(M)
      CALL SHAPEF(MM,XSJ,R,S)
      DV=XSJ*WG(N)*WG(M)
C
C  *** FOR EACH J NODE COMPUTE CB=C*B  ***
C
      DO 100 J=1,NP
      CB11=C(1,1)*SHP(1,J)*DV+C(1,3)*SHP(2,J)*DV
      CB12=C(1,2)*SHP(2,J)*DV+C(1,3)*SHP(1,J)*DV
      CB21=C(1,2)*SHP(1,J)*DV+C(2,3)*SHP(2,J)*DV
      CB22=C(2,2)*SHP(2,J)*DV+C(2,3)*SHP(1,J)*DV
      CB31=C(1,3)*SHP(1,J)*DV+C(3,3)*SHP(2,J)*DV
      CB32=C(2,3)*SHP(2,J)*DV+C(3,3)*SHP(1,J)*DV
C
C  *** FOR EACH I NODE COMPUTE S=BT*CB  ****
C
      DO 100 I=1,J
      I1=2*I-1
      J1=2*J-1
      I2=2*I
      J2=2*J
C
      A(I1,J1)=A(I1,J1)+SHP(1,I)*CB11+SHP(2,I)*CB31
      A(I1,J2)=A(I1,J2)+SHP(1,I)*CB12+SHP(2,I)*CB32
      A(I2,J1)=A(I2,J1)+SHP(2,I)*CB21+SHP(1,I)*CB31
      A(I2,J2)=A(I2,J2)+SHP(2,I)*CB22+SHP(1,I)*CB32
  100 CONTINUE
C
C  *** COMPUTE LOWER TRIANGULAR PART BY SYMMETRY  ***
C
      NL=NP*2
      DO 200 I=1,NL
      DO 200 J=1,I
  200 A(I,J)=A(J,I)
C
      RETURN
```

```
 0040            END                                                    1157.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = STIFF    , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       40,PROGRAM SIZE =     1516
  *STATISTICS*  NO DIAGNOSTICS GENERATED
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```
MICHIGAN TERMINAL SYSTEM FORTRAN G(21.8)             CFAIL          06-01-82        16:15:07         PAGE P001

 0001               SUBROUTINE CFAIL (KT,MT,FAL)                                       1158.000
 0002               IMPLICIT REAL*8(A-H,O-Z)                                           1159.000
 0003               COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N  1160.000
                   1RAN                                                                1160.000
 0004               COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3  1161.000
                   1)                                                                  1161.000
             C                                                                         1162.000
             C                                                                         1163.000
             C    *** FAILURE CRITERIAN  ******                                        1164.000
             C                                                                         1165.000
 0005               FAL=0.0                                                            1166.000
 0006               DO 100 K=1,NLY                                                     1167.000
 0007               DO 100 M=1,18                                                      1168.000
             C                                                                         1169.000
 0008               FL=(STRSS(K,M,1)/XX)**2+(STRSS(K,M,3)/SS)**2                       1170.000
 0009               IF (FL .LT. FAL) GO TO 100                                         1171.000
 0010               FAL=FL                                                             1172.000
 0011               KT=K                                                               1173.000
 0012               MT=M                                                               1174.000
 0013               IF(FAL .GT. 1.0) GO TO 200                                         1175.000
 0014           100 CONTINUE                                                           1176.000
             C                                                                         1177.000
 0015           200 RETURN                                                             1178.000
 0016               END                                                                1179.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = CFAIL    , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =        16,PROGRAM SIZE =       584
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE  STRESS(DIS)
0002          IMPLICIT REAL*8(A-H,O-Z)
        C
0003          COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
             1RAN
0004          COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
0005          COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0006          COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
             1               ,SI(4),RI(4),P(20),Q(20)
0007          COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
             11)
0008           COMMON /ALOA/ XLC(20),YLC(20)
        C
        C
        C
0009          DIMENSION HP(3,4),LEB(20),XI(4),YI(4),DIS(2,400),STRAS(11,3)
0010          DO 10 I=1,18
0011          DO 10 J=1,3
0012          STRA(I,J)=0.0
0013       10 CONTINUE
        C
0014          DO 13 I=1,20
0015          XLC(I)=0.0
0016       13 YLC(I)=0.0
        C
0017          CALL POINT(LEB)
        C
0018          DO 200 M=1,18
0019          II=LEB(M)
0020          II1=IJK(1,II)
0021          II2=IJK(2,II)
0022          II3=IJK(3,II)
0023          II4=IJK(4,II)
        C
0024          XI(1)=X(II1)
0025          XI(2)=X(II2)
0026          XI(3)=X(II3)
0027          XI(4)=X(II4)
        C
0028          YI(1)=Y(II1)
0029          YI(2)=Y(II2)
0030          YI(3)=Y(II3)
0031          YI(4)=Y(II4)
        C
0032          AP=P(M)
0033          AQ=Q(M)
        C
0034          CALL NEWTON (II,AP,AQ,R,S)
        C
0035          CALL SHAPEF(II,XSJ,R,S)
        C
0036          DO 100 N=1,4
0037          NN=IJK(N,II)
0038          STRA(M,1)=STRA(M,1)+SHP(1,N)*DIS(1,NN)
0039          STRA(M,2)=STRA(M,2)+SHP(2,N)*DIS(2,NN)
```

```
0040              STRA(M,3)=STRA(M,3)+SHP(1,N)*DIS(2,NN)+SHP(2,N)*DIS(1,NN)
            C
0041          100 CONTINUE
            C
0042              DO 50 K=1,NLY
0043              DO 40 L=1,3
0044              STRSS(K,M,L)=0.0
0045              DO 30 I=1,3
0046              DO 20 J=1,3
            C
0047              STRSS(K,M,L)=STRSS(K,M,L)+T(K,L,I)*QA(K,I,J)*STRA(M,J)
            C
0048           20 CONTINUE
0049           30 CONTINUE
0050           40 CONTINUE
0051           50 CONTINUE
            C
0052          200 CONTINUE
0053          250 DO 11 I=1,11
0054              DO 11 J=1,3
0055           11 STRAS(I,J)=0.0
            C
0056              MX=148
0057              MY=154
0058              MT=0
0059              DO 400 IN=MX,MY
0060              MT=MT+1
0061              II1=IJK(1,IN)
0062              II2=IJK(2,IN)
0063              II3=IJK(3,IN)
0064              II4=IJK(4,IN)
            C
0065              XI(1)=X(II1)
0066              XI(2)=X(II2)
0067              XI(3)=X(II3)
0068              XI(4)=X(II4)
            C
0069              YI 1)=Y(II1)
0070              YI(2)=Y(II2)
0071              YI(3)=Y(II3)
0072              YI(4)=Y(II4)
            C
0073              AP=RG(1)
0074              AQ=SG(2)
            C
0075              CALL SHAPEF(IN,XSJ,AP,AQ)
            C
0076              DO 500 NT=1,4
0077              NN=IJK(NT,IN)
            C
0078              XLC(MT)=XLC(MT)+SHP(3,NT)*X(NN)
0079              YLC(MT)=YLC(MT)+SHP(3,NT)*Y(NN)
            C
            C
0080              STRAS(MT,1)=STRAS(MT,1)+SHP(1,NT)*DIS(1,NN)
```

```
0081              STRAS(MT,2)=STRAS(MT,2)+SHP(2,NT)*DIS(2,NN)
0082              STRAS(MT,3)=STRAS(MT,3)+SHP(1,NT)*DIS(2,NN)+SHP(2,NT)*DIS(1,NN)
            C
0083          500 CONTINUE
            C
0084              DO 520 J=1,3
0085          520 ST(MT,J)=0.0
0086              DO 550 J=1,3
0087              ST(MT,1)=ST(MT,1)+C(1,J)*STRAS(MT,J)
0088              ST(MT,2)=ST(MT,2)+C(2,J)*STRAS(MT,J)
0089              ST(MT,3)=ST(MT,3)+C(3,J)*STRAS(MT,J)
            C
0090          550 CONTINUE
            C
            C
0091          400 CONTINUE
            C
            C
0092              MX=164
0093              MY=166
0094              MT=7
0095              DO 401 IN=MX,MY
0096              MT=MT+1
0097              II1=IJK(1,IN)
0098              II2=IJK(2,IN)
0099              II3=IJK(3,IN)
0100              II4=IJK(4,IN)
            C
0101              XI(1)=X(II1)
0102              XI(2)=X(II2)
0103              XI(3)=X(II3)
0104              XI(4)=X(II4)
            C
0105              YI(1)=Y(II1)
0106              YI(2)=Y(II2)
0107              YI(3)=Y(II3)
0108              YI(4)=Y(II4)
            C
0109              AP=RG(1)
0110              AQ=SG(2)
            C
0111              CALL SHAPEF(IN,XSJ,AP,AQ)
            C
0112              DO 501 NT=1,4
0113              NN=IJK(NT,IN)
            C
0114              XLC(MT)=XLC(MT)+SHP(3,NT)*X(NN)
0115              YLC(MT)=YLC(MT)+SHP(3,NT)*Y(NN)
            C
            C
0116              STRAS(MT,1)=STRAS(MT,1)+SHP(1,NT)*DIS(1,NN)
0117              STRAS(MT,2)=STRAS(MT,2)+SHP(2,NT)*DIS(2,NN)
0118              STRAS(MT,3)=STRAS(MT,3)+SHP(1,NT)*DIS(2,NN)+SHP(2,NT)*DIS(1,NN)
            C
0119          501 CONTINUE
```

```
             C
 0120                DO 521 J=1,3
 0121            521 ST(MT,J)=0.0
 0122                DO 551 J=1,3
 0123                ST(MT,1)=ST(MT,1)+C(1,J)*STRAS(MT,J)
 0124                ST(MT,2)=ST(MT,2)+C(2,J)*STRAS(MT,J)
 0125                ST(MT,3)=ST(MT,3)+C(3,J)*STRAS(MT,J)
             C
 0126            551 CONTINUE
             C
             C
 0127            401 CONTINUE
             CC       DO 666 I=1,10
             C       WRITE(6,677) I,(STRAS(I,J),J=1,3)
             C 677 FORMAT(/,5X,'POINT=',I5,5X,'STRA11=',E15.8,5X,'STRA22=',E15.8,
             C      15X,'STRA12=',E15.8,/)
             C 666 CONTINUE
             C
 0128            300 RETURN
 0129                END
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = STRESS   , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       129,PROGRAM SIZE =      4434
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE  OUTPUT(KT,MT,FAL)
0002          IMPLICIT REAL*8(A-H,O-Z)
       C
0003          COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
             1RAN
0004          COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
0005          COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
0006          COMMON / AFORE/ PF,DP,F(706)
0007          COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0008          COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0009          COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
             1                ,SI(4),RI(4),P(20),Q(20)
0010          COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
             1)
0011           COMMON /ALOA/ XLC(20),YLC(20)
       C
       C
       C
0012          HIH=2.0*HI
       C
0013          SXS1=STRSS(KT,MT,1)*DSQRT(1.0/FAL)
0014          SXS2=STRSS(KT,MT,2)*DSQRT(1.0/FAL)
0015          SXS3=STRSS(KT,MT,3)*DSQRT(1.0/FAL)
       C
0016          TP=2000.0*HI
       C
       C
0017          WRITE(6,15)
0018       15 FORMAT(///,'--------------------------------------------------------
             1------------------------------------------------------',/)
0019          IF(ITEST .EQ. 0) WRITE(6,7)
0020        7 FORMAT(/,10X,'****************** THE STRENGTH PREDICTION OF
             1LAMINATE CONTAINING A OPEN HOLE     ********************'/)
       C
0021          IF(ITEST .EQ. 1) WRITE(6,100)
0022      100 FORMAT(/,10X,'**************    THE STRENGTH PREDICTION OF
             1FASTENED COMPOSITE JOINTS    **********'/)
0023          WRITE(6,16)
0024       16 FORMAT(/,'--------------------------------------------------------
             1--------------------------------------------------',/)
0025          WRITE (6,901)
0026          WRITE (6,311) E1,V12,G12
0027          WRITE (6,888)
0028      888 FORMAT(//,20X,'MATERIAL PROPERTIES')
0029          WRITE(6,889) XX,SS,RT,RC
0030      889 FORMAT(/20X,'X(PLY T-STRENGTH)=',F15.3,5X,'SC(LAMINATE S-STRENGTH)
             1=',
             1F15.3,//,20X,'RT(CHAR. - TEN.)=',F7.4,5X,'RC(CHAR. - COMP.)=',
             2F7.4,/)
0031          WRITE (6,902)
0032          IF(NRAN .EQ. 0) WRITE(6,213) ANG,DANG
0033          IF(NRAN .EQ. 1) WRITE(6,903) (I,ANT(I),THICK(I),I=1,NLY)
0034          WRITE(6,904)
0035          WRITE (6,400) D,WD,ED,ALD,HIH
0036          WRITE(6,905)
```

AD-A121 407 STRENGTH OF MECHANICALLY FASTENED COMPOSITE JOINTS(U) 2/2
MICHIGAN UNIV ANN ARBOR DEPT OF MECHANICAL ENGINEERING
AND APPLIED MECHANICS F CHANG ET AL. JUL 82

UNCLASSIFIED AFWAL-TR-82-4095 F33615-81-C-5050 F/G 13/5 NL

END

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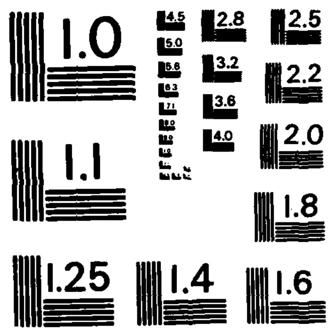
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
              C
0037            901 FORMAT (//,20X,'MATERIAL PROPERTIES OF SINGLE PLY')
0038            311 FORMAT (/,20X,'E1=',F15.3,5X,'V12=',F10.3,
                   15X,'G12=',F15.3/)
0039            213 FORMAT (/,20X,'**  THE MAXIMUM ORIENTATION ANGLE=',
                   11X,F6.2,5X,'** THE INCREMENTAL ORIENTATION ANGLE=',
                   21X,F6.2/)
0040            400 FORMAT (//,20X,'DIAMETER=',F10.4,5X,'W/D=',F10.4,/,20X,'E/D=',
                   1F10.4,5X,'L/D=',F10.4/,20X,'THICKNESS=',F10.5//)
0041            902 FORMAT (//,20X,'PLY ORIENTATION ',10X,'PLY THICKNESS')
0042            903 FORMAT (/,20X,'PLY',I3,'=',F7.3,11X,F8.5,5X)
0043            904 FORMAT (///,20X,'GEOMETRY')
0044            905 FORMAT('---------------------------------------------------------
                   1-------------------------------------------------------',///)
              C
0045                WRITE(6,101) PF
0046            101 FORMAT(/,10X,'******   THE MAXIMUM LOAD=',F10.3,2X,'    *******',1
                   1X)
0047                IF(MT .LE. 5) WRITE(6,91)
0048                IF(MT .LE. 11 .AND. MT .GE. 8) WRITE(6,92)
0049                IF(MT .GE. 14 .AND. MT .LE. 18) WRITE(6,93)
0050                IF(MT .LE. 7 .AND. MT .GE. 6) WRITE(6,94)
0051                IF(MT .GE. 12 .AND. MT .LE. 13 ) WRITE(6,95)
0052                WRITE(6,102)
0053            102 FORMAT(/,3X,'FAILURE LAYER',3X,'FAILURE POINT',
                   13X,'FAILURE POSITION',3X,'STRESS11',3X,'STRESS22',3X,'STRESS12',/)
0054                WRITE(6,103) KT,MT,P(MT),Q(MT),SXS1,SXS2,SXS3
0055            103 FORMAT (/,7X,I3,12X,I3,7X,'(',1X,F5.3,2X,F5.3,')',3X,E10.4,
                   13X,E10.4,3X,E10.4,//)
              C
0056                WRITE(6,96)
0057                WRITE(6,97) TP
0058                WRITE(6,98) FAL
0059                WRITE(6,99)
              C
              C
0060             91 FORMAT(/,10X,'******','   THE FAILURE MODE = BEARING MODE',//)
0061             92 FORMAT(/,10X,'******','   THE FAILURE MODE = SHEAROUT MODE',//)
0062             93 FORMAT(/,10X,'******','   THE FAILURE MODE = TENSION MODE',//)
0063             94 FORMAT(/,10X,'******','   THE FAILURE MODE = BEARING AND SHEAROUT
                   1MODE',//)
0064             95 FORMAT(/,10X,'******','   THE FAILURE MODE = TENSION AND SHEAROUT
                   1MODE',//)
0065             96 FORMAT(/,'--------------------------------------------------------
                   1----------------------------------------------------------',/)
0066             97 FORMAT(/,10X,'******','   THE INITIAL LOAD=',F15.5,/)
0067             98 FORMAT(/,10X,'******','   THE FAILURE INDICTOR =',F7.4,//)
0068             99 FORMAT(///,'*****************   THE STRESS DISTRIBUTIONS DUE TO
                   1THE INITIAL LOAD ON THE CHARACTERISTIC CURVE FOR EACH PLY   *',//)
0069                DO 200 K=1,NLY
0070                WRITE(6,104)
0071            104 FORMAT(/,3X,'LAYER',7X,'NO.',5X,'X1',8X,'X2',8X,'TXX',10X,'TYY',10
                   1X,'TXY',/)
0072                DO 150 N=1,18
0073                WRITE(6,105) K,N,P(N),Q(N),STRSS(K,N,1),STRSS(K,N,2),STRSS(K,N,3)
```

```
 0074          105 FORMAT(/,5X,I3,5X,I3,5X,F6.3,3X,F6.3,6X,E10.4,5X,E10.4,5X,E10.4,/)   1466.000
 0075          150 CONTINUE                                                            1467.000
             C                                                                         1468.000
 0076          200 CONTINUE                                                            1469.000
 0077              WRITE(6,17)                                                         1470.000
 0078           17 FORMAT (///,'********************   THE STRESS DISTRIBUTIONS        1471.000
                  1ACROSS THE LIGAMENT OF THE PLATE   ***********************',/)     1472.000
             C                                                                         1473.000
 0079          300 DO 301 I=1,10                                                       1474.000
 0080              WRITE(6,305) I,XLC(I),YLC(I),(ST(I,J),J=1,3)                        1475.000
 0081          305 FORMAT(/,5X,'POINT=',I5,5X,'X1=',F7.4,5X,'X2=',F7.4,5X,'T11=',     1476.000
                  1E15.8,5X,'T22=',E15.8,5X,'T12=',E15.8,/)                           1477.000
 0082          301 CONTINUE                                                            1478.000
             C                                                                         1479.000
 0083              GO TO 500                                                           1480.000
 0084          500 RETURN                                                              1481.000
 0085              END                                                                 1482.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = OUTPUT   , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       85,PROGRAM SIZE =     4414
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001               SUBROUTINE  FORMK
0002               IMPLICIT REAL*8 (A-H,O-Z)
0003               COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0004               COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0005               COMMON /AMAX/ SK(706,90),R1(706)
           C
           C
           C
0006               DIMENSION A(8,8)
           C
           C
0007               DO 300 N=1,NEQ
0008               DO 300 M=1,NBAND
0009           300 SK(N,M)=0.0
           C
           C
0010               DO 400 N=1,NELX
           C
0011               CALL STIFF(N,A)
           C
0012               DO 350 JJ=1,NP
0013               NROWB=(IJK(JJ,N)-1)*2
0014               DO 350 J=1,2
0015               NROWB=NROWB+1
0016               I=(JJ-1)*2+J
0017               DO 330 KK=1,NP
0018               NCOLB=(IJK(KK,N)-1)*2
0019               DO 320 K=1,2
0020               L=(KK-1)*2+K
0021               NCOL=NCOLB+K+1-NROWB
           C
0022               IF(NCOL) 320,320,310
           C
0023           310 SK(NROWB,[illegible])=SK(NROWB,NCOL)+A(I,L)
           C
0024           320 CONTIN
0025           330 CONTIN
0026           350 CONTI
0027           400 CONTIN
           C
0028               RETURN
0029               END
 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = FORMK    , LINECNT =       57
 *STATISTICS*    SOURCE STATEMENTS =        29,PROGRAM SIZE =     1428
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001              SUBROUTINE BOUND
0002              IMPLICIT REAL*8 (A-H,O-Z)
0003              COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
                 1RAN
0004              COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
0005              COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0006              COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0007              COMMON /AMAX/ SK(706,90),R1(706)
            C
0008              NB=37
0009              IF(ITEST .EQ. 1) NB=33
            C
0010              DO 500 N=1,NB
0011              NX=10
0012              I=NBC(N)
0013              NROWB=(I-1)*2
            C
            C
            C   **** EXAMINE EACH DEGREE OF FREEDOM  *****
            C
0014              DO 490 M=1,2
0015              NROWB=NROWB+1
0016              ICON=NFIX(N)/NX
            C
0017              IF(ICON) 450,450,420
            C
0018          420 SK(NROWB,1)=1.0
0019              DO 430 J=2,NBAND
0020              SK(NROWB,J)=0.0
0021              NR=NROWB+1-J
            C
0022              IF(NR) 430,430,425
            C
0023          425 SK(NR,J)=0.0
0024          430 CONTINUE
            C
0025              NFIX(N)=NFIX(N)-NX*ICON
0026          450 NX=NX/10
0027          490 CONTINUE
0028          500 CONTINUE
            C
0029              RETURN
0030              END
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = BOUND    , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       30,PROGRAM SIZE =      740
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001           SUBROUTINE SOLVE
0002           IMPLICIT REAL*8 (A-H,O-Z)
0003           COMMON / AFORE/ PF,DP,F(706)
0004           COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0005           COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0006           COMMON /AMAX/ SK(706,90),R1(706)
         C
0007           DO 100 I=1,NEQ
0008           R1(I)=F(I)
0009       100 CONTINUE
         C
0010           DO 300 N=1,NEQ
0011           I=N
0012           DO 290 L=2,NBAND
0013           I=I+1
         C
0014           IF(I-NEQ) 230,230,290
         C
0015       230 IF(SK(N,L)) 240,290,240
         C
0016       240 C1=SK(N,L)/SK(N,1)
0017           J=0
0018           DO 270 K=L,NBAND
0019           J=J+1
         C
0020           IF(SK(N,K)) 260,270,260
         C
0021       260 SK(I,J)=SK(I,J)-C1*SK(N,K)
0022       270 CONTINUE
         C
0023       280 SK(N,L)=C1
0024           R1(I)=R1(I)-C1*R1(N)
         C
0025       290 CONTINUE
         C
0026       300 R1(N)=R1(N)/SK(N,1)
         C
         C
         C    **** BACK SUSTITUTION   ******
         C
         C
0027           N=NEQ
0028       350 N=N-1
0029           IF(N) 500,500,360
         C
0030       360 L=N
0031           DO 400 K=2,NBAND
0032           L=L+1
         C
0033           IF(SK(N,K)) 370,400,370
         C
0034       370 R1(N)=R1(N)-SK(N,K)*R1(L)
         C
0035       400 CONTINUE
0036           GO TO 350
```

```
0037          500 RETURN                                                   1622.000
0038              END                                                      1623.000
 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = SOLVE    , LINECNT =       57
 *STATISTICS*    SOURCE STATEMENTS =       38,PROGRAM SIZE =      1068
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE   SHAPEF(MM,XSJ,R,S)
0002          IMPLICIT REAL*8 (A-H,O-Z)
0003          COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0004          COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0005          COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
             1               ,SI(4),RI(4),P(20),Q(20)
0006          COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
             1)
        C
        C
0007          DIMENSION XS(2,2)
        C
        C
        C   **** COMPUTE SHAPE FUNCTION AND DEVIVATIVE IN LOCAL COORD. ****
        C
0008          DO 100 I=1,NP
        C
0009          SHP(3,I)=0.25*(1.0+R*RI(I))*(1.0+S*SI(I))
0010          SHP(1,I)=0.25*RI(I)*(1.0+S*SI(I))
0011          SHP(2,I)=0.25*SI(I)*(1.0+R*RI(I))
        C
0012     100   CONTINUE
        C
        C
        C   **** COMPUTE JOCOBIAN MATRIX XS: DX/DR   *****
        C
0013          DO 200 I=1,2
0014          DO 200 J=1,2
0015     200 XS(I,J)=0.0
        C
        C
0016          DO 210 J=1,2
0017          DO 210 K=1,NP
0018          NN=IJK(K,MM)
0019          XS(1,J)=XS(1,J)+X(NN)*SHP(J,K)
0020          XS(2,J)=XS(2,J)+Y(NN)*SHP(J,K)
        C
0021     210 CONTINUE
        C
        C
        C   *** COMPUTE DEETERMINENT OF JOCOBIAN MATRIX    *****
        C
0022          XSJ=XS(1,1)*XS(2,2)-XS(1,2)*XS(2,1)
        C
        C   **** TRANSFER NATURAL DERIVATIVE TO X,Y DERIVATIVE  *****
        C
0023          DO 300 I=1,4
        C
0024          TEMP=(XS(2,2)*SHP(1,I)-XS(2,1)*SHP(2,I))/XSJ
0025          SHP(2,I)=(-XS(1,2)*SHP(1,I)+XS(1,1)*SHP(2,I))/XSJ
        C
0026     300 SHP(1,I)=TEMP
        C
0027          RETURN
0028          END
```

```
*OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = SHAPEF   , LINECNT =       57
*STATISTICS*    SOURCE STATEMENTS =       28,PROGRAM SIZE =      1028
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE NEWTON(IN,AP,AQ,R,S)
0002          IMPLICIT REAL*8 (A-H,O-Z)
0003          COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
             1               ,SI(4),RI(4),P(20),Q(20)
0004          COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0005          COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0006          DIMENSION RP(2)
0007          F(R,S)=A3*R*S+A2*S+A1*R+A0-AP
0008          G(R,S)=B3*R*S+B2*S+B1*R+B0-AQ
        C
0009          FR(R,S)=A3*S+A1
0010          FS(R,S)=A3*R+A2
        C
0011          GR(R,S)=B3*S+B1
0012          GS(R,S)=B3*R+B2
        C
0013          A0=0.0
0014          A1=0.0
0015          A2=0.0
0016          A3=0.0
        C
0017          B0=0.0
0018          B1=0.0
0019          B2=0.0
0020          B3=0.0
        C
        C
0021          DO 100 I=1,4
0022          N=IJK(I,IN)
        C
0023          A0=A0+0.25*X(N)
0024          A1=A1+0.25*X(N)*RI(I)
0025          A2=A2+0.25*X(N)*SI(I)
0026          A3=A3+0.25*X(N)*RI(I)*SI(I)
        C
0027          B0=B0+0.25*Y(N)
0028          B1=B1+0.25*Y(N)*RI(I)
0029          B2=B2+0.25*Y(N)*SI(I)
0030          B3=B3+0.25*Y(N)*RI(I)*SI(I)
        C
0031      100 CONTINUE
        C
        C
0032          R=0.0
0033          S=0.0
        C
0034          AA=F(R,S)
0035          BB=G(R,S)
        C
        C     WRITE(6,3) R,S,AA,BB
        C
0036          DO 10 I=1,20
0037          XJ=FR(R,S)*GS(R,S)-FS(R,S)*GR(R,S)
0038          IF(XJ .EQ. 0.0)  WRITE(6,6)
0039          IF(XJ .EQ. 0.0) GO TO 150
```

```
              C
0040                DELTAX=(-F(R,S)*GS(R,S)+G(R,S)*FS(R,S))/XJ
0041                DELTAY=(-G(R,S)*FR(R,S)+F(R,S)*GR(R,S))/XJ
              C
0042                R=R+DELTAX
0043                S=S+DELTAY
              C
0044                AA=F(R,S)
0045                BB=G(R,S)
              C
              C     WRITE(6,4) I,R,S,AA,BB
0046                IF(DABS(DELTAX) .LT. 1.E-7 .AND. DABS(DELTAY) .LT. 1.E-7)
                   1GO TO 150
              C
0047             10 CONTINUE
0048                WRITE(6,5)
              C   3 FORMAT(//,10X,'R',15X,'S',15X,'F(R,S)',15X,'G(R,S)',/,
              C    15X,F10.7,5X,F10.7,5X,F12.7,5X,F12.7/)
              C   4 FORMAT(/,'I=',I3,F10.7,5X,F10.7,5X,F12.7,5X,F12.7/)
0049              5 FORMAT(//,'FAIL TO CONVERGE IN 20 ITERATION ',/)
0050              6 FORMAT(//,'***  JOCOBIAN=0.0  ***',/)
              C
0051            150 RETURN
0052                END
 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = NEWTON   , LINECNT =       57
 *STATISTICS*    SOURCE STATEMENTS =       52,PROGRAM SIZE =     2348
 *STATISTICS*  NO DIAGNOSTICS GENERATED

NO STATEMENTS FLAGGED IN THE ABOVE COMPILATIONS.
```

```
C    ***************************************************************
C
C                              TEST SAMPLE
C
C    ****************************************************************
C
C          CONSIDER A SINGLE FASTENER COMPOSITE JOINT MADE OF
C       T300/SP286 GRAPHITE EPOXY LAMINATE.  THE MATERIAL PRO-
C       PERITIES ARE GIVEN AS E1=18.70 E06 PSI, G12=0.719 E06
C       PSI, V12=0.30, X(PLY TENSILE STRENGTH)=0.15 E 06 PSI,
C       SS(LAMINATE SHEAR STRENGTH--CROSS  PLY)=0.018 E06 PSI.
C       QUASI-ISOTROPIC LAMINATE IS USED IN THIS CALCULATIONS.
C
C          THE GEOMETRY OF THE JOINT ARE DESCRIBED AS BELOW:
C                      D(DIAMETER)=0.25 IN
C                      W/D(WIDTH RATIO)=6.0
C                      E/D(EDAGE RATIO)=3.0
C                      L/D(LENGTH RATIO)=14.0
C                      H(THICKNESS)=0.021 IN
C          THE CHARACTERISTIC LENGTH FOR TENSION IS EQUAL TO
C      0.043 IN AND FOR COMPRESSION IS 0.120 IN.
C
C
C
C_____________________________________________________________________
C
C          THE FORMAT OF INPUT DATA IS GIVEN IN THE FOLLOWING:
C
C          (SEE INPUT INSTRUCTIONS IN PROGRAM )
CC
C
   1
   1
         0.0           1.0
  18700000.0          0.30       719000.0
      0.2500         6.000          3.000          14.0
     150000.       18000.0          0.043        0.1200
       0.021
   4
         0.0       0.00525           45.0       0.00525
       -45.0       0.00525           90.0       0.00525
```

Output

```
-------------------------------------------------------------------------------
     ***************   THE STRENGTH PREDICTION OF       FASTENED COMPOSITE JOINTS    ***********

-------------------------------------------------------------------------------
                  MATERIAL PROPERTIES OF SINGLE PLY
                  E1=   18700000.000      V12=      0.300     G12=      719000.000
                  MATERIAL PROPERTIES
                  X(PLY T-STRENGTH)=       150000.000    SC(LAMINATE S-STRENGTH)=       18000.000
                  RT(CHAR. - TEN.)= 0.0430     RC(CHAR. - COMP.)= 0.1200
                  PLY ORIENTATION             PLY THICKNESS
                  PLY   1=  0.0               0.00525
                  PLY   2= 45.000             0.00525
                  PLY   3=-45.000             0.00525
                  PLY   4= 90.000             0.00525
                  GEOMETRY
                  DIAMETER=     0.2500     W/D=     6.0000
                  E/D=     3.0000     L/D=    14.0000
                  THICKNESS=   0.04200

-------------------------------------------------------------------------------
      ******   THE MAXIMUM LOAD=  1161.505      *******
      ******   THE FAILURE MODE = BEARING AND SHEAROUT MODE
 FAILURE LAYER   FAILURE POINT   FAILURE POSITION   STRESS11    STRESS22    STRESS12
       2               6         ( 0.102  0.215)   -.1448E+06  0.1820E+00  -.4687E+04

-------------------------------------------------------------------------------
      ******   THE INITIAL LOAD=         42.00000
      ******   THE FAILURE INDICTOR = 0.0013
*****************  THE STRESS DISTRIBUTIONS DUE TO   THE INITIAL LOAD ON THE CHARACTERISTIC CURVE FOR EACH PLY   *
```

LAYER	NO.	X1	X2	TXX	TYY	TXY
1	1	0.004	0.245	-.4941E+04	0.8567E-02	0.3624E+02
1	2	0.012	0.245	-.4906E+04	0.8634E-02	0.4183E+02
1	3	0.036	0.242	-.4525E+04	0.7968E-02	0.1229E+03
1	4	0.059	0.235	-.3780E+04	0.6694E-02	0.1953E+03
1	5	0.081	0.226	-.2723E+04	0.4975E-02	0.2530E+03
1	6	0.102	0.215	-.3422E+04	-.2146E-03	0.3091E+03
1	7	0.120	0.201	-.1855E+04	-.2014E-02	0.3317E+03
1	8	0.137	0.185	-.1380E+03	-.3547E-02	0.3319E+03
1	9	0.151	0.167	0.1630E+04	-.4515E-02	0.3095E+03
1	10	0.163	0.148	0.2002E+04	-.6198E-02	0.2530E+03
1	11	0.172	0.127	0.2841E+04	-.7092E-02	0.2235E+03
1	12	0.178	0.107	0.3552E+04	-.6921E-02	0.1816E+03
1	13	0.182	0.086	0.4095E+04	-.5661E-02	0.1340E+03
1	14	0.183	0.065	0.4461E+04	-.3454E-02	0.8795E+02
1	15	0.181	0.045	0.4572E+04	-.3443E-02	0.1065E+03
1	16	0.177	0.026	0.4909E+04	0.2985E-03	0.6738E+02
1	17	0.172	0.008	0.5220E+04	0.3879E-02	0.4397E+02
1	18	0.169	0.003	0.5407E+04	0.5019E-02	0.2622E+02
LAYER	NO.	X1	X2	TXX	TYY	TXY
2	1	0.004	0.245	-.1400E+04	-.4691E-02	-.3086E+03
2	2	0.012	0.245	-.1454E+04	-.4289E-02	-.3073E+03
2	3	0.036	0.242	-.2437E+04	0.1522E-03	-.2835E+03
2	4	0.059	0.235	-.3237E+04	0.4662E-02	-.2371E+03
2	5	0.081	0.226	-.3777E+04	0.8923E-02	-.1719E+03
2	6	0.102	0.215	-.5237E+04	0.6581E-02	-.1695E+03
2	7	0.120	0.201	-.5151E+04	0.1032E-01	-.7824E+02
2	8	0.137	0.185	-.4696E+04	0.1352E-01	0.1860E+02

2	9	0.151	0.167	-.3876E+04	0.1610E-01	0.1139E+03
2	10	0.163	0.148	-.3168E+04	0.1316E-01	0.1447E+03
2	11	0.172	0.127	-.2575E+04	0.1318E-01	0.1930E+03
2	12	0.178	0.107	-.1765E+04	0.1298E-01	0.2273E+03
2	13	0.182	0.086	-.8389E+03	0.1281E-01	0.2454E+03
2	14	0.183	0.065	0.9469E+02	0.1289E-01	0.2478E+03
2	15	0.181	0.045	-.1070E+03	0.1407E-01	0.2533E+03
2	16	0.177	0.026	0.8699E+03	0.1542E-01	0.2432E+03
2	17	0.172	0.008	0.1618E+04	0.1736E-01	0.2330E+03
2	18	0.169	0.003	0.2021E+04	0.1770E-01	0.2342E+03

LAYER	NO.	X1	X2	TXX	TYY	TXY
3	1	0.004	0.245	-.4572E+03	-.8219E-02	0.3086E+03
3	2	0.012	0.245	-.3658E+03	-.8361E-02	0.3073E+03
3	3	0.036	0.242	0.7596E+03	-.1181E-01	0.2835E+03
3	4	0.059	0.235	0.1843E+04	-.1436E-01	0.2371E+03
3	5	0.081	0.226	0.2802E+04	-.1570E-01	0.1719E+03
3	6	0.102	0.215	0.2802E+04	-.2351E-01	0.1695E+03
3	7	0.120	0.201	0.3476E+04	-.2197E-01	0.7824E+02
3	8	0.137	0.185	0.3936E+04	-.1880E-01	-.1860E+02
3	9	0.151	0.167	0.4173E+04	-.1403E-01	-.1139E+03
3	10	0.163	0.148	0.3411E+04	-.1147E-01	-.1447E+03
3	11	0.172	0.127	0.3237E+04	-.8575E-02	-.1930E+03
3	12	0.178	0.107	0.2957E+04	-.4693E-02	-.2273E+03
3	13	0.182	0.086	0.2647E+04	-.2402E-03	-.2454E+03
3	14	0.183	0.065	0.2382E+04	0.4328E-02	-.2478E+03
3	15	0.181	0.045	0.2663E+04	0.3700E-02	-.2533E+03
3	16	0.177	0.026	0.2622E+04	0.8859E-02	-.2432E+03
3	17	0.172	0.008	0.2761E+04	0.1308E-01	-.2330E+03
3	18	0.169	0.003	0.2703E+04	0.1514E-01	-.2342E+03

LAYER	NO.	X1	X2	TXX	TYY	TXY
4	1	0.004	0.245	0.3084E+04	-.2148E-01	-.3624E+02
4	2	0.012	0.245	0.3086E+04	-.2128E-01	-.4183E+02
4	3	0.036	0.242	0.2848E+04	-.1963E-01	-.1229E+03
4	4	0.059	0.235	0.2386E+04	-.1639E-01	-.1953E+03
4	5	0.081	0.226	0.1747E+04	-.1176E-01	-.2530E+03
4	6	0.102	0.215	0.9863E+03	-.1671E-01	-.3091E+03
4	7	0.120	0.201	0.1799E+03	-.9631E-02	-.3317E+03
4	8	0.137	0.185	-.6218E+03	-.1735E-02	-.3319E+03
4	9	0.151	0.167	-.1333E+04	0.6576E-02	-.3095E+03
4	10	0.163	0.148	-.1760E+04	0.7885E-02	-.2530E+03
4	11	0.172	0.127	-.2179E+04	0.1170E-01	-.2235E+03
4	12	0.178	0.107	-.2360E+04	0.1521E-01	-.1816E+03
4	13	0.182	0.086	-.2287E+04	0.1823E-01	-.1340E+03
4	14	0.183	0.065	-.1984E+04	0.2067E-01	-.8795E+02
4	15	0.181	0.045	-.2015E+04	0.2121E-01	-.1065E+03
4	16	0.177	0.026	-.1417E+04	0.2398E-01	-.6738E+02
4	17	0.172	0.008	-.8404E+03	0.2656E-01	-.4397E+02
4	18	0.169	0.003	-.6836E+03	0.2782E-01	-.2622E+02

******************** THE STRESS DISTRIBUTIONS ACROSS THE LIGAMENT OF THE PLATE ***********************

POINT=	1	X1= 0.1271	X2= 0.0026	T11= 0.10461911E+03	T22= 0.43780118E+04	T12= 0.10341868E+02
POINT=	2	X1= 0.1397	X2= 0.0029	T11= 0.29470390E+03	T22= 0.33064404E+04	T12=-0.26063698E+02
POINT=	3	X1= 0.1627	X2= 0.0034	T11= 0.34292025E+03	T22= 0.22448220E+04	T12=-0.79359774E+02
POINT=	4	X1= 0.1962	X2= 0.0041	T11= 0.29215934E+03	T22= 0.15173410E+04	T12=-0.11360285E+03
POINT=	5	X1= 0.2401	X2= 0.0050	T11= 0.21404444E+03	T22= 0.10787318E+04	T12=-0.12442130E+03
POINT=	6	X1= 0.2944	X2= 0.0061	T11= 0.14530340E+03	T22= 0.81755422E+03	T12=-0.11835797E+03
POINT=	7	X1= 0.3591	X2= 0.0075	T11= 0.93844418E+02	T22= 0.65169154E+03	T12=-0.10077351E+03
POINT=	8	X1= 0.4401	X2= 0.0091	T11= 0.55440756E+02	T22= 0.53468656E+03	T12=-0.71020255E+02
POINT=	9	X1= 0.5513	X2= 0.0091	T11= 0.25849230E+02	T22= 0.42057408E+03	T12=-0.36908192E+02
POINT=	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]