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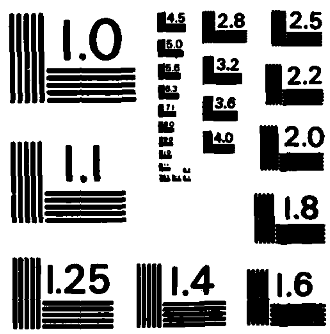
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DEPARTMENT OF STATISTICS

Carnegie-Mellon University
PITTSBURGH, PENNSYLVANIA 15213

STATISTICAL ANALYSIS OF MULTIPLE SOCIOMETRIC RELATIONS

by

Stephen E. Fienberg

Michael M. Meyer*

Stanley S. Wasserman**

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Department of Statistics

Carnegie-Mellon University

Pittsburgh, PA 15213

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*Department of Statistics, University of Wisconsin-Madison

**Departments of Psychology and Mathematics, University of Illinois at Urbana-Champaign

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ABSTRACT

Loglinear models are adapted for the analysis of multivariate social networks, a set of sociometric relations among a group of actors. Models that focus on the similarities and differences between the relations and models that concentrate on individual actors are discussed. This approach allows for the partitioning of the actors into blocks or subgroups. Some ideas for combining these models are described, and the various models and computational methods are applied to the analysis of data for a corporate interlock network of the 25 largest organizations in Minneapolis/St. Paul and for a classic network of eighteen monks in a cloister.

Key Words: Loglinear model; Directed graph; Social network; Sociometric data; Iterative proportional fitting; GLIM model.

1. INTRODUCTION

Sociometric relations are typically defined for a set of social actors. A social network is a construct describing these actors and the various relations that exist among them. As used in the social sciences, actors have been individuals in groups, organizations, cities, or even nation states; relations have ranged from kinship to friendship to transfers of scarce resources to corporate board of directors interlocks.

Moreno (1934) was the first social scientist to study individual networks in a systematic manner, and was apparently the first network researcher to use mathematics. Much of his terminology, including such phrases as "sociogram," sociomatrix," and "sociometric test," is still in use today. Festinger (1949) and Katz (1947, 1953, 1955) developed Moreno's ideas, focussing on matrix representations of sociometric data, the popularity of actors, mutuality of relationships in social groups, and even the representation of interpersonal relations as a stochastic process (see Katz and Proctor, 1959). Formal graph theory, as reviewed in this paper in Section 2, was introduced to social network research by Cartwright and Harary (1958), in an attempt to quantify the social psychological theories of Heider (1958).

Since these pioneering efforts, sociologists, social psychologists, and social anthropologists have repeatedly used the social network paradigm. Davis and Leinhardt (see Davis, 1970) scanned the "sociometry" literature and found nearly 900 examples of social networks from diverse small groups. Since 1970, social network analysis has grown rapidly in popularity. Leinhardt (1977) presents a collection of twenty-four previously published papers which provide an historical perpsective on social network analysis, and a collection of papers in a volume edited by Holland and Leinhardt (1979) summarizes the state-of-the-art as of about 1975. Burt (1980) discusses more recent sociological developments, Wasserman (1978) reviews alternative mathematical models for small group behavior, and Frank (1981) summarizes some of the statistical theory on random graphs. Almost none of this research on the analysis of social networks has appeared in statistical journals, with the exception of some of the work by Katz and Wasserman (1980). There are just a few papers with substantial statistical content.

In a landmark statistical paper for network analysis, Holland and Leinhardt (1981) proposed an exponential family of probability distributions for the analysis of a single sociometric relation. Fienberg and Wasserman (1981a) discussed simple computational procedures for fitting these models, and proposed some extensions to model networks in which the actors fall into natural subgroups. These distributions include parameters that relate characteristics of individual actors (e.g., popularity) to differential rates for entering into or severing sociometric relations. In Fienberg, Meyer, and Wasserman (1981), we described a related class of models for multiple relations, extending Holland and Leinhardt's family to more than one relation by focussing on the associations among the relations rather than on influences of individual actors. Here we bring these two types of analyses together, and present some "combined models" for the analysis of multivariate directed graphs. These models incorporate actor and subgroup parameters, and quantities to measure the degree of interrelatedness of the different relations.

Methods to study a multivariate directed graph which focus solely on the relations and ignore individual social actors are forms of *macroanalysis*. Data for such analyses consist of aggregate counts of the different structural patterns which occur within the network. The methods for studying local structure in a network by using the *triad census* (Holland and Leinhardt, 1975; Wasserman, 1977) can be labelled macroanalytic. Alternatively, we could study the attributes of the actors, and how these attributes affect the existing ties between them. Such a study is a *microanalysis*, and promises a more fine-grained investigation.

Both the macroanalysis and microanalysis approaches have substantive value. A macroanalysis of a group centers on the global structure of its relations, asking questions such as: Which relation exhibits the strongest "reciprocity," or is most likely to have symmetric flows? Are there any "multiplex" patterns, flows of different relations in the same direction? Are there any patterns of "exchange," in which a flow in one direction for one relation is reciprocated by a flow in the opposite direction for a different relation? Are there any higher order interactions, involving three or more flows for two or more relations?

A microanalysis of a group is a local study, turning attention to the level at which data are actually gathered. Most microanalyses have been limited to groups with data on just a single relation. The primary concern of such studies is the individual group member: Which actors have the most prestige or popularity? Which actors are involved in many relations, which in few? Do actors enter into mutual, symmetric relationships at different rates? Such questions, while concerned with individual actor effects, are often answered by examining dyadic or triadic relationships.

As an example, we consider the now classic study of 18 monks in an isolated American monastery, conducted by Sampson (1969) and partially analyzed by Holland and Leinhardt (1981), Breiger (1981), and many others. Sampson studies four types of relations: Affect, Esteem, Influence, and Sanction. Actors were asked to give three positive choices — e.g., which three brothers do you like best (positive affect) — and three negative choices — e.g., which three brothers are you most antagonistic towards (negative affect) — for each of the four types. In this way, data were gathered on eight relations: (1) Like and (2) Antagonism (Affect), (3) Esteem and (4) Disesteem, (5) Influence and (6) Negative Influence, and (7) Praise and (8) Blame (Sanction).

We define

$$x_{ijr} = \begin{cases} 1, & \text{if actor } i \text{ chooses actor } j \text{ on relation } r \\ 0, & \text{otherwise} \end{cases}$$

$$i,j = 1,2,\ldots,18; \; r = 1,2,\ldots,8$$

and arrange these data into 8 binary sociomatrices, $\mathcal{X} = \{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \ldots, \underset{\sim}{x}_8\}$, each of dimensions 18 × 18. Versions of these arrays are given in Table 1, where the rows and columns have been permuted to reflect constructed subgroupings of the actors. Since the 1's, 2's and 3's in Table 1 refer to order of choices, we set all non-zero entries equal to 1 to obtain binary sociomatrices.

TABLE 1
Sampson's (1969) Data

LIKE

10	11 3		2
5	1 32		
9	1 3	2	
6	12 3		
4	3 1 2		
11	2 3	1	
8	123		
12		32 1	
1		1 2	3
2		23 1	
14		13 2	
15		2 3 1	
7		1 3 2	
16		3 12	
13	2	1	3
3		3	2 12
17		1	2 3
18		1	23

ESTEEM

10			
5	1 32		
9	3 2	1	
6	32		
4	2 1 3		
11	23		
8	1 23		
12		321	
1	1 3	2	
2	1 2	3	
14		132 2	
15		132	
7		1 3 1 2	
16		1 3 22	
13	3 1	2	
3		3	2 11
17		1	12 3
18		2	31

INFLUENCE

10			
5	32	1	
9	1	32	
6	2 3	1	
4	2 1 3		
11	32 1		
8	1 32	2	
12		32 1	
1		2 3 1	
2		23 1	
14		132 2	
15		32 1	
7		1 3 2	
16		3 12	
13	3 1	2	
3		3	2 1
17		1	2 3
18		3	21

PRAISE

10			
5			
9			
6	1 32		
4	2 1 3		
11	3 2	1	
8	231		
12			
1		2 3	1
2		23 1	
14		33 2 1	
15		132	
7		3 1 2	
16		2 13	
13	3 2		
3		3	2 1
17			
18		11	1

ANTAGONISM

10			
5			
9			321
6		1 2	32
4		231	
11		13	2
8		31	2
12			
1	1	2 3	
2	3 2		1
14	3 2		1
15	1		32
7	23 1		
16	3		12
13	2	1 3	
3	23	2 1	
17	31 2		
18	2 1 2		

DISESTEEM

10			
5			132
9			1 32
6	1	3	22
4		231	
11		13 1	2
8		1	23
12			
1		2 3	1
2	3		12
14	22		3 11
15	31		211
7	32 2		21
16	3		221
13	3	2	1
3	23	1	
17	32 1		
18	22 3		

NEGINFL

10			
5			132
9		1	32
6		3	21
4		231	
11		3	12
8		1 3	2
12			
1	2	1	3
2	3		2
14	2		3 11
15	3		21
7	32 2		21
16	32		1
13	2	1 3	
3	23	1	
17	32 1		
18	132		

BLAME

10			
5			
9			
6		3	1 21
4		32 1	
11		1	23
8			123
12		2	13
1	2		3 1
2			3
14	32		1
15	1		32 1
7	12		3
16	3		1222
13		1	23
3	3	21	
17			
18			

Holland and Leinhardt (1981) used the "Like" relation to illustrate their new methods. Other researchers (White, Boorman, and Breiger, 1976; Brieger, Boorman, and Arabie, 1975) have studied all the relations, but in a non-statistical attempt to aggregate the 18 monks in a substantively meaningful manner. In later sections of this paper, we analyze a version of the network which aggregates over positive and negative affects, searching for both macro- and micro-models that provide good statistical descriptions of the relationships among the actors.

Most sociometric research, both empirical and mathematical, is preoccupied with overly simplistic descriptions of group structure. This is very apparent in Burt's (1980) review. The goal of this paper is to build upon the ideas of Holland and Leinhardt to develop models for the simultaneous macro- and micro-analysis of multiple relational networks. These models aid in the formulation and testing of theories concerning group dynamics. In the next section we review Holland and Leinhardt's model and our extensions of it, and then in Section 3 we illustrate these ideas in an analysis of a 1976 corporate interlock network from the Twin Cities (Minneapolis/St. Paul). We emphasize the many substantive findings that can be obtained from this form of statistical modelling. In Section 4, we present several models for the analysis of data from multivariate directed graphs, and conclude by demonstrating these ideas on Sampson's network.

2. BACKGROUND: MODELS FOR SINGLE RELATIONAL DATA

A directed graph, or digraph, consists of a set of g nodes, and sets of directed arcs or "choices" connecting pairs of nodes. Digraphs are natural mathematical representations of social networks, where the nodes represent individuals, organizations or other social actors, and the arcs represent relations: directed attitudes, feelings, or transfers, such as friendship. A digraph is frequently summarized by $g \times g$ sociomatrices, $\underset{\sim}{X}_r$, one for each of the R defined relations. The g diagonal terms of each sociomatrix, X_{iir}, are defined to be zero.

First consider a digraph with a single relation, $R = 1$. The row total, X_{i+}, is referred to as the out-degree of node i, and the corresponding column total, X_{+i}, as the in-degree of node i. A matrix $\underset{\sim}{x}$ can be thought of as the realization of a matrix of random variables, $\underset{\sim}{X}$, where we assume that the $\binom{g}{2}$ pairs or dyads,

$$D_{ij} = (X_{ij}, X_{ji}),\ i < j,$$

are independent bivariate random variables, with $2^2 = 4$ possible realizations, only 3 of which are distinguishable:

$$D_{ij} = \begin{cases} (1,1) : \text{mutual} \\ (1,0) \text{ or } (0,1) : \text{asymmetric} \\ (0,0) : \text{null.} \end{cases}$$

A multivariate directed graph, or multigraph, is described by a collection of random sociomatrices $X = \{\underset{\sim}{X}_1, \underset{\sim}{X}_2, \ldots, \underset{\sim}{X}_R\}$, and we assume that the $\binom{g}{2}$ dyads,

$$\underset{\sim}{D}_{ij} = \begin{pmatrix} X_{ij1}, X_{ji1} \\ X_{ij2}, X_{ji2} \\ \cdot \\ \cdot \\ \cdot \\ X_{ijR}, X_{jiR} \end{pmatrix}, \quad i < j,$$

are independent 2R-variate random variables with 2^{2R} possible realizations. For both digraphs and multigraphs, the assumption that the dyads are independent random variables is a crucial one, and is not subject to examination by the framework developed in this paper.

Holland and Leinhardt (1981) introduced a class of models, labelled p_1, to model micro-behavior in a social group, on which only one relation has been defined. We now describe these models, and explain how their analysis can be accomplished by using standard computational approaches to the analysis of loglinear models for categorical data. We then outline some extensions of these models that allow for grouping of individual actors. Further details can be found in Fienberg and Wasserman (1981a). In Section 4 we extend this approach to the analysis of multigraph data.

Consider a network of g nodes and a single relation, and represent the sociomatrix $\underset{\sim}{X}$ as a four-dimensional $g \times g \times 2 \times 2$ cross-classification $\underset{\sim}{Y} = (Y_{ijk\ell})$, where the subscripts i and j refer to the two actors in a dyad, and k and ℓ refer to the dyad state:

$$Y_{ijk\ell} = \begin{cases} 1, \text{ if } D_{ij} = (X_{ij},X_{ji}) = (k,\ell) \\ 0, \text{ otherwise.} \end{cases} \tag{2.1}$$

For example, $Y_{ij11} = 1$ if D_{ij} is a mutual dyad. Note that the 2×2 tables $\underset{\sim}{Y}_{ij} (i \neq j)$ contain one 1 and three 0's. Furthermore, $Y_{ijk\ell} = Y_{ji\ell k}$, and the marginal totals of these 2×2 tables correspond to indicator variables for X_{ij} and X_{ji}. Because each margin is either (0,1) or (1,0), the interior of the table is completely determined by its marginal totals.

We denote a realization of $\underset{\sim}{Y}$ by $\underset{\sim}{y} = (y_{ijk\ell})$, and let $\pi_{ijk\ell}$ be the probability of the observation (k,ℓ) for the dyad (i,j), where

$$\sum_{k,\ell} \pi_{ijk\ell} = 1, \tag{2.2}$$

and we define $\mu_{ijk\ell} = \log \pi_{ijk\ell}$. The Holland-Leinhardt p_1 class of models is as follows:

$$\begin{aligned}
\mu_{ij00} &= \lambda_{ij}, \\
\mu_{ij10} &= \lambda_{ij} + \alpha_i + \beta_j + \theta, \\
\mu_{ij01} &= \lambda_{ij} + \alpha_j + \beta_i + \theta, \\
\mu_{ij11} &= \lambda_{ij} + \alpha_i + \alpha_j + \beta_i + \beta_j + 2\theta + \rho_{ij},
\end{aligned} \tag{2.3}$$

where

$$\sum_{i=1}^{g} \alpha_i = \sum_{j=1}^{g} \beta_j = 0, \tag{2.4}$$

and $\rho_{ij} \equiv \rho$. The sufficient statistics for the parameters of p_1 are easily expressed as margins of $\underset{\sim}{y}$:

$$\begin{aligned}
\tfrac{1}{2} y_{++11} &= M, && \text{Number of mutuals,} \\
y_{i+1+} &= x_{i+}, && \text{Out-degree of node i,} \\
y_{+j1+} &= x_{+j}, && \text{In-degree of node j,} \\
y_{++1+} &= x_{++}, && \text{Total number of choices.}
\end{aligned} \tag{2.5}$$

Through the use of the full $\underset{\sim}{y}$ array, and its redundancies, one can show that fitting p_1 to the $\underset{\sim}{x}$ array is equivalent to fitting the "no three-factor" interaction loglinear model to $\underset{\sim}{y}$. A proof of this equivalence is given in Meyer (1981). Thus we can fit p_1 to data by using the standard iterative proportional fitting procedure (IPFP) applied to $\underset{\sim}{y}$. Furthermore, the special cases of p_1, listed in Table 1 of Holland and Leinhardt (1981), all have equivalent loglinear models for $\underset{\sim}{y}$, and thus can also be fit using the standard IPFP. The equivalent models are given in Table 2 of Fienberg and Wasserman (1981a).

An important generalization of p_1 starts with the equations (2.3) with constraints (2.4) and further postulates that

$$\rho_{ij} = \rho + \rho_i + \rho_j, \quad i < j \tag{2.6}$$

where the $\{\rho_i\}$ are normalized to sum to zero. The effect of reciprocity now depends additively on the individual actors in a dyad, and the $\{\rho_i\}$ measure the rates at which actors are likely to enter into mutual, symmetric relationships. This model provides an important goodness-of-fit test for p_1 (see Fienberg and Wasserman, 1981b) since it contains p_1 as a special case, when $\rho_1 = \rho_2 = \ldots = \rho_g = 0$.

We now describe a variant on p_1 for single relational sociometric data that assumes that the g actors have been partitioned into K subgroups. Of substantive interest is how likely it is that actors in one subgroup have relations with actors in other subgroups, and how structurally similar are actors in a given subgroup. We label the subgroups $G_1, G_2, \ldots, G_K$, where the partition of actors is mutually exclusive and exhaustive, and assume that subgroup G_k contains g_k actors, such that $g_1 + g_2 + \ldots + g_K = g$. For example, White, Boorman and Brieger (1976) (see also, Breiger, 1981) aggregate the 18 monks from Sampson's cloister into 3 "blocks" or subgroups, containing $g_1 = 7$, $g_2 = 7$, $g_3 = 4$ actors. This aggregation is reflected in Table 1, where the rows and columns of the $\underset{\sim}{X}$ matrices have been rearranged so that the first 7 rows and columns refer to actors in G_1, and so forth. Brieger, Boorman, and Arabie (1975) construct a slightly different partition. We note that these partitions, called "blockmodels," were accomplished by grouping together all actors that are "structurally equivalent," relating to the other actors in the group in identical fashion (see Lorrain and White, 1971).

We modify equations (2.3) by introducing inter- and intra-subgroup choice and reciprocity parameters:

$$\begin{aligned} \mu_{ij00} &= \lambda^{(rs)} \\ \mu_{ij10} &= \lambda^{(rs)} + \theta^{(rs)} \\ \mu_{ij01} &= \lambda^{(rs)} + \theta^{(sr)} \\ \mu_{ij11} &= \lambda^{(rs)} + \theta^{(rs)} + \theta^{(sr)} + \rho^{(rs)} \end{aligned} \qquad i \in G_r \text{ and } j \in G_s \qquad (2.7)$$

The parameters $\{\theta^{(rs)}\}$ are choice effects, and the $\{\rho^{(rs)}\}$, reciprocity effects. The parameters $\{\lambda^{(rs)}\}$ are included to insure that the $y_{ijk\ell}$ sum to 1 for each dyad. One special case of the subgroup model (2.7) sets $\rho^{(rs)} = 0$ for all r and s. Holland and Leinhardt (1981) note that, if we further define

$$\pi^{(rs)} = P\{X_{ij} = 1 \mid i \in G_r \text{ and } j \in G_s\}, \qquad (2.8)$$

then, in this special case,

$$\theta^{(rs)} = \log \frac{\pi^{(rs)}}{1 - \pi^{(rs)}} = \text{logit}\,(\pi^{(rs)}). \tag{2.9}$$

A second special case of (2.7) is also a special case of p_1 in which we have a simple additive model for $\theta^{(rs)}$. All actors in subgroup G_r have a common α, $\alpha^{(r)}$, and a common β, $\beta^{(r)}$. We set

$$\begin{aligned} \theta^{(rs)} &= \theta + \alpha^{(r)} + \beta^{(s)} \\ \rho^{(rs)} &= \rho. \end{aligned} \tag{2.10}$$

This model is equivalent to p_1 if K = g, and is a simplification, in the sense that we reduce the number of α's (and β's) from g-1 to K-1.

For details about these and other generalizations and specializations of p_1, and for comments on fitting these subgroup models to single relational data, see Fienberg and Wasserman (1981a). In Section 4 we give a multivariate generalization of this model.

3. ANALYSIS OF A SINGLE RELATION IN A CORPORATE NETWORK

To illustrate these models, and to present some additional methods, we consider 1976 data on a network of the twenty-five largest publicly-owned corporations headquartered in the Twin Cities of Minneapolis and St. Paul. A firm is included in the network if it is among *Fortune* magazine's 500 largest industrials, 50 largest commercial banks, 50 largest life insurance companies, 50 largest financial companies, 50 largest retailers, 50 largest transportation companies, and 50 largest utilities. These companies are listed in Table 2, along with their ranks and location.

A preliminary analysis and thorough discussion of this network is given by Galaskiewicz and Wasserman (1981). An arc (or a "corporate interlock") exists from firm i to firm j if an officer of firm j is on the corporate board of directors of firm i. An interesting feature of this network is the exclusion of dyadic interactions in which the two firms of the dyad have the same Standard Industrial Code. These "competitive" dyads have been excluded because of SEC anti-trust regulations that prevent interlocks between firms in the same industry. There are 27 of these "structurally zero" dyads.

A variety of models was fitted to two versions of this network. One version included all 25 firms, and the other included only 20 firms, excluding four firms that do not interact with the others (have zero in-degrees and out-degrees) — American Hoist and Derrick, IDS, Gamble-Skogmo, and North Central Airlines — and a firm, Land O'Lakes, which is a cooperative, and hence not strictly publicly owned. The calculation of degrees of freedom (df) is tricky because of the structural zeros and the zero in-degrees and out-degrees. In general we follow an approach similar to that suggested by Bishop, Fienberg, and Holland (1975, pp. 115–116). Below, we report likelihood ratio (G^2) statistics and degrees of freedom for just 2 models.

Manufacturers	Fortune Rank (1976)	City
Minnesota Mining & Manufacuring (3M)	56	St. Paul
Honeywell	67	Minneapolis
General Mills	84	Minneapolis
Control Data	170	Minneapolis
Pillsbury	173	Minneapolis
Land O'Lakes	180	Minneapolis
International Multifoods	233	Minneapolis
Bemis	318	Minneapolis
Peavy	361	Minneapolis
Heorner-Waldorf	382	St. Paul
American Hoist and Derrick	434	St. Paul
Economics Laboratory	500	St. Paul
Commerical Banks		
Northwest Bankcorporation	18	Minneapolis
First Bank System	20	Minneapolis
Life Insurance Companies		
Minnesota Mutual Life Insurance	41	St. Paul
Northwestern National Life Insurance	42	Minneapolis
Diversified Financial Companies		
St. Paul Companies	20	St. Paul
Investors Diversified Services (IDS)	28	Minneapolis
Retailing Companies		
Dayton Hudson	20	Minneapolis
Gamble-Skogmo	22	Minneapolis
Transportation Companies		
Burlington Northern R.R.	10	St. Paul
Northwest Orient Airlines	18	St. Paul
North Central Airlines	48	Minneapolis
Soo Line R.R.	49	Minneapolis
Utilities		
Northern States Power	28	Minneapolis

Table 2. Twin Cities Corporate Network

	g = 25		g = 20	
Model	G^2	*df*	G^2	*df*
$p_1(\theta,\rho,\{\alpha\},\{\beta\})$	186.69	192	182.89	176
$\theta(\rho = \alpha_i = \beta_j = 0)$	324.66	545	276.46	341

As can be seen, the very simple model with a single parameter provides an adequate description of both versions. This implies that the actors in neither version exhibit differential productivity or attractiveness, and that there is no tendency toward reciprocity. We conclude that the elements in $\underset{\sim}{X}$ are independent identically-distributed Bernoulli random variables with p = $P\{X_{ij} = 1\}$ and log odds ratio $\theta = \log(p/(1-p))$. Maximum likelihood estimates (MLEs) of θ are -2.49 (g = 25) and -2.01 (g = 20). This yields $\hat{\rho} = 0.0906$ (g = 25) and $\hat{\rho} = 0.1553$ (g = 20).

We now discuss some additional and new methods for the analysis of single relational data. We first describe tests for the adequacy of partitions of actors into subgroups, and then show how to estimate main effects for and interactions between the discrete variables used to partition the actors. These ideas, along with the methods described in Holland and Leinhardt (1981) and Fienberg and Wasserman (1981a), should provide a more complete "package" for single relational data. We intend the remainder of this section to fill the existing gaps in this methodology, and will use the 1976 Twin Cities corporate network simply for illustrative purposes.

Suppose we have two possible mutually exclusive and exhaustive partitions of a set of g actors, $\mathcal{G} = \{G_1, G_2, ..., G_K\}$ and $\mathcal{H} = \{H_1, H_2, ..., H_L\}$, such that $K < L$, and the G's are unions of the H's. For example, let g = 6, and define $G_1 = \{1,2,3\}$, $G_2 = \{4,5,6\}$, and $H_1 = \{1,2\}$, $H_2 = \{3\}$, and $H_3 = \{4,5,6\}$; then, $G_1 = H_1 \cup H_2$ and $G_2 = H_3$. Thus, $\mathcal{G}$ is an aggregation of $\mathcal{H}$.

We consider whether or not to further aggregate the actors into K subgroups, assuming that the actors are already partitioned into L subgroups; i.e., can we combine some of the L existing subgroups to form K larger ones? Note that if $L = g$, then we ask whether or not we should do any aggregation at all. We test

$$H_0 : p_1 \text{ applied to K subgroups is appropriate}$$

versus

$$H_A : p_1 \text{ applied to L subgroups is appropriate}$$

The version of p_1 applied to subgroups is given by equations (2.6) and (2.7). In terms of the model parameters, there are L-1 each of the $\alpha^{(r)}$ and $\beta^{(s)}$ effects under H_A and K-1 each under H_0. The α's and β's for the subgroups that are aggregated under H_0 are equated. Since H_0 is a special case of H_A, if we assume that the model under H_A is correct, then the conditional likelihood ratio statistic $G^2(H_0 | H_A) = G^2(H_0) - G^2(H_A)$, with $g(g-1) - 2K - [g(g-1) - 2L] = 2(L-K)$ degrees of freedom can be used to test H_0 versus H_A. If $L = g$, then the test statistic has $2(g - K)$ degrees of freedom.

For the 1976 Twin Cities corporate network, we focus on three partitions using the information in Table 2:

$$\mathcal{G}_1 = \{G_{11} = \text{Mpls. firms}, G_{21} = \text{St. Paul firms}\}$$

$$\mathcal{G}_2 = \{G_{12} = \text{Large firms}, G_{22} = \text{Small firms}\}$$

$$\mathcal{H} = \{H_1 = \text{Large Mpls. firms}, H_2 = \text{Large St. Paul firms},$$
$$H_3 = \text{Small Mpls. firms}, H_4 = \text{Small St. Paul firms}\}$$

"Size" of a firm is determined by the *Fortune* ratings: "large" firms rank among the larger 250 (or 25). The 25×25×2×2 y array, aggregated to a 4×4×2×2 array to reflect the $\mathcal{H}$ partition, is given as Table 3. Note that both $\mathcal{G}_1$ and $\mathcal{G}_2$ are aggregations of $\mathcal{H}$.

The following hierarchy lists the three aggregations and gives the associated likelihood and conditional likelihood ratio statistics for testing the significance of aggregations:

		H_1 LARGE		H_2		H_3 SMALL		H_4	
		Minneapolis		St. Paul		Minneapolis		St. Paul	
L	Minneapolis	54	7	31	0	53	5	35	2
A		7	8	4	3	4	1	1	0
R									
G	St. Paul	31	4	6	3	25	0	11	3
E		0	3	3	0	0	0	0	0
S	Minneapolis	53	4	25	0	38	2	25	0
M		5	1	0	0	2	0	0	0
A									
L	St. Paul	35	1	11	0	25	0	8	1
L		2	0	3	0	0	0	1	0

Table 3. 1976 Twin Cities Corporate Network Relations aggregated into 4 subgroups based on Location and Size

Aggregation	G^2	*df*	ΔG^2	Δdf
p_1 – no aggregation; 25 actors	186.69	192		
H – aggregation by size & location; L=4	401.80	538	215.11	346
G_1 – aggregation by location; K_1=2	461.62	542	59.82	4
G_2 – aggregation by size; K_2=2	525.63	542	124.01	4

Note that $G^2(H|p_1)$ = 401.80 − 181.54 = 220.26, is less than the corresponding difference in df, 346, so that one could argue that aggregating the 25 actors into 4 subgroups is not necessary. The statistic $G^2(H) \sim \chi^2_{538}$ is clearly small, however, and simplicity of the H aggregation is so desirable that it is a very attractive model. Both statistics $G^2(G_1|H)$ = 59.82 and $G^2(G_2|H)$ = 124.01 yield p-values less than 10^{-4}, so further aggregation is not advisable.

There is one substantial advantage in using aggregated versions of these models. Besides the ease with which the maximum likelihood cell estimates can be computed (we need only a K × K × 2 × 2 table, where K is usually quite a bit smaller than g), the standard χ^2 distributions are more appropriate as reference distributions for the resulting test statistics. This is because the number of parameters (2K with the p_1–subgroup model) is fixed and does not increase in the limit, as $g \to \infty$. There are problems that arise in testing when using models with parameters for each actor (see Haberman (1981)). Fortunately, these problems are attenuated when actors are aggregated.

In the following section we generalize this approach to the case of multiple relations.

4. MODELS FOR MULTIPLE RELATION DATA

We now turn our attention to networks of actors on which several relations are defined. We discuss three types of models: (1) Models with neither actor nor group parameters; (2) models with only group parameters; and (3) models with both actor and group parameters. The first type is a family of models for the macroanalysis of the multiple relations that ignores any differences between actors. These models are briefly described in Fienberg, Meyer and Wasserman (1981), and were used implicitly by Galaskiewicz and Marsden (1978) to study resource flows between organizations in a midwestern community.

The most useful models for multiple relations are those that include parameters to reflect different choice tendencies of the actors, particularly when they have been partitioned into groups. If each group is a singleton, then we have a different set of parameters for each actor; however, in practice this is likely to be a very large number. Thus, the assumption of a specific partition, chosen as a consequence of extra-relational information, allows us to parsimoniously limit the number of parameters, and (as is the case with single relational data) use standard X^2 asymptotic distributions for testing.

The last type of model is a generalization of the family of models for multiple relational data sets in which the actors have been partitioned into mutually exclusive and exhaustive groups. The assumption that all actors in a specific group relate to actors in other groups and to other actors in the same groups in identical ways may not always be the case. There may be subtle individual differences among the actors in a subgroup. Thus, the third type of models allows us to add individual actor parameters to study these differences to the second type of models with just group parameters.

We conclude this paper by illustrating these models on Sampson's network of 18 monks, for which we have 4 positive relations and 4 negative relations, and three subgroups, empirically determined by the use of clustering algorithms.

4.1 Models for the Macroanalysis of Multiple Relations

In order to model the macro-aspects of multigraphs we need to develop a notation for the 2^{2R} possible realizations of the $\{D_{ij}\}$ and a representation for the table of summary counts of these realizations obtained by adding across dyads. Since these models assume no individual actor differences, the sufficient statistics for the model parameters are margins of this table.

Table 4 contains summaries of Sampson's data, shown in Table 1, in the form of two 2^8 tables of counts of pairs of monks, one for the four positive relations and the other for the four negative relations. Within each table, each pair is counted twice, once from the perspective of each member, yielding a total count of $2 \times \binom{18}{2} = 306$. We refer to these tables as $\underset{\sim}{w}$-arrays, with entries $\{w_{ii'jj'kk'\ell\ell'}\}$. Here R = 4.

There are other ways to arrange these summary counts in tabular form. One way eliminates the cells which occur twice. In general, a 2^{2R} $\underset{\sim}{w}$-array contains $2^{R-1}(2^R + 1)$ unique cells. Among these, are 2^R cells whose counts are duplicated; i.e., occur twice in $\underset{\sim}{w}$. If we eliminate the doubling and duplication in the 8-dimensional $\underset{\sim}{w}$-arrays given in Table 4, we get two arrangements of 136 cells, whose counts correctly total 153. In Table 5, we give one possible arrangement of these 136 cells in a form resembling a four dimensional $3 \times 3 \times 3 \times 3$ cross-classification, in which some of the 81 cells have more than 1 count. We denote the counts in Table 5 by $\underset{\sim}{z} = \{z_{abcd}: a,b,c,d = M,A,\bar{A},N\}$ (the use of the subscripts A and $\bar{A}$ is described in the caption to the table).

We wish to model p_{abcd}, the probability that a randomly selected dyad would be assigned to cell (a,b,c,d) of Table 5, where

$$\sum_{\text{all cells}} p_{abcd} = 1. \tag{4.1}$$

Table 4

Sampson's Cloister Data Aggregated Over Actors

(a) the order of the variables is {like, esteem, influence, praise} with the index on the first variable changing fastest.

180	6	6	2	6	2	1	0	6	1	2	0	0	0	0	0
4	1	0	0	3	8	0	1	0	0	0	0	0	0	0	0
4	0	1	0	0	0	0	0	3	0	8	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0
2	1	0	0	2	13	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
3	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
2	0	1	0	0	0	0	0	2	0	13	0	0	0	1	0
0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0

(b) the order of the variables is {antagonism, disesteem, neg influence, blame} with the index in the first variable change fastest.

164	3	3	0	3	7	0	0	3	0	7	0	0	0	0	0
7	0	0	0	7	5	0	1	0	0	0	0	0	1	0	0
7	0	0	0	0	0	0	0	7	0	5	1	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
7	0	1	0	6	1	0	0	1	0	0	0	0	0	0	0
2	1	0	0	1	7	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
7	1	0	0	1	0	0	0	6	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	1	0	7	0	0	0	0	0
0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	2	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4

We define

$$\ell_{abcd} = \begin{cases} \log p_{abcd}, \text{ if a,b,c, and d are each equal to either M or N} \\ \log (p_{abcd}/2), \text{ if one of a,b,c, or d equals A ,} \end{cases} \tag{4.2}$$

and we develop a class of linear models for the $\{\ell_{abcd}\}$ which yields an affine translation of a class of loglinear models for the $\{p_{abcd}\}$. The reasons for this approach are discussed by Fienberg, Meyer, and Wasserman (1981); primarily, we introduce the factor of ½ for A cells to make our models consistent with the univariate model of Holland and Leinhardt (1981).

The models for the $\{\ell_{abcd}\}$ are linear in sets of parameters that reflect the various distinct types of dyadic patterns. In Fienberg, Meyer, and Wasserman (1981) we considered R = 3 relations as displayed in Figure 1. The $\{\ell_{abc}\}$ were modeled with up to 36 (one per cell in $\underset{\sim}{z}$) parameters with hierarchical structure reflecting 13 distinct types. When R = 2, there are only 7 distinct types and at most, 10 parameters are necessary. When R = 4, there are 22 distinct types and 81 parameters.

FIGURE 1 PATTERNS OF FLOW DEPENDENCY IN DYADIC PATTERNS

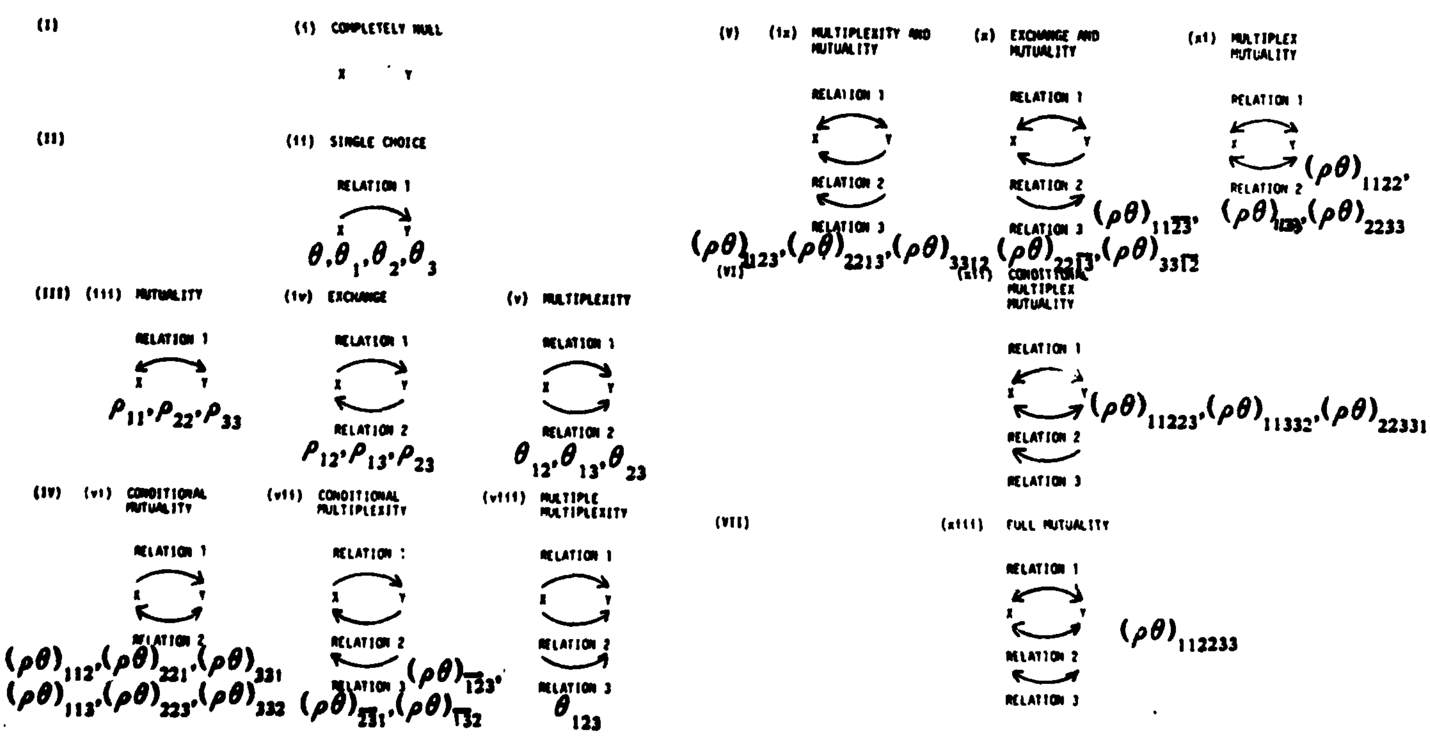

Table 5

Nonredundant Arrangement of Cells for Positive Relations from Sampson's Data (see Table 4a)

		Praise M				Praise A				Praise N			
		Influence				Influence				Influence			
Like		M	A		N	M	A		N	M	A		N
M	Esteem M	z_{MMMM} 0	z_{MMAM} 0		z_{MMNM} 0	z_{MMMA} 0	z_{MMAA} 0	$z_{MMA\bar{A}}$ 0	z_{MMNA} 0	z_{MMMN} 0	z_{MMAN} 0		$z_{M\ldots}$ 0
	Esteem A	z_{MAMM} 0	z_{MAAM} 0	$z_{MA\bar{A}M}$ 0	z_{MANM} 0	z_{MAMA} 0	z_{MAAA} 0	$z_{MAA\bar{A}}$ 0	z_{MANA} 1	z_{MAMN} 0	z_{MAAN} 1	$z_{MA\bar{A}N}$ 0	$z_{MA\ldots}$ 0
						$z_{MAM\bar{A}}$ 1	$z_{MA\bar{A}A}$ 0	$z_{MA\bar{A}\bar{A}}$ 0	$z_{MAN\bar{A}}$ 0				
	Esteem N	z_{MNMM} 0	z_{MNAM} 0		z_{MNNM} 0	z_{MNMA} 0	z_{MNAA} 0	$z_{MNA\bar{A}}$ 0	z_{MNNA} 0	z_{MNMN} 0	z_{MNAN} 0		$z_{M\ldots}$ [illegible]
A	Esteem M	z_{AMMM} 0	z_{AMAM} 0	$z_{AM\bar{A}M}$ 0	z_{AMNM} 0	z_{AMMA} 0	z_{AMAA} 1	$z_{AM\bar{A}A}$ 0	z_{AMNA} 0	z_{AMMN} 0	z_{AMAN} 0	$z_{AM\bar{A}N}$ 0	$z_{A\ldots}$ 0
						$z_{AMM\bar{A}}$ 0	$z_{AMA\bar{A}}$ 0	$z_{AM\bar{A}\bar{A}}$ 0	$z_{AMN\bar{A}}$ 0				
	Esteem A	z_{AAMM} 0	z_{AAAM} 0	$z_{AA\bar{A}M}$ 0	z_{AANM} 0	z_{AAMA} 1	z_{AAAA} 13	$z_{AA\bar{A}A}$ 0	z_{AANA} 0	z_{AAMN} 0	z_{AAAN} 8	$z_{AA\bar{A}N}$ 0	$z_{A\ldots}$ 2
						$z_{AAM\bar{A}}$ 0	$z_{AAA\bar{A}}$ 0	$z_{AA\bar{A}\bar{A}}$ 0	$z_{AAN\bar{A}}$ 0				
		$z_{A\bar{A}MM}$ 1	$z_{A\bar{A}AM}$ 0	$z_{A\bar{A}\bar{A}M}$ 0	$z_{A\bar{A}NM}$ 0	$z_{A\bar{A}MA}$ 0	$z_{A\bar{A}AA}$ 0	$z_{A\bar{A}\bar{A}A}$ 0	$z_{A\bar{A}NA}$ 0	$z_{A\bar{A}MN}$ 0	$z_{A\bar{A}AN}$ 0	$z_{A\bar{A}\bar{A}N}$ 0	$z_{A\bar{A}\ldots}$ 1
						$z_{A\bar{A}M\bar{A}}$ 0	$z_{A\bar{A}A\bar{A}}$ 1	$z_{A\bar{A}\bar{A}\bar{A}}$ 0	$z_{A\bar{A}N\bar{A}}$ 0				
	Esteem N	z_{ANMM} 0	z_{ANAM} 0	$z_{AN\bar{A}M}$ 0	z_{ANNM} 0	z_{ANMA} 0	z_{ANAA} 1	$z_{AN\bar{A}A}$ 0	z_{ANNA} 1	z_{ANMN} 0	z_{ANAN} 1	$z_{AN\bar{A}N}$ 0	$z_{AN\ldots}$ 6
						$z_{ANM\bar{A}}$ 0	$z_{ANA\bar{A}}$ 0	$z_{AN\bar{A}\bar{A}}$ 0	$z_{ANN\bar{A}}$ 0				
N	Esteem M	z_{NMMM} 0	z_{NMAM} 0		z_{NMNM} 0	z_{NMMA} 0	z_{NMAA} 0	$z_{NMA\bar{A}}$ 0	z_{NMNA} 0	z_{NMMN} 0	z_{NMAN} 0		$z_{NM\ldots}$ 0
						z_{NAMA} 0	z_{NAAA} 2	$z_{NAA\bar{A}}$ 0	z_{NANA} 1				
	Esteem A	z_{NAMM} 0	z_{NAAM} 0	$z_{NA\bar{A}M}$ 0	z_{NANM} 1	$z_{NAM\bar{A}}$ 0	$z_{NA\bar{A}A}$ 0	$z_{NA\bar{A}\bar{A}}$ 0	$z_{NAN\bar{A}}$ 0	z_{NAMN} 0	z_{NAAN} 3	$z_{NA\bar{A}N}$ 0	$z_{NA\ldots}$ 6
	Esteem N	z_{NNMM} 0	z_{NNAM} 0		z_{NNNM} 0	z_{NNMA} 0	z_{NNAA} 2	$z_{NNA\bar{A}}$ 0	z_{NNNA} 3	z_{NNMN} 0	z_{NNAN} 4		$z_{NN\ldots}$ 90

The parameters in this 'amily of models are GLIM-like in structure (see Nelder and Wedderburn, 1972). A parameter is included in the model if and only if the corresponding effect (such as choice, conditional multiplexity, etc.) is present. The parameters are also hierarchical: if we set some parameters equal to zero, all related higher-order terms are also zero.

To fit these models to multivariate networks, we apply the general results for fitting loglinear models given in Haberman (1974) or Appendix II of Fienberg (1980). The minimal sufficient statistics (MSS's) are linear combinations of the elements of the $\underset{\sim}{z}$ array, with coefficients of 0, 1, or 2. The fitted values of these elements are found by solving the likelihood equations, which set the MSS's equal to their estimated expected values. We can either use a version of generalized iterative proportional fitting due to Darroch and Ratcliff (1972), or a "trick," given in Fienberg, Meyer, and Wasserman (1981), which relies on the following two results:

Result 1: For the class of affine translations of hierarchical loglinear models described above, each set of MSS's is equivalent to a set of marginal totals for the 2^{2R} table (i.e., the $\underset{\sim}{w}$-table) with doubled and duplicated counts.

Result 2: For each affi.e translation of a loglinear model for the $\underset{\sim}{z}$-table, there is a corresponding loglinear model for the $\underset{\sim}{w}$-table, with equivalent estimated expected values, once we take account of the duplication and doubling.

The estimated expected values for the elements of the $\underset{\sim}{w}$-array can be computed using the standard IPFP and the estimates of the parameters calculated from the fitted values. We note that the degrees of freedom for any model must be calculated using the model for the $\underset{\sim}{z}$-array, and values of goodness-of-fit statistics computed using the $\underset{\sim}{w}$-array, dividing by 2 to adjust for the doubling and duplications.

4.2 Models for Both Microanalysis and Macroanalysis: Actor and Group Effects

We now consider models for multiple relations that allow the actors in the network to engage in relations at possibly different rates, and include both actor and group effects. To review, we suppose that the R sociometric relations defined for a group of g actors, are binary, and the presence/absence of directed links between actors is recorded in the form of R sociomatrices. As before, we concentrate on the dyadic relationships between the $\binom{g}{2}$ pairs of actors i and j, represented by the 2R–variate $\underset{\sim}{D}_{ij}$, with realization $\underset{\sim}{d}_{ij}$.

Primarily to limit the number of parameters, we now assume that the actors have been partitioned into K mutually exclusive and exhaustive subgroups, G_1, G_2, ..., G_K. In practice, it is very useful to allow for the inherent differences in the actors in this manner. If there are single actors that behave contrary to the group as a whole (or to the collection of subgroups), then they can be placed into their own singleton subgroups. Thus, their individual differences can still be modeled directly.

In this section we outline models which can include both actor and group effects. These models contain all the previous models as special cases. The R sociomatrices are used to construct a table of pseudo–counts, of size $g \times g \times (2 \times 2)^R$. From this multivariate version of the $\underset{\sim}{y}$–array, we can aggregate $(2 \times 2)^R$ tables to form a $K \times K \times (2 \times 2)^R$ table, whose entries are the frequencies of the different dyadic relationship patterns between actors of a group partitioned into K subgroups. As in the earlier cases it is most convenient to work with the full $g \times g \times (2 \times 2)^R$ data table but to describe models in terms of the unduplicated data array. This approach also grants us a considerable degree of flexibility in fitting the models. For many of the models it is possible to consider collapsed or aggregated versions of the data which would result in smaller data tables. We believe that the unification which is introduced by always considering the full data table outweighs the occassional advantage of having a smaller table.

We will begin our discussion by concentrating on the choice parameters in an R = 3 relation network. As a starting point we contemplate the model

$$\log P(\underset{\sim}{D}_{ij} = \underset{\sim}{d}_{ij}) = \lambda^{(ij)} + \Sigma_{r=1}^{R} \theta_r^{(ij)} X_{ijr} + \Sigma_{r=1}^{R} \theta_r^{(ji)} X_{jir}, \text{ for all } i > j, \tag{4.3}$$

which includes different choice parameters for each pair of individuals. The parameters $\lambda^{(ij)}$ are normalizing constants and are required so as to meet the sampling constraints of the problem.

Initially, we focus our attention on just the first relation. If we wished to consider a model which asserted that the response depended only on the chooser we would allow $\theta_1^{(ij)} = \theta_1^{(i)}$. Similarly, dependence only on the chosen actor would lead to $\theta_1^{(ij)} = \theta_1^{(j)}$. Obviously we could allow chooser and chosen actor effects (but excluding the interaction) by specifying $\theta_1^{(ij)} = \theta_1^{(i)} + \theta_1^{(j)}$. Another version of this model would be to suppose that individual actors assert influence only through the groups to which they belong. In this case we could write $\theta_1^{(ij)} = \theta_1^{(\gamma(i))}$ for the choosing group i, $\theta_1^{(ij)} = \theta_1^{(\gamma(j))}$ for the chosen group j, $\theta_1^{(ij)} = \theta_1^{(\gamma(i))} + \theta_1^{(\gamma(j))}$ for both, or even $\theta_1^{(i,j)} = \theta_1^{(\gamma(i),\gamma(j))}$ to indicate group by group choice interactions. If we aggregate over groups then these models are just "actor" models where the actors are the groups. In summary we have just described four basic classes of models; those which involve parameters ($\theta^{(ij)}$) for each pair of actors, those with individual parameters ($\theta^{(i)}$ and $\theta^{(j)}$) for actors and the corresponding notions, $\theta^{(\gamma(i),\gamma(j))}$ and $\{\theta^{(\gamma(i))}, \theta^{(\gamma(j))}\}$, for groups. Thus some possible "choice-only" models for one relation are:

$\log P(D_{ij} = d_{ij}) =$

1)	θ	constant
2)	$\theta^{(i)}$	chooser
3)	$\theta^{(j)}$	chosen
4)	$\theta^{(i)} + \theta^{(j)}$	chooser and chosen
5)	$\theta^{(ij)}$	interaction
6)	$\theta^{(\gamma(i))}$	group chooser
7)	$\theta^{(\gamma(j))}$	group chosen
8)	$\theta^{(\gamma(i))} + \theta^{(\gamma(j))}$	group chooser and chosen
9)	$\theta^{(\gamma(i),\gamma(j))}$	group interaction.

It is possible to mix and match among these models to consider, for example, the model $\theta^{(i)} + \theta^{(j)} + \theta^{(\gamma(i),\gamma(j))}$, which allows individual actor parameters and a group interaction. As soon as we contemplate such models we need to note that there is a partial hierarchy to the models listed above, which we represent in Figure 2.

Figure 2. Hierarchy Displaying Levels of Parameters in the Loglinear Models

```
Level 4              5
                    / \
                   /   \
Level 3           4     9
                 /|     |
                / |     |
Level 2        2  8     3
                \ |\   /
                 \| \ /
Level 1           6  7
                   \/
Level 0            1
```

The diagram indicates that any parameter at level i implies all those parameters at levels less than i. The modelling strategy we have outlined above can be used for other types of flows (e.g. mutual, reciprocal) and multiple relationships. In these cases we need to be concerned about the hierarchical structure between parameter types as well as within parameter types.

4.3 Fitting Models

If we restrict ourselves to actor parameters then we can fit the models described above using the IPFP to adjust simple margins of the symmetric $g \times g \times (2\times2)^R$ data array. When models with group parameters are included it is still possible to use the IPFP but now a more general notion of margin is needed.

Let us consider two relations and the model $\log P(\underset{\sim}{D}_{ij} = \underset{\sim}{d}_{ij}) = \lambda_1^{(ij)} + \theta_1^{(i)}$, i.e. a choice parameter on only the first relationship. The sufficient statistics for this model in the symmetric data array are

$$[12] \qquad \sum_{k\ell mn} X_{ijk\ell mn} \qquad \text{for all } i,j,$$

$$[13] \qquad \sum_{j\ell mn} X_{ijk\ell mn} \qquad \text{for all } i,k,$$

$$[24] \qquad \sum_{ikmn} X_{ijk\ell mn} \qquad \text{for all } j,\ell.$$

Now consider the model $\lambda^{(ij)} + \theta_1^{\gamma(i)}$. For this model the sufficient summary is

$$\sum_{k\ell mn} X_{ijk\ell mn} \qquad \text{for all } i,j,$$

$$\sum_{i\in G_d} \sum_{j\ell mn} X_{ijk\ell mn} \qquad \text{for } d = 1,\dots,G \text{ and for all } k,$$

$$\sum_{j\in G_d} \sum_{ikmn} X_{ijk\ell mn} \qquad \text{for } d = 1,\dots,G \text{ and for all } \ell.$$

We extend the usual square bracket notation to this situation. Recall that [12] indicates that for each value of i and j we should sum over all other dimensions in the table. We shall use the notation [-1 -2] to indicate that for each G_d and G_e we should sum over all entries in the table. A simple example should help to explain the notation.

Consider the following 3×3 table:

	1	2	3
1	a	b	c
2	d	e	f
3	g	h	i

Let G_1 = {1,2} and G_2 = {3}. Then the [1] margin is the triple (a+b+c, d+e+f, g+h+i), the [-1] margin is the pair (a+b+c+d+e+f, g+h+i) and the [-1 -2] margin is the table

a+b+d+e	c+f
g+h	i

In effect we have collapsed over the groups. It is an easy application of the IPFP to fit models which use this generalized notion of margin. We note, however, that most standard packages, which contain an IPFP routine, cannot be cajoled into fitting such models without some tinkering.

We now use the notation to show which models correspond to certain parametrizations for R = 2. Table 6 lists some of the choice models, and a small selection of other possible models. There are many possible models with many possible combinations of population, group, and individual parameters.

4.4 An Example: Sampson's Data

In order to demonstrate the ubiquity and apparent complexity of these social network models, we have taken a somewhat unusual (and bold) approach to the analysis of Sampson's network of eighteen monks. We view the four positive attributes (like, esteem, influence, and praise) as realizations of a single positive affect process, and the four negative relations (antagonism, disesteem, negative influence, and blame) in a similar manner. There is substantial justification for this pooling. White, Boorman, and Breiger (1976) found that when the eighteen actors are aggregated into three blocks, the concrete social structure of this network is

Table 6. A Selection of Possible Models for R=2

	Parameters	*Margins to be Fit*	
choice	θ_1	[12] [3] [4]	simple choice
	$\theta_1^{(i)}$	[12] [13] [24]	
	$\theta_1^{(j)}$	[12] [14] [23]	
	$\theta_1^{(i)} + \theta_1^{(j)}$	[12] [13] [14] [23] [24]	
	$\theta_1^{(\gamma(i))}$	[12] [-13] [-24]	
	$\theta_1^{(\gamma(j))}$	[12] [-14] [-23]	
	$\theta_1^{(\gamma(i), \gamma(j))}$	[12] [-1 -2 3] [-1 -2 4]	
mutuality	$\rho_{11}^{(i)}$	[12] [134] [234]	
	$\rho_{22}^{(\gamma(i))}$	[12] [-156] [-256]	
	$\rho_{11}^{(\gamma(i), \gamma(j))}$	[12] [-1 -2 34]	
multiplex	$\theta_{12}^{(i)}$	[12] [135] [246]	
	$\theta_{12}^{(j)}$	[12] [-146] [-235]	
	$\theta_{12}^{(\gamma(i), \gamma(j))}$	[12] [-1 -2 35] [-1 -2 46]	
reciprocity	$\rho_{12}^{(i)}$	[12] [136] [245]	
	$\rho_{12}^{(\gamma(j))}$	[12] [-145] [-236]	
	$\rho_{12}^{(\gamma(i), \gamma(j))}$	[12] [-1 -2 36] [-1 -2 45]	

etc.

much the same across the four pairs of positive/negative relations: "A top-esteemed block (consisting of 7 actors) unambivalently positive toward itself, in conflict with ... a second, more ambivalent block (also of 7 actors) to which is attached a block of lasers (of size 4). We label these blocks or subgroups as

$$G_1 = \{1, 2, ..., 7\}, \quad G_2 = \{8, 9, ..., 14\}, \quad G_3 = \{15, 16, 17, 18\} .$$

We therefore aggregate over both sets of relations by summing the four sociomatrices for the positive relations, and the four negative relations, to obtain one positive and one negative relation matrix. These arrays, given in Table 7, have entries indicating the number of times actor i chooses actor j, either on the positive or negative choices.

The techniques we have used to analyze 0–1 sociomatrices are directly applicable here. Furthermore, with multiple observations on each actor, the asymptotic basis for the goodness-of-fit statistics stands on firmer ground. In our analysis we have examined the 18 × 18 × (2×2) × (2×2) (corresponding to actor × actor × positive × negative) version of this table and have used the three groups given above.

A priori, some choices are unlikely to be reciprocated across relations, and we should find a simple choice or group choice model to be an adequate summarization of the flows of attitudes, both positive and negative, across and between these three, substantively different, subgroups. A summary of some of the models that we fit to this network is given in Table 8.

The difference in goodness of fit between models 2 and 3 (which in a sense is a measure of the impact of the grouping effect) is statistically significant at any reasonable level of significance and is typical of the improvement resulting from the addition of simple grouping parameters. Similarly the difference in G^2 values for models 4 and 5 is also large but is less than the difference in degrees of freedom. The small number of degrees of freedom for models is caused by a large number of fitted zeros. Indeed, any model which includes even an overall multiplex (θ_{12}) parameter induces at least 2142 fitted zeros out of the 18 × 18 × 4 × 4 = 5184 cells in the table, and the goodness of fit statistics are not dramatically improved

Table 7

Sampson's Cloister Data Aggregated Over Relations

Aggregated positive

0	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	2	0	3	3	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	2	0	2	1	2	1	0	0	0	0	0	0	0	0
0	1	3	0	4	2	0	0	0	1	0	0	0	0	0	0	0	0
3	1	0	4	0	4	0	0	0	0	0	0	0	0	0	0	0	0
0	3	2	0	2	0	2	0	0	0	1	0	0	1	0	0	0	0
0	1	2	3	4	2	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	3	3	1	0	2	0	0	0	0	0
0	1	0	0	0	0	1	4	0	1	2	0	1	0	0	1	1	0
0	1	0	0	0	0	1	3	4	0	0	0	3	0	0	0	0	0
0	0	0	0	0	0	0	3	4	3	0	4	0	1	0	0	0	0
0	0	0	0	0	0	0	1	2	4	3	0	2	0	0	0	0	0
0	0	0	0	0	0	0	3	0	4	0	2	0	4	0	0	0	0
0	0	0	0	0	0	0	1	0	4	0	4	4	0	0	0	0	0
0	4	0	0	0	3	0	0	0	0	0	0	3	0	0	0	0	1
0	0	0	0	0	0	0	0	4	0	0	0	0	0	4	0	4	2
0	0	0	0	0	0	0	0	0	3	0	0	0	0	1	3	0	3
0	0	0	0	0	0	0	0	1	4	0	0	0	0	0	3	4	0

Aggregated negative

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2
0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	3	3
0	0	0	0	0	0	1	0	0	1	0	0	4	0	1	2	4	2
0	0	0	0	0	0	0	0	3	4	4	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	2	2	1	0	1	1	0	2	2	2
0	0	0	0	0	0	0	1	0	1	2	0	0	1	1	2	2	2
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1
2	0	0	0	0	1	0	0	0	2	0	0	2	1	0	2	0	2
0	0	0	0	3	0	1	0	0	0	0	0	0	0	3	2	0	0
0	0	0	0	4	2	1	0	0	0	0	0	0	0	2	1	3	2
0	0	0	0	4	1	0	0	0	0	0	0	0	0	3	4	1	2
0	0	0	4	4	0	3	0	0	0	0	0	0	0	0	3	2	0
0	0	0	0	4	1	0	0	0	0	0	0	0	0	1	4	3	2
0	0	0	3	0	0	0	2	0	4	0	0	0	0	0	1	1	1
0	0	0	3	4	0	0	0	0	4	1	0	0	0	0	0	0	0
0	0	0	3	3	0	3	0	0	0	0	0	0	0	0	0	0	0
2	0	0	2	3	1	2	0	0	0	0	0	0	0	0	0	0	0

Table 8

Summary of Fit of Several Models on Sampson's Data

	Model	Margins	d.f.	G^2	X^2
1.	$\lambda^{(ij)}$	[12]	2295	2835	6252
2.	$\lambda^{(ij)} + \theta_1 + \theta_2$	[12] [3] [4] [5] [6]	2291	1453	3392
3.	$\lambda^{(ij)} + \theta_1^{(\gamma(i))} + \theta_1^{(\gamma(j))}$ $+ \theta_2^{(\gamma(i))} + \theta_2^{(\gamma(j))}$	[12] [-13] [-14] [-15] [-16] [-23] [-24] [-25] [-26]	2271	1395	3350
4.	$\lambda^{(ij)} + \rho_{12}^{(\gamma(i))} + \rho_{12}^{(\gamma(j))}$	[12] [-136] [-236] [-145] [-245]	2259	1368	3088
5.	$\lambda^{(ij)} + \rho^{(\gamma(i),\gamma(j))}$	[12] [-1-236] [-1-245]	<1800	1180	2158

by the inclusion of these parameters. We note the very large differences between the G^2 and X^2 values in Table 8, which go in the opposite direction from that suggested by the argument given in Larntz (1978). The only explanation we can offer is the presence of the large proportion of observed zero cells.

It appears that model 4, which includes different reciprocity effects for each group, provides a reasonable description of the data.

A more thorough analysis of the data for this network should include a detailed study of the similarities of the four pairs of positive/negative relations, and should experiment with other, more refined partitions of the actors, as suggested by Breiger, Boorman, and Arabie (1975). We have just touched the surface of a rather large, and certainly rich, set of longitudinal data. We have studied the monastery structure only at the midpoint of a 12-month period, during which a crisis over theology occurred, and the group split up.

5. CONCLUSION

In this paper, we have considered a variety of loglinear models for micro and macro analysis of binary social network data, and we have demonstrated how these models can be treated in a unified manner. The models we have considered describe important aspects of the data, and we have had the good fortune to be able to take advantage of relatively easy estimation methods for model fitting.

Unfortunately, large data sets and corresponding large models are almost axiomatic with the type of data we have described here. Our modelling has been consciously and unconsciously influenced by what it is possible for us to compute. The models with separate group effects seem to be at the limits of the computational methodology we have presented. Other models which could be considered interesting (e.g. additional relationships between the groups, akin to ordered category models for contingency tables) have not been mentioned. This is not because we find them uninteresting, but rather because the prospect of numerically fitting such models is daunting.

We believe we have indicated how more general models could be formulated, and have presented some of the techniques that are appropriate for fitting the models to actual data. Further advances in methodology in this area are likely to be as dependent upon advances in numerical algorithms or computer hardware, as they will be on new statistical ideas.

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Loglinear models are adapted for the analysis of multivariate social networks, a set of sociometric relations among a group of actors. Models that focus on the similarities and differences between the relations and models that concentrate on individual actors are discussed. This approach allows for the partitioning of the actors into blocks or subgroups. Some ideas for combining these models are described, and the various models and computational methods are applied to the analysis of data for a corporate interlock network of the 25 largest organizations in Minneapolis/St. Paul and for a classic network of eighteen monks in a cloister.

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