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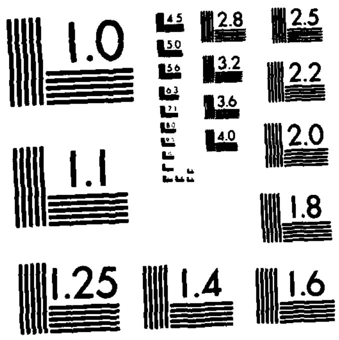


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# ADMIRALTY MARINE TECHNOLOGY ESTABLISHMENT

## FREE-WAVE PROPAGATION IN AND SOUND RADIATION BY LAYERED MEDIA WITH FLOW

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DECEMBER 1982

AMTE(N) TM82102

# FREE-WAVE PROPAGATION IN AND SOUND RADIATION BY LAYERED MEDIA WITH FLOW

BY

W J SPICER


## Summary

The layered system is composed of elastic, viscous and acoustic layers bounded by upper and lower acoustic half-spaces. Uniform subsonic flows may be present in the acoustic layers and half-spaces. The method of dynamic stiffness coupling is used to obtain the system dynamic stiffness matrix which connects 'spectral' displacements and external stresses at the interfaces. Far-field sound radiation and the wavenumbers of free-waves are found from this matrix relation. Numerical results include wavenumber-frequency plots which exhibit the characteristics of instabilities.


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26 pages
6 figures

# LIST OF SYMBOLS

| | |
|---|---|
| $(x, y, z)$ | Cartesian coordinates |
| $(R, \theta, \varphi)$ | spherical coordinates |
| $u_x(x, y, z)$ etc. | displacements |
| $\bar{u}_x(\alpha, \beta, z)$ etc. | transforms of displacements |
| $\alpha, \beta$ | Fourier transform wavenumbers |
| $\lambda, \mu$ | Lamé constants |
| $\rho$ | density |
| $p$ | pressure in fluid |
| $w$ | radian frequency, $=2\pi f$ |
| $c$ | sound velocity in fluid |
| $k$ | acoustic wavenumber, $=2\pi f/c$ |
| $k_L$ | wavenumber of longitudinal waves in elastic solid, $=w/c_L=w\surd[(\lambda+2\mu)/\rho]$ |
| $k_T$ | wavenumber of shear waves in elastic solid, $=w/c_T=w/\surd(\mu/\rho)$ |
| $\gamma$ | $=\surd(k^2[1-\alpha M/k]^2-\alpha^2-\beta^2)$, $\text{Im}(\gamma)\geqslant 0$ |
| $\gamma_L$ | $=\surd(k_L{}^2-\alpha^2-\beta^2)$, $\text{Im}(\gamma_L)\geqslant 0$ |
| $\gamma_T$ | $=\surd(k_T{}^2-\alpha^2-\beta^2)$, $\text{Im}(\gamma_T)\geqslant 0$ |
| $V$ | flow velocity in fluid |
| $M$ | Mach number, $=V/c$ |
| $h$ | thickness of layer |
| $R_0$ | $=\surd[(x-x_0)^2+(y-y_0)^2+(z-z_0)^2]$ |
| $\delta$ | Dirac delta function |

## INTRODUCTION

Pestell and James [1] have described the procedure necessary to calculate the far-field sound radiation of layered media composed of solids and fluids excited by time-harmonic point forces. They use the Lamé decomposition of a vector, viz.

$$\underline{u} = \text{grad}(\varphi)+\text{curl}(\underline{\psi}) \tag{1.1}$$

together with the gauge condition

$$\text{div}(\underline{\psi}) = 0 \tag{1.2}$$

to facilitate solution of the differential equations satisfied by the displacements of the elastic and viscous-fluid layers. The method of dynamic stiffness coupling is then used to assemble a system matrix, from finite element matrices, that relates interface 'spectral' stresses and displacements. The far-field pressure in the upper half-space is simply related to the normal 'spectral' displacement of the uppermost interface. Fortran programs, written in single precision complex arithmetic, have been used successfully to calculate far-field sound radiation and free-wave dispersion plots.

The analysis contained herein is based substantially on the work of Pestell and James, but important differences are present. First, alternative decompositions of the elastic and viscous displacement fields, viz.

$$\underline{u}=\text{grad}(F)+(\partial G/\partial y-\partial^2H/\partial x\partial z,-\partial G/\partial x-\partial^2H/\partial y\partial z,\partial^2H/\partial x^2+\partial^2H/\partial y^2) \tag{1.3}$$

are used which simplify the programming and effectively halve computation times. Secondly, because of a recent interest in the effect of fluid flow on wave propagation in, and sound radiation by, a 'thin' plate [2], it was decided to include uniform mean subsonic flows in the acoustic layers and half-spaces. Finally, because numerical ill-conditioning was expected at the low frequencies of interest to a stability analysis, it was felt necessary to develop Fortran subroutines to simulate double precision complex arithmetic on a PDP-11/34A computer, whose arithmetic word length is 32 bits.

The description of the problem is given in Section 2 and the 'spectral' dynamic stiffness matrices of the elastic and viscous-fluid layers are derived in Sections 3 and 4. The 'spectral' dynamic stiffness matrix of an acoustic layer with a uniform mean subsonic flow is given in Section 5, and the acoustic half-spaces with a mean flow are considered in Section 6. The Fortran programs, particularly the subroutines used to simulate double precision complex arithmetic, are discussed in Section 7. Finally, some numerical results are discussed in Section 8: these results include wavenumber-frequency plots of layered media with flow, which are expected to exhibit the characteristics of instabilities.

## 2. PROBLEM FORMULATION

### a. General

The layered system is composed of an arbitrary number of elastic, viscous and acoustic layers sandwiched between upper and lower acoustic half-spaces.

The elastic, viscous and acoustic media satisfy respectively the exact linearized equations of elasto-dynamics, visco-dynamics and acoustics. Uniform one-dimensional flows may be present in the acoustic layers and half-spaces. The layered system is excited either by time-harmonic stresses applied at the interfaces or by a time-harmonic point source located in the lower half space. The *time factor* exp(-iwt) is omitted from all equations and the geometry is shown in Figure 1.

b. Fourier Transforms

It is convenient to represent the variables as Fourier transforms. For example

$$u_x(x, y, z) = (1/4\pi^2)\iint_{-\infty}^{+\infty} \bar{u}_x(\alpha, \beta, z) \exp(i\alpha x + i\beta y)\, d\alpha d\beta \tag{2.1}$$

where the 'spectral' displacement $\bar{u}_x$ is given by the inverse transform

$$\bar{u}_x(\alpha, \beta, z) = \iint_{-\infty}^{+\infty} u_x(x, y, z) \exp(-i\alpha x - i\beta y)\, dxdy \tag{2.2}$$

The Fourier transforms facilitate solution of the differential equations by reducing them to algebraic equations.

c. Spectral Dynamic Stiffness

The 'spectral' dynamic stiffnesses of the layers and half-spaces are matrices which relate 'spectral' stresses or pressures to 'spectral' displacements at the interfaces. These matrices,

$$\underset{6\times6}{[S(\alpha,\beta)]},\quad \underset{6\times6}{[S_V(\alpha,\beta)]},\quad \underset{2\times2}{[F(\alpha,\beta)]},\quad \underset{1\times1}{[F_U(\alpha,\beta)]} \text{ and } \underset{1\times1}{[F_L(\alpha,\beta)]},$$

for the elastic, viscous and acoustic layers, and upper and lower half-spaces, are obtained in Sections 3-6.

The assembly of the element matrices to form the system dynamic stiffness matrix relation

$$[Z(\alpha, \beta)][U(\alpha, \beta)] = [E(\alpha, \beta)] \tag{2.3}$$

is a standard finite element procedure that reflects continuity of displacement and equilibrium of stresses at the interfaces. Computer implementation is straightforward. The matrix $[U(\alpha, \beta)]$ is a column matrix of interface displacements and $[E(\alpha, \beta)]$ is a column vector of externally applied 'spectral' stresses which are defined to be positive when acting in the positive direction of the coordinate axes. With reference to Figure 2, this sign convention requires changes of sign, before assembly of the elements, to: (i) the last three rows of the elastic and viscous matrices; (ii) the first row of the acoustic layer matrix and (iii) the lower half-space element. An example of an assembled system matrix is shown elsewhere [1].

d. Spectral Excitation

A point force $F_0$ at a layer interface may be represented as an external stress

$$E(x,y) = F_0 \delta(x-x_0)\delta(y-y_0) \tag{2.4}$$

whose 'spectral' form is obtained via equation (2.2) as

$$\bar{E}(\alpha,\beta) = F_0 \exp(-i\alpha x_0 - i\beta y_0) \tag{2.5}$$

For the case of a point source, of free-field pressure $p_0 \exp(ikR_0)/R_0$, located at distance H below the lowest interface, the z-direction 'spectral' excitation in the absence of flow is stated without proof as

$$\bar{E}_{zn}(\alpha,\beta) = 4\pi i p_0 \exp(-i\alpha x_0 - i\beta y_0 + i\gamma |H|) \tag{2.6}$$

e. Free-Wave Propagation

In the absence of external forces, the system of homogeneous equations (2.3) has a non-trivial solution only if det[$Z(\alpha,\beta)$] vanishes. For a given value of w, there will be real and imaginary values of $\alpha$ and $\beta$ at which the determinant vanishes. Real values of $(\alpha,\beta)$ are the wavenumbers at which free waves propagate. Complex values of $(\alpha,\beta)$ describe the evanescent waves whose effect decreases exponentially with distance. Here, the real branches only are of interest. Plots of real wavenumber $\alpha$ versus frequency for a selected value of wavenumber $\beta$ are called wavenumber-frequency or dispersion plots.

f. Far-Field Pressure

The Fourier transform representation enables the far-field radiated sound in the upper half-space to be given explicitly in terms of the 'spectral' normal displacement of the uppermost interface as [2]

$$p(R,\theta,\varphi) = -\rho w^2 [\bar{u}_{z1}(\alpha,\beta,h) \exp(ikRg)/2\pi R][(1-\alpha M/k)^2/q] \tag{2.7}$$

where

$$g=[-M.\sin(\theta)\cos(\varphi)+q]/(1-M^2), \qquad q^2=1-M^2[1-\sin^2(\theta)\cos^2(\varphi)] \tag{2.8}$$

and the stationary phase point is given by

$$\begin{aligned} \alpha &= k[-M+\sin(\theta)\cos(\varphi)/q]/(1-M^2) \\ \beta &= k.\sin(\theta)\sin(\varphi)/q \end{aligned} \tag{2.9}$$

## 3. THE ELASTIC LAYER

### a. General

Figure 2(a) shows a section through an elastic layer with lower boundary z=0 and upper boundary z=h. The surfaces are subject to prescribed normal and tangential 'spectral' stresses

$$[\bar{\tau}(\alpha,\beta)] =$$

$$[\bar{\tau}_{zx}(\alpha,\beta,h), \bar{\tau}_{zy}(\alpha,\beta,h), \bar{\tau}_{zz}(\alpha,\beta,h), \bar{\tau}_{zx}(\alpha,\beta,0), \bar{\tau}_{zy}(\alpha,\beta,0), \bar{\tau}_{zz}(\alpha,\beta,0)]^T \tag{3.1}$$

which produce 'spectral' surface displacements

$$[\bar{u}(\alpha,\beta)] =$$

$$[\bar{u}_x(\alpha,\beta,h), \bar{u}_y(\alpha,\beta,h), \bar{u}_z(\alpha,\beta,h), \bar{u}_x(\alpha,\beta,0), \bar{u}_y(\alpha,\beta,0), \bar{u}_z(\alpha,\beta,0)]^T \tag{3.2}$$

A matrix $[S(\alpha,\beta)]$ is required, which relates the surface 'spectral' stresses and displacements by the equation

$$\underset{6\times 6}{[S(\alpha,\beta)]}\underset{6\times 1}{[\bar{u}(\alpha,\beta)]} = \underset{6\times 1}{[\bar{\tau}(\alpha,\beta)]} \tag{3.3}$$

### b. Equations of Motion

The linear elastic equations of motion [4]

$$(\lambda+\mu)\,\mathrm{grad}[\mathrm{div}(\underline{u})]+\mu\nabla^2\underline{u} = \rho\partial^2\underline{u}/\partial t^2 \tag{3.4}$$

may be reduced to three wave equations

$$\nabla^2 F + k_L{}^2 F = 0 \qquad \nabla^2 G + k_T{}^2 G = 0 \qquad \nabla^2 H + k_T{}^2 H = 0 \tag{3.5}$$

by means of the substitutions

$$\begin{aligned} u_x &= \partial F/\partial x + \partial G/\partial y - \partial^2 H/\partial x \partial z \\ u_y &= \partial F/\partial y - \partial G/\partial x - \partial^2 H/\partial y \partial z \\ u_z &= \partial F/\partial z + \partial^2 H/\partial x^2 + \partial^2 H/\partial y^2 \end{aligned} \tag{3.6}$$

Substituting the Fourier transform representations of F, G and H into equations (3.5) and integrating the resulting equations gives

$$\begin{aligned} \bar{F}(\alpha,\beta,z) &= A_1(\alpha,\beta)\exp(i\gamma_L z) + A_2(\alpha,\beta)\exp(-i\gamma_L z) \\ \bar{G}(\alpha,\beta,z) &= A_3(\alpha,\beta)\exp(i\gamma_T z) + A_4(\alpha,\beta)\exp(-i\gamma_T z) \\ \bar{H}(\alpha,\beta,z) &= A_5(\alpha,\beta)\exp(i\gamma_T z) + A_6(\alpha,\beta)\exp(-i\gamma_T z) \end{aligned} \tag{3.7}$$

where $A_1$ to $A_6$ are the unknown constants of integration.

c. <u>Spectral Displacements</u>

The 'spectral' displacements are obtained by substituting the Fourier transform representations of F, G and H into equations (3.6). They are

$$\begin{aligned} \bar{u}_x(\alpha,\beta,z) &= i\alpha\bar{F} + i\beta\bar{G} - i\alpha\partial\bar{H}/\partial z \\ \bar{u}_y(\alpha,\beta,z) &= i\beta\bar{F} - i\alpha\bar{G} - i\beta\partial\bar{H}/\partial z \\ \bar{u}_z(\alpha,\beta,z) &= \partial\bar{F}/\partial z - \alpha^2\bar{H} - \beta^2\bar{H} \end{aligned} \tag{3.8}$$

$\bar{F}$, $\bar{G}$ and $\bar{H}$ may be eliminated from equations (3.8) by the use of equations (3.7) to give

$$\begin{aligned} \bar{u}_x = \; & i\alpha A_1(\alpha,\beta)\exp(i\gamma_L z) + i\alpha A_2(\alpha,\beta)\exp(-i\gamma_L z) \\ & + i\beta A_3(\alpha,\beta)\exp(i\gamma_T z) + i\beta A_4(\alpha,\beta)\exp(-i\gamma_T z) \\ & + \alpha\gamma_T A_5(\alpha,\beta)\exp(i\gamma_T z) - \alpha\gamma_T A_6(\alpha,\beta)\exp(-i\gamma_T z) \end{aligned}$$

$$\bar{u}_y = i\beta A_1(\alpha,\beta)\exp(i\gamma_L z) + i\beta A_2(\alpha,\beta)\exp(-i\gamma_L z)$$
$$- i\alpha A_3(\alpha,\beta)\exp(i\gamma_T z) - i\alpha A_4(\alpha,\beta)\exp(-i\gamma_T z) \qquad (3.9)$$
$$+\beta\gamma_T A_5(\alpha,\beta)\exp(i\gamma_T z) - \beta\gamma_T A_6(\alpha,\beta)\exp(-i\gamma_T z)$$

$$\bar{u}_z = i\gamma_L A_1(\alpha,\beta)\exp(i\gamma_L z) - i\gamma_L A_2(\alpha,\beta)\exp(-i\gamma_L z)$$
$$- \alpha^2 A_5(\alpha,\beta)\exp(i\gamma_T z) - \alpha^2 A_6(\alpha,\beta)\exp(-i\gamma_T z)$$
$$- \beta^2 A_5(\alpha,\beta)\exp(i\gamma_T z) - \beta^2 A_6(\alpha,\beta)\exp(-i\gamma_T z)$$

d. Spectral Stresses

It is straightforward to show that the stresses

$$\tau_{zx} = \mu(\partial u_z/\partial x + \partial u_x/\partial z)$$
$$\tau_{zy} = \mu(\partial u_z/\partial y + \partial u_y/\partial z) \qquad (3.10)$$
$$\tau_{zz} = \lambda \operatorname{div}(\underline{u}) + 2\mu\partial u_z/\partial z$$

have the following 'spectral' representations in terms of the unknowns $A_1$ to $A_6$,

$$\bar{\tau}_{zx} = -2\mu\alpha\gamma_L A_1\exp(i\gamma_L z) + 2\mu\alpha\gamma_L A_2\exp(-i\gamma_L z)$$
$$- \mu\beta\gamma_T A_3\exp(i\gamma_T z) + \mu\beta\gamma_T A_4\exp(-i\gamma_T z)$$
$$+i\mu\alpha(\gamma_T^2-\alpha^2-\beta^2)A_5\exp(i\gamma_T z) + i\mu\alpha(\gamma_T^2-\alpha^2-\beta^2)A_6\exp(-i\gamma_T z)$$

$$\bar{\tau}_{zy} = -2\mu\beta\gamma_L A_1\exp(i\gamma_L z) + 2\mu\beta\gamma_L A_2\exp(-i\gamma_L z)$$
$$+ \mu\alpha\gamma_T A_3\exp(i\gamma_T z) - \mu\alpha\gamma_T A_4\exp(-i\gamma_T z) \qquad (3.11)$$
$$+i\mu\beta(\gamma_T^2-\alpha^2-\beta^2)A_5\exp(i\gamma_T z) + i\mu\beta(\gamma_T^2-\alpha^2-\beta^2)A_6\exp(-i\gamma_T z)$$

$$\bar{\tau}_{zz} = -(\lambda k_L^2+2\mu\gamma_L^2)A_1\exp(i\gamma_L z) - (\lambda k_L^2+2\mu\gamma_L^2)A_2\exp(-i\gamma_L z)$$
$$-2i\mu\gamma_T(\alpha^2+\beta^2)A_5\exp(i\gamma_T z) + 2i\mu\gamma_T(\alpha^2+\beta^2)A_6\exp(-i\gamma_T z)$$

e. Matrix Equation

Equations (3.9) and (3.11), evaluated at $z=h$ and $z=0$, give the matrix equations

$$\underset{6x6}{[R(\alpha,\beta)]}\underset{6x1}{[A(\alpha,\beta)]} = \underset{6x1}{[\bar{u}(\alpha,\beta)]}$$

$$\underset{6x6}{[P(\alpha,\beta)]}\underset{6x1}{[A(\alpha,\beta)]} = \underset{6x1}{[\bar{\tau}(\alpha,\beta)]} \qquad (3.12)$$

from which $[A(\alpha,\beta)]$ may be eliminated to give the required relation between surface 'spectral' stresses and displacements, viz.

$$\underset{6x6}{[P(\alpha,\beta)]}\underset{6x6}{[R(\alpha,\beta)]^{-1}}\underset{6x1}{[\bar{u}(\alpha,\beta)]} = \underset{6x1}{[\bar{\tau}(\alpha,\beta)]} \qquad (3.13)$$

The elements of the matrices $[P(\alpha,\beta)]$ and $[R(\alpha,\beta)]$ are listed in Appendix A.

f. Special Case

In the special case $\alpha=\beta=0$, the matrix $[R(\alpha,\beta)]$ is singular, so the matrix inversion is not possible. However, it is not difficult to show that the required 'spectral' stress-displacement relation is

$$\frac{w^2\rho/k_L}{\sin(k_L h)}\begin{bmatrix}\cos(k_L h) & -1\\ 1 & -\cos(k_L h)\end{bmatrix}\begin{bmatrix}\bar{u}_z(0,0,h)\\ \bar{u}_z(0,0,0)\end{bmatrix} = \begin{bmatrix}\bar{\tau}_{zz}(0,0,h)\\ \bar{\tau}_{zz}(0,0,0)\end{bmatrix} \qquad (3.14)$$

which as expected is independent of the shear wave speed, $c_T$. The 'spectral' displacements $\bar{u}_x$, $\bar{u}_y$ are identically zero.

4. THE VISCOUS FLUID LAYER

a. General

Figure 2(a) shows a section through a viscous fluid layer with lower boundary $z=0$ and upper boundar ' $z=h$ The surfaces are subject to prescribed normal and tangential 'spectral' stresses $[\bar{\tau}(\alpha,\beta)]$ which produce 'spectral' surface displacements $[\bar{u}(\alpha,\beta)]$. A matrix $[S_V(\alpha,\beta)]$ is required which relates the surface 'spectral' stresses and displacements by the equation

$$\underset{6\times6}{[S_V(\alpha,\beta)]}\underset{6\times1}{[\bar{u}(\alpha,\beta)]} = \underset{6\times1}{[\bar{\tau}(\alpha,\beta)]} \qquad (4.1)$$

b. Equations

The linearized Navier-Stokes equations of a viscous fluid [4]

$$\begin{aligned} &-\mathrm{grad}(p)+\mu\nabla^2\underline{\dot{u}}+(\mu/3)\,\mathrm{grad}[\mathrm{div}(\underline{\dot{u}})] = \rho\partial\underline{\dot{u}}/\partial t \\ &\partial\rho/\partial t+\rho\,\mathrm{div}(\underline{\dot{u}}) = 0 \\ &\partial p/\partial t = c^2\partial\rho/\partial t \end{aligned} \qquad (4.2)$$

may be reduced to the equations

$$\begin{aligned} &\nabla^2 F+[w^2/(c^2-4iw\mu/3\rho)]F = 0 \\ &\nabla^2 G+(iw\rho/\mu)G = 0 \\ &\nabla^2 H+(iw\rho/\mu)H = 0 \\ &p = iw\rho F+(4\mu/3)\nabla^2 F \end{aligned} \qquad (4.3)$$

by means of the substitutions

$$\begin{aligned} \dot{u}_x &= \partial F/\partial x+\partial G/\partial y-\partial^2 H/\partial x\partial z \\ \dot{u}_y &= \partial F/\partial y-\partial G/\partial x-\partial^2 H/\partial y\partial z \\ \dot{u}_z &= \partial F/\partial z+\partial^2 H/\partial x^2+\partial^2 H/\partial y^2 \end{aligned} \qquad (4.4)$$

Equations (4.3) and (4.4) together with the stress-velocity relations

$$\begin{aligned} \tau_{zx} &= \mu(\partial\dot{u}_z/\partial x+\partial\dot{u}_x/\partial z) \\ \tau_{zy} &= \mu(\partial\dot{u}_z/\partial y+\partial\dot{u}_y/\partial z) \\ \tau_{zz} &= -p-(2\mu/3)\,\mathrm{div}(\underline{\dot{u}})+2\mu\partial\dot{u}_z/\partial z \end{aligned} \qquad (4.5)$$

enable the analysis to proceed as in Section 3, to give the matrix relation

$$[P(\alpha,\beta)][A(\alpha,\beta)] = [\bar{\tau}(\alpha,\beta)]$$
$$\quad 6x6 \qquad 6x1 \qquad\qquad 6x1$$

(4.6)

$$[R(\alpha,\beta)][A(\alpha,\beta)] = [\bar{\dot{u}}(\alpha,\beta)]$$
$$\quad 6x6 \qquad 6x1 \qquad\qquad 6x1$$

from which $[A(\alpha,\beta)]$ may be eliminated to give the required relation between 'spectral' stresses and displacements, viz.

$$-iw[P(\alpha,\beta)][R(\alpha,\beta)]^{-1}[\bar{u}(\alpha,\beta)] = [\bar{\tau}(\alpha,\beta)] \qquad (4.7)$$
$$\quad 6x6 \qquad 6x6 \qquad 6x1 \qquad\qquad 6x1$$

The elements of $[P(\alpha,\beta)]$ and $[R(\alpha,\beta)]$ are the same as for the elastic layer, except that

$$\gamma_L{}^2 \equiv [w^2/(c^2-4iw\rho/3\mu)]-\alpha^2-\beta^2$$

$$\gamma_T{}^2 \equiv (iw\rho/\mu)-\alpha^2-\beta^2$$

$$\lambda \equiv (i\rho c^2/w)-2\mu/3 \qquad (4.8)$$

$$\mu \equiv \mu$$

The special case of the singular matrix $\alpha=\beta=0$ is also treated in the same way as in Section 3, except that $k_L{}^2 = w^2/(c^2-4iw\mu/3\rho)$.

## 5. THE ACOUSTIC FLUID LAYER

### a. General

Figure 2(b) shows a section through a fluid layer with lower boundary $z=0$ and upper boundary $z=h$. The fluid flows with constant subsonic velocity V, parallel to the x-axis. The surfaces are subject to prescribed normal 'spectral' pressures

$$[\bar{p}(\alpha,\beta)] = [\bar{p}(\alpha,\beta,h), \bar{p}(\alpha,\beta,0)]^T \qquad (5.1)$$

which produce, normal to the surface, the 'spectral' displacements

$$[\bar{u}_z(\alpha,\beta)] = [\bar{u}_z(\alpha,\beta,h), \bar{u}_z(\alpha,\beta,0)]^T \qquad (5.2)$$

A matrix $[F(\alpha,\beta)]$ is required, which relates the 'spectral' pressures and displacements by the equation

$$\underset{2x2}{[F(\alpha,\beta)]}\underset{2x1}{[\bar{u}_z(\alpha,\beta)]} = \underset{2x1}{[\bar{p}(\alpha,\beta)]} \tag{5.3}$$

b. <u>Equations of Motion</u>

The linearized convected form of the wave equation

$$\nabla^2\varphi = (1/c^2)[\partial/\partial t+V\partial/\partial x]^2\varphi \tag{5.4}$$

may be put into its 'spectral' form by substituting the Fourier transform representation of $\varphi$ into equation (5.4) and then integrating the resulting equation to give

$$\bar{\varphi}(\alpha,\beta,z) = A_1(\alpha,\beta)\exp(i\gamma z)+A_2(\alpha,\beta)\exp(-i\gamma z) \tag{5.5}$$

where

$$\gamma = \sqrt{[k^2(1-\alpha M/k)^2-\alpha^2-\beta^2]}$$

c. <u>Spectral Displacements</u>

The condition of continuity of normal displacement between the fluid and a fixed point on the boundary is

$$\partial\varphi/\partial z = \partial u_z/\partial t+V\partial u_z/\partial x \tag{5.6}$$

from which, using equation (5.5), the displacement can be shown to have the following 'spectral' representation

$$\bar{u}_z(\alpha,\beta,z) = -[\gamma/w(1-\alpha M/k)]A_1(\alpha,\beta)\exp(i\gamma z)$$
$$+[\gamma/w(1-\alpha M/k)]A_2(\alpha,\beta)\exp(-i\gamma z) \tag{5.7}$$

d. <u>Spectral Pressures</u>

It is straightforward to show that the pressure

$$p = -\rho(\partial\varphi/\partial t+V\partial\varphi/\partial x) \tag{5.8}$$

has the following 'spectral' representation in terms of the unknowns $A_1$ and $A_2$.

$$\bar{p}(\alpha,\beta,z) = iw\rho(1-\alpha M/k)A_1(\alpha,\beta)\exp(i\gamma z)$$
$$+iw\rho(1-\alpha M/k)A_2(\alpha,\beta)\exp(-i\gamma z) \tag{5.9}$$

e. Matrix Equation

Equations (5.7) and (5.9), evaluated at $z=h$ and $z=0$, give the matrix equations

$$\underset{2x2}{[R(\alpha,\beta)]}\underset{2x1}{[A(\alpha,\beta)]} = \underset{2x1}{[\bar{u}_z(\alpha,\beta)]}$$

$$\underset{2x2}{[P(\alpha,\beta)]}\underset{2x1}{[A(\alpha,\beta)]} = \underset{2x1}{[\bar{p}(\alpha,\beta)]} \tag{5.10}$$

from which $[A(\alpha,\beta)]$ may be eliminated to give the required relation between surface 'spectral' pressure and normal displacement, viz.

$$\frac{-w^2\rho(1-\alpha M/k)^2}{\gamma\sin(\gamma h)}\begin{bmatrix}\cos(\gamma h) & -1\\ 1 & -\cos(\gamma h)\end{bmatrix}\begin{bmatrix}\bar{u}_z(\alpha,\beta,h)\\ \bar{u}_z(\alpha,\beta,0)\end{bmatrix} = \begin{bmatrix}\bar{p}(\alpha,\beta,h)\\ \bar{p}(\alpha,\beta,0)\end{bmatrix} \tag{5.11}$$

6. THE ACOUSTIC INFINITE HALF-SPACES

a. Upper Half-Space

Figure 2(c) shows a section through the upper acoustic half-space with boundary $z=0$. The fluid flows with constant subsonic velocity $V$, parallel to the x-axis. A relation is required between the prescribed 'spectral' pressure, $\bar{p}(\alpha,\beta,0)$, and the normal displacement, $\bar{u}_z(\alpha,\beta,0)$.

The mathematical analysis is similar to that used in Section 5, except that $A_2$ is identically zero because outgoing waves only are allowed. The boundary condition equation (5.6) and the pressure-potential relation, equation (5.8), give the required relation as

$$[F_U(\alpha,\beta)][\bar{u}_z(\alpha,\beta)] = [\bar{p}(\alpha,\beta)] \quad (6.1)$$
$$\quad 1\times1 \qquad\quad 1\times1 \qquad\quad 1\times1$$

where

$$F_U(\alpha,\beta) = -i\rho w^2(1-\alpha M/k)^2/\gamma$$

b. Lower Half-Space

Figure 2(c) shows a section through the lower acoustic half-space with boundary z=0. The fluid flows with constant subsonic velocity V, parallel to the x-axis. In this case, $A_1$ is identically zero because outgoing waves only are allowed. Thus the required relation is

$$[F_L(\alpha,\beta)][\bar{u}_z(\alpha,\beta)] = [\bar{p}(\alpha,\beta)] \quad (6.2)$$
$$\quad 1\times1 \qquad\quad 1\times1 \qquad\quad 1\times1$$

where

$$F_L(\alpha,\beta) = +i\rho w^2(1-\alpha M/k)^2/\gamma$$

## 7. FORTRAN COMPUTER PROGRAMS

Fortran computer programs have been written for a PDP-11/34A computer running under the RT-11 operating system. The programs calculate the far-field sound radiation due to point-source or point-force excitation, and also the real branches of the wavenumber $\alpha$ versus frequency plots for a selected value of $\beta$. A wavenumber is found by stepping through a range of $\alpha$-values until a sign change occurs in $\det[Z(\alpha,\beta)]$: it is refined to a selected accuracy by repeated interval halving.

Programs have been written, using complex arithmetic, in both single precision (32 bit accuracy) and double precision (64 bit accuracy). The double precision version was necessary because the single precision arithmetic gave numerical problems of ill-conditioning at the low frequencies which were of special interest in the wavenumber versus frequency plots. Because double precision complex arithmetic is not a standard feature of RT-11 Fortran, it was necessary to write a series of subroutines that simulated double precision complex arithmetic by operating on double precision real arrays of leading dimension two. Details of these subroutines together with some equivalent assembler level routines are to be published elsewhere [5].

8. NUMERICAL RESULTS

a. Constants

The material and geometric constants, in SI units, used in the computations are as follows:

steel plate: $\lambda$=10.44E10, $\mu$=7.56E10, $\rho$=7700.0, h=0.01
water: $\rho$=1000.0, c=1500.0

Damping was included in the sound radiation computations by setting $\lambda=\lambda(1-i\eta_\lambda)$ and $\mu=\mu(1-\eta_\mu)$, where the hysteretic loss factors were chosen as 0.02.

b. Wavenumber-Frequency: Single Layer

Figures 3 and 4 are wavenumber versus frequency plots of a steel plate loaded on one side only with water which moves in the positive x-direction with velocities 7.5 and 22.5m/s, respectively. For comparison purposes, wavenumber versus frequency plots without flow are also given. In Figure 3, there are no numerical results below 0.25Hz, due to ill-conditioning despite double precision arithmetic being used for the calculations.

The plots with flow show the characteristics of instabilities, viz., infinite group velocity $(d\alpha/dw)^{-1}$ in Figure 4 and a multivalued wavenumber branch in Figure 3. The plots are in agreement with those obtained by Atkins [2] who has used 'thin-plate' theory in his analysis which shows that the instabilities are of the type convective (growth downstream) and absolute (growth at a point), respectively.

c. Wavenumber-Frequency: Three Layers

Figure 5 contains wavenumber versus frequency plots of a system comprising a 5cm layer of water which is sandwiched between two steel plates. Omitted from the plots is a wavenumber branch that is close to a fluid wave of wavenumber k. The plot with a flow speed of 22.5m/s contains a multivalued wavenumber branch, but further work using a root-locus technique [2] would be necessary to confirm the presence of the instability and to determine its nature.

d. Sound Radiation

Figures 6(a) and 6(b) show the far-field sound radiation due to point force excitation of the lower facing of a system composed of a 5cm layer of water sandwiched between two 1cm steel plates: the upper and lower half-spaces are water and vacuum respectively. The sound levels are expressed in dB reference 1 micropascal at 1m for 1kN rms force.

Figure 6(a) shows the sound level variation with frequency at $\theta=0^\circ$: the first resonant peak is caused by a coupled plate-fluid resonance, while subsequent peaks are close to the frequencies at which 'half-wavelength' transmission is possible by plane waves at 'normal' incidence. Figure 6(b) shows the angular variation of the radiated sound at a frequency of 40kHz: the broad lobes centred at $\theta=62^\circ$ are the well-known coincidence lobes [3] of a 1cm plate; the small peaks at $\theta=17^\circ$ are caused by longitudinal waves in the plates; other peaks are caused by coupled layer/plate resonances corresponding to oblique incidence of plane waves - their physical interpretation via wavenumber-frequency plots is to be discussed elsewhere.

## 9. ACKNOWLEDGEMENTS

Thanks are due to J H James for his helpful supervision of the project and its write-up, and also to E J Clement for his help with the graphical presentations.

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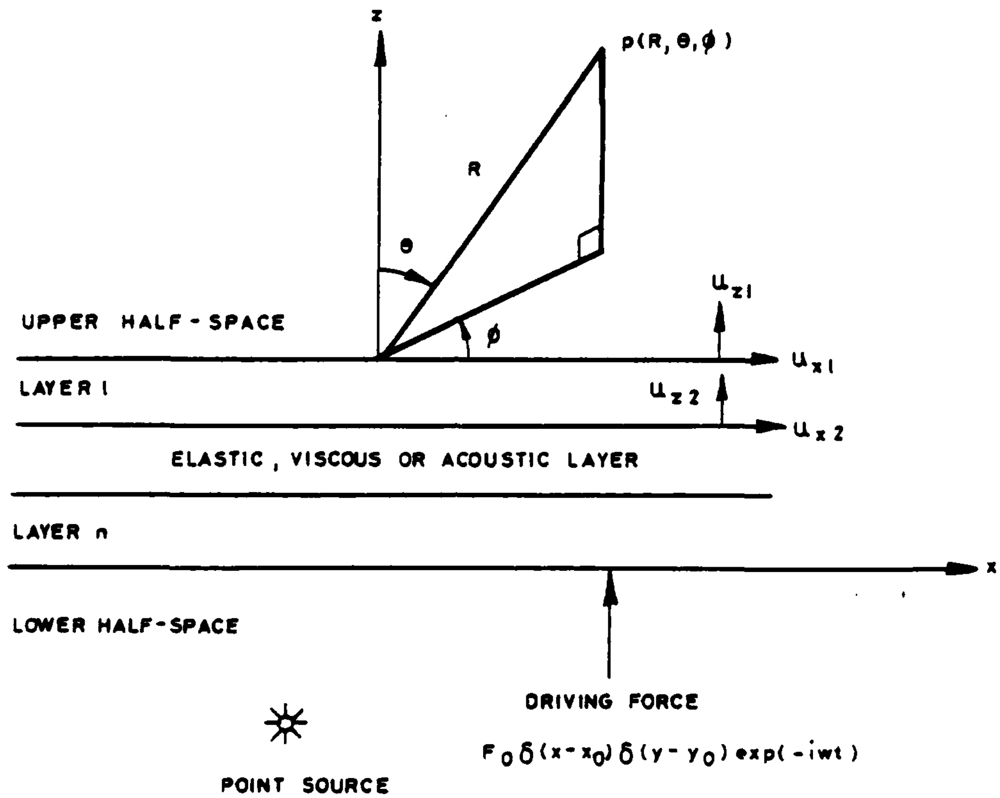


FIG. 1 SOUND RADIATION FROM LAYERED MEDIA

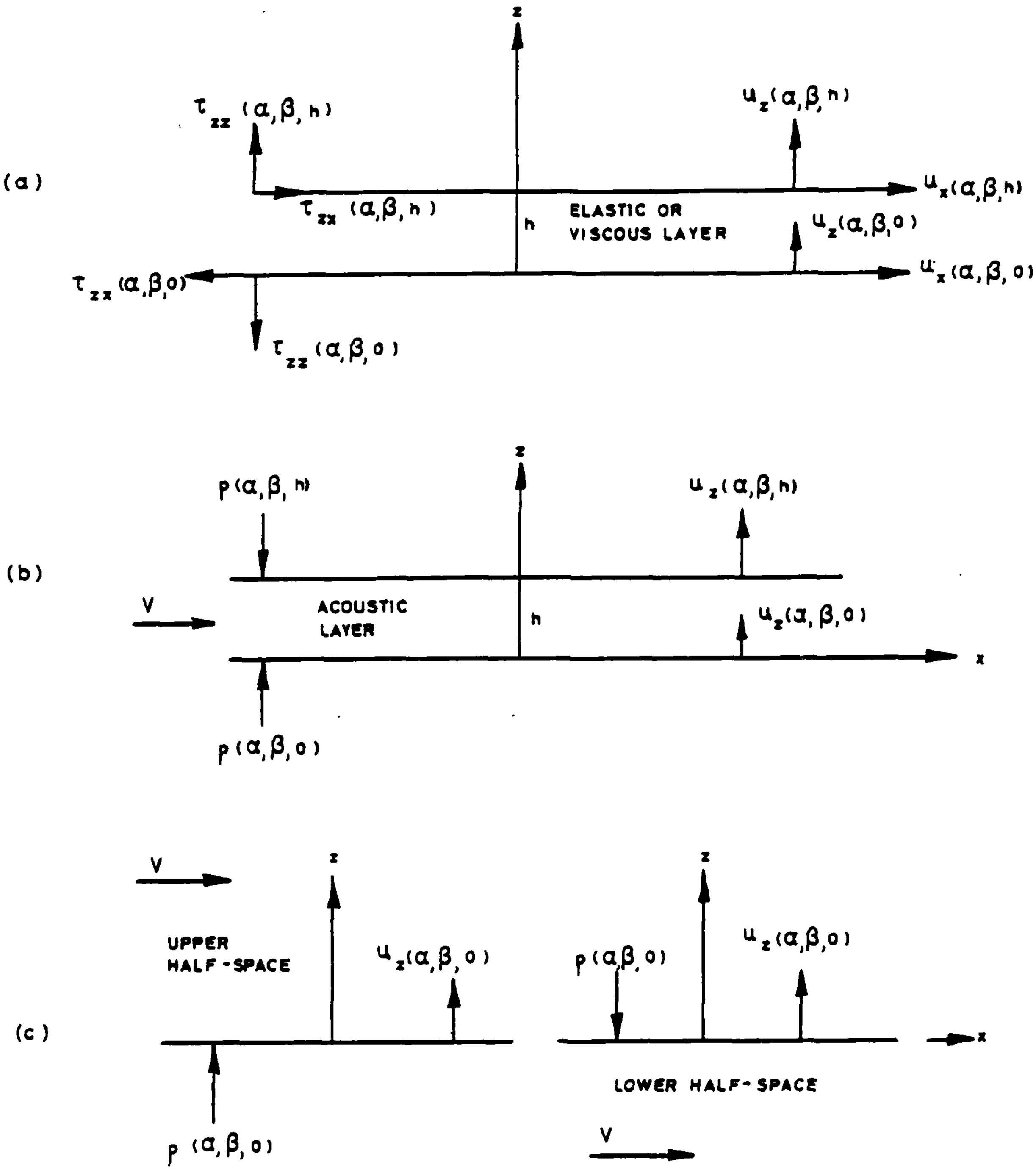


FIG. 2 SPECTRAL STRESSES AND DISPLACEMENTS ON ELEMENTS

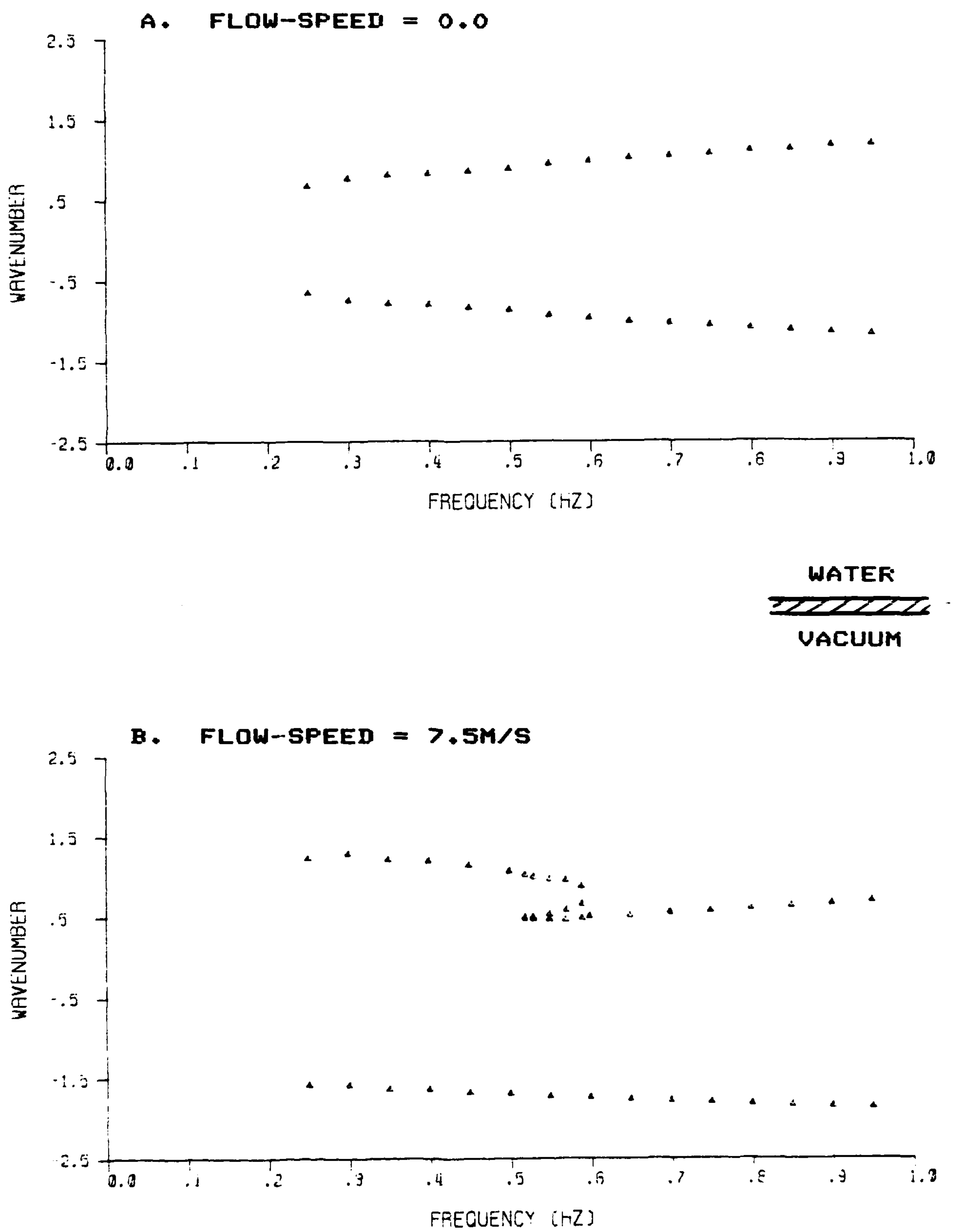


FIG. 3 WAVENUMBER ALPHA VERSUS FREQUENCY. BETA=0.0.
1CM STEEL PLATE. WATER ABOVE. VACUUM BELOW.

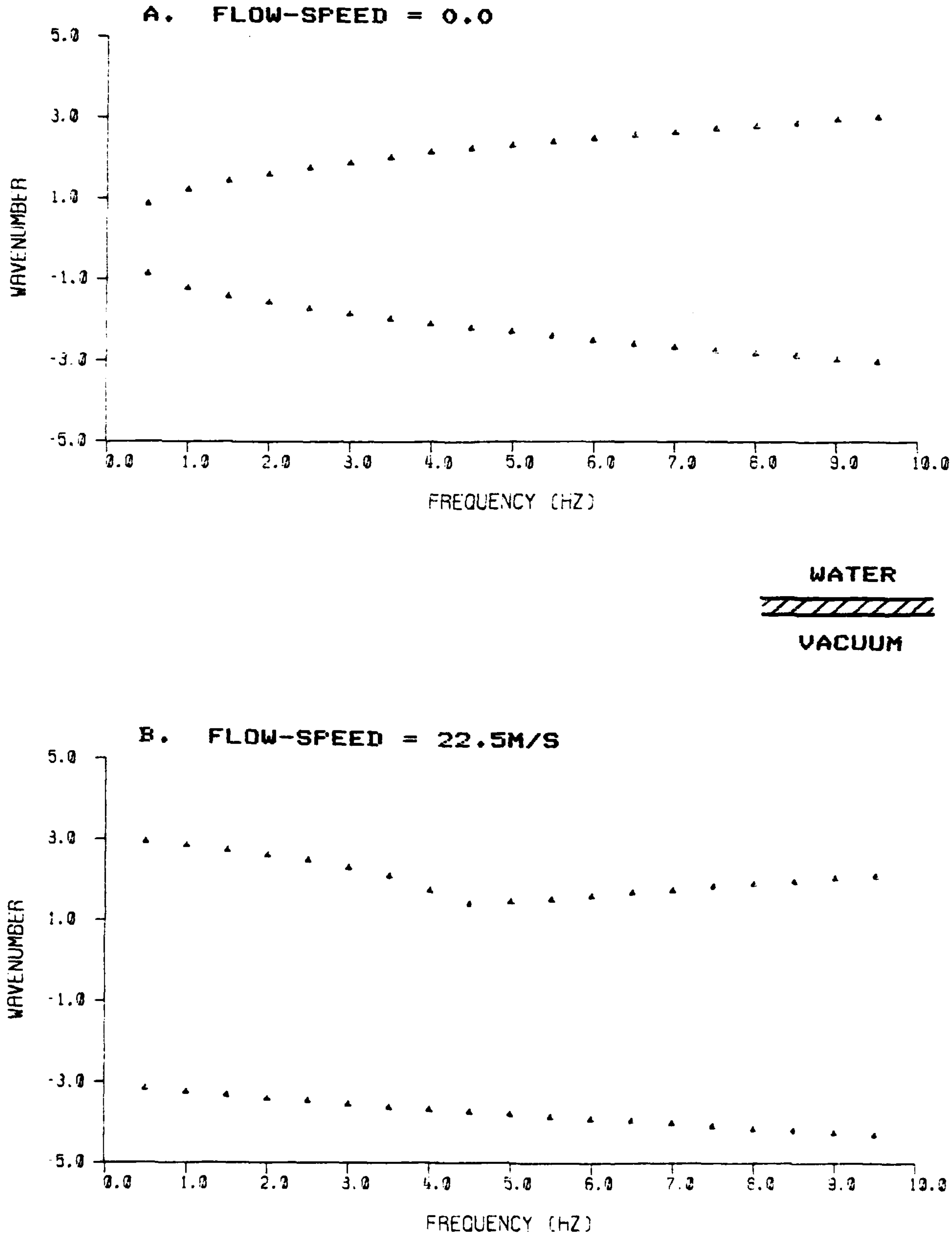


FIG. 4 WAVENUMBER ALPHA VERSUS FREQUENCY. BETA=0.0.
1CM STEEL PLATE. WATER ABOVE. VACUUM BELOW.

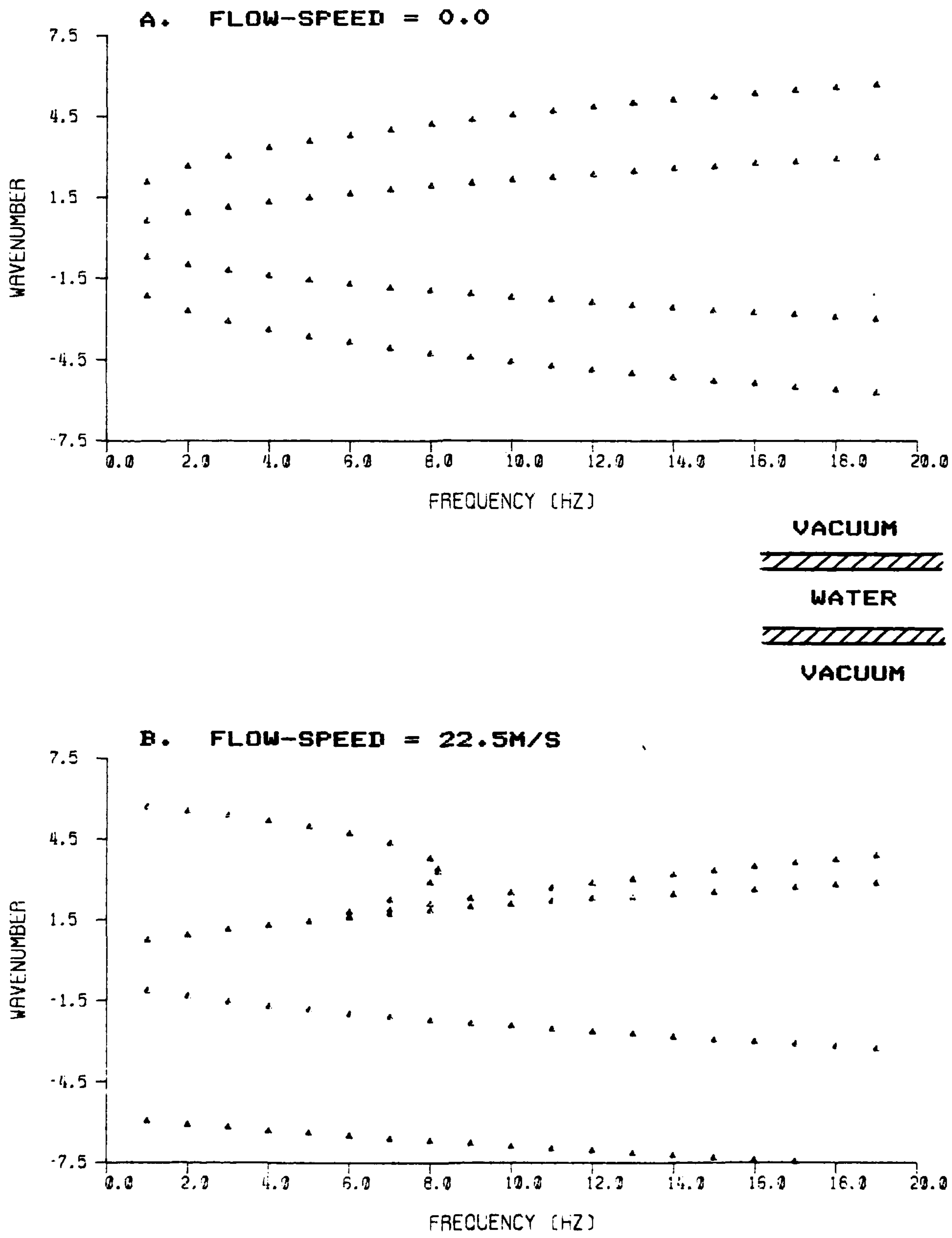


FIG. 5 WAVENUMBER ALPHA VERSUS FREQUENCY. BETA=0.0.
TWO 1CM STEEL PLATES SEPARATED BY 5CM WATER.

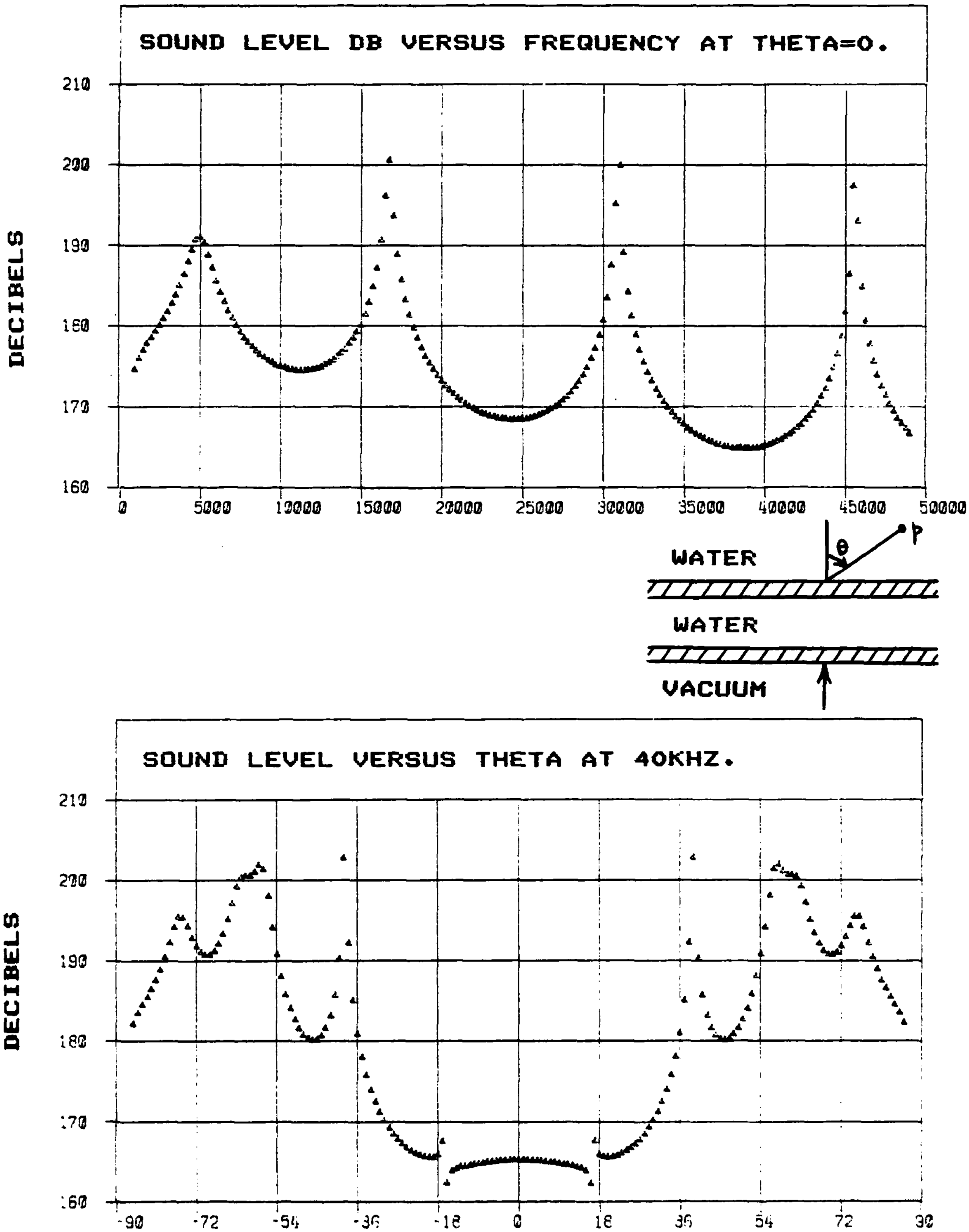


FIG. 6 SOUND RADIATION FROM POINT FORCE EXCITATION.
TWO 1CM STEEL PLATES SEPARATED BY 5CM WATER.

# APPENDIX A

The Elastic Layer Matrices $[P(\alpha,\beta)]_{6x6}$ and $[R(\alpha,\beta)]_{6x6}$

$P_{11} = -2\mu\alpha\gamma_L \exp(i\gamma_L h)$

$P_{12} = 2\mu\alpha\gamma_L \exp(-i\gamma_L h)$

$P_{13} = -\mu\beta\gamma_T \exp(i\gamma_T h)$

$P_{14} = \mu\beta\gamma_T \exp(-i\gamma_T h)$

$P_{15} = i\mu\alpha(\gamma_T{}^2-\alpha^2-\beta^2)\exp(i\gamma_T h)$

$P_{16} = i\mu\alpha(\gamma_T{}^2-\alpha^2-\beta^2)\exp(-i\gamma_T h)$

$P_{21} = -2\mu\beta\gamma_L \exp(i\gamma_L h)$

$P_{22} = 2\mu\beta\gamma_L \exp(-i\gamma_L h)$

$P_{23} = \mu\alpha\gamma_T \exp(i\gamma_T h)$

$P_{24} = -\mu\alpha\gamma_T \exp(-i\gamma_T h)$

$P_{25} = i\mu\beta(\gamma_T{}^2-\alpha^2-\beta^2)\exp(i\gamma_T h)$

$P_{26} = i\mu\beta(\gamma_T{}^2-\alpha^2-\beta^2)\exp(-i\gamma_T h)$

$P_{31} = [-\lambda k_L{}^2-2\mu\gamma_L{}^2]\exp(i\gamma_L h)$

$P_{32} = [-\lambda k_L{}^2-2\mu\gamma_L{}^2]\exp(-i\gamma_L h)$

$P_{33} = 0$

$P_{34} = 0$

$P_{35} = -2i\mu\gamma_T(\alpha^2+\beta^2)\exp(i\gamma_T h)$

$P_{36} = 2i\mu\gamma_T(\alpha^2+\beta^2)\exp(-i\gamma_T h)$

Rows 4, 5 and 6 are obtained by setting $h=0$ in rows 1, 2 and 3.

$R_{11} = i\alpha \exp(i\gamma_L h)$

$R_{12} = i\alpha \exp(-i\gamma_L h)$

$R_{13} = i\beta \exp(i\gamma_T h)$

$R_{14} = i\beta \exp(-i\gamma_T h)$

$R_{15} = \alpha\gamma_T \exp(i\gamma_T h)$

$R_{16} = -\alpha\gamma_T \exp(-i\gamma_T h)$

$R_{21} = i\beta \exp(i\gamma_L h)$

$R_{22} = i\beta \exp(-i\gamma_L h)$

$R_{23} = -i\alpha \exp(i\gamma_T h)$

$R_{24} = -i\alpha \exp(-i\gamma_T h)$

$R_{25} = \beta\gamma_T \exp(i\gamma_T h)$

$R_{26} = -\beta\gamma_T \exp(-i\gamma_T h)$

$R_{31} = i\gamma_L \exp(i\gamma_L h)$

$R_{32} = -i\gamma_L \exp(-i\gamma_L h)$

$R_{33} = 0$

$R_{34} = 0$

$R_{35} = [-\alpha^2-\beta^2]\exp(i\gamma_T h)$

$R_{36} = [-\alpha^2-\beta^2]\exp(-i\gamma_T h)$

Rows 4, 5 and 6 are obtained by setting $h=0$ in rows 1, 2 and 3.