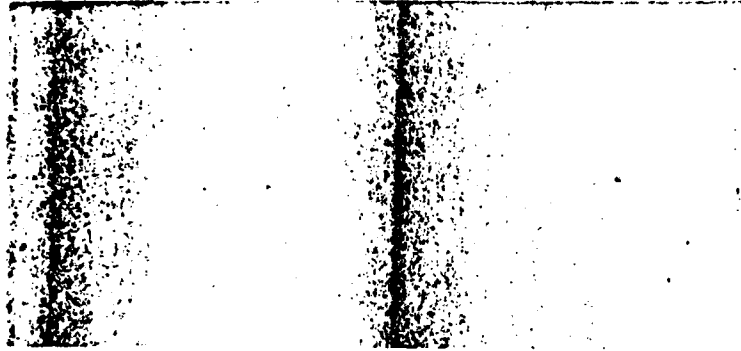


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SCATTERING COEFFICIENT ESTIMATION: AN
EXAMINATION OF THE NARROW-BEAM
APPROXIMATION

by

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ABSTRACT

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One quick and easy way of estimating scattering coefficient is to assume that the antenna beam is confined in a finite cone with constant gain inside and the range to the antenna is constant for all the cells inside the illuminated area. Also the variation of the σ^0 is assumed to be small enough inside the illuminated area so one can estimate the σ^0 with only the power return measurements.

The above may not be true depending on the antenna gain shapes or the target characteristic. In this report, the so-called narrow-beam approximation is examined using a little more realistic antenna gain functions and range variation and also a specific σ^0 variation function. The numerical integration shows that the narrow beam approximation can give as much as 3.6 dB error for normal incidence to the target with rapidly varying σ^0 when the beamwidths are 9^{deg} wide. For the incidence angles of 10^{deg} or larger, the error is in the order of a dB or less depending on the beamwidths and target characteristics.

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1.0 BACKGROUND

Accurate estimation of scattering coefficient for an area-extensive target using the average return power is not a simple task because it involves an integral equation. Hence, it is usually simplified by using several approximations and assumptions. The purpose of this report is to investigate the validity of these commonly used approximations and to evaluate the possible error when these simplifying assumptions are used to calculate the scattering coefficient, for example antenna beamwidths and target scattering coefficient angular variations.

Axline [1] developed a matrix inversion technique to invert the integral equation for various cases of antenna gain shapes and σ^0 variation functions. This technique requires power return measured as a function of incidence angle at a small angular interval and is not readily applicable when this is not true.

Stiles et al. [2] described an empirical procedure to estimate the error, when the narrow-beam approximation is used for small incidence angles, by initially hypothesizing the theoretical $\sigma^0(\theta)$ curve.

Moore [3] suggested an iteration scheme to estimate the scattering coefficient by using the constant- σ^0 approximation to get a first estimate of the σ^0 variation function and then continuously refining that function to get better estimate of σ^0 . This method is used in this report, and the integral is evaluated numerically using the real system parameters of the HELOSCAT III FM-CW scatterometer and a data set taken from sea ice.

2.0 NARROW-BEAM APPROXIMATION

Consider the radar equation

$$P_r = \int \int_{\substack{\text{area} \\ \text{illuminated}}} \frac{P_t G_t^2 \lambda^2 \sigma^o}{(4\pi)^3 R^4} dA \quad (1)$$

where identical transmit- and receive-antenna gains are used.

The first problem here is how to define the illuminated area. Actually the antennae have lobes everywhere, so the area has to be from $-\infty$ to $+\infty$ in the plane of illumination. Usual practice for this is to assume that the antenna beam is confined in a finite cone within which the maximum value of gain applies, with zero gain elsewhere. The effective beamwidth, β_{eff} , is defined in such a way that the pattern solid angles for the two-way real antenna beam and the cone-shaped beam are the same (see Figure 1). Therefore,

$$\begin{aligned} \int \int_{4\pi} G^2(\theta, \phi) d\Omega &= \int_0^{2\pi} \int_0^{\pi} G^2(\theta, \phi) \sin\theta d\theta d\phi \\ &\triangleq \int_0^{2\pi} \int_0^{\left(\frac{\beta_{\text{eff}}}{2}\right)} G_0^2 \sin\theta d\theta d\phi \end{aligned} \quad (2)$$

where G_0^2 is the maximum gain for the two-way pattern. This is not a simple task unless $G^2(\theta, \phi)$ is completely known. If the gain function is assumed to be a Gaussian and if it is ϕ -independent, then

$$2\pi \int_0^{\pi} G_0^2 e^{-2.773\theta^2/\beta_{3\text{dB}}^2} \sin\theta d\theta \triangleq 2\pi \int_0^{\frac{\beta_{\text{eff}}}{2}} G_0^2 \sin\theta d\theta \quad (3)$$

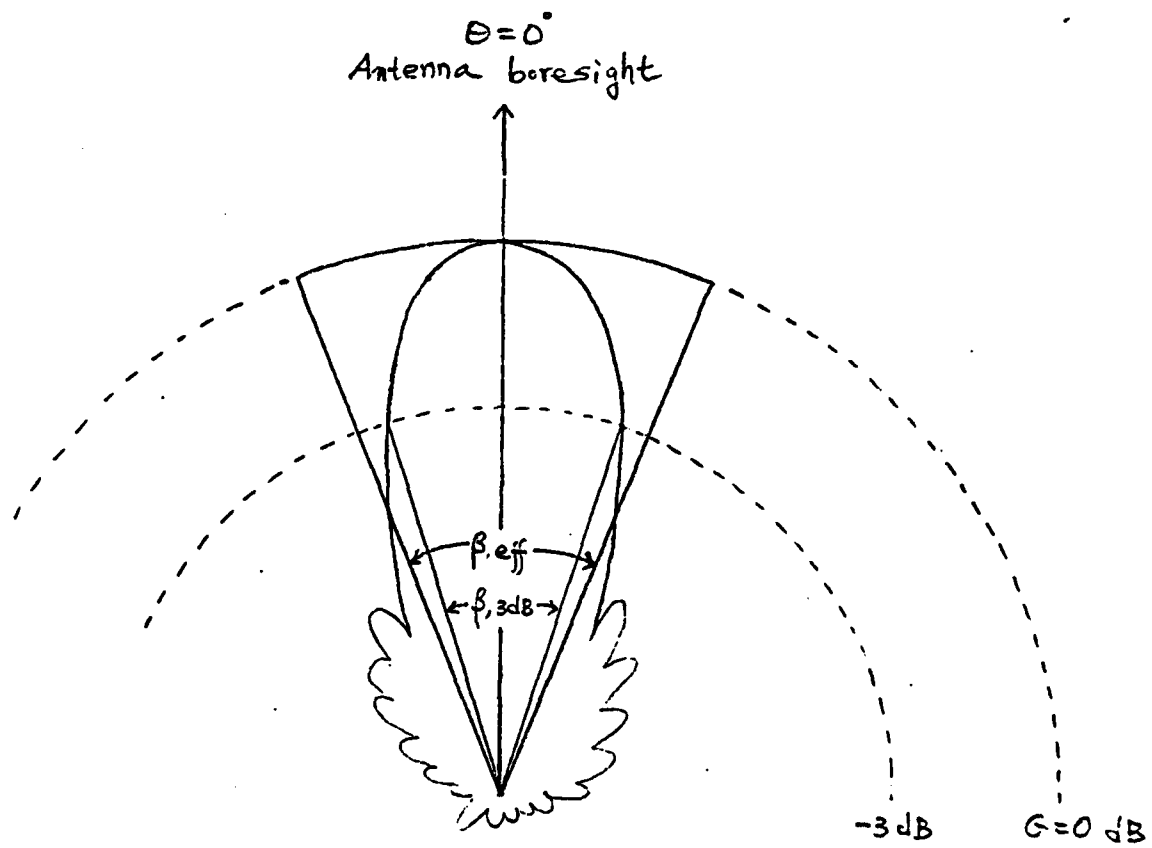


Figure 1: Effective Beamwidth and Half-Power Beamwidth

which reduces to

$$\beta_{\text{eff}} = 2 \cos^{-1} \left(1 - \int_0^{\pi} e^{-2.773\theta^2/\beta_{3\text{dB}}^2} \sin\theta d\theta \right) \quad (4)$$

This equation can be solved numerically and for the $\beta_{3\text{dB}}$ of up to 15° , the following relation can be used.

$$\beta_{\text{eff}} = 1.2 \beta_{3\text{dB}} \quad (5)$$

Antenna beamwidths are usually measured in azimuth and elevation planes. When the measured patterns are considerably different from each other, separate effective beamwidths are defined.

$$\beta_{\text{a,eff}} = 1.2 \beta_{\text{a,3dB}} \quad (6)$$

$$\beta_{\text{e,eff}} = 1.2 \beta_{\text{e,3dB}} \quad (7)$$

The illuminated area is then the ground projection (Figure 2) of the conical beam and the area of the ellipse is approximately

$$A_i = \frac{\pi h^2}{2 \cos \theta_p} \left[\tan \left(\theta_p + \frac{\beta_{\text{e,eff}}}{2} \right) - \tan \left(\theta_p - \frac{\beta_{\text{e,eff}}}{2} \right) \right] \tan \left(\frac{\beta_{\text{a,eff}}}{2} \right) \quad (8)$$

If the beamwidths are narrow enough, R^4 inside the integral in Eq. (1) can be treated to be constant; and if σ^0 varies slowly inside the illuminated area given by Eq. (8), one can remove the σ^0 from the integral and solve for σ^0 .

$$\sigma^0 = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t} \right) \left(\frac{R_0}{G_0} \right)^4 \frac{1}{A_i} \quad (9)$$

This is the normal practice. The variations in transmitted power, P_t and G_0^2 are found using a delay line calibration scheme and standard point-target measurements, respectively. However, these approximations of constant-gain,

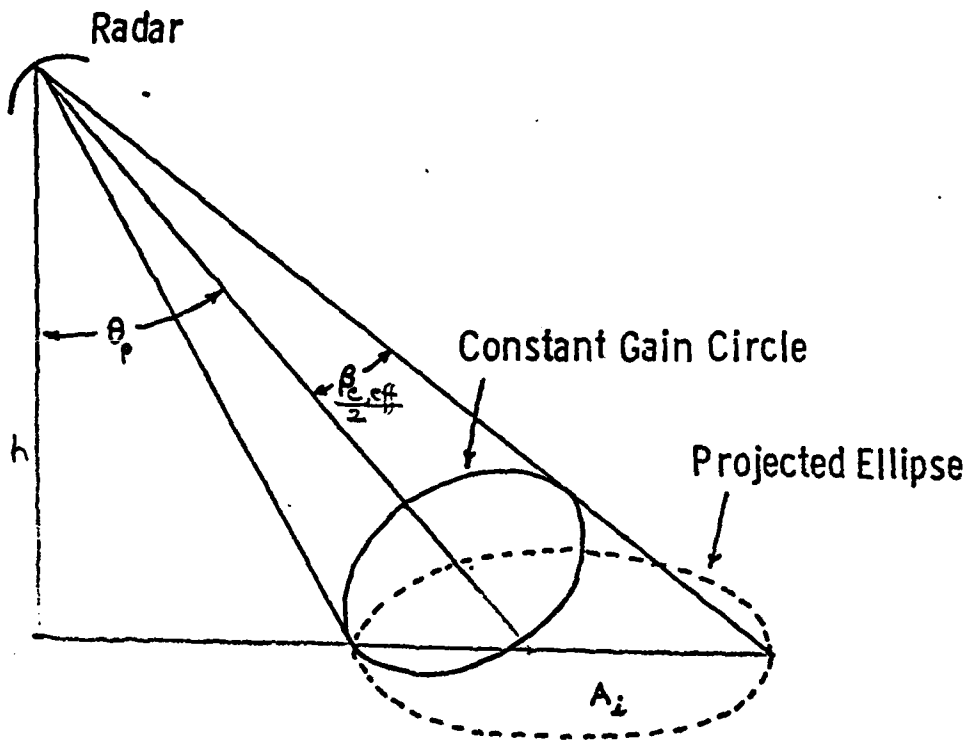


Figure 2: Effective Illuminated Area

finite-beam, constant-range, constant- σ^0 inside the integral may not be true for a wide-beam, an antenna pattern with high sidelobes, near-nadir or near grazing angles of incidence.

3.0 EFFECT OF ANTENNA GAIN AND RANGE VARIATION INSIDE THE INTEGRAL

As a first attempt, the effects of a more realistic antenna gain and R^4 inside the integral are evaluated. The effect of σ^0 variation across the illuminated area will be treated in Section 4.0.

If a full set of antenna patterns in all directions is available, these numerical values can be entered as a gain function. But this is not usually the situation and some kind of functional approximation of the antenna gain is inevitable.

3.1 Gaussian Antenna Gain

If the antenna gain can be represented as a Gaussian, then

$$G^2 = G^2(\theta_a, \phi_a) = G_0^2 \exp(-2.773(\theta_a - \theta_p)^2 / \beta_{e,3dB}^2) \exp(-2.773\phi_a^2 / \beta_{a,3dB}^2) \quad (10)$$

where:

θ_p = antenna boresight pointing angle off vertical (in x-z plane)

θ_a = angle off vertical in elevation plane for observation direction

ϕ_a = angle off x-z plane for observation direction

$\beta_{e,3dB}$ = two-way 3 dB beamwidth in elevation plane

$\beta_{a,3dB}$ = two-way 3 dB beamwidth in azimuth plane

For point P(x,y) on the ground (see Figure 3),

$$\theta_a = \tan^{-1}(x/h) \quad (11)$$

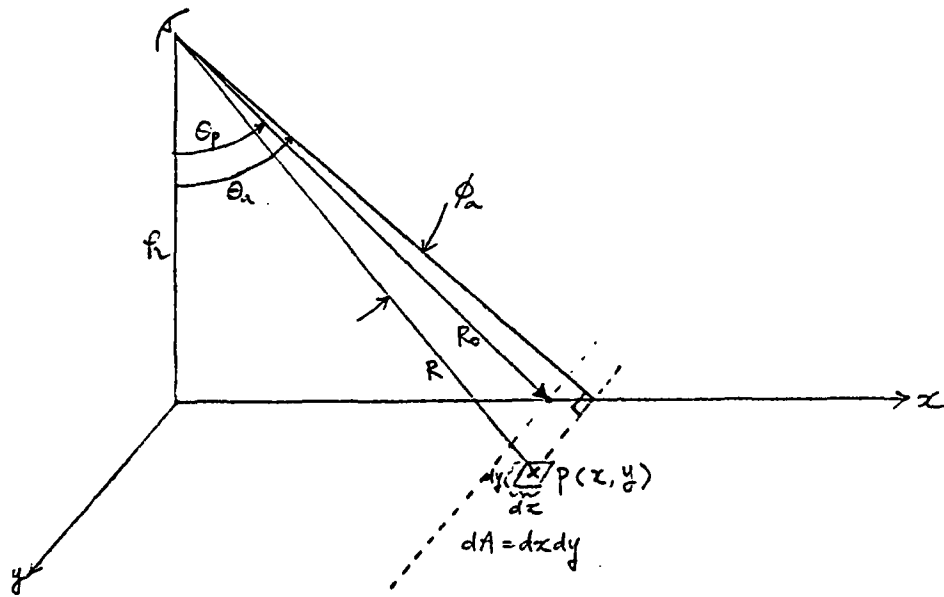


Figure 3: Coordinate System for the Antenna Gain Functions

$$\phi_a = \tan^{-1}(y/\sqrt{x^2+h^2}) \quad (12)$$

This coordinate system is not a spherical coordinate system, but this is used because the antenna patterns are available in these azimuth and elevation planes.

3.2 $(\sin x/x)^4$ Antenna Gain

One-way antenna gain can also be represented by $(\sin x/x)^2$. In terms of one-way 3 dB beamwidth, β'_{3dB} ,

$$G(\theta) = G_o \left[\frac{\sin(2.7831\theta/\beta'_{3dB})}{2.7831\theta/\beta'_{3dB}} \right]^2 \quad (13)$$

Therefore, in the coordinate system shown in Figure 3, two-way antenna gain can be represented as

$$G^2(\theta_a, \phi_a) = G_o^2 \left(\frac{\sin x_\theta}{x_\theta} \right)^4 \left(\frac{\sin x_\phi}{x_\phi} \right)^4 \quad (14)$$

where:

$$x_\theta = 2.7831 (\theta_a - \theta_p) / \beta'_{a,3dB}$$

$$x_\phi = 2.7831 \phi_a / \beta'_{a,3dB}$$

$\beta'_{e,3dB}$ = one-way 3 dB beamwidth in elevation plane

$\beta'_{a,3dB}$ = one-way 3 dB beamwidth in azimuth plane

θ_a, ϕ_a = given in Eqs. (11) and (12)

This two-way gain function given by Eq. (14) has many sidelobes and nulls, with the highest sidelobes 26.5 dB down from mainlobe (see Figure 4). The location of the first sidelobes will be about $1.6 \beta'_{3dB}$ away from the center of the mainlobe.

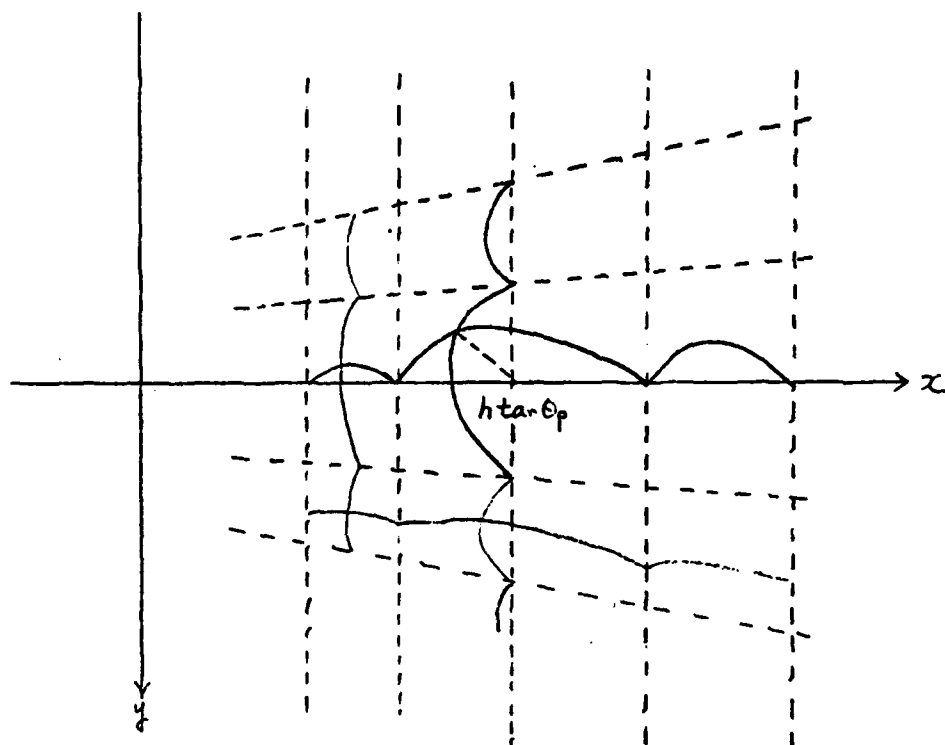


Figure 4: $\left(\frac{\sin x}{x}\right)^4$ Antenna Gain Projected on the Ground

3.3 Range Variation and the Effect of the Bandpass Filter

The range R from the antenna to the infinitesimal area element dA on the ground is

$$R = \sqrt{x^2 + y^2 + h^2} \quad (15)$$

In the actual HELOSCAT III system (an FM radar) the range to the target is limited by the bandpass filter in the final stage. Each area element on the ground has different range to the antenna and thereby gives a different IF frequency. Therefore a bandpass filter with finite bandwidth would limit the range from which the radar receives the scattered field. In the narrow-beam approximation, the bandpass filter plays a role only when the effective beamwidth is very wide and the incidence angle is large enough. In such a case, the filter cuts the area of the ellipse from which σ^0 is calculated.

The transfer function of the bandpass filter shown in Figure 5 can be represented as a function of range as follows.

$$\begin{aligned} f(R) &= 1, && \text{when } R_0(1 - B'/2f_0) \leq R \leq R_0(1 + B'/2f_0) \\ &= 10 \frac{145}{4} \left(\frac{R}{R_0} - (1 - B'/2f_0) \right) && \text{when } R \leq R_0(1 - B'/2f_0) \\ &= 10 \frac{-30}{4} \left(\frac{R}{R_0} - (1 + B'/2f_0) \right) && \text{when } R \geq R_0(1 + B'/2f_0) \end{aligned} \quad (16)$$

where:

R_0 is the distance to the center of the illuminated area

B' is the bandwidth of the filter (Figure 5),

f_0 is the center frequency of the filter.

Theoretically in a FM-CW radar, the range R to the target and the IF frequency output (F_{if}) of the mixer have the following relationship.

$$F_{if} = \frac{4R\Delta F F_m}{c} \quad (17)$$

where:

ΔF = RF frequency sweep width

F_m = modulating frequency

c = velocity of light

A sample of the transmitted waveform long before it goes out of the antenna is usually used as the local oscillator signal to be mixed with the received signal. Hence the actual IF frequency is different from what one expects from Eq. (17), especially when long antenna cables are used. The discrepancy in range should be obtained experimentally as a system characteristic. When this additional range, R_c , is not considered to be negligible, Eq. (17) and the bandpass filter function given by Eq. (16) should be modified accordingly (see Figure 5).

$$F_{if} = \frac{4(R+R_c)\Delta F F_m}{c} \quad (18)$$

$$f(R) = 1, \quad \text{when } R_o(1-B'/2f_o) - (B'/2f_o)R_c \leq R \leq (1+B'/2f_o)R_o + (B'/2f_o)R_c$$

$$= 10^{-145} \left(\frac{R+R_c}{R_o+R_c} - (1-B'/2f_o) \right), \quad \text{when } R \leq (1-B'/2f_o)R_o - (B'/2f_o)R_c$$

$$= 10^{-30} \left(\frac{R+R_c}{R_o+R_c} - (1+B'/2f_o) \right), \quad \text{when } R \geq (1+B'/2f_o)R_o + (B'/2f_o)R_c \quad (19)$$

Examining Eq. (19), one can see that having some additional range, R_c , has effectively widened the bandwidth of the filter.

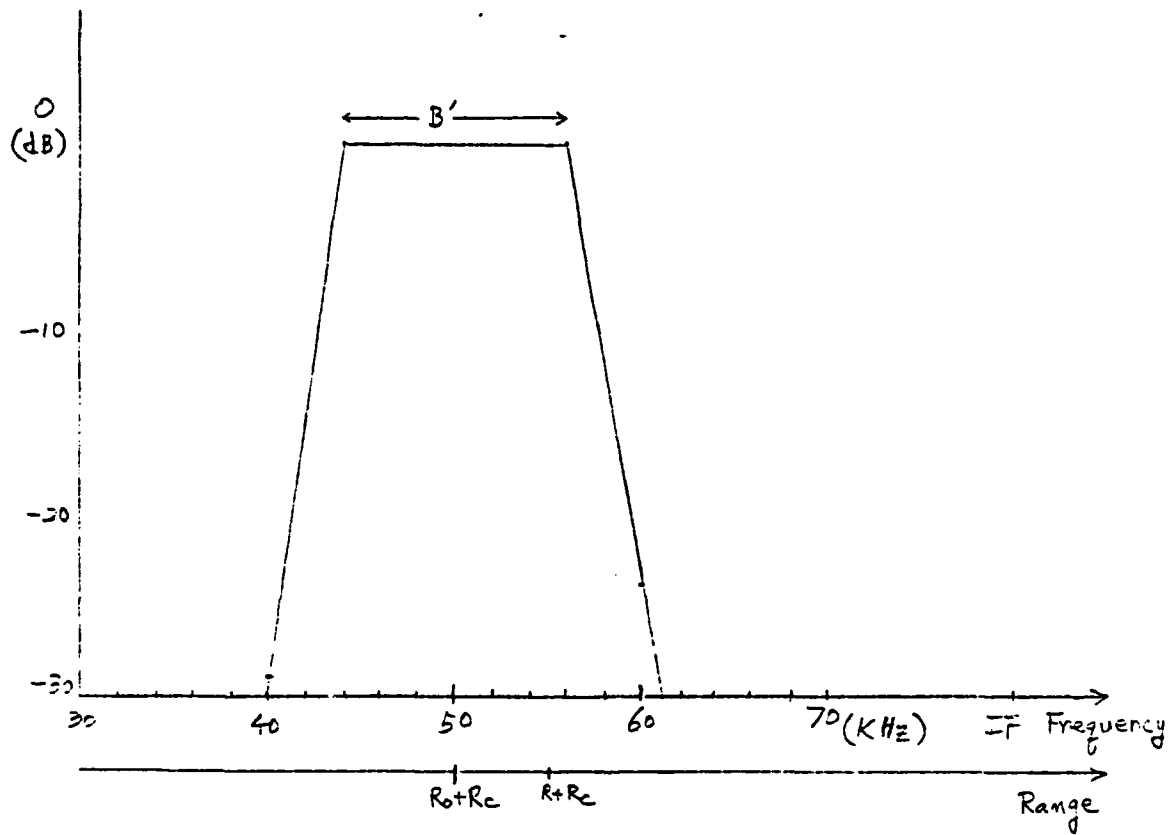


Figure 5: Bandpass Filter Function

3.4 Integration Limits

In this section, σ^0 will be assumed to be constant across the integration limits and evaluated using the following equation.

$$\sigma^0 = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right) \frac{1}{\iint \frac{G^2(\theta_a, \phi_a) f(R)}{R^4} dA} \quad (20)$$

In view of the narrow beam approximation given by Eq. (9), one can rewrite this equation as

$$\sigma^0 = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right) \left(\frac{R_o}{G_o}\right)^4 \frac{1}{\iint \frac{g^2(\theta_a, \phi_a) f(R)}{(R/R_o)^4} dA} \quad (21)$$

where $G^2(\theta_a, \phi_a) = G_o^2 g^2(\theta_a, \phi_a)$.

The integral can be treated as a weighted area [4] compared to the illuminated area described in Section 2.0.

$$A_w \triangleq \iint \frac{g^2(\theta_a, \phi_a) f(R)}{(R/R_o)^4} dA \quad (22)$$

The integration is done using a rectangular coordinate system. Therefore the infinitesimal area $dA = dx dy$ and the integration limits should be the whole ground plane and the filter function will effectively set the limit. To simplify the numerical integration, the integration limits were set to be the points where the antenna gain function reduces to small enough numbers (about -30 dB) for the Gaussian gain, and for the $(\sin x/x)^4$ gain function the limits were up to the first sidelobes. In this case the integral is also evaluated for the mainlobe only to see the effect of sidelobes. Figure 6 shows the integration limits (square area marked -30 dB on four sides)

together with the elliptical (shaded) illuminated area normally used for the narrow-beam approximation. Also, the equal-range circles which effectively limit the area by the filter function can be seen.

The weighted area now becomes

$$A_w = \int_{y^-}^{y^+} \int_x^{x^+} \frac{g^2[\theta_a(x,y), \phi_a(x,y)] f[R(x,y)]}{[R(x,y)/R_0]^4} dx dy \quad (23)$$

where g^2 , R and f are given by Eqs. (10-12), (14), (15), (19) and

$$\begin{aligned} x_{\pm} &= h \tan(\theta_p \pm 1.6 \beta_{e,3dB}) \text{ for Gaussian gain} \\ &= h \tan(\theta_p \pm 3.2 \beta_{e,3dB}) \text{ for } (\sin x/x)^4, \text{ up to first sidelobes} \\ &= h \tan(\theta_p \pm 1.6 \beta_{e,3dB}) \text{ for } (\sin x/x)^4, \text{ mainlobe only} \end{aligned}$$

and

$$\begin{aligned} y_{\pm} &= R_0 \tan(\pm 1.6 \beta_{a,3dB}) \text{ for Gaussian gain} \\ &= R_0 \tan(\pm 3.2 \beta_{a,3dB}) \text{ for } (\sin x/x)^4, \text{ up to first sidelobes} \\ &= R_0 \tan(\pm 1.6 \beta_{a,3dB}) \text{ for } (\sin x/x)^4, \text{ mainlobe only} \end{aligned}$$

and

$$R_0 = h/\cos\theta_p$$

3.5 Results

In this section, the possible error when using constant range and constant gain across the effective beamwidth is defined as follows.

$$\text{ERROR1} = A_i/A_w \quad (24)$$

where:

A_w is the weighted area given by Eq. (23) and

A_i is the area of the ellipse given by Eq. (8)

then

$$\text{ERROR1 (dB)} = 10 \log(A_i/A_w) \quad (25)$$

and this ERROR1 (dB) should be added to the σ^0 (dB) calculated using a narrow-beam approximation.

The numerical integration was done using Gauss' formula [5] and a total of 1600 (40x40) points inside the integration limit shown in Figure 6. Because the number of points evaluated was fixed for all the incidence angles between 0° to 70° while the actual integration limits in x- and y-coordinate grew tremendously from 0° to 70° , the accuracy of the numerical integration tends to get worse at large incidence angles.

Table 1 is a summary of ERROR1 for various antenna beamwidths and antenna gain functions. As can be seen in the table, the constant-range and constant-gain across the effective beamwidths are not bad approximations to use up to the pointing angle of 60° , except when the two-way beamwidths are as large as 9° , if the variations of σ^0 inside the integral can be neglected as assumed here.

4.0 EFFECT OF σ^0 VARIATION INSIDE THE INTEGRAL

Up to now, σ^0 was assumed to be constant within the limits of integration, so σ^0 was outside the integral. Actually, σ^0 itself is a function of the local incidence angle and therefore a function of x- and y-coordinates of the point evaluated (see Figure 7).

TABLE 1
ERRORS

	Pointing Angle $\theta_p(^{\circ})$	Two-Way 3dB Beamwidths			
		$\beta_e = 9.0^{\circ}$ $\beta_a = 9.0^{\circ}$	$\beta_e = 6.3^{\circ}$ $\beta_a = 5.7^{\circ}$	$\beta_e = 5.0^{\circ}$ $\beta_a = 5.0^{\circ}$	$\beta_e = 2.4^{\circ}$ $\beta_a = 2.0^{\circ}$
Gaussian Gain	0	0.055 (dB)	0.025 (dB)	0.017 (dB)	0.004 (dB)
	10	0.057	0.025	0.018	0.004
	20	0.060	0.027	0.019	0.004
	30	0.069	0.031	0.021	0.005
	40	0.095	0.038	0.026	0.006
	50	0.208	0.063	0.035	0.008
	60	0.591*	0.222	0.094	0.012
	70	0.366*	0.616	0.588	0.033
$(\frac{\sin x}{x})^4$ up to first sidelobes	0	0.059	0.029	0.022	0.008
	10	0.060	0.029	0.022	0.008
	20	0.067	0.031	0.023	0.008
	30	0.076	0.037	0.026	0.009
	40	0.094	0.046	0.033	0.010
	50	0.228	0.070	0.044	0.012
	60	0.583*	0.206	0.091	0.021
	70	0.484*	0.484	0.528	0.042
$(\frac{\sin x}{x})^4$ mainlobe only	0	0.079	0.049	0.043	0.030
	10	0.080	0.050	0.043	0.030
	20	0.084	0.052	0.044	0.030
	30	0.092	0.056	0.047	0.030
	40	0.111	0.063	0.051	0.031
	50	0.208	0.080	0.060	0.033
	60	0.561*	0.213	0.102	0.038
	70	0.340*	0.582	0.557	0.052

* These values may not be very accurate due to the large spread between points actually evaluated.

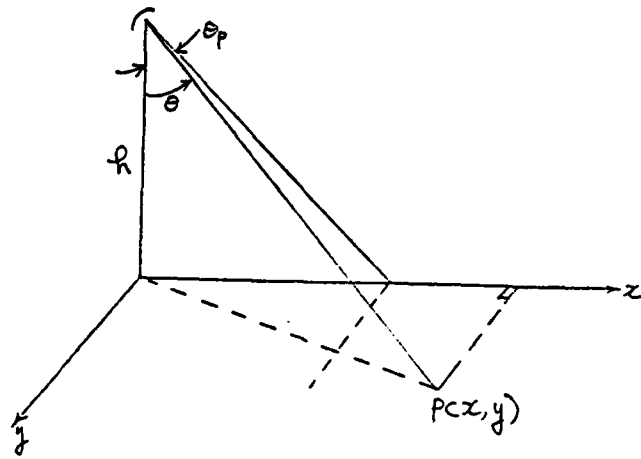


Figure 7: Local Incidence Angle

$$\sigma^0 = \sigma^0(\theta) = \sigma^0(x, y)$$

where θ is the local incidence angle and

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{h} \quad (26)$$

One way of evaluating σ^0 suggested in [3] is to apply Eq. (21) to a set of measurements to get a rough estimate of σ^0 variation with incidence angle, then use this function in the integral (Eq. (1)), and continuously refine the σ^0 variation function to get better estimate of σ^0 .

Variation of σ^0 with incidence angle will take many forms depending on the kind of target. In many cases, including sea ice, it fits quite well with the exponential function used here. From Eqs. (1) and (21),

$$P_r = \frac{\lambda^2}{(4\pi)^3} P_t \frac{G_o^2}{R_o^4} \iint \frac{g^2(\theta_a, \phi_a) f(R) \sigma^0(\theta)}{(R/R_o)^4} dA \quad (27)$$

and $\sigma^0(\theta)$ for some particular angle of incidence can be evaluated as,

$$\sigma_2^0(\theta_p) = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right) \left(\frac{R_o}{G_o}\right)^4 \frac{1}{\iint \frac{g^2 f [\sigma_1(\theta)/\sigma_1(\theta_p)]}{(R/R_o)^4} dA} \quad (28)$$

where $\sigma_1^0(\theta)$ is the function one gets by applying Eq. (21) to a set of measurements. The next iteration becomes

$$\sigma_3^0(\theta_p) = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right) \left(\frac{R_o}{G_o}\right)^4 \frac{1}{\iint \frac{g^2 f [\sigma_2(\theta)/\sigma_2(\theta_p)]}{(R/R_o)^4} dA} \quad (29)$$

and so on.

If we assume the form $\sigma^0(\theta) = ae^{-b\theta}$, then

$$\sigma_1^o(\theta)/\sigma_1^o(\theta_p) = e^{-b_1(\theta-\theta_p)} \text{ and } \sigma_2^o(\theta)/\sigma_2^o(\theta_p) = e^{-b_2(\theta-\theta_p)}$$

and σ^o variation function around the pointing angle, θ_p , can be defined as

$$S_n(\theta, \theta_p) \triangleq \frac{\sigma_n^o(\theta)}{\sigma_n^o(\theta_p)} = e^{-b_n(\theta-\theta_p)} \quad (30)$$

and the n-th refined σ^o becomes

$$\sigma_n^o(\theta_p) = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right) \left(\frac{R_o}{G_o}\right)^4 \frac{1}{\iint \frac{g^2 f S_{n-1}(\theta, \theta_p)}{(R/R_o)^4} dA} \quad (31)$$

Hence the integral can be treated as another weighted area similar to that of Eq. (23).

$$A_{ws} \triangleq \iint \frac{g^2 f S_{n-1}}{(R/R_o)^4} dA \quad (32)$$

The possible error when the σ^o variation function is neglected is defined as

$$\text{ERROR2 (dB)} = 10 \log(A_w/A_{ws}) \quad (33)$$

and numerically evaluated (Table 2) for several antenna gain functions, beam-widths and σ^o variation across the integration limits (assuming that the variation is known in advance). In Table 2, the functions of $e^{-\theta/12.9^\circ}$ and $e^{-\theta/9.4^\circ}$ were derived from linear regression (in dB scale) of the data points obtained from sea ice using the narrow beam approximation and the steep variation function of $e^{-\theta/5^\circ}$ is appropriate to some open water.

In the next section, the iteration will be tried. The error caused by not considering steep σ^o variation can be as large as 3.6 dB for normal

TABLE 2

ERROR2

Pointing Angle θ_p (°)	σ° -variation and Two-way 3 dB Beamwidths				
	$\sigma^\circ = e^{-\theta/5^\circ}$	$\sigma^\circ = e^{-\theta/12.9^\circ}$	$\sigma^\circ = 3^{-\theta/5^\circ}$	$\sigma^\circ = e^{-\theta/9.4^\circ}$	
	$\beta_e = 9.0^\circ$ $\beta_a = 9.0^\circ$	$\beta_e = 6.3^\circ$ $\beta_a = 5.7^\circ$	$\beta_e = 5.0^\circ$ $\beta_a = 5.0^\circ$	$\beta_e = 2.4^\circ$ $\beta_a = 2.0^\circ$	
Gaussian Gain	0	3.647 (dB)	1.040 (dB)	2.140 (dB)	0.532 (dB)
	10	-0.484	0.004	-0.193	-0.010
	20	-0.994	-0.060	-0.314	-0.020
	30	-1.170	-0.087	-0.365	-0.025
	40	-1.301	-0.108	-0.401	-0.029
	50	-1.456	-0.133	-0.438	-0.033
	60	-1.386	-0.190	-0.505	-0.038
	70	-0.398	-0.184	-0.548	-0.050
$\left(\frac{\sin x}{x}\right)^4$ up to first sidelobes	0	3.613	1.026	2.114	0.526
	10	-0.450	0.008	-0.184	-0.010
	20	-1.015	-0.061	-0.326	-0.020
	30	-1.330	-0.089	-0.377	-0.025
	40	-1.405	-0.111	-0.417	-0.029
	50	-1.359	-0.122	-0.437	-0.033
	60	-1.366	-0.176	-0.458	-0.040
	70	-0.633	-0.205	-0.555	-0.044
$\left(\frac{\sin x}{x}\right)^4$ mainlobe only	0	3.574	1.012	2.090	0.517
	10	-0.466	0.003	-0.177	-0.009
	20	-0.904	-0.055	-0.289	-0.019
	30	-1.063	-0.080	-0.336	-0.023
	40	-1.177	-0.099	-0.369	-0.027
	50	-1.339	-0.120	-0.402	-0.030
	60	-1.388	-0.177	-0.459	-0.035
	70	-1.010	-0.186	-0.535	-0.044

incidence and in the order of 1 dB for other incidence angles when the beamwidths are 9° . When the beamwidths are 5° , the error is 2.1 dB for normal incidence and less than 0.5 dB for other incidence angles. When the σ^0 variation is not so rapid ($e^{-\theta/12.9^\circ}$ or $e^{-\theta/9.4^\circ}$), the error reduces to 1 dB or less for normal incidence and becomes very small for other incidence angles. Recall from Section 3.5 that the numerical integration may have some error for large incidence angles. Table 2 also shows that the $(\sin x/x)^4$ gain function has such a low sidelobe level (-26.5 dB) that integrating up to first sidelobes did not differ significantly from integrating only the mainlobe. Furthermore, the $(\sin x/x)^4$ antenna gain function and the Gaussian gain function did not show very much difference.

4.1 Iteration

As explained in the previous section, iteration is tried with real data and some arbitrary data which fits well with the exponential function. Because the real data points are from 10° to 70° , only these points are evaluated to get an exponential fit and then iterated several times. Table 3 is a summary of the result. The first row shows the σ^0 calculated using the narrow beam approximation (σ_{nb}^0) and the second row is the modified σ^0 computed using gain function and range variations ($\sigma_{nb}^0 + \text{ERROR1}$). These points are regressed to get an exponential fit, and this exponential function is included in the integration to give the third row ($\sigma_{nb}^0 + \text{ERROR1} + \text{ERROR2}$). From these points we get new slope and therefore new ERROR2, and for the fourth row, $\sigma^0 = \sigma_{nb}^0 + \text{ERROR1} + \text{ERROR2}$, and so on.

TABLE 3
 σ^0 DATA, ORIGINAL AND UP TO 4th REFINED

1. 13.6 GHz Sea Ice Data ($\sigma^0 = e^{-\theta/9.4^\circ}$)
 $\beta_e = 2.4^\circ$ $\beta_a = 2.0^\circ$

	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE CORRELATION	
Gaussian Gain		2.70	2.60	-1.70	-6.80	-10.00	-14.90	-26.00	-0.4621	-0.9643
		2.71	2.61	-1.69	-6.79	-9.99	-14.88	-25.96	-0.4617	-0.9644
		2.70	2.59	-1.72	-6.82	-10.02	-14.92	-26.00	-0.4622	-0.9645
		2.70	2.59	-1.72	-6.82	-10.02	-14.92	-26.00	-0.4622	-0.9645
		2.70	2.59	-1.72	-6.82	-10.02	-14.92	-26.00	-0.4622	-0.9645
$\left(\frac{\sin \tau}{\tau}\right)^4$ up to first sidelobes	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE CORRELATION	
		2.70	2.60	-1.70	-6.80	-10.00	-14.90	-26.00	-0.4621	-0.9643
		2.70	2.60	-1.70	-6.79	-9.99	-14.89	-25.97	-0.4618	-0.9644
		2.69	2.58	-1.72	-6.82	-10.03	-14.93	-26.02	-0.4623	-0.9645
		2.69	2.58	-1.72	-6.82	-10.03	-14.93	-26.02	-0.4623	-0.9645

2. 4.8 GHz Sea Ice Data ($\sigma^0 = e^{-\theta/12.9^\circ}$)
 $\beta_e = 6.3^\circ$ $\beta_a = 5.7^\circ$

	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE CORRELATION	
Gaussian Gain		-2.70	-4.90	-10.50	-12.30	-14.80	-18.00	-24.00	-0.3371	-0.9886
		-2.67	-4.87	-10.47	-12.26	-14.74	-17.78	-23.38	-0.3293	-0.9893
		-2.67	-4.93	-10.55	-12.36	-14.87	-17.96	-23.56	-0.3324	-0.9901
		-2.67	-4.93	-10.55	-12.37	-14.87	-17.97	-23.56	-0.3324	-0.9901
		-2.67	-4.93	-10.55	-12.37	-14.87	-17.97	-23.56	-0.3324	-0.9901
$\left(\frac{\sin \tau}{\tau}\right)^4$ up to first sidelobes	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE CORRELATION	
		-2.70	-4.90	-10.50	-12.30	-14.80	-18.00	-24.00	-0.3371	-0.9886
		-2.67	-4.87	-10.46	-12.25	-14.73	-17.79	-23.52	-0.3309	-0.9894
		-2.66	-4.93	-10.55	-12.36	-14.85	-17.97	-23.72	-0.3341	-0.9896
		-2.66	-4.93	-10.55	-12.36	-14.85	-17.97	-23.72	-0.3341	-0.9896

3. Arbitrary Data ($\sigma^0 = e^{-\theta/5^\circ}$ $\beta_e = \beta_a = 9^\circ$)

	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE CORRELATION	
Gaussian Gain		-8.70	-17.00	-26.60	-33.00	-42.00	-53.00	-60.00	-0.8618	-0.9987
		-8.64	-16.94	-26.53	-32.91	-41.79	-52.41	-59.63	-0.8542	-0.9989
		-9.11	-17.90	-27.67	-34.17	-43.22	-53.78	-60.62	-0.8636	-0.9989
		-9.13	-17.93	-27.69	-34.20	-43.25	-53.80	-60.63	-0.8636	-0.9989
		-9.13	-17.93	-27.69	-34.20	-43.25	-53.80	-60.63	-0.8636	-0.9989
$\left(\frac{\sin \tau}{\tau}\right)^4$ up to first sidelobes	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE CORRELATION	
		-8.70	-17.00	-26.60	-33.00	-42.00	-53.00	-60.00	-0.8618	-0.9987
		-8.64	-16.93	-26.52	-32.91	-41.77	-52.42	-59.52	-0.8530	-0.9988
		-9.07	-17.92	-27.81	-34.27	-43.10	-53.76	-60.14	-0.8577	-0.9987
		-9.08	-17.93	-27.83	-34.29	-43.11	-53.77	-60.14	-0.8577	-0.9987

For all the cases in Table 3, the data seems to converge after three iterations, so it did not require further iteration. Also from Table 1 and Table 2, it can be seen that the ERROR1 and ERROR2 tend to cancel out each other to make σ_{nb}^0 a tolerable estimation at least for the incidence angles of 10° to 70° and when the beamwidths are not very wide.

5.0 SUMMARY

Three kinds of so-called illuminated areas are treated in this report:

- (1) the area of an ellipse with constant gain and constant range to the antenna for all the cells within the ellipse, A_i , (Eq. (8)).
- (2) the area weighted with antenna gain and range, A_w (Eq. (23)).
- (3) the area weighted with antenna gain and range and also σ^0 variation, A_{ws} , (Eq. (32)).

The possible error when using A_i instead of A_w was defined as ERROR1 and summarized in Table 1. When the σ^0 variation with incidence angle is small, the result shows that the constant range and constant gain approximation gives a result within 0.6 dB or less.

Next, σ^0 variation is included and the possible error of using A_w instead of A_{ws} was defined as ERROR2 and listed in Table 2. For normal incidence, one can have up to 3.6 dB error when the beamwidth is 9° for rapidly varying $(e^{-\theta/5^\circ})\sigma^0$. The big error at normal incidence is always present and one must consider this problem when σ^0 for normal incidence is evaluated. The error will be bigger than the values obtained here if the real σ^0 variation near vertical is steeper than the exponential function, which is often true for very smooth targets.

The more realistic σ^0 values corrected with these estimated errors were further refined using iterative scheme. Actually, however, the iteration did not show any further improvement because it converged almost instantly for the exponential σ^0 variation function assumed in this report. This may not be true for other kinds of σ^0 variation functions and can be further investigated using the technique described in this report.

REFERENCES

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- [3] Moore, R.K., "Radar Return from Ground," Chapter 25 of Radar Handbook (M.I. Skolnik, ed.), New York: McGraw-Hill, 1970.
- [4] Ulaby, F.T., R.K. Moore and A.K. Fung, Microwave Remote Sensing: Active and Passive, Vol. II, Chapter 10, Reading, Massachusetts: Addison-Wesley, 1982.
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APPENDIX
PROGRAM LISTING

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10C THIS PROGRAM IS TO EVALUATE THE POSSIBLE ERROR WHEN
20C USING THE NARROW BEAM APPROXIMATION IN CALCULATING
30C THE SCATTERING COEFFICIENT.
40C ANTENNA GAIN FUNCTIONS, GAUSSIAN AND (SIN(X)/X)**4,
50C AND REAL RANGE AND TRAPEZOIDAL BAND PASS FILTER FUNCTION
60C AND THE EXPONENTIAL SIGMA VARIATION FUNCTION IS EVALUATED
70C IN THE INTEGRAL USING GAUSS' FORMULA. 40 POINT BY 40 POINT
80C TOTAL 1600 POINTS ARE EVALUATED IN THE INTEGRATION LIMIT.
90C

```

FUNCTIONS REFERENCED

```

100C GAIN1=GAUSSIAN ANTENNA GAIN
110C GAIN2=(SIN(X)/X)**4 GAIN
120C BPF1=BANDPASS FILTER FUNCTION
130C ASIGMA=EXPONENTIAL SIGMA VARIATION FUNCTION
140C REGRES=LINEAR REGRESSION ROUTINE
150C AVG=AVERAGING ROUTINE
160C AREA=EFFECTIVE AREA CALCULATING ROUTINE FOR THE
170C NARROW BEAM APPROXIMATION CASE
180C

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VARIABLES USED

```

190C BE=TWO WAY 3 DB ELEVATION BEAMWIDTHS
200C BA=TWO WAY 3 DB AZIMUTH BEAMWIDTHS
210C ANGLE=POINTING ANGLE
220C THEO=ANGLE IN RADIAN
230C W(40),X(40)=GAUSS' CONSTANTS FOR INTEGRATION
240C RO=RANGE TO THE BEAM CENTER
250C UPX,UPY=UPPER LIMITS IN X AND Y COORDINATES
260C LOWX,LOWY=LOWER LIMITS
270C H=ANTENNA HEIGHT
280C ERROR1=ERROR FOR NOT COUNTING GAIN AND RANGE VARIATION
290C ERROR2=ERROR FOR NOT COUNTING SIGMA
300C EAREA=EFFECTIVE AREA OF THE ELLIPSE
310C WAREA=GAIN AND RANGE WEIGHTED AREA
320C RAREA=SIGMA,GAIN,RANGE WEIGHTED AREAS
330C AL=CONST TO SET THE LIMIT OF INTEGRATION
340C
350C

```

COMMON BE,BA

```

360 REAL BE,BA
370 REAL ANGLE,THEO,PI,W(40),X(40),RO,UPX,LOWX,UPY,LOWY
380 REAL HALFV,HALFY,AVGX,AVGY,ISUM,JSUM,XX,YY,H,G,F
390 REAL ISUM1,JSUM1,PROD1,ERROR1(7),ERROR2(7),RAREA
400 REAL PROD,RANGE,INTGRL,ERRDR,ERRDS
410 REAL WAREA,EAREA,AREA,ABE,ABA,EBE,EBA
420 REAL FILTER,BPFO,BPF1
430 REAL BCONST,CCONST,SIGMA,AL,ASIGMA
440 REAL SIGMA0(7),SIGMA1(7)
450 INTEGER I,II,J,K
460 DATA H/3.6/
470 DATA (W(I),I=1,20)/.07750594,.07703981,.07611036,.07472316,
480 .07288658,.07051164,.06791204,.06480401,
490 .06130624,.05743976,.05322784,.04869580,
500 .04387090,.03878216,.03346019,.02793700,
510 .02224584,.01642105,.01049828,.00452127,
520 DATA (X(I),I=1,20)/.03877241,.11608407,.19269758,.26815218,
530 .34199409,.41377920,.48307580,.54946712,
540 .61255388,.67195068,.72731825,.77830565,
550 .82461223,.86595950,.90209880,.93281280,
560 .95791581,.97725994,.99072623,.99823770/
570
580
590
600

```

SYSTEM CONST FOR INTERNAL RANGE
DATA RCONST/2.6/

```

610C
620C CONSTANT TO CONVERT SLOPE IN DB SCALE INTO EXPONENTIAL SLOPE
630 BBB=10.*ALOG10(EXP(1.))
640 PI=4.*ATAN(1_0)
650C
660C TWO WAY 3 DB BEAMWIDTHS IN DEGREES
670 ABE=9.0
680 ABA=9.0
690C
700 BE=ABE*PI/180.
710 BA=ABA*PI/180.
720C TWO WAY EFFECTIVE BEAMWIDTHS IN DEGREES
730 EBE=1.201*ABE
740 EBA=1.201*ABA
750C
760C INPUT SIGMAO AND OUTPUT THESE INPUT PARAMETERS
770 DATA SIGMAO/-8.7,-17.,-26.6,-33.,-42.,-53.,-60./
780 N=7
790 CALL REGRES(SIGMAO,N,SLOPE,CORR)
800 WRITE(6,699)
810 699 FORMAT(/1X,'SIGMAO DATA, ORIGINAL AND UP TO 4TH REFINED')
820 WRITE(6,698)
830 698 FORMAT(/1X,'ANGLE 10 20 30 40 50 60
840 70 SLOPE CORRELATION')
850 WRITE(6,697) (SIGMAO(I),I=1,7),SLOPE,CORR
860 697 FORMAT(4X,7F7.2,1X,F7.4,2X,F7.4)
870C
880C
890C ASSIGN INTEGRATION CONSTS
900 DO 100 I=21,40
910 W(I)=W(I-20)
920 X(I)=-X(I-20)
930 100 CONTINUE
940C
950C DO UNTIL 4 TH ITERATION
960C
970 DO 89 II=1,4
980 SLOPP=-BBB/SLOPE
990C
1000 DO 90 K=1,7
1010 ANGLE=FLOAT(K)*10.
1020 THEO=PI*ANGLE/180.
1030C
1040C
1050 RO=H/COS(THEO)
1060C
1070C LIMITS UP TO FIRST SIDELOBES OF (SIN(X)/X)**4
1080 AL=PI*1.414/2.7831
1090 ALU=AL
1100 IF(K.EQ.6.OR.<.EQ.7) ALU=1.5
1110 UPX=H*(TAN(THEO+ALU*BE))
1120 LOWX=H*(TAN(THEO-AL*BE))
1130 UPY=RO*TAN(AL*BA)
1140 LOWY=-UPY
1150 HALFX=(UPX-LOWX)/2.
1160 HALFY=(UPY-LOWY)/2.
1170 AVGX=(UPX+LOWX)/2.
1180 AVGY=(UPY+LOWY)/2.
1190C
1200 ISUM1=0.

```

```

1210 ISUM=0.
1220 DO 101 I=1,20
1230 YY=X(I)*HALFY+AVGY
1240 JSUM=0
1250 JSUM1=0.
1260 C
1270 DO 102 J=1,40
1280 XX=X(J)*HALFX+AVGX
1290 CALL GAIN1 (XX,YY,H,THEO,G)
1300 RANGE=SQRT (XX**2+YY**2+H**2)
1310 FILTER=BPF1 (RANGE,RO,RCONST)
1320 RANGE4=RANGE**4
1330 SIGMA=ASIGMA (XX,YY,H,ANGLE,SLOPP)
1340 PROD=G*FILTER*SIGMA/RANGE4
1350 C
1360 PROD1=G*FILTER/RANGE4
1370 JSUM1=JSUM1+W(J)*PROD1
1380 JSUM=JSUM+W(J)*PROD
1390 102 CONTINUE
1400 C
1410 ISUM=ISUM+W(I)*JSUM
1420 ISUM1=ISUM1+W(I)*JSUM1
1430 101 CONTINUE
1440 C
1450 INTGRL=2.*HALFX*HALFY*ISUM1
1460 C
1470 C WEIGHTED AREA USING 3DB BEAMWIDTHS WITHOUT SIGMA VARIATION
1480 WAREA=RO**4*INTGRL
1490 C
1500 C WEIGHTED AREA INCLUDING SIGMA VARIATION
1510 RAREA=2.*HALFX*HALFY*ISUM*(RO**4)
1520 C
1530 C EFFECTIVE AREA USING EFFECTIVE BEAMWIDTHS
1540 C AND CONST GAIN AND CONST RANGE
1550 EAREA=AREA (EBE,EBA,ANGLE,H,RCONST)
1560 C
1570 ERROR1(K)=10.*ALOG10 (WAREA/EAREA)
1580 ERROR2(K)=10.*ALOG10 (RAREA/WAREA)
1590 C
1600 C WRITE(6,999) ANGLE,ERROR1,ERROR2
1610 C 999 FORMAT (1X,'ANGLE=' ,F5.1,2X,'ERROR1=' ,F6.3,' DB',2X,
1620 C & 'ERROR2=' ,F6.3,' DB')
1630 C
1640 C WRITE(6,997) EAREA,WAREA,RAREA
1650 C 997 FORMAT (12X,'EAREA=' ,F9.3,2X,'WAREA=' ,F9.3,2X,'RAREA=' ,F9.3)
1660 C
1670 C 90 CONTINUE
1680 C
1690 IF (II.NE. 1) GO TO 112
1700 DO 111 I=1,7
1710 SIGMA1(I)=SIGMA(I)-ERROR1(I)
1720 111 CONTINUE
1730 C
1740 CALL REGRES (SIGMA1,N,SLOPE,CORR)
1750 WRITE (6,697) (SIGMA1(I),I=1,7),SLOPE,CORR
1760 GO TO 114
1770 112 CONTINUE
1780 C
1790 DO 113 I=1,7
1800 SIGMA(I)=SIGMA1(I)-ERROR2(I)

```

```

1810      113 CONTINUE
1820 C
1830      CALL REGRES(SIGMAO,N,SLOPE,CORR)
1840      WRITE(6,697) (SIGMAO(I),I=1,7),SLOPE,CORR
1850      114 CONTINUE
1860      89 CONTINUE
1870      STOP
1880      END
1890 C
1900 C *****
1910      SUBROUTINE GAIN1(X,Y,HH,TD,GG)
1920 C
1930 C GAUSSIAN ANTENNA GAIN FUNCTION
1940 C WITH TWO WAY BEAMWIDTHS=BE,BA
1950 C
1960      COMMON BE,BA
1970      REAL BE,BA
1980      REAL X,Y,HH,TD,GG
1990      REAL THEA,PHIA,G1,G2,AA
2000 C
2010      THEA=ATAN(X/HH)
2020      PHIA=ATAN(Y/(SQRT(X*X+HH*HH)))
2030      AA=4.*ALOG(2.)
2040      G1=EXP(-AA*((THEA-TD)**2)/(BE**2))
2050      G2=EXP(-AA*(PHIA**2)/(BA**2))
2060      GG=G1*G2
2070      RETURN
2080      END
2090 C
2100 C *****
2110      SUBROUTINE GAIN2(X,Y,HH,TD,GG)
2120 C
2130 C (SIN(X)/X)**4 TWO WAY ANTENNA GAIN PATTERN
2140 C WITH TWO WAY BEAMWIDTHS=BE,BA
2150 C
2160      COMMON BE,BA
2170      REAL BE,BA
2180      REAL X,Y,HH,TD,GG
2190      REAL THEA,PHIA,G1,G2,BEONE,BAONE,A1,A2,G11,G22
2200 C
2210 C ONE WAY 3DB BEAMWIDTHS
2220      BEONE=BE*SQRT(2.)
2230      BAONE=BA*SQRT(2.)
2240 C
2250      THEA=ATAN(X/HH)
2260      PHIA=ATAN(Y/(SQRT(X*X+HH*HH)))
2270      A1=2.7831*(THEA-TD)/BEONE
2280      A2=2.7831*PHIA/BAONE
2290      G1=ABS((SIN(A1))/A1)
2300      IF(G1.LE.(1.0E-10)) GO TO 400
2310      G11=G1**4
2320      G2=ABS((SIN(A2))/A2)
2330      IF(G2.LE.(1.0E-10)) GO TO 400
2340      G22=G2**4
2350      GG=G11*G22
2360      RETURN
2370 C
2380      400 CONTINUE
2390      GG=0.
2400      RETURN

```

```

2410      END
2420 C *****
2430      REAL FUNCTION BPF1(R,RO,CONST)
2440 C
2450 C      TRAPEZOIDAL BANDPASS FILTER FUNCTION
2460 C
2470      REAL R,RO,CONST
2480      REAL FIL,BW1,CF,LOWR,HIGHR,RRO,HF,LF
2490      DATA BW1,CF/12.0,50.0/
2500 C
2510      FIL=BW1/(2.*CF)
2520      HF=1.+FIL
2530      LF=1.-FIL
2540      LOWR=RO*LF-FIL*CONST
2550      HIGHR=RO*HF+FIL*CONST
2560      RRO=(R+CONST)/(RO+CONST)
2570      IF(R.GT.HIGHR .AND. RRO.LE.(1.2*HF)) BPF1=10.**(-30.*(RRO-HF))
2580      IF(RRO.GT.(1.2*HF)) BPF1=0.
2590      IF(R.LE.HIGHR .AND. R.GE.LOWR) BPF1=1.
2600      IF(R.LT.LOWR) BPF1=10.**(-145.*(RRO-LF)/4.)
2610      RETURN
2620      END
2630 C *****
2640      REAL FUNCTION BPF0(R,RO,CONST)
2650 C
2660 C      RECTANGULAR BANDPASS FILTER FUNCTION
2670 C
2680      REAL R,RO,CONST,FIL
2690      REAL BW,CF,LOWR,HIGHR
2700      DATA BW,CF/13.5,50.0/
2710 C
2720      FIL=BW/(2.*CF)
2730      LOWR=RO*(1.-FIL)-CONST*FIL
2740      HIGHR=RO*(1.+FIL)+CONST*FIL
2750 C
2760      IF(R.GE.LOWR .AND. R.LE.HIGHR) GOTO 800
2770      BPF0=0.
2780      RETURN
2790 800 CONTINUE
2800      BPF0=1.0
2810      RETURN
2820      END
2830 C *****
2840      REAL FUNCTION ASIGMA(X,Y,HH,ANGL,B)
2850 C
2860 C      THIS FUNCTION IS THE SIGMA VARIATION FUNCTION FOR POINT
2870 C      P(X,Y) WITH POINTING ANGLE=ANGL AND SLOPE=B.
2880 C
2890 C
2900      REAL X,Y,HH,ANGL,B
2910      REAL THETA,ATHETA,PI
2920      PI=4.*ATAN(1.)
2930      THETA=ATAN(SQRT(X*X+Y*Y)/HH)
2940      ATHETA=THETA+180./PI
2950      ASIGMA=EXP(-(ATHETA-ANGL)/B)
2960      RETURN
2970      END
2980 C *****
2990      SUBROUTINE REGRES(Y,NN,SLOP,COR)
3000 C

```

3010 C THIS SUBROUTINE CALCULATES THE SLOPE AND THE CORRELATION
3020 C COEFFICIENT OF AN ARRAY Y(NN). THE X-VARIABLE IS THE
3030 C INCIDENCE ANGLE FROM 10 TO 10*NN DEGREES.
3040 C

3050 C INTEGER NN
3060 C REAL Y(NN), SLOP, SLOPE, COR, X(10), SY(10), SX(10)
3070 C REAL SXY(10), AVG, AVX, AVY, BBB
3080 C

3090 C DO 10 I=1, NN
3100 C X(I)=FLOAT(I)*10.
3110 C 10 CONTINUE
3120 C NNN=NN
3130 C AVX=AVG(X, NNN)
3140 C AVY=AVG(Y, NNN)
3150 C DO 11 I=1, NNN
3160 C SY(I)=(Y(I)-AVY)**2
3170 C SX(I)=(X(I)-AVX)**2
3180 C SXY(I)=(X(I)-AVX)*(Y(I)-AVY)
3190 C 11 CONTINUE
3200 C SLOP=AVG(SXY, NNN)/AVG(SX, NNN)
3210 C COR=SLOP*SQRT(AVG(SX, NNN)/AVG(SY, NNN))
3220 C RETURN
3230 C END

3240 C *****
3250 C REAL FUNCTION AVG(A, N)

3260 C
3270 C THIS FUNCTION CALCULATES THE AVERAGE OF AN ARRAY WITH N
3280 C ELEMENTS.
3290 C

3300 C INTEGER N
3310 C REAL A(N), SUM
3320 C SUM=0.
3330 C DO 12 I=1, N
3340 C SUM=SUM+A(I)
3350 C 12 CONTINUE
3360 C AVG=SUM/FLOAT(N)
3370 C RETURN
3380 C END

3390 C *****
3400 C REAL FUNCTION AREA(ELB, AZB, ANGL, H, RCONST)

3410 C
3420 C
3430 C THIS FUNCTION AREA CALCULATES THE EXACT ILLUMINATED AREA
3440 C FOR AN FM-CW RADAR. CALCULATION INCLUDES AREA LIMITING
3450 C BY THE BAND PASS FILTER .
3460 C

3470 C INPUT PARAMETERS
3480 C ELB: ELEVATION BEAMWIDTH IN DEGREES
3490 C AZB: AZIMUTH BEAMWIDTH IN DEGREES
3500 C ANGL: INCIDENCE ANGLE IN DEGREE
3510 C H: HEIGHT OF THE ANTENNA
3520 C RCONST: SYSTEM CONST FOR INTERNAL RANGE
3530 C

3540 C REAL PI, H
3550 C REAL ANGL, ELB, AZB, RCONST, ELBR, AZBR
3560 C REAL THE, FC, BW, FIL, RE, RA
3570 C REAL R, C, ALPA1, ALPA2, GAM, A, B, K, L1, L2, X1, X2
3580 C

3590 C FUNCTIONS REFERENCED
3600 C

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3610 REAL LAREA1,LAREA2
3620 C
3630 DATA FC,BW/50.0,13.5/
3640 PI=3.1415926
3650 C
3660 CONVERT BEAMWIDTHS TO RADIAN
3670 C
3680 ELBR=PI*ELB/180.
3690 AZBR=PI*AZB/180.
3700 C
3710 IF(IFIX(ANGL) .EQ. 0) GO TO 301
3720 C
3730 THE=ANGL*PI/180.
3740 R=H/COS(THE)
3750 C=H*TAN(THE)
3760 FIL=BW/(2.*FC)
3770 ALPA1=R*(1.-FIL)-RCONST*FIL
3780 ALPA2=R*(1.+FIL)+RCONST*FIL
3790 GAM=H*(TAN(THE-ELBR/2.))
3800 RA=2.*R*(TAN(AZBR/2.))
3810 RE=H*(TAN(THE+ELBR/2.))-GAM
3820 A=RE/2.
3830 B=RA/2.
3840 K=GAM+A
3850 L1=C-GAM
3860 L2=RE-L1
3870 IF(ALPA1-H) 120,120,122
3880 120 X1=C
3890 GO TO 124
3900 122 X1=C-SQRT(ALPA1*ALPA1-H*H)
3910 124 CONTINUE
3920 X2=SQRT(ALPA2**2-H*H)-C
3930 C
3940 CHECK IF THERE IS ANY BEAM LIMITING DUE TO NARROW BPF
3950 AND IF ANY, GO TO APPROPRIATE CASES
3960 C
3970 IF(X1-L1) 200,201,201
3980 200 IF(X2-L2) 205,203,203
3990 201 IF(X2-L2) 204,202,202
4000 C
4010 202 CONTINUE
4020 CASE 1, NO LIMITING OF THE BEAM
4030 AREA=PI*A*B
4040 RETURN
4050 C
4060 203 CONTINUE
4070 CASE 2, NEAR SIDE OF THE BEAM IS LIMITED BY THE FILTER
4080 AREA=PI*A*B-LAREA1(A,B,C,K,X1)
4090 RETURN
4100 C
4110 204 CONTINUE
4120 CASE 3, FAR SIDE OF THE BEAM IS LIMITED BY THE FILTER
4130 AREA=LAREA2(A,B,C,K,X2)
4140 RETURN
4150 C
4160 205 CONTINUE
4170 CASE 4, BOTH SIDES OF THE BEAM IS LIMITED BY THE FILTER
4180 AREA=LAREA2(A,B,C,K,X2)-LAREA1(A,B,C,K,X1)
4190 RETURN
4200 C

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4210 301 CONTINUE
4220 AREA=PI*H*H*TAN(ELBR/2.)*TAN(AZBR/2.)
4230 RETURN
4240 END
4250C
4260C *****
4270C REAL FUNCTION INTSEC(A1,A2,A3,A4)
4280C
4290C THIS FUNCTION CALCULATES THE INTERSECTION OF THE ELLIPSE
4300C (X-K)**2/A**2+Y**2/B**2=1 AND THE CIRCLE X**2+Y**2=A4**2
4310C
4320C REAL A1,A2,A3,A4
4330C LOCAL VARIABLES
4340C REAL B1,B2,B3,B4,B5,B6
4350C B1=(A2+A1)*(A2-A1)
4360C B2=(A4+A2)*(A4-A2)
4370C B3=(A1*A2*A3)**2
4380C B4=A2*A2*A3
4390C B5=A1*A1
4400C B6=SQRT(B3-B5*B1*B2)
4410C INTSEC=(B4-B6)/B1
4420C RETURN
4430C END
4440C
4450C *****
4460C REAL FUNCTION MISC(THETA)
4470C
4480C THIS FUNCTION CALCULATES (2*THETA-SIN(2*THETA))/4
4490C
4500C REAL THETA
4510C MISC=(2.*THETA-SIN(2.*THETA))/4.
4520C RETURN
4530C END
4540C
4550C *****
4560C REAL FUNCTION THEX(MG,MP)
4570C
4580C THIS FUNCTION CALCULATES ARCTAN(SQRT(MG**2-MP**2)/MP)
4590C
4600C REAL MG,MP
4610C REAL MGP
4620C MGP=SQRT(MG*MG-MP*MP)
4630C THEX=ATAN(MGP/MP)
4640C RETURN
4650C END
4660C
4670C *****
4680C REAL FUNCTION LAREA1(AA,BB,CC,KK,XX1)
4690C
4700C THIS FUNCTION CALCULATES THE NEAR-SIDE-BEAM LIMITED AREA
4710C WHICH SHOULD BE SUBTRACTED FROM AREA CALCULATION
4720C
4730C REAL AA,BB,CC,KK,XX1
4740C REAL ITH1,IINT1,KCMP,TTHE2,IINT2,GG,PPSI
4750C REAL THEX,MISC,INTSEC
4760C GG=CC-XX1
4770C PPSI=INTSEC(AA,BB,KK,GG)
4780C
4790C INTEGRATE ALONG THE CIRCLE
4800C

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4810      TTH1=THEX(GG,PPSI)
4820      IINT1=GG+GG*MISC(TTH1)
4830C
4840C INTEGRATE ALONG THE ELLIPSE
4850C
4860      KKMP=KK-PPSI
4870      TTHE2=THEX(AA,KKMP)
4880      IINT2=AA*BB*MISC(TTHE2)
4890C
4900      LAREA1=2.*(IINT1+IINT2)
4910      RETURN
4920      END
4930C
4940C *****
4950C REAL FUNCTION LAREA2(AA,BB,CC,KK,XX2)
4960C
4970C THIS FUNCTION CALCULATES THE AREA FOR THE CASE WHEN
4980C THE FAR SIDE OF THE BEAM IS LIMITED BY THE FILTER
4990C
5000      REAL AA,BB,CC,KK,XX2
5010      REAL G,PSI,THE1,THE3,INT1,INT2,PI,KMP
5020      REAL THEX,MISC,INTSEC
5030      DATA PI/3.1415926/
5040C
5050      G=CC+XX2
5060      PSI=INTSEC(AA,BB,KK,G)
5070C
5080C INTEGRAL FROM PSI TO G THROUGH THE CIRCLE
5090C
5100      THE1=THEX(G,PSI)
5110      INT1=G*G*MISC(THE1)
5120C
5130C INTEGRAL FROM GAMMA TO PSI THROUGH THE ELLIPSE
5140C
5150      IF(KK-PSI) 300,301,302
5160 300 KMP=PSI-KK
5170      THE3=THEX(AA,KMP)
5180      INT2=AA*BB*(.5*PI-MISC(THE3))
5190      GO TO 303
5200 301 INT2=PI*AA+BB/4.
5210      GO TO 303
5220 302 THE3=THEX(AA,KK-PSI)
5230      INT2=AA*BB*MISC(THE3)
5240C
5250 303 LAREA2=2.*(INT1+INT2)
5260      RETURN
5270      END

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END

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