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THE THEORY OF PACKET BROADCASTING

BY

NORMAN ABRAMSON

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University of Hawaii
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Abstract

Packet broadcasting is a form of data communications architecture which can combine the features of packet switching with those of broadcast channels for data communication networks. Much of the basic theory of packet broadcasting has been presented as a byproduct in a sequence of papers with a distinctly practical emphasis. In this paper we provide a unified presentation of packet broadcasting theory.

In Section 2 we introduce the theory of packet broadcasting data networks. In Section 3 we provide some theoretical results dealing with the performance of a packet broadcasting network when the users of the network have a variety of data rates. In Section 4 we deal with packet broadcasting networks distributed in space and in Section 5 we derive some properties of power limited packet broadcasting channels, showing that the thruput of such channels can approach that of equivalent point-to-point channels.

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1. INTRODUCTION

1.1 Packet Switching and Packet Broadcasting

The transition of packet switched computer networks from experimental [1] to operational [2] status during 1975 provides convincing evidence of the value of this form of communications architecture. Packet switching, or statistical multiplexing [3], can provide a powerful means of sharing communications resources among large numbers of data communications users when those users can be characterized by a high ratio of peak to average data rates. Under such circumstances data from each user is buffered, address and control information is added in a "header," and the resulting bit sequence, or "packet", is routed through a shared communications resource by a sequence of node switches [4], [5].

Packet switched networks however still employ point-to-point communication channels and large multiplexing switches for routing and flow control in a fashion similar to conventional circuit switched networks. In some situations [6]-[10] it is desirable to combine the efficiencies achievable by a packet communications architecture with other advantages obtained by use of broadcast communication channels. Among these advantages are elimination of routing and network switches, system modularity and overall system simplicity. In addition certain kinds of channels available to the communications systems designer, notably satellite channels, are basically broadcast in their structure. In such cases use of these channels in their natural broadcast mode can lead to significant system performance advantages [11], [12].

1.2 Outline of Results

Packet broadcasting is a form of data communications architecture which can combine the features of packet switching with those of broadcast channels for data communication networks. Much of the basic theory of packet broadcasting has been presented as a byproduct in a sequence of papers with a distinctly practical emphasis. In this paper we provide a unified presentation of packet broadcasting theory.

In Section 2 we introduce the theory of packet broadcasting as implemented in THE ALOHA SYSTEM at the University of Hawaii; also in Section 2 we explain a modification of the basic ALOHA method, called slotting. In Section 3 we provide some theoretical results dealing with the performance of a packet broadcasting channels when the users of the channel have a variety of data rates. In Section 4 we deal with packet broadcasting networks distributed in space and present some incomplete results on the theoretical properties of such networks. Finally in Section 5 we derive some properties of power limited packet broadcasting channels showing that the thruput of such channels can approach that of equivalent point-to-point channels. This result is of importance in satellite systems using small earth stations since it implies that the multiple access capability and the complete connectivity (in the topological sense) of packet broadcasting channels can be obtained at no price in average thruput.

2. PACKET BROADCASTING CHANNELS

2.1 Operation of a Packet Broadcasting Channel

Consider a number of widely separated users, each wanting to transmit short packets over a common high speed channel. Assume that the rate at which users generate packets is such that the average time between packets from a single user is much greater than the time needed to transmit a single packet. In Figure 1 we indicate a sequence of packets transmitted by a typical user.

Figure 1. PACKETS FROM A TYPICAL USER

Conventional time or frequency multiplexing methods or some kind of polling scheme could be employed to share the channel among the users. Some of the disadvantages of these methods for users with high peak-to-average data rates are discussed by Carleial and Hellman [13].

In a packet broadcasting system the simplest possible solution to this multiplexing problem is employed. Each user transmits its packets over the common broadcast channel in a completely unsynchronized (from one user to another) manner. If each individual user of a packet broadcasting channel is required to have a low duty cycle the probability of a packet from one user interfering with a packet from another user is small as long as the total number of users on the common channel is not too large. As the number of users increases however the number of packet overlaps increases and the probability that a packet will be lost due to an overlap also increases. The question of how many users can share such a channel and the analysis of various methods of dealing with packets lost due to overlap are the primary concerns of this paper. In Figure 2 we show a packet broadcasting channel with two overlapping packets. Since the first packet broadcasting channel was put into operation in THE ALOHA SYSTEM radio-linked computer network at the University of Hawaii [6], they have been referred to as ALOHA channels.

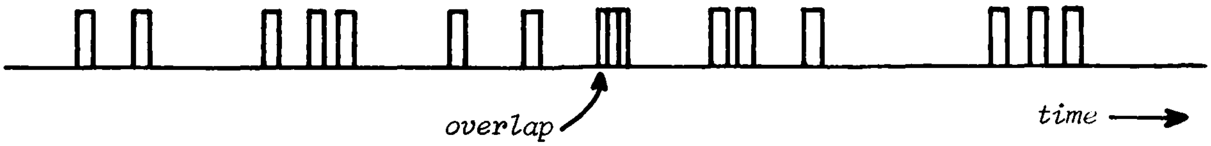

Figure 2. PACKETS FROM SEVERAL USERS ON AN ALOHA CHANNEL

2.2 ALOHA Capacity

A transmitted packet can be received incorrectly or lost completely because of two different types of errors: (1) random noise errors and (2) errors caused by packet overlap. In this paper we assume the first type of error can be ignored and we shall be concerned only with errors caused by packet overlap. In Section 2.4 we describe several methods of dealing with the problem of packets lost due to overlap, but first we derive the basic results which tell us how many packets can be transmitted with no overlap.

Assume the start times of packets in the channel comprise a Poisson point process with parameter λ packets/second. If each packet lasts τ seconds we can define the normalized channel traffic, G, where

$$G = \lambda\tau \tag{1}$$

If we assume only those packets which do not overlap with any other packet are received correctly we may define $\lambda' < \lambda$ as the rate of occurrence of those packets which are received correctly. Then we define the normalized channel thruput, S, by

$$S = \lambda'\tau \tag{2}$$

The probability that a packet will not overlap a given packet is just the probability that no packet starts τ seconds before or τ seconds after the start time of the given packet. Then, since the point process formed from the start times of all packets in the channel was assumed Poisson, the probability that a packet will not overlap any other packet is $e^{-2\lambda\tau}$, or e^{-2G}. Therefore

$$S = Ge^{-2G} \tag{3}$$

and we may plot the channel thruput versus channel traffic for an ALOHA channel (Figure 3).

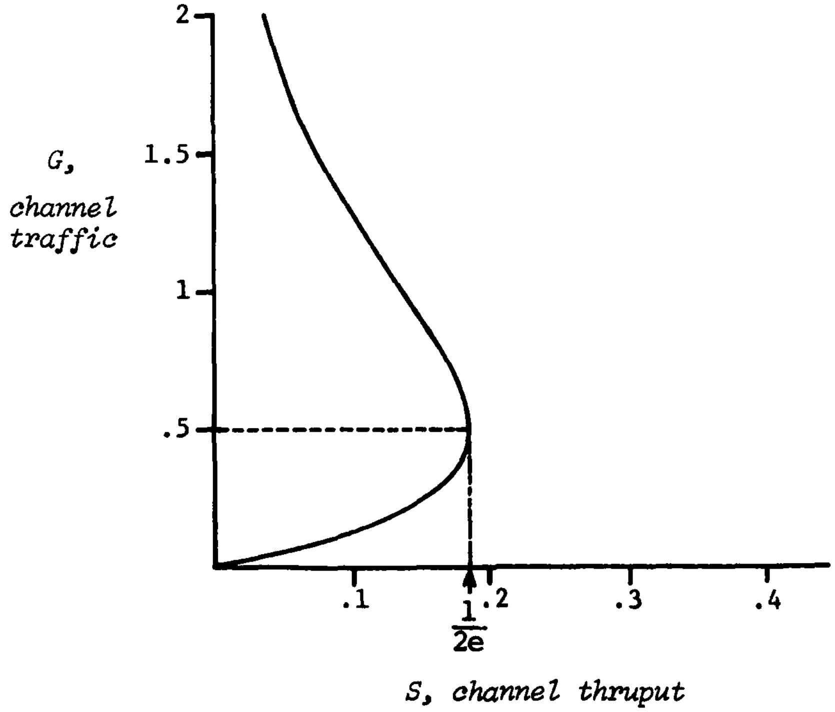

Figure 3
CHANNEL THRUPUT VS CHANNEL TRAFFIC FOR AN ALOHA CHANNEL

From Figure 3 we see that as the channel traffic increases the thruput also increases until it reaches its maximum at $S = 1/2e = 0.184$. This value of thruput is known as the capacity of an ALOHA channel, and it occurs for a value of channel traffic equal to 0.5. If we increase the channel traffic above 0.5 the thruput of the channel will decrease.

2.3 Application of an ALOHA Channel

In order to indicate the capabilities of such a channel for use in an interactive network of alphanumeric computer terminals, consider the 9600 bits/second packet broadcasting channel used in THE ALOHA SYSTEM. From the results of Section 2.2 we see that the maximum average thruput of this channel is 9600 bits/second 2e, or about 1600 bits/second. If we assume the conservative [14] figure of 5 bits/second as the average data rate (including overhead) from each active* terminal in the network, this channel can handle the traffic of over 300 active terminals and each terminal will operate at a peak data rate of 9600 bits/second. Of course the total number of terminals in such a network can be much larger than 300 since only a fraction of all terminals will be active and a terminal consumes no channel resources when it is not active.

2.4 Recovery of Lost Packets

Since the packet broadcasting technique we have described will result in some packets being lost due to packet overlaps it is necessary to introduce some technique to compensate for this loss. We may list four different packet recovery techniques for dealing with the problem of lost packets. The first three make use of a feedback channel to the packet transmitter and the repetition of lost packets, while the fourth is based on coding.

---

* A terminal is defined as active from the time a user transmits an attempt to log on until he transmits a log off message.

2.4.1 Positive Acknowledgements (POSACKS)

Perhaps the most direct way to handle lost packets is to require the receiver of the packet to acknowledge correct receipt of the packet. Each packet is transmitted and then stored in the transmitter's buffer until a POSACK is received from the receiver. If a POSACK is not received in a given amount of time the transmitter can repeat the transmission and continue to repeat until a POSACK is received or until some other criterion is met. The POSACK can be transmitted on a separate channel (as in the ALOHANET [6]) or transmitted on the same channel as the original packets (as in the ARPA packet radio system [15]). An error detection code and a packet numbering system can be used to increase the reliability of this technique.

2.4.2 Transponder Packet Broadcasting

Certain communication channels -- notably communication satellite channels -- transmit packets on one frequency to a transponder which retransmits the packets on a second frequency. In such cases all units in a packet broadcasting network can receive their own packet retransmissions, determine whether a packet overlap has occurred and repeat the packet if necessary. This technique has been employed in ATS-1 satellite experiments in the Pacific Educational Computer Network (PACNET) [16], and in the ARPA Atlantic INTELSAT IV packet broadcasting experiments [17].

2.4.3 Carrier Sense Packet Broadcasting

For ground based packet broadcasting networks where the signal propagation time over the furthest transmission path is much less than the packet duration it is feasible to provide each transmission unit with a device to inhibit packet transmission while another unit is detected transmitting. A carrier sense capability can increase the channel thruput, even if these conditions are not met, when used in conjunction with other packet recovery methods. Carrier sense systems have been analyzed by Tobagi [18] and by Kleinrock and Tobagi [19].

2.4.4 Packet Recovery Codes

When a user employs a packet broadcasting channel to transmit long files by breaking them into large numbers of packets it is possible to encode the files so that packets lost due to broadcasting overlap can be recovered. It is clear that some of the existing classes of multiple burst error correcting codes [20] and cyclic product codes [21] can be used for packet recovery in transmissions of long files. It is also clear that these codes are not as efficient as possible for packet recovery and that considerable work remains to be done in this area.

2.5 Slotted Channels

It is possible to modify the completely unsynchronized use of the ALOHA channel described above in order to increase the maximum thruput of the channel. In the pure ALOHA channel each user simply transmits a packet when ready without any attempt to coordinate his transmission with those of other users. While this strategy has a certain elegance it does lead to somewhat inefficient channel utilization. If we establish a time base and require each user to start his packet only at certain fixed instants it is possible to increase the maximum value of the channel thruput. In this kind of channel, called a Slotted ALOHA channel, a central clock establishes a time base for a sequence of "slots" of the same duration as a packet transmission. Then when a user has a packet to transmit he synchronizes the start of his transmission to the start of a slot. In this fashion, if two messages conflict they will overlap completely, rather than partially.

To analyze the Slotted ALOHA channel define G_i as the probability that the i'th user will transmit a packet in some slot. Assume that each user operates independently of all other users and that whether a user transmits a packet in a given slot does not depend upon the state of any previous slot. If we have n users we can define the normalized channel traffic for the slotted channel, G, where

$$G = \sum_{i=1}^{n} G_i \tag{4}$$

Note that G may be greater than 1.

As before we can also consider the rate at which a user sends packets which do not experience an overlap with other user's packets. Define $S_i \leq G_i$ as the probability that a user sends a packet and that this packet is the only packet in its slot. If we have n users, then we define the normalized channel thruput for the slotted channel, S, where

$$S = \sum_{i=1}^{n} S_i \tag{5}$$

Note that S is less than or equal to 1, and $S \leq G$.

For the Slotted ALOHA channel with n independent users, the probability that a packet from the i'th user will not experience an interference from one of the other users is

$$\prod_{\substack{j=1 \\ j \neq i}}^{n} (1-G_i)$$

Therefore we may write the following relationship between the message rate and the traffic rate of the i'th user.

$$S_i = G_i \prod_{\substack{j=1 \\ j \neq i}}^{n} (1-G_j) \tag{6}$$

If all users are identical we have

$$S_i = \frac{S}{n} \tag{7}$$

and

$$G_i = \frac{G}{n} \tag{8}$$

so that (6) can be written

$$S = G\left(1 - \frac{G}{n}\right)^{n-1} \tag{9}$$

and in the limit as $n \rightarrow \infty$, we have

$$S = Ge^{-G} \tag{10}$$

Equation (10) is plotted in Figure 4 (curve labeled Slotted ALOHA). Note that the message rate of the Slotted ALOHA channel reaches a maximum value of $1/e=0.368$, twice the capacity of the pure ALOHA channel.

This result for Slotted ALOHA channels was first derived by Roberts [22] using a different method.

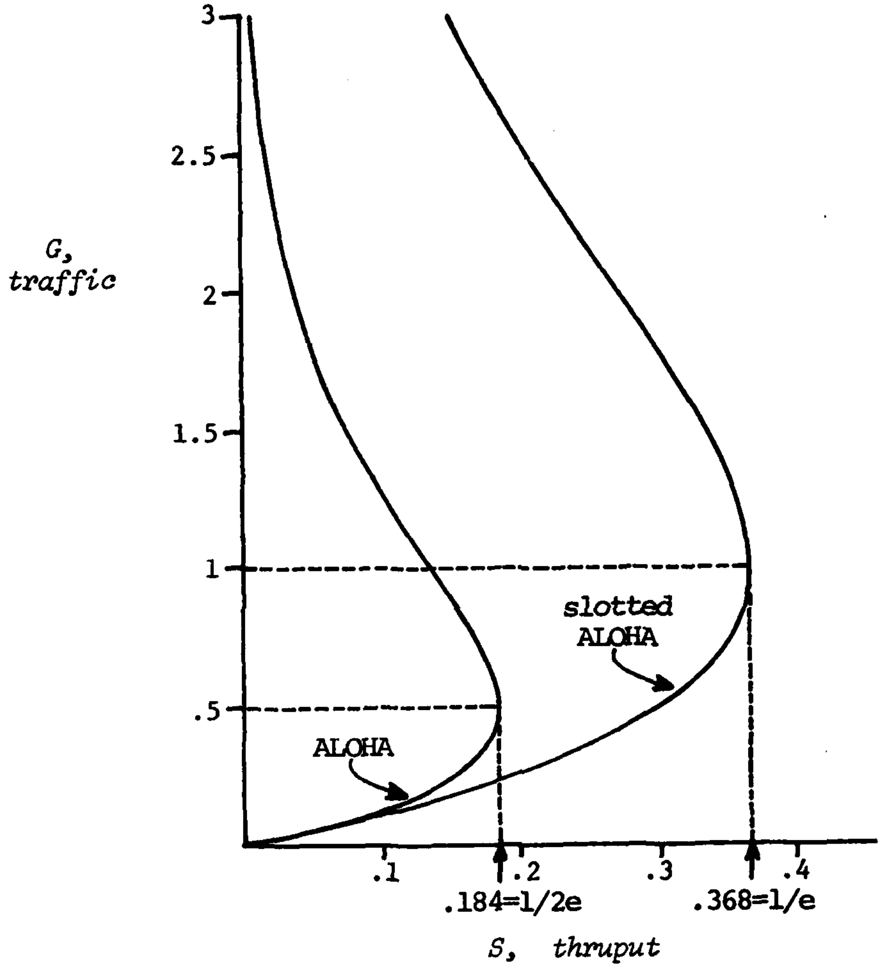

Figure 4

TRAFFIC VS THRUPUT FOR AN ALOHA CHANNEL AND A SLOTTED ALOHA CHANNEL

3. PACKET BROADCASTING WITH MIXED DATA RATES

3.1 Unslotted Case: Variable Packet Lengths

In Section 2 we were concerned with the analysis of ALOHA channels carrying a homogeneous mix of packets. If some channel users have a higher average data rate than others however, the high rate users must either transmit packets more frequently or transmit longer packets. In this section we shall analyze the Unslotted ALOHA channel when carrying packets of different lengths, and we shall analyze the Slotted ALOHA channel when the probability of transmitting in a given slot varies from user to user.

Let us assume an Unslotted ALOHA channel with two different possible packet durations, τ_1 and τ_2. Assume $\tau_2 \geq \tau_1$, and therefore we refer to the two different length packets as long packets and short packet respectively. Assume also the start times of the long packets and short packets form two Poisson point processes with parameters λ_2 and λ_1 packets/second, and that the two Poisson point processes are mutually independent. Then we can define the normalized channel traffic for those packets of duration τ_i.

$$G_i = \lambda_i \tau_i \qquad i=1,2 \tag{11}$$

Again assume only those packets which do not overlap with any other packet are received correctly and define $\lambda_i' < \lambda_i$ as the rate of occurrence of those packets of duration τ_i which are received correctly. Define the normalized thruput of packets of duration τ_i as

$$S_i = \lambda_i' \tau_i \qquad i=1,2 \tag{12}$$

Since we assumed two independent Poisson point processes the probability that a short packet will be received correctly is

$$\exp[-\lambda_1(2\tau_1) - \lambda_2(\tau_1+\tau_2)] \tag{13}$$

and if we define

$$G_{12} \triangleq \lambda_1\tau_2 \tag{14a}$$

$$G_{21} \triangleq \lambda_2\tau_1 \tag{14b}$$

(13) becomes

$$\exp[-2G_1 - G_{21} - G_2] \tag{15}$$

Therefore

$$S_1 = G_1 \exp[-2G_1 - G_{21} - G_2] \tag{16a}$$

and, by a similar argument, the thruput of long packets is

$$S_2 = G_2 \exp[-G_{12} - G_1 - 2G_2] \tag{16b}$$

For any given values of λ_1 and λ_2 we may calculate G_1, G_2, G_{12} and G_{21}; substitution of these values into (16a) and (16b) will allow calculation of the thruputs, S_1 and S_2. Therefore Equations (16a) and (16b) may be used to define an allowable set of thruput pairs (S_1,S_2) in the (S_1,S_2) plane.

To determine the boundary of this region we define

$$\alpha \triangleq \frac{\tau_2}{\tau_1} \tag{17}$$

Note that $\alpha \geq 1$. We may rewrite Equations (16a) and (16b) in terms of α, the ratio of long packet duration to short packet duration.

$$S_1 = G_1 \exp[-2G_1 - (1+\tfrac{1}{\alpha})G_2] \tag{18a}$$

$$S_2 = G_2 \exp[-(1+\alpha)G_1 - 2G_2] \tag{18b}$$

The boundary of the set of allowable (S_1,S_2) pairs in the (S_1,S_2) plane is defined by setting the Jacobian

$$J = \left| \frac{\partial S_i}{\partial G_j} \right| \qquad i,j = 1,2 \tag{19}$$

equal to zero. A simple calculation shows that the Jacobian is zero when

$$G_2 = \frac{1 - 2G_1}{\left(\frac{(\alpha-1)^2}{\alpha} G_1 + 2 \right)} \tag{20}$$

Note that this checks for $G_1=0$ and for $\alpha=1$.

We need only substitute this expression for G_2 into Equations (18a) and (18b) to obtain two equations for S_1, the short packet thruput, and S_2, the long packet thruput, in terms of the single parameter G_1; and as G_1 varies from 0 (all long packets) to 1/2 (all short packets) we will trace out the boundary of the achievable values of thruput in the (S_1,S_2) plane. These achievable thruput regions are indicated for several values of α in Figure 5.

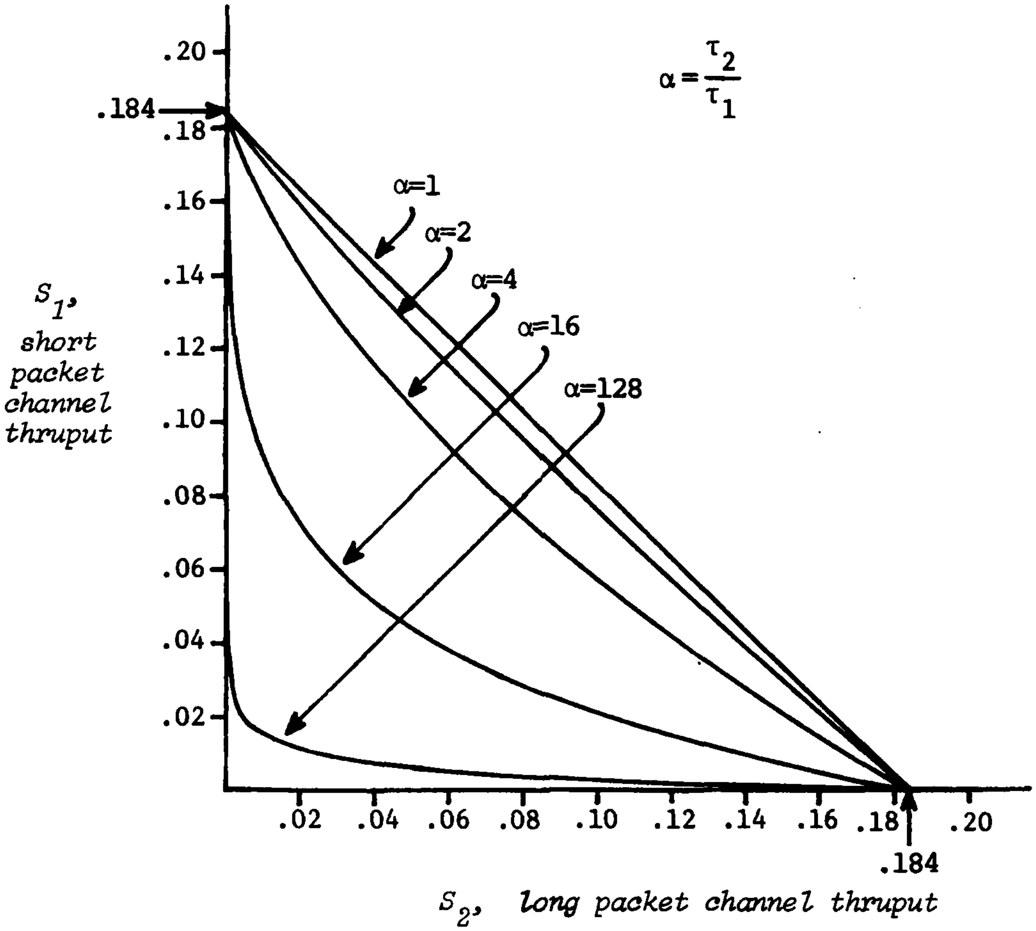

Figure 5

ACHIEVABLE THRUPUT REGIONS IN AN UNSLOTTED ALOHA CHANNEL

The basic conclusion of this analysis is that the total channel thruput can undergo a significant decrease if all packets are not of the same length. Thus if the two different packet lengths differ by a large factor it is often preferable to break up long packets into many shorter packets as long as the overhead necessary to transmit the text in each packet is small. Ferguson [23] has generalized these results to show that channel thruput is maximized over all possible packet length distributions with fixed length packets.

In view of this discouraging result we might conclude that an inhomogeneous mix of users inevitably leads to a decrease in the maximum value of channel thruput. Surprisingly, this conclusion is not warranted, and we shall show in Section 3.2 that a mix of users of varied data rates can lead to an increase in the maximum value of channel thruput.

3.2 Slotted Case: Variable Packet Rates

In this section we shall consider a Slotted ALOHA channel used by n users, possibly with different values of channel traffic, G_i. From Equation (6) we have a set of n nonlinear equations relating the channel traffics and the channel thruputs for these n users.

$$S_i = G_i \prod_{\substack{j=1 \\ j\neq i}}^{n} (1-G_j) \qquad i=1,2,\ldots,n \tag{21}$$

Define

$$\alpha = \prod_{j=1}^{n} (1-G_j) \tag{22}$$

then (21) can be written

$$S_i = \frac{G_i}{1-G_i}\alpha \qquad i=1,2,\ldots,n \tag{23}$$

For any set of n acceptable traffic rates $G_1, G_2, \ldots, G_n$ these n equations define a set of channel thruputs $S_1, S_2, \ldots, S_n$, or a region in an n-dimensional space whose coordinates are the S_i. In order to find the boundary of this region we calculate the Jacobian,

$$J = \left| \frac{\partial S_j}{\partial S_k} \right| \qquad j,k = 1,2,\ldots,n \tag{24}$$

Since

$$\frac{\partial S_j}{\partial G_k} = \begin{cases} \prod\limits_{\substack{i=1 \\ i \neq j}}^{n} (1-G_i) & j=k \\ \\ - G_j \prod\limits_{\substack{i=1 \\ i=j,k}}^{n} (1-G_i) & j \neq k \end{cases} \tag{25}$$

after some algebra we may write the Jacobian as

$$J = \alpha^{n-2} \begin{vmatrix} (1-G_1) & -G_1 & -G_1 & \\ -G_2 & (1-G_2) & -G_2 & \cdots \\ -G_3 & -G_3 & (1-G_3) & \\ & \vdots & & \end{vmatrix}$$

$$= \alpha^{n-2}[1 - G_1 - G_2 - \ldots - G_n] \tag{26}$$

Thus the condition for maximum channel thruputs is

$$\sum_i G_i = 1 \tag{27}$$

This condition can then be used to define a boundary to the n dimensional region of allowable thruputs $S_1, S_2, \ldots, S_n$.

Consider the special case of two classes of users with n_1 users in class 1 and n_2 users in class 2.

$$n_1 + n_2 = n \tag{28}$$

Let S_1 and G_1 be the thruputs and traffic rates for users in class 1, and S_2 and G_2 be the thruputs and traffic rates for users in class 2. Then the n Equations (21) can be written as the two equations

$$S_1 = G_1(1-G_1)^{n_1-1}(1-G_2)^{n_2} \tag{29a}$$

$$S_2 = G_2(1-G_2)^{n_2-1}(1-G_1)^{n_1} \tag{29b}$$

For any pair of acceptable traffic rates G_1 and G_2 these two equations define a pair of channel thruputs, S_1 and S_2, or a region in the (S_1,S_2) plane.

From (27) we know that the boundary of this region is defined by the condition

$$n_1G_1 + n_2G_2 = 1 \tag{30}$$

We can use (30) to substitute for G_1 in Equations (29a) and (29b) and obtain two equations for S_1 and S_2 in terms of a single parameter G_2. Then as G_2 varies from 0 to 1 the resulting (S_1,S_2) pairs define the boundary of the region we seek. These achievable regions are indicated for various values of n_1 and n_2 in Figures 6 and 7.

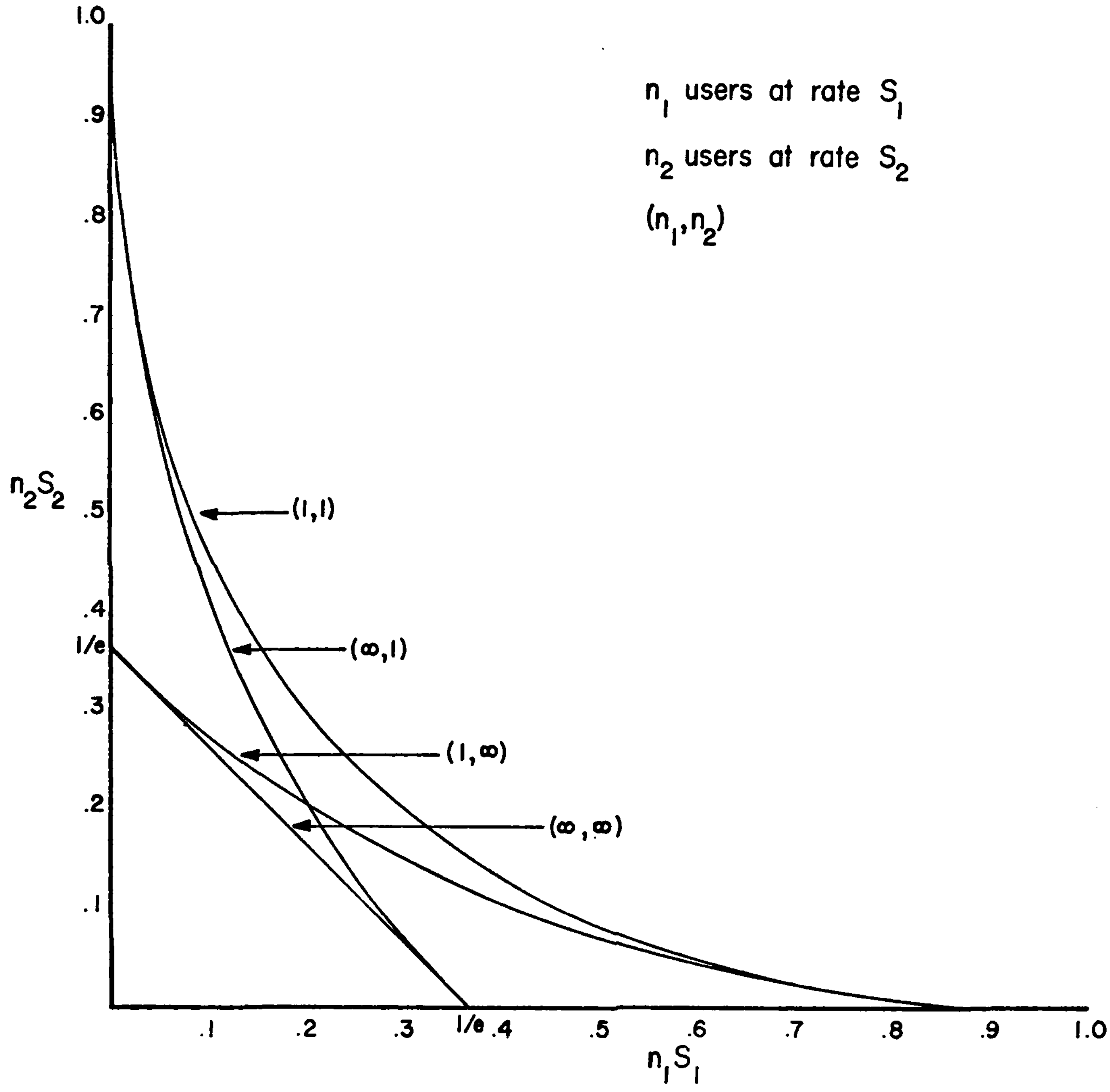

Figure 6. ALLOWABLE CHANNEL THRUPUTS

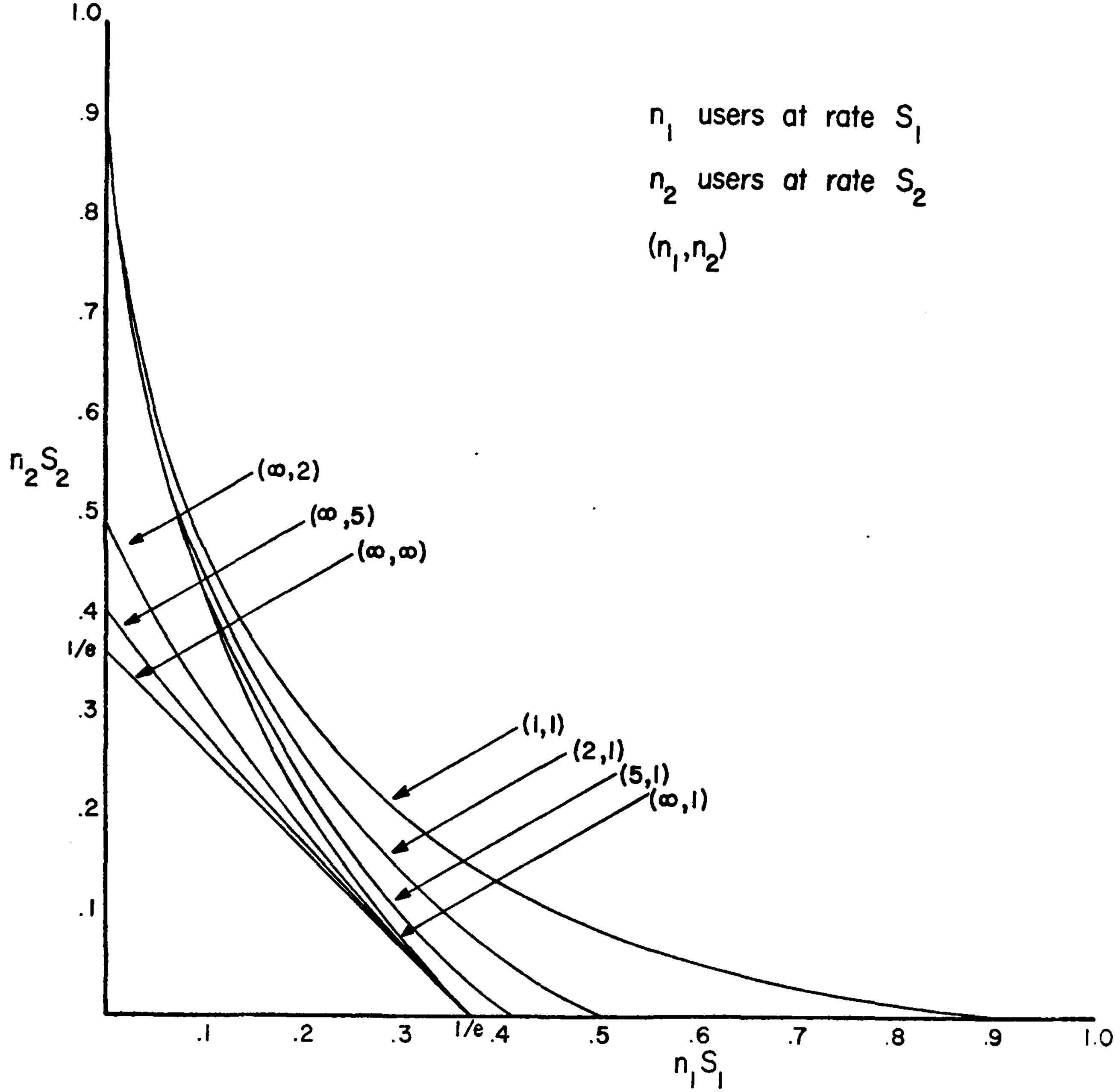

Figure 7. ALLOWABLE CHANNEL THRUPUTS

The important point to notice from Figures 6 and 7 is that in a lightly loaded Slotted ALOHA channel, a single large user can transmit data at a significant percentage of the total channel data rate, thus allowing use of the channel at rates well above the limit of 1/e or 37 percent obtained when all users have the same message rate. A thruput data rate above the 1/e limit has been refered to as "excess capacity" [24]. Excess capacity is important for a lightly loaded packet broadcasting network consisting of many interactive terminal users and a small number of users who send large but infrequent files over the channel. Operation of the channel in a lightly loaded condition of course may not be desirable in a bandwidth limited channel. For a communications satellite where the average power in the satellite transponder limits the channel however operation in a lightly loaded packet switched mode is an attractive alternative. Since the satellite will transmit power only when it is relaying a packet, the duty cycle in the transponder will be small and the average power used will be low (see Section 5).

Finally we note it is possible to deal with certain limiting cases in more detail, to obtain equations for the boundary of the allowable (S_1,S_2) region.

(a) for $n_1=n_2=1$

Upon using (30) in (29) we obtain

$$S_1 = G_1^2 \tag{31a}$$

$$S_2 = (1 - G_1)^2 \tag{31b}$$

(b) for $n_2 \to \infty$

$$S_1 = G_1(1-G_1)^{n_1-1} \cdot \exp[-(1-n_1G_1)] \tag{32a}$$

$$S_2 = (1-n_1G_1)(1-G_1)^{n_1-1} \cdot \exp[-(1-n_1G_1)] \tag{32b}$$

(c) for $n_1 = n_2 \to \infty$

$$S_1 = \frac{G_1}{e} \tag{33a}$$

$$S_2 = \frac{1 - G_1}{e} \tag{33b}$$

Additional details dealing with excess capacity and the delay experienced with this kind of use of a Slotted ALOHA channel may be found in [11] and [25].

4. SPATIAL PROPERTIES OF PACKET BROADCASTING NETWORKS

4.1 Packet Repeaters

In this section we deal with certain spatial properties of packet broadcasting networks. Not long after the initial units of THE ALOHA SYSTEM went into operation it was realized that the range of the network could be extended beyond the range of a single radio link in the network (about 200 kilometers) by the use of packet repeaters. A packet repeater operates in much the same manner as a conventional radio repeater with one major exception. Since radio transmission in a packet broadcasting network is intermittent, a packet repeater can receive a packet and retransmit that packet in the same frequency band by turning off its receiver during a retransmission burst. Thus a packet repeater can sidestep many of the frequency allocation and spatial cell problems [26] of conventional land based repeater networks.

The use of packet repeaters leads to the consideration of packet broadcasting networks with more than one central station, distributed over very large areas. Users transmit a packet, and if the packet cannot be received directly by its destination, it is forwarded to its destination by one or more packet repeaters according to some routing algorithm [27]. The study of such networks has led to the analysis of two communication theory issues related to the performance of the networks: (a) capture effect and (b) the distribution of packet traffic and packet thruput in space.

4.2 Capture Effect

Up to this point we have analyzed packet broadcasting channels under the pessimistic assumption that if two packets overlap at the receiver both packets are lost. In fact this assumption provides a lower bound to the performance of real packet broadcasting channels, since in many receivers the stronger of two overlapping packets may capture the receivers and may be received without error. In order to include the effect of capture in a packet broadcasting network, we consider a distribution of packet generators over a two dimensional plane and a single packet broadcasting receiver which receives packets from these generators. The receiver then may be viewed as a "packet sink" and the packet generators as a distribution of "packet sources" in the plane. We assume that the rate of generation of packets in a given area depends only on r, the distance from the packet sink, and is independent of direction, θ.

Then we may define a traffic density and a thruput density analogous to the normalized traffic, G, and normalized thruput, S, defined in Section 2.2.

G(r) = normalized packet traffic per unit area at a distance r

S(r) = normalized packet thruput per unit area at a distance r

The traffic due to all packet generators in a differential ring of width dr at a radius r is

$$G(r)\ 2\pi r\ dr \tag{34}$$

We assume that packets from different users are generated so that the packet starting times of all packets generated in the differential ring constitute a Poisson point process. Then since the sum of two independent Poisson processes is a Poisson point process, if users in different rings are independent, the start times of all packets generated in a circle of radius r also constitute a Poisson point process and the total traffic generated by all users within a distance r of the center is

$$\int_0^r G(r)\ 2\pi r\ dr \tag{35}$$

If we assume that a packet from a user at a distance r from the center will be received correctly unless it is overlapped by a packet sent from a user at a distance ar or less ($a \geq 1$), then using the results of Section 2.2, the probability that such a packet will be received correctly is

$$\exp[-4\pi \int_0^{ar} G(r) r\ dr] \quad . \tag{36}$$

Any packet generated from a packet source in the circle of radius ar shown in Figure 8 will interfere with packets generated from a source in the circle of radius r. A packet generated outside the circle of radius ar will not interfere with packets generated from a source in the circle of radius r.

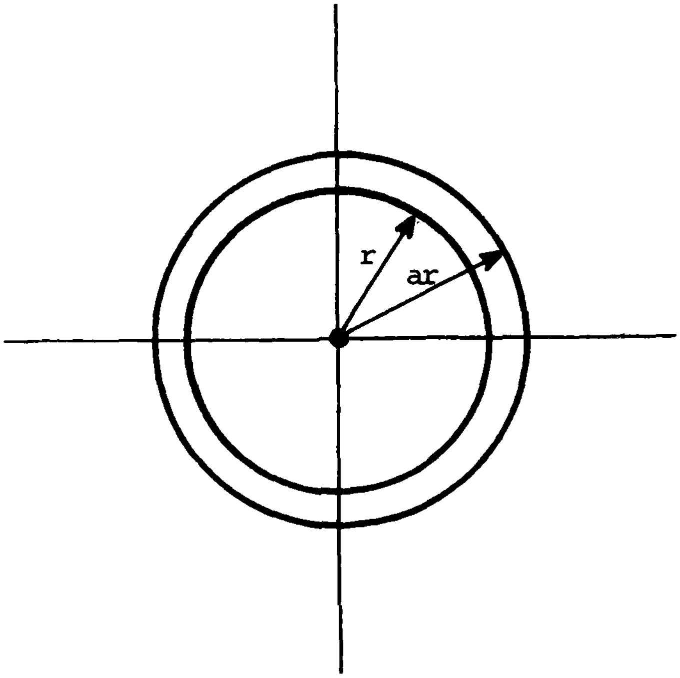

Figure 8

REGIONS OF INTERFERING PACKETS

We can relate the normalized packet throughput to the normalized packet packet traffic in the usual way

$$2\pi r\ S(r)\ dr = 2\pi r\ G(r)\ \exp[-4\pi \int_0^{ar} G(r) r\ dr] dr$$

or

$$S(r) = G(r)\ \exp[-4\pi \int_0^{ar} G(r) r\ dr] \tag{37}$$

If we take a derivative of (3) with respect to r and use (37) to substitute for the exponential, we get

$$S'(r)G(r) = G'(r)S(r) - 4\pi ra^2 S(r)G(r)G(ar) \tag{38}$$

We have not found a general solution of (38) for relating S(r) to G(r) in the presence of capture. We have been able to analyze two special cases however.

4.3 Two Solutions

In the first of these special cases we assume a constant traffic density, G(r). We can then show that the thruput density, S(r), has a Gaussian form, due to the fact that those packets generated further from the receiver will be received correctly less frequently than those packets generated close to the receiver.

In the second special case analyzed we assume a constant packet thruput density, S(r) and perfect capture (a=1). Under these assumptions the packet traffic density will increase as the distance from the receiver increases. We show that there exists a radius r_0 such that the packet traffic density is finite within a circle of radius r_0 around the receiver, while the packet traffic density becomes unbounded on the circle of radius r_0.

For the important case of a packet broadcasting channel distributed over some geographical area and using a packet retransmission policy (Section 2.4) this result has an interesting interpretation. In such a situation any packet transmitted from a terminal located within the circle of radius r_0 will be received correctly with probability one (after a finite number of retransmissions) while the expected number of retransmissions required for a packet transmitted from a terminal further from the center than r_0 will be unbounded. Thus there exists a circle of radius r_0 such that terminals transmitting from within this circle can get their packets into the central receiver, while terminals transmitting from outside this circle spend all their time retransmitting their packets in vain. We call r_0 the Sisyphus distance of the ALOHA channel.

4.3.1 Constant Packet Traffic Density

Assume the density of normalized packet traffic is constant over the plane

$$G(r) = G_0 \tag{39}$$

and define the distance r_1 as the radius of a circle within which the total packet traffic is unity.

$$\pi r_1^2 G_0 \triangleq 1 \tag{40}$$

Then (38) reduces to

$$S'(r) = -\frac{4ra^2}{r_1^2} S(r) \tag{41a}$$

with the boundary condition

$$S(0) = G_0 \tag{41b}$$

so that the packet thruput density is

$$S(r) = G_0 \exp[-2a^2(\frac{r}{r_1})^2] \tag{42}$$

and the total normalized packet thruput from a circle of radius r is

$$S = \int_0^r S(r')2\pi r'\,dr'$$

$$= \frac{1}{2a^2}\{1 - \exp[-2(\frac{ar}{r_1})^2]\} \tag{43}$$

and

$$\lim_{r\to\infty} S = \frac{1}{2a^2} \tag{44}$$

Note that the total thruput which can be supported by a single packet sink with "perfect capture" (a=1) is equal to one-half.

4.3.2 Constant Packet Thruput Density

Another case of interest where we have found a solution for Equation (38) is that of constant packet thruput density in the plane. Assume

$$S(r) = S_0 \tag{45}$$

over the region in the plane where S(r) and G(r) are bounded.

Then (38) becomes

$$G'(r) = 4\pi ra^2G(r)G(ar) \tag{46}$$

For the case of a=1 (perfect capture), (46) becomes

$$G'(r) = 4\pi r\,G^2(r) \tag{47}$$

with the boundary condition

$$G(0) = S_0 \tag{48}$$

so that

$$G(r) = \frac{S_0}{1-2\pi r^2 S} \tag{49}$$

Note that the normalized packet traffic per unit area is finite for

$$0 \leq r < r_0 \tag{50}$$

where

$$r_0 \triangleq [2\pi S_0]^{-1/2} \tag{51}$$

and r_0 is the Sisyphus distance mentioned in Section 4.3. Note that the Sisyphus distance also has the property that

$$\pi r_0^2 S_0 = \frac{1}{2} \tag{52}$$

As in the previous case the total packet thruput which can be supported by a single packet sink operating with perfect capture is one-half.

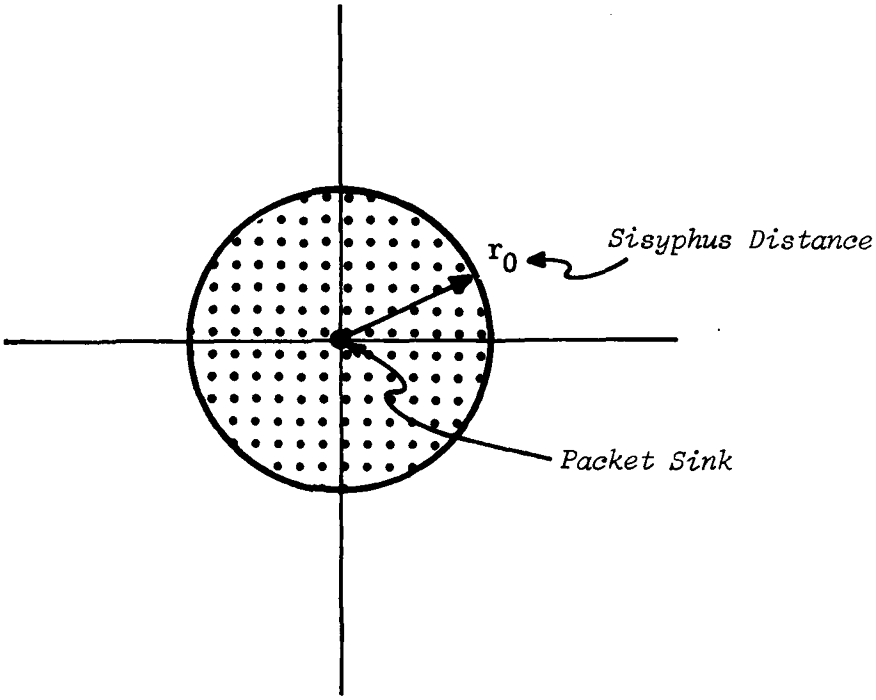

Figure 9

REGION OF CONSTANT PACKET THRUPUT, S_0, FOR A SINGLE PACKET SINK

5. PACKET BROADCASTING WITH AVERAGE POWER LIMITATIONS

5.1 Satellite Packet Broadcasting

In previous sections we have analyzed the performance of packet broadcasting channels and compared the performance of these channels to that of conventional point-to-point channels operating at the same peak data rate. Such a comparison is of interest in the case of channels limited by multiple access interference rather than noise, since an increase in the transmitted power of such channels will not lead to improved performance. But just as the average data rate of a packet broadcasting channel can be well below its peak data rate when it is operated at a low duty cycle, the average transmitted power of a packet broadcasting channel can be well below its peak transmitted power.

In this section we analyze the thruput of a packet broadcasting channel when compared to that of a conventional point-to-point channel of the same average power. This analysis is of interest in the case of satellite information systems employing thousands of small earth stations. For a satellite system the fundamental limitation in the downlink is the average power available in the satellite transponder rather than the peak power. Our results show that in the limit of large numbers of small earth stations the packet thruput approaches 100% of the point-to-point capacity. Thus the multiple access capability and the complete connectivity (in the topological sense) of an ALOHA channel can be obtained at no price in average thruput. Furthermore, since our results suggest the use of higher peak power in the satellite transponder (while the average power is kept constant) the small earth stations may use smaller antennas and simpler receivers and modems than would be necessary in a conventional system.

In existing satellite systems the TWT output power in each transponder cannot be varied dynamically. In such systems the advantages implied by our analysis may be realized by frequency division sharing a single transponder among several voice users and a single channel, operating in an ALOHA mode or some other burst mode and occupying a frequency band equivalent to one or more voice users. The type of operation implied by our analysis also suggests investigation of high peak power satellite burst transponders (perhaps employing power devices similar to those used in radar systems) for use in information systems composed of large numbers of ultra-small earth stations.

5.2 Burst Power and Average Power

The capacity of a satellite channel can be calculated by the classical Shannon equation

$$C = W\log\left(1 + \frac{P}{N}\right) \qquad (53)$$

where C is the capacity in bits (if the log is a base two logarithm), W is the channel bandwidth, P is the average received signal power at the eatth station and N is the average noise power at the earth station. Equation (53) expresses the capacity of the satellite channel under the assumption that the transponder transmits continuously.

If the channel is used in burst mode the transponder will emit power only when a data burst occurs and the average power out of the transponder will be less than the burst power. Let D be the ratio of the average power transmitted to the power transmitted during a data burst. For a linear transponder D will equal the channel traffic, G, and for a hard limiting transponder D will equal the duty cycle of the channel. For both the Unslotted and Slotted ALOHA channel the duty cycle is $1-e^{-G}$. Thus, for a linear transponder*

$$D = G \qquad (54a)$$

---

* Our analysis is of significance only for $G < 1$. The analysis is formally correct however for all G, even though the designation of the power transmitted during bursts as "peak power" becomes inappropriate for the linear transponder case when $G > 1$. (In such a situation the "peak power" is less than the average power.)

While for a hard limiting transponder

$$D = 1-e^{-G} \tag{54b}$$

Note that in the case of a hard limiting transponder with small values of channel traffic, the duty cycle approaches that of a linear transponder.

If we retain P as the notation for the average signal power received at the earth station, the power received during a data burst will be P/D. Thus Equation (53) should be modified in two ways:

a. We replace W by SW to account for the fact that the channel is only used intermittently.

b. We replace P in (53) by P/D, to keep the average power of the channel fixed at P.

We should note that when we make these changes we are assuming that the packet length of the system is long enough so that the asymptotic assumptions which are used to derive (53) still apply. In practice this is not a problem.

With these two changes then we have four different cases:

1. unslotted channel, linear transponder

$$C_1 = Ge^{-2G}W\log(1+\frac{P}{GN}) \tag{55a}$$

2. unslotted channel, limiting transponder

$$C_2 = Ge^{-2G}W\log\left(1+\frac{P}{(1-e^{-G})N}\right) \tag{55b}$$

3. slotted channel, linear transponder

$$C_3 = Ge^{-G}W\log(1+\frac{P}{GN}) \tag{55c}$$

4. slotted channel, limiting transponder

$$C_4 = Ge^{-G}W\log 1 + \frac{P}{(1-e^{-G})N} \qquad (55d)$$

We have calculated the normalized capacities, C_i/C for i=1,2,3,4, for different values of P/N the signal-to-noise ratio of the earth station when the transponder operates continuously. The normalized capacities are plotted in Figures 10, 11, 12 and 13 for P/N equal to -20, -10, 0, 10, 20 db. Of particular interest in these curves is the fact that the highest values of C_i/C occur just where we would want them to occur -- for small values of channel traffic (G) and for small earth stations (low P/N). In the limit we have (for a fixed value of G)

$$\lim_{\frac{P}{N}\to 0} \frac{C_i}{C} = \frac{S}{D} \qquad i=1,2,3,4 \qquad (56)$$

so that

1. unslotted channels, linear transponder

$$\lim_{\frac{P}{N}\to 0} \frac{C_1}{C} = e^{-2G} \qquad (57a)$$

2. unslotted channels, limiting transponder

$$\lim_{\frac{P}{N}\to 0} \frac{C_2}{C} = \frac{Ge^{-2G}}{(1-e^{-G})} \qquad (57b)$$

3. slotted channel, linear transponder

$$\lim_{\frac{P}{N}\to 0} \frac{C_3}{C} = e^{-G} \qquad (57c)$$

4. slotted channel, limiting transponder

$$\lim_{\frac{P}{N}\to 0} \frac{C_4}{C} = \frac{Ge^{-G}}{(1-e^{-G})} \qquad (57d)$$

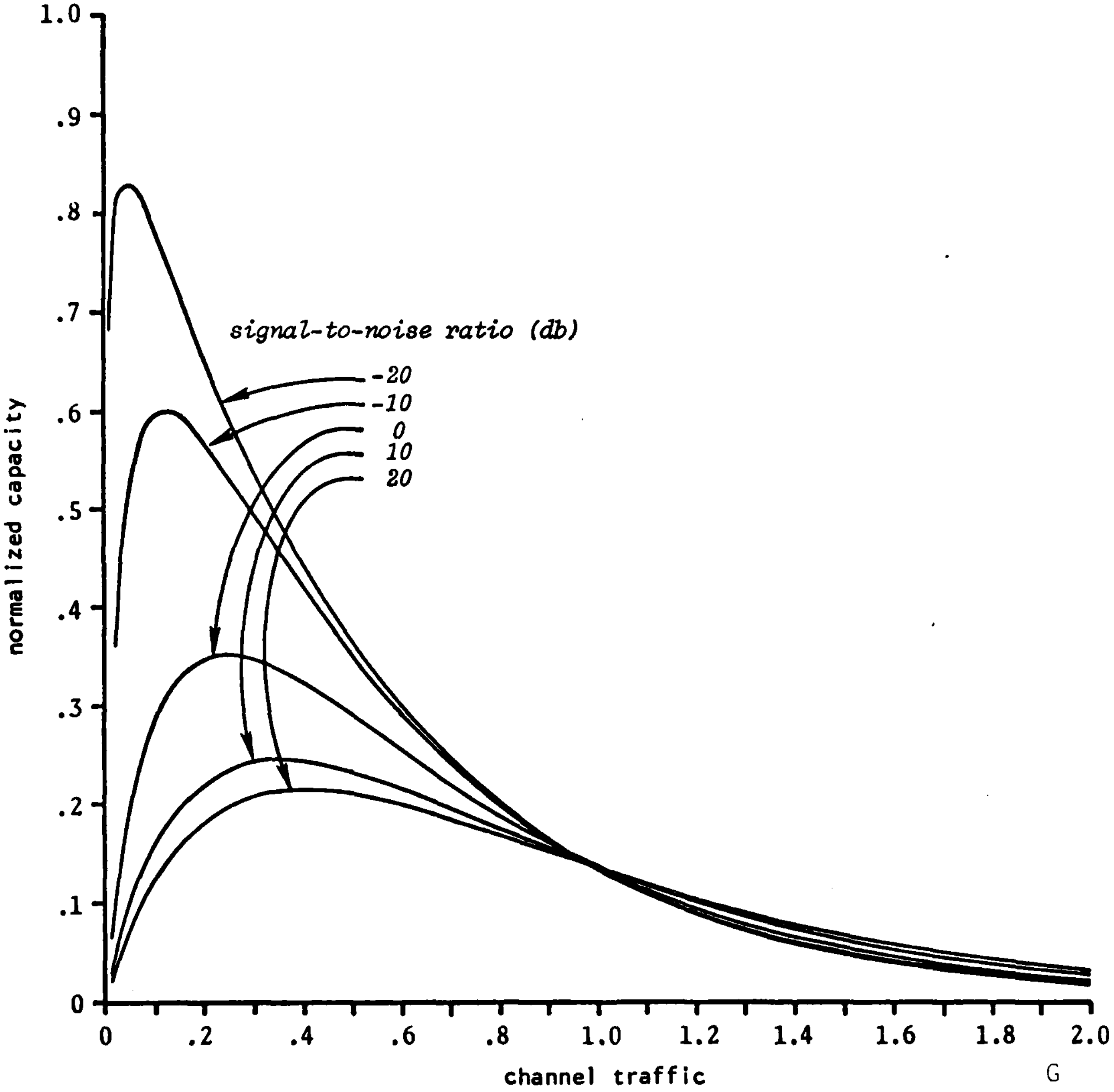

Figure 9. LINEAR TRANSPONDER; UNSLOTTED CHANNEL

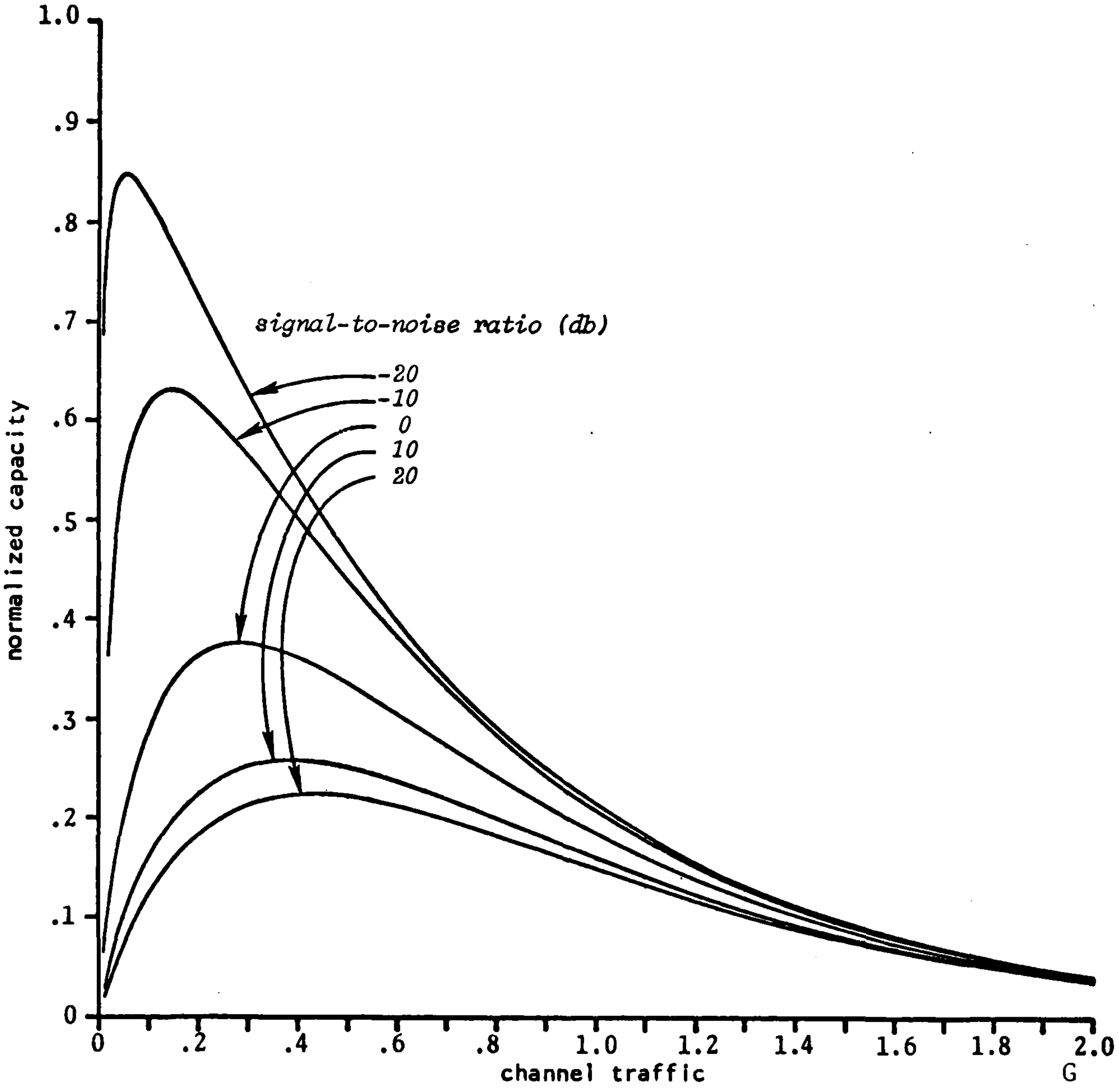

Figure 10. LIMITING TRANSPONDER; UNSLOTTED CHANNEL

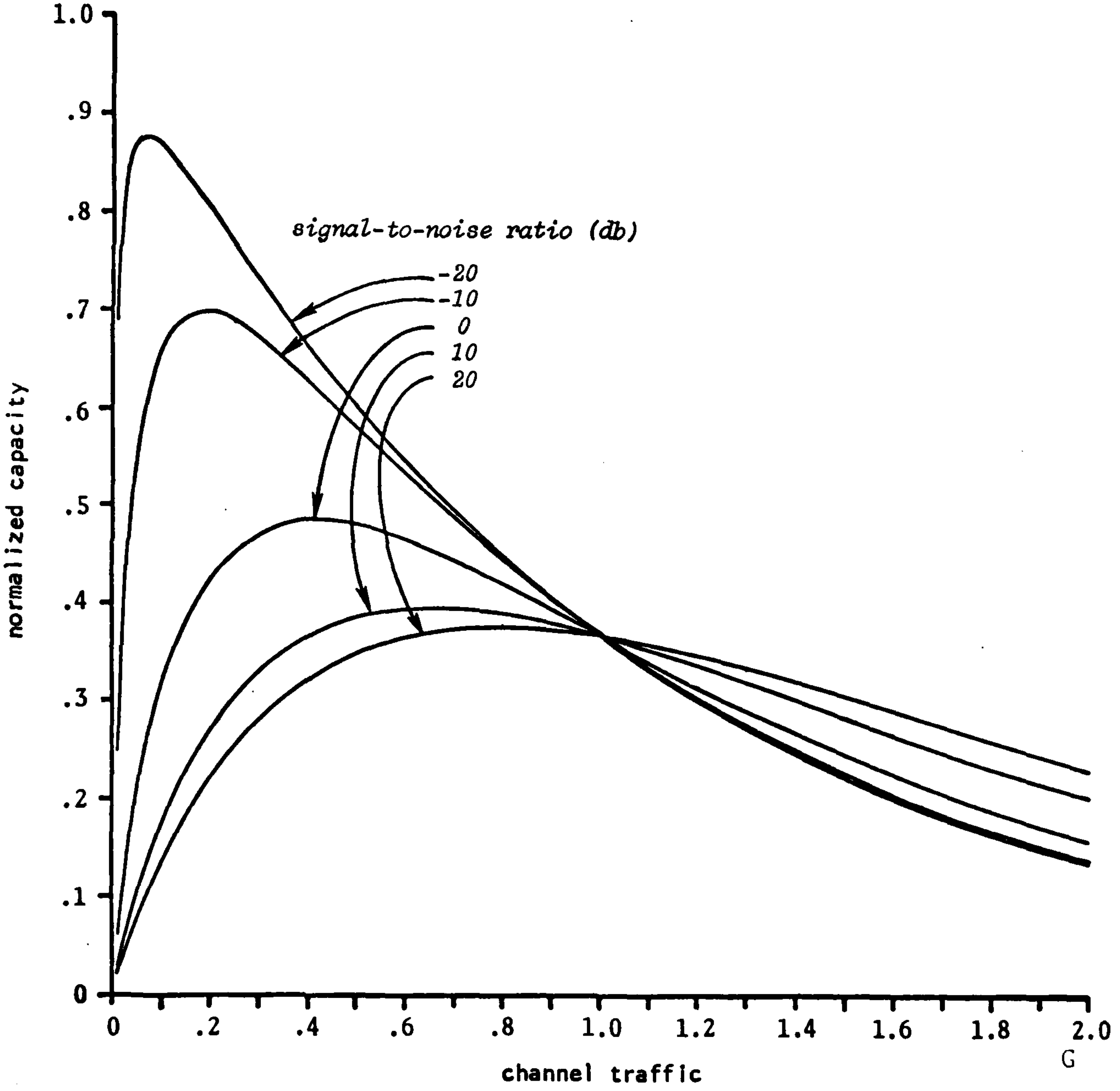

Figure 11. LINEAR TRANSPONDER; SLOTTED CHANNEL

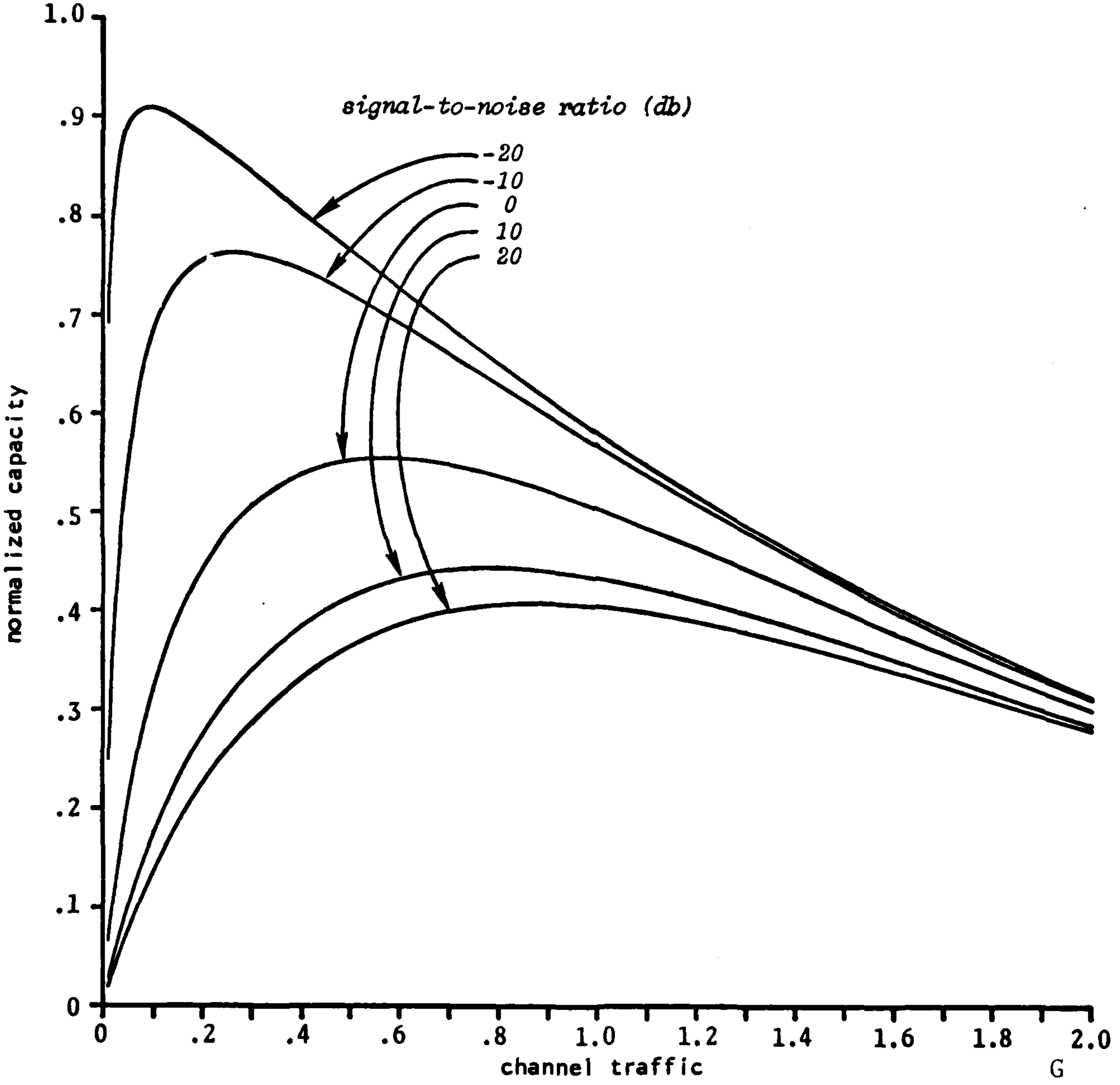

Figure 12. LIMITING TRANSPONDER; SLOTTED CHANNEL

and in all cases

$$\lim_{G \to 0} \lim_{\frac{P}{N} \to 0} \frac{C_i}{C} = 1 \tag{58}$$

Thus this multiplexing technique allows a network of small inexpensive earth stations to achieve the maximum value of channel capacity, at the same time providing complete connectivity and multiple access capability.

6. BACKGROUND AND ACKNOWLEDGEMENTS

The term "packet broadcasting" was first coined by Robert Metcalfe in his Ph.D. thesis [28]. As is often the case with simple ideas, the concept of combining burst transmission and Poisson user statistics to provide random access to a channel has occurred independently to a number of investigators. The first attempt at an analysis of such a system of which I am aware is contained in an internal Bell Laboratories memorandum by Schroeder [29], suggested by an earlier paper by Pierce and Hopper [30]. Two other early related papers were written by Costas [31] and Fulton [32]. Of course, a theoretical analysis is not necessary in order to build such a system and anyone who has sat in a taxi listening to the staccato voice bursts of a radio dispatcher and a set of taxi drivers sharing a single voice channel will recognize the operation of a voice packet broadcasting channel using a carrier sense protocol. And even after an analysis is available the concept of packet broadcasting may be suggested without reference to the theory [33].

The first papers analyzing packet broadcasting in the form implemented in THE ALOHA SYSTEM assumed fixed packet thruput and a retransmission protocol as described in 2.4.1 [6]. This approach leads to a number of questions involving optimum retransmission policy [28], the behavior of the channel with a finite number of users [39], stability of the channel [13], and transmission of long files by means of various reservation schemes [34]. In this paper we have taken a different approach by assuming a fixed packet traffic rather than thruput. With such a starting point the questions mentioned above do not assume key importance in the theory, although their practical importance is not diminished.

Much of the theory of packet broadcasting was developed in two working groups sponsored by the ARPA Advanced Research Projects Agency of the Department of Defense. These groups circulated a private series of working papers -- the ARPANET Satellite System notes (ASS notes) and the Packet Radio Temporary notes (PRT notes) -- where many of the theoretical results described or referenced in this paper appeared for the first time. Unfortunately the several references to ASS notes in papers subsequently published in the open literature may have produced some confusion in the minds of those trying to trace the references. Among the most significant of the ASS note and PRT note results was the first derivation of the capacity of a Slotted ALOHA channel and the first analysis of the use of the capture effect in packet broadcasting, both by Larry Roberts. The results of Section 3.1 dealing with two different packet length were suggested by an ASS note written by Tom Gaarder and the results of Section 3.2 dealing with the excess capacity of a slotted channel were suggested by an ASS note written by Randy Rettberg. Other problems which were first analyzed in ASS notes or PRT notes but not emphasized in this paper include various packet broadcasting reservation systems [22,35,36], carrier sense packet broadcasting [18,19], and questions dealing with packet routing and protocol issues in a network of repeaters [37]. The reader interested in theoretical network protocol questions should also see Gallagher [38], although this work did not originate in an ASS note or PRT note.

The first system to employ packet broadcasting techniques was THE ALOHA SYSTEM computer network at the University of Hawaii in 1970. Subsequently packet repeaters were added to the network and packet broadcasting by satellite was demonstrated in the system. Some of the people involved in the implementation and development of the system were Richard Binder, Chris Harrison, Alan Okinaka and David Wax.

The historical relevance of references [29] and [32] was pointed out to me by Joe Aein to whom I am indebted, in spite of my embarrassment at having forgotten I was thesis supervisor on the second of these papers.

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