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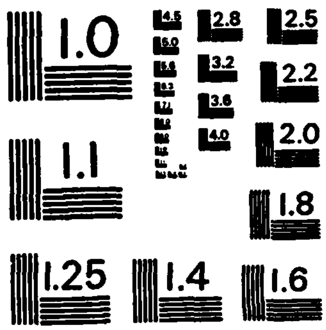

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STOCHASTIC AIRCRAFT AVAILABILITY SENSITIVITY MODEL (SAASY MODEL)

A Probabilistic Technique to Translate DO 41 Recoverable Spares Data into Aircraft Availability Forecasts

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(SAASY)

A Probabalistic Technique to Translate DO041 Recoverable Spares Data into Aircraft Availability Forecasts

I. INTRODUCTION

There currently exists a variety of aircraft spares requirement calculation methods. In its simplest form, spares requirements may be forecast as an extension of historic consumption trends. The AFLC DO041 system employs such time series forecasting techniques for requirements calculations on some recoverable (reparable) spares. An improved method currently being used in AFLC for recoverable assets employs a marginal analysis technique. In this case, spares are bought such that each increment of funding is sequentially applied to those spares that provide the greatest reduction per dollar to system backorder rates. Models in use that employ marginal analysis include METRIC, Mod METRIC, and the DO041 VSL (variable safety level) computation.

All else being equal, the current marginal analysis calculations will always buy the less costly spares. For example, if two different spares would each "buy" an equal reduction in expected backorders, the marginal analysis technique would always buy the less expensive item. The DO041 VSL system constrains this cost impact by forcing the procurement of some minimum quantity of each item and prohibiting procurement of on items assets that result in an expected backorder rate for the item less than some minimum value.

All of these techniques are limited because their measure of effectiveness (expected backorders) does not translate to terms of aircraft availability. The most logical measure of effectiveness for the logistics system is the number of aircraft that can be supported in an operationally ready state for a given level of flying activity. The discontinuity between back-orders and availability can be illustrated through an oversimplified hypothetical example. Assume that the world-wide inventory of a particular aircraft type (MDS) is 40, and that during the last month 100 supply requisitions were submitted and 90 of them were filled from available stock (10% backorder, 90% fill rate). It is possible that all ten of the unfilled requisition were for the same part, and thus ten aircraft could be in an NMCS (not mission capable-supply) status. In this case the availability rate would be 75% even though the supply fill rate was 90%.

There are several models that use aircraft availability as a measure of effectiveness. One of these models is extremely detailed and comprehensive, but bulky and costly to run, and therefore not convenient for a variety of quick turn around "what if" questions. Another model provides a quick reaction capability, but is oversimplified in some areas. Therefore, the need existed to develop a quick reaction model that addresses all or most of the major variables in the aircraft availability problem. The SAASY model, which is the focus of this paper, attempts to provide a stochastic technique that translates a given stock position into the number of aircraft expected to be in an operationally ready status, at least not NMCS. The model has been programmed on the AFLC CREATE computer system and is being used on an experimental basis.

II. GENERAL DESCRIPTION OF THE MODEL

A. Overview: The SAASY model begins with the assumption of a stock position. It does not matter which requirements calculation method is used (i.e.: DO041, METRIC, VSL, etc.), only that a given quantity of spares are available or on order with an expected due-in date. Next, using selected DO041 historical data of consumption, repair times, and shipping times, the expected number of items of each stock number in each segment of the logistics pipeline can be calculated. The result is a daily time-dependent series of expected assets in the pipeline. These expected values are used as the mean of a probability distribution of assets not available to meet daily demands. The form of the distribution will be discussed in the next section.

The distribution and the current total asset position is then used to calculate the expected number of backorders on a daily basis for each reparable line item on the weapon system. Questions of mission essential items are handled by limiting the stock numbers included in the analysis. (i.e. the analysis assumes all items analyzed are 'mission essential'.) Finally, the expected backorders are randomly distributed to each aircraft in the inventory, and the fleet availability rate calculated using methods similar to those used in the LMI Availability Model. Note that, since the expected pipeline assets and backorders were calculated as a function of time, the projected fleet availability is also time-dependent.

B. <u>Assumptions</u>. The assumptions needed for analysis are listed below.

(1) The logistics pipeline is defined to have three segments: base repair cycle, depot repair cycle (includes retrograde shipment time), and order and ship time for servicables.

(2) No unservicable assets are sitting idle at either the base or depot. All unservicables enter the appropriate repair cycle without delay other than nominal batching delays, which are included in the cycle time data. Extraordinary waiting

times imposed through management action or deficit repair capacity can be accounted for through arbitrary extension of repair cycle times.

(3) No servicable assets remain in the depot. Thus, servicable parts are immediately requisitioned by or pushed to the operating base.

(4) Servicable assets exist at the base where they are needed. This gives rise to the assumption of a single world-wide base (and implies perfect lateral support). This assumption will be relaxed and a multi-base concept developed in later sections of this paper.

(5) No cannibalization. This is probably the greatest limitation of the model. There is, however, an empirical conversion formula developed by LMI that can be used to account for cannibalization: actual availability equals 0.31 plus 0.69 times computed availability without cannibalization. Note also that the assumption on cannibalization (pessimistic support) tends to counteract the one on inactive servicable and unservicable assets (optimistic support). The degree of counteraction is uncertain, and to some degree scenario-dependent.

(6) Asset availability. Asset availability is determined at some asset cut-off date preceeding the assumed start of a war. The asset position for a reparable spare is defined as

the sum of servicable and unservicable spare assets. Note that the asset position could be negative if there are no servicable or unservicable spares to satisfy installed requirements.

(7) The distribution of assets unavailable for aircraft needs (pipeline plus condemnations) is assumed to follow a negative binomial distribution. This distributions requires two parameters: the mean and variance, or mean and variance-to-mean ratio. In our case the mean is the expected number of assets not available as calculated from standard DO41 data. A number of techniques can be used to estimate variance-to-mean ratios. The one used here is based on empirical observations of mean and variance trends in the DO41 data. The negative binomial was selected as it has been shown that this distribution often serves as a good descriptor of the demand for aircraft spare parts. Further study may indicate that other distribuions are either more appropriate or easier to use. For example, as the variance-to-mean ratio approaches one, the negative binomial approaches a poisson distribution. For large mean values, the negative binomial, poisson, and normal distributions are similar.

(8) Reparable spare backorders are randomly (uniformly) distributed to all aircraft. This is the mathematical expedient that prohibits quantification of the effects of cannibalization.

If cannibalization occured, backorders would tend to accumulate on a subset of all aircraft, and the assumption of randomness would be violated.

(9) Depot demands are considered to remain constant as a function of time and programmed flying rate. Thus, the variable demand in this analysis is related to Organizational Intermediate Maintenance (OIM) generated demands. Although increased flying rates could imply some increases in depot-generated demands, there could be conflicting pressures of reduced programmed depot maintenance (PDM) and modification programs that would keep depot demands at a constant or even reduced rate during wartime.

C. <u>Organization of the analysis</u>. The following is an overview of the order in which the several steps in the analysis will be discussed. The time line description of Figure 1 helps define some of the terms and concepts outlined.

(1) The first section of the analysis will address the basic aircraft availability calculations. It assumes that expected backorders have already been determined. The calculation of backorders is discussed in subsequent paragraphs.

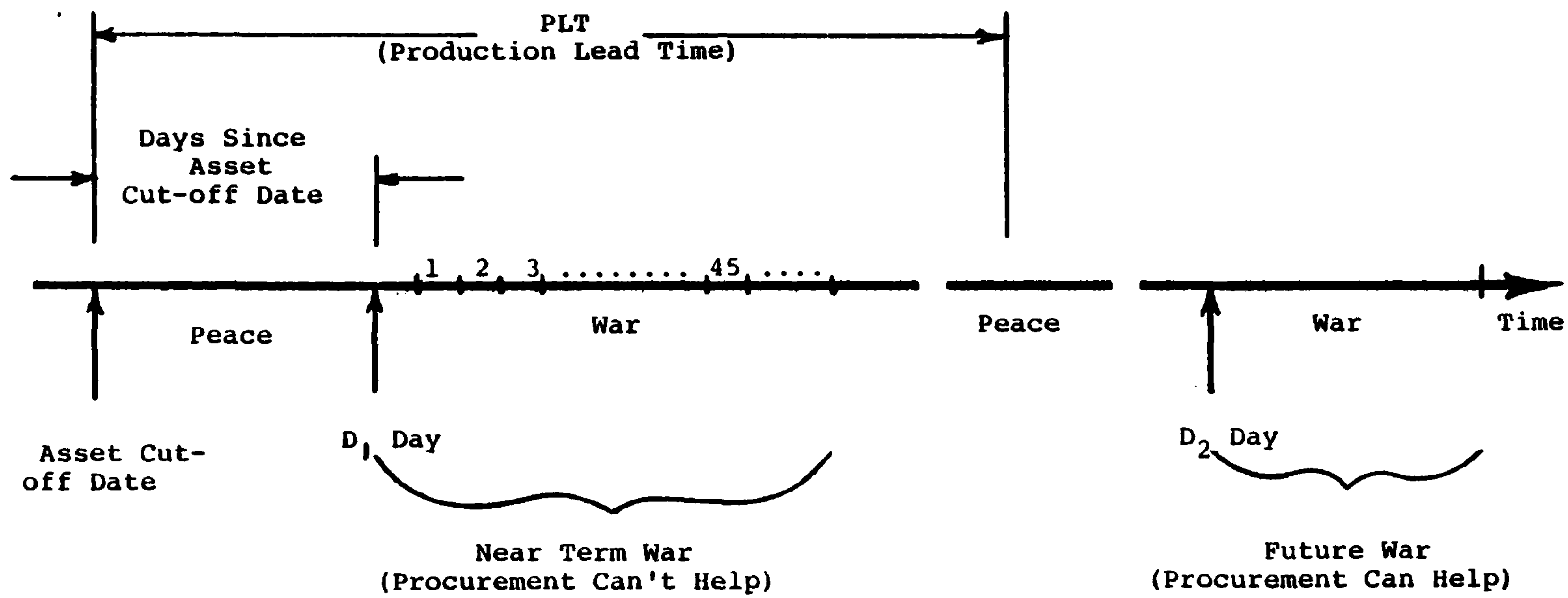


FIGURE 1. LOGISTIC SUPPORT TIME LINE

(2) The next section contains the backorder calculation for a near-term war. It includes a sample calculation of expected pipeline assets

(3) The backorder calculation expanded to include procurement assets for a future war is in the next section.

(4) Next, the procedure used to handle situations where the application percent is less than unity is demonstrated.

(5) Finally, the single-base assumption is relaxed and the method used to calculate expected backorders in a multi-base environment is presented.

III. MODEL ANALYSIS

A. Aircraft Availability Calculation.

(1) Theory. Basic probability theory teaches that if a number of ~~things~~ components can fail in a system, and if the system is successful only if all of its components are also successful, and if individual component failures are independent of other component failures; then the overall probability of success can be determined by multiplying individual component probabilities of success (one minus the probability of failure).

$$P_s = \prod_{j=1}^{N} (1 - P_{fj})$$

where P_s = Probability of overall success

P_{fj} = Probability of failure of the jth component

N = Total number of components

j = Index for a specific component

(2) Application. In our case, success is defined as the probability that an aircraft is operationally ready with no outstanding backorders (holes). Note that this model considers only failures to provide serviceable spares. Failures in maintenance operations to repair spare parts in a normal span of time are not considered. It will be seen later that average maintenance cycle times remain constant and as determined by historical DO041 data. Thus, a component failure exists when an unserviceable part is removed from the aircraft and no serviceable part is available for replacement, thus creating a backorder. The total number of aircraft available (OR) is just the product of P_s and the number of aircraft in the inventory (AC).

$$OR = AC * P_s$$

$$OR = AC * \prod_{j=1}^{N} (1 - P_{fj})$$

The probability that backorders exists for the j-th component is the total expected number of backorders (B_j) divided by the number of aircraft. Thus

$$OR = AC * \prod_{j=1}^{N} \left(1 - \frac{B_j}{AC}\right)$$

The previous equation is true if there is only one of each component on each aircraft. The concept of quantity per application (QPA) is handled by distributing all backorders to each location where ~~it~~ the item can be used and raising the term in the parenthesis to the QPA power.

$$OR = AC * \prod_{j=1}^{N} \left(1 - \frac{B_j / QPA_j}{AC}\right)^{QPA_j}$$

Note that this equation would reduce to the preceeding one if each QPA location on the aircraft were treated as a separate component. Note also that backorders will be allowed to vary day by day, so the OR equation will be applied differently for each day in the analysis (i.e., a day subscript (i) could be added so that $OR = OR_i$ and $B_j = B_{ij}$).

(3) Example: The following is an example that illustrates the use of the availability equation. Assume that there are 100 aircraft in the inventory (AC = 100) and that there are but three mission essential components (N = 3) with the QPA and expected number of backorders for each component as listed below.

j	Component (j)	QPA_j	Exp Backorders (B_j)
1	A	3	27
2	B	2	10
3	C	1	12

If the total expected backorders are uniformly distributed to all of the QPA locations on the aircraft, the aircraft probability status would be as illustrated in Figure 2. For this example, the availability equation would be:

$$OR = 100\left(1-\frac{9}{100}\right)^3\left(1-\frac{5}{100}\right)^2\left(1-\frac{12}{100}\right)$$

$$OR = 60$$

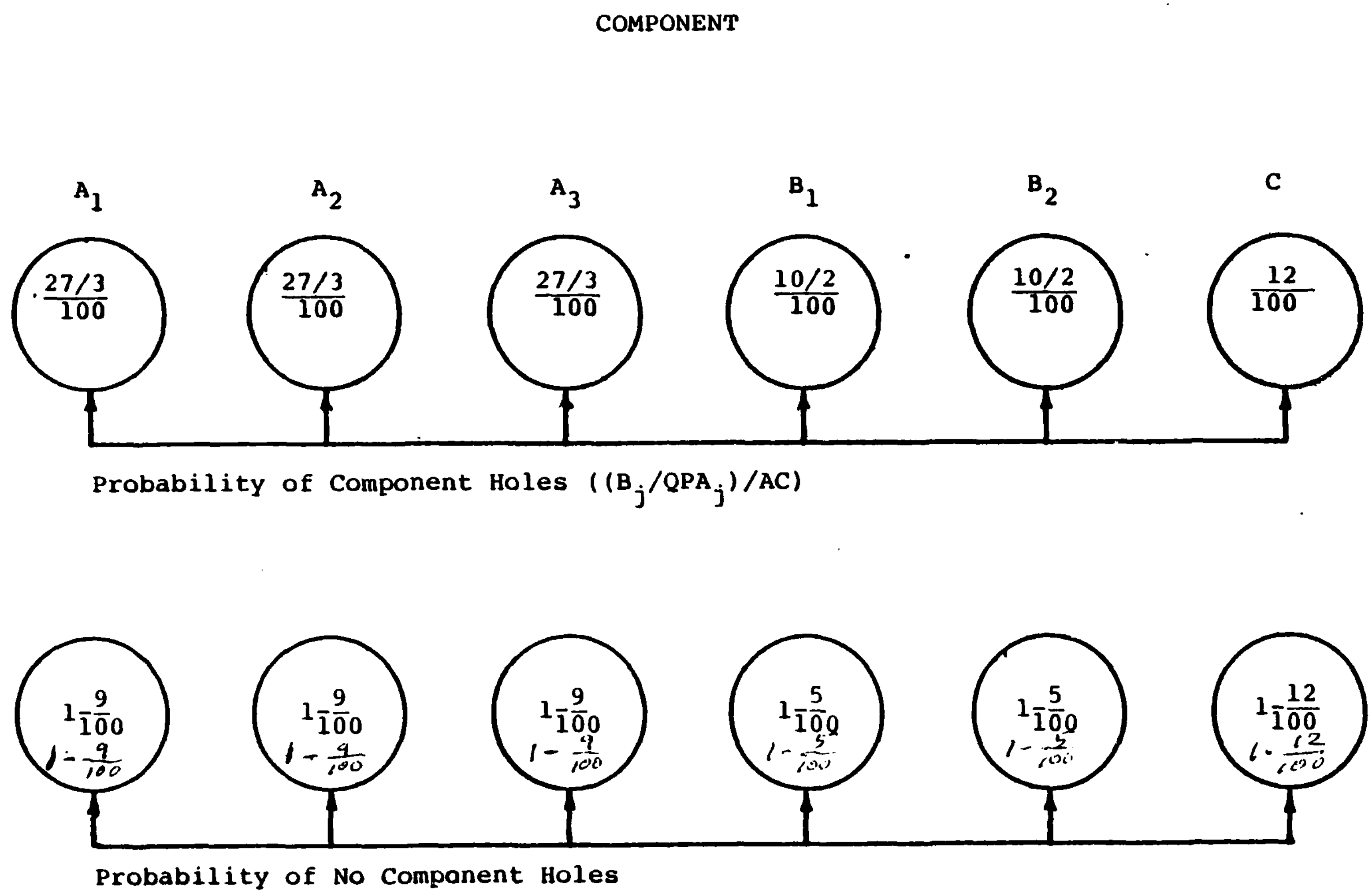


FIGURE 2. AIRCRAFT COMPONENT STATUS

(4) Aircraft Attrition. The preceeding discussion assumed that the number of aircraft (AC) was constant. This implies no attrition or losses in peace or in war. This assumption can be relaxed to some degree by allowing AC to vary as a function of time (AC_i) in the aircraft availability equation. The various AC_i values can be assumed or extracted from some program planning factor document. Accounting for attrition in this manner is equivalent to attempting to fly a predetermined flying hour program with reduced aircraft. If the flying hour program is assumed to be independent of attrition and failure rates linearly related to flying hours, then the expected number of backorders is also independent of attrition and the number of surviving aircraft. In this case, the inclusion of attrition into the availability equation would result in reduced aircraft availability. Note that this condition is only valid for small attrition values, for at some point, as attrition increases, the remaining aircraft would have to be flown at impossible sortie rates to maintain the flying hour program.

(5) Aircraft Availability Equation. The final form of the basic aircraft availability equation is as follows.

$$OR_i = AC_i \prod_{j=1}^{N} \left(1 - \frac{B_{ij}/QPA_j}{AC_i}\right)^{QPA_j}$$

B. Backorder Calculation - Near-Term War.

(1) Backorder Equation. Assume for the moment that the expected number of assets of the j-th component not available for aircraft installation (assets in the pipeline) has been calculated from available DO41 data. Call this expected value μ_j. The various μ_j are then used as the means of a negative binomial probability distribution of assets not available. Since the pipeline can vary from day to day, daily pipeline means also require a time subscript. Thus, μ_{ij} is the mean number of assets of component j not available during the i-th day.

The expected number of backorders at the end of the i-th day is

$$B_{ij} = \sum_{x=A_j}^{\infty} (X - A_j)\ p(x \geq x_m \mid \mu = \mu_{ij})$$

where B_{ij} = Number of expected backorders of component j on the i-th day

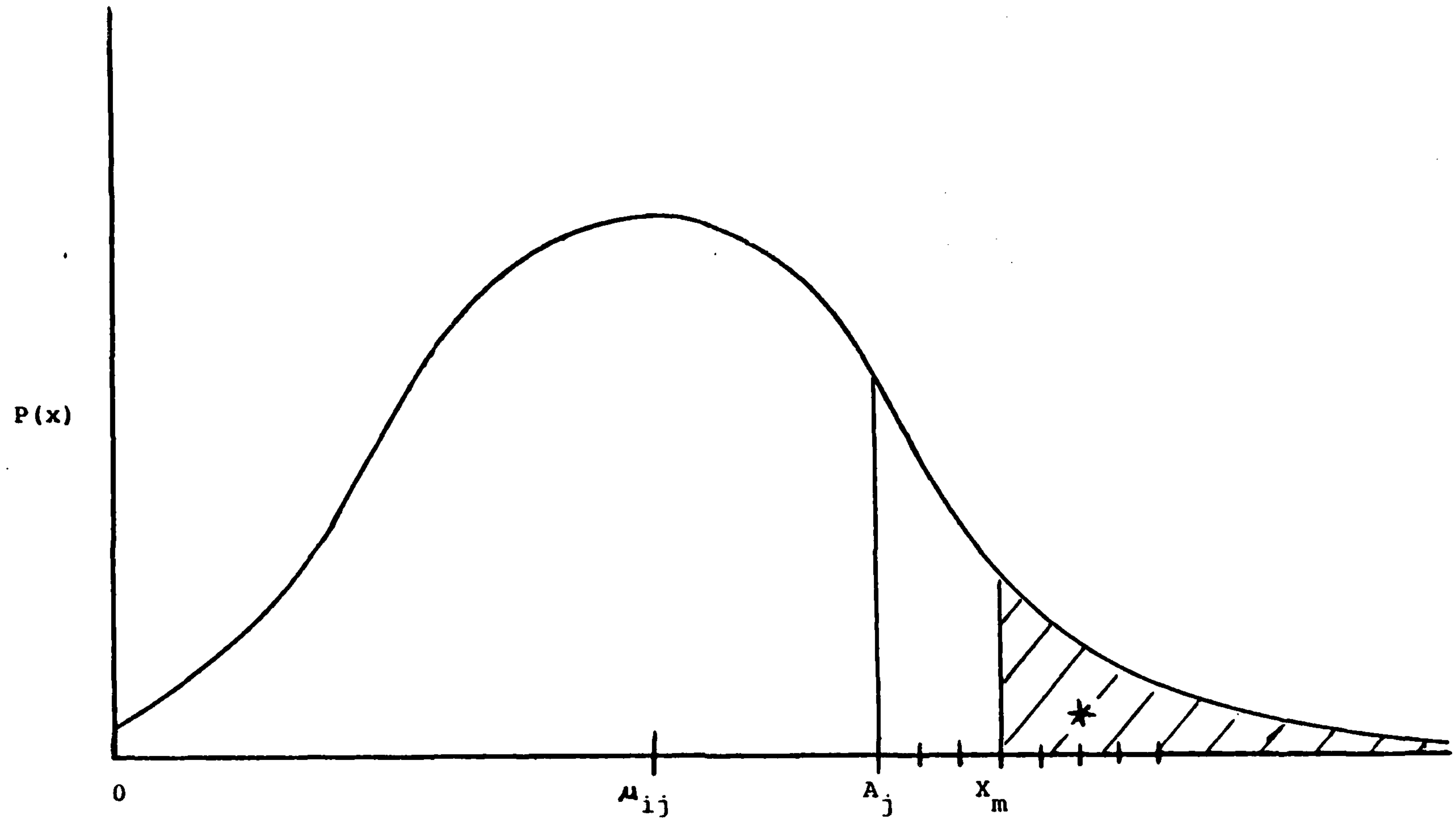
A_j = Asset position of component j

x = Negative binomial random variable

For example, if the random variable x (assets not available) is less than the asset position (A_j) then no backorders exist (serviceable spares exist). If $x = x_1$ ($x_1 = A_j + 1$), then assets not available exceeds the asset position by one, and one backorder exists. $x = x_2$ yields two backorders, etc. Thus, the backorder equation can be read as one times the probability that one backorder exists, plus two times the probability that two backorders exist, plus three times...etc. Recall that the probabilities are taken from the negative binomial distribution ~~which~~ and have been tabulated in statistics texts. The probability distribution might look like figure 3.

(2) Calculation of μ_{ij}. The calculation of the expected value of assets of the j-th component in the pipeline on the i-th day is basically straight forward and uses data available in the DO041 system. In order to avoid developing a number of equations with involved subscripts, a sample calculation for one component on one day will be presented to illustrate the procedure. Assume the following data:

- Daily failures in peace = 3
  - 1 repaired at base
  - 1 repaired at depot
  - 1 condemned



Random Variable = X

X= Assets Not Available (In Pipeline)

If $X \leq A_j$ No Backorders Exist

If $X > A_j$ Backorders Will Exist

* = Probability $X \geq X_m$

FIGURE 3. DISTRIBUTION OF ASSETS NOT AVAILABLE

- War flying rate = 3 times peace flying rate.

  Daily failures in war = 9

    3 repaired at base

    3 repaired at depot

    3 condemned

- Assets at cut-off date = A = 300
- Base repair cycle time = BRCT = 5 days
- Depot repair cycle time = DRCT = 50 days
- Order and ship time = OST = 15 days
- Asset position determined 30 days before D-day.

At the end of the second day of war, the pipeline status under the assumed data conditions would be as described below:

a. Base Repair Cycle Assets. Since the base repair cycle time is five days, three days' worth of peace demand rate and two days' worth of war demand rate assets will be in base repair.

BRCT assets = (3 * 1) + (2 * 3) = 9

b. Depot Repair Cycle Assets. Since the depot repair cycle time is 50 days, 48 days' worth of peace demand rate and two days' worth of war demand rate assets will be in depot repair.

DRCT assets = (48 * 1) + (2 * 3) = 54

c. Order and Ship Time Assets. For an asset to be in the OST category, it must have failed between 50 and 65 days ago. Since we are considering only the second day of the war, all assets entering this category do so at the peacetime rate of one per day

OST assets = 15 * 1 = 15

d. Condemnation Assets. Since the asset position was determined 30 days before the start of the war, one spare was condemned (peace rate) during each of those 30 days, and three spares were condemned (war rate) for each of the two days of war.

Condemnations = (30 * 1) + (2 * 3) = 36

e. Total pipeline assets. The expected value of assets of the j-th component in the pipeline on the second day of war is the sum of the four segments described above.

μ_{2j} = 9 + 54 + 15 + 36 = 114

Thus, for the second war day, the backorder equation would take the form

$$B_{2j} = \sum_{X=300}^{\infty} (X-300)\, p(X \geq X_m \mid \mu = 114)$$

This equation may be completely evaluated using tables of the negative binomial distribution or mathematical evaluation using available computer routines. Once the B_{ij} values are determined, they are substituted into the basic aircraft availability equation in paragraph III. A.(2) above.

C. Backorder Calculation - War in Future. War in the future differs from a near-term war only in that procurement actions are allowed to modify the asset position. The calculation of the μ_{ij} values is similar to that described in paragraph III. B.(2) and in fact, exactly the same for the example presented, except for the number of condemnations since the asset cut off date. The backorder calculation is also similar except that the asset position (A_j) is increased by the number of items of the j-th component added to the inventory from production by the i-th day (P_{ij}). Thus,

$$B_{ij} = \sum_{X=A_j+P_{ij}}^{\infty} \left(X-(A_j+P_{ij})\right) p\left(X \geq X_m \mid \mu = \mu_{ij}\right)$$

Therefore, for any amount of procurement dollars, a requirements computation can be made (for example, DO 41 VSL) and the P_{ij} values determined. This in turn allows the calculation of backorders and aircraft availability as previously described. The inclusion of P_{ij} values in the model permits investigation of the relationship between program funding and aircraft availability.

D. Availability Calculation - Application Percent. To this point in the analysis, we have assumed that the quantity per application for each component (QPA_j) in a given aircraft type (MDS) is a constant. This is often a valid assumption for airframe and engine components. Conversely, this is often not the case for avionics equipment, especially in fighter aircraft. The question is then, how can a variable QPA_j be reflected in the aircraft availability calculation?

Note that the QPA was not a consideration in the calculation of pipeline assets or the expected backorders. Thus, in this analysis, the QPA consideration is only applicable to the aircraft availability calculation. In order to avoid further complicating the aircraft availability equation with additional subscripts, a simple example will be presented to demonstrate how to calculate one of the terms (the j-th of N terms) of the product in the availability equation.

Recall that the j-th term is the following form

$$\left(1 - \frac{B_{ij}/QPA_j}{AC_i}\right)^{QPA_j}$$

Assume that an MDS fleet of 100 aircraft contains a particular component that can exist in one of four forms as shown in the following table.

MDS Sub Class	# Aircraft	QPA_j
1	20	0
2	30	1
3	30	2
4	20	3

Assume also that on the i-th day, the calculation of expected backorders resulted in $B_{ij} = 15$. The procedure used to account for the variable QPA simply distributes the expected backorders in a pro-rata share to each of the MDS sub-classes, and then procedes to calculate the probability of no holes (1 - probability of backorder) in the sub-class. Finally, the probability of no holes for the j-th component is calculated for the entire fleet through an expected value calculation (the sum of four terms of fraction of aircraft in the sub-class times the probability of no backorders in the sub-class). The calculation is shown below.

Sub-Class	Pro Rata Share of Backorders	Probability of No Backorders in Sub-Class
1	0	1.0
2	$15\left[\frac{(1*30)}{(1*30)+(2*30)+(3*20)}\right]=3$	$\left(1-\frac{3}{30}\right)=0.90$
3	$15\left[\frac{(2*30)}{(1*30)+(2*30)+(3*20)}\right]=6$	$\left(1-\frac{6}{60}\right)^2=0.81$
4	$15\left[\frac{(3*20)}{(1*30)+(2*30)+(3*20)}\right]=6$	$\left(1-\frac{6}{60}\right)^3=0.73$

The expected value calculation for the probability of no backorders for the j-th component is then

$$\left(\frac{20}{100} * 1.0\right) + \left(\frac{30}{100} * 0.9\right) + \left(\frac{30}{100} * 0.81\right) + \left(\frac{20}{100} * 0.73\right) = 0.86$$

The value 0.86 is then substituted for the j-th term of the product in the basic aircraft availability equation.

E. Backorder Calculation - Multiple Bases. There are at least four sub-sets of the multiple base problem. The simplest mathematically, and the first to be presented, is the case of multiple bases of equal size, equal priority, and independent of each other. In the succeeding paragraphs each of the limiting conditions will be relaxed until, finally, the most general form of the model will be presented. Note that dividing the world up into more than one base that may or may not be allowed lateral resupply, results in a less optimistic approach than a single-base model, because all available stock is apportioned to the various locations. We still assume that there are no inactive unserviceables and that all serviceable assets have been shipped to a base. Now, however, it is possible to send a serviceable spare to the wrong base. In the sample calculations to come, we will be using the same data set assumed for the single-base calculation in paragraph III. B.(2).

(1) Bases Equal Size, Equal Priority, Independent. Assume that aircraft of a given type are located at K bases throughout the world where each base is of equal size (same spares demand rate due to equal flying programs), equal priority (each base gets an identical share of serviceable assets), and independent (no lateral resupply allowed among the K bases). Now that we are dealing with more than one base, it is necessary to provide notation to distinguish each base. Let k denote the k-th base where k=1,2,3...K. Thus μ_{ijk} represents the expected value of pipeline assets of the j-th component on the i-th day that were generated by the k-th base. In a similar manner, a base designator subscript must be added to the asset position (A_{jk}), the production quantity (P_{ijk}), the expected backorders (B_{ijk}), and the available aircraft (AC_{ik}).

a. Theory. In the case at hand, if all bases are equal size, then the μ_{ijk} are equal:

$$\mu_{ij1} = \mu_{ij2} = \mu_{ij3} \cdots = \mu_{ijK} = \frac{1}{K}\mu_{ij}$$

If they are of equal priority and assets are divided equally, then the A_{jk} and P_{ijk} are equal:

$$A_{j1} = A_{j2} = A_{j3} \ldots\ldots = A_{jK} = \frac{1}{K} A_j$$

$$P_{ij1} = P_{ij2} = P_{ij3} \cdots\cdots = P_{ijK} = \frac{1}{K} P_{ij}$$

If they are independent, there are no interactions among the bases and the backorders for each base may be calculated separately:

$$B_{ijk} = \sum_{X=A_{jk}+P_{ijk}}^{\infty} \left(X-(A_{jk}+P_{ijk})\right) p\left(X \geq X_m \mid \mu = \mu_{ijk}\right)$$

Finally, the worldwide availability is simply the sum of the independent base aircraft availabilities:

$$OR_i = \sum_{k=1}^{K} AC_{ik} \prod_{j=1}^{N} \left(1 - \frac{B_{ijk}/QPA_j}{AC_{ik}}\right)^{QPA_j}$$

Note that, as long as the K bases are of equal size, have equal priority, and are independent, the B_{ijk} and AC_{ik} are equal, and the previous equation may be simplified to

$$OR_i = K * AC_{ik} \prod_{j=1}^{N} \left(1 - \frac{B_{ijk}/QPA_j}{AC_{ik}}\right)^{QPA_j}$$

b. Example. Using the data assumed in paragraph III.B.(2) for the second day of war, a production value of $P_{2j} = 75$ and three bases for the MDS in question (K=3); the following results;

$$\mu_{2j1} = \mu_{2j2} = \mu_{2j3} = \frac{1}{3}\mu_{2j} = \frac{114}{3} = 38$$

$$A_{j1} = A_{j2} = A_{j3} = \frac{1}{3} A_j = \frac{300}{3} = 100$$

$$P_{2j1} = P_{2j2} = P_{2j3} = \frac{1}{3} P_{2j} = \frac{75}{3} = 25$$

Thus

$$B_{2j1} = B_{2j2} = B_{2j3}$$

$$= \sum_{X=125}^{\infty} (X-125)\, p(X \geq X_m \mid \mu = 38)$$

and

$$OR_i = 3\,\frac{100}{3} \prod_{j=1}^{N} \left(1 - \frac{B_{ijk}/QPA_j}{100/3}\right)^{QPA_j}$$

These equations are evaluated as described previously.

(2) Accounting for Lateral Resupply. The condition of base independence is removed as soon as lateral resupply between or among bases is allowed. In this analysis, perfect lateral resupply can be allowed among selected bases while prohibiting lateral support among others. For the time being, we still require bases of equal priority and size. Lateral support is accounted for simply by mathematically considering two or more locations as a single integrated base and reducing the number of bases by an equivalent amount. This approach to lateral resupply is closest to the physical case where two or more bases are located close together, or where the bases enjoy particularly good inter-base communications, transportation, and cooperation. Thus, this analysis is limited to a binary view of lateral support: either perfect lateral cooperation exists among selected bases, or no lateral resupply exists at all.

Continuing on with the previous example, assume that base one and two will enjoy lateral resupply. Mathematically we simply renumber the bases so that bases one and two are now base one, which is twice as large as before, and base three becomes base two, with the total number of bases now being two:

$$\mu^{*}_{2j1} = \mu_{2j1} + \mu_{2j2} = 38 + 38 = 76$$

$$\mu^{*}_{2j2} = \mu_{2j3} = 38$$

After the P_{2jk} and A_{jk} are similarly combined to P^*_{2j1}, P^*_{2j2} and A^*_{j1}, A^*_{j2} the backorder equations are

$$B^*_{2j1} = \sum_{\chi = A^*_{j1}+P^*_{2j1}}^{\infty} \left(\chi - (A^*_{j1}+P^*_{2j1})\right) p\left(\chi \geq \chi_m \mid \mu = \mu_{2j1}\right)$$

$$= \sum_{\chi = 200+50}^{\infty} \left(\chi - (200+50)\right) p\left(\chi \geq \chi_m \mid \mu = 76\right)$$

$$B^*_{2j2} = \sum_{\chi = A^*_{j2}+P^*_{2j2}}^{\infty} \left(\chi - (A^*_{j2}+P^*_{2j2})\right) p\left(\chi \geq \chi_m \mid \mu = \mu^*_{2j2}\right)$$

$$= \sum_{\chi = 100+25}^{\infty} \left(\chi - (100+25)\right) p\left(\chi \geq \chi_m \mid \mu = 38\right)$$

It is important to recognize that from the three original partially interdependent bases, two mathematically independent bases have been created. Because the mathematical independence has been preserved, it is possible to employ the basic aircraft availability equation using the modified backorder values (B^*_{ijk}),

$$OR_i = \sum_{k=1}^{K} AC_{ik} \prod_{j=1}^{N} \left(1 - \frac{B^*_{ijk}/QPA_j}{AC_{ik}}\right)^{QPA_j}$$

where K=2 and i=2 for two bases on the second day of war.

(3) Bases of Unequal Priority. Bases may be afforded a variable priority by providing them with asset levels (initial stock and production spares) differing from their activity-based

pro-rated share. For example, the two bases with lateral support provide each other with an added protection not available to the third base, that is conceptually isolated from the other two. Therefore it might be desirable to provide an increased share of the assets to the third base. This concept can be implemented mathematically by use of a weighting factor on the A^{*}_{jk} and P^{*}_{ijk} variables

$$A^{**}_{jk} = W_k \, A^{*}_{jk}$$

$$P^{**}_{ijk} = W_k \, P^{*}_{ijk}$$

Although the selection of the weighting factor is completely arbitrary and at the discretion of the analyst, one possible factor that provides protection to small or isolated bases is

$$W_k = \frac{\mu^{*}_{ijk} + \sqrt{3\,\mu^{*}_{ijk}}}{\sum_{k=1}^{K}\left(\mu^{*}_{ijk} + \sqrt{3\,\mu^{*}_{ijk}}\right)}$$

Evaluation of the weighting factor for $\mu_{2j1} = 76$ and $\mu_{2j2} = 38$ results in $A^{**}_{j1} = 196$ where the unweighted share for the combined base was 200, and $A^{**}_{j2} = 104$ where the unweighted share for the isolated base was 100.

The A** and P** values are used in place of A* and P* in the backorder equation and the evaluation completed as previously described. Note that the weighting factor described above attempts to restore a level of protection for an isolated base to that enjoyed by two or more bases that are capable of lateral resupply. Other weighting factors could be constructed to reflect mission priorities, so that selected bases would be provided with an increased share of total assets at the expense of bases of lesser priority.

(4) Bases of Unequal Size or Activity. In all ~~previous~~ the above analyses, it has been assumed that all bases were equal in activity and thus all μ_{ijk} values were equal. This was done only for temporary convenience and is not necessary. It is only required to note that the μ_{ijk} values need not be equal and, in fact, may be different for each of the K bases. Both the backorder equation and the aircraft availability equation work equally well for unequal values of μ_{ijk}.

IV SUMMARY

The following summarizes the analysis procedure and displays the basic equations in their most general form:

A. Calculate μ_{ijk} values from applicable DO041 system data.

B. Convert μ_{ijk} to μ^{*}_{ijk} to account for those bases where lateral resupply is to be allowed (para III. E.(2)).

C. Determine A_{jk} and P_{ijk} from applicable stock availability and procurement records.

D. Convert A_{jk} and P_{ijk} to A^{*}_{jk} and P^{*}_{ijk} to account for lateral resupply (para III. E.(2)).

E. Convert A^{*}_{jk} and P^{*}_{ijk} to A^{**}_{jk} and P^{**}_{ijk} to account for unequal distribution of assets to the bases using some selected weighting factor (para III. E.(3)).

F. Calculate expected backorders using

$$B^{*}_{ijk} = \sum_{X=A^{**}_{jk}+P^{**}_{ijk}}^{\infty} \left(X-(A^{**}_{jk}+P^{**}_{ijk})\right) p\left(X \geq X_m \mid \mu = \mu^{*}_{ijk}\right)$$

and an appropriate probability distribution.

G. Calculate aircraft availability using

$$OR_i = \sum_{k=1}^{K} AC_{ik} \prod_{j=1}^{N} \left(1 - \frac{B^{*}_{ijk}/QPA_j}{AC_{ik}}\right)^{QPA_j}$$

END

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