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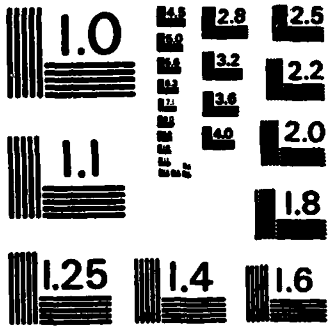

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ANALYSIS OF TURBULENT NEAR WAKES

by

A. Prabhu and V. C. Patel

Sponsored by

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of the Naval Sea Systems Command
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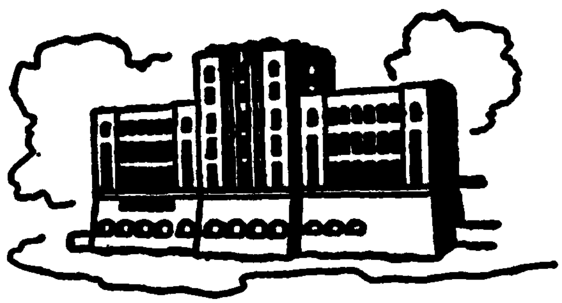

IIHR Report No. 253

Iowa Institute of Hydraulic Research
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Square root of C sub f X R sub Sigma as it approaches infinity

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report is concerned with the analytical treatment of the flow in the near wake behind sharp trailing edges with a fully-developed zero-pressure gradient turbulent boundary layer just upstream of the trailing edge. The three cases considered are the flows behind two-dimensional, axisymmetric and infinitely-yawed sharp trailing edges. The principal limits that govern the near wake seem to be $c_f^{1/2} \to 0$ and $c_f^{1/2} R_\delta \to \infty$, where c_f and R_δ are the skin friction in the upstream boundary layer and the Reynolds number based on shear layer thickness, respectively. This is the same limit as that for the local

C sub f

R sub sigma

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) asymptotic analysis of a fully-developed turbulent boundary layer under zero pressure gradient at large Reynolds numbers. These limiting conditions lead to the governing equations for the inner and outer near-wake and a logarithmic variation of the velocity in the overlapping region. It is argued that the same limiting conditions also lead to a logarithmic variation of the centerline velocity in the near wake. Similarity solutions have been obtained for the governing equations of the inner and outer near-wakes. Limited comparisons with experiments for a two-dimensional flow show that the variation of the scales and profiles predicted agree reasonably well with experiments.

Analysis of Turbulent Near Wakes

by

A. Prabhu and V.C. Patel

Abstract

This report is concerned with the analytical treatment of the flow in the near wake behind sharp trailing edges with a fully-developed zero-pressure gradient turbulent boundary layer just upstream of the trailing edge. The three cases considered are the flows behind two-dimensional, axisymmetric and infinitely-yawed sharp trailing edges. The principal limits that govern the near wake seem to be $c_f^{1/2} \rightarrow 0$ and $c_f^{1/2} R_\delta \rightarrow \infty$, where c_f and R_δ are the skin friction in the upstream boundary layer and Reynolds number based on shear layer thickness, respectively. This is the same limit as that for the local asymptotic analysis of a fully-developed turbulent boundary layer under zero pressure gradient at large Reynolds numbers. These limiting conditions lead to the governing equations for the inner and outer near-wake and a logarithmic variation of the velocity in the overlapping region. It is argued that the same limiting conditions also lead to a logarithmic variation of the centerline velocity in the near wake. Similarity solutions have been obtained for the governing equations of the inner and outer near-wakes. Limited comparisons with experiments for a two-dimensional flow show that the variation of the scales and profiles predicted agree reasonably well with experiments.

1. Introduction

Close to the trailing edge of an arbitrary body the flow is usually dominated by separation and reattachment or trailing edge singularities and this complex flow is not clearly understood. In the case of laminar flow on a flat plate the boundary layer on the plate develops into Goldstein's near wake beyond the trailing edge through a transition in a small region, known as the triple-deck region (Stewartson, 1965) around the trailing edge. This triple-deck region, whose order of magnitude in Reynolds number is higher than that of the boundary layer (imbedded inside the boundary layer) arises in order to avoid the singularity of the boundary-layer equations at the trailing edge. In the case where the boundary layer is turbulent the extent of the triple-deck region becomes very small as the Reynolds number is necessarily large and therefore, probably, is of little consequence for the development of the near wake beyond it. Evaluation of experimental data in the near wakes behind round and two-dimensional bodies have been recently made by Patel (1981a, 1981b). The present report is concerned with the analytical treatment of near wakes beyond the triple-deck region of sharp-edged bodies.

If the oncoming turbulent boundary layer is fully developed, it has two distinct layers or regions governed by two different length scales. Since the turbulence structure is known to retain memory it is reasonable to expect the flow in the near wake also to retain the two-layered structure. Once this two-layer structure (an inner and an outer layer) similar to that of a turbulent boundary layer is assumed to exist, the velocity profile in the overlapping or matching region must vary logarithmically as in a turbulent boundary layer. The logarithmic velocity variation will then be the boundary condition for describing the inner and outer wake development. Further, the changes along the streamwise direction x near the trailing edge in the inner wake are largely due to the mixing produced by eddies whose characteristics are determined by the viscous scale of the boundary layer near the wall. Therefore, the relevant length scale for development in the x direction for the inner wake will also be the viscous scale of the upstream boundary layer. These arguments have been incorporated into the study of two-dimensional near wakes by Alber (1980), Andreopoulos and Bradshaw (1980), Ramaprian et al. (1981), and Robinson (1967). They have all shown that the centerline velocity varies logarithmically for large x in the near wake. Alber made further similarity analysis and obtained detailed velocity profiles in the inner wake which he compared with the velocity profiles measured by Chevray and Kovasznay (1969). He also considered a region of development very close to the trailing edge similar to the Goldstein wake, where the laminar sublayer of the upstream boundary layer is consumed. There exists no analysis for the outer wake. Also, it is not clear whether similar analyses can be made for near wakes behind infinite-swept sharp trailing edges and pointed axisymmetric bodies.

In this report, Goldstein's near-wake solution, and the inner and outer wake analysis, are considered for the three cases: two-dimensional, infinite-swept and axisymmetric wakes. Goldstein's 1/3 power-law for the variation of the centerline velocity seems to be valid for all the cases in the laminar region very near the trailing edge. Similarity solutions for the inner and outer wake which have logarithmic velocity variation at the match boundaries have also been obtained. Existence of similarity solutions in the inner layer requires that the centerline velocity should vary logarithmically for large x in all three cases. This variation is in support of a more general dimensional argument, similar to that of Millikan (1938), which leads to a logarithmic velocity variation in the overlap region between the near wake and the far wake. Outer layer similarity requires that the square of the total wake width vary logarithmically with x, and this is supported by the recent measurements of Sastry (1981).

2. The Model and the Equations

The various flow regions in the near wake are shown in figure 1. The nomenclature "near wake" here signifies the region beyond the trailing edge over which the memory of the viscous length scale of the boundary layer is lost or where the log region of the boundary layer is consumed. It is not clear at this stage whether the asymptotic or the far wake is far beyond the end of the near wake or matches with it in an overlapping region. The inner layer itself has a laminar region, very near the trailing edge, where the laminar sublayer of the upstream boundary layer is consumed due to the mixing produced by the eddies of the viscous scale and a region beyond it where the flow is turbulent but characterized still by the dynamics of the eddies which retain the memory of the viscous scale of the upstream boundary layer. The relevant length scale for both streamwise and normal direction is ν/u_τ, where ν is the kinematic viscosity and u_τ the friction velocity of the boundary layer just upstream of the trailing edge.

The coordinates chosen and the corresponding velocity components are shown in figure 2 for all three cases. The governing equations for the outer and inner flows can be obtained by making a local asymptotic analysis, similar to that made for a fully-developed turbulent boundary layer (see, for example, Yajnik (1970), and Mellor (1972)). The limiting conditions that lead to the equations given below are the same as that for the turbulent boundary layer. They are $u_\tau/U_\infty \to 0$ and $\delta u_\tau/\nu \to \infty$. where δ is the total wake thickness. In the outer layer the variables $\frac{U_\infty - u}{u_\tau}$, τ/u_τ^2, y/δ and x/L are held fixed, and in the inner layer the variables u/u_τ, τ/u_τ^2, xu_τ/ν, and yu_τ/ν are fixed while the above limits are taken. Note that for the boundary layer asymptotic analysis (made by Yajnik, 1970, Mellor, 1972), the stretched variable for the x coordinate in the inner layer is x/L which is different from the xu_τ/ν used for the wake here. With these inner and outer variables fixed under the limiting condition $u_\tau/U_\infty \to 0$ the following equations which govern the inner and outer wake are obtained.

Inner wake

$$\frac{\partial(uy^J)}{\partial x} + \frac{\partial(vy^J)}{\partial y} = 0 , \tag{1a}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{1}{\rho y^J}\frac{\partial(y^J\tau_1)}{\partial y} = 0 \tag{1b}$$

$$k\ [u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} - \frac{1}{\rho}\frac{\partial}{\partial y}(\tau_2)] = 0 \tag{1c}$$

J = 1 only for axisymmetric and k = 1 only for yawed wake, otherwise J and k are zero.

Outer wake

$$U_\infty\frac{\partial u_d}{\partial x} + \frac{1}{\rho}\frac{1}{y^J}\frac{\partial\ (y^J\tau_1)}{\partial y} = 0, \tag{2a}$$

$$k(U_\infty\frac{\partial w_d}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}(\tau_2)) = 0. \tag{2b}$$

Here u_d and w_d are the defect velocities defined by $u_d = U_\infty - u$ and $w_d = (W_\infty - w)$, and τ_1 and τ_2 are the shear stresses in the xy and yz planes, respectively. Matching between these two layer leads to the boundary condition in the overlapping region as

$$u/u_\tau = A_1 \ln\left(\frac{y\,u_\tau}{\nu}\right) + B_1, \quad w/w_\tau = A_2 \ln\left(\frac{y\,u_\tau}{\nu}\right) + B_2. \tag{2c}$$

where u_τ and w_τ are the friction velocities in the x and z direction.

3. Laminar Near Wake Region

In this region the laminar sublayer of the upstream boundary layer develops like a laminar wake before it is consumed by the mixing due to the convected small scale eddies of the turbulent boundary layer on either side of the plate. The variable in equations (1) are normalized appropriately using the velocity scale u_τ and the length scale ν/u_τ. Superscript + denotes these normalized variables. The shear stresses τ_1 and τ_2 are given, in this region, by the viscous stresses $\mu\partial u/\partial y$ and $\mu\partial w/\partial y$, respectively. The equations which govern this region can then be written as

$$\frac{\partial(u^+ y^{+J})}{\partial x^+} + \frac{\partial(v^+ y^{+J})}{\partial y^+} = 0 , \tag{3a}$$

$$u^+ \frac{\partial u^+}{\partial x^+} + v^+ \frac{\partial u^+}{\partial y^+} - \frac{1}{y^{+J}} \frac{\partial}{\partial y^+}\left(y^{+J} \frac{\partial u^+}{\partial y^+}\right) = 0 , \tag{3b}$$

and

$$k\left(u^+ \frac{\partial w^+}{\partial x^+} + v^+ \frac{\partial w^+}{\partial y^+} - \frac{\partial^2 w^+}{\partial y^{+2}}\right) = 0 . \tag{3c}$$

At the edge of the laminar sublayer u^+ and w^+ vary linearly with y^+. This is the boundary condition for large y^+ used for obtaining solutions of equations (3) in the laminar region of the near wake. At the center-line

$\frac{\partial u^+}{\partial y^+} = 0$. In view of the linearly increasing velocity far away from the centerline Goldstein's (1930) asymptotic expansion for the laminar wake for small distances from the trailing edge seem to be quite appropriate for the solution of these equations. The first term in the Goldstein power series expansion for the wake flow close to the trailing edge constitutes the similarity solutions for these equations. The variables for the similarity solution are given by

$$u^+ = \frac{1}{3} x^{+\,1/3} f'(\eta),$$

$$w^+ = A \frac{1}{3} x^{+\,1/3} g(\eta), \tag{4}$$

and $$\eta = \frac{1}{3} y^+/(x^+)^{1/3},$$

where A is given by the relation $\frac{w}{w_\tau} = Ay^+$ which is satisfied in the laminar sublayer of the boundary layer. If v^+ is obtained from the continuity equation (2a) and the above variables from (4) are substituted into equation (3), the following equations result

$$\eta^J f'^2 - (2+J)\, f'' \int_0^\eta \eta^J f'\, d\eta - (\eta^J f'')' = 0\ , \tag{5a}$$

$$k[(f'g - 2fg') - g''] = 0\ . \tag{5b}$$

Here, and later, the primes represent differentiation with respect to the appropriate variables. For a two-dimensional flow (5a) reduces to the equation

$$f'^2 - 2ff'' - f''' = 0. \tag{6}$$

If $f' = g$ then equation (5b) can also be reduced to (6). Equation (6) is the same as that obtained by Goldstein (1930) for a two-dimensional flow. For axisymmetric flow the similarity equation is slightly different and is, from (5a),

$$\eta f'^2 - 3f'' \int_0^\eta \eta f' d\eta - (\eta f'')' = 0 \, . \tag{7}$$

Equations (5a), (6) and (7) have linearly increasing solutions for large η as the most dominating term in an asymptotic expansion for large η. The boundary condition for large η, in view of the definition (4), is given by

$$f' = 9\eta = g \tag{8}$$

and the inner boundary condition is

$$f(0) = f''(0) = g'(0) = 0. \tag{9}$$

For small η it is possible to develop a series solution for these equations. In the two-dimensional and yawed wakes it is the same as that given by Goldstein, namely

$$f'(\eta) = g = a_0 + a_0^2\eta^2/2 - 2a_0^3\,\eta^4/4 + \ldots \tag{10a}$$

In the axisymmetric case the series solution for small η assumes the form

$$f'(\eta) = a_o + \frac{a_o^2}{4}\eta^2 - \frac{a_o^3}{64}\eta^3 + \ldots \tag{10b}$$

where a_o is a free parameter to be determined by the requirement that $f'(\eta)$ and $g(\eta)$ increase like 9η at the edge of this layer. Equations (5a) and (5b) have a certain invariant characteristic which leads to the following property. If $F_1(\eta)$ and $F_2(\eta)$ are solutions of (5a) and (5b) then $f = aF_1(\zeta)$ and $g = aF_2(\zeta)$ with $\zeta = a\eta$ are also solutions of equation (5a) and (5b), respectively. In view of this, $a^2 = f'(\eta)/F_1'(\zeta)$. Since, for large η, $f'(\eta)$ varies as 9η and $F_1'(\zeta)$ as $F_1''(\infty)\zeta$ it is easily seen that

$$a^3 = 9/F_1''(\infty). \tag{11}$$

$F_1''(\infty)$ can be obtained by constructing a solution starting with $F_1(0) = F_1''(0) = 0$ and $F_1'(0) = 1$. Since $a_o = f'(0) = a^2F_1'(0)$, equation (11) leads to a relation between a_o and $F_1''(\infty)$ as

$$a_o = \{9/(F_1''(\infty)\}^{2/3}. \tag{12}$$

Once a_o is found from (12), the centerline velocity is obtained from

$$u^+(0) = w^+(0)/A = a_o \frac{1}{3} x^{+\ 1/3}. \tag{13}$$

Equation (13) leads to the centerline velocity variation very close to the trailing edge, and equation (10a) and (10b) lead to the velocity profiles near the centerline.

4. Turbulent Inner Near Wake

In this region the flow is completely turbulent. It is therefore assumed that the viscous stress is negligible compared to the turbulent stress. The latter is obtained by an eddy-viscosity assumption. Then if equations (1) are normalized as in section 3 the following governing equations are obtained

$$\frac{\partial(u^+ y^{+J})}{\partial x^+} + \frac{\partial(v^+ y^{+J})}{\partial y^+} = 0, \tag{14a}$$

$$u^+ \frac{\partial u^+}{\partial x^+} + v^+ \frac{\partial u^+}{\partial y^+} - \frac{1}{y+J} \frac{\partial}{\partial y^+} \left(y^{+J} \nu_T \frac{\partial u^+}{\partial y^+}\right) = 0, \tag{14b}$$

where ν_T is the ratio of eddy viscosity to molecular viscosity. The assumptions on the form of the eddy viscosity will be made later. Since $\delta u_\tau/\nu$ tends to infinity in the limit for which equations (1) and (2) are derived, it is reasonable to assume that, even for the wake, a Reynolds number based on u_τ and a length scale h describing the extent of the inner wake, also tends to infinity. The limit $u/u_\tau \to 0$ corresponds to $\ln(\delta u_\tau/\nu) \to \infty$ and hence $[\ln h\, u_\tau/\nu]^{-1} \to 0$. Here u^+ is expanded in an asymptotic expansion in terms of the small parameter $\varepsilon = (\ln h\, u_\tau/\nu)^{-1}$ in the form

$$u^+ - S(x^+) = \frac{1}{k} (\phi_0' + \varepsilon\, \phi_1' + \varepsilon^2 \phi_2' + \ldots), \tag{15}$$

where k is the Karman constant, ϕ_0, ϕ_1, etc. are functions of x^+ and y^+, and $S(x^+)$ is related to the centerline velocity. $S(x^+)$ tends to become large, as will be seen later because eventually at the end of the near wake it must be of the same order as the velocity ratio u^+ at the edge of the log layer of the boundary layer. It is also argued later on that if there exist two length scales for describing the wake development in the x direction, the viscous length in the near wake and the distance from the trailing edge in the far wake, then matching between the two regions requires that the centerline velocity should become large compared to the friction velocity in the overlapping region. The right-hand side of (15) then represents, in some sense, small departures from the centerline velocity. It is further assumed here that ϕ_o and ϕ_1 are functions of the similarity variable $\zeta = y^+/h^+$ only, where h^+ is hu_τ/ν. Then substituting (15) into the continuity equation (14a) gives

$$v^+ = -\frac{S'h^+\zeta}{2^J} + \frac{h^{+'}}{k\zeta^J}\int_0^\zeta \zeta^J\zeta(\phi_o'' + \epsilon\phi_1'' + ..)\,d\zeta + \frac{h^{+'}}{k}\epsilon^2\int_0^\zeta \zeta^J(\phi_1 + 2\epsilon\phi_2' + ..)\,d\zeta. \tag{16}$$

The momentum equation (14b) then reduces to

$$h^+SS' - \frac{Sh^{+'}}{k}\zeta\phi_o'' + \frac{h^+}{k}S'\phi_o' - \frac{Sh^{+'}}{k}\epsilon\zeta\phi_1'' - \frac{S'h^+\zeta}{2^Jk}\phi_o''$$
$$- \frac{h^{+'}}{k^2}\frac{\phi_o''}{\zeta^J}\int_0^\zeta (1+J)\,\zeta^J\phi_o'\,d\zeta$$
$$= \frac{1}{\zeta^J}\left(\zeta^J\frac{\nu_T}{kh^+}\phi_o''\right)' + \frac{\epsilon}{\zeta^J}\left(\frac{\zeta^J\nu_T}{kh^+}\phi_1''\right)' . \tag{17}$$

The above equation contains terms of $O(1)$ and $O(\epsilon)$. The terms that are neglected are of $O(\epsilon^2)$ or smaller (see also discussion below). In order to separate the terms in the above equation the order of magnitude of the coefficient of each term has to be first assessed. The order of magnitude of each term will be clear after certain conditions for similarity solutions are evaluated. If ϕ_0 and ϕ_1 in equation (17) are to be functions of ζ only, all the functions of x^+ appearing as coefficients in equation (17) must be constants. In particular

$$h^+SS' = c_1, \qquad \frac{Sh^{+'}}{k} = c_2 \tag{18}$$

c_1 and c_2 can be set equal to unity without loss of generality. Then the solution of (18) leads to

$$S = \frac{1}{k}\ln h^+ + c_3, \qquad h^+[\ln h^+ + c_3 - 1] = k^2 x\ , \tag{19}$$

where c_3 is a constant which could be used to define a suitable initial h^+. These solutions reveal the character of the centerline velocity variations and the limits in which these approximations are valid. S varies like ε^{-1} and $h^{+'}$ like ε. Under these conditions $h^+S' = k\varepsilon$ and $h^{+'} = k^2\varepsilon$. With these orders of magnitude for h^+S' and $h^{+'}$, it is clear that the terms that are neglected in equation (17) are of $O(\varepsilon^2)$ or smaller. An equation for ϕ_0 can be obtained by collecting terms of O(1), as

$$\frac{1}{\zeta^J}\left(\zeta^J \frac{\nu_T}{kh^+}\phi_0''\right)' + \zeta\phi_0'' - 1 = 0 \tag{20a}$$

Collecting coefficients of order $O(\varepsilon)$ leads to the equation for ϕ_1, as

$$\frac{1}{\zeta^J}\left(\zeta^J \frac{\nu_T}{kh^+}\phi_1''\right)' + \zeta\phi_1'' = \phi_0' - \frac{\zeta\phi_0''}{2^J} - \frac{\phi_0''}{\zeta^J}\int_0^\zeta (1+J)\zeta^J\phi_0'\,d\zeta. \tag{20b}$$

Equations (20a) and (20b) reduce to those obtained by Alber (1980) for two-dimensional flow. However, the present derivation is more consistent than that made by Alber (1980) since he assumed an asymptotic expansion of the form $\phi = \phi_o + \varepsilon(x)\phi_1$ after obtaining a governing equation for ϕ assuming that ϕ is a function of ζ only (which is inconsistent). In view of the particular nature of ε in this problem the derivative of ε contributes to terms of the order $O(\varepsilon^2)$. Hence the equation for ϕ_1 remains the same as that obtained by Alber. Equation (20a) can be further simplified to

$$\phi_o''' + \phi_o'' \left[\frac{kh^+\zeta}{\nu_T} + \frac{\nu_T'}{\nu_T} + \frac{J}{\zeta}\right] - \frac{kh^+}{\nu_T} = 0 \; . \tag{21}$$

A solution for the above equation can be written down as

$$\phi_o''(\zeta) = \left[\frac{1}{\zeta^J \nu_T} \exp\left(-\int^{\zeta} \frac{kh^+\zeta}{\nu_T} d\zeta\right)\right]\left[A_3 + \int_0^{\zeta} kh^+ \zeta_1^J \exp \int_0^{\zeta_1} \frac{kh^+\zeta_2}{\nu_T} d\zeta_2 \, d\zeta_1\right], \tag{22}$$

where A_3 is an arbitrary constant. Since $\phi_0'' = 0$ at $\zeta = 0$, equation (22) can be further simplified to

$$\phi_o''(\zeta) = \frac{kh^+}{\nu_T\zeta^J} \int_o^{\zeta} \zeta_2^J \left(\exp - \int_{\zeta_2}^{\zeta} \frac{kh^+\zeta_3}{\nu_T} d\zeta_3\right) d\zeta_2 \; . \tag{23}$$

Integration of (23) once gives

$$\phi_o'(\zeta) = \phi_o'(0) + \int_o^{\zeta} \frac{kh^+}{\nu_T\zeta_1^J} \left(\int_o^{\zeta_1} \zeta_2^J \left(\exp - \int_{\zeta_2}^{\zeta_1} \frac{kh^+\zeta_3}{\nu_T} d\zeta_3\right) d\zeta_1\right) d\zeta_1 . \tag{24}$$

For a yawed wake, if an expansion for w^+ given by

$$w^+ = A\left[s_1 + \frac{1}{k}\left(\psi_o(b) + \varepsilon\, \psi_1(\zeta) + ..\right)\right], \tag{25}$$

is used, it can be easily shown that $\psi_0(\zeta)$ is equal to $\phi_0'(\zeta)$ given by equation (24) with $J = 0$.

No assumption has been made so far concerning the form of the eddy viscosity. This should be such that it reflects the turbulent structure of the inner wake and the resulting velocity profiles obtained from (24) and (15) should be such that the matching condition (boundary condition) given by (26) is satisfied. In the inner boundary layer ν_T varies like ky^+. Therefore it is reasonable to assume in the wake, at least in the logarithmic region, that ν_T/kh^+ varies like ζ. If this form is assumed throughout the inner region of the wake it leads to the condition that the eddy viscosity is zero at the centerline. In fact, as the wake develops the eddy viscosity at the centerline should increase gradually so that at the end of the near wake development it is more or less constant across the entire wake width. Assumption of a linearly varying eddy viscosity all across was made by Alber. In such a case, even though the velocity profile can be shown to vary logarithmically for large ζ, ϕ_0'' becomes singular as $\zeta \to 0$. In order to avoid the singular nature of ϕ_0'' for small ζ a non-zero eddy viscosity is required at $\zeta = 0$. One alternative is to assume $\nu_T/kh^+ = (\zeta + \alpha)$ where α is of the order unity. A slightly general form which gives the required behavior for ν_T is given by

$$\frac{\zeta}{\nu_T/kh^+} = 1 - p'/p \;, \tag{26}$$

where p is chosen such that $p'/p \to 1$ as $\zeta \to 0$ and $p'/p \to 0$ rapidly as $\zeta \to \infty$. The detailed form of p is still unspecified. This form is assumed since it facilitates the asymptotic expansion of the integrals in equation (24). Under this assumption an asymptotic expansion for large ζ can be obtained for ϕ_0' as

$$\phi'(\zeta) = \phi'(0) + \int_0^\zeta \frac{(1-p'/p)}{\zeta_1} p \; I(\zeta_1)d\zeta_1 + J \int_0^\zeta \frac{(1-p'/p)p}{\zeta_1\zeta_1^J} \left(\int_0^\infty \int_\infty^F \frac{e^{-F_1}}{p(\zeta_1-F_1)} dF_1 dF\right) d\zeta_1$$

$$- \int_0^\zeta \frac{(1-p'/p)p}{\zeta_1\zeta_1^J} \left(\int_{\zeta_1}^\infty \frac{e^{-F}(\zeta_1-F)^J}{p(\zeta_1-F)} dF\right) d\zeta_1, \qquad (27a)$$

where
$$I(\zeta_1) = \int_0^\infty \frac{e^{-F}}{p(\zeta_1-F)} dF \; . \qquad (27b)$$

The only term which gives a logarithmic behavior for $\phi'(\zeta)$ as $\zeta \to \infty$ is the first integral on the right hand side of (27). The expansion of the first integral for large ζ is given by

$$\int_0^\zeta \frac{(1-p'/p)pI}{\zeta_1} d\zeta_1 = [(1-p'/p)pI] \ln \zeta - \int_0^\zeta \frac{d}{d\zeta_1}\{1-p'/p)pI\} \ln \zeta \; d\zeta \qquad (28)$$

Since p' tends to zero for large ζ , p tends to constant for large ζ , it can easily be seen from (27b) that pI tends to unity for large ζ. Therefore, the first term on the right side of (28) is $\log \zeta$. The other two integrals in (27) contribute to the constant term in the expansion of $\phi'(\zeta)$ for large ζ. Thus, the expansion for $\phi'(\zeta)$ for large ζ is of the form

$$\phi'(\zeta) = \frac{1}{k} \ln \zeta + \text{constant} + O(\zeta^{-1}) \qquad (29)$$

It is shown here that the solution of equation (14b) has the required logarithmic behavior which matches with the boundary condition in the overlapping region, irrespective of the detailed behavior of p. The velocity profile thus satisfies both the conditions, namely zero gradient at $\zeta = 0$ and the logarithmic variation for large ζ.

Equations (24) and (19) and (15) give the complete development of the velocity distribution. The centerline velocity variation, u_{cL} as given by (15) and (19) show that for large x^+, u_{cL}^+ can be expanded in an asymptotic form (only the dominant terms are given here) as

$$u_{cL}^+ = \frac{1}{k} \ln x^+ + C_4 \tag{30}$$

where C_4 is a constant (which depends on C_3 and $\phi_0'(0)$).

This logarithmic behavior for large x^+ could be obtained purely from dimensional arguments in exactly the same way as the logarithmic behaviour in y^+ for a turbulent boundary layer is obtained (see Millikan, 1938). It is a consequence of the fact that in the near wake u_{cL}^+ is a function of x^+ and away from the near wake u_{cL}/U_∞ is a function of x/L. If there exists an overlapping region where these two functional dependancies match, then by the same arguments used by Millikan, it can be shown that u_{cL}^+ varies as given in equation (30) with the constants of the log law undetermined. The logarithmic variation of the centerline velocity is therefore a requirement of the constraints imposed by the two different similarity coordinates which individually govern the near wake and the wake beyond it. The fact that the dynamics reflect this constraint requirement suggests that the wake development in the streamwise direction also can possibly be described by a two-layer hypothesis.

5. Turbulent Outer Near Wake

Equations (2) that govern the flow in this region are in fact the small-defect wake equations (Townsend, 1956). Denli and Landweber (1979) studied these equations in connection with the development of the defect layer in a thick axisymmetric turbulent boundary layer. The particular cases which they disregarded in their analysis as not relevant to thick axisymmetric boundary layers seem to be the most relevant for the present case. The analysis carried out here follows the lines of Denli and Landweber (1979). Because of the similarities in equations (2a) and (2b) the general features of the analysis and conclusions on the development of u_d and w_d are similar. Hence, only equation (2a) will be treated in the subsequent analysis. In terms of the nondimensional quantities given by

$$\bar{u}_d = u_d/U_\infty,\ \bar{x} = x/L,\ \bar{y} = y/\delta_1,\ \bar{\tau} = \tau/\rho u_\tau^2, \tag{31a}$$

(where L and δ_1 are the length scales for the initial wake in the x and y directions) the mean flow equation (2a) reduces to

$$\frac{U_\infty^2}{u_\tau^2}\frac{\delta_1}{L}\frac{\partial \bar{u}_d}{\partial \bar{x}} = -\frac{1}{\bar{y}^J}\frac{\partial(\bar{y}^J\bar{\tau})}{\partial \bar{y}}. \tag{31b}$$

Using the transformation $\xi^2 = \bar{x}$, equation (31b) can be written as

$$\frac{\beta^2}{\xi}\frac{\partial \bar{u}_d}{\partial \xi} = -\frac{1}{\bar{y}^J}\frac{\partial}{\partial \bar{y}}(\bar{y}^J\bar{\tau}), \tag{32}$$

where $\beta^2 = (\frac{1}{2}\frac{U_\infty^2}{U_\tau^2}\frac{\delta}{L})$.

If now u_d is assumed to be given by a similarity solution of the form

$$u_d = q(\xi)\,\psi(\eta_1),$$

where $\eta_1 = \overline{y}/\delta$, the equation (32) can be transformed to

$$\frac{\beta^2 \delta' q}{\xi^2} \left[\frac{q'\delta}{q\delta'} \psi - \Psi'\right) = - \frac{1}{\eta_1^J} \frac{\partial}{\partial \eta_1} (\eta_1^J \overline{\tau}) . \tag{34}$$

Integrating (34) once with respect to η_1 from η_1 to ∞ and using the boundary conditions $U_d(\infty) = \overline{\tau}(\infty) = 0$, yields

$$\frac{\beta^2 \delta' q}{\xi} [\eta_1^{J+1} \psi + \{(J+1) + \frac{\delta q'}{\delta' q}\} \int_{\eta_1}^{\infty} \eta_1^J \psi d\eta_1] = \eta_1^J \overline{\tau} . \tag{35}$$

If now an eddy viscosity is assumed to obtain the turbulent shear stress, $\overline{\tau}$ can be written

$$\overline{\tau} = 2 \frac{\nu_T \beta^2}{R} \frac{1}{8} q \psi', \tag{36}$$

where R is the Reynolds number based on the freestream velocity U_∞ and length L. Substituting (36) into (35) and rearranging gives

$$- \frac{1}{\eta_1^J \psi'} [\eta_1^{J+1} \psi + \{J+1 + \frac{\delta q'}{\delta' q}\} \int_{\eta_1}^{\infty} \eta_1^J \psi \, d\eta_1] = 2 \frac{\nu_T}{R} \frac{\xi}{\delta\delta'}. \tag{37}$$

If ψ is to be a function of η_1 only then it is required that

$$(J+1) + \frac{\delta q'}{\delta' q} = (J+1) C_5, \tag{38}$$

and

$$2 \frac{\nu_T}{R} \frac{\xi}{\delta\delta'} = C_6^2 \tag{39}$$

where C_5 and C_6 are constants. The right-hand side of (38) is chosen to be $(J+1)\ C_5$ for convenience in handling both two-dimensional and axisymmetric flows in a similar way. If the eddy viscosity is assumed to be given by the relation

$$\nu_T/R = D\xi^{m-1} \tag{40}$$

a solution of equation (38) and (39) can be written down as

$$\delta^2 \sim A_4\xi^{(m+1)}, \qquad q^2 \simeq A_5\ \xi^{(m+1)(J+1)(C_5-1)} \tag{41}$$

for $m \neq -1$ and

$$\delta^2 \sim A_6 \ln \xi, \qquad q^2 \simeq A_7(\ln \xi)^{J+1(C_5-1)}, \tag{42}$$

for $m = -1$,

where A_4, A_5, A_6, A_7 are all constants.

Using the constraints (38) and (39) in (37) and differentiating (37) with respect to η_1 once leads to

$$C_6^2\ \eta_1^J\ \psi'' + (C_6^2 + \eta_1^2)\ \psi' + (J+1)(1-C_5)\eta_1\psi = 0. \tag{43}$$

The above equation can be transformed to the well known confluent hypergeometric differential equation by changing the variables to

$$z = \left(\frac{\eta_1}{C_6}\right)^2/2, \qquad \psi_1(z) = e^{\tau}\psi. \tag{44a}$$

Then equation (43) reduces to

$$z\,\psi_1''(z) + (1-z)\,\psi_1'(z) - b_1\psi_1 = 0 \qquad (44b)$$

where

$$b_1 = [(J-1) - (J+1)C_5]/2\ . \qquad (44c)$$

A general solution for ψ_1 can be written down (see Abromowitz and Stegun, 1972) in terms of Kummer's functions as

$$\psi = k_1 M(1_1,b,z) + k_2\,U(b_1,1,z) \qquad (45)$$

where k_1 and k_2 are arbitrary constants, M is the Kummers function and U is related to Kummers function. M and U are given by

$$M\,(b_1,1\ z) = \sum_0^\infty (b_1)_n \frac{z^n}{n!}, \qquad (46a)$$

and

$$U(b_1 1,z) = \frac{1}{\Gamma(b_1)}\{M(b_1,1,z)\ \ln z + \sum_0^\infty \frac{(b_1)_n z^n}{(n!)^2}[\psi_2(b+n) + \psi_2(1+n)]\}, \qquad (46b)$$

where $\psi_2(b_1+n)$ is known as the Digamma function, a logarithmic derivative of the Gamma function $\Gamma\ (b_1+n)$, given by

$$\psi_2(b_1+n) = \frac{1}{n-1+b_1} + \frac{1}{n-2+b_1} + \ldots.\ \frac{1}{1+b_1} + \frac{1}{b_1} + \psi_2(b_1). \qquad (47a)$$

Here

$$(b_1)_n = b_1(b_1+1)\ldots.(b_1+n-1)\ , \qquad (47b)$$

and

$$\psi_2(b_1) = \frac{d}{db_1}\{\ln \Gamma(b_1)\}. \qquad (47c)$$

For large z the behavior of M and U are given by

$$M(b_1,1,z) = \frac{1}{\Gamma(b_1)} e^z z^{b_1-1}[1 + O(z^{-1})] \qquad (48)$$

and $$U(b_1,1,z) = z^{-b_1}[1 + O(z^{-1})] . \qquad (49)$$

The above trend in M for large z makes the velocity profile ψ(see (44a)) decay algebraically at large η_1 for $b_1 < 1$ and blow up for $b_1 > 1$. Since it is required that the solutions decay exponentially to the freestream values at the edge of the shear layer k_1 in equation (45) must be set equal to zero. Because of the algebraic behavior of U for large z, ψ tends exponentially to zero at large z. The solution for $\psi(\eta_1)$ can be written down as

$$\psi(\eta_1) = k_2 e^{-z}\, U(b_1,1,z),\; z = (\eta_1^2/2C_6^2) . \qquad (50)$$

The behavior of this solution for small η_1 is given by

$$\frac{\bar{u}_d}{q(\xi)} = \psi(\eta_1) = k_1[-\frac{2}{\Gamma(b_1)}\{\ln \eta_1 + C_7 + O(\eta_1^2 \ln \eta_1)\}]. \qquad (51)$$

Here C_7 is another constant. The above solution has the required logarithmic behavior for small η_1 to match with inner wake solutions for large ζ. Since for small η_1 the inner wake solution gives the behavior for u as $u/u_\tau = \frac{1}{k} \ln \eta_1 + C_8$, $q(\xi)$ must be proportional to u_τ and hence must be a constant given by $2k_1 q = (u_\tau/U_\infty)/k$. The value of C_5 therefore turns out to be 1 from equations (41) and (42). Hence b_1 will be equal to 1 for both two-dimensional and axisymmetric flow as can be seen from (44b). The value of C_6 can be arbitrarily set to unity without loss of generality because it only serves to determine the definition of δ.

6. Discussion

No attempt is made here to compute the detailed velocity profiles either in the inner wake or in the outer wake. Comparison of the analysis of Alber with the two-dimensional experimental data has been made by Alber (1980) and Ramaprian et al. (1981). These comparisons show that, for the two-dimensional wake flow, the limiting variations of the velocity profile like the log law at the outer edge of the inner wake and logarithmic variation of the centerline velocity agree reasonably well (see figures 3 and 4) inspite of the fact that Alber's analysis did not give a zero velocity gradient at the centerline. The present analysis has an additional option of choosing $p(\zeta)$, the function which determines the departure of the eddy viscosity from its linear variation with ζ at the centerline. By properly choosing p it is possible to obtain the correct variations of the velocity profiles near the centerline in addition to the correct limiting variation for large ζ and large x^+. Since well-established experimental data are not available for wakes behind an axisymmetric body and a yawed plate with zero pressure gradient fully-developed turbulent boundary layers, no comparison can be made at present, even with respect to the limiting variations of the velocity profiles and their scales.

Since no predictions have been made earlier for the outer wake development it will be interesting to examine the two-dimensional wake data in the light of the analysis made here. A cursory examination of the data of

Sastry (1) in the wake of a flat plate show that the total wake width varies very slowly in the near wake region. Equation (41) therefore suggests that (m+1) should be very nearly zero. If (m+1) is zero then the variation of δ is given by equation (42) which shows that δ^2 is proportional to $\ln \xi$ or $\ln x^+$. Figure 5 shows the variation of the total width of the wake with x^+ The wake width is defined by the condition $u = 0.995\ U_\infty$. Figure 5 shows clearly the logarithmic variation as predicted by the present analysis (equation (42)). Since (m+1) is very near zero it would mean that the eddy viscosity in the outer near wake region is constant even with x and is determined by the outer layer of the upstream boundary layer. Departures from the logarithmic variation occur when the maximum defect velocity has reduced appreciably (to around 20% of the freestream velocity) and probably there the limit considered in the present analysis is no longer valid.

The agreement of the predictions with the experimental data with respect to the variations of the different scales and the velocity profile in the overlapping region, suggests that there is only one limiting condition that leads to the description of the near wake flow. This limiting condition, $u_\tau/U_\infty \to 0$ as $\frac{u_\tau \delta}{\nu} \to \infty$ is the same as that used for an asymptotic description of the fully developed turbulent boundary layer. Since the maximum defect velocity in the wake becomes small at the end of the near wake, it is possible to describe the region beyond the near wake by the linearized wake equations which are the same as the outer near wake equations. The initial condition for the solution of the linearized wake equation could be obtained from the asymptotic near wake solutions for large x^+. Before the linearized wake develops into a self preserving asymptotic far wake (Narasimha and Prabhu, 1972), a relaxation in the turbulence closure hypothesis from the boundary-layer type to the far-wake type would be required. The differences observed in the development of the scales of the velocity profile in this transition region (Ramaprian et al. 1981) as compared to the far wake are essentially due to this relaxation in the shear stress.

Conclusions

Analysis of turbulent near wakes developing from zero pressure-gradient fully-developed turbulent boundary layer at two-dimensional, axisymmetric and infinitely-swept trailing edges have been considered. It is shown that the same limiting condition ($u_\tau/U_\infty \to 0$) as used for an asymptotic treatment of the under-determined equations (with no assumption on the closure hypothesis) of a fully-developed turbulent boundary layer for large Reynolds numbers in zero pressure gradients, is also applicable to the analysis of the near wake. It is argued that such an analysis leads to the governing equations for different regions of the near wake and the limiting logarithmic velocity variations in both the streamwise and normal directions in the overlapping regions. It is shown that similarity solutions can be obtained for both the inner and outer regions which tend to logarithmic velocity variations in the overlapping region. Comparison with recent experiments in two-dimensional wakes behind a flat plate shows that the most relevant solutions for the outer wake corresponds to that in which the square of the total wake width varies slowly like log x in the near wake. Finally, it is argued that the flow in the region beyond the near wake can be obtained by using linearized far-wake equations with the initial conditions for the velocity and shear stress obtained from the near-wake solution.

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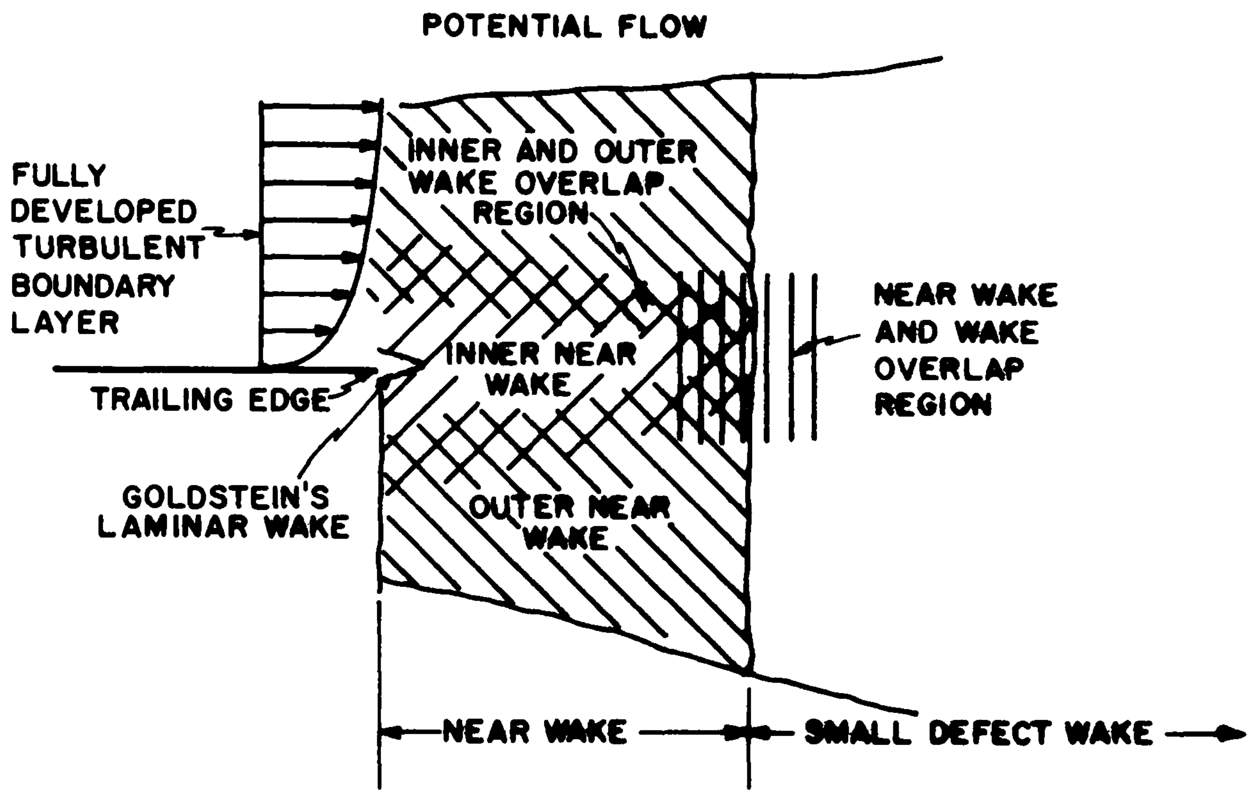
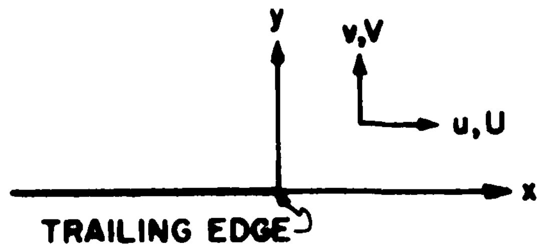
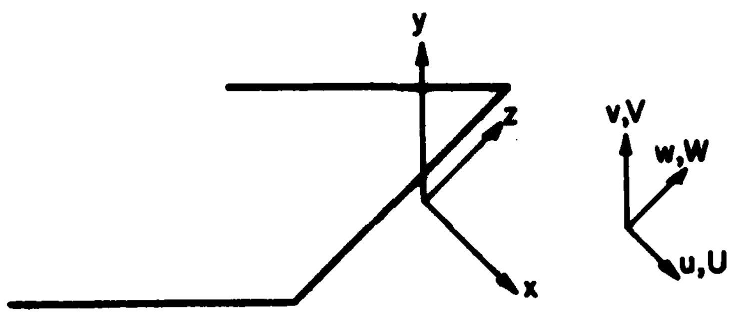


FIGURE 1. FLOW REGIONS IN THE NEAR WAKE.



(a) TWO DIMENSIONAL AND AXISYMMETRIC.



(b) YAWED TRAILING EDGE.

FIGURE 2. COORDINATES AND VELOCITY COMPONENTS.

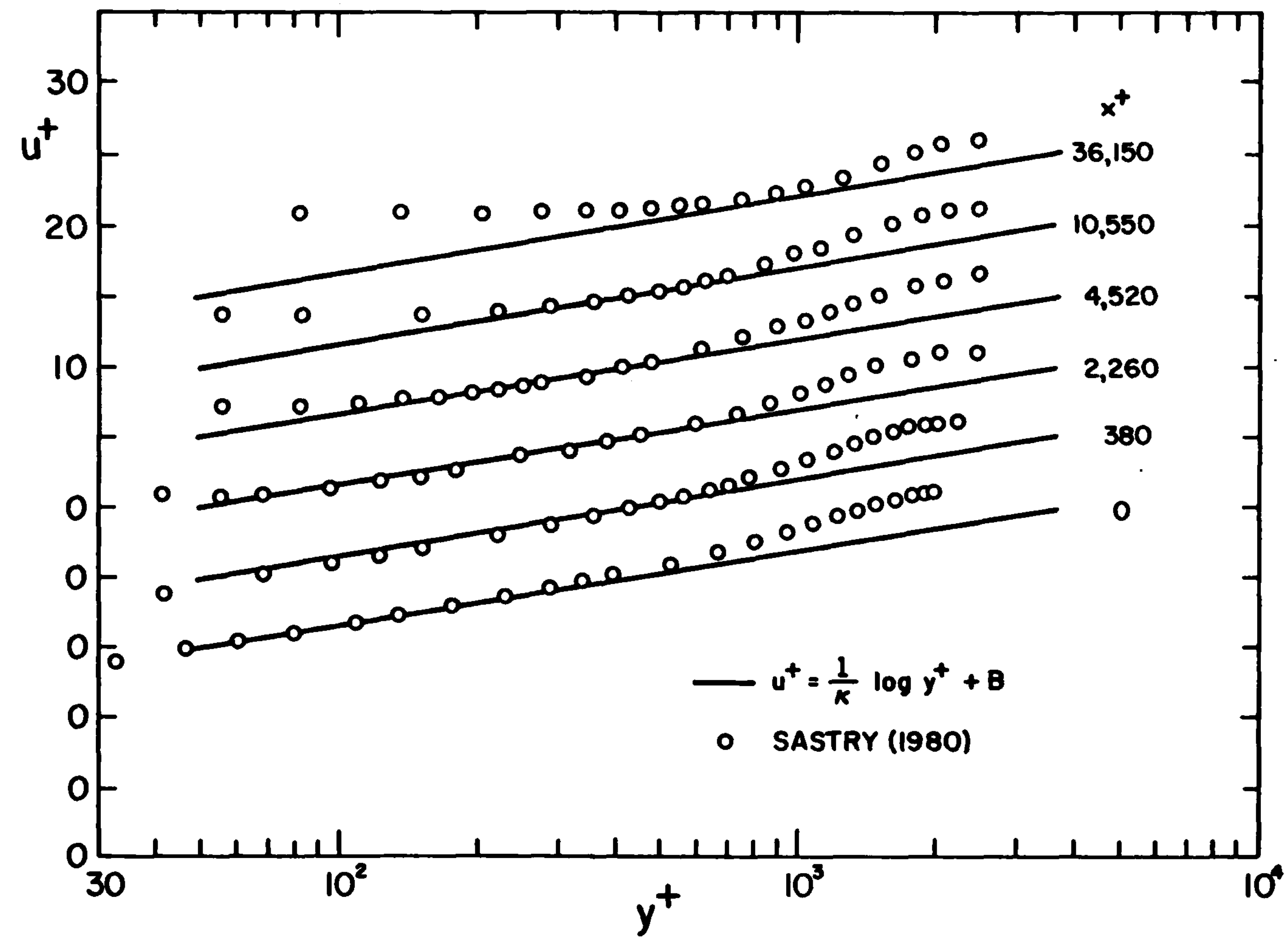


FIGURE 3. VELOCITY PROFILES IN INNER-LAYER COORDINATES.

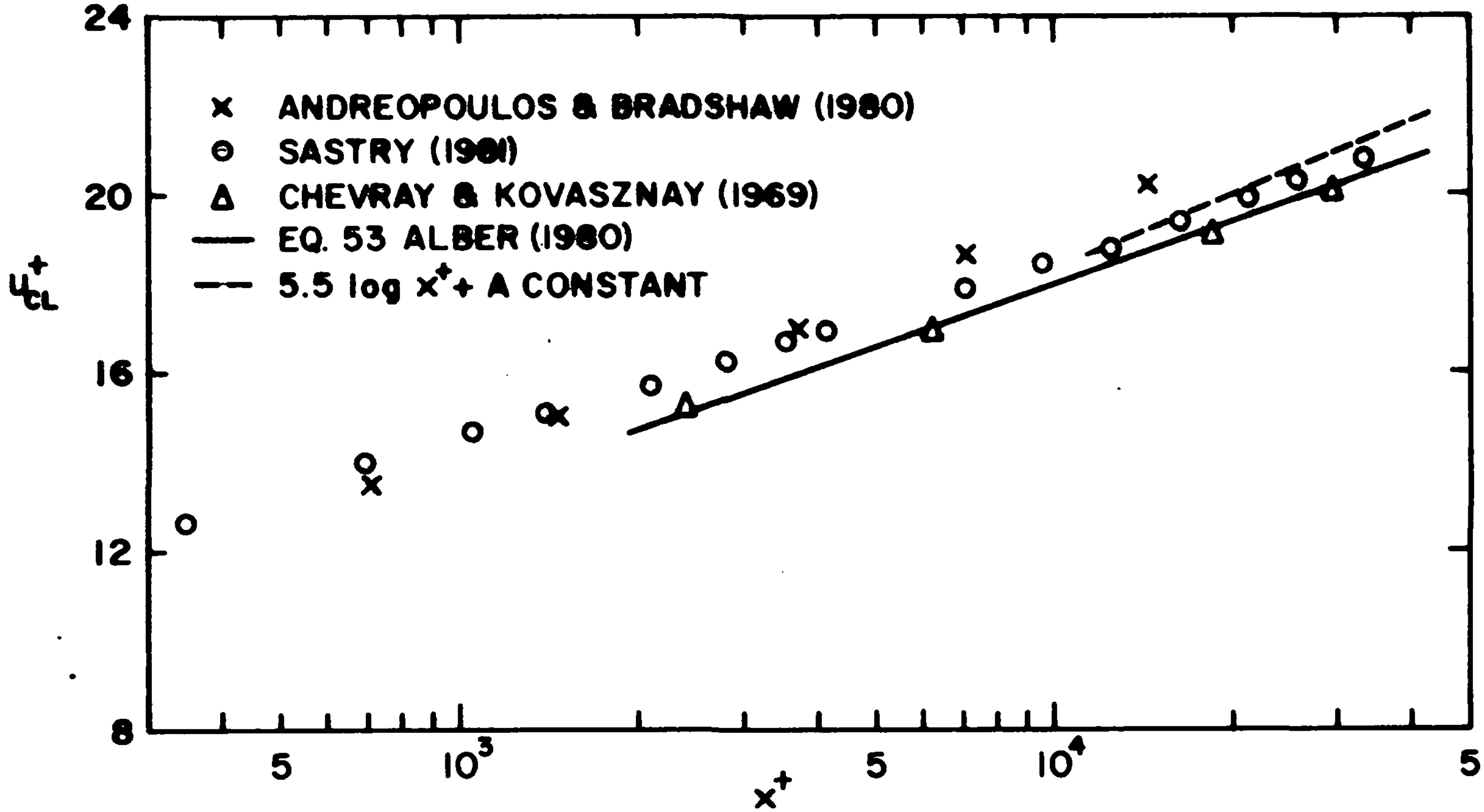


FIGURE 4. DEVELOPMENT OF CENTERLINE VELOCITY IN THE NEAR WAKE.

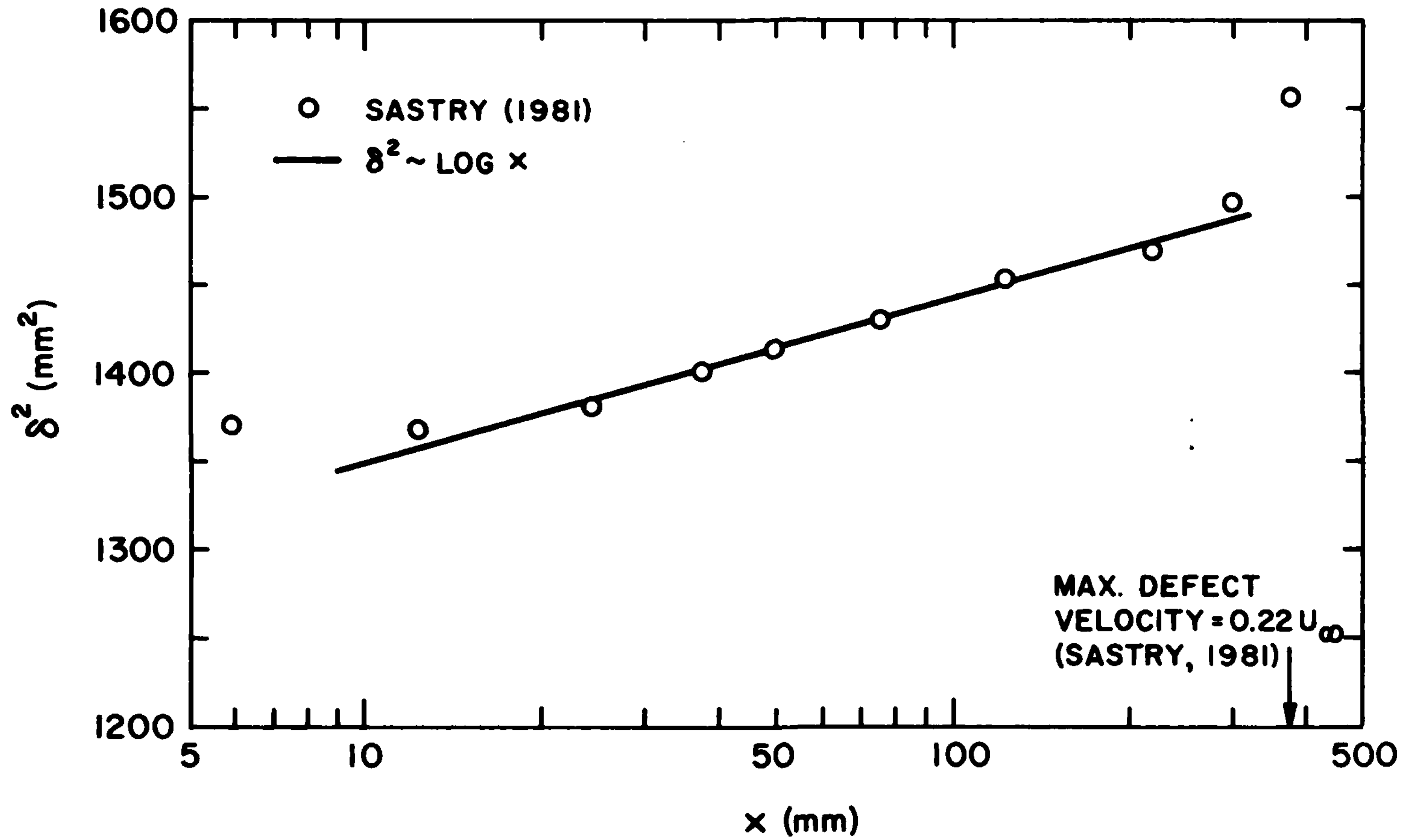


FIGURE 5. WAKE WIDTH DEVELOPMENT IN THE NEAR WAKE.

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