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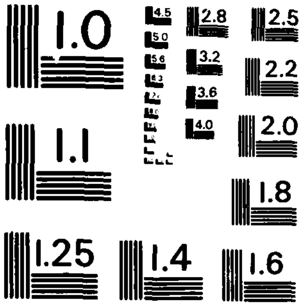

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VERTICAL INTEGRATION, CONTESTABLE MARKETS, AND
THE MISFORTUNES OF THE MISSHAPED U

Herman C. Quirmbach

December 1981
Revised June 1982

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The Rand Corporation
and
The University of Southern California

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ABSTRACT

This paper models the vertical integration of an "upstream" monopolist who sells an "intermediate" good to firms in a contestable "downstream" market. The downstream firms combine that good with other inputs--according to a production function with U-shaped average costs--to produce a "final" good which is sold to consumers at minimum average cost.

The paper has two main themes.

The first is to compare the incentives for and results of vertical integration in the case where the upstream market is protected from entry with those in the case where the upstream market is contestable. The results suggest that vertical mergers should be encouraged in the latter case but tolerated in the former only under specific guidelines.

The second theme is to explore the effects on the scale of the firms in the downstream industry of the monopolization of the upstream market and of vertical integration. I find that monopolization upstream may cause distortions in the scale of downstream firms and that such scale distortions create incentives for integration. The use of a non-constant returns downstream technology also helps to explain partial forward integration.

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CONTENTS

I. INTRODUCTION AND SUMMARY [0]

This paper models the vertical integration of an "upstream" monopolist who sells an "intermediate" good to firms in a contestable[1] "downstream" market. The downstream firms combine that good with other inputs--according to a production function with U-shaped average costs--to produce a "final" good which is sold to consumers at minimum average cost.

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The process of vertical integration is modelled by the following conceptual experiment. I suppose that the upstream monopolist enters the downstream industry by setting up a downstream subsidiary which at

[0] Due to software limitations, footnote references are in square brackets on the text line. Bibliographic references are in { } brackets.

[1] A market is "contestable" in the sense of Baumol, Panzar, and Willig {2} if it allows both free entry (i.e., no cost disadvantages to a new entrant) and free exit (i.e., no sunk costs). (This reference will hereafter be designated B-P-W.)

first faces the same input and output prices and operates at the same scale as the other downstream firms. Since there is still free entry downstream, the final good price is still the minimum average cost of an independent downstream firm. Thus, adding the downstream subsidiary initially earns the integrated firm no additional profit. The integrated firm now can adjust three variables: the price of internal sales of the intermediate good (hereafter, the internal price), the price of external sales of the intermediate good to the remaining downstream firms (hereafter the external price), and the output of its new downstream subsidiary. If there is any profit incentive to change any of these three variables from their initial post-integration values, then there is positive incentive to integrate.

The source of the incentives to integrate is the same whether the upstream market is protected or contestable: the incentives arise when non-marginal cost-pricing of the intermediate good causes cost distortions in the downstream industry. When the upstream monopolist is protected from entry, his price is set above marginal cost in the usual monopoly way. If the upstream market is contestable, a monopoly must price at average cost to avoid attracting entry. I will assume that said monopoly results from scale economies to the extent of the market.[2] Since average costs are declining, the intermediate good price again exceeds marginal cost. When, in either case, such non-marginal cost pricing causes downstream cost distortions, integration incentives arise.

[2] In such a market, all we actually need in order to get a monopoly which can survive against undercutting entrants is to have average costs be lower at the market demand than at any lesser output level. This implies that average costs decline in the (left) neighborhood of the market demand.

Two types of downstream cost distortions are possible: "substitution" distortions and "scale" distortions. A substitution distortion is the inefficient substitution among inputs which occurs when the relative input prices a firm faces do not reflect the inputs' relative social marginal costs. This type of distortion and the integration incentive it generates have been studied by Vernon and Graham {15}, among many others. The other type of distortion--*a scale distortion*--has *not* been widely recognized in the literature.[3] It occurs when the mark-up on the intermediate good causes the bottom of the downstream average cost curve to shift. The combination of free entry and the "misshaped U" causes downstream firms to operate at a socially inefficient scale. Counting the distortion in total downstream output that occurs when the intermediate price mark-up is passed on to final consumers,[4] we find that there are now three sources of social welfare loss created by the monopolization of the input market: output, substitution, and scale distortions.

Distinguishing substitution distortions from scale distortions requires an elaboration of the distinction between "fixed proportions" and "variable proportions." I define a new category of input, a "fixed schedule" input, as one whose derived input demand curve depends only on the level of output and not at all on the input prices. A "fixed proportions" input is then a fixed schedule input where the derived

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[3] Blair-Kaserman {3} and Carlton-Lowry {4} have recognized that lump sum transfers to or from a firm will distort its perception of optimal scale. That input prices affect optimal scale is discussed more generally in Bassett and Borcherding {1}, Quirmbach {8} and Silberberg {13}, but none of these papers make the connection with vertical integration or the choice of an optimal intermediate good price.

[4] See for instance {5}.

input demand is linear in output. The derived demand for a "variable proportions" input is by definition sensitive to input prices.

The type of distortion (if any) caused by pricing the intermediate good above marginal cost depends on how the input is used in downstream production. Substitution distortions occur when the intermediate good is a variable proportions input but not when it is a fixed schedule input. Scale distortions occur unless the input demand is linear in output in the relevant range. Thus, substitution and scale distortions are both avoided only if the intermediate good is a fixed proportions input. In that case alone, there is no incentive to integrate.

For either type of upstream monopoly, immediately after integration the monopolist changes the internal price and its downstream output to reduce the cost distortions of the downstream industry. By adjusting the internal price to upstream marginal cost, the integrated firm eliminates any substitution distortion in its downstream subsidiary. Further, the integrated firm initially has the incentive to adjust its downstream output to reduce excess costs from scale distortions.[5] By altering its downstream output toward the socially optimal scale the integrated firm crowds out (or entices in) production by independent downstream firms. They then adjust their demands for the intermediate good. The net adjustment in intermediate good usage reduces total industry costs. Since neither of these moves affects total industry revenue and since all the independent downstream firms make zero profit, the cost savings show up as higher profits for the integrated firm.

[5] As the internal price and the price on sales to other downstream firms (if any remain) are adjusted, the downstream output incentives change.

With U-shaped average costs downstream, the integrated firm often does not take over the entire downstream market. In the previously studied case of downstream constant returns (e.g., Schmalensee {12}), the integrated firm finds it profitable to extend its monopoly to the downstream market: once it eliminates its subsidiary's substitution distortions, the monopoly enjoys a unit cost advantage over non-integrated downstream rivals to the extent of that market. In the current case, by contrast, the integrated firm eventually encounters decreasing returns in its own downstream production. When these are sufficiently severe, it becomes advantageous for the firm to continue taking part of its profit from sales of the intermediate good to independent downstream producers, rather than to expand its own downstream production further.

In several special cases, comparisons can be made between the downstream subsidiary's output level and the pre- and post-integration output levels of its non-integrated rivals. The comparisons are generated by considering sequentially the effects of intermediate good price changes on the downstream subsidiary's incentives to adjust output. Initially, the post-integration incentive is to reverse scale distortions, either up or down. Then, as the internal price is lowered and with it downstream costs, it usually becomes more attractive to increase the subsidiary's output. Finally, an external price rise forces up the final goods price, making it still more attractive to expand output; a fall in the external price does the opposite. Interestingly enough, if the intermediate good is a fixed or inferior input downstream, then, after integration in the protected case, the downstream subsidiary winds up being <u>smaller</u> than its rivals.

Stark differences between the integration results of the contestable and the protected upstream cases occur in the changes in the external price and the final good price. When there is free entry upstream by firms which also can integrate forward, the external and final good prices fall[6]. When the upstream monopoly is protected--and when, in addition, forward integration is partial and upstream marginal costs constant in the relevant range--the external and final good prices always rise.

In the contestable upstream market case, the reduction of cost distortions after integration creates positive profit for the integrated firm and thus may attract entry by another integrated firm. To avoid entry, the integrated firm must dissipate this profit by offering a lower external price. Downstream costs fall. Since the price of the final good is the minimum average cost of the independent downstream firms, the final good price must fall in turn.

When the upstream firm is protected from entry (and with the other conditions mentioned), the advantage of raising the external price after integration stems from its effect on the net profit contribution of the downstream subsidiary. Two forces are at work. First, the revenue of the downstream subsidiary rises as the increase in the external price causes the final good price to rise. A second force works through the "crowding out" effect of downstream production: each unit of its own downstream output "costs" the integrated firm profit by crowding out production by--and therefore intermediate good sales to--the independent

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[6] When there is free entry upstream but only the incumbent upstream firm can integrate, both prices stay the same after integration.

downstream firms. While these lost external sales tend to be replaced by internal sales, the latter are sold at cost while the former earn a profit. Raising the external price may raise or lower this foregone profit. I demonstrate that it is always advantageous to raise the external price by showing that the rise in downstream revenue always dominates the effect on foregone profit, whatever the direction of the latter effect. Hence, the final good price rises, too.[7]

All of the above conclusions hold even if the partial integration scenario is changed a bit. Above I assume that an integrated firm is limited to the same downstream technology choices as any other firm producing downstream--in essence that the upstream firm can only set up a single downstream division. Decreasing returns eventually set in perhaps because of some problem in managing a large enterprise. If, on the other hand, the decreasing returns part of the downstream average cost curve were the result of plant-specific diseconomies, then the integrated firm might be able to avoid decreasing returns by buying up several downstream plants. In that case, partial integration would result either if the integrated firm feared that buying up all the downstream firms would bring antitrust action or if the costs of coordinating multiple downstream plants made it unprofitable to acquire all of them. In this case, integration occurs for the same reasons as above. And, each time another downstream plant is acquired, the external and final good prices fall further if the upstream market is contestable and rise further otherwise.[8]

[7] In Appendix B, I revise Schmalensee's (constant returns downstream) model and demonstrate that the external and final price always rise in his partial integration scenario as well.

[8] Given, again, constant upstream marginal cost in the protected case. Appendix B analyzes the analogous case when the downstream industry has constant returns to scale. The results are the same.

The welfare performance of such vertically related industries is not the best. In the protected upstream case, both before and after integration, welfare is not maximized, nor are production costs minimized (partial integration), nor does the industry satisfy any of several Ramsey (second best) criteria. This is perhaps not surprisng.

What is surprising is that almost the same can be said of the contestable upstream case. The contestability literature (see B-P-W {2}, Chap. 11, Proposition 11B2.) leads one to expect that industry costs are minimized even if extensive scale economies lead some goods to be produced only by a single firm and priced above marginal cost. Here, unfortunately, the monopolized good priced above marginal cost is used as an input by yet other firms. The result is that those independent downstream firms use the intermediate good insufficiently intensively to achieve industry cost minimization. The partial integration studied does not (completely) solve the problem, since the industry production is not subadditive in such a case: some independent downstream firms remain.

The contestability literature does not generate clear expectations as to the second best performance of a contestable market. On the one hand, if a single firm dominates all product lines, an equilibrium satisfying Ramsey conditions occurs in certain circumstances {2, Chap. 8}. On the other hand, if only certain product lines are monopolized while others are shared, the best that can be said is a conjecture that some weak Ramsey conditions are met.[9] The vertical case--with partial integration--resembles the latter situation: only the weakest of Ramsey conditions is fulfilled.

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[9] {2, Chap. 11}

Does vertical integration at least improve performance? For the contestable upstream case, the answer is unambiguously yes. For the protected upstream case, the answer is a definite yes only if the external price falls as a result. Since, as above, its tendency is to rise, the welfare effect of integration in general depends on the parameters involved. However, it could be stipulated as a condition of integration that the external price remain fixed and that all intermediate demand at that price be met. Such a stipulation insures that any integration is socially beneficial, while it does not eliminate the cost reducing incentives which make integration attractive in the first place.

These results suggest some antitrust policy guidelines for vertical integration in markets as described here. The first is not to worry about the market share acquired in the downstream industry, as long as there is still the opportunity for downstream entry. The second is to determine whether the upstream industry is contestable. If so, no antitrust intervention is needed. If not, then the only intervention needed to insure socially beneficial results is to require that all remaining intermediate good customers be served at the old price. Such guidelines are simple and reasonably applicable and serve to focus antitrust proceedings on the key determinants of performance.

As the reader has already discovered, the two main themes--the comparison of the cases of protected and contestable upstream markets and the issue of downstream scale--are thoroughly intertwined. I will present the formal material as follows. Section II details the assumptions of the model and deals with some analytic preliminaries.

Section III analyzes market performance before integration. It explains the sources of excess costs and welfare losses before integration and defines input types. Section IV analyzes the case of integration where the upstream firm faces no entry threat. Section V deals with the contestable upstream case. Section VI analyzes the case of multiple downstream acquisitions. Section VII returns to welfare analysis, post-integration, to offer conclusions and policy analysis. Appendix A describes rigorously the assumptions made about the technology and proves the theorems underlying the representation that is used throughout the paper for an integrated firm's cost function. Appendix B examines the effect of partial integration on external and final prices when the downstream technology has constant returns.

II. THE MODEL

This section describes the technological and market conditions which characterize the upstream and downstream industries and determine the economic options available to an integrated firm.

II.A. The Downstream Industry

I assume the downstream industry is a contestable, single-product industry. Following Baumol, Panzar, and Willig {2}, I mean by contestability both that the production technology is costlessly available to all comers and that there are no sunk costs. In such a market, and before integration, B-P-W have shown that, in equilibrium, all firms (provided there are at least two) must produce at the output level which minimizes average cost and price their output at that average cost. They thus earn zero profit.

The downstream average cost function is assumed to be U-shaped for all (including integrated) firms producing in the industry. The assumption of a U shape is maintained for all relevant input prices. Note that marginal cost is upward sloping at the "bottom of the U". I will assume a bit more: that it is upward sloping in the relevant range on either side of this point. The cost function will also be assumed to be twice continuously differentiable in output and continuously differentiable in prices. (See Appendix A for a complete discussion of the technology assumptions.

A generic problem in such a market is that for equilibrium to exist market demand must be an exact integer multiple of the efficient firm scale. I assume away this problem by treating the number of firms as a continuous variable. Before integration, it is equal to total final good demand divided by efficient firm size. After integration, the number of independent downstream firms is found by subtracting the final good output of the integrated firm from total final demand and then dividing by the efficient scale. (Endogenous changes in efficient scale are accounted for.)

Downstream firms are represented as price-takers with regard to inputs. Raw inputs (i.e., all inputs other than the intermediate good)[1] are assumed to be perfectly elastically supplied at their social marginal costs: the prices of these inputs are not affected by the events under study. As for the intermediate good, price-taking by the downstream firms is an "as if" representation of the actual input acquisition process. In equilibrium, any downstream firm which wishes to bid a bit more for the input can easily acquire all his neighbor's share.[2] On the other hand, if he bids a bit less than the going rate, he will lose his supply to an entrant who will pay the going rate. Thus, the supply of the intermediate good--as an incumbent (or entrant) experiences it--is locally perfectly elastic, and downstream firms are properly represented as price-takers for all inputs.

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[1] The term "raw inputs" will be used to refer to all inputs not produced within the pair of industries in question. It is intended to refer primarily to outputs of the household sector, but may include other intermediate goods as long as they are perfectly elastically supplied at social marginal cost.

[2] If there is only one downstream firm, price-taking must be assumed explicitly.

A long-term supply contract would present an entry barrier; such barriers are presumed not to exist in the downstream industry.

II.B The Upstream Industry

Two cases will be considered, both of which generate an upstream monopoly. In the first case, there is a single upstream firm, and upstream entry is impossible. In the second, the upstream market is contestable, and the monopoly results from scale ecomomies. Specific restrictions and extensions include:

In the case with no entry upstream, the monopoly can be replaced by a joint profit maximizing oligopoly. The monopoly also need not be a single-product firm, nor need it hold a monopoly in the other markets it supplies. What I will require is that it supplies only the one intermediate good to the downstream industry in question and that all other markets it serves are unrelated in demand or supply to the downstream market.

In the case where the upstream industry is contestable, I assume, if it is a single product industry, everywhere decreasing average costs. This generates a monopoly as a result [3]. If the upstream industry is instead multiproduct, the decreasing average cost assumption is replaced by an assumption of decreasing average incremental costs[4] in the intermediate good in question. This again indicates that the intermediate good product line is monopolized, though the firm monopolizing that line may well face competition in its other product

[3] Were the upstream market contestable but not monopolized, then a result due to B-P-W {2, chapter 11} indicates that marginal cost pricing is a natural consequence. This would remove all cost distortions and incentives to integrate.

[4] Average incremental cost of a product line is the cost of adding that product line to the others the firm produces, divided by the volume in that line. See B-P-W, {2, chapter 11} for proof of the various assertions of this paragraph.

lines. In the single (multi-) product case, the price must cover average (incremental) costs. This in turn implies that price exceeds marginal cost. It is again assumed in the multiproduct case that the product line monopolist sells only one intermediate good to the downstream industry in question and that all other markets it serves are unrelated in demand or supply.

For either type of upstream market, the upstream firm is modelled as a price-taker in the markets for its raw inputs. As a supplier, however, the upstream firm is assumed to anticipate fully the effect of price on the demand for the intermediate good. This includes anticipating the possibility of inducing entry or exit downstream.

II.C The Technology of an Integrated Firm

The purpose of this subsection is to establish the technological conditions under which, in later sections, I can represent an integrated firm as a pair of subsidiaries, one upstream and one downstream. Specifically, I will represent the raw input costs of an integrated firm as the sum of the upstream and downstream subsidiaries' cost functions, netting out internal sales. The general outline of the argument is given here, with the mathematical details reserved for Appendix A.

I assume throughout the paper that the technology available to an integrated firm is a simple concatenation of the upstream and downstream technologies. Let us use $x > 0$ to represent a vector of raw inputs, y for the intermediate good (positive if output, negative if input), and $z > 0$ for the final good. Then the technological assumption is that any production possibility $(-x, y, z)$ available to an integrated firm $(y > 0)$ can equally well be produced by an upstream firm paired with a downstream firm and vice versa. Said upstream firm can find a vector

$(-x^U, y + y^N, 0)$ (with $y^N \geq$), and the downstream firm a vector $(-x^D, -y^N, z)$ for some y^N, x^U, x^D such that $x^U + x^D = x$. Similarly, for any $(-x^U, y + y^N, 0)$ produced by an upstream form, and any $(-x^D, -y^N, z)$ produced downstream, $(-(x^U + x^D), y, z)$ can be produced by an integrated firm.

That this technology suffices for the desired representation is established in two steps. First, it is shown in Appendix A:

> Theorem 2.1 (Informal Statement). If the integrated firm's technology is a simple concatenation of upstream and downstream technologies, then, for a given net output of final and intermediate goods, the integrated firm's minimum raw input cost is exactly equal to the minimum achievable by an optimally coordinated upstream-downstream pair of firms.

By "optimally coordinated" I mean that the production vectors of the pair of firms are coordinated centrally to minimize raw input costs.

The second step is to show that the optimal degree of central coordination can in fact be achieved by a decentralized price mechanism:

> Theorem 2.2 (Informal Statement).[5] Suppose that an upstream and a downstream firm individually minimize their costs. Then, for any given net output of intermediate and final goods, the decentralized pair will achieve the minimum possible total raw input costs
> i) if the upstream firm meets the downstream firm's demand for the intermediate good (in addition to producing the net output amount) and
> ii) if that good is priced at its marginal cost.
> Marginal cost pricing is also necessary to achieve the minimum, unless the derived downstream demand for the intermediate good is price inelastic.

[5] Formal statement and proof in Appendix A.

Putting these two theorems together, we find that, for the described technology, we may represent the costs of an integrated firm as the sum of the raw input costs of upstream and downstream subsidiaries operating "at arm's length." The only thing required is that the internal price be upstream marginal cost.

To put it another way, what this technology rules out is the case where a vertically integrated firm has cost saving advantages not available to any arrangement of non-integrated firms. For instance, we shall not consider here the classic example {18} of making steel ingots and sheet steel, where a firm which produces both can save reheating costs by rolling the hot ingots directly into sheet.

II.D The Integration Scenario

We are now in a position to describe the framework for vertical integration.[6] Once the upstream firm sets up its downstream subsidiary, it has three variables to choose. The internal price is adjusted to marginal cost,[7] the output of the downstream subsidiary is chosen directly, and the external price may be readjusted.

The external price determines the market price of the final good by determining the minimum average cost available to independent downstream firms. Any final good price above this minimum induces downstream entry by independent firms, while any price below induces exit.[8] Thus, the final good price is established in the same way after integration as before.

[6] Consistent with the plan for Sec. II, what follows is only a description of the assumptions made about the integration process. The actual results of integration are deferred until after Sec. III's welfare analysis of the non-integrated market.

[7] Following Sec. II.C., the integrated firm's choices upstream and downstream production plus can be modeled "as if" made at arm's length with the intermediate good priced at marginal cost.

[8] In those cases where the upstream firm fully integrates into

As for the downstream output choice, I assume the downstream subsidiary can sell as much as it chooses (up to market demand) at the reigning final good price. As a technical nicety, one could think of the downstream subsidiary undercutting this price by just a smidgeon to guarantee winning the desired sales from its independent rivals. Since all functions are assumed continuous, the resulting change in profit could be made arbitrarily small. For this reason, I will ignore the technical point and assume the downstream subsidiary sells at the market price.

I will, though, observe that the expansion of the subsidiary's final goods output, by "crowding out" independent downstream firms, indirectly affects external sales of the intermediate good. I assume that the integrated firm considers this effect in choosing its downstream output.

The cumulative effect of the technological assumptions in Secs. II.A. and II.C. should be considered carefully. Since the technology available to an integrated firm is a simple concatenation of upstream and downstream technologies, the integrated firm faces (eventually) decreasing returns to scale downstream, just as any other firm producing downstream.[9] Thus, it will appear--and will be represented until Sec.VI--as though the upstream firm integrates by setting up only a single downstream subsidiary.

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the downstream industry, the external price in the intermediate market is defined to be the value which would cause the minimum average cost of an independent downstream firm (if one existed) to equal the actual final price chosen.

[9] Some difference results from the differences in intermediate good price, but the U-shape of average costs downstream is a hypothesis maintained for all relevant input prices.

I wish to emphasize that this appearance of an institutional limitation (i.e., on the number of downstream subsidiaries that can be set up) in reality grows out of a limitation imposed by the underlying technology. "Technology" is meant in the broadest sense: it includes the management of production as well as production itself. Thus, a downstream firm runs into decreasing returns not only because of diseconomies of plant scale, but also because of the increasing *complexity of managing a large organization*. As the firm's scale grows, the number of management layers increases. Thus, the weight of the corporate bureaucracy pyramid grows more than proportionately with the base. Decisions require more approvals, oversight is more complex, and the coordination facility of top management becomes congested. The result is that unit costs eventually rise, no matter how cleverly the institutional arrangements are handled.

The assumption that decreasing downstream returns set in at exactly the same scale for an integrated firm as for an independent[10] can be relaxed. One could suppose, for instance, that the upstream firm finds it profitable to set up K downstream divisions (for some K>1), but not K+1. The adjustments in the model's results are minor and are discussed in Sec. VI. What matters is that decreasing returns eventually set in downstream for the integrated firm as well as for the independents.

Of course, if there are institutional limitations on forward integration--in addition or instead--then the model applies "without apology." For instance, such a limitation might be imposed by the actual or feared response of the antitrust authorities. If the

[10] More precisely: Whether any differences are not fully explained by differences in the intermediate good price.

downstream "U"'s are plant costs--and managerial diseconomies do not apply--then it can be shown that the upstream monopolist has the *incentive* to own all downstream plants. But, this is clearly an "attempt to monopolize" violation of the Sherman Act, Section 2. Thus, the upstream firm may stop or be stopped short of full integration. For purposes of convenience, I will assume that the limit is just one downstream acquisition. (The adjustments for a K-firm limit will be noted in Sec. VI).

For practical policy, the fact that, in this alternative, "institutional limitation" scenario, partial (rather than full) forward integration is exogenously imposed does not matter. Most often, the antitrust authorities will have to take a position on each downstream acquisition (i.e., each partial integration) without being privy to the upstream firm's overall ambitions. *For this purpose*, the results provide substantial guidance.

III. BEFORE INTEGRATION: WELFARE ANALYSIS

This section examines the efficiency properties of an upstream-downstream pair of markets absent integration. Efficiency, I demonstrate, generally requires marginal cost pricing in the intermediate market. This holds whether the efficiency criterion is maximizing welfare or minimizing aggregate input costs. Unfortunately, before integration, either regime for upstream entry entails non-marginal cost pricing and thus usually leads to inefficient operations by both of these criteria. Distinguishing the types of distortions caused by non-marginal cost pricing leads to a sharpening of the fixed proportions-variable proportions distinction. Among the three sources of welfare distortion identified is a new one: scale distortions in the downstream industry. Finally, I examine several specifications of non-negative profit constrained (or "Ramsey") optima. In general, only the weakest of these is satisfied when the upstream market is contestable, and none at all are when the upstream market is protected.

There is, unfortunately, a large amount of notation. For ease of reference, the notation common to all sections is defined in Table 3.1. (Some additional notation specific to certain sections is defined later when needed.)

Table 3.1

NOTATION CONVENTIONS

Notation Conventions: Small case letters refer to firm variables (inputs and outputs). Capital letters refer to market aggregates. Post-integration variables are usually designated with a "^". A star "*" designates the value of a function when evaluated at the efficient scale of a downstream firm. The letter "y" refers to amounts of the intermediate good, and "p" to its price. Similarly, "x" refers to amounts of raw inputs and "w" is the vector of their prices. (These are usually suppressed.) The "z" refers to outputs of the final good. I define the following specific variables and functions.

$C^D(z,p,w)$	=	cost function of a downstream firm.
$AC^D(z,p)$	=	downstream firm's average cost (w suppressed).
$z^*(p)$	=	$\underset{z}{\text{argmin}}\ AC^D(z,p)$ = efficient scale of a downstream firm as a function of intermediate good price.
$AC^*(p)$	=	$AC^D(z^*(p),p)$ = minimum value of downstream average cost. This is also the market price of the final good except in the Ramsey analysis where said price is chosen explicitly.
f	=	price of the final good (for Ramsey analysis only).
$Q(AC^*(p))$	=	market demand for final good as a function of intermediate good price.
$N(p)$	=	number of downstream firms without integration = $Q(AC^*(p))/z^*(p)$. This is treated as a continuous variable.

$y^*(p)$ = $y(z^*(p),p)$ = intermediate good demand by downstream firm operating at efficient scale.

$Y(p)$ = total sales of intermediate good (without integration) $\equiv N(p)\cdot y^*(p)$.

$C^U(Y)$ = upstream cost function.

p^m = monopoly price of intermediate good (before integration)

e_{yz} = $\frac{z^*}{y^*}\cdot\frac{\partial y^*}{\partial z}$ = output elasticity of a downstream firm's input demand for the intermediate good, at efficient scale.

$\pi^U(p)$ = profit of non-integrated upstream monopolist.

<u>Post-integration variables</u>:

p^I = price of internal sales of intermediate good to downstream subsidiary; the "internal price."

$\hat{z}$ = output of downstream subsidiary of integrated firm.

$\hat{y}$ = $y(\hat{z},p^I)$ = intermediate good demand of downstream subsidiary.

$\hat{p}$ = price on external sales of intermediate good to remaining downstream independent firms; the "external price."

$\hat{N}$ = $\hat{N}(\hat{p},\hat{z})$ = number of independent downstream firms after integration = $\frac{Q(AC^*(\hat{p})) - \hat{z}}{z^*(\hat{p})}$

$\hat{Y}(\hat{p},\hat{z})$ = external sales of intermediate good (post-integration) $= \hat{N}(\hat{p},\hat{z})\cdot y^{*}(\hat{p})$.

$\hat{p}^{m}$ = post-integration monopoly price on external sales of intermediate good. This represents the profit maximizing choice when the upstream market is protected, and the equilibrium value in the contestable case.

$\pi^{Int}(\hat{p},p^{I},\hat{z})$ = profit of integrated firm with monopoly in intermediate market.

III.A. Welfare Maximization, Welfare Distortions, and Input Types

To show when welfare maximization requires marginal cost pricing, I derive the appropriate first order condition. Welfare (W) here is defined as the sum of final good consumer surplus (CS) plus upstream and downstream industry profits (π^{U} and π^{D}, respectively):

(3.1) $W \equiv CS + \pi^{U} + \pi^{D}$, where $\pi^{D} = 0$ by assumption

(3.2) $CS \equiv \int_{AC^{*}(p)}^{\infty} Q(s)ds$, and

(3.3) $\pi^{U} \equiv pY - C^{U}(Y)$

Then, using $dAC^*/dp = y^*/z^*$ (itself derived from Shephard's Lemma and the fact that $\partial AC/\partial z = 0$ at the efficient downstream scale) and $Y = N \cdot y^* = (Q(AC^*)/z^*)y^*$ I get:

$$(3.4) \qquad \frac{dW}{dp} = -Q \cdot \frac{dAC^*}{dp} + Y + (p - \partial C^U/\partial y) \cdot \frac{dY}{dp} = (p - \frac{\partial C^U}{\partial y})\frac{dY}{dp}$$

Setting Eq. (3.4) to zero, we see that marginal cost pricing of the intermediate good is necessary for welfare maximization unless the market demand for that good is completely inelastic.

Computing the price response of intermediate market demand yields:

$$(3.5) \qquad \frac{dY}{dp} = Q' \cdot (y^*/z^*)^2 + Q \cdot \frac{d(y^*/z^*)}{dp}$$

$$= [Q' \cdot (y^*/z^*)^2] + N \cdot [\frac{\partial y^*}{\partial p} - \frac{\partial^2 C^D(z^*)}{\partial z^2} \cdot (\frac{dz^*}{dp})^2]$$

which uses

$$\frac{d(y^*/z^*)}{dp} = \frac{1}{z^*}[\frac{\partial y^*}{\partial p} + \frac{\partial y^*}{\partial z^*} \cdot \frac{dz^*}{dp} - \frac{y^*}{z^*} \cdot \frac{dz^*}{dp}]$$

(3.5a)

$$= \frac{1}{z^*}[\frac{\partial y^*}{\partial p} - \frac{\partial^2 C^D(z^*)}{\partial z^2} \cdot (\frac{dz^*}{dp})^2 ,$$

the second equality of which in turn uses

$$(3.6) \qquad \frac{dz^*}{dp} = \frac{y^*}{z^*}\left(\frac{1 - e_{yz}}{\partial^2 C^D(z^*)/\partial z^2}\right).$$

Eq. (3.6) is a result due to Bassett and Borcherding {1}. Of course, the substitution response of a firm's input demand is always non-positive when the input price rises:

$$(3.7) \qquad \partial y^*/\partial p \leq 0$$

Further, downstream marginal cost is upward sloping at the "bottom of the U":

$$(3.8) \qquad \partial^2 C^D(z^*)/\partial z^2 > 0$$

Then, if final demand is not upward sloping ($Q' \leq 0$), [1]

$$(3.9) \qquad dY/dp \leq 0.$$

---

[1] Downward sloping total market demand for the input does not imply downward sloping firm market demand. Certainly the firm's substitution response is negative when the input price rises, but at the same time the firm may be induced to operate at a higher scale. The output effect on input demand is positive for a normal input. It may swamp the substitution effect and cause the firm's demand to rise. But, if efficient scale expands while total *final good demand* shrinks (due to passing on the cost increase to the final consumers), exit is induced downstream. What guarantees (3.9) is that the exit is always sufficient to cause total demand for the intermediate good to fall.

That $d(y^*/z^*)/dp$ and (3.9) are non-positive was recognized by Silberberg {13}, among others. However, he did not break the former into substitution and scale effects, as in Eq. (3.5a).

To identify more easily the case where $dY/dp = 0$ (that is, where marginal cost pricing is not necessary), let us define the following input types:

Definition 3.1. A firm's demand for a variable proportions input is sensitive to the input's price. (Specifically, (3.7) holds strictly.)

Definition 3.2. A fixed schedule input is one whose firm level derived demand is not sensitive to its price.[2] ((3.7) holds with equality.)

Definition 3.3. A fixed proportions input is a fixed schedule input whose firm demand is linear in firm output.

If $y(z,p)$ is linear in z at z^*, then $e_{yz} = 1$ and $dz^*/dp = 0$. Thus, if the intermediate good is a locally fixed proportions input at z^* and p, then $\partial y^*/\partial p = 0$ and $dz^*/dp = 0$.

I have thus established the following theorem:

> Theorem 3.1. Welfare maximization (without integration) requires marginal cost pricing of the intermediate good unless
> i) final demand is perfectly inelastic, and
> ii) the intermediate good is a locally fixed proportions input (LFPI, hereafter) downstream. [3]

---

[2]Neither this nor the next definition should be confused with a fixed input, which is simply an input which has zero output elasticity; i.e., one which is only an element of fixed costs.

[3] To streamline the verbiage, I will usually drop the expressions "locally" and "in the relevant range of input prices and outputs," in connection with input types. They may be assumed to apply to all theorems below except where global requirements are stated explicitly.

If the intermediate price exceeds marginal cost, Eqs. (3.4) and (3.5) together show clearly the three sources of welfare loss. The first two are well-known. The first term on the right side of (3.5) is the "output effect": the effect on total final demand when the higher intermediate price is passed on to final consumers. The second term represents another well-known welfare loss: the "substitution effect." The downstream firms face input prices which do not reflect relative social marginal costs and inefficiently substitute away from the intermediate good.

The third term in (3.5) is the "scale effect," an effect not capturable in a model with constant returns downstream. Except when the intermediate good demand is locally linear in output, a rise in the intermediate price shifts the privately efficient scale away from the socially efficient scale. Downstream firms thus are induced to operate away from the socially efficient output level. The resultant welfare loss is the "misfortune of the misshaped U."

It is interesting to note a way in which the scale effect resembles a substitution effect. The substitution effect causes the firm to use less y at a given z as p rises. Thus, y/z falls: the input is used less intensively relative to output. Input intensity also falls with the scale effect. When e_{yz} is less (greater) than one, y/z falls if output grows (falls). But from Eq. (3.6), output expands (falls) when p rises exactly when e_{yz} is less (greater) than one. (The two parts of the shift toward lower y-intensity are also shown in Eq. (3.5a).) In general, then, as the price of the intermediate good rises, an individual firm at a given output substitutes to an input mix less intensive in that input, and the market, via the scale effect, substitutes to a firm size which uses that input less intensively.

III. B. Cost Minimization

The other efficiency criterion I wish to apply is the minimization of the aggregate cost of raw inputs used in producing a given total final output. The raw input usage is induced by the conditions in the market, principally by the intermediate good price. Subtracting out downstream expenditures on the intermediate good, we define an "induced cost function," representing the raw input costs of producing a fixed final output $\bar{Q}$:

$$(3.10) \quad IC(\bar{Q},p) \equiv \bar{N}\cdot C^D(z^*,p) + C^U(\bar{N}y^*) - p\cdot(\bar{N}y^*)$$

$$\text{where} \qquad \bar{N} \equiv \bar{Q}/z^*$$

is the induced number of downstream firms.

I prove the following theorem:

Theorem 3.2. The necessary condition for minimizing induced raw input costs is that the intermediate good price be set at upstream marginal cost, unless the intermediate good is a LFPI.

Proof: Substituting the identities $\bar{N}\cdot C^D(z^*) = \bar{Q}\cdot AC^*$ and $\bar{N}y^* = \bar{Q}\cdot(y^*/z^*)$ into Eq. (3.10) and using the expression for $d(y^*/z^*)/dp$ from Eq.(3.5a), I derive the effect of p on IC:

$$(3.11)\quad \frac{dIC}{dp} = \bar{Q}\cdot(y^*/z^*) + (\partial C^U/\partial y - p)\cdot\bar{Q}\cdot \frac{d(y^*/z^*)}{dp} - \bar{Q}(y^*/z^*)$$

$$= -\ (p - \frac{\partial C^U}{\partial y})\cdot\bar{N}\cdot[\frac{\partial y^*}{\partial p} - (\frac{dz^*}{dp})^2 \cdot \frac{\partial^2 C^D(z^*)}{\partial z^2}]$$

The necessary condition for cost minimization is that Eq. (3.11) equal zero. The last factor is zero only if the intermediate good is a LFPI. The only other way (3.11) can be zero is for p to equal upstream marginal cost.

E.O.P.

The message from (3.11) is clear: substitution and scale distortions cause higher costs. Indeed, Eq. (3.4), the overall welfare effect of the intermediate price, and Eq. (3.11), the cost effect, would be identical if final demand were inelastic.

In the operation of the actual market, neither welfare criterion is usually satisfied for either type of upstream monopoly. When the upstream firm is protected from entry, it sets price above marginal cost in the usual monopoly way. When there is free entry upstream, the upstream monopolist gets zero profit, setting price at average cost. But, since average cost is by assumption declining, it exceeds marginal cost. Thus, marginal cost pricing fails in either entry situation. This leads to an easy corollary:

Corollary 3.3. In either upstream entry regime, a non-integrated market will fail to be welfare optimal unless both conditions of Thm. 3.1 hold and will fail to minimize raw input costs unless the intermediate good is a L.F.P.I.

The finding that raw input costs are not minimized even when the upstream market is contestable runs counter to the findings of B-P-W {2, Chap. 11} for a horizontal industry. Their finding is that in a contestable market--as is either the upstream or downstream market in isolation--not only does each firm minimize its own cost of production, but output is allocated among firms in such a way as to minimize aggregate costs. This minimization is relative to the input prices faced, which are supposed to represent social marginal costs. The problem for the downstream industry here is that one of those prices, the intermediate good price, is endogenous and thus may not represent a social marginal cost. The combined effects upstream of scale economies and the need to cover costs create too great a burden: the wrong price is chosen. And, given the wrong signal, neither individual downstream firms nor the downstream contestable industry as a whole can usually accomplish its cost minimizing task.

III.C. Ramsey Optima

If the upstream firm's need to cover costs prevents the contestable case from achieving first best results, does this market at least satisfy a profit-constrained second best--or "Ramsey"--optimum? The answer depends in part on how one defines a Ramsey optimum. There are several possibilities, depending on the degree to which market forces are accommodated.

The best of the second best is the case where inputs and outputs of all firms are chosen optimally, subject only to the constraint that the revenue generated by sales of the final product cover the total industry raw input costs. For whatever total final output is produced, raw input costs are naturally minimized. Clearly, by Corollary 3.3, neither the contestable nor, for that matter, the protected upstream case satisfies such a Ramsey optimum (except for L.F.P.I.).

A weaker Ramsey-type optimum would allow downstream firms to choose their own inputs. In this case, only prices and downstream firm outputs would be chosen optimally, subject to the constraint that all firms have non-negative cash flow. If, in this case, the intermediate good price is above marginal cost, then substitution distortions arise even in the Ramsey optimum. Scale distortions, however, are at least partially eliminated[4] by the fact that downstream firm outputs are optimally chosen. Thus, the market outcome will be inconsistent with this Ramsey optimum if the former entails scale distortions.

It is easy to establish this formally. To set up the problem, first note that downstream firm output and final good price are going to be chosen directly. Let these be z and f, respectively, and note that they are not in general equal to $z^*(p)$ and $AC^*(p)$. Then, the Lagrangean for the problem is

$$(3.12) \qquad L(z,p,f,\lambda,\mu,) = \int_f^\infty Q(s)ds + (1+\lambda)\pi^D + (1+\lambda)\pi^U ,$$

[4] To be precise, downstream firms will use the "wrong" mix of inputs at any output level if there are substitution distortions. The Ramsey downstream firm output level will be that which minimizes social unit costs--not private unit costs--along this "wrong" expansion path The scale chosen may thus differ both from the market outcome and from the scale which minimizes unit costs along the "right" expansion path.

where λ and μ are the multipliers, respectively, for the downstream and upstream profit constraints. With $Q(f)/z$ identical downstream firms, the downstream industry profit can be written.

$$\text{(3.13)} \qquad \pi^D(z,p,f) = (Q(f)/z)\cdot(f\cdot z - C^D(z,p)) = Q(f)\cdot(f - AC^D(z,p)).$$

Market demand for the intermediate good is now

$$\text{(3.14)} \qquad Y(z,p,f) = (Q(f)/z)\cdot y(z,p) = Q(f)\cdot(y(z,p)/z)$$

so that upstream profit is

$$\text{(3.15)} \qquad \pi^U(z,p,f) = p\cdot Q(f)\cdot(y(z,p)/z) - C^U(Q(f)(y(z,p)/z))$$

Then, a first order condition for this Ramsey optimum is

$$\text{(3.16)} \qquad \frac{\partial L}{\partial z} = -(1+\lambda)Q\cdot\frac{\partial AC^D}{\partial z} + (1+\mu)(p - \frac{\partial C^U}{\partial y})Q\,\frac{\partial(y/z)}{\partial z} = 0.$$

Dividing by Q, writing out d(y/z)/dz, and rearranging yields

$$\text{(3.17)} \qquad (1+\lambda)\frac{\partial AC^D}{\partial z} = \frac{(1+\mu)}{z}(p - \frac{\partial C^U}{\partial y})(\frac{\partial y}{\partial z} - \frac{y}{z}).$$

Since μ and λ are both non-negative, if p exceeds upstream marginal cost, then

$$\text{(3.18)} \qquad \text{sign}\,(\frac{\partial AC^D(z,p)}{\partial z}) = \text{sign}\,(\frac{\partial y(z,p)}{\partial z} - \frac{y(z,p)}{z})$$

The inconsistency of market and Ramsey outcomes is as follows. In the market outcome, the left side of (3.18) is always zero: downstream firms always operate in the market at the bottom of their (perceived) average cost curves. But, the right side of (3.18) at the market outcome equals $(e_{yz} - 1)(y^*/z^*)$. Thus, if there are scale distortions by Eq. (3.6) in the market outcome (i.e., if $e_{yz} \neq 1$) then (3.18) will not hold.

The Ramsey adjustment implied by (3.18) is easy to understand. The right side of (3.18) will be positive (negative) if y is used more (less) intensively (in the sense established above) at higher levels of output. The Ramsey output, by (3.18), will then be on the upward (downward) sloping section of the downstream average cost curve. That is, it will be higher (lower) than the level the market would choose at the same p. Thus, the Ramsey outcome will use the intermediate good more intensively downstream than the market outcome at the same p, unless the intensity does not vary with downstream scale, that is, unless there are no scale distortions.

Note, conversely, that if there are scale distortions the Ramsey optimum cannot be a market outcome: downstream Ramsey firms are operating--and therefore pricing--above minimum perceived average cost. Sustaining this Ramsey optimum in the market would thus require restrictions on entry.

The weakest form of a Ramsey optimum, the one that makes greatest accommodation to market forces, is the one in which only the intermediate good price is chosen optimally. Downstream output, price, and input demands are determined in the usual market way. Equations

(3.4) and (3.9) then dictate that the optimal p, if not marginal cost, is the lowest consistent with upstream cost coverage. Since this is the price chosen when the upstream market is contestable, said case satisfies the weakest of Ramsey criteria. The protected upstream case, with neither marginal cost pricing or zero profit upstream, does not.

IV. VERTICAL INTEGRATION WHEN THERE IS NO UPSTREAM ENTRY

This section analyzes the incentives for forward integration and the local and global changes that result when the upstream monopolist faces no threat of entry. Generally speaking, either scale or substitution distortions generate incentive to integrate. I suppose that a firm integrates by setting up a downstream subsidiary which initially faces the same input and output prices and produces at the same scale as the other downstream firms. The newly integrated firm tends locally to correct for substitution and scale distortions in its downstream subsidiary. It also (at least as a local change) raises the external price. Globally, the internal price falls all the way to marginal cost. Conditions are discussed which determine whether the integrated firm has profit incentive to expand output to 100 percent of the downstream market. Frequently the answer is no. When it does not (and assuming upstream marginal costs are constant), the external price always rises. This forces up the final good price. Analyzing the incentives for adjusting downstream output generates comparisons among the size of the downstream subsidiary and those of its pre- and post-integration rivals.

IV.A. The Incentives to Integrate: A Conceptual Experiment

The existence of incentives to integrate is determined by the following conceptual experiment. Suppose that integration begins with the upstream monopolist setting up a downstream subsidiary as above.[1] Immediately after integration--before any price or output adjustments--

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[1] As discussed in Sec. II, although it appears in the mathematics that there is an institutional limitation on the number of downstream

the integrated firm's profits are unchanged, since its downstream subsidiary is just breaking even. The firm now has three variables to adjust: the internal and external prices and the output of the downstream subsidiary. If it can increase profit by adjusting any one of these, then there is positive incentive for forward integration. Profit-raising adjustments in turn exist if a derivative of integrated firm profits by any one of these three variables is non-zero.

The derivative of integrated profits with respect to the internal price is negative unless the intermediate good is used as a locally fixed schedule input downstream. Integrated firm profits are given by

$$\text{(4.1)} \qquad \pi^{Int}(\hat{p},p^I,\hat{z}) = \underbrace{\left[\hat{z} \cdot AC^*(\hat{p}) - C^D(\hat{z}, p^I)\right]}_{\text{downstream subsidiary profit}} + \underbrace{\left[\hat{p}\hat{Y} + p^I\hat{y} - C^U(\hat{y} + \hat{Y})\right]}_{\text{upstream subsidiary profit}}$$

with the profits of the upstream and downstream branches as indicated. Using Shephard's Lemma, which implies $\hat{y} = \partial C^D(\hat{z}, p^I)/\partial p^I$, it is straightforward to compute:

$$\text{(4.2)} \qquad \frac{\partial \pi^{Int}}{\partial p^I} = \left(p^I - \frac{\partial C^U}{\partial y}\right)\left(\frac{\partial \hat{y}}{\partial p^I}\right)$$

Since the immediate post-integration internal price is the former

subsidiaries set up, the source of the limitation is really in the underlying technology available for downstream production. Of course, the analysis is equally applicable when the limitation is in fact institutional (e.g., antitrust considerations). See Sec. VI for the relaxation of this limitation.

upstream monopoly price (and thus exceeds marginal cost), Eq. (4.2) is indeed negative unless y is a local fixed schedule input downstream.

A substitution distortion--which occurs if the intermediate good is not a fixed schedule input--thus provides incentive to integrate.[2] Lowering the internal price to marginal cost increases profit by eliminating any substitution distortion in the downstream subsidiary. Thus, the local and global changes in p^I are determined.

The existence of scale distortions also creates an incentive to integrate forward. This fact becomes clear by computing the derivative of integrated profit with respect to downstream output and evaluating it at the immediate post-integration values of the choice variables. The first step is to compute

(4.3)

$$\frac{\partial \pi^{Int}}{\partial \hat{z}} = \underbrace{\left[AC^* - \frac{\partial C^D(\hat{z}, p^I)}{\partial \hat{z}}\right]}_{\text{price cost margin on final good sales}} + \underbrace{\left[\hat{p} - \frac{\partial C^U}{\partial y}\right] \cdot \left(\frac{\partial \hat{Y}}{\partial \hat{z}}\right)}_{\text{price cost margin on external intermediate good sales}} + \underbrace{\left[p^I - \frac{\partial C^U}{\partial y}\right] \frac{\partial \hat{y}}{\partial \hat{z}}}_{\text{price cost margin on internal intermediate good sales}}$$

where

(4.4) $\partial \hat{Y}/\partial \hat{z} = -y^*/z^*$ = "crowding out" effect.

Immediately after integration, internal and external prices are both still at the old monopoly level p^m and downstream output at the old efficient scale. Thus, downstream average and marginal costs are still equal, and the price-marginal cost margin on final sales is zero. The only local effect of $\hat{z}$ then is on upstream subsidiary profit. The last two terms of Eq. (4.3) capture that effect. At the indicated prices and outputs, those terms collapse to yield

[2] In the case of downstream constant returns, this is a very well-known result. See (6), (12), (15), (16), (17), and (18).

(4.5)

$$\left.\frac{\partial\pi^{Int}}{\partial\hat{z}}\right|_{\substack{\hat{p} = p^I = p^m \\ \hat{z} = z^*(p^m)}} = -(p^m - \partial C^U/\partial y)\ (y^*/z^* - \partial\hat{y}/\partial\hat{z})$$

$$= -\left(p^m - \frac{\partial C^U}{\partial y}\right)\left(\frac{\partial^2 C^D(z^*)}{\partial z^2}\right)\frac{dz^*(p^m)}{dp}\begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0$$

$$\text{as } \frac{dz^*(p^m)}{d\hat{p}}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0,$$

where the second equality uses Eq. (3.6). Thus, as claimed, if the downstream efficient scale is subject to distortion when the intermediate good price is above marginal cost (i.e., if $dz^*/dp \neq 0$), there is an incentive to integrate.

The initial adjustment in $\hat{z}$ partially redresses any previous downstream scale distortion. For example, suppose the intermediate good is a fixed input[3] downstream, so that $e_{yz} = 0$ and $dz^*/dp > 0$. Then, setting the intermediate good price above marginal cost induces the pre-integration downstream firms to be "too large." But, in that case, the local profit incentive after integration is to lower $\hat{z}$, per Eq. (4.5). This tends to "correct" the scale distortion in the downstream subsidiary. Indeed Eq. (4.5) indicates that, in general, the integrated firm's immediate incentive is to correct any scale distortion in its own subsidiary.

The real incentive for the adjustment in downstream output is not to save this integrated firm's costs, but to reduce the costs of the total downstream output by increasing the usage intensity of the

[3] Recall that a fixed input is one with zero output elasticity--not necessarily a fixed schedule input.

intermediate good. As described in Sec. III, setting the intermediate good price above marginal cost causes downstream firms to operate at a scale (as well as an input mix) where the intermediate good is used less intensively than if the price were at its optimal value. By adjusting its subsidiary's output against this distortion, the integrated firm increases the intensity of the intermediate good usage at least for the output the firm controls.

That this increase in intensity reduces total downstream costs is argued by process of elimination. Since no independent downstream firm makes a profit, the integrated firm's profit is what is left over from total final good revenue after subtracting upstream and downstream raw input costs. Adjusting $\hat{z}$ does not affect the final good price [4] and thus does not change final good revenue. As noted, the adjustment raises upstream output and costs. For the output change to be profitable, it must therefore reduce total downstream costs even more.

Either scale or substitution distortions also create an integration incentive by providing a profitable local opportunity to raise the intermediate good price to the remaining downstream firms. Again, this result is established by computing and evaluating the appropriate derivative of integrated profits.

(4.6)

$$\frac{\partial \pi^{Int}}{\partial \hat{p}} = \underbrace{\left[\left(\hat{p} - \frac{\partial C^U}{\partial y}\right)\frac{\partial \hat{Y}}{\partial \hat{p}} + \hat{Y}\right]}_{\substack{\text{effect on}\\ \text{upstream}\\ \text{profit}}} + \underbrace{\left[\hat{z} \cdot \frac{dAC^*}{d\hat{p}}\right]}_{\substack{\text{effect on}\\ \text{downstream}\\ \text{subsidiary's}\\ \text{profit}}}$$

---

[4] Recall that the final good price AC^* depends only on the external price. Any change in $\hat{z}$ is compensated by an induced change in the other firms' output, so that the total output remains $Q(AC^*)$.

Using first the identities

$$\hat{Y} = \hat{N}\cdot y^* = \left(\frac{Q-\hat{z}}{z^*}\right)\cdot y^* = Y - \hat{z}(y^*/z^*), \tag{4.7}$$

and $dAC^*/dp = y^*/z^*$, then the first order conditions for the maximization of non-integrated upstream profit, and finally the expression for $d(y^*/z^*)/dp$ in Eq.(3.5a), it can be shown that, at the immediate post-integration values, Eq. (4.6) simplifies to:

$$\left.\frac{\partial\pi^{Int}}{\partial\hat{p}}\right|_{\substack{\hat{p} = p^I = p^m \\ \hat{z} = z^*(p^m)}} = (p^m - \partial C^U/\partial y)\left[dY/d\hat{p} - \hat{z}\cdot\frac{d(y^*/z^*)}{d\hat{p}}\right] + Y$$

$$= \left.(p^m - \frac{\partial C^U}{\partial y})\left[\frac{\partial^2 C^D}{\partial z^2}\cdot\left(\frac{dz^*}{dp}\right)^2 - \frac{\partial y^*}{\partial p}\right]\right|_{\hat{p} = p^m} \geq 0 \tag{4.8}$$

Equality holds in (4.8) only if the intermediate good is a L.F.P.I. Thus, the local incentive is to raise the external price if there are any cost distortions downstream.

The incentive to raise the external price is purely to benefit the downstream subsidiary. Such a price rise increases downstream profits in Eq. (4.6), since the only effect on the downstream subsidiary is to induce a rise in the price of its output. Using Eq. (4.7) and, again, the non-integrated first order profit conditions, it can be shown that the first term in (4.6) is negative at the initial post-integration values. Thus, the effect of the price rise on upstream profit is negative. The net effect on integrated profits is positive whenever there are cost distortions to aggravate among the downstream independents.

In sum, this subsection has established the following theorems:

> <u>Theorem 4.1</u>. The upstream monopolist can increase profit by forward integration anytime there are substitution or scale distortions downstream; that is, unless the intermediate good is a L.F.P.I.
>
> <u>Proof</u>: Eqs. (4.2), (4.5), (4.8).
>
> <u>Theorem 4.2</u>. After integration, the integrated firm's local incentives are to adjust the internal intermediate good price and the downstream output to correct for, respectively, substitution and scale distortions in the downstream subsidiary.
>
> <u>Proof</u>: Eqs. (4.2), (4.5), (3.6).
>
> <u>Theorem 4.3</u>. Unless the intermediate good is a L.F.P.I., the local post-integration incentive is to raise the price on external sales of the intermediate good.
>
> <u>Proof</u>: Eq. (4.8)

The principal contributions of these theorems to what is already known about the constant returns downstream case are the identification of scale distortions, the more precise definition of input types, and the linkage of each to integration incentives. Compare, for instance, Schmalensee {12}.

IV.B. Global Changes

This subsection deals with the global changes that result from integration. It is obvious from Eq. (4.2) that the internal price is adjusted to marginal cost if there are any substitution distortions at all. However, as the choice variables are adjusted away from their initial post-integration values, the incentives to alter the external price and the downstream output change in somewhat complicated ways. I describe these incentives after discussing the issue of whether the integrated firm takes over the entire downstream market.

The integrated firm's downstream subsidiary may or may not take over the entire downstream market, depending on the cost and demand functions involved. The incentives to adjust the downsteam subsidiary's output are captured in Eq. (4.3). Before the internal and external prices are adjusted, Eq. (4.3) simplifies to Eq. (4.5). As p^I is adjusted to upstream marginal cost, the third term in (4.3) disappears, while, if y is a normal input, the downstream subsidiary's marginal costs fall. If the external price rises, as is the local incentive, AC* also rises. Since AC* is the output price received by the downstream subsidiary, the tendency of these two price changes is to create a positive price-cost margin for the output of the downstream subsidiary (i.e., in the first term of (4.3)). If said margin reinforces (or overcomes) the initial output adjustment incentive in Eq. (4.5), then the downstream subsidiary will expand.[5] As it expands, however, its marginal cost (by assumption) rises, causing the price-cost margin to evaporate. Whether the subsidiary takes over the whole downstream market depends on whether

[5] If it does not expand, then taking over the whole downstream market is rather hard.

said margin disappears before the subsidiary's output reaches market demand.[6] If demand is deep enough (and marginal cost rises quickly enough), the subsidiary will stop short, and integration will be partial.[7]

Consider an illustrative example. Let upstream marginal and average costs be unity. Suppose there are only two inputs downstream. Let downstream costs and final demand be, respectively,

$$C^D(z,p,w) = p + wz^2,$$

$$Q(AC^*) = g \cdot (AC^*)^{-2},$$

where w is the price of the raw downstream input and g is a depth of demand parameter. Note that the intermediate good is a fixed input downstream. If w is unity, then it can be shown for any value of g exceeding 63.73, *final demand is sufficiently deep as to make full* integration less desirable than partial integration.

For the balance of the paper, I will limit my attention to partial integration, which I consider the more usual case. Also, to proceed with a global analysis of the external price and downstream output changes, I have found it necessary to assume that <u>upstream</u> marginal costs are

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[6] Note, too, that the second term in (4.3), the crowding out effect, is negative. Expanding downstream output crowds out independent downstream firms, causing a loss of (profitable) intermediate good sales. (The internal sales which tend to replace them have a zero profit margin.) This effect creates an additional drag on the incentive to expand.

[7] If the downstream average and marginal cost curves are only very slightly bent, approximating arbitrarily closely the downstream constant returns case of Warren-Boulton {16} and successors {6} and {17}, then the integrated firm may in fact take over the entire downstream market. In that case, the internal price will still be set to marginal cost, and the downstream output, of course, expands. Following the earlier results, final product price could rise or fall, depending on the parameter values.

constant in the relevant range. For the balance of Sec. IV. (and for designated parts of Secs. V. and VI.), I will therefore assume that $\partial C^U/\partial y = \bar{c}$, a constant, for the range of intermediate outputs considered. I allow for the possibility of upstream scale economies by including non-negative "fixed costs," F^U.[8]

IV. B.1. The external and final good price changes. Given the assumption of constant upstream marginal costs, when the forward integration is only partial, the external price always rises, pushing the final good price up with it.[9]

Before proving a formal theorem to this effect, I first rearrange the expression for the integrated firm's profit. Substituting into Eq. (4.1) the constant marginal cost representation of upstream costs, $C^U(\hat{y} + \hat{Y}) = F^U + \bar{c} \cdot (\hat{y} + \hat{Y})$, and using Eq. (4.7) for $\hat{Y}$, I get

$$\pi^{Int}(\hat{p}, p^I, \hat{z}) = \hat{z}\left[AC^* - \frac{y^*}{z^*}\left(\hat{p} - \bar{c}\right)\right] + \left[\hat{p}Y - \bar{c}Y - F^U\right] + \left[\left(p^I - \bar{c}\right)\hat{y} - C^D(\hat{z}, p^I)\right] \tag{4.9}$$

Recall that Y represents the non-integrated intermediate market demand. Setting the internal price to upstream marginal cost (as is always

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[8] Technically, $F^U \equiv C^U(\hat{y} + \hat{Y}) - \bar{c} \cdot (\hat{y} + \ddot{Y})$. It thus behaves as a fixed cost amount in the range for which $\partial C^U/\partial y = \bar{c}$.

[9] Schmalensee {12} is unable to determine the direction of these price changes for partial integration. His model, which has constant returns to scale, both upstream and down, and a protected upstream monopoly, results in partial integration only when there is an institutional constraint on the degree of forward integration. By slightly reformulating his model in Appendix B, I am able to show that external and final prices do in fact rise as the integration constraint is relaxed. Thus, partial integration by a protected monopolist results in higher intermediate and final prices for either constant returns or U-shaped average costs downstream.

optimal) and recognizing that the second square bracket in Eq. (4.9) is functionally identical to π^U, the non-integrated upstream profit [10], the result is

$$\pi^{Int}(\hat{p},\bar{c},\hat{z}) = \hat{z}\cdot\Theta(\hat{p}) + \pi^U(\hat{p}) - C^D(\hat{z},\bar{c}) \tag{4.10}$$

where

$$\Theta(p) \equiv AC^*(\hat{p}) - (\hat{p} - \bar{c})(y^*/z^*) \tag{4.11}$$

We are now in a position to prove the following theorem:

Theorem 4.4. If it is profitable (due to scale or substitution distortions) to integrate forward--but only partially--and if upstream marginal costs are constant in the relevant range, then the external price unambiguously rises, and with it the price of the final good.

Proof: Setting $p^I = \bar{c}$ and downstream output to its optimal value (whatever it may be), integrated firm profit is given by Eq. (4.10) evaluated at the optimal $\hat{z}$. Note that, from Eq. (3.5a)

$$\Theta'(\hat{p}) = -\frac{(\hat{p} - \bar{c})}{z^*}\left[\partial y^*/\partial\hat{p} - \left(\frac{\partial z^*}{\partial p}\right)^2\cdot\left(\partial^2 C^D(z^*)/\partial z^2\right)\right] \geq 0 \tag{4.12}$$

[10] It should be noted that said expression does not represent the profits of the upstream subsidiary or any other post-integration entity. It is only a functional identity.

with the inequality being strict if there are substitution or scale distortions. Thus, in adjusting the last variable, the external price, to its optimal value, integrated profit is the sum of--left to right in (4.10)--an increasing function, plus a function which reaches a global maximum at the pre-integration intermediate good price, minus a constant (with respect to $\hat{p}$). The first two terms are lower for external prices below the pre-integration level and the third is the same. Since (4.10) is increasing in $\hat{p}$ at the pre-integration level (if there are cost distortions), the global maximum must occur at a higher value of $\hat{p}$. The final good price rises with it, as before.
E.O.P.

The key to this result is the θ function. The θ function (Eq. (4.11)) is the difference between the final good price the downstream subsidiary receives and the intermediate good profits it "crowds out," on average, per unit of its own final output. Equation (4.12) indicates that, if there are cost distortions, the former rises faster than the latter as the external price rises. Thus, if cost distortions make integration profitable in the first place, then raising the external price makes producing the integrated firm's own final output even more profitable, relative to selling the intermediate good to the downstream independents.

IV. B.2. The output of the downstream subsidiary. The global change in the integrated firm's downstream output is found by adjusting the internal and external prices to their optimal values and examining the integrated firm's profits as a function of downstream output alone. After the price changes, integrated profit is concave in $\hat{z}$. Thus, the direction of global adjustment in $\hat{z}$ is given by the sign of $\partial\pi^{Int}/\partial\hat{z}$ when evaluated at the optimal $\hat{p}$, the original $\hat{z}$ and $p^I = \bar{c}$. The value of the derivative at this point is the derivative's value at the initial post-integration point plus the influences of the price changes.

The formal analysis begins with the following lemma:

Lemma 4.5. Once the internal and external prices have been set to the optimal values, integrated profit is a concave function of downstream output.

Proof: When the internal price has been set to $\bar{c}$, integrated profit is given by Eq. (4.10). Then, by direct computation,

(4.13)
$$\left.\frac{\partial^2 \pi^{Int}}{\partial \hat{z}^2}\right|_{p^I=\bar{c}} = -\frac{\partial^2 C^D(\hat{z},\bar{c})}{\partial z^2}$$

which is negative by our assumption that downstream marginal costs are incresing in the relevant range.

E.O.P.

The value of the output derivative of integrated profits at the optimal internal and external prices (and original downstream output) is computed from said derivative's value at the intermediate post-integration price using the line integral:

(4.14)
$$\left.\frac{\partial \pi^{Int}}{\partial \hat{z}}\right|_{\substack{p^I=\bar{c}\\ \hat{p}=\hat{p}^m\\ \hat{z}=z^*(p^m)}} = \left.\frac{\partial \pi^{Int}}{\partial \hat{z}}\right|_{\substack{p^I=\hat{p}=p^m\\ \hat{z}=z^*(p^m)}} + \int_{p^m}^{\bar{c}} \frac{\partial^2 \pi^{Int}(p^m,p^I,z^*(p^m))}{\partial p^I \partial \hat{z}}\, dp^I$$
$$+ \int_{p^m}^{\hat{p}^m} \frac{\partial^2 \pi^{Int}(\hat{p},\bar{c},z^*(p^m))}{\partial \hat{p} \partial \hat{z}}\, d\hat{p} \,,$$

where p^m is, as before, the pre-integration price for the intermediate good, and $\hat{p}^m$ is the optimal post-integration external price. The first term in (4.14) is evaluated in Eq. (4.5). Note that the limits of integration in the second term of (4.14) are inverted.

In Eq. (4.9), only the last term is a function of p^I and $\hat{z}$. Thus, using Shephard's Lemma,

$$(4.15) \qquad \frac{\partial^2 \pi^{Int}}{\partial \hat{z} \partial p^I} = (p^I - \tilde{c}) \frac{\partial}{\partial p^I}\left(\frac{\partial \hat{v}}{\partial \hat{z}}\right)$$

Only the first term in Eq.(4.9) is a function of both $\hat{p}$ and $\hat{z}$. Then

$$(4.16) \qquad \left.\frac{\partial^2 \pi^{Int}}{\partial \hat{z} \partial \hat{p}}\right|_{p^I=\tilde{c}} = \theta'(\hat{p}) \geq 0$$

The following theorem now establishes sufficient conditions for the downstream subsidiary's output to rise.

Theorem 4.6. If the immediate post-integration incentive is to raise the downstream subsidiary's output, and if

$$(4.17) \qquad \frac{\partial}{\partial p^I}\left(\frac{\partial \hat{v}}{\partial \hat{z}}\right) \leq 0$$

then the downstream subsidiary's output does indeed rise above its immediate post-integration level.

Proof: By hypothesis, the first term in Eq. (4.14) is here positive. Eq. (4.17) implies (4.15) is non-positive for all internal prices between p^m and $\tilde{c}$. Thus, the second term in

(4.14) is non-negative. (Recall the reversed integration limits.) Eq. (4.16) implies the third term of (4.14) is always non-negative. Thus, once the prices have been adjusted, the local incentive here is to increase downstream output. Lemma (4.5) guarantees that this is the direction of the global change.

E.O.P.

What is the economic content of Eq. (4.17)? The cross-partial of the input demand represents the effect of raising the input price on the <u>slope</u> of the input demand graphed as a function of output. When the input price rises, the demand for the input must fall at all output levels. Thus, we know that the output slope cannot get steeper at <u>all</u> output levels.[11] It seems reasonable (though not necessary) that the expansion path will in fact become flatter at all output levels. Thus, Eq. (4.17) should be characteristic of most inputs.

Condition (4.17) encourages the expansion of downstream output. The reason is that (4.17) implies the downstream subsidiary uses the intermediate good more intensively at the margin as the internal price falls. Since both substitution and scale distortions result from insufficiently intensive use of the intermediate good (see Sec. IV.A), condition (4.17) means expanding downstream output is more effective in reducing these cost distortions--and thus more likely to be profitable--once p^I has been adjusted to $\bar{c}$.

The conditions of Theorem (4.6) are satisfied by two special cases:

<u>Corollary</u> (4.7). If the intermediate good is a fixed schedule, "luxury" input downstream, then the downstream subsidiary's post-integration output rises, while that of each other downstream firm falls.

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[11] I assume here that y(z, p) is continuously differentiable in z and that y(0, p) = 0.

Proof: Reversing the order of the partial derivatives, we see that (4.17) is satisfied with equality for any fixed schedule input. By definition, a luxury input's output elasticity exceeds unity, which implies $dz^*/d\hat{p} < 0$. By Eq. (4.5), the immediate post-integration incentive is to raise downstream output. Then, by Theorem (4.6), this is also the direction of global change. Finally, note that $dz^*/d\hat{p} < 0$ implies that, when the external price rises, the size of the independent downstream firms falls from the pre-integration scale.
E.O.P.

Corollary (4.8). If the downstream production function is homothetic and integration is profitable because of cost distortions, then the downstream subsidiary's output rises, while that of each remaining downstream firm stays fixed. The downstream industry thus becomes more concentrated.

Proof: As is well-known[12], each input demand for a homothetic production function can be written as the product of an increasing function of the output level and a function of the input prices. In this case, let

$$y(z, p^I, w) = g(z)h(p^I, w) \tag{4.18}$$

Then,

$$\partial y/\partial z = g' \cdot h = (g'(z)/g(z)) \cdot y(z, p^I, w), \tag{4.19}$$

so that

$$\frac{\partial^2 y}{\partial p^I \partial z} = \frac{g'(z)}{g(z)} \cdot \frac{\partial y}{\partial p^I} \leq 0, \tag{4.20}$$

[12] For instance, apply Shephard's Lemma to the results of Exercise (1.11) in Varian {14, p. 49}.

since $g(z)$ is increasing. Therefore, (4.17) is satisfied.

Further, as has been shown elsewhere, {8} and {13}, $dz^*/dp = 0$ for a homothetic production function. Thus, the initial incentive is not to change downstream output. Since cost distortions exist by presumption, they must be substitution distortions. Consequently, (4.20) must hold strictly at least at the initial post-integration point. Thus, the internal price adjustment creates a positive incentive to expand. For the remaining downstream firms, however, $dz^*/d\hat{p} = 0$ indicates no scale responses to the rise in external price. Since the subsidiary expands, everyone else who stays in the market stays the same size, and total downstream demand falls, the downstream market must get more concentrated.

E.O.P.

A last theorem:

Theorem (4.9). If the intermediate good is a fixed or inferior input (with or without being a fixed schedule input) then the downstream subsidiary's output is strictly less than that of its post-integration rivals.

Proof: Suppose first that the intermediate good is a fixed input. Set the internal price to marginal cost and the external price to its optimal value. Because the intermediate good is a fixed input, these changes in internal and external prices leave the marginal cost curves of botn the downstream subsidiary and its rivals unchanged. The independent rival chooses its output where its marginal and average cost curves intersect. Since the downstream subsidiary has the same marginal cost curve, then--at its rival's output--the first and third terms in Eq. (4.3) are zero. The subsidiary's incentive is thus to set output below its rivals' because of the "crowding out" term. Invoking Lemma (4.5) gives the required result.

Now if the intermediate good is an inferior input (negative output elasticity), the effects of the intermediate price changes are to lower the independents' marginal cost curves

and raise the subsidiary's. (Recall $\partial(\partial C^D/\partial z)/\partial p = \partial y/\partial z$ by Shephard's Lemma.) Thus, at the rivals' output level, both of the first two terms of Eq. (4.3) are negative (and the last zero). Thus, the optimal subsidiary output is lower by Lemma (4.5).

E.O.P

An application: A hamburger chain franchiser buys up one of the chain stores (to whom he sells only the franchise rights). My result would suggest that he would raise the franchise fee (a fixed input) for the non-owned stores. The fact that his own store winds up being smaller than the non-owned stores should not mislead us about the profit incentive behind the integration.

As a final comment, let me note that what makes integration profitable is not simply price discrimination. It has been suggested by various authors that one motive for vertical integration is to facilitate price discrimination: the upstream firm buys into those customer industries with more elastic input demand (to whom he sells at marginal cost) and raises prices to the remaining less elastic demanders. While it is true that the upstream subsidiary here does discriminate between his subsidiary and the downstream independents, price discrimination alone is not sufficient to maximize profit. The downstream subsidiary, if left to its own devices, will expand output until its marginal cost rises to the market price. This result is not optimal, since it fails to consider the "crowding out" effect of downstream output on external intermediate sales. (That is, the downstream subsidiary by itself would set the first term of Eq. (4.3) to zero and ignore the second.) To maximize profit, the integrated firm must dictate output to its downstream subsidiary, leaving it to operate "at arm's length" only with regard to input choices. Price discrimination thus is not the sole motive for integration.

Nor is the opportunity for price discrimination necessary for integration to be profitable. Scale distortions make integration profitable, even if the only change that can be made is in downstream output. (Eq. (4.5)) Of course, scale distortions also create the incentive to raise external price, but, if the intermediate good is a fixed schedule input, the internal price could be set nondiscriminatorily (i.e., also high) without distorting the internal demand for the intermediate good.

V. INTEGRATION WHEN THE UPSTREAM MARKET IS CONTESTABLE

This section considers the case where the upstream market is contestable. I assume that there are economies of scale (declining average costs) to the extent of the intermediate market.[1] Neglecting the possibility of integration, we know entry will occur upstream unless that market is monopolized by a firm pricing at average cost. I examine the incentives for such a monopoly to integrate and the likely results. Cost distortions again provide opportunities for profitable integration. In this case, however, the pressure of potential entry prohibits the integrated firm from raising the external price and, indeed, forces the external price lower unless an upstream entrant is unable to integrate forward. Thus, the final product price does not rise, and usually falls.

Before presenting these results formally, let me specify the conditions for entry and equilibrium in this pair of vertically related markets. As before, I will presume that the upstream firm fully recognizes the effect of its choice of intermediate price on the downstream market, including inducing downstream entry or exit. Moreover, I make the same presumption about an upstream entrant. An upstream entrant may win over downstream customers (or entice entry by new ones) simply by offering a slightly lower price.[2] An integrated entrant must correspondingly offer lower prices on both intermediate and

[1] I temporarily drop the assumption of constant upstream marginal costs. It will be reimposed later to do downstream output comparisons.

[2] An upstream entrant's ability to entice additional downstream entry prevents an integrated firm from using its control of its own downstream subsidiary to foreclose part of the intermediate market from such an upstream entrant. This seems a reasonable approach, since in a contestable market, the integrated firm has no way of credibly committing itself to that downstream output.

final goods. Equilibrium exists when there is no opportunity for profitable entry.

Correcting substitution or scale distortions again provides a profit motive for forward integration. To establish this proposition, we perform the conceptual experiment of Sec. IV.A: the upstream firm sets up a downstream subsidiary identical (initially) to the other downstream firms. The local incentives for adjusting the internal price and the downstream subsidiary's output are again given by Eqs. (4.2) and (4.5), respectively.[3] Clearly, cost distortions produce the same incentives for changing these variables as before:

<u>Theorem</u> <u>5</u>.<u>1</u>. Theorems 4.1 and 4.2 hold for the case of a contestable upstream market.

For just a moment, suppose that the upstream <u>incumbent</u> is the only firm able to integrate. To deter upstream entry after integration, he need only leave his external price at the pre-integration level: the (lowest) value at which the intermediate good demand of an independent downstream industry intersects the upstream average cost curve.[4] Pricing below this level, an upstream entrant would be unable to generate enough demand to cover costs, even by inducing downstream entry. Adjustments by the integrated firm in the internal price and downstream output again generate industry cost savings which increase

[3] Note that the first order condition for maximizing (protected) upstream profit was used in Sec.IV.A only in evaluating the <u>external</u> price derivative, Eq. (4.8).

[4] I presume, of course, that the intersection is from above.

the firm's profit. These profits are rents which accrue to the firm's unique ability to integrate and are not dissipated by the threat of entry, even though upstream and downstream markets are each individually contestable.

For the rest of this paper, I will assume that any upstream entrant may also integrate; that is, that the pair of markets are jointly contestable by integrated entrants.

The next theorem draws on this stronger free entry concept:

Theorem 5.2. In equilibrium, an integrated firm must maximize profit with respect to internal price and downstream output. Specifically, it must price internal sales at marginal cost unless the intermediate good is a fixed schedule input.

Proof: Suppose an integrated firm did not maximize profit with respect to these two variables. Then an integrated entrant could come in, choose the optimal internal price and downstream output, use part of the higher profit to cut the external price, and thus take over the intermediate market. In equilibrium, no such opportunity can exist. Thus, a stable incumbent must be choosing the optimal values. The internal price requirement is then a consequence of Eq. (4.2).

E.O.P.

Before any change in the external (or final) price, the profits of the newly integrated firm result purely from saving industry cost. This is one of the benefits of the pressure of entry. Indeed, one can state the case a bit more strongly:

Theorem 5.3. **There is no non-integrated equilibrium unless the intermediate good is a L.F.P.I.**

Proof: From Theorem 5.1, we know that unless the intermediate input is a L.F.P.I., there are profitable opportunities to integrate forward. If an upstream incumbent refuses to integrate, an integrated entrant can come in, earn positive profit, and take away the upstream firm's customers by offering a lower extenal price. Thus, an upstream firm must integrate to stay in business.

E.O.P.

Not only does the threat of entry promote these cost saving adjustments, but it forces the integrated firm to pass on the benefits to its downstream (and, ultimately, final) customers: [5]

Theorem 5.4. Unless the intermediate good is a L.F.P.I., the post-integration external and final prices fall. The intermediate price, however, does not fall to marginal cost.

Proof: If the intermediate good is not a L.F.P.I, the integrated firm increases profit by adjusting the internal price and downstream output without changing the external (and thus final) prices. But, if it tries to keep any of this profit, it will attract entry by another integrated firm who will dissipate some of the profit in a lower external price. However, it cannot lower the external price to marginal cost. If it tried to do so, the upstream subsidiary would run a loss (due to scale economies), while the downstream subsidiary would just break even, since it would then be on equal footing with its rivals.

E.O.P.

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[5] This theorem assumes (as throughout the paper) that the final goods transactions all take place at a single price, namely AC*, the only price available to the downstream independents without inducing entry. It is, I suppose, technically possible for an integrated firm to disperse some of its profits by selling its own downstream output below AC*. What the equilibrum would look like then depends on how that market reacts to trading at two prices. If it is subject to arbitrage, then the trading price to consumers would be raised to AC*. The opportunities for entry would then be the same as before, and Theorem 5.4 would hold.

The last part of Theorem 5.4 foreshadows a result to be discussed in more detail in Sec. VII: even with integration, some cost distortions may presist.

Finally, we can again make some comparisions among the output of the downstream subsidiary and those of its pre- and post-integration rivals. To do so, I reimpose the assumption that upstream marginal costs are constant in the relevant range.[6] Then, the requirements of Theorem 5.2 that $p^I = \bar{c}$ and that $\hat{z}$ be profit optimal make the analytic process of Sec. IV.B.2 applicable. In particular, after the internal and external prices have been adjusted, integrated profit is concave in downstream output (Lemma 4.5). Then the direction of global change in $\hat{z}$ is given by $\partial\pi^{Int}/\partial\hat{z}$. Equation (4.14), *mutatis mutandis*, [7] gives the adjusted value of $\partial\pi^{Int}/\partial\hat{z}$. Of course, in the contestable upstream case, the integration limits are reversed in both the second and third terms of Eq. (4.14). Equations (4.15) and (4.16) again hold.

With these observations in mind, we hardly need to prove formally the following analog to Cor. 4.7:

> Theorem 5.5. If upstream marginal costs are constant, if integration is partial, and if the intermediate good is a fixed schedule normal, fixed, or inferior input downstream, then the output of the downstream subsidiary will fall after integration while that of each of its rivals will increase.

No work at all is needed to establish Theorem 4.9 for this case: its proof is independent of the direction of change of the external price. Finally, Corollary 4.8 holds again also, but the proof is sufficiently different that I state it as a separate theorem:

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[6] Positive fixed costs are also assumed, in order to maintain upstream scale economies.

[7] That is, where p^m represents the pre-integration intermediate good price (= average cost upstream) and $\hat{p}^m$ the post-integration equilibrium external price.

Theorem 5.6. If the downstream production function is homothetic and integration is profitable because of cost distortions, then the downstream subsidiary's output rises, while that of each independent downstream firm stays fixed. The downstream industry becomes more concentrated.

Proof: That $dz^*/d\hat{p} = 0$ for a homothetic production function means that the first term of Eq. (4.14) is zero (using Eq. (4.5)). It also implies (using Eqs. (4.12) and (4.16)) that the integrand for the third term reduces to:

$$(5.1)\quad \partial^2\pi^{Int}(\hat{p},\bar{c},z^*(p^m))/\partial\hat{p}d\hat{z} = -(\hat{p}-\bar{c})(\partial y^*/\partial\hat{p})/z^*(\hat{p})$$

Equation (4.20) for a homothetic production function and Eq. (4.15) together imply that the integrand in the second term of (4.14) is

$$(5.2)\quad \partial^2\pi^{Int}(p^m,\ p^I,z^*(p^m))/\partial p^I\partial\hat{z} = (p^I-\bar{c})\cdot(\partial\hat{y}/\partial p^I)\cdot(g'(z^*(p^m)))/g(z^*(p^m))$$

I now show that these two integrands are of equal magnitude but opposite sign when $\hat{p} = p^I$. First note that all of the input demands for a homothetic production function are of the form of Eq. (4.18), for the same $g(\cdot)$ function. Multiplying them by their respective input prices and summing, I get

$$(5.3)\quad C^D(\hat{z},\ p,\ w) = g(\hat{z})\cdot H(p,\ w),\quad \text{for some function } H.$$

At $\hat{z} = z^*$, average and marginal cost are equal. Thus,

$$(5.4)\quad g'(z^*)/g(z^*) = 1/z^*$$

Since $dz^*/d\hat{p} = 0$ implies $\hat{z} = z^*(p^m) = z^*(\hat{p})$ for all $\hat{p}$, $\partial\hat{y}/\partial p^I$ and $\partial y^*/\partial\hat{p}$ are thus evaluated at the same output levels. Equation (5.4) then implies Eqs. (5.1) and (5.2) are equal and opposite in sign if $p^I = \hat{p}$.

The above discussion establishes that the second and third integrands of Eq. (4.14) cancel each other out on the range $[\hat{p}^m, p^m]$. Equation (4.14) then reduces to:

$$(5.5) \qquad \left.\frac{\partial\pi^{Int}}{\partial\hat{z}}\right|_{\substack{\hat{p}=\hat{p}^m \\ p^I=\bar{c} \\ \hat{z} = z^*(p^m)}} = \left(\frac{g'(z^*)}{g(z^*)}\right) \cdot \int_{\hat{p}^m}^{\bar{c}} (p^I - \bar{c}) \cdot \frac{\partial\hat{y}}{\partial p^I}\, dp^I$$

From Thm. 5.4, $\hat{p}^m > \bar{c}$. Thus, Eq.(5.5) is positive. Lemma 4.5 then indicates that $\hat{z}$ rises globally. That the independent downstream firms stay the same size is a result of $dz^*/d\hat{p} = 0$.

E.O.P.

VI. MULTIPLE DOWNSTREAM SUBSIDIARIES

In this section, I allow the upstream monopolist to acquire (or set up) multiple downstream subsidiaries. As explained in Sec. II.D., it has been assumed to this point that the decreasing returns eventually incurred downstream are the result of managerial or other diseconomies not avoidable by replicating plants. In this section, I suppose instead that upstream monopoly can establish multiple downstream plants, each of which has U-shaped average costs. I further presume that the monopolist does not take over the entire downstream market, either for fear of antitrust action or because of costs of coordinating multiple plants not related to output level.

For either type of upstream market, correcting cost distortions motivates each additional acquisition as it did the first. For the protected upstream case, external and final goods prices rise with each acquisition, whereas for the contestable upstream case, they fall. The plant output of each downstream subsidiary moves in the same direction as the external price as the number of plants grows.

The motivations for further downstream acquisitions are made plain by another conceptual experiment. Suppose the integrated firm already owns K downstream subsidiaries. Let it establish a (K+1)st subsidiary by setting up a downstream plant facing the same intermediate and final price and operating at the same output level as the independent downstream firms. So far, this acquisition has added exactly zero to the integrated firm's profits.[1] Now, examining the incentives for changing

[1] It may actually have lowered profits by the additional coordination cost. If so, one must assume the acquisition is beneficial. The results of this paragraph indicate that the source of the gain, if there is one, is correcting cost distortions.

the internal price to and the output level of the new plant, I can derive equations isomorphic to Eqs. (4.2) and (4.5), respectively. These incentives are non-zero--and thus integration is profitable--if there are substitution or scale distortions (respectively). Thus, correcting cost distortions provides the motivation for further downstream penetration, regardless of whether upstream entry is possible.

To derive global changes in prices and output, we need some special notation. Let $\hat{p}_K$ be the optimal (protected) or equilibrium (contestable) external price after the Kth acquisition. Let $\hat{z}$ now be the output of each of the K downstream plants and $\hat{z}_K$ the optimal or equilibrium value. (Equality of output of the subsidiaries is guaranteed by upward sloping downstream marginal costs.) Profit of the integrated firm with K subsidiaries is given by π_K^{Int}. Finally, let the intermediate good demand by the <u>independent</u> downstream firms (when the integrated firm has K downstream subsidiaries) be:

$$(6.1) \qquad \hat{Y}_K \equiv ((Q-K\hat{z})/z^*).y^* \equiv \hat{N}_K \cdot y^* ,$$

where $\hat{N}_K$ is the number of independent downstream firms.

For the protected upstream case, I now prove the following theorem

Theorem 6.1. Suppose that upstream entry is impossible and that upstream marginal costs are constant in the relevant range. Then, provided the intermediate good is not a L.F.P.I., the external and final prices and downstream plant output all rise with each acquisition. That is:

$$\hat{p}_K < \hat{p}_{K+1} \text{ and } \hat{z}_K < \hat{z}_{K+1} \text{ for } K = 1,2,\ldots$$

Proof: Suppose the integrated firm owns K downstream plants and is currently choosing the output of each and the external and internal prices optimally. Let it now establish a (K+1)st plant. As always, it is optimal to sell the intermediate good to the new plant at marginal cost. Let the new plant operate initially at $\hat{z}_K$, the output level of the other K plants. Then, using the assumption of constant upstream marginal cost, $\bar{c}$, the profit of the integrated firm is the sum of its downstream and upstream profits:

$$(6.2) \quad \pi^{Int}_{K+1} \left(\hat{p},\bar{c},\hat{z}\right) = (K+1)\cdot(AC^*\cdot\hat{z}-C^D(\hat{z},\bar{c})) + (\hat{p}-\bar{c})\cdot\hat{Y}_{K+1} - F^U,$$

where $(\hat{p}, \hat{z})$ are evaluated at $(\hat{p}_K, \hat{z}_K)$ initially and F^U is again upstream fixed cost.

With internal price set to marginal cost, a necessary condition for profit maximization is, using (6.1) and (6.2),

$$(6.3) \quad \frac{\partial\pi^{Int}_{K+1}\left(\hat{p}_{K+1}, \bar{c}, \hat{z}_{K+1}\right)}{\partial\hat{z}} = (K+1)\left(\theta\left(\hat{p}_{K+1}\right) - \frac{\partial C^D\left(\hat{z}_{K+1},\bar{c}\right)}{\partial\hat{z}}\right) = 0,$$

where $\theta(\cdot)$ is given by Eq. (4.11). Note Eq. (6.3) says that for a $(\hat{p}, \hat{z})$ pair to be optimal for some K, it is necessary that

$$(6.4) \qquad \theta(\hat{p}) = \frac{\partial C^D(\hat{z}, \bar{c})}{\partial \hat{z}},$$

a condition independent of K. Equation (6.4) defines $\hat{z}$ implicitly as a function of $\hat{p}$: $\hat{z} = \hat{z}(\hat{p})$. Observe that

$$(6.5) \qquad \frac{d\hat{z}}{d\hat{p}} = \frac{\theta'(\hat{p})}{\partial^2 C^D / \partial \hat{z}^2} > 0,$$

with the inequality being strict since there are cost distortions (by assumption). (See Eq. (4.12).)

The problem of maximizing $\pi_{K+1}^{Int}(\hat{p},\bar{c},\hat{z})$ (or π_K^{Int} for that matter) over $(\hat{p}, \hat{z})$ can thus be "condensed"[2] to maximizing $\pi_{K+1}^{Int}(\hat{p}, \bar{c}, \hat{z}(\hat{p}))$ over the choice of $\hat{p}$.

Using

$$(6.6) \qquad \hat{Y}_{K+1} = \hat{Y}_K - (y^*/z^*)\cdot\hat{z},$$

I rewrite Eq. (6.2) to get

$$(6.7) \qquad \pi_{K+1}^{Int}(\hat{p},\bar{c},\hat{z}(\hat{p})) = \pi_K^{Int}(\hat{p},\bar{c},\hat{z}(\hat{p})) + [(\hat{z}(\hat{p})\cdot\theta(\hat{p}) - C^D(\hat{z}(\hat{p}),\bar{c})]$$

The derivative by $\hat{p}$ of the term in square brackets is:

[2] As in the sense of a condensed likelihood function.

$$(6.8) \qquad \left(\theta(\hat{p}) - \frac{\partial C^D(\hat{z}(\hat{p}),\bar{c})}{\partial z}\right) \cdot \frac{d\hat{z}}{d\hat{p}} + \hat{z}(\hat{p}) \cdot \Theta'(p) > 0$$

The bracketed term in Eq. (6.8) is zero by definition of $\hat{z}(\hat{p})$, and the inequality is strict because of cost distortions. (See Eq.(4.12).) Thus $\pi^{Int}_{K+1}(\hat{p},\bar{c},\hat{z}(\hat{p}))$ in Eq. (6.7) is the sum of a first term which reaches a global maximum at $\hat{p}_K$ and a second which is strictly increasing in $\hat{p}$. Clearly, $\hat{p}_{K+1} > \hat{p}_K$.

Since $\hat{z}_{K+1} = \hat{z}(\hat{p}_{K+1})$, Eq. (6.5) then indicates $\hat{z}_{K+1} > \hat{z}_K$. Final price must also rise, of course.

E.O.P.

For the case of a contestable [3] upstream market, the pricing results are reversed. Note that these results do not depend on constant upstream marginal costs.

Theorem 6.2. If the upstream market is contestable, and if further integration seems initially profitable, then external--and hence, final--prices fall with each acquisition.

Proof: To prevent entry by a similarly integrated firm, the incumbent must dissipate any temporary acquisition profits by offering a lower external price. The final good price falls correspondingly.

An output comparison requires constant upstream marginal costs.

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[3] Contestable here means "jointly contestable" in the sense of Sec. V: integrated entry is possible.

<u>Corollary 6.3</u>: If the upstream market is contestable and upstream marginal costs locally constant, then the output of each downstream plant of the integrated firm falls with each acquisition.

<u>Proof</u>: Per Theorem 5.2, the integrated firm must maximize profit over the choice of $\hat{z}$. Equation (6.3) again provides the necessary condition. Thus, $\hat{z}_{K+1} = z(\hat{p}_{K+1})$. Equation (6.5) and Theorem 6.2 then imply $\hat{z}_{K+1} < \hat{z}_K$

E.O.P.

VII. VERTICAL INTEGRATION: WELFARE AND POLICY ANALYSIS

This section, building on the analysis of Sec. III, examines the welfare effects of vertical integration. For partial integration, the absolute results are again largely negative. The protected upstream case still does not maximize welfare, minimize costs, nor achieve any type of Ramsey optimum. The contestable case is only a little better, being consistent only with the weakest Ramsey optimum.

The before-and-after comparisons are, however, brighter. When the upstream market is contestable, vertical integration always increases welfare. When the upstream market is protected, the gains to rationalized production within the integrated firm may be offset by potential rises in the external price. If that price does rise, as in the special partial integration case of Sec. IV., then independent downstream production becomes less efficient and consumer prices rise. The net welfare effect of integration depends in the protected case on the parameters involved. However, if integration proceeds on condition that all demand for the intermediate good still be served at the pre-integration price, then any integration that take place increases welfare. Importantly, all these comparative results hold for full as well as partial integration and for general technologies as well as for the concatenated technology used above. (I retain the concatenated technology until explicitly dropped in the last subsection.)

VII.A Welfare Maximization

The failure of welfare maximization in either upstream case results from the failure to price external sales of the intermediate good at marginal cost.

Theorem 7.1: Under either entry scenario, social welfare is not maximized after integration except if both conditions of Theorem 3.1 hold.

Proof: As in Sec. III, welfare (W) is the sum of consumer surplus plus profit, with the profit of the independent downstream firms being zero. Using Eq. (3.2), I get

$$(7.1) \qquad W = \int_{AC^*(\hat{p})}^{\infty} Q(s)ds + \pi^{Int}(\hat{p}, p^I, \hat{z})$$

To establish the theorem, I calculate, using Eqs. (3.5a),(4.6), and (4.7):

$$(7.2) \qquad \frac{\partial W}{\partial \hat{p}} = - Q(AC^*)(y^*/z^*) + \partial\pi^{Int}/\partial\hat{p}$$

$$= (\hat{p} - \frac{\partial C^U}{\partial y}) \frac{\partial \hat{Y}}{\partial \hat{p}},$$

where

$$(7.3) \qquad \frac{\partial \hat{Y}}{\partial \hat{p}} = (\frac{y^*}{z^*}) \cdot Q' \cdot \frac{dAC^*}{d\hat{p}} + (Q - \hat{z}) \frac{d(y^*/z^*)}{d\hat{p}}$$

$$= Q' \cdot (\frac{y^*}{z^*})^2 + \hat{N}\{\frac{\partial y^*}{\partial \hat{p}} - (\frac{\partial^2 C^D}{\partial z^2}) (\frac{dz^*}{d\hat{p}})^2\} \leq 0$$

Equality holds in Eq. (7.3) only if final demand is completely inelastic and the intermediate good is a L.F.P.I., the conditions of theorem (3.1). Since external price exceeds marginal cost in either upstream case, welfare will not be maximized in either case except when said conditions hold.

E.O.P.

VII.B Cost Minimization

Industry total input costs are still not minimized after (partial) integration.[1] To analyze the raw input costs, I modify the "induced cost function" from Section III to include the explicit choice of the output of the downstream subsidiary:

$$(7.4) \quad IC(\bar{Q},\hat{p},p^I,\hat{z}) = \hat{\bar{N}}\cdot C^D(z^*,\hat{p}) + C^D(\hat{z},p^I) + C^U(\hat{\bar{N}}y^*+\hat{y}) - \hat{p}\hat{\bar{N}}y^* - p^I\hat{y}$$

where $$N \equiv \frac{\bar{Q} - \hat{z}}{z^*}$$

is the number of independent firms. Then, the formal theorem is:

[1] I here analyze the case of a single downstream acquisition. The results are the same for any number of acquisitions, as long as integration is partial.

<u>Theorem</u> 7.2. Partial integration fails to minimize total raw input costs for either type of upstream market if there are cost distortions.

<u>Proof</u>: To minimize raw input costs requires $\partial IC/\partial\hat{p}$ be zero. To compute this derivative, use the identity $\hat{N}\cdot C^D(z^*,\hat{p}) = (\bar{Q}-\hat{z})(y^*/z^*)$ and Eq. (3.5a):

$$(7.5)\quad \frac{\partial IC}{\partial\hat{p}} = (\bar{Q}-\hat{z})(y^*/z^*) - \hat{N}y^* - (\hat{p}-\partial C^U/\partial y)\cdot(\bar{Q}-\hat{z})\cdot(d(y^*/z^*)/d\hat{p})$$

$$= -(\hat{p} - \partial C^U/\partial y)\,\hat{N}\left[\frac{\partial y^*}{\partial\hat{p}} - \left(\frac{dz^*}{d\hat{p}}\right)^2 \cdot \frac{\partial^2 C^D(z^*)}{\partial z^2}\right]$$

As argued above, the post-integration external price exceeds marginal cost when the upstream market is contestable (Thm. 5.4). This, of course, is also true when there is no upstream entry. (See Eqs. (7.3) and (4.6).) Thus, with cost distortions, Eq. (7.5) is positive after integration, and induced costs are not minimized.

E.O.P.

Thus, the B-P-W result of horizontal industry cost minimization does not carry over to vertically related contestable markets, even when integration is allowed. Sustainable equilibria exist with persistent cost distortions. The invisible hand is insufficiently strong in any system limited to a simple price system.[2]

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[2] In a companion paper {9}, I show that a royalty payment scheme <u>will</u> achieve both efficiency and cost coverage.

VII.C Ramsey Optima

As discussed in Sec. III, three types of profit-constrained or Ramsey optima can be defined. The differences among them depend on the degree to which market forces are accommodated. The results here are very similar to those of Sec. III and will be presented only briefly.

The strongest Ramsey optimum is one in which all firms' inputs and outputs are chosen optimally, subject only to total revenue covering total costs. Clearly, the optimal choice of inputs at each stage is that which minimizes raw input costs. Per Sec. VII.B, this requirement is not met by any market outcome in which there are cost distortions.

The second type of Ramsey optimum defined in Sec. III allows each firm to determine its own input demands with only outputs and prices dictated optimally. The first order conditions for this optimum yield results identical to Eq. (3.18): independent downstream firms do not optimally produce at the bottom of their perceived average cost curves except in the absence of scale distortions. Thus, if there are scale distortions, the market outcomes are inconsistent with the Ramsey requirements, and, conversely, this Ramsey optimum cannot be sustained in the market without entry restrictions.

The weakest Ramsey optimum is one in which inputs and outputs of independent downstream firms and the final good price are determined by market forces. Only the external and internal prices and the downstream output of the integrated firm are chosen optimally. Since only the choice of the external price affects either the final price or the efficiency of the independent downstream firms, the internal price and integrated firm final output can be chosen simply to maximize integrated profits. External price, if above upstream marginal cost, is then set as low as possible consistent with non-negative integrated profits. In short, the market outcome when the upstream market is contestable is consistent with this weakest Ramsey optimum, while the protected upstream case is not.

VII.D The Welfare Effects of Vertical Integration

As a matter of antitrust policy, the proper question is not whether a market after vertical integration performs first best or even second best but whether it performs better than before integration.

It is possible to attack this question from a more general framework than we have been using to this point. The assumption of a concatenated tecnology was primarily useful in exposing the links between cost distortions and the incentives to integrate. Here, I presume integration is attractive to the upstream firm. I therefore can drop the concatentation assumption and represent the integrated firm's costs more generally by a single function, C^{Int}, of its net intermediate and final outputs. The profits of the integrated firm are thus

$$(7.6) \quad \pi^{Int}(\hat{p},\hat{z}) = \hat{p}\cdot\hat{Y}(\hat{p},\hat{z}) + \hat{z}\cdot AC^{*}(\hat{p}) - C^{Int}(\hat{Y},\hat{z}),$$

where $\hat{Y}$ is defined as before. In addition, I presume that the integrated firm still offers to serve any potential intermediate good demand even if the integrated firm takes over the whole downstream market. The effect of this assumption is that the final good price is still AC*, since entry will occur downstream for any final price above AC* and the integrated firm has no incentive to charge less.[3] Then, Eq. (7.6) defines integrated firm profits for full as well as partial integration.[4] Since the profits of independent downstream firms (if any) are still zero, the expression for post-integration welfare becomes (using (7.6)):

$$(7.7) \qquad W = \int_{AC^*(\hat{p})}^{\infty} Q(s)ds + \pi^{Int}(\hat{p},\hat{z}).$$

If the upstream market is protected, then integration occurs if π^{Int} rises above the preintegration value of π^{U}. The welfare effect will surely be positive if $\hat{p}$ in the process falls. (Compare Eq. (3.1).) However, we have reason to believe--as in the case of the concatenated technology, constant upstream marginal costs, and partial integration--that this price will rise. The welfare effect then involves the integrated firm profiting at the expense of lost surplus of final consumers. The net effect depends on the parameters involved.

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[3] Alternatively, in the cases of full forward integration, the external price could be defined implicitly as the value which equates AC* with the final price.

[4] $\hat{z} = Q$, and thus $\hat{Y} = 0$, for full integration.

There is a way to guarantee that integration will be socially beneficial. That is to require that the external price not be increased as a result of integration and that all intermediate good demand continue to be met at that old price. Such a requirement, say as part of a consent decree, would protect consumers from any harm. Yet, as established by Theorems 4.1 and 4.2, the requirement would not eliminate the incentives for integration in the concatenated technology case. Correcting cost distortions through adjustments of the internal price and its own downstream output still would provide profit gains for the integrated firm. Further, under the restriction on external price, there would be no need to oppose the multiple downstream acquisitions of Sec. VI or indeed to oppose full integration as long as intermediate good supplies were truly available to potential downstream entrants.

If the upstream market is contestable (in the sense of free entry by integrated firms) then any integration is socially beneficial. No restrictions are needed on the external price. The pressure of entry keeps π^{Int} at zero after integration just as π^{U} was before. Thus, welfare changes in the opposite direction from the external price. But, the external price cannot rise, since this would create an entry opportunity for an upstream firm operating at pre-integration price and output. Indeed, any temporary profit gains must be dissipated via a lower external price. Thus, the incentives of private profit and social welfare coincide when the upstream market is contesable.

The guidance for antitrust policy is clear. Provided that supplies of the intermediate good remain available after integration, what matters for welfare is not how much of the downstream market is taken over by the integrated firm but rather what happens to the external price of the intermediate good. A key to this price change is the contestability of the upstream market. If the upstream market is contestable, the external price cannot rise. Then, all integration is welfare improving, and no antitrust intervention is needed. When the position of the upstream firm is protected, however, it is likely--and, in a special case, assured--that the external price will rise. The welfare effect is then not a priori determined. However, for the policy authorities to guarantee that integration is welfare (and Pareto) improving, they need only require the integrated firm not raise the external price. For its simplicity and efficacy, this rule merits serious consideration any time intervention is required.

Finally, any policy which improves the contestability of the upstream market would seem to reduce the need for intervention in vertical merger cases.

APPENDIX A

Technology and Cost Representation of Vertically Related Production Processes

The purpose of this appendix is first to specify the formal properties of the technology available to upstream, downstream, and integrated firms. For the particular technology used in this paper, I then show the cost function of an integrated firm can be represented as the sum of costs of an upstream division and a downstream division, netting out intrafirm sales.

Let us begin by defining the appropriate technology sets. Using $x \epsilon R_+^{\ell}$ to denote a vector of raw inputs, $y \epsilon R^1$ to denote the intermediate good, and $z \epsilon R_+^1$ for the final good, I define the technology available to the industry by the set

(A1) $T \equiv \{(-x,y,z)$ s.t. z can be produced from inputs $(-x,y)$ for $y \leq 0$; or (y,z) can be produced from inputs x, for $y \geq 0.\}$

An "upstream" firm is then defined to be a firm which produces no final output, while a "downstream" firm produces only final output. An "integrated" firm produces non-negative amounts of both intermediate and final outputs. Formally, an upstream firm chooses a production vector from the subset of T given by

(A2) $$T^U \equiv \{(-x,y,z) \epsilon T \text{ s.t. } y \geq 0 = z\}$$

A downstream firm chooses a vector from a different subset of T, namely

(A3) $$T^D \equiv \{(-x,y,z)\varepsilon T \text{ s.t. } y \leq 0 \leq z\}$$

Finally, an integrated firm chooses a vector from the subset

(A4) $$T^I \equiv \{(-x,y,z)\varepsilon T \text{ s.t. } y \geq 0,\ z \geq 0\}.$$

For each type of firm, let us define the corresponding cost function. For an upstream firm,

(A5) $$C^U(y,w) \equiv \min_{(-x,y,0)\varepsilon T^U} w\cdot x\ ,$$

where w is an ℓ-vector of raw input prices.[1] For a downstream firm,

(A6) $$C^D(z,w,p) \equiv \min_{(-x,-y,z)\varepsilon T^D} w\cdot x + py\ ,$$

where p is the intermediate good price, taken as fixed by a downstream firm. The costs of an integrated firm are given by

(A7) $$C^I(y,z,w) = \min_{(-x,y,z)\varepsilon T^I} w\cdot x$$

Some assumptions need to be made for these cost functions (and the attendant input demands) to have the usual "nice" properties. Specifically, it is assumed for T^U (and, mutatis mutandis, for T^D and T^I):

T1) <u>Regularity</u>. T^U is closed and nonempty. $(0,y,0)\varepsilon T^U$ iff $y = 0$.

T2) <u>Monotonicity</u>. $(-x,y,0)\varepsilon T^U$ implies $(-(x+\gamma),(y-\delta),0)\varepsilon T^U$ for all $\gamma\varepsilon R_+^\ell, \delta\varepsilon R_+^1$.

[1] It is assumed throughout that w is taken as given by all firms.

T3) Marginal Costs. $\partial C^U/\partial y$ is well-defined for all $y > 0$.

Further, we assume for T^U, and correspondingly for T^D, [2]

T4) Strictly Convex Input Requirement Sets. Define the input requirement set $A^U(y) \equiv \{x \text{ s.t. } (-x,y,0) \varepsilon T^U\}$. Then for any $x,x' \varepsilon A^U(y)$ and any $0 < \theta < 1$, $\theta x + (1 - \theta)x'$ ε interior of $A^U(y)$.

Given these assumptions, the cost function is well-defined, monotonic in w and y, and concave in w. (See McFadden { 7 , pp. 6-8, 16-18} and Rosse { 10 }.) T4) guarantees single-valued input demand functions (cost-minimizing input bundles are unique) and (McFadden { 7 , p. 20}) that the cost function is continuously differentiable in w for all positive w. [3] Shephard's Lemma then implies $\partial C^U/\partial w_i = x_i(y,w)$, where $x_i(y,w)$ is the ith input demand. Finally, each input demand is non-increasing in its own input price.

The emphasis in this paper is on the gains to integration from improved allocation of inputs within the integrated firm and of final output among the final good producers. To focus on these allocation issues requires a production technology which entails no technological bias for (or against) integration. Such bias would exist, for instance, if an upstream firm and a downstream firm each required a fixed cost item which would not need to be bought twice by an integrated firm. To rule out such bias, I will assume (except in Sec. VII):

T5) Upstream/Downstream Concatenation. [4] $T^I = (T^U \oplus T^D)_{-++}$

[2] Note that I do not make this assumption for T^I. See comment below on "backward" integration.

[3] The latter property and the existence of well-defined marginal costs hold almost everywhere for any technology satisfying T1) and T2) alone. See Royden {11}, Theorems 5.2 and 5.16.

[4] The symbol $\oplus$ indicates set sum: $A \oplus B \equiv \{(a+b)$ for some $a \varepsilon A$, $b \varepsilon B\}$. Also $A_{-++} \equiv A \cap (R^{\ell}_{-} \times R^1_{+} \times R^1_{+})$.

Assumption T5) indicates that the technological opportunities available to an integrated firm are equivalent to a concatenation of the technological opportunities of a pair of single-stage firms producing a net output of intermediate good. The balance of this appendix is devoted to showing that the raw input costs of an integrated firm using such a technology can be represented by the sum of the raw input costs of an upstream subsidiary and a downstream subsidiary, and that these subsidiaries can be appropriately coordinated simply by means of an appropriate price (i.e., marginal cost) for internal sales of the intermediate good.

The first step, Theorem 2.1, is to establish (not surprisingly) that, for a given net ouput (y,z) and a technology characterized by T5), expenditures on raw inputs are the same for an optimally coordinated upstream-downstream pair as for a single integrated firm. The raw input costs of the former are given by a "paired firm" cost function.

$$\text{(A8)} \qquad C^P(y,z,w) \equiv \min_{\substack{(-x^U,\ y+y^N,\ 0)\in T^U \\ (-x^D,\ -y^N,\ z)\in T^D}} w(x^U + x^D)$$

for all non-negative (y,z). Note that the amount of intermediate good produced by the upstream division includes a non-negative amount y^N to be used by the downstream division. The theorem then is

Theorem 2.1. Given T1), T2), and T5), $C^P(y,z,w) \equiv C^I(y,z,w)$.

Proof:

(i) $T^I \subset (T^U \oplus T^D)_{-++}$ implies $C^P(y,z,w) \leq C^I(y,z,w)$. To see this, let $(-x^I,y,z)$ be any of the vectors (not necessarily unique) which achieve C^I for a given (y,z). By hypothesis, there are vectors $(-x^U,y^U,0)\epsilon T^U$ and $(-x^D,-y^D,z)\epsilon T^D$ such that $x^I = x^D + x^U$ and $y = y^U - y^D$. The raw input cost of these vectors is $w(x^U + x^D)$. But C^P minimizes just such costs over the class of such vectors. Thus $C^P(y,z,w) \leq C^I(y,z,w)$.

(ii) That $(T^U \oplus T^D)_{-++} \subset T^I$ implies $C^I(y,z,w) \leq C^P(y,z,w)$ is established by a similar argument.

E.O.P.

The second step, a bit harder, is to show that, for the optimal coordination of an upstream-downstream pair to be achieved, one need only set the proper price on sales of the intermediate good between the pair and require that the downstream division's demand be met. The proper price--upstream marginal cost--is sufficient to guide this choice, and is indeed necessary unless the downstream input demand for the intermediate good is locally insensitive to said price. More formally,

Theorem 2.2. Suppose that an upstream firm and a downstream firm--not centrally coordinated--produce a net output $(\bar{y},\bar{z})$, each seeking only to minimize its expenditures on inputs (i.e., to achieve C^U and C^D). Let y^N be the amount of intermediate good transferred between an optimally coordinated pair producing a net output $(\bar{y},\bar{z})$, and let T1)-T4) hold. Then, for the non-coordinated pair to spend exactly $C^P(\bar{y},\bar{z},w)$ on raw inputs, it is sufficient that

(i) the upstream firm meets the downstream firm's demand for the intermediate good (in addition to producing $\bar{y}$), and

(ii) the price p for such sale is $\partial C^U(\bar{y}+\bar{y}^N)/\partial y$.

Where the downstream derived demand for the intermediate good is differentiable with respect to its own price, then, unless said derivative is zero, it is necessary that the intermediate good is priced at marginal cost.

Proof: The raw input costs of a non-coordinated pair are given by

$$\text{(A9)} \qquad C^U(\bar{y} + y^I,w) + C^D(\bar{z},w,p) - py^I ,$$

where $y^I = y^I(\bar{z},p,w)$ is the amount of intermediate good demanded by the downstream firm and p is its price. By definition, (A9) is at least as great as $C^P(\bar{y},\bar{z},w)$. The theorem establishes the necessary and sufficient conditions for (A9) to equal C^P.

Let us begin with the necessary conditions. If (A9) equals C^P, then (A9) is at a global minimum with respect to p. Provided $\partial y^I/\partial p$ exists, the necessary condition for such a minimum is:

$$\text{(A10)} \qquad \frac{\partial C^U}{\partial y} \cdot \frac{\partial y^I}{\partial p} + \frac{\partial C^D}{\partial p} - y^I - p\,\frac{\partial y^I}{\partial p} = (\partial C^U/\partial y - p)\partial y^I/\partial p = 0$$

where the simplification uses Shephard's Lemma. Clearly, either y^I is inelastic with respect to p or y^I must be priced at marginal cost.

The sufficiency of (i) and (ii) is established in two stages. Let $(-x^{PU}, \bar{y} + y^N, 0)\varepsilon T^U$ and $(-x^{PD}, -y^N, \bar{z})\varepsilon T^D$ be the pair of vectors which achieve $C^P(\bar{y},\bar{z},w)$. Then the first stage is to show that an upstream firm which produces $\bar{y} + y^N$ at minimum cost (given w) uses raw inputs x^{PU}. The second stage is to show that a downstream firm desiring to produce $\bar{z}$ at minimum cost and facing raw input prices w will choose inputs (x^{PD}, y^N) provided that it also is quoted $p = \partial C^U(\bar{y} + y^N)/\partial y$.

To show the first stage, note that by definition of C^U,

$$\text{(A11)} \qquad C^U(\bar{y} + y^N, w) \leq w \cdot x^{PU} .$$

If the inequality were strict, then $(-x^{PU}, \bar{y} + y^N, 0)$ could not be part of the solution of C^P, since the cheaper bundle of raw inputs which solves C^U could be substituted for x^{PU} without reducing the amount of intermediate good available to the downstream process. Thus, (A11) holds with equality. Since, by T4), cost minimizing bundles are unique, an upstream firm seeking only to minimize its <u>own</u> costs in producing $\bar{y} + y^N$ (i.e., seeking only to achieve C^U) will choose raw inputs x^{PU}.

To establish the second stage, begin by defining the input requirement set for the downstream technology as

$$\text{(A12)} \qquad A^D(\bar{z}) \equiv \{x,y) \text{ s.t. } (-x,-y,z)\varepsilon T^D\}.$$

If, at (x^{PD},y^N), the ℓ-dimensional iso-expenditure hyperplane

$$\text{(A13)} \qquad H \equiv \{(x,y) \text{ s.t. } w\cdot x + p\cdot y = wx^{PD} + py^N \equiv k\}$$

supports $A^D(\bar{z})$ (where $p = \partial C^U(\bar{y} + y^N)/\partial y$), then (x^{PD},y^N) will be demanded by a downstream firm seeking only to minimize its own expenditures on x and y when faced with input prices (w,p).

Suppose H does not support $A^D(\bar{z})$ at (x^{PD},y^N). There is thus a point $(x'',y'')\varepsilon A^D(\bar{z})$ such that

(A14) $$wx'' + py'' \equiv k'' < wx^{PD} + py^N \equiv k \ .$$

Since $A^D(\bar{z})$ is convex by assumption T4), the line segment connecting (x^{PD},y^N) and (x'',y'') lies entirely in $A^D(\bar{z})$. Let us parameterize that line segment as

(A15) $$L \equiv \{x(\theta),y(\theta)) = \theta\cdot(x'',y'') + (1-\theta)\cdot(x^{PD},y^N), \text{ for } 0\leq\theta\leq 1\}$$

On L,

(A16) $$[w\cdot x(\theta) + p\cdot y(\theta)] = [\theta k'' + (1-\theta)k] = [k-\theta(k-k'')] = [wx^{PD} + py^N - \theta(k-k'')].$$

Thus, since $k > k''$ by (A14), for $0<\theta<1$,

(A17) $$w\cdot(x(\theta) - x^{PD}) + p(y(\theta)-y^N) = - \theta(k-k'') < 0$$

The implication of (A17) is that the downstream expenditure on all inputs at prices (w,p) is less along the interior of L than at (x^{PD},y^N). The difference in actual raw input costs is

(A18) $$w(x(\theta) - x^{PD}) + [C^U(y(\theta) + \bar{y}) - C^U(y^N + \bar{y})] \ .$$

If (A18) were negative for some θ, then that would countradict the optimality of x^{PD} and y^N, since $(x(\theta),y(\theta))$ would represent lower raw input costs and still be able to produce $\bar{z}$ (recall $L\subset A^D(\bar{z})$). This contradiction would occur if $(C^U(y(\theta) + \bar{y}) - C^U(y^N + \bar{y}))$ gets close to $p(y(\theta) - y^N)$ faster than $\theta(k - k'')$ goes to zero. But, this is exactly what happens when p is equal to upstream marginal cost.

Specifically, by definition of $\partial C^U(\bar{y}+y^N)/\partial y$, for any $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

(A19) $$\left| \frac{C^U(y(\theta) + \bar{y}) - C^U(y^N + \bar{y})}{y(\theta) - y^N} - \frac{\partial C^U(y^N + \bar{y})}{\partial y} \right| < \varepsilon$$

if $|y(\theta) - y^N| = \theta|y'' - y^N|<\delta(\varepsilon)$. That is, since $p = \partial C^U/\partial y$,

(A20) $|\{C^U(y(\theta) + \bar{y}) - C^U(y^N + \bar{y})\} - p(y(\theta) - y^N)|< \varepsilon \cdot \theta|y'' - y^N|$

if $\theta<\delta(\varepsilon)/|y'' - y^N|$. Now choose $\varepsilon = (k-k'')/|y'' - y^N|$. Then, for θ small enough

(A21) $[C^U(y(\theta)+\bar{y}) - C^U(y^N+\bar{y})] - \theta(k-k'') < p(y(\theta)-y^N)$

Then (A17) implies, for θ small enough,

(A22) $w(x(\theta)-x^{PD}) + [C^U(y(\theta) + \bar{y}) - C^U(y^N+\bar{y})] < 0$,

the desired contradiction. Thus, H must support $A^D(\bar{z})$ at (x^{PD},y^N). Therefore, by T4), (x^{PD},y^N) would be the unique input demand of a downstream firm required to produce $\bar{z}$ and facing input prices (w,p).

The conclusion is that an upstream-downstream pair required to produce a net output of $(\bar{y},\bar{z})$ will duplicate the input choices of an optimally coordinated pair given no other coordination than setting $p = \partial C^U(y^N + \bar{y})/\partial y$, provided the upstream firm meets the downstream firm's input demand.

E.O.P.

A heuristic argument for H supporting $A^D(\bar{z})$ at (x^{PD},y^N) can be made from Figure A1. Let

(A23) $G \equiv \{(w,y) \text{ s.t. } C^U(y+\bar{y}) + w\cdot x = C^U(y^N+\bar{y}) + wx^{PD} = w(x^{PU}+x^{PD})\}$

be the iso-raw input cost surface through (x^{PD},y^N). By the optimality of (x^{PD},y^N), G must support $A^D(\bar{z})$ at (x^{PD},y^N). Since $p \equiv \partial C^U(y^N+\bar{y})/\partial y$, H is tangent to G at (x^{PD},y^N). Thus, since $A^D(\bar{z})$ is convex, H must also support $A^D(\bar{z})$ at that point.

Finally, a few comments on the possibility of "backward" integration. The case of interest in this paper is "forward" integration; that is, where an integrated firm has net positive production of the intermediate

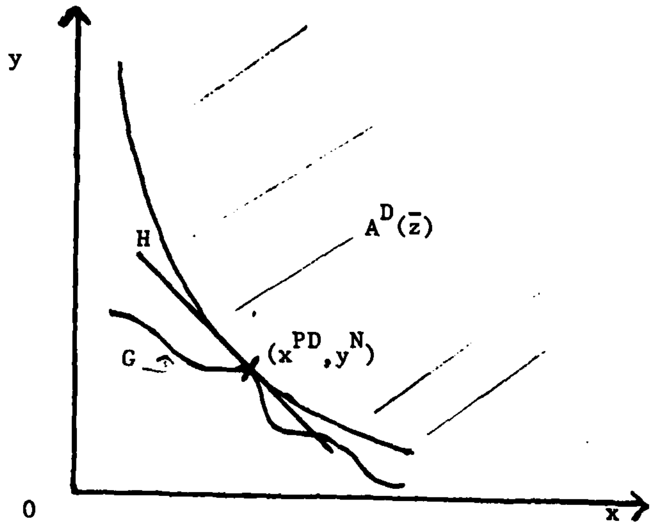

Figure A1

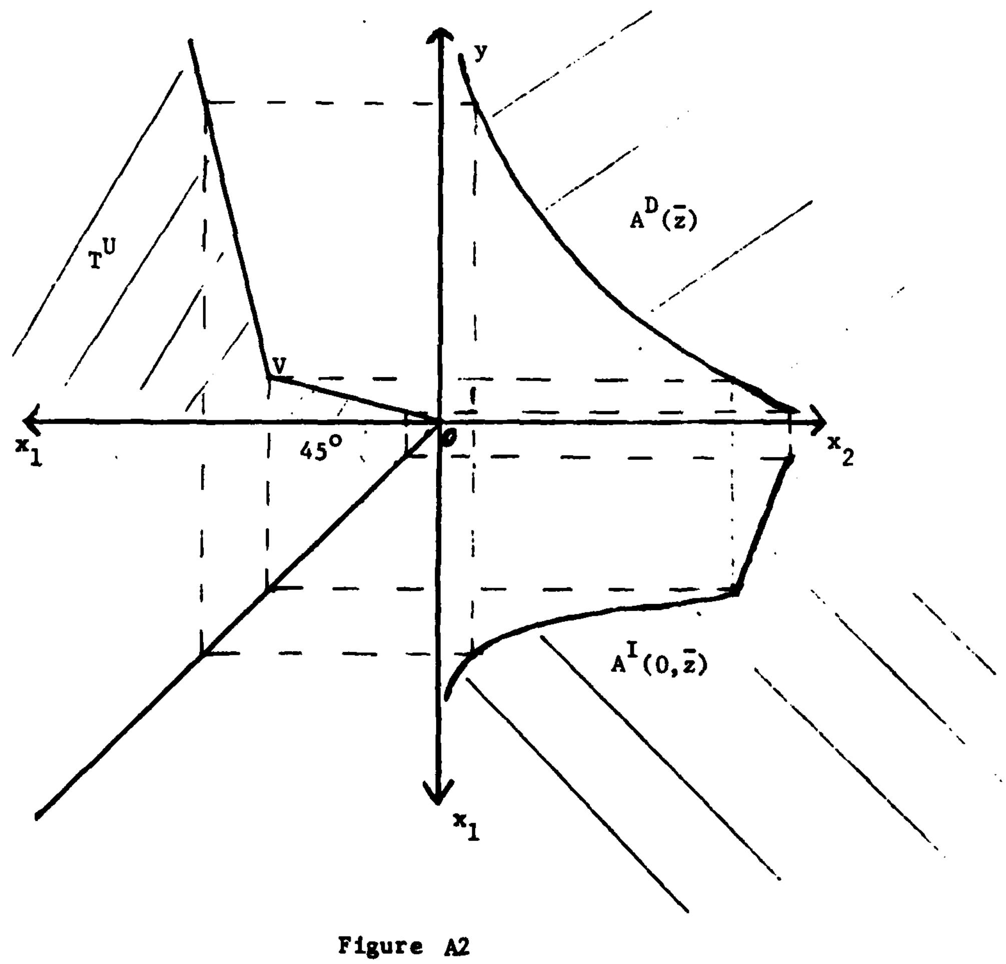

Figure A2

good. A "backward" integrated firm produces some or all of its internal needs for y but none for external sale. Such a firm is indistinguishable from a purely downstream firm on the basis of net output vector alone.

A problem is created for the representation of the costs of a forward integrated firm if its intermediate good production is in reality split between its upstream subsidiary and such a backward integrated "downstream" subsidiary. The problem arises where the proof of Theorem 2.2 relies on the convexity of $A^D(\bar{z})$, since the input requirement sets of T^I are not necessarily convex, even if T4) holds for T^U and T^D!

The input sets of T^I are not convex when upstream marginal cost is falling "too fast;" that is, when T^U is not "convex enough" in output of y, as at point V in Figure A2. The upper left quadrant in the figure represents T^U, which converts the (single) input x_1 into the intermediate good. The upper right quadrant represents $A^D(\bar{z})$, using the additional input x_2. Then, the lower right quadrant gives the implied requirement set $A^I(0,\bar{z})$ in terms of (x_1,x_2) for an integrated firm under T5). Clearly, A^I is not convex despite the convexity of A^U and A^D.

This apparent representational problem is not, however, troublesome in the special cases investigated in this paper. First, when the upstream market is contestable, it is assumed that the upstream technology has global scale economies. In that case, an integrated firm would never split its intermediate good production between its two subsidiaries. Second, in the case where upstream marginal costs are assumed constant,

$$p(y'' - y^N) = C^U(y'' + \bar{y}) - C^U(y^N + \bar{y}) \ . \qquad \text{(A24)}$$

Then, using (x'',y'') for $(x(\theta),y(\theta))$, (A22) follows directly from (A14) without using the convexity of $A^D(\bar{z})$.

In the general case, T^D would have to be defined more precisely than in terms of net vectors in order to separate production of the intermediate good for internal use from true "downstream" production. Assumption T5) then would not allow for multiple centers of intermediate good production.

APPENDIX B

Price Changes in the Downstream Constant Returns Case

This Appendix determines the price changes which result from partial vertical integration when the downstream technology is linearly homogeneous. The results are the same as for the U shaped average cost case examined in Secs. IV - VI: the external and final prices rise or fall as the upstream market is protected or contestable, respectively.

Partial integration can in this case result only from institutional constraints such as fear of antitrust action. As Schmalensee {12} has shown, when there are substitution distortions[1], the upstream firm always has economic incentive to take over the entire downstream industry. The analysis here is therefore comparatively static: I examine the direction of price changes as the constraint on the integrated firm's final output is relaxed.

The case of a contestable upstream industry hardly merits formal analysis. As the integrated firm expands its final output, it crowds out production by independent downstream firms. The former is produced efficiently while the latter is not because of substitution distortions. To avoid entry by a similarly integrated entrant, the integrated firm must pass the resulting cost savings on to its customers. Thus, the external and final prices must fall with each additional unit of final good produced by the integrated firm.

The protected upstream case is analyzed by Schmalensee {12} under the particular assumption that upstream marginal costs are constant in the relevant range. He also represents the output of the downstream subsidiary as a percentage of the final market, rather than in absolute units. With this representation, he is unable to sign the direction of price changes as the integrated firm's percentage of the final market increases and conjectures that it

[1] There are, of course, no scale distortions here.

depends on parameter values {12, p. 446}.

Representing the downstream subsidiary's output in absolute units (and assuming constant upstream marginal costs, $\bar{c}$), I find the external price rises with said output. Downstream average (= marginal) cost is a function M(p) of the intermediate good price alone. The amount of intermediate good used per unit of final output is (by Shephard's Lemma) M'(p). The intermediate good demand of the independent downstream firms is thus $(Q-\hat{z})\cdot M'(\hat{p})$. With the internal price set at upstream marginal cost, the profit of the integrated firm is

$$\text{(B.1)}\quad \Pi^{Int}(\hat{p}, \bar{c}, \hat{z}) = \hat{z}\cdot(M(\hat{p}) - M(\bar{c})) + \hat{p}\cdot(Q-\hat{z})\cdot M'(\hat{p}) - \bar{c}\cdot(Q-\hat{z})\cdot M'(\hat{p}) - F^U,$$

where $\hat{z}$ is treated as a parameter. The optimal external price is a function of $\hat{z}$, defined implicitly by

$$\text{(B.2)}\quad \frac{\partial \Pi^{Int}}{\partial \hat{p}} = Q \cdot M'(\hat{p}) + (\hat{p}-\bar{c})\{(Q-\hat{z})M''(\hat{p}) + Q'\cdot(M'(\hat{p}))^2\} = 0$$

Then, at the optimal, $\hat{p}$,

$$\text{(B.3)}\quad \frac{d\hat{p}}{d\hat{z}} = -\frac{\partial^2\Pi^{Int}/\partial\hat{p}\partial\hat{z}}{\partial^2\Pi^{Int}/\partial\hat{p}^2} = +\frac{(\hat{p}-\bar{c})M''(\hat{p})}{\partial^2\Pi^{Int}/\partial\hat{p}^2} > 0,$$

where the denominator is negative by the second order condition and $M''(\hat{p})$, being the price response of input demand (per unit of output), is also negative. Thus, the external and, hence, final prices rise as the integrated firm expands its final output.

The policy conclusions from the constant returns case are thus the same as for the U-shaped average cost case. When integration is partial (or total, for that matter) and the upstream market is contestable, cost savings resulting from integration will be fully passed on to final consumers. When the upstream monopolist is protected, partial integration will result in higher prices for final consumers unless the monopolist is actively prevented from raising the external price.

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