

AD-A124 420

STOCHASTIC IMAGE MODELS AND APPLICATIONS(U) CALIFORNIA
UNIV DAVIS SIGNAL AND IMAGE PROCESSING LAB A K JAIN
DEC 82 SIPL-82-15 ARO-16222. 10-EL DAG29-78-G-0206

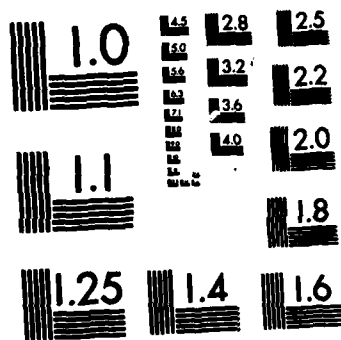
1/1

UNCLASSIFIED

F/G 12/1

NL

												END FILMED BY DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

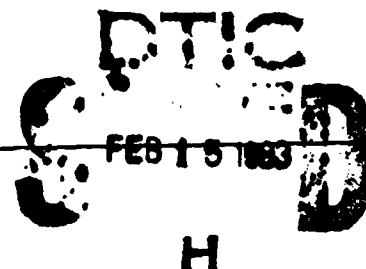
61

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 16222.10-EL	2. GOVT ACCESSION NO. AD-A124420	3. RECIPIENT'S CATALOG NUMBER
TITLE (and Subtitle) Stochastic Image Models and Applications		5. TYPE OF REPORT & PERIOD COVERED Final: 28 Sep 78 - 30 Jun 82
AUTHOR(s) Anil K. Jain		6. PERFORMING ORG. REPORT NUMBER
PERFORMING ORGANIZATION NAME AND ADDRESS University of California Davis, CA 95616		8. CONTRACT OR GRANT NUMBER(s) DAAG29 78 G 0206
CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE Dec 82
		13. NUMBER OF PAGES 19
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)



18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

stochastic processes	estimating
data compression	signal processing
algorithms	
images	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Research is reported in the following areas:

- (1) Stochastic Image Modeling,
- (2) Image Data Compression,
- (3) Spectral Estimation and Signal Extrapolation,
- (4) Fast Algorithms.

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

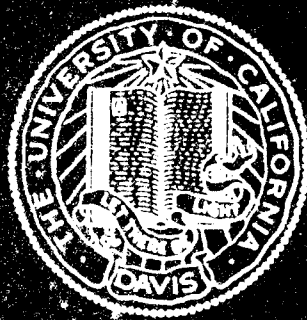
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

83 02 012 100

AD A 124420
DTIC FILE COPY

UCD

University of California, Davis



6

DTIC
COPY
INSPECTED
S

Accession For	
NTIS GRA&I	☒
NTIS TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Avail and/or	
Dist	Special
A	

STOCHASTIC IMAGE MODELS AND APPLICATIONS
FINAL REPORT

by

Anil K. Jain

Report No. SIPL-82-15 ✓

December 1982

This work is supported by the United States Army
Research Office under Grant DAAG29-78-G-0206

DISTRIBUTION STATEMENT I
Approved for public release;
Distribution Unlimited

STOCHASTIC IMAGE MODELS AND APPLICATIONS

FINAL REPORT

ARO GRANT DAAG29-78-G-0206

September 1978 to June 1982

Anil K. Jain

Signal & Image Processing Lab
Department of Electrical and Computer Engineering
University of California
Davis, California 95616

December 1982

TABLE OF CONTENTS

	Page
Acknowledgment	3
Final Report Summary Page	4
I. List of Publications	5-7
II. Personnel Supported	7-8
III. AWARDS	8
IV. SUMMARY OF RESEARCH FINDINGS	8
1. STOCHASTIC IMAGE MODELING PROJECT	8-9
2. IMAGE DATA COMPRESSION PROJECT	10-14
3. SPECTRAL ESTIMATION AND SIGNAL EXTRAPOLATION PROJECT	15-16
4. FAST ALGORITHMS PROJECTS	17-18
5. SUMMARY	18

ACKNOWLEDGMENT

The principal investigator, Professor Anil K. Jain is grateful to the U.S. Army Research Office for its support of the research summarized in this final report. Special thanks are due to Dr. William A. Sander and Dr. Jagdish Chandra of ARO for their continuing interest in this research throughout the duration of this grant.

FINAL REPORT

1. ARO PROPOSAL NUMBER: DRXRD-PR, P-16222-EL
2. PERIOD COVERED BY REPORT: Sept. 28, 1978 - June 30, 1982
3. TITLE OF PROPOSAL: Stochastic Image Models
4. CONTRACT OR GRANT NUMBER: DAAG29-78-G-0206
5. NAME OF INSTITUTION: University of California, Davis
6. AUTHOR(S) OF REPORT: Anil K. Jain
7. LIST OF MANUSCRIPTS: Submitted or Published Under ARO SPONSORSHIP
Attached.
8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED
DURING THIS REPORTING PERIOD: Attached.

I. LIST OF PUBLICATIONS

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

A. JOURNAL PAPERS

1. A.K. Jain and J.R. Jain, "Partial Differential Equations and Finite Difference Methods in Image Processing--Part II: Image Restoration," IEEE Trans. Auto. Contr., Vol. AC-23, No. 5, pp. 817-834, Oct. 1978.
2. E. Angel and A.K. Jain, "Frame to Frame Restoration of Diffusion Images," IEEE Trans. Auto. Contr., Vol. AC-23, No. 5, pp. 850-854, Oct. 1978.
3. A.K. Jain, "A Sinusoidal Family of Unitary Transforms," IEEE Trans. in Pattern Analysis and Machine Intelligence, Vol. PAMI-1, No. 4., pp. 356-365, Oct. 1979.
4. A.K. Jain, Book Review of Two Dimensional Digital Signal Processing by M.P. Ekstrom and S.K. Mitra (Eds.), IEEE Trans. ASSP, Vol. ASSP, NO. 2, p. 266, April 1980.
5. A.K. Jain, "Image Data Compression: A Review," Proceedings of the IEEE, Vol. 69, No. 3, pp. 349-389, March 1981.
6. A.K. Jain, "Advances in Mathematical Models in Image Processing," Proceedings IEEE, Vol. 69, No. 5, pp. 502-528, May 1981.
7. A.K. Jain and S. Ranganath, "Extrapolation of Discrete Time Signals with Application in Spectral Estimation," IEEE Trans. ASSP, Vol. ASSP-29, No. 4, pp. 830-845, 1981.
8. J.R. Jain and A.K. Jain, "Motion Analysis and Its Application in Inter-frame Image Coding", IEEE Trans. Commun., Vol. COM-29, No. 12, pp. 1799-1808, 1981.
9. A.K. Jain and J. Jasiulek: "Fast Fourier Transform Algorithms for Linear Estimation, Smoothing, and Riccati Equations", (submitted for publication).

B. BOOK

A.K. Jain, Fundamentals of Digital Image Processing, (in progress).

C. CONFERENCE PAPERS AND PRESENTATIONS

1. A.K. Jain, "Application of Stochastic Models in Image Processing," presented at ARO workshop Stochastic Models in Fire Control, Night Vision and Electro Optics Laboratory, Ft. Belvoir, Virginia, Nov. 1978.
2. A.K. Jain, "Two Dimensional Data Sets and Image Processing," Proc. Workshop on Advanced Concepts in Ocean Measurements I, Baruch Institute, University of So. Carolina, Georgetown, S.C., Nov. 1978.

3. A.K. Jain and S. Ranganath, "Image Coding by Synthesis," Proc. International Picture Coding Symposium, Ipswich, United Kingdom, July 9-11, 1979.
4. A.K. Jain and S.H. Wang, "Image Coding by Stochastic Decomposition of Features," Proc. International Picture Coding Symposium, Ipswich, United Kingdom, July 9-11, 1979.
5. J.R. Jain and A.K. Jain, "Optimization of Transform Coding for Transmission of Images Over Noisy Channels," Proc. International Picture Coding Symposium, Ipswich, United Kingdom, July 9-11, 1979.
6. A.K. Jain, "Spectral Estimation and Signal Extrapolation in One and Two Dimensional," Proceedings Spectral Estimation Workshop, Rome Air Dev. Center, pp. 195-214, Oct. 1979.
7. A.K. Jain, "PDE Models in Image Processing," First ASSP Workshop on Two Dimensional Digital Signal Processing, Berkeley, CA, Oct. 1979.
8. A.K. Jain, "New Mathematical Concepts for Image Modeling and Image Processing," presented at EASCON, Wash. D.C., Oct. 1979.
9. A.K. Jain and K.W. Au, "On Linear Estimation via the Fast Fourier Transform" Conf. Decision and Control, Florida, Dec. 1979.
10. A.K. Jain and S. Ranganath, "Image Coding by Autoregressive Synthesis," 1980 Int. Conf. on Acoustics Speech and Signal Processing, Denver, Colorado, April 1980.
11. A.K. Jain, "Fast Fourier Transform Algorithms for Linear Estimation and Solution of Riccati Equations," ARO Workshop on Fast Algorithms, Dover, N.J.
12. A.K. Jain, "A New Algorithm for Two Dimensional Spectral Estimation," IEEE Comm Theory Workshop, Cyprus Garden, FLA., April 1980.
13. A.K. Jain and S. Ranganath, Application of Two Dimensional Spectral Estimation in Image Restoration, Proc. 1981, ICASSP, pp. 1113-1116, Atlanta, GA, 1981.
14. A.K. Jain and J. Jasiulek, "A Class of FFT Based Algorithms for Linear Estimation and Boundary Value Problems", Proc. Joint Aut. Contr. Conf., Charlottesville, Va., June 1981.
15. A.K. Jain and S. Ranganath, High Resolution Spectrum Estimation in Two Dimensions, Proc. IEEE Workshop on Spectral Estimation, Hamilton, Ontario, Aug. 1981.
16. A.K. Jain and S. Ranganath, "High Resolution Spectral Estimation using Two-Dimensional Prediction Models", Second ASSP Workshop on Two-Dimensional Signal Processing, New Paltz, N.Y., Oct. 1981.
17. A.K. Jain, "Models for Image Coding and Image Restoration", Second ASSP Workshop on Two-Dimensional Signal Processing, New Paltz, N.Y., Oct. 1981.
18. A.K. Jain and S. Ranganath, "Image Restoration and Edge Extraction Based on 2-D Stochastic Models", Proc. 1982 ICASSP, Paris, France, May 1982.

D. REPORTS

1. A.K. Jain and S. Ranganath, "Extrapolation and Spectral Estimation Techniques for Discrete Time Signals," Signal and Image Processing Laboratory, Electrical and Computer Engineering Dept., Technical Report SIPL-78-1, U.S. Davis, Nov. 1978.
2. J.R. Jain and A.K. Jain, "Interframe Data Compression Techniques for Images," Signal and Image Processing Laboratory, Department of Electrical and Computer Engineering, U.C. Davis, Calif., 95616, Aug. 1979.
3. S.H. Wang and A.K. Jain, "Applications of Stochastic Models for Image Data Compression," Signal and Image Processing Laboratory, Department of Electrical and Computer Engineering, U.C. Davis, Calif., 95616, Sept. 1979.
4. A.K. Jain and J. Jasiulek, "A Class of FFT Based Algorithms for Linear Estimation," Tech. Rep. Signal and Image Processing Lab, University of California, Davis, April 1982.

II. PERSONNEL SUPPORTED AND DEGREES AWARDED

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

1. Anil K. Jain, Project Director
2. S. Ranganath, Research Assistant
3. S.H. Wang, Research Assistant
4. J.R. Jain, Research Assistant
5. K.W. Au, Research Assistant
6. J. Jasiulek, Post Doctoral Research Associate
7. S. Sugimoto, Visiting Assistant Professor
8. S. Ansari, Research Assistant
9. P. Farrelle, Research Assistant
10. S. Louton, Research Assistant

E. DEGREES AWARDED OR IN PROGRESS UPON THE COMPLETION OF PROJECT

Ph.D.

1. S.H. Wang, "Applications of Stochastic Image Models in Image Data Compression", Sept. 1979.
2. J.R. Jain, "Interframe Adaptive Data Compression Techniques for Images", Sept. 1979.
3. S. Ranganath, "Spectral Factorization, Spectral Estimation and Application in Image Processing", (in progress-expected to be completed in March 1983).
4. S. Ansari, "Radon Transform Theory for Random Fields with Application in Image Reconstruction", (in progress).

M.S.

5. K.W. Au, "Linear Estimation via Fast Fourier Transform", Sept. 1979.
6. P. Farrelle, "Block Recursive Coding via Noncausal Models", (in progress, expected completion date, Dec. 1982).

III. AWARDS

Anil K. Jain has been named the recipient of the 1983 Donald G. Fink Prize Award for the paper:

"Image Data Compression: A Review", Proc. IEEE, March 1981.

This paper is a comprehensive review of the state of the art in data compression and includes much of the work supported by ARO on data compression. According to IEEE this award "is given for an outstanding survey, review, or tutorial paper published in any of the IEEE TRANSACTIONS, JOURNALS, MAGAZINES or PROCEEDINGS of the IEEE during the period January 1 and December 31".

IV. SUMMARY OF RESEARCH FINDINGS

The entire research program from September 1978 to June 1982 was divided into four projects, viz,

1. Stochastic Image Modeling
2. Image Data Compression
3. Spectral Estimation and Signal Extrapolation
4. Fast Algorithms

The progress made in each of these areas is summarized below.

1. STOCHASTIC IMAGE MODELING PROJECT

In 1978 [Refs. A-1, A-2, C-1, C-2] we introduced three canonical forms of representations for two dimensional discrete random fields. These representations were called causal, semicausal and noncausal. Our causal models were the same as those studied earlier by other authors. However, the semicausal and noncausal models were new. In our initial attempts it was suggested that these models could be realized by considering stable, finite difference approximations of partial differential equations. Using the concept of minimum variance, the spectral densities and prediction

properties of such models could be related.

In subsequent developments [A-6, C-6, C-7, C-8], the theory of these models was extended to arbitrary order models without requiring their origin in partial differential equations. This work has led to the following results.

- i) A method of determining causal, semicausal or noncausal filters which realize a given spectrum density function or magnitude frequency response. This yields a class of algorithms for spectral factorization of a two-dimensional positive, analytic function $S(\omega_1, \omega_2)$.
- ii) The algorithm above yields irrational factors, i.e., infinite order models in general. We have developed methods of finding finite order approximations of the factors such that, a) the filters are stable, b) the filters can match the spectral density as closely as desired while remaining finite order and c) the filters converge uniformly to their unique, limit (irrational factors) as their orders go to infinity.

Details of these results will appear in a forthcoming Ph.D. dissertation (see Ranganath, E-3) and some related papers currently under preparation.

- iii) This theory, which has now been rigorously established will enable design of algorithms for two-dimensional signal and image processing problems such as data compression, spectral estimation, restoration, edge detection, etc. Figs. 1 to 3 show an application of semicausal models for developing an edge extraction algorithm for noisy images. Comparison with some of the common edge extraction operators (having the same computational complexity) shows the advantage of our technique. These models have also been found useful in developing fast and effective filters for image restoration problems. Details are discussed in [Ref. C-18].



i) Noiseless Image



ii) Noisy Image



iii) Noiseless Image



iv) Noisy Image

Figure 1 : Original and noisy images.



i) Sobel operator edge detection on noiseless image



iii) Sobel operator edge detection on noisy image, probability of error is about 24%

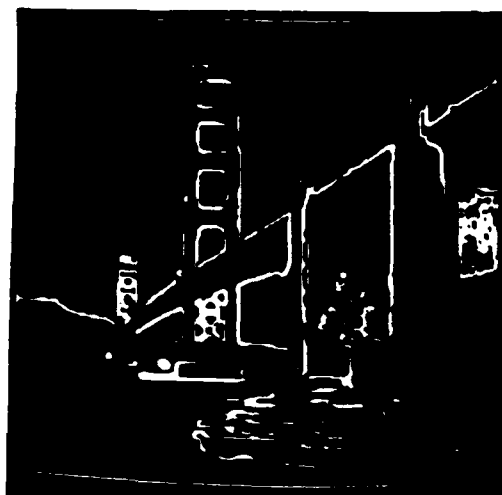


ii) Semicausal model operator edge detection on noiseless image

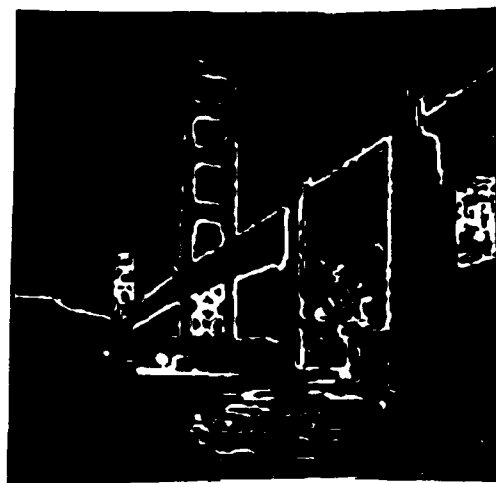


iv) Semicausal model operator edge detection on noisy image, probability of error is about 2%

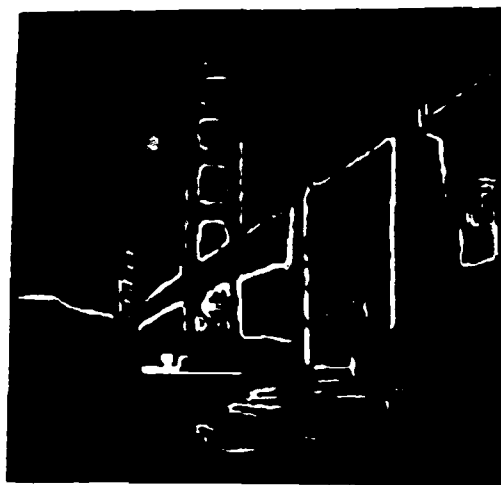
Figure 2 : An edge detection operator has been developed using a semicausal prediction model. This figure shows its performance on noiseless as well as noisy images. Other common operators such as Prewitt, isotropic, etc. have performance similar to or worse than the Sobel operator.



i) Sobel operator edge detection on noiseless image



iii) Sobel operator edge detection on noisy image, probability of error is about 24%



ii) Semicausal model operator edge detection on noiseless image



iv) Semicausal model operator edge detection on noisy image, probability of error is about 2%

Figure 3 : An edge detection operator has been developed using a semicausal prediction model. This figure shows its performance on noiseless as well as noisy images. Other common operators such as Prewitt, isotropic, etc. have performance similar to or worse than the Sobel operator.

2. IMAGE DATA COMPRESSION PROJECT

Results achieved in this effort have appeared in [A-5, A-8, C-3, C-4, C-5, C-10, D-2, D-3] and in the dissertations E-1, E-2. In particular we have shown

- i) The causal, semicausal, and noncausal models developed in Image Modeling Project lead to predictive, hybrid and transform coding techniques, respectively. Extension to noisy images and adaptive techniques is possible in the framework of these models. (Refs. A-5, A-6,, D-3). One application of these models is the following study.

- a) Image Coding by Stochastic Decomposition of Features

Here we consider an image U to be a composition of two types of processes U_0 and U_f , i.e.,

$$U = U_0 + U_f$$

where U_0 represents a stationary random process in two dimensions (or a random field) and U_f represents a deterministic or a nonstationary part of the image called features. The stationary part can be decomposed from the other by using the knowledge of the power spectrum of the image. Specifically, knowing the image power spectrum, we can find an operator L such that

$$L[U] \triangleq \epsilon \triangleq \epsilon_0 + \epsilon_f$$

where ϵ_0 is a white noise process whose mean and variance are known. If the given image were a stationary random field, then ϵ would be ϵ_0 . However, due to presence of nonstationarities in the image e.g., edges and corners, the field $\epsilon(i,j)$ is a signal $\epsilon_f(i,j)$ immersed in a white noise field $\epsilon_0(i,j)$. Classical techniques for detection of signal (ϵ_f) in noise

(ϵ_0) can be employed to discriminate ϵ_0 and ϵ_f or equivalently U_0 and U_f .

The stationary component U_0 is coded by (optimum) transform coding method and U_f is coded by a simple PCM combined with run-length coding. The component U_f is seen to contain image features such as sharp boundaries, edges, etc. At the receiver the encoded images U_0^* and U_f^* are combined to reproduce the original image as

$$U^* = U_0^* + U_f^* .$$

By varying the allocation of bits between U_0^* and U_f^* , for a fixed overall rate, several encoded images differing in visual quality are obtained. If no bits are allocated to U_f^* , the technique reduces to ordinary transform coding. Results show, for a fixed signal to noise ratio (based on mean square error) and fixed average rate, encoded images differing in visual quality can be achieved. In this way, images with higher visual quality than in transform coding are obtained. Theoretical details and experimental results are given in Ref. [D-3].

b) Image Coding by Autoregressive Synthesis

A new method of image data compression by autoregressive synthesis (Refs. C-3, C-10) has been developed. Results on line by line image coding show this method is superior to transform coding techniques (in the visual sense). It is well known that image transform coding methods do not perform well for high contrast and facsimile (printed document) images. This AR Synthesis method is a universal transform coding system which performs well for monochrome as well as binary images. This method would be suitable for transform coding of composite images as well as high resolution radar data.

It is believed that these results can be extended to two-dimensional data compression by considering image synthesis via two-dimensional semicausal models developed in the Image Modeling Project.

This method is based on the physics of image formation which suggests that each line may be considered as a sample function of a power spectrum. This formulation directly yields the autocorrelations as the coefficients of a Cosine transform. An autoregressive (AR) model may then be used to approximate the given power spectrum with fewer coefficients.

If a one-dimensional object is illuminated by coherent or non-coherent light and is imaged through a lens system, an intensity detector such as film will record

$$s(x) = \left| \int h(x-\xi)a(\xi)d\xi \right|^2 \text{ in the coherent case}$$

or

$$s(x) = \int h^2(x-\xi)|a(\xi)|^2 d\xi \text{ in the non-coherent case}$$

where $h(x)$ represents the "impulse response" of the lens and $a(x)$ the object field distribution. In both cases, phase information is lost and $s(x)$ may be regarded as a power spectrum. Hence,

$$s(x) = \sum_{k=-\infty}^{\infty} r_k e^{-j2\pi xk} \quad ; \quad -1/2 \leq x \leq 1/2$$

will satisfy the usual properties of a power spectrum such as positivity (implying a positive definite autocorrelation sequence r_k) and evenness. In the image model we assume, each line is regarded as a spectrum and

satisfies these properties.

By appropriate sampling of $s(x)$, the autocorrelation sequence r_k is given by

$$r_k = 2 \sum_{m=0}^{N-1} s(m) \cos\left(\frac{m+1}{2} \pi k\right) \frac{1}{N}, \quad 0 \leq k \leq N-1; \quad r_k = r_{-k}.$$

implying that r_k is the Cosine transform of the sampled image $s(n)$.

In transform coding a zonal filter is applied to the transform domain coefficients (autocorrelations in this case) retaining only p coefficients ($0 \leq p \leq N-1$) for storage or transmission.

In AR Synthesis we assume that the object amplitude distribution is generated by an AR model:

$$a(k) = \sum_{m=1}^p \alpha_m a(k-m) + \epsilon(k) \quad 0 \leq p \leq N-1$$

where $\{\epsilon(k)\}$ is a white noise sequence and

$$r_k = E[a(m)a^*(m+k)] ; \quad k \in [-p, p]$$

α_k may be obtained by solving the well known "normal equations" using the efficient Levinson algorithm. The α_k may then be coded instead of the r_k .

In conclusion, the image $\hat{s}(x)$ synthesized by AR coding is continuous (filter interpolates). Moreover, the AR model provides a maximum entropy extrapolation in transform domain (for 1-D case). Images encoded by this method show no block effects and superior resolution

compared to transform domain coding.

c) An Area Correlation Method for Motion Estimation

Area correlation and image registration techniques have been developed for measurement and analysis of motion using interframe image data. Applications in image data compression have been studied, (Refs. A-5, A-8).

The method we developed is based on an efficient algorithm for searching the direction of displacement of an object from successive frames of image data. This method requires a search for the direction of minimum (DMD) between pixel neighborhoods in successive frames. The distortion referred to is the mean square value of the prediction error. The search algorithm converges logarithmically (under the assumption of local spatial-stationarity of the image correlation statistics). With this algorithm we found 8-10dB reduction in the prediction error which led to improved compression by a factor of 3 to 1. Details are given in Ref. [A-8].

d) Transform Coding in the Presence of Channel Errors

We have developed a new technique for optimization of transform coding in the presence of channel errors or jamming. Results show the performance of conventional transform coding in the presence of channel errors can be improved by several dB in signal to noise ratio.

Most transform image coding techniques have been designed for noiseless channels. When these coded images are transmitted over a noisy channel generally some error correcting codes are used to reduce the probability of channel errors. In many cases no error correction is done. An obvious drawback of using traditional error correcting codes is that

they provide equal protection to all the bits in the code. However, it is well known that error in different bits of the code (which may represent different transform coefficients) do not have equal effect on the quality of the image.

We give the optimum solution to the problem of bit assignment to different transform coefficients for quantization as well as error protection which takes into account the distortions due to channel errors for the minimum mean square error criterion. Rate distortion curves as well as actual experiments on images show that by including the channel characteristics in optimization of transform coding, large gains are possible when channel errors are present. Robustness of the present approach to channel characteristics seems to be high. These and other related results are given in Refs. [A-5, D-2].

3. SPECTRAL ESTIMATION AND SIGNAL EXTRAPOLATION PROJECT

Some new advances have been made in extrapolation and spectral estimation of one and two dimensional discrete time/space signals. For one-dimensional signals, we have shown that several existing and some new algorithms for extrapolation of discrete time bandlimited signals arise when one considers the minimum norm least squares criterion (MNLS). The new results are as follows [Refs. A-7, A-6].

- a) The MNLS problem can be solved to obtain the extrapolated estimate by a gradient algorithm with linear convergence. We show that the iterative algorithm proposed earlier by Papoulis when applied to discrete signals is a special case of this gradient algorithm and converges to the MNLS solution of $Ax = b$.
- b) It is shown the convergence properties of the iterative extrapolation

algorithms can be improved by considering the steepest descent and two step conjugate gradient algorithm.

- c) It is shown that the MNLS solution obtained via the singular value decomposition of A gives rise to the discrete prolate spheroidal wave functions (DPSWF).
- d) In the presence of noise or clutter, we show that by simply considering the minimum mean square (MNS) criterion, one can obtain the extrapolation filter equations.
- e) A recursive Kalman filter type extrapolation algorithm is easily derived by using the formulation above.

For two-dimensional data, we have shown how the above algorithms could be extended.

2-Dimensional Spectral Estimation

We have also shown that the stochastic models developed in the Image Modeling Project are very useful in high resolution radar imaging applications where spectral estimation techniques are employed to resolve say, two point targets which are close to each other. An example of this result is shown in Figs. 4 and 5. In particular, the Minimum Variance Semicausal (MV-SC) model yields a special Autoregressive Moving Average (ARMA) representation of random fields. It is different from the two-dimensional so called "causal filters" and is not the maximum entropy model often cited in the literature. However, it is linear and is experimentally shown to perform similar to or better than the maximum entropy spectral estimation methods. Experiments show the applicability of this algorithm to real world spectral estimation problems. Details are given in Refs. [A-6, C-6, C-12, C-15, C-16, E-3].

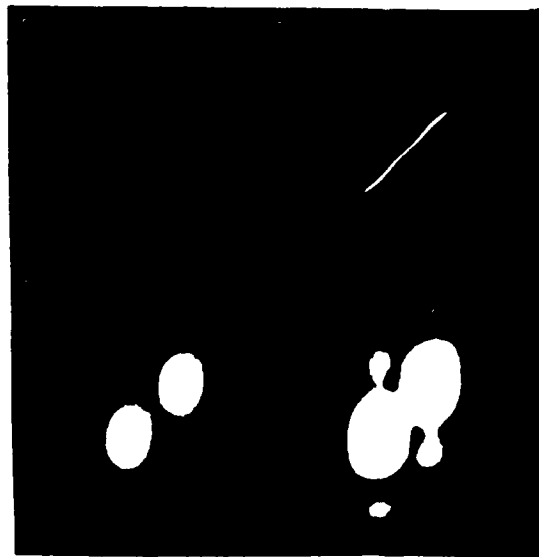


Figure 4 : Top row, left; detected targets using a 9x4 element array antenna followed by an FFT processor for spectral estimation. The two targets in each pair are not resolved. Top row, right; detected targets are resolved when a semicausal model based spectral estimation algorithm is used. Bottom row; images shown in top row are now shown on dB scale.



Figure 5 : Top row, left; detected targets using causal model. Top row, right; detected targets using semicausal model. Bottom row; images shown in top row are now shown on dB scale. Causal model has better resolution than FFT processing, but has false alarms. In general, semicausal model spectra are found to be more accurate.

Application of two dimensional spectral estimation in high resolution target detection. There are actually two pairs of point targets present. In each pair the targets are closely spaced.

4. FAST ALGORITHMS PROJECT

a) Fast Sinusoidal Transforms [Ref. A-3]

We introduced a new family of unitary transforms. It is shown that the well known discrete Fourier, Cosine, Sine and the Karhunen-Loeve (for first order stationary Markov processes) transforms are members of this family. All the member transforms of this family are sinusoidal sequences which are asymptotically equivalent. For finite length data, these transforms provide different approximation to the Karhunen Loeve transform of the said data. From the theory of these transforms some well known facts about orthogonal transforms are easily explained and some widely misunderstood concepts are brought to light. For example, the near-optimal behavior of the even discrete Cosine transform to the KL transform of first order Markov processes is explained and, at the same time, it is shown that this transform is not always such a good (or near optimal) approximation to the above mentioned KL transform. It is also shown that each member of the Sinusoidal family is the KL transform of a unique, first order, nonstationary (in general), Markov process. Asymptotic equivalence and other interesting properties of these transforms can be studied by analyzing the underlying Markov processes.

b) Linear Estimation via the Fast Fourier Transform (FFT):

A new fast non-recursive smoothing algorithm for a linear, discrete, time-invariant state variable system has been developed. It is shown that smoothing problem can be solved by an FFT algorithm without solving the Riccati matrix equation. Moreover, the Riccati Matrix at several instances of time can also be evaluated via the FFT approach. Three methods, including a doubling algorithm, have been developed. Finally,

a block recursive filtering algorithm, where data is received and smoothed recursively block by block is shown to be achieved by combining the above techniques. This algorithm is suitable for real-time, batch processing applications such as image processing and array processing of signals. Details are given in Refs. A-9, C-9, C-11, C-14, D-4.

Several of the results described above have been received favorably and have resulted in several invited papers and invited conference presentations and invited conference session chairmanships as indicated below.

Invited Journal Papers: A-5, A-6

Invited Conference Papers: C-1, C-2, C-6, C-8, C-12, C-14, C-15, C-16,
C-17

5. SUMMARY

In summary, our research efforts have resulted in

- i) 9 Journal Articles of which 8 have been published to date
- ii) 18 conference papers and presentations
- iii) 4 Technical Reports
- iv) 4 Ph.D's (2 completed and 2 in progress)
- v) 2 M.S. thesis (one completed and one in progress)
- vi) One IEEE Award; Donal G. Fink Prize to A.K. Jain (to be awarded in 1983).

Copies of papers, reports and other documents have been submitted to ARO earlier and are, therefore, not included in this report.

END