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WAVE PROPAGATION-NONLINEAR BOUNDARY VALUE PROBLEMS(U)
MIAMI UNIV CORAL GABLES FLA DEPT OF MATHEMATICS
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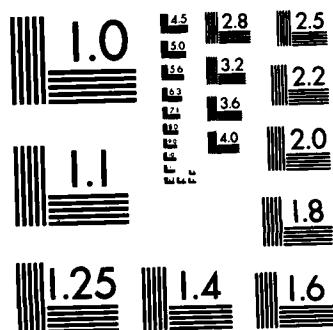


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Annual Report

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"Wave Propagation-Nonlinear Boundary Value Problems"

Principal Investigator: R.A. Goldstein

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Work was completed on a paper entitled "Perturbed Bifurcation of Stationary Striations in a Contaminated, Non-Uniform Plasma." It was accepted by SIAM APPLIED MATH and is currently in press. The paper is attached.

On going research in collaboration with J. Magnan (Northwestern) continues with respect to moving situations or modulated travelling waves in reaction diffusion systems.

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Moung Striations

Problem

$$\frac{\partial N}{\partial t} = D_- \frac{\partial^2 N}{\partial z^2} + \frac{\mu_+}{\mu_-} \frac{J}{e} \left(\frac{P}{N^2} \frac{\partial N}{\partial z} - \frac{1}{N} \frac{\partial P}{\partial z} \right) + F,$$

$$\frac{\partial P}{\partial t} = D_+ \frac{\partial^2 P}{\partial z^2} + \frac{\mu_+}{\mu_-} \frac{J}{e} \left(\frac{P}{N^2} \frac{\partial N}{\partial z} - \frac{1}{N} \frac{\partial P}{\partial z} \right) + G,$$

$$\frac{\partial M}{\partial t} = D^* \frac{\partial^2 M}{\partial z^2} + H,$$

where it has been assumed that $\mu_- N \gg \mu_+ P$

B.C. (see Fig 1)

$$N(0) - \alpha \frac{\partial N(0)}{\partial z} = A, \quad N(L) + \alpha \frac{\partial N(L)}{\partial z} = A$$

$$P(0) - \beta \frac{\partial P(0)}{\partial z} = B, \quad P(L) + \beta \frac{\partial P(L)}{\partial z} = B$$

$$M(0) - \gamma \frac{\partial M(0)}{\partial z} = C, \quad M(L) + \gamma \frac{\partial M(L)}{\partial z} = C$$

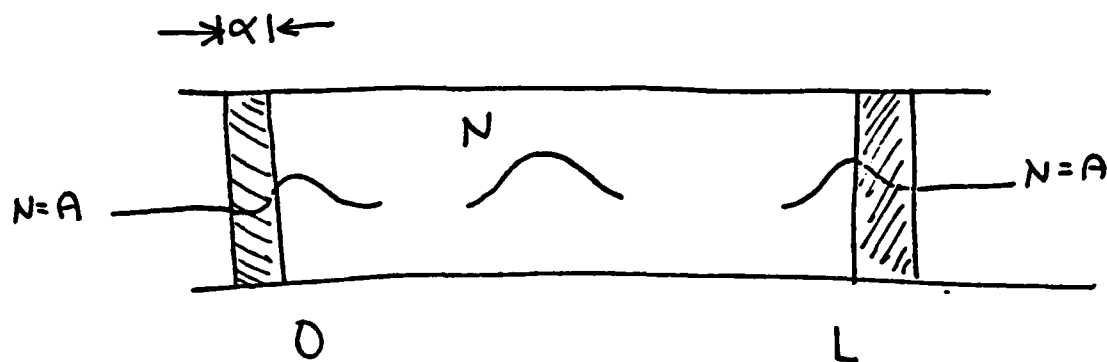
I.C.

$$N(z, 0) = N_0$$

$$P(z, 0) = P_0$$

$$M(z, 0) = M_0$$

Fig 1



Scaled equations

$$\frac{\partial n}{\partial \tau} = \epsilon \frac{\partial^2 n}{\partial \xi^2} + \frac{p}{n^2} \frac{\partial n}{\partial \xi} - \frac{1}{n} \frac{\partial p}{\partial \xi} + f$$

$$\frac{\partial p}{\partial \tau} = \epsilon \delta \frac{\partial^2 p}{\partial \xi^2} + \frac{p}{n^2} \frac{\partial n}{\partial \xi} - \frac{1}{n} \frac{\partial p}{\partial \xi} + g$$

$$\frac{\partial m}{\partial \tau} = \epsilon \theta \frac{\partial^2 m}{\partial \xi^2} + h$$

B.C.

$$n(0, \tau) - \epsilon \frac{\partial n(0, \tau)}{\partial \xi} = a = n(l, \tau) + \epsilon \frac{\partial n(l, \tau)}{\partial \xi}$$

$$p(0, \tau) + \epsilon \frac{\partial p(0, \tau)}{\partial \xi} = b = p(l, \tau) + \epsilon \frac{\partial p(l, \tau)}{\partial \xi}$$

$$m(0, \tau) - \epsilon \frac{\partial m(0, \tau)}{\partial \xi} = c = m(l, \tau) + \epsilon \frac{\partial m(l, \tau)}{\partial \xi}$$

I.C.

$$n(\frac{1}{2}, 0) = 1$$

$$p(\frac{1}{2}, 0) = 1$$

$$m(\frac{1}{2}, 0) = 1$$

where

$$z = \left(\frac{L^2}{D_-} \right) \frac{\mu_+}{\mu_-} \frac{J}{e N_0} \frac{1}{\xi} ,$$

$$t = \frac{L^2}{D_-} \tau ,$$

$$\epsilon = \left(\frac{\frac{D_- N_0}{L}}{\frac{\mu_+}{\mu_-} \frac{J}{e}} \right)^2 = \left(\frac{\Gamma_0}{\Gamma_+} \right)^2 ,$$

$$N = n N_0 , P = p N_0 , M = m M_0 ,$$

$$A = a N_0 , B = b N_0 , C = c M_0 ,$$

$$\alpha = \beta = \gamma = \sqrt{\epsilon} L ,$$

$$\delta = \left(\frac{D_+}{D_-} \right)^2 , \theta = \left(\frac{D^*}{D_-} \right)^2 , \ell = \sqrt{\epsilon} .$$

Xform to homogeneous B.C.

$$\text{let } n = n' + a$$

$$p = p' + b$$

$$m = m' + c$$

substitute into the scaled equations and drop the primes.

$$\frac{\partial n}{\partial \tau} = \epsilon \frac{\partial^2 n}{\partial \xi^2} + \frac{(p+b)}{(n+a)^2} \frac{\partial n}{\partial \xi} - \frac{1}{(n+a)} \frac{\partial p}{\partial \xi} + f$$

$$\frac{\partial p}{\partial \tau} = \epsilon \delta \frac{\partial^2 p}{\partial \xi^2} + \frac{(p+b)}{(n+a)^2} \frac{\partial n}{\partial \xi} - \frac{1}{(n+a)} \frac{\partial p}{\partial \xi} + g$$

$$\frac{\partial m}{\partial \tau} = \epsilon \theta \frac{\partial^2 m}{\partial \xi^2} + h$$

BC:

$$n(0, \tau) - \epsilon \frac{\partial n(0, \tau)}{\partial \xi} = 0 = n(l, \tau) + \epsilon \frac{\partial n(l, \tau)}{\partial \xi}$$

$$p(0, \tau) - \epsilon \frac{\partial p(0, \tau)}{\partial \xi} = 0 = p(l, \tau) + \epsilon \frac{\partial p(l, \tau)}{\partial \xi}$$

$$m(0, \tau) - \epsilon \frac{\partial m(0, \tau)}{\partial \xi} = 0 = m(l, \tau) + \epsilon \frac{\partial m(l, \tau)}{\partial \xi}$$

IC:

$$n(\xi, 0) = 1 - a$$

$$p(\xi, 0) = 1 - b$$

$$m(\xi, 0) = 1 - c$$

Try a solution

$$\text{let } u(\xi, \tau) = e^{-\frac{\phi(\xi, \tau)}{2\epsilon}} x(\xi, \tau, \epsilon),$$

$$p(\xi, \tau) = e^{-\frac{\phi(\xi, \tau)}{2\epsilon}} y(\xi, \tau, \epsilon),$$

$$w(\xi, \tau) = e^{-\frac{\phi(\xi, \tau)}{2\epsilon}} z(\xi, \tau, \epsilon),$$

substitute into equations and get
to order $\frac{1}{\epsilon}$:

$$\phi_\tau + \frac{1}{2} \phi_\xi^2 = 0$$

with solution

$$\phi(\xi, \tau) = \frac{1}{2} \left(\frac{\xi - \xi_0}{\tau} \right)^2$$

$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ will satisfy the 1st order unperturbed problem (hence a travelling wave)

We will seek "modulated" travelling wave solutions