

AD-A 122 279

AD

TECHNICAL REPORT ARLCB-TR-82033

A FINITE DIFFERENCE PROGRAM FOR COMPUTING THE  
THERMO-ELASTIC-PLASTIC RESPONSE OF LINED GUN BARRELS

John D. Vasilakis

TECHNICAL  
LIBRARY

October 1982



US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND  
LARGE CALIBER WEAPON SYSTEMS LABORATORY  
BENÉ WEAPONS LABORATORY  
WATERVLIET, N. Y. 12189

AMCMS No. 61110191A0011

DA Project No. 1L161101A9A

PRON No. 1A2231491A1A

DTIC QUALITY INSPECTED 2

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

#### DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

The use of trade name(s) and/or manufacture(s) does not constitute an official indorsement or approval.

#### DISPOSITION

Destroy this report when it is no longer needed. Do not return it to the originator.

| REPORT DOCUMENTATION PAGE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                       | READ INSTRUCTIONS<br>BEFORE COMPLETING FORM                                                                                                      |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. REPORT NUMBER<br>ARLCB-TR-82033                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER                                                                                                                    |
| 4. TITLE (and Subtitle)<br>A FINITE DIFFERENCE PROGRAM FOR COMPUTING THE<br>THERMO-ELASTIC-PLASTIC RESPONSE OF LINED GUN<br>BARRELS                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                       | 5. TYPE OF REPORT & PERIOD COVERED<br>Final                                                                                                      |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                       | 6. PERFORMING ORG. REPORT NUMBER                                                                                                                 |
| 7. AUTHOR(s)<br>John D. Vasilakis                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                       | 8. CONTRACT OR GRANT NUMBER(s)                                                                                                                   |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>US Army Armament Research & Development Command<br>Benet Weapons Laboratory, DRDAR-LCB-TL<br>Watervliet, NY 12189                                                                                                                                                                                                                                                                                                                                                                                                                                        |                       | 10. PROGRAM ELEMENT, PROJECT, TASK<br>AREA & WORK UNIT NUMBERS<br>AMCMS No. 61110191A0011<br>DA Project No. 1L161101A9A<br>PRON No. 1A2231491A1A |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>US Army Armament Research & Development Command<br>Large Caliber Weapon Systems Laboratory<br>Dover, NJ 07801                                                                                                                                                                                                                                                                                                                                                                                                                                                |                       | 12. REPORT DATE<br>October 1982                                                                                                                  |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                       | 13. NUMBER OF PAGES<br>27                                                                                                                        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                       | 15. SECURITY CLASS. (of this report)<br>UNCLASSIFIED                                                                                             |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                       | 15a. DECLASSIFICATION/DOWNGRADING<br>SCHEDULE                                                                                                    |
| 16. DISTRIBUTION STATEMENT (of this Report)<br><br>Approved for public release; distribution unlimited.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                       |                                                                                                                                                  |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                       |                                                                                                                                                  |
| 18. SUPPLEMENTARY NOTES<br>Presented at 1982 Army Numerical Analysis and Computers Conference, US Army<br>Engineers Waterways Experiment Station, Vicksburg, MS, 3-4 February 1982.<br>Published in proceedings of the conference.                                                                                                                                                                                                                                                                                                                                                                      |                       |                                                                                                                                                  |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)<br>Transient Temperature Response<br>Thermo-Elastic-Plastic Stresses<br>Multi-Layered Cylinders<br>Repeated Firing Loads                                                                                                                                                                                                                                                                                                                                                                                             |                       |                                                                                                                                                  |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>A finite difference computer program for computing the thermo-elastic-plastic<br>response of multi-layered cylinders due to repeated firing loads was discussed<br>at the 27th Conference of Army Mathematicians. The multi-layered cylinder is a<br>representation of a lined gun barrel. The program can accommodate several<br>layers and can compute the transient temperatures and/or the stresses. It has<br>been upgraded to include an initial program which computes heat transfer<br>(CONT'D ON REVERSE) |                       |                                                                                                                                                  |

20. ABSTRACT (CONT'D)

coefficients, pressures and gas temperatures in the firing cycle for input to the main program. The effect of contact resistance between layers is now included. Results are shown for the behavior of a TZM liner in a steel tube.

## TABLE OF CONTENTS

|                                           | <u>Page</u> |
|-------------------------------------------|-------------|
| ACKNOWLEDGEMENT                           | ii          |
| INTRODUCTION                              | 1           |
| PROCEDURE                                 | 2           |
| Theory                                    | 2           |
| Boundary Conditions                       | 8           |
| Numerical Procedure                       | 8           |
| Mechanical and Thermo-Physical Properties | 11          |
| RESULTS                                   | 12          |
| SUMMARY                                   | 16          |
| REFERENCES                                | 17          |

## LIST OF ILLUSTRATIONS

|                                                                                                      |    |
|------------------------------------------------------------------------------------------------------|----|
| 1. Typical Multi-Layered Geometry.                                                                   | 18 |
| 2. Firing Cycle Characteristics.                                                                     | 19 |
| 3. Bore Temperature vs. Time.                                                                        | 20 |
| 4. Bore Temperature vs. Time.                                                                        | 21 |
| 5. Temperature vs. Radius Contact Resistance Effects.                                                | 22 |
| 6. Stress vs. Radius Temperature Dependence Effects for Steel Tube.                                  | 23 |
| 7. Hoop Stress vs. Radius at Peak Mechanical Load Thermal, Mechanical, and Combined Loading Effects. | 24 |
| 8. Hoop Stress vs. Radius at Peak Thermal Load Thermal and Combined Loading Effects.                 | 25 |

#### ACKNOWLEDGEMENT

The author would like to thank Pat Vottis for allowing the use of his program to generate proper boundary conditions.

## INTRODUCTION

This report describes a finite difference computer program for investigating the response of multilayered gun barrels subject to some firing cycle. Results, typical of which the program is capable of generating, are presented for a tube model which has a TZM liner and a steel jacket. The application is to a large caliber weapon, but the program can be used for small caliber also. The program was written to coincide with a development program which is examining the feasibility of fabricating and firing multilayered gun tubes. One of the main factors limiting the life of gun tubes is the excessive wear and erosion which occurs especially at the forcing cone area of the gun tube. The experimental program, which has shown success for 20 mm weapons, is to insert liners fabricated from refractory materials into the forcing cone area of the gun tube. Since refractory materials have high melting points, there is a strong indication that they will experience less wear and thus increase the life of the weapon.

Earlier versions of the computer program have been used to describe other behavior.<sup>1-3</sup> Preliminary work on the current problem was presented in reference 3. That work has been improved by inputting the thermo-physical properties as functions of temperature and by allowing contact resistance

---

<sup>1</sup>Vasilakis, J. D., "Temperatures and Stresses Due to Quenching of Hollow Cylinders," Transactions of the 24th Conference of Army Mathematicians, ARO Report 79-1, January 1979.

<sup>2</sup>Vasilakis, J. D. and Chen, P. C. T., "Thermo-Elastic-Plastic Stresses on Hollow Cylinders Due to Quenching," Transactions of the 25th Conference of Army Mathematicians, ARO Report 80-1, January 1980.

<sup>3</sup>Vasilakis, J. D., "Thermo-Elastic Plastic Stresses in Multi-Layered Cylinders," Transactions of the 27th Conference of Army Mathematicians, ARO Report 82-1, January 1982.

between layers. The boundary conditions have also been improved so they are now generated for the current problem at hand, i.e., for specific configuration and bore material, whereas previously they were empirically generated for another system and simply used in the program as typical input.

The computer program consists of three parts which can be run as a single program or as three separate stand alone programs. The first is an internal ballistics program which generates the boundary conditions, i.e., heat transfer coefficients, pressures, and gas temperatures as a function of time for a single firing pulse, for input to the next two programs. The next program section computes the transient temperatures due to some firing cycle and can be used to show the thermal response of the system over several firing cycles indicating the temperature buildup, and/or the temperatures can be used as input to the third program section for the computation of stresses. This can be done as the thermo-mechanical program is treated as uncoupled.

Results from each of the three sections using a TZM liner/steel jacket configuration for a 105 mm large caliber weapon system are presented.

#### PROCEDURE

The equations used to describe the behavior are discussed below followed by the numerical work.

#### Theory

The partial differential equation for describing the axisymmetric transient temperature distribution in multilayered cylinders is given by, for layer L,

$$\frac{1}{r} \frac{\partial}{\partial r} (k^L(T)r \frac{\partial T^L}{\partial r}) = c^L(T)\rho^L(T) \frac{\partial T^L}{\partial t} \quad (1)$$



where  $r$  represents the radial distance,  $T$  the temperature, and  $t$  the time. The thermal conductivity, specific heat, and density are given by  $k$ ,  $c$ , and  $\rho$  respectively. These properties are assumed to be functions of temperature. Axial effects are ignored in the program. The geometry is shown in Figure 1.

The initial condition is given by

$$T(r,0) = T_0 \quad (2)$$

where  $T_0$  would normally represent some ambient temperature. A temperature other than ambient, say due to some environmental condition, could also be used. The boundary conditions are of the type

$$k(T) \frac{\partial T}{\partial r} - h(T - T_g) = -g \quad (3)$$

where  $h$  is convection type heat transfer coefficient,  $g$  would represent some heat input if it existed, and  $T_g$  is the temperature of the propellant gases when the boundary condition is applied on the inside or bore diameter and the ambient temperature when applied on the outside surface of the gun tube.

The thermo-physical properties are made dimensionless with regard to their respective values for steel at the ambient temperature. The temperatures are made dimensionless with regard to the maximum gas temperature achieved during the interior ballistic cycle. The dimensionless time is defined by

$$\tau = \frac{k_0 t}{\rho_0 c_0 b^2} \quad (4)$$

where  $k_0$ ,  $\rho_0$ , and  $c_0$  are the values for steel as mentioned previously. In the boundary conditions,

$$\hat{h} = \frac{hb}{k} \quad (5)$$

becomes the Nusselt number.

Also required are continuity conditions between the concentric cylinders. Normally one requires continuity of temperature and heat flux. However, contact resistance does exist between surfaces and it was decided to include this effect here. The resistance results from the true nature of surfaces.<sup>4</sup> Conduction occurs at the discrete points of contact between the surfaces and is therefore a function of pressure, surface conditions, fluids in the voids, etc. It is treated here as a thin layer resisting the flow of heat. For the finite difference formulation, there is a jump in temperature at node  $i$

$$T_i^L = T_i^{L+1} + \Delta T \quad (6)$$

while the heat flux

$$\frac{q}{A} = k^L(T) \frac{T_{i-1}^L - T_i^L}{\Delta r} = h_c \Delta T = k^{L+1}(T) \frac{T_i^{L+1} - T_{i+1}^{L+1}}{\Delta r} \quad (7)$$

remains constant as one passes across the layer from cylinder  $L$  to cylinder  $L+1$ . Equations (6) and (7) allow the computation of  $\Delta T$  and  $T_i^{L+1}$ .

The use of finite difference equations to solve the thermo-elastic-plastic stress problem requires expressing the equilibrium equation and the equation of compatibility at each node at which the finite difference equations are desired. The Prandtl-Reuss flow rule is used to eliminate the incremental stresses so that what results is a matrix for evaluating the incremental radial and tangential strains at each node. The required equations follow, written in dimensionless form. The problem is treated as plane strain.

---

<sup>4</sup>Fenech, H. and Rohsenow, W. M., "Prediction of Thermal Conductance of Metallic Surfaces in Contact," Journal of Heat Transfer, February 1963, pp. 15-24.

The equation of equilibrium is written

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (8)$$

where

$$\sigma_r (= \frac{\sigma_r}{\sigma_0}) \text{ is the dimensionless radial stress}$$

$$\sigma_\theta (= \frac{\sigma_\theta}{\sigma_0}) \text{ is the dimensionless tangential stress}$$

and  $\sigma_0$  is the yield stress in tension, and the compatibility equation

$$\frac{\partial \varepsilon_\theta}{\partial r} + \frac{\varepsilon_\theta - \varepsilon_r}{r} = 0 \quad (9)$$

where

$$\varepsilon_\theta (= E \frac{\varepsilon_\theta}{\sigma_0}) \text{ is dimensionless tangential strain}$$

$$\varepsilon_r (= E \frac{\varepsilon_r}{\sigma_0}) \text{ is dimensionless radial strain}$$

and  $\sigma_0/E$  is yield strain in tension when  $E$  is Young's Modulus. The compressibility of the material is expressed by

$$\varepsilon = \alpha T + \frac{\sigma}{3K} \quad (10)$$

$$\varepsilon = \frac{1}{3} (\varepsilon_r + \varepsilon_\theta) \text{ is mean strain}$$

$$\sigma = \frac{1}{3} (\sigma_r + \sigma_\theta + \sigma_z) \text{ is mean stress}$$

$$K (= \frac{K}{\sigma_0}) \text{ is dimensionless bulk modulus}$$

$$\alpha (= \alpha T_1) \text{ is dimensionless coefficient of thermal expansion}$$

and

$$\epsilon_z = 0 \text{ for plane strain}$$

Traction free boundary conditions are always used in the outside radius and on the bore when only thermal stresses are required. When mechanical loads are desired, the pressure pulse is applied to the bore.

It was desirable to write the finite difference equations in terms of strain alone; hence, the stresses in the equations of equilibrium had to be expressed in terms of the strains. This was accomplished by modifying a plastic stress-strain matrix<sup>5</sup> which was derived by inverting the Prandtl-Reuss equations. The inverted Prandtl-Reuss equation is

$$\{d\sigma\} = [D^P]\{d\epsilon\} - \frac{E\alpha dT}{(1-2\nu)\sigma_0} \{1\} \quad (11)$$

where the stress vector is  $\{d\sigma\} = \{d\sigma_r, d\sigma_\theta, d\sigma_z\}^T$ , the strain vector  $\{d\epsilon\} = \{d\epsilon_r, d\epsilon_\theta, 0\}^T$ , and  $\{1\}$  represents a unit vector. The plastic stress-strain matrix  $[D^P]$  is given by

$$[D^P] = \frac{1}{1+\nu} \begin{bmatrix} \frac{1-\nu}{1-2\nu} - \frac{\sigma_r'^2}{S} & & & \\ \frac{\nu}{1-2\nu} - \frac{\sigma_r'\sigma_\theta'}{S} & \frac{1-\nu}{1-2\nu} - \frac{\sigma_\theta'^2}{S} & & \\ \frac{\nu}{1-2\nu} - \frac{\sigma_r'\sigma_z'}{S} & \frac{\nu}{1-2\nu} - \frac{\sigma_\theta'\sigma_z'}{S} & \frac{1-\nu}{1-2\nu} - \frac{\sigma_z'^2}{S} & \\ & & & \text{SYMMETRIC} \end{bmatrix} \quad (12)$$

<sup>5</sup>Yamada, Y., Yoshimura, N., and Sakuri, T., "Plastic Stress-Strain Matrix and Its Application For the Solution of Elastic-Plastic Problems by the Finite Element Method," International Journal of Mechanical Sciences, 1968, Vol. 10, pp. 343-354.

The primed stresses are deviatoric stresses,

$$\sigma_i' = \sigma_i - \frac{1}{3} \sum \sigma_i \quad i = r, \theta, z \quad (13)$$

At each node during a computation, the von Mises' yield criterion

$$\frac{1}{2} [(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_r)^2 + (\sigma_z - \sigma_r)^2] = 1 \quad (14)$$

is checked to see if plastic deformation has progressed to that node. If not, the stresses remain elastic and can still be computed using Eq. (12) by setting the deviatoric stresses equal to zero. The matrix  $[D^P]$  then becomes the same matrix as would exist if linear elastic behavior had been assumed.

The quantity  $S$  is given by

$$S = \frac{2}{3} \bar{\sigma}^2 \left( 1 + \frac{H'}{3G} \right) \quad (15)$$

where

$$\bar{\sigma} = \frac{3}{2} \sigma_{ij}' \sigma_{ij}' = \frac{3}{2} (\sigma_r'^2 + \sigma_\theta'^2 + \sigma_z'^2) \quad (16)$$

is the equivalent stress and

$$H' = \frac{d\sigma}{d\epsilon_p} \quad (17)$$

is the slope of the equivalent stress/equivalent plastic strain curve and is a measure of hardening. The increment in equivalent plastic strain is given by

$$d\epsilon_p = \frac{2}{3} d\epsilon_{ij}^P d\epsilon_{ij}^P \quad (18)$$

### Boundary Conditions

The boundary conditions that are used as input for the calculation of temperatures and of stresses are generated using a computer program based on reference 6. In that paper, the burning of a specific propellant for the purpose of firing a projectile from a gun tube is modeled. The equations used are based on Corner's work<sup>7</sup> and represent a first order interior ballistics solution. Lagrange's approximation is assumed, i.e., the velocity of the gas at any instant increases linearly with distance along the bore from zero at the breech to the full shot velocity at the base of the projectile. Exponential decay is assumed during the blowdown cycle, i.e., after the projectile has left the gun tube. Based upon the rate equations and heat balances involved, the heat transfer coefficients, the pressure pulse, and the gas temperature can be found as a function of time during the firing cycle. The quantity of heat that goes into the heating of the gun tube can be computed. The bore surface can be specified to be the liner material. Figure 2 shows the output of this program for a gun tube with steel at the bore surface.

### Numerical Procedure

The Crank-Nicolson representation for finite differences of the partial differential equation governing the temperatures in time is<sup>1</sup>

---

<sup>1</sup>Vasilakis, J. D., "Temperatures and Stresses Due to Quenching of Hollow Cylinders," Transactions of the 24th Conference of Army Mathematicians, ARO Report 79-1, January 1979.

<sup>6</sup>Vottis, P. M., "Digital Computer Simulation of the Interior Ballistic Process in Guns," WVT-6615, Watervliet Arsenal, Watervliet, NY, October 1966.

<sup>7</sup>Corner, J., Theory of Interior Ballistics of Guns, John Wiley and Sons, Inc., New York, 1950.

$$\begin{aligned}
& [a+i\Delta r]k_{i+1/2,n+1/2}]T_{i+1,n+1} + [-(a+i\Delta r)k_{i+1/2,n+1/2} \\
& -(a+(i-1)\Delta r)k_{i-1/2,n+1/2}]\rho_{i,n+1/2}(\frac{2\Delta r^2}{\Delta t})(a+(i-1/2)\Delta r)T_{i,n+1} \\
& + [(a+(i-1)\Delta r)k_{i-1/2,n+1/2}]T_{i-1,n+1} = [-(a+i\Delta r)k_{i+1/2,n+1/2}]T_{i+1,n} + \\
& + [(a+i\Delta r)k_{i+1/2,n+1/2} + (a+(i-1)\Delta r)k_{i-1/2,n+1/2} - \\
& c_{i,n+1/2}\rho_{i,n+1/2}(\frac{2\Delta r^2}{\Delta t})(a+(i-1/2)\Delta r)]T_{i,n} + [-(a+(i-1)\Delta r)k_{i-1/2,n+1/2}]T_{i-1,n}
\end{aligned} \tag{19}$$

The equation is solved twice:

1. At  $n+1/2$  step, allowing  $k, \rho, c$  etc. to take on the values at  $t = n$  step.
2. The new temperatures are then used to evaluate  $k, c, \rho$  at  $n+1/2$  step and the set of equations re-evaluated for the temperatures at the  $n+1$  step.

The computed temperature distributions at each full-time step are saved on disk and eventually called in when required by the stress program.

The finite difference equations are within any layer.

Compatibility:

$$\begin{aligned}
& -r_i \Delta \epsilon_{\theta_{i-1}} + (2r_i - r_{i-1}) \Delta \epsilon_{\theta_i} - (r_i - r_{i-1}) \Delta \epsilon_{r_i} = \\
& -r_i (\epsilon_{\theta_i} - \epsilon_{\theta_{i-1}}) - (r_i - r_{i-1}) (\epsilon_{\theta_i} - \epsilon_{\theta_i})
\end{aligned} \tag{20}$$

Equilibrium:

$$\begin{aligned}
& -r_i \Delta \sigma_{r_{i-1}} - (r_i - r_{i-1}) \Delta \sigma_{\theta_i} + (2r_i - r_{i-1}) \Delta \sigma_{r_i} \\
& -r_i (\sigma_{r_i} - \sigma_{r_{i-1}}) - (r_i - r_{i-1}) (\sigma_{r_i} - \sigma_{\theta_i})
\end{aligned} \tag{21}$$

Substituting the Prandtl-Reuss equations into that of equilibrium

$$\begin{aligned}
 & -r_i D(r, \theta) \Delta \epsilon_{\theta_{i-1}} - r_i D(r, r) \Delta \epsilon_{r_{i-1}} + [-(r_i - r_{i-1}) D(\theta, \theta) + (2r_i - r_{i-1}) D(r, \theta)] \Delta \epsilon_{\theta_i} \\
 & + [-(r_i - r_{i-1}) D(\theta, r) + (2r_i - r_{i-1}) D(r, r)] \Delta \epsilon_{r_i} \\
 & r_i [\sigma_{r_{i-1}} - \sigma_{r_i}] + (r_i - r_{i-1}) (\sigma_{\theta_i} - \sigma_{r_i}) + r_i \frac{E\alpha}{1-2\nu} [\Delta T_i - \Delta T_{i-1}] \quad (22)
 \end{aligned}$$

Equations (20) and (21) are in backward difference equations. The actual computations are performed by averaging backward and forward difference schemes. At the interface between cylinders, continuity of the radial stress and radial displacement is specified and on the boundary,  $i = 1$ ,

$$D(r, \theta) \Delta \epsilon_{\theta_1} + D(r, r) \Delta \epsilon_{r_1} = \frac{E\alpha \Delta T_1}{1-2\nu} - \Delta p_1 \quad (23)$$

where  $\Delta p_1$  represents a pressure increment at the bore or inside diameter.

The solution procedure for the transient temperature problem is as follows. The temperature problem is solved, and the temperature distributions at their computation times are stored on disk. These distributions are called into the thermo-elastic-plastic stress program one at a time. The corresponding thermal stresses are calculated and each node checked to see if the yield criterion is satisfied. If not, the problem is still assumed to be elastic, a new temperature distribution is called in, and new stress increments calculated. The stresses are updated, and the yield criterion checked again. When the stresses at a point are found to satisfy the yield criterion, the node is identified, and the stress increments at that node from the next set of temperatures are computed using the Prandtl-Reuss equation or  $[D^P]$  matrix identified earlier. This procedure is continued with new sets of temperature called in and with the tracking of the elastic-plastic boundary with time.



The mechanical properties are evaluated at the existing temperatures. However, the yield stress has not yet been incorporated as a function of temperature in the program.

#### Mechanical and Thermo-Physical Properties

The properties used in the calculations were found in reference 8. The nominal values are given in Table I. It is always one of the more difficult tasks to find properties as functions of temperature. The steel properties used were those of 4340 and 4150. "Gun Steel" is typically 4340 or a modification thereof. The thermo-physical properties for TZM used were those for molybdenum itself since they were readily available as functions of temperature and the same properties for TZM, only given at one or two specified temperatures tended to fall on or near the same property for molybdenum.

TABLE I. ROOM TEMPERATURE PROPERTIES

|       | k                     | c       | $\rho$            | $\alpha$             | E                | $\sigma_{yp}$     | $\nu$ |
|-------|-----------------------|---------|-------------------|----------------------|------------------|-------------------|-------|
|       | BTU/#in°F             | BTU/#°F | #/in <sup>3</sup> | in/in°F              | Psi              | Psi               |       |
| Steel | $5.01 \times 10^{-4}$ | .105    | .289              | $6.2 \times 10^{-6}$ | $30 \times 10^6$ | $160 \times 10^3$ | .3    |
| TZM   | $1.87 \times 10^{-4}$ | .06     | .369              | $3.0 \times 10^{-6}$ | $45 \times 10^6$ | $130 \times 10^3$ | .314  |

---

<sup>8</sup>Aerospace Structural Metals Handbook, AFML-TR-68-115.

## RESULTS

The interior ballistic code was first run to set up the input data (heat transfer for coefficients and gas temperatures during firing cycle) for the position of the program which computes the transient temperature distribution and pressure time curve for the mechanical load contribution to the stress part of the program. Two data sets were established, one for steel at the bore and one for TZM at the bore. Figure 2 shows the results for steel. The interior ballistic code computes these results at several stations along the tube, but only the section at which the maximum pressure was generated was considered at this time. The temperature portion of the program serves two separate purposes. The program can be run over several firing pulses based on some specific firing cycle. This will show the buildup in temperature during the firing. It can also be run to provide input data in the form of temperature distributions throughout the wall of the tube for specific times during the firing cycle. This data set is then used in the stress program for the computation of thermal stresses or thermo-mechanical stresses when the pressure-time curve is also applied. There were four types of material problems considered, a single (monobloc) steel tube with constant properties, monobloc steel tube with temperature dependent properties, a TZM liner/steel jacket with constant properties in each cylinder, and a TZM liner/steel jacket with temperature dependent properties. Results from some of these cases are presented below.

Figure 3 shows the change of the bore temperature with time over four firing cycles. The configuration is the TZM liner/steel jacket with temperature dependent properties. The firing cycle depicted represents a projectile being fired at the rate of four rounds per minute. The temperature buildup at

the bore can easily be seen. Figure 4 shows the temperature response at the bore for a monobloc steel tube with temperature dependent properties. This is shown on an expanded scale and represents the thermal response used on one of the data sets for the stresses. When these data sets for stresses are established, a finer time increment is used over that which simply computes the thermal response for temperature buildup. The finer time increment decreases the temperature difference,  $\Delta T$ , from time step to time step which is used in the stress program. This helps in approaching yield in the stress program even though the temperature difference is further divided near yielding and after yielding has begun. When only the temperatures themselves are desired, it was previously shown<sup>3</sup> that larger time increments can be used. As the finite difference program for temperatures is implicit, the time increment between pulses is increased at a rapid rate until the next firing pulse comes along. It took approximately 90 time steps to complete the temperature response to the four cycles.

When investigating the effect of contact resistance, it becomes obvious why many papers ignore it. The difficulty is not with the computation, but rather the uncertainty of the physical constant to use in the evaluation. The property  $h$  described earlier, is treated as a constant here but in reality would be a function of pressure, temperature, the roughness of the surfaces in contact, the hardness of the materials involved, etc. One would actually need the true area of contact as opposed to the apparent area and how this changed

---

<sup>3</sup>Vasilakis, J. D., "Thermo-Elastic-Plastic Stresses in Multi-Layered Cylinders," Transactions of the 27th Conference of Army Mathematicians, ARO Report 82-1, January 1982.

in time. Table II shows the effect of varying  $h$  on the bore temperature for five firing pulses. The  $h$  is dimensionless. The table shows that when  $h = 1000$ , the system is equivalent to having no resistance to heat flow and when  $h = .00001$ , the effect is equivalent to zero heat flow at the interface. As can be seen by converting some of the values from reference 4, one can get coefficients which result in measurable effects. To better show the effect, the resulting temperature distribution as a function of radius as shown in Figure 5 for  $h = 1$ . The second, third, and fourth pulses are displaced from the first for clarity of viewing. The bore temperature increases substantially compared to zero contact resistance (as can also be seen from Table II). The temperature jump also increases substantially. The line indicating the jump should be a vertical line from the point on the inner cylinder. The fact that it is not is due to the plotting routine. A final remark on this section would be that with the uncertainty in computing or experimentally determining an actual  $h$  for a system, Table II shows little difference between an  $h = 1000$  and  $h = 100$  as a threshold for zero contact resistance and again little difference between  $h = .1$  and  $h = .00001$  for an insulating barrier.

---

<sup>4</sup>Fenech, H. and Rohsenow, W. M., "Prediction of Thermal Conductance of Metallic Surfaces in Contact," Journal of Heat Transfer, February 1963, pp. 15-24.

TABLE II. EFFECT OF H ON BORE TEMPERATURE

|      |         | 1000   | 100    | 1      | .1     | .00001 |
|------|---------|--------|--------|--------|--------|--------|
| Temp | Pulse 1 | 0      | 0      | 0      | 0      | 0      |
|      | Pulse 2 | .00352 | .00356 | .00837 | .00999 | .01021 |
|      | Pulse 3 | .00580 | .00582 | .01493 | .01952 | .02020 |
|      | Pulse 4 | .00758 | .00756 | .02013 | .02861 | .02998 |
|      | Pulse 5 | .00904 | .00901 | .02424 | .03728 | .03596 |

The remaining results relate to the output from the stress portion of the program. Figure 6 shows the effect of including temperature dependence of material properties in the analysis. The variation of radial stress and tangential stress across the wall thickness is shown. There is little effect except at the peak tangential stress at the bore which is about three to four percent higher for a tube with temperature dependent material properties. This represents a stress difference of about 5000 psi.

The stress results being shown are only the response to the first firing pulse. Hence, the stresses are mainly due to a steep thermal gradient at the bore of the tube. If inelastic response does not occur, the stress response would not change significantly from pulse to pulse as the mechanical load vanishes and there remains only a slight thermal gradient throughout the tube wall. Depending on the configuration of the system, the firing rate, etc., the rise in bore temperature before the next round is fired is small,  $\sim 3^{\circ}\text{F}$  to  $9^{\circ}\text{F}$ .

Figure 7 shows the variation in tangential stress versus radius for the multi-cylinder configuration, TZM/steel, with the temperature dependent properties at the time when the internal pressure peaks. The mechanical loading due to the pressure-time curve alone is shown as is the thermal stress distribution due to the temperature distribution. The combined effects are also shown. These combined effects are not arrived at by adding the separate ones but are recomputed. This is important, especially in the case shown, because the mechanical load alone was sufficient to cause the inelastic deformation at the bore. While this inelastic zone was concentrated at the bore and there was little depth of plastic zone penetration into the wall, it was still incorrect to linearly superimpose solutions by adding the separate stress distributions. Under the combined loads, the solution remained elastic throughout. Figure 8 shows the results from the same problem at peak thermal load.

#### SUMMARY

The computer program described is capable of predicting the thermal response and the thermo-elastic-plastic response of liner/jacket gun tube designs with temperature dependent thermo-physical properties. Realistic input to this problem is generated using an interior ballistics code. While only a two cylinder system is described, allowance is made for up to five cylinders.

Improvements can always be made and in this case, the following could be included. By using a variable space increment in the thermal program, the effect of a deposited thin layer on the bore surface can be investigated. Initial stresses due to interference should be incorporated as should a temperature dependent yield stress.

#### REFERENCES

1. Vasilakis, J. D., "Temperatures and Stresses Due to Quenching of Hollow Cylinders," Transactions of the 24th Conference of Army Mathematicians, ARO Report 79-1, January 1979.
2. Vasilakis, J. D. and Chen, P. C. T., "Thermo-Elastic-Plastic Stresses on Hollow Cylinders Due to Quenching," Transactions of the 25th Conference of Army Mathematicians, ARO Report 80-1, January 1980.
3. Vasilakis, J. D., "Thermo-Elastic-Plastic Stresses in Multi-Layered Cylinders," Transactions of the 27th Conference of Army Mathematicians, ARO Report 82-1, January 1982.
4. Fenech, H. and Rohsenow, W. M., "Prediction of Thermal Conductance of Metallic Surfaces in Contact," Journal of Heat Transfer, February 1963, pp. 15-24.
5. Yamada, Y., Yoshimura, N., and Sakuri, T., "Plastic Stress-Strain Matrix and Its Application for the Solution of Elastic-Plastic Problems by the Finite Element Method," International Journal of Mechanical Sciences, 1968, Vol. 10, pp. 343-354.
6. Vottis, P. M., "Digital Computer Simulation of the Interior Ballistic Process in Guns," WVT-6615, Watervliet Arsenal, Watervliet, NY, October 1966.
7. Corner, J., Theory of the Interior Ballistics of Guns, John Wiley and Sons, Inc., New York, 1950.
8. Aerospace Structural Metals Handbook, AFML-TR-68-115.



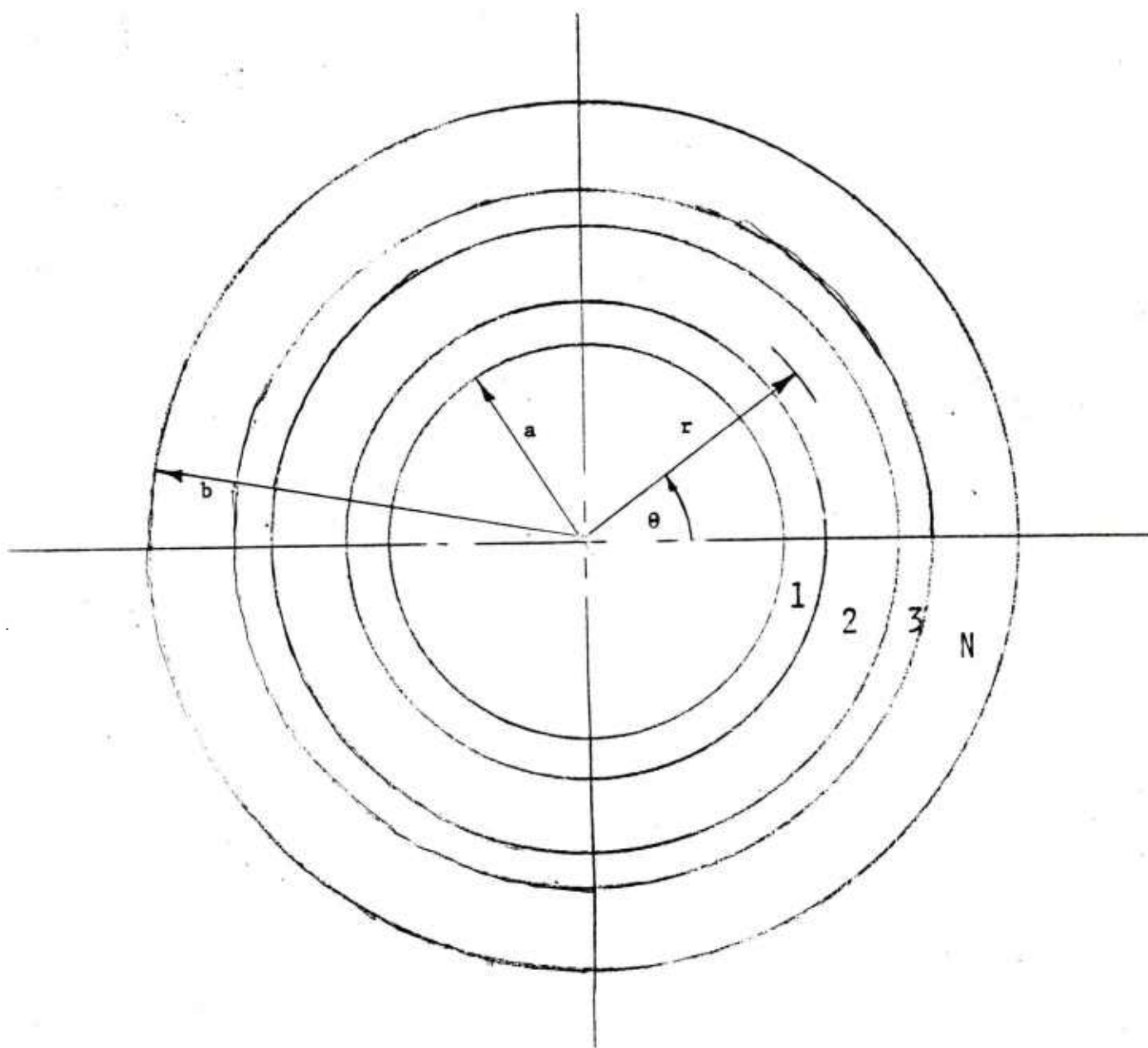
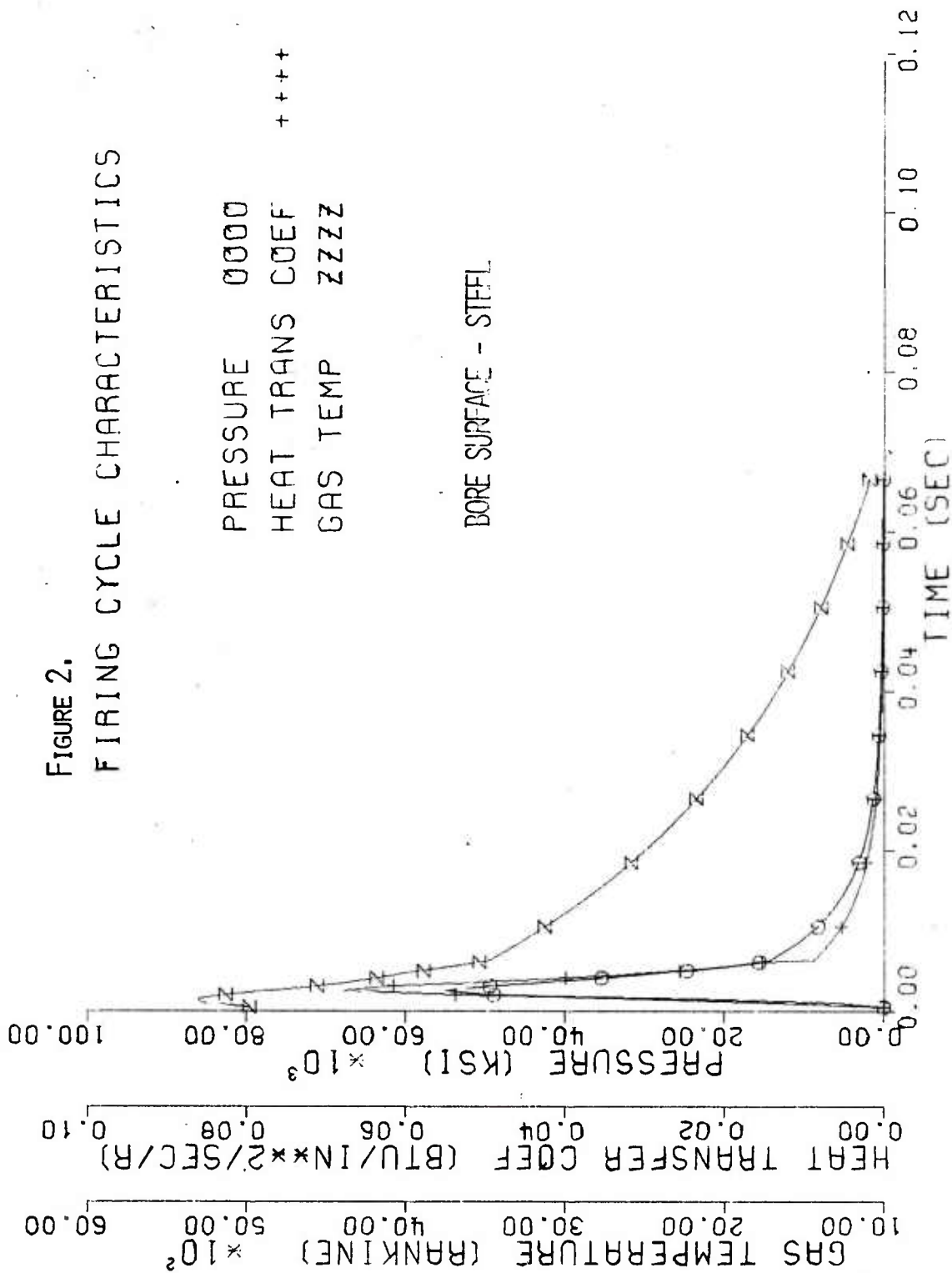


FIGURE 1. TYPICAL MULTI-LAYERED GEOMETRY



FIGURE 2.  
FIRING CYCLE CHARACTERISTICS



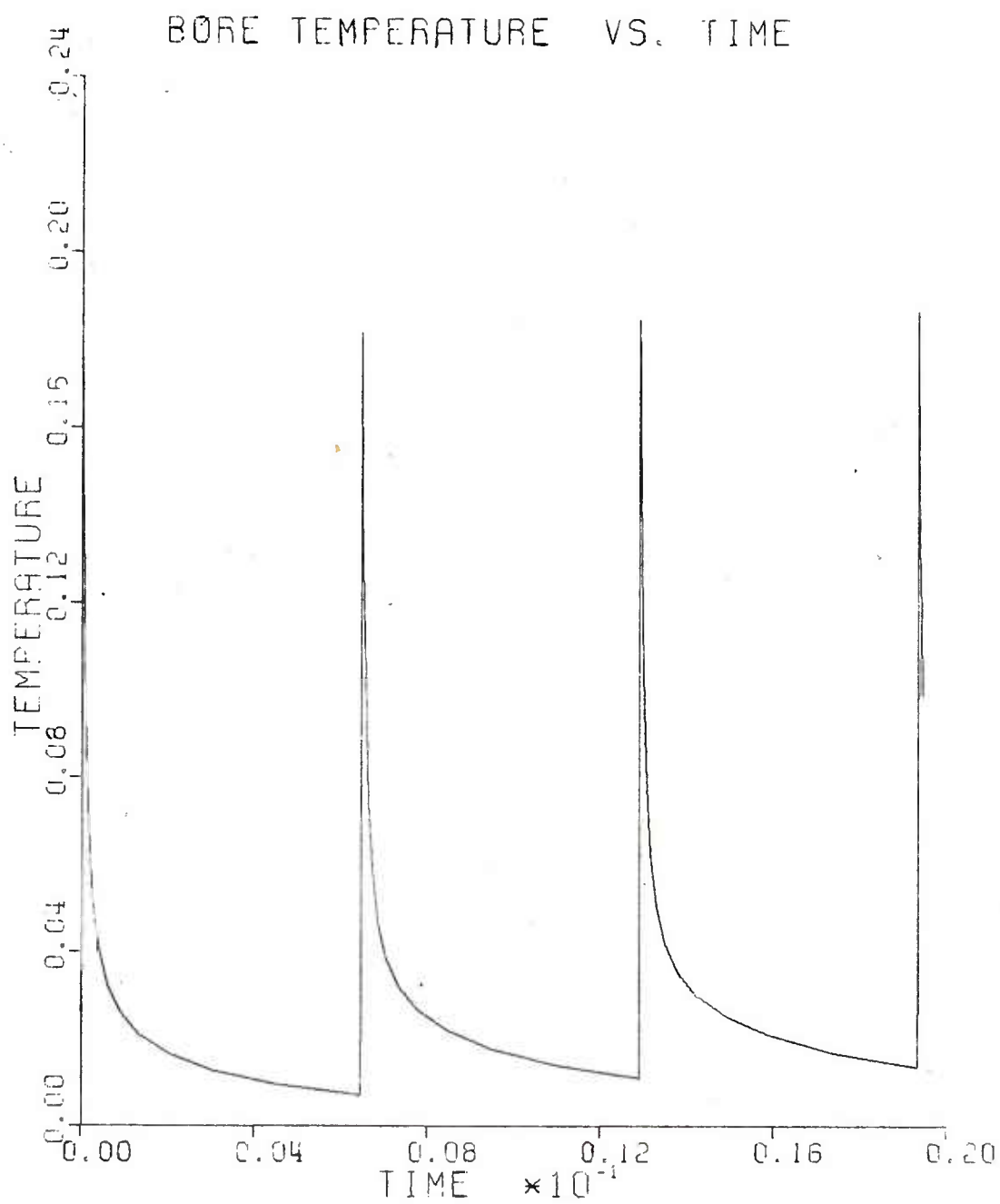


FIGURE 3. BORE TEMPERATURE RESPONSE OVER SEVERAL FIRING CYCLES.

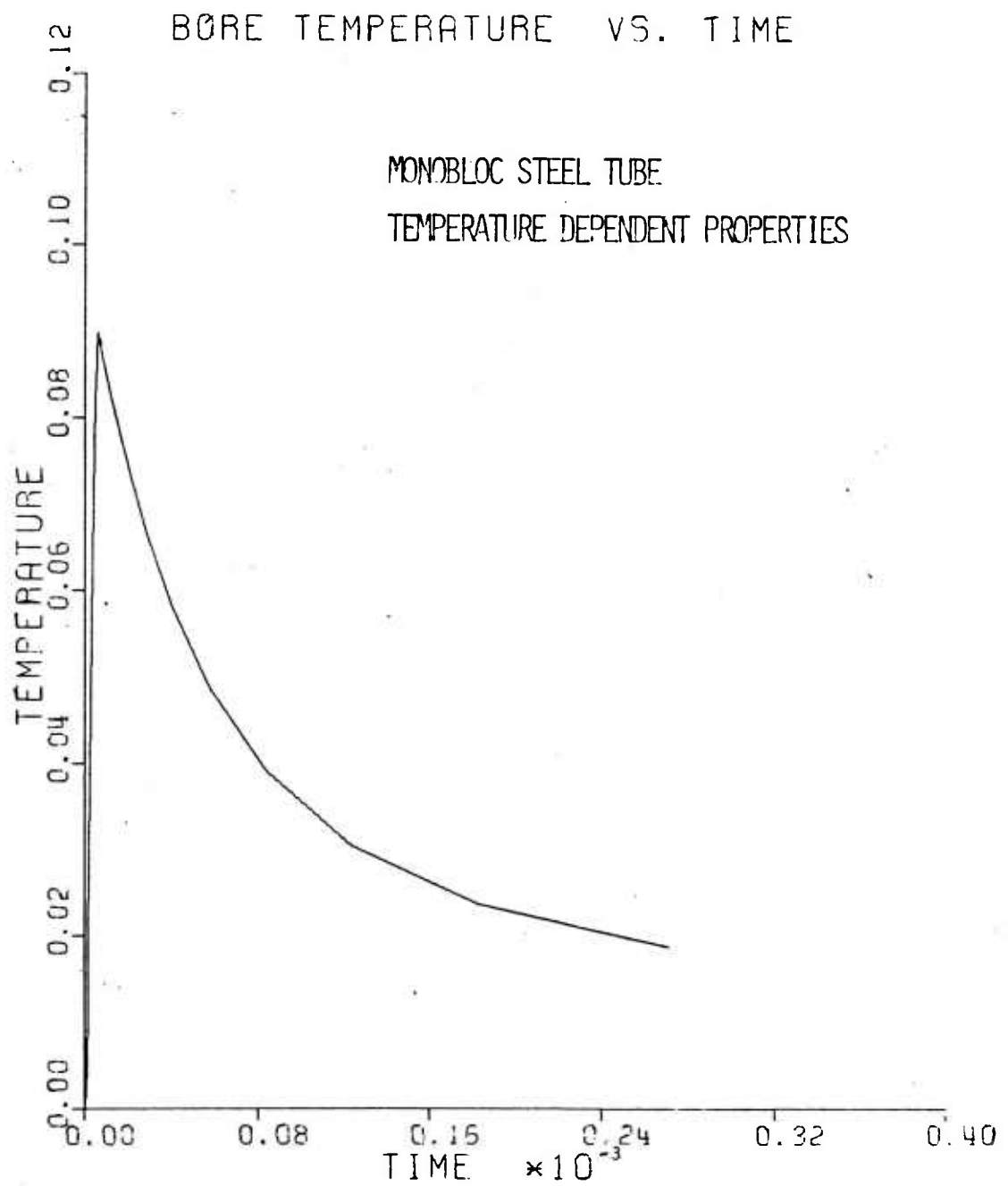


FIGURE 4. BORE TEMPERATURE VS TIME.  
RESPONSE FOR INPUT TO STRESS PROGRAM.

FIGURE 5. TEMPERATURE VS RADIUS.  
CONTACT RESISTANCE EFFECTS

$H = 1.0$

1ST PULSE . . . . . Z

2ND PULSE . . . . . +

3RD PULSE . . . . . X

4TH PULSE . . . . . Y

2ND, 3RD AND 4TH PULSE ARE  
DISPLACED FROM 1ST FOR VIEWING

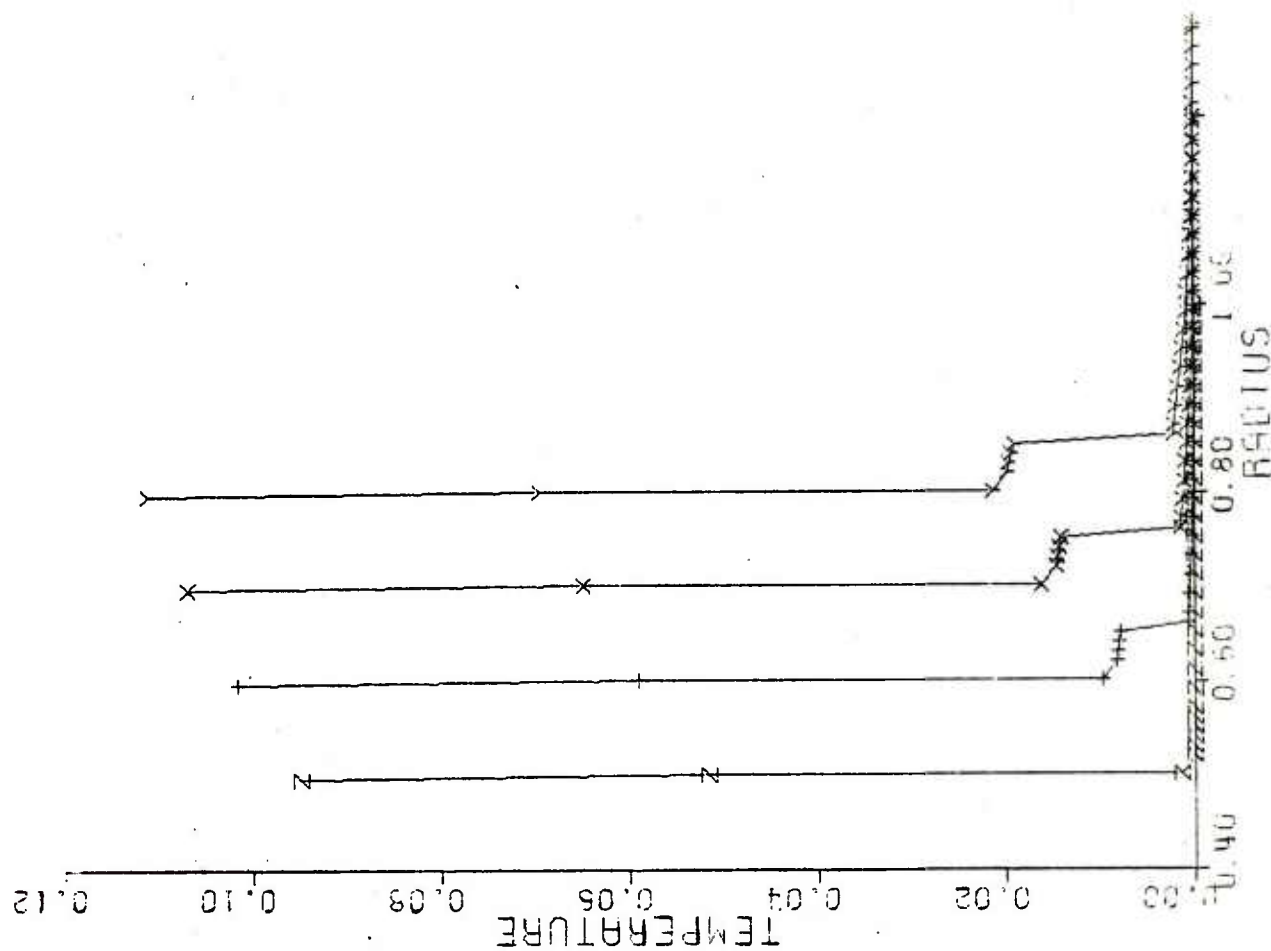


FIGURE 6. STRESS VS RADIUS  
TEMPERATURE DEPENDENCE  
EFFECTS FOR STEEL TUBE

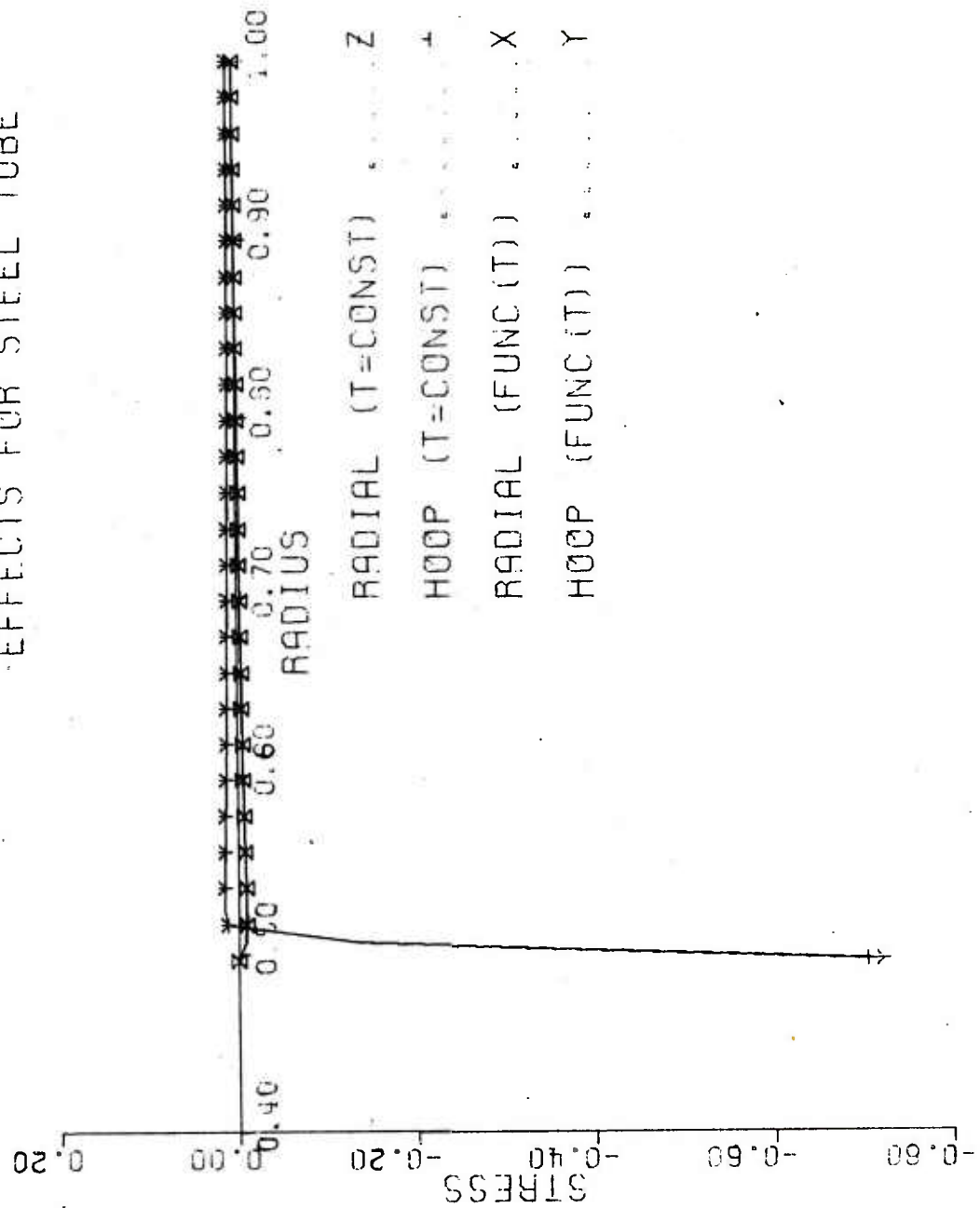


FIGURE 7. HOOP STRESS VS RADIUS AT  
PEAK MECHANICAL LOAD  
THERMAL, MECHANICAL AND  
COMBINED LOADING EFFECTS  
TZM/STEEL TUBE

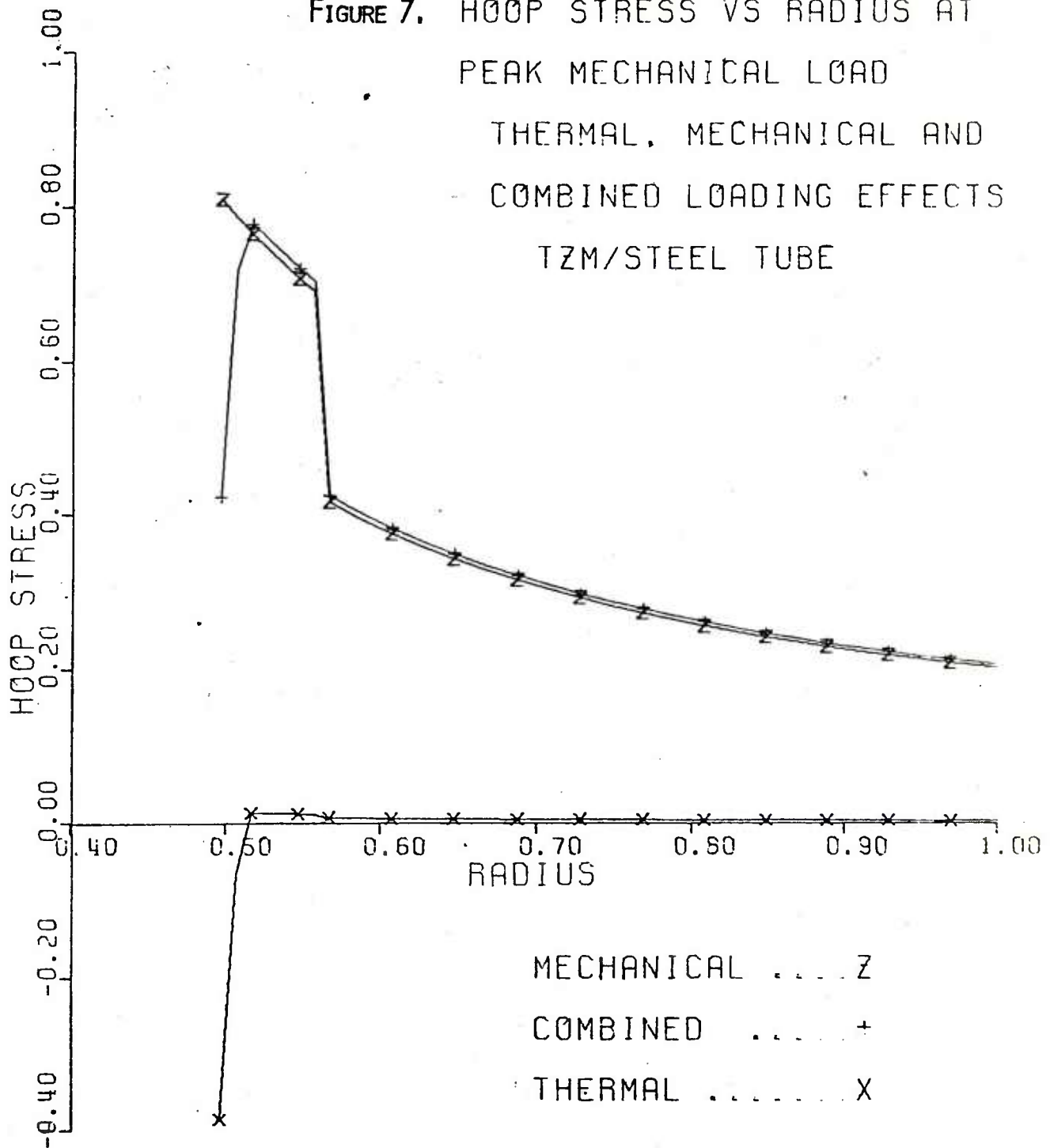
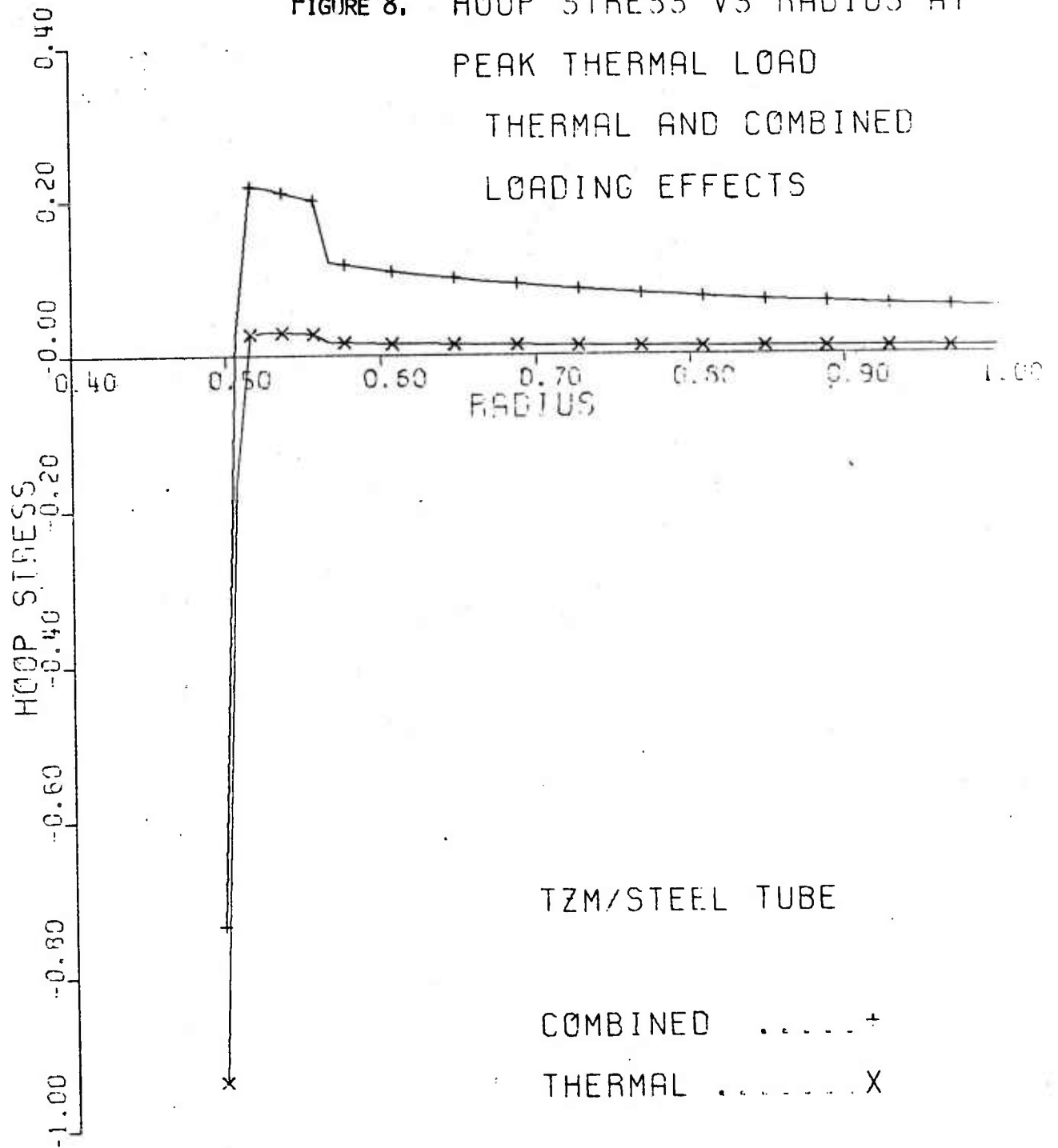


FIGURE 8. HOOP STRESS VS RADIUS AT  
PEAK THERMAL LOAD  
THERMAL AND COMBINED  
LOADING EFFECTS



# TECHNICAL REPORT INTERNAL DISTRIBUTION LIST

|                                         | <u>NO. OF<br/>COPIES</u> |
|-----------------------------------------|--------------------------|
| CHIEF, DEVELOPMENT ENGINEERING BRANCH   | 1                        |
| ATTN: DRDAR-LCB-DP                      | 1                        |
| -DR                                     | 1                        |
| -DS (SYSTEMS)                           | 1                        |
| -DS (ICAS GROUP)                        | 1                        |
| -DC                                     | 1                        |
| CHIEF, ENGINEERING SUPPORT BRANCH       | 1                        |
| ATTN: DRDAR-LCB-SE                      | 1                        |
| CHIEF, RESEARCH BRANCH                  | 2                        |
| ATTN: DRDAR-LCB-R (ELLEN FOGARTY)       | 1                        |
| -RA                                     | 1                        |
| -RM                                     | 1                        |
| -RP                                     | 1                        |
| -RT                                     | 1                        |
| TECHNICAL LIBRARY                       | 5                        |
| ATTN: DRDAR-LCB-TL                      |                          |
| TECHNICAL PUBLICATIONS & EDITING UNIT   | 2                        |
| ATTN: DRDAR-LCB-TL                      |                          |
| DIRECTOR, OPERATIONS DIRECTORATE        | 1                        |
| DIRECTOR, PROCUREMENT DIRECTORATE       | 1                        |
| DIRECTOR, PRODUCT ASSURANCE DIRECTORATE | 1                        |

NOTE: PLEASE NOTIFY DIRECTOR, BENET WEAPONS LABORATORY, ATTN: DRDAR-LCB-TL,  
OF ANY REQUIRED CHANGES.



# TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST

|                                                                                                                      | <u>NO. OF<br/>COPIES</u> |                                                                                                                     | <u>NO. OF<br/>COPIES</u> |
|----------------------------------------------------------------------------------------------------------------------|--------------------------|---------------------------------------------------------------------------------------------------------------------|--------------------------|
| ASST SEC OF THE ARMY<br>RESEARCH & DEVELOPMENT<br>ATTN: DEP FOR SCI & TECH<br>THE PENTAGON<br>WASHINGTON, D.C. 20315 | 1                        | COMMANDER<br>ROCK ISLAND ARSENAL<br>ATTN: SARRI-ENM (MAT SCI DIV)<br>ROCK ISLAND, IL 61299                          | 1                        |
| COMMANDER<br>DEFENSE TECHNICAL INFO CENTER<br>ATTN: DTIC-DDA<br>CAMERON STATION<br>ALEXANDRIA, VA 22314              | 12                       | DIRECTOR<br>US ARMY INDUSTRIAL BASE ENG ACT<br>ATTN: DRXIB-M<br>ROCK ISLAND, IL 61299                               | 1                        |
| COMMANDER<br>US ARMY MAT DEV & READ COMD<br>ATTN: DRCDE-SG<br>5001 EISENHOWER AVE<br>ALEXANDRIA, VA 22333            | 1                        | COMMANDER<br>US ARMY TANK-AUTMV R&D COMD<br>ATTN: TECH LIB - DRSTA-TSL<br>WARREN, MICHIGAN 48090                    | 1                        |
| COMMANDER<br>US ARMY ARRADCOM.<br>ATTN: DRDAR-LC<br>DRDAR-LCA (PLASTICS TECH<br>EVAL CEN)                            | 1                        | COMMANDER<br>US ARMY TANK-AUTMV COMD<br>ATTN: DRSTA-RC<br>WARREN, MICHIGAN 48090                                    | 1                        |
| DRDAR-LCE                                                                                                            | 1                        | COMMANDER<br>US MILITARY ACADEMY<br>ATTN: CHMN, MECH ENGR DEPT<br>WEST POINT, NY 10996                              | 1                        |
| DRDAR-LCM (BLDG 321)                                                                                                 | 1                        |                                                                                                                     |                          |
| DRDAR-LCS                                                                                                            | 1                        | US ARMY MISSILE COMD<br>REDSTONE SCIENTIFIC INFO CEN<br>ATTN: DOCUMENTS SECT, BLDG 4484                             | 2                        |
| DRDAR-LCU                                                                                                            | 1                        | REDSTONE ARSENAL, AL 35898                                                                                          |                          |
| DRDAR-LCW                                                                                                            | 1                        |                                                                                                                     |                          |
| DRDAR-TSS (STINFO)                                                                                                   | 2                        | COMMANDER<br>US ARMY FGN SCIENCE & TECH CEN<br>ATTN: DRXST-SD<br>220 7TH STREET, N.E.<br>CHARLOTTESVILLE, VA 22901  | 1                        |
| DOVER, NJ 07801                                                                                                      |                          |                                                                                                                     |                          |
| DIRECTOR<br>US ARMY BALLISTIC RESEARCH LABORATORY<br>ATTN: DRDAR-TSB-S (STINFO)<br>ABERDEEN PROVING GROUND, MD 21005 | 1                        | COMMANDER<br>US ARMY MATERIALS & MECHANICS<br>RESEARCH CENTER<br>ATTN: TECH LIB - DRXMR-PL<br>WATERTOWN, MASS 02172 | 2                        |
| COMMANDER<br>US ARMY ARRCOM<br>ATTN: DRSAR-LEP-L<br>ROCK ISLAND ARSENAL<br>ROCK ISLAND, IL 61299                     | 1                        |                                                                                                                     |                          |

NOTE: PLEASE NOTIFY COMMANDER, ARRADCOM, ATTN: BENET WEAPONS LABORATORY, DRDAR-LCB-TL, WATERVLIET ARSENAL, WATERVLIET, N.Y. 12189, OF ANY REQUIRED CHANGES.

# TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST (CONT.)

|                                                                                                                | <u>NO. OF<br/>COPIES</u> |                                                                                                            | <u>NO. OF<br/>COPIES</u> |
|----------------------------------------------------------------------------------------------------------------|--------------------------|------------------------------------------------------------------------------------------------------------|--------------------------|
| COMMANDER<br>US ARMY RESEARCH OFFICE<br>ATTN: CHIEF, IPO<br>P.O. BOX 12211<br>RESEARCH TRIANGLE PARK, NC 27709 | 1                        | DIRECTOR<br>US NAVAL RESEARCH LAB<br>ATTN: DIR, MECH DIV<br>CODE 26-27 (DOC LIB)<br>WASHINGTON, D.C. 20375 | 1<br>1                   |
| COMMANDER<br>US ARMY HARRY DIAMOND LAB<br>ATTN: TECH LIB<br>2800 POWDER MILL ROAD<br>ADELPHIA, MD 20783        | 1                        | METALS & CERAMICS INFO CEN<br>BATTELLE COLUMBUS LAB<br>505 KING AVE<br>COLUMBUS, OHIO 43201                | 1                        |
| COMMANDER<br>NAVAL SURFACE WEAPONS CEN<br>ATTN: TECHNICAL LIBRARY<br>CODE X212<br>DAHLGREN, VA 22448           | 1                        | MATERIEL SYSTEMS ANALYSIS ACTV<br>ATTN: DRSXY-MP<br>ABERDEEN PROVING GROUND<br>MARYLAND 21005              | 1                        |

NOTE: PLEASE NOTIFY COMMANDER, ARRADCOM, ATTN: BENET WEAPONS LABORATORY,  
DRDAR-LCB-TL, WATERVLIET ARSENAL, WATERVLIET, N.Y. 12189, OF ANY  
REQUIRED CHANGES.