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A SENSITIVITY ANALYSIS OF THE LOGARITHMIC-POISSON-GAMMA
DISTRIBUTION(U) DECISION SYSTEMS BEAVERCREEK OH
W S DEMMY ET AL. DEC 81 WP-81-05 F33600-80-C-0530

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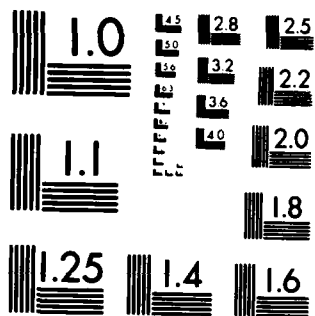
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A Sensitivity Analysis
of
The Logarithmic-Poisson-Gamma Distribution

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) ✓ The Logarithmic-Poisson-Gamma (LPG) distribution describes the probability distribution for demand in a lead time for the situation in which customers arrive according to a Poisson process, requisition sizes are described by the Logarithmic distribution, and lead times are Gamma distributed. This paper presents plots of the probability mass functions for the LPG and for an associated scaled binomial approximation to the LPG.		

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Section I

Introduction

In Reference 1, we develop the probability distribution for demand in a lead time for the situation in which customers arrive according to a Poisson process and each customer requires X units, where X is described by the logarithmic distribution; i.e. the probability mass function of X is

$$f(x) = \frac{\Theta^x}{x \ln \Theta} \quad x = 1, 2, \dots$$

In addition, lead times are assumed to be Gamma Distributed. We call the resulting distribution of demand in a lead time the *Logarithmic-Poisson-Gamma (LPG)* distribution. This distribution appears to be a useful approximation to the demand processes of a number of Air Force EOQ-type (consumable) items.

Analytic details of the LPG distribution are presented in Reference 1. In addition, reference 1 presents a scaled negative binomial approximation to the LPG. We assume the reader is familiar with these results. In this paper, we present a sensitivity analysis of the LPG distribution and of the associated scaled negative binomial approximation.

SECTION II

Sensitivity Analysis

To get a feel for the shape of the LPG distribution, and to evaluate the accuracy of the scaled negative Binomial approximation, we computed the cumulative distribution function values for both of these distributions for 40 different parameter sets. The BASIC computer code used for these calculations is presented in Appendix B, while Figure II-1 illustrates the results of our calculations for one of these data sets. This figure presents the cumulative distribution function (CDF) and probability density function (PDF) for both the LPG and scaled negative binomial distributions for the case in which the average requisition size is 2 units, the average monthly demand rate is .5, the average lead time is 8 months, and the coefficient of variation of lead time is .25. For this situation, the mean, coefficient of variation, skewness, and kurtosis of the distribution of demand in a leadtime equals 4.00, 0.97, 1.46, and 6.40, respectively. These values are shown on the left hand side of Figure II-1. The specific parameters utilized in the LPG distribution calculations are also shown on the left hand side of this figure, as well as specific CDF values. The column labeled "FX" presents cumulative distribution function values for the LPG distribution, while the column labeled "FNBX" presents the corresponding cumulative distribution function values for the scaled negative binomial approximation. Finally, the column labeled "DIFF" presents the difference between the LPG distribution values and the corresponding negative binomial CDF values.

The right hand side of Figure II-1 plots both the cumulative and density function values associated with the LPG and scaled negative binomial distributions. As shown in

DEMAND IN LEAD TIME PARAMETERS

Avg Req Size 2.00
 Avg Monthly Demand 0.50
 Avg Lead Time 2.00
 C of U of LY 0.25

LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C of U of DLT 0.97
 SKEWNESS 1.46
 KURTOSIS 6.40

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 2.00

X	FX	FMAX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.55	0.43	0.13
4.	0.65	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.91	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.99	0.99	-0.00
16.	0.99	0.99	-0.01

Scaled Negative Binomial CDF

LPG CDF

----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE 0.126 FOR X = 3

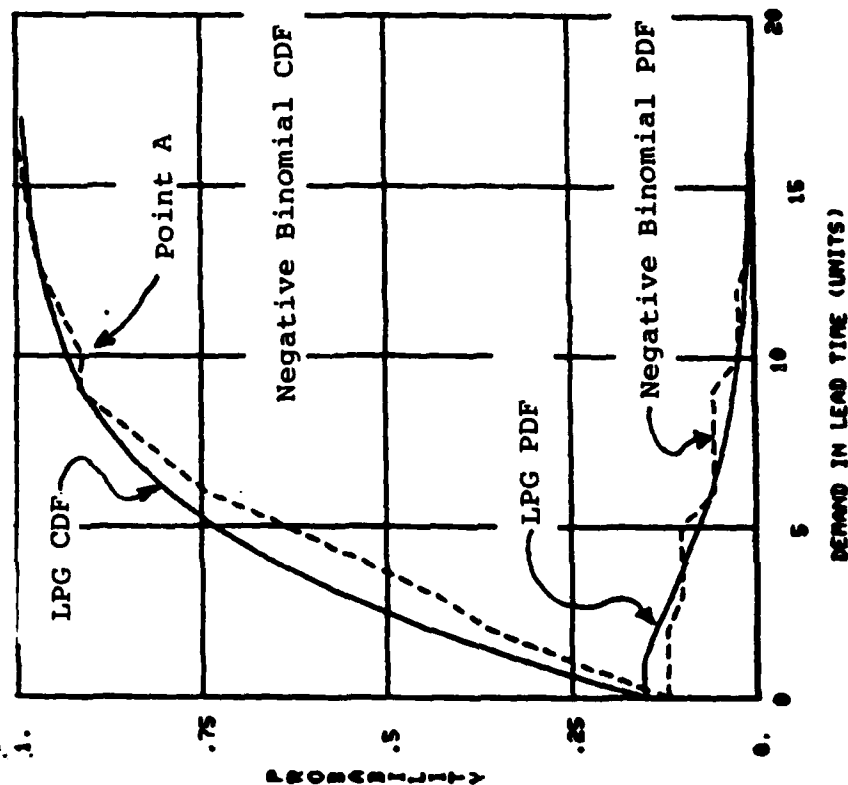


Figure II-1. LPG and Scaled Negative Binomial Distributions for the LO Demand Base Case.

the Figure, the scaled negative binomial approximation underestimates the corresponding LPG CDF values for low levels of demand in the leadtime, but the approximation becomes much better for demand values exceeding 10 units. Note the slight down-turn labeled as Point A. This is caused by the rounding rules employed in the scaled negative binomial code. A slight change in these rules may be used to make the CDF monotonic.

Figure II-2 presents a similar set of calculations in which the monthly demand rate is equal to 2 units per month, and the other parameters are the same as used in Figure II-1. In this case, the scaled negative binomial distribution is a much closer approximation to the LPG curve.

We will refer to the specific data sets plotted in Figures II-1 and II-2 as the "LO" and "HI" base cases, respectively, Table II-1 presents the specific parameter values selected for these calculations. The data set numbers assigned to each of these curves are shown in the right hand side of Table II-1 and detailed plots associated with each of the data sets are presented in Appendix A. Let us now discuss our results.

Results for LO Base Case.

As noted above, the LO Base Case has the following characteristics:

	<u>Value</u>
Average Requisition Size (Units)	2
Average Units Demanded per Month	.5
Average Lead Time (Months)	8
Coefficient of Variation of Lead Time	.25

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
AVE MONTHLY DEMAND 2.00
AVE LEAD TIME 8.69
C OF U OF LT 0.25

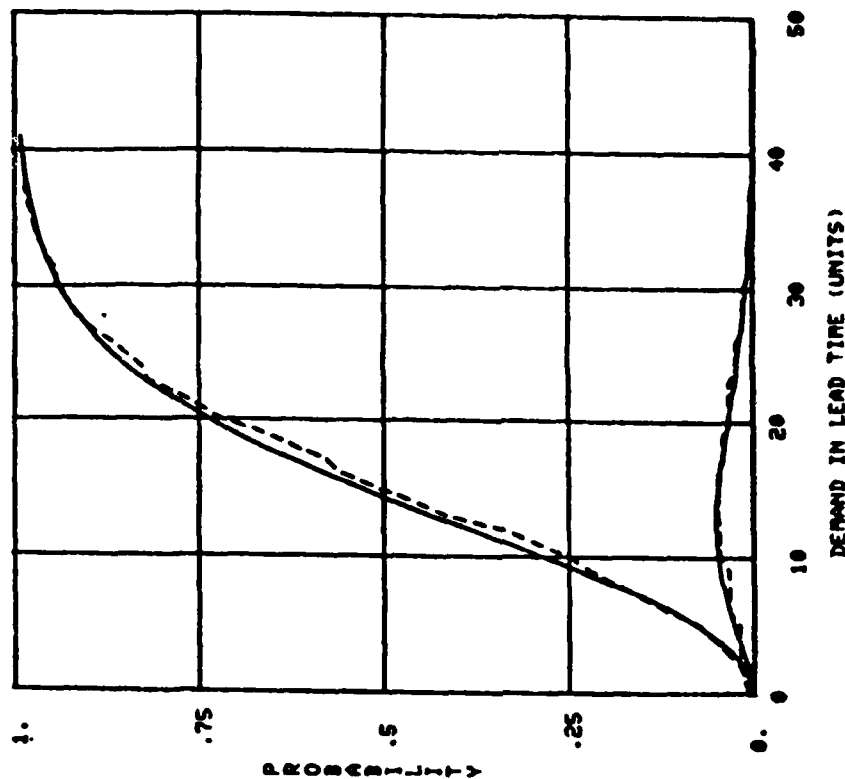
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
C OF U OF DLT 0.53
SKEWNESS 0.60
KURTOSIS 3.64

LPG PARAMETERS

THETA 0.72
LAMBDA 1.00
ALPHA 16.00
BETA 2.00

X	FX	FNDX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	-0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	-0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.03
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.58	0.56	0.02
17.	0.62	0.58	0.05
18.	0.67	0.62	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



----- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE 0.047 FOR X = 17

Figure II-2. LPG and Scaled Negative Binomial Distributions for the HI Demand Base Case.

TABLE II-1

Sensitivity Analysis Parameter Values

<u>Variable</u>	<u>Parameter Settings</u>	<u>Data Set Number</u>	
		<u>LO</u>	<u>HI</u>
Average Requisition size	<u>1</u> .01, <u>2</u> , 4, 8, 16	1-5	21-25
Average Lead Time in Months	4, 6, <u>8</u> , 16	6-10	26-30
Demand Rate in Units per Month	.1, .5, 1., 1.5, 2.0	11-15	31-35
Coefficient of Variation of Leadtime	.01, .25, .50, .75, 1.0	16-20	36-40

Let us now consider the impact of variations in these characteristics.

Requisition Size Sensitivity. In performing our sensitivity analyses, three of the major distribution characteristics defined in Table II-1 were held constant, while the fourth characteristic was varied. For example, data set numbers 1 thru 5 were constructed by holding average leadtime, demand rate, and the coefficient of variation of leadtime equal to the values for the LO demand base case, while varying the average requisition size. Data set numbers 21 thru 25 were performed similarly, but in this case the HI demand base case was used.

Details of our results for all 40 data sets are presented in Appendix A, while Table II-2 and Figures II-3 thru II-7 illustrate our results. Table II-2 summarizes the primary calculations associated with data sets 1 thru 5, while Figures II-3 thru II-7 present the associated plots of the LPG CDF. The table summarizes the sensitivity of the primary LPG distribution parameters (i.e. Theta Lambda, Alpha, and Beta) to changes in average requisition size for the low demand case base. This table displays the requisition size, demand rate, mean leadtime and leadtime variability coefficients that were used to drive the data set calculations, as well as the associated LPG parameters and the moments of the LPG leadtime demand distribution. Finally, at the bottom of Table II-2, we present the specific demand values associated with given percentage points of the LPG distribution and of the associated scaled negative binomial approximation. For example, consider the data shown in the first column at the bottom of Table II-2. For data set number 1, a reorder point of eight units is required to insure a 95% probability of no stockouts during a leadtime. Using the scaled negative binomial approximation for data set 1, a reorder level of 9 units is required to produce the same 95%

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
Ave. Req. Size	1.01	2	4	8	16
Ave. Demand/Mo.	.5	.5	.5	.5	.5
Mean Lead Time	8.0	8.0	8.0	8	8
CV of Lead Time	.25	.25	.25	.25	.25
<u>LPG Parameter</u>					
Theta	.02	.72	.90	.96	.99
Lambda	.49	.25	.12	.06	.03
Alpha	16	16	16	16	16
Beta	2	2	2	2	2
<u>LPG Moments</u>					
Mean	4	4	4	4	4
CV	.56	.97	1.63	2.63	4.15
Skewness	.41	1.46	2.96	5.08	8.18
Kurtosis	3.23	6.40	16.45	42.13	103.79
<u>Percentage Points</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
	<u>LPG</u> <u>NB</u>	<u>LPG</u> <u>NB</u>	<u>LPG</u> <u>NB</u>	<u>LPG</u> <u>NB</u>	<u>LPG</u> <u>NB</u>
.50	4 5	3 4	1 4	0 8	0 18
.60	4 5	4 5	2 5	0 10	0 22
.70	5 6	5 6	4 6	2 11	0 26
.80	6 7	7 7	7 10	4 13	1 29
.85	6 7	8 8	9 12	7 14	3 31
.90	7 8	9 9	12 14	12 22	7 33
.95	8 9	12 12	17 16	22 33	22 43
.97	9 10	13 13	21 22	31 38	38 68
.99	10 11	17 16	31 26	52 42	82 93

Table II-2. Requisition Size Sensitivity for LO Base Case

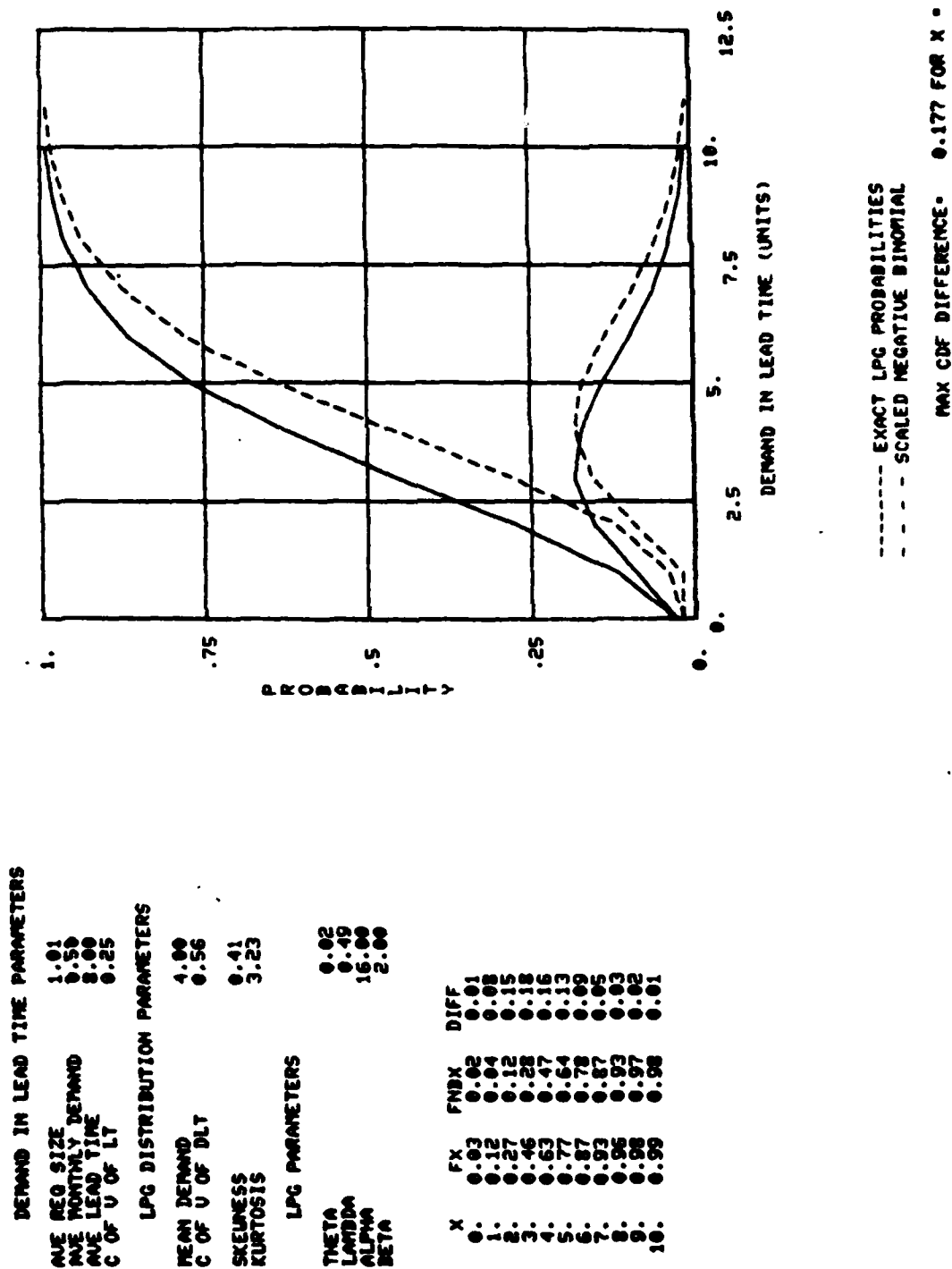


Figure II-3. Data Set No. 1 LPG Curves.

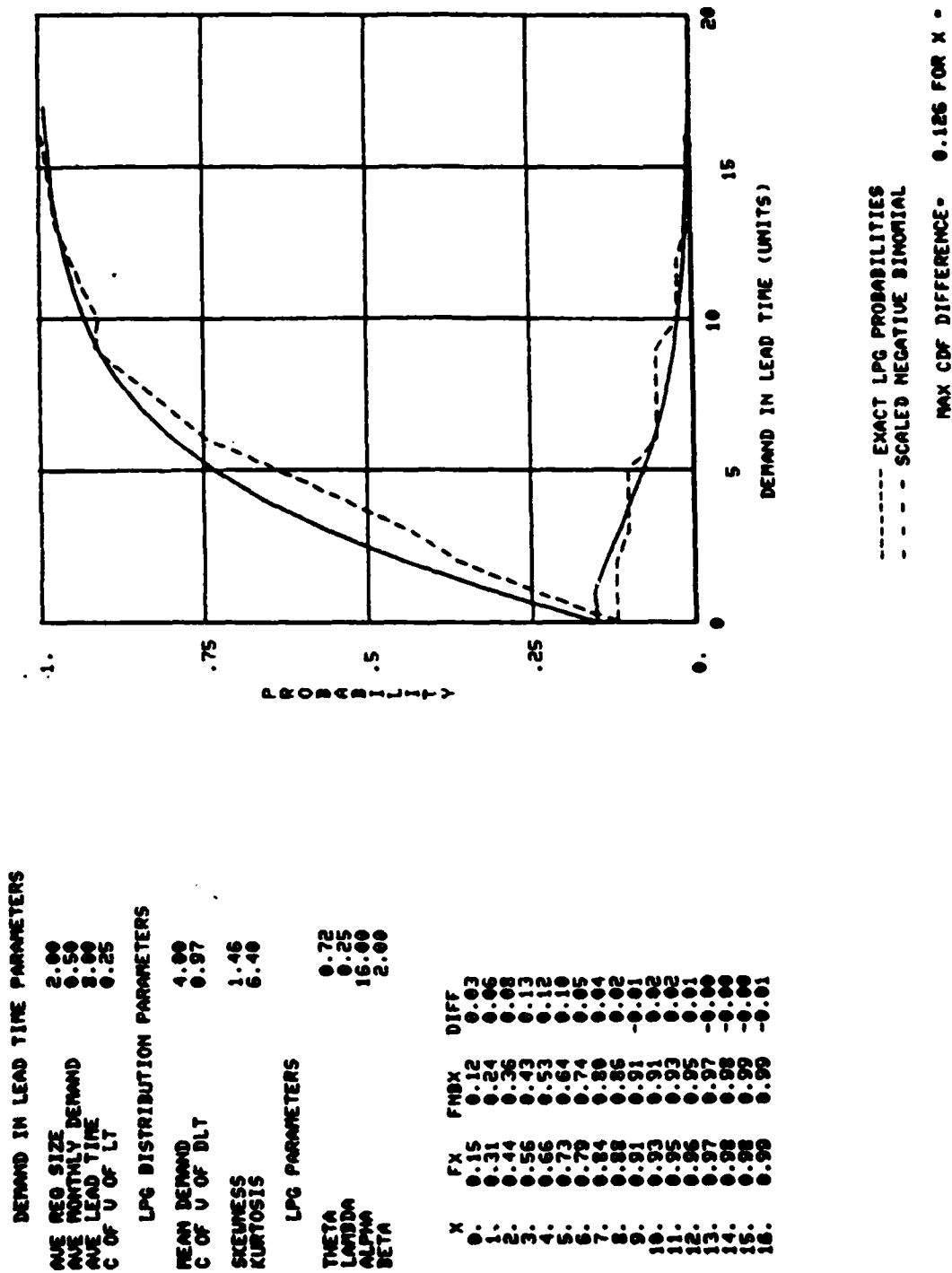


Figure II 4. Data Set No. 2 LPG Curves.

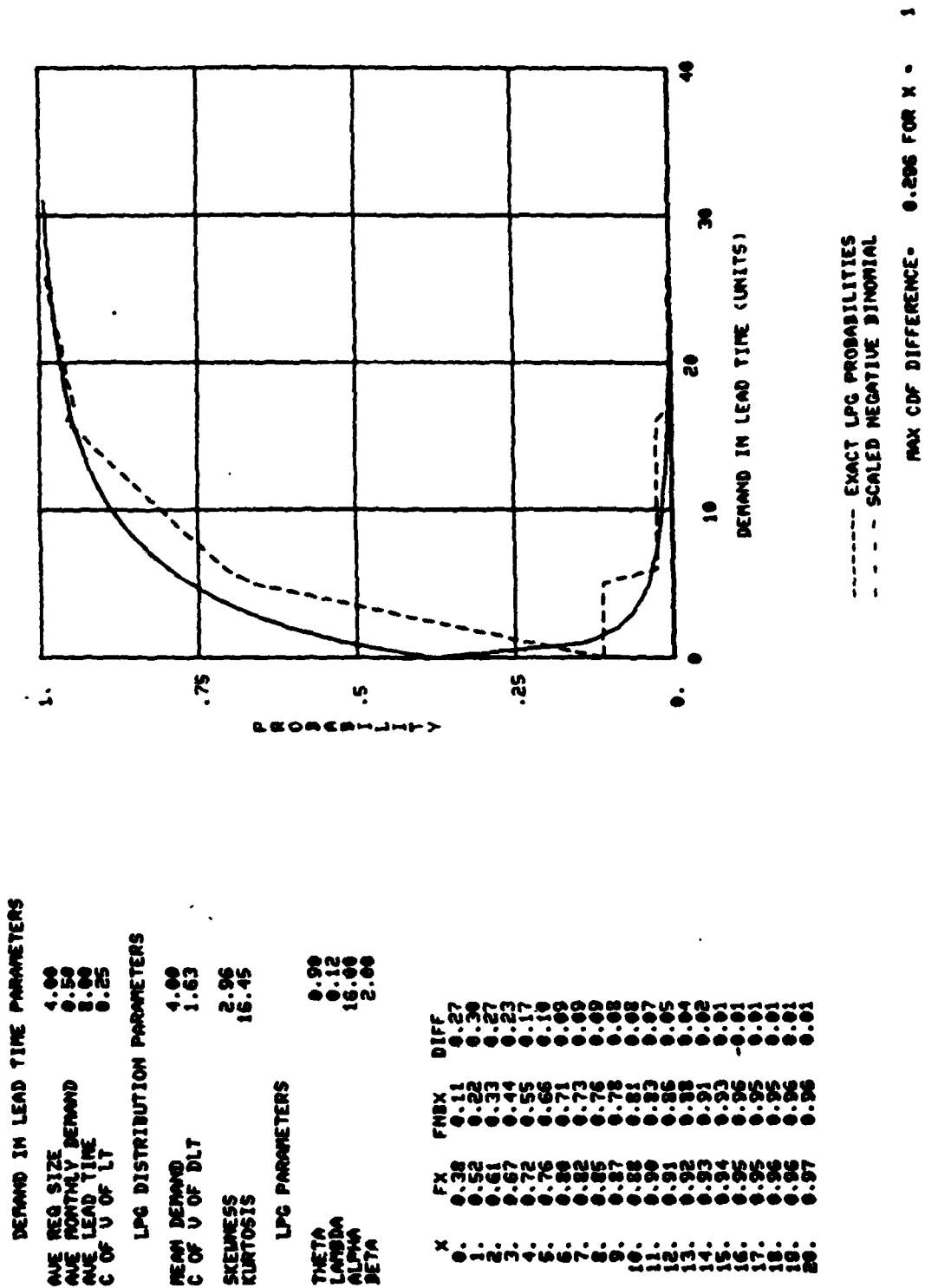


Figure II-5. Data Set No. 3 LPG Curves.

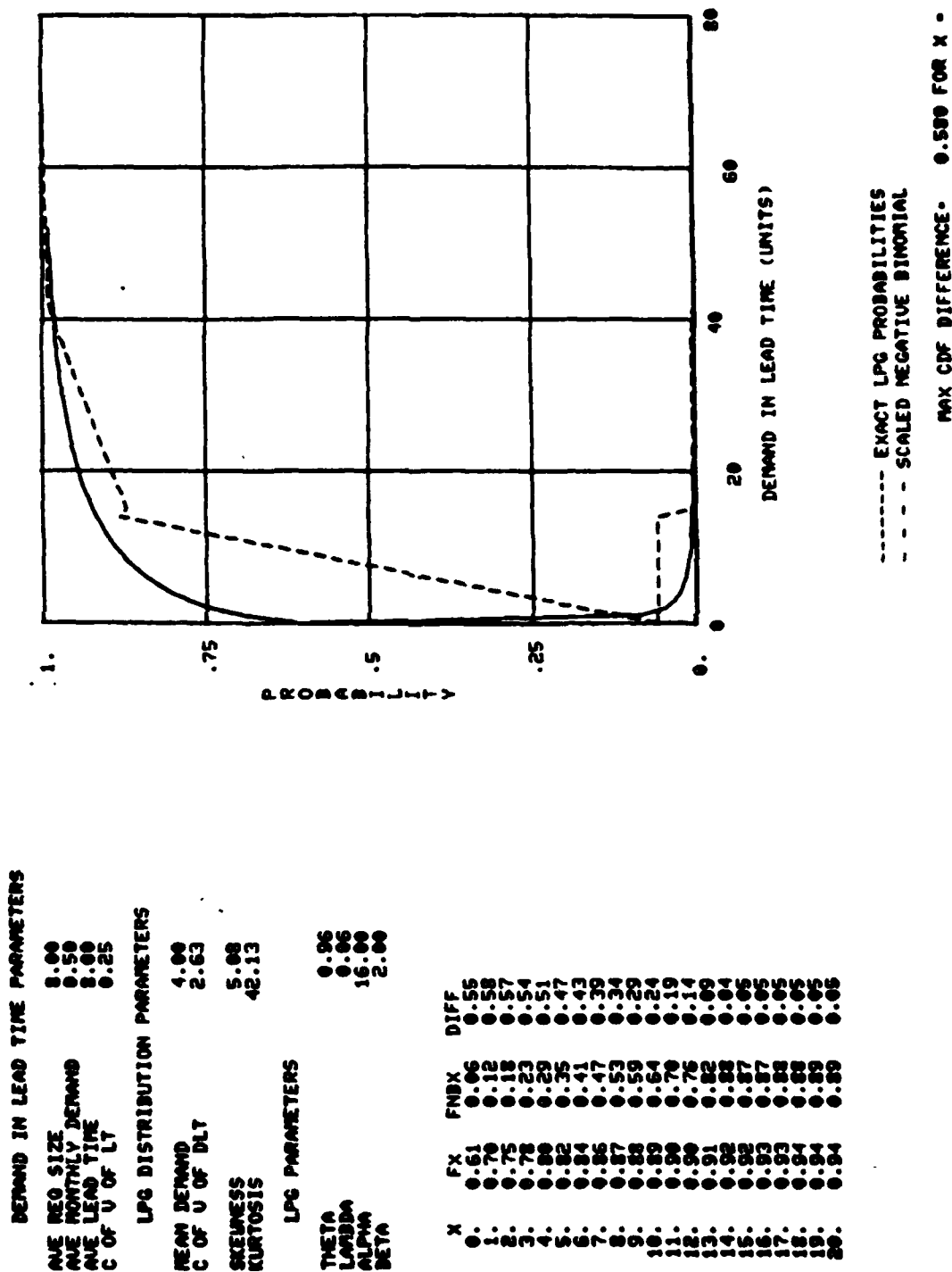


Figure II-6. Data Set No. 4 LPG Curves.

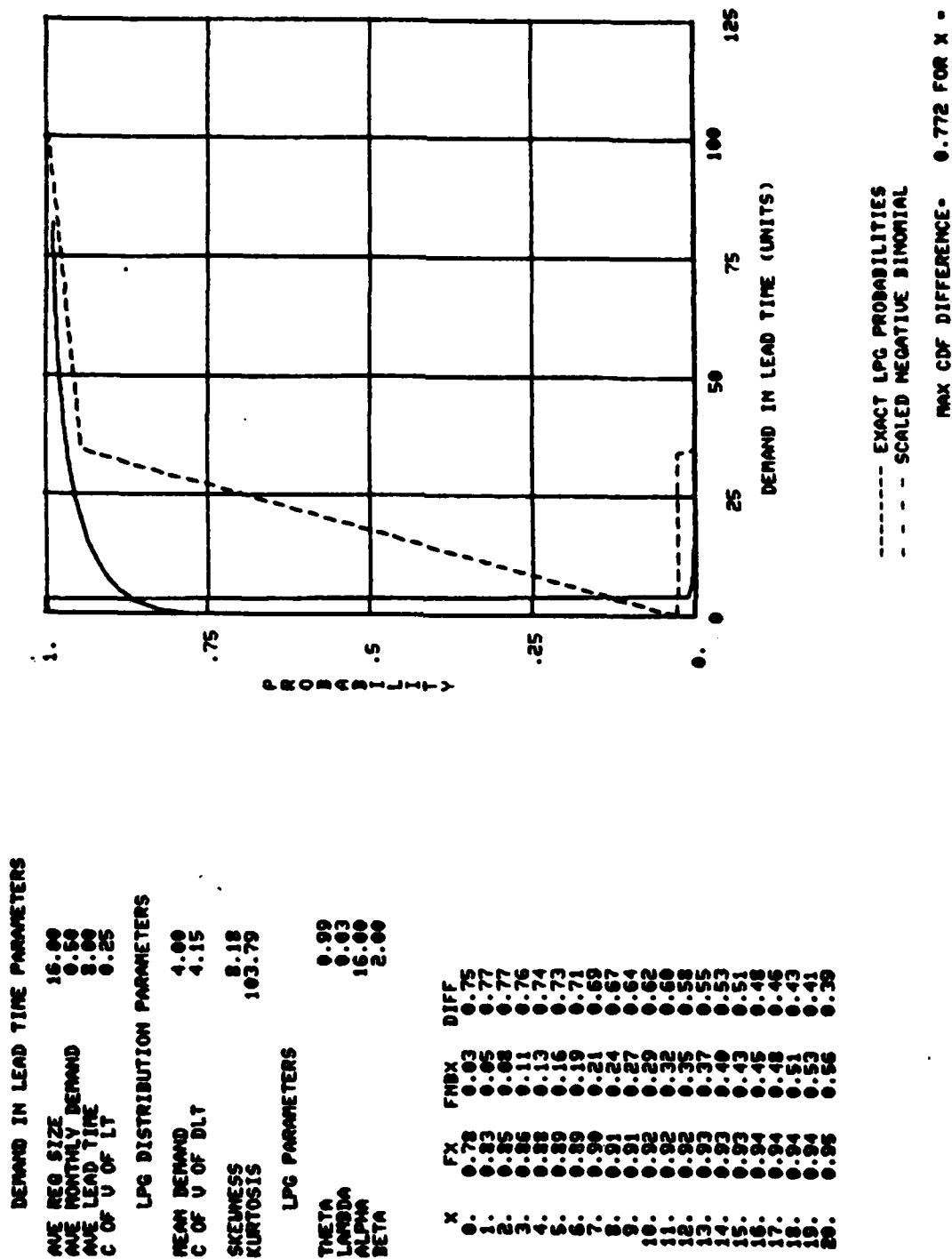


Figure II-7. Data Set No. 5 LPG Curves.

probability. Scanning the results for all five data sets, we observe that the scaled negative binomial provides reorder point results very similar to the LPG distribution for average requisition sizes of 4 or less, but provides a very poor approximation for the high average requisition size values.

Lead Time Sensitivity. Table II-3 presents percentage points for the LPG and Scaled Negative Binomial (SNB) approximations for the LO Base case and Data Sets 6 thru 10. This Table shows the effect of mean lead times of 4, 6, 8, 10, and 12 months, respectively. Note that the SBN yields percentage points which are very close to the LPG values, and that the approximation improves as the lead time increases.

Demand Rate Sensitivity. Table II-4 presents results of varying the LO Base case using demand rates of .1, .5, 1.0, 1.5, and 2.0 units per month, respectively. Note that the LPG and SNB values are very similar for all but the demand =.1 case.

Lead Time Variability Sensitivity. Table II-5 presents the results of increasing lead time variability (Data Sets 16-20). Observe the percentage points are within 1 unit for all but the CV=.75, 60% point. Hence, the approximation is excellent for all five data sets.

Results for the HI Base Case. The HI Base Case has the following characteristics:

	<u>Value</u>
Average Requisition Size	2
Average Units Demanded per Month	2
Average Lead Time (Months)	8
Coefficient of Variation of Lead Time	.25

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo	.50	.50	.50	.50	.50
Mean Lead Time	4.00	6.00	8.00	10.00	12.00
CV of Lead Time	.25	.25	.25	.25	.25

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	.25	.25	.25	.25	.25
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	4.00	2.67	2.00	1.60	1.33

LPG Moments

Mean	2.00	3.00	4.00	5.00	6.00
Skewness	2.16	1.72	1.46	1.28	1.14
Kurtosis	10.22	7.67	6.40	5.65	5.15

<u>Percentage Points</u>	<u>6</u>		<u>7</u>		<u>8</u>		<u>9</u>		<u>10</u>	
	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	1	2	2	3	3	4	4	5	5	6
.60	2	2	3	4	4	5	5	6	6	7
.70	2	4	4	5	5	6	6	7	8	8
.80	3	5	5	6	7	7	8	9	9	11
.85	4	5	6	7	8	8	9	9	11	12
.90	5	6	7	8	9	9	11	12	12	13
.95	7	8	10	9	12	12	14	14	15	15
.97	9	9	11	12	13	13	15	15	17	16
.99	12	9	15	14	17	16	19	19	22	20

Table II-3. Lead Time Sensitivity for LO Base Case

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo	.10	.50	1.00	1.50	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	.05	.25	.50	.75	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00

LPG Moments

Mean	.80	4.00	8.00	12.00	16.00
CV	2.11	.97	.71	.60	.53
Skewness	3.52	1.46	.95	.73	.60
Kurtosis	21.73	6.40	4.53	3.93	3.64

<u>Percentage Points</u>	<u>11</u>		<u>12</u>		<u>13</u>		<u>14</u>		<u>15</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	0	1	3	4	7	8	11	12	15	15
.60	0	2	4	5	8	9	13	13	17	18
.70	1	2	5	6	10	11	15	15	19	20
.80	1	2	7	7	12	13	17	19	23	23
.85	2	2	8	8	14	14	19	20	26	26
.90	3	5	9	9	16	16	22	22	27	28
.95	4	5	12	12	19	19	25	26	32	32
.97	5	6	13	13	21	21	28	28	35	35
.99	8	8	17	16	26	25	34	33	41	40

Table II-4. Demand Rate Sensitivity for LO Base Case

SAMPLE LPG CALCULATION

Data Set No.	<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	0.5	0.5	0.5	0.5	0.5
Mean Lead Time	8	8.0	8.0	8.0	8.0
CV of Lead Time	.01	.25	.50	.75	1.0

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	.25	.25	.25	.25	.25
Alpha	1,000	16.00	4.00	1.78	1.00
Beta	1,250	2.00	.50	.22	.13

LPG Moments

Mean	4.00	4.00	4.00	4.00	4.00
CV	.94	.97	1.06	1.20	1.37
Skewness	1.61	1.46	1.21	1.13	1.29
Kurtosis	6.84	6.40	5.55	5.15	5.76

<u>Percentage Points</u>	<u>16</u>		<u>17</u>		<u>18</u>		<u>19</u>		<u>20</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	3	4	3	4	3	4	2	3	2	2
.60	4	5	4	5	4	5	3	5	3	4
.70	5	6	5	6	5	6	5	6	5	6
.80	7	7	7	7	7	8	7	8	7	8
.85	8	8	8	8	8	9	8	9	9	9
.90	9	9	9	9	10	11	10	11	11	12
.95	11	12	12	12	12	13	14	14	15	15
.97	13	13	13	13	14	14	16	17	18	19
.99	17	15	17	16	19	19	21	21	25	25

Table 11-5. Lead Time Variability Sensitivity for LO Base Case

Recall that Figure II-2 plots the LPG and SNB curves for this case, and that the detailed plots for each data set are presented in Appendix A.

Requisition Size Sensitivity. Table II-6 presents the results for Data Sets, 21-25, reflecting average requisition sizes of 1.01, 2.0, 4.0, 8.0, and 16.0, respectively. Note that the LPG and SNB curves yield similar results for average requisition sizes of 4 or less, but differ significantly for average requisition sizes of 8 and 16.

Lead Time Sensitivity. Table II-7 presents results for Data Sets 26-30, reflecting lead times of 4, 6, 8, 10, and 12 months, respectively. For this case, the LPG and SNB yield similar results in all cases.

Demand Rate Sensitivity. The results for Data Sets 31-35 are presented in Table II-8. These Data Sets have average demands of .1, .5, 1, 1.5, and 2 units per month, respectively. Observe that the SNB approximation is a particularly good fit to the LPG except for the 50 and 60% points for the .1 units per month case.

Lead Time Variability Sensitivity. Table II-9 presents results for coefficients of variation of .01, .25, .50, .75, and 1.0, respectively (Data Sets 36-40). Note that the LPG and SNB distributions are very similar for all of these cases.

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>
Ave. Req. Size	1.01	2.00	4.00	8.00	16.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25

LPG Parameter

Theta	.02	.72	.90	.96	.99
Lambda	1.98	1.00	.50	.25	.13
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00

LPG Moments

Mean	16.00	16.00	16.00	16.00	16.00
CV	.36	.53	.84	1.33	2.09
Skewness	.27	.60	1.35	2.44	4.02
Kurtosis	3.13	3.64	5.97	12.32	27.79

<u>Percentage Points</u>	<u>21</u>		<u>22</u>		<u>23</u>		<u>24</u>		<u>25</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	15	16	15	15	13	14	8	13	2	22
.60	17	18	17	18	16	19	12	18	5	26
.70	19	19	19	20	20	23	18	26	11	31
.80	21	21	23	23	25	27	27	35	23	37
.85	22	22	25	26	29	31	33	39	33	56
.90	24	24	27	28	34	35	43	47	50	75
.95	26	27	32	32	42	42	59	61	80	93
.97	28	28	35	34	49	46	72	67	105	101
.99	31	32	41	40	61	57	99	88	163	148

Table II-6. Requisition Size Sensitivity for HI Base Case

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>26</u>	<u>27</u>	<u>28</u>	<u>29</u>	<u>30</u>
Ave. Req Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	4.00	6.00	8.00	10.00	12.00
CV of Lead Time	.25	.25	.25	.25	.25

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	1.00	1.00	1.00	1.00	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	4.00	2.67	2.00	1.60	1.33

LPG Moments

Mean	8.00	12.00	16.00	20.00	24.00
CV	.71	.60	.53	.49	.46
Skewness	.95	.73	.60	.52	.47
Kurtosis	4.53	3.92	3.64	3.48	3.38

<u>Percentage Points</u>	<u>26</u>		<u>27</u>		<u>28</u>		<u>29</u>		<u>30</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	7	8	11	12	15	15	19	20	23	23
.60	8	9	13	13	17	18	21	22	25	26
.70	10	11	15	15	19	20	24	25	29	29
.80	12	13	17	19	23	23	28	28	33	33
.85	14	14	19	20	25	26	30	30	35	36
.90	16	16	22	22	27	28	33	34	39	39
.95	19	19	25	26	32	32	38	37	44	44
.97	21	21	28	28	35	34	41	41	48	48
.99	26	25	34	33	41	40	48	48	55	55

Table II-7. Lead Time Sensitivity for HI Base Case

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>31</u>	<u>32</u>	<u>33</u>	<u>34</u>	<u>35</u>
Ave. Req Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	.10	.50	1.00	1.50	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	.05	.25	.50	.75	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00

LPG Moments

Mean	.80	4.00	8.00	12.00	16.00
CV	2.11	.97	.71	.60	.53
Skewness	3.52	1.46	.95	.73	.60
Kurtosis	21.73	6.40	4.53	3.92	3.64

<u>Percentage Points</u>	<u>31</u>		<u>32</u>		<u>33</u>		<u>34</u>		<u>35</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	0	1	3	4	7	8	11	12	15	15
.60	0	2	4	5	8	9	13	13	17	18
.70	1	2	5	6	10	11	15	15	19	20
.80	1	2	7	7	12	13	17	19	23	23
.85	2	2	8	8	14	14	19	20	25	26
.90	3	5	1	9	16	16	22	22	27	28
.95	4	5	12	12	19	19	25	26	32	32
.97	5	6	13	3	21	21	28	28	35	34
.99	8	8	17	16	26	25	34	33	41	40

Table II-8. Demand Rate Sensitivity for HI Base Case

SAMPLE LPG CALCULATIONS

<u>Date Set No.</u>	<u>36</u>	<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.01	.25	.50	.75	1.00

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	1.00	1.00	1.00	1.00	1.00
Alpha	10,000	16.00	4.00	1.78	1.00
Beta	1,250	2.00	.50	.22	.13

LPG Moments

Mean	16.00	16.00	16.00	16.00	16.00
CV	.47	.53	.69	.88	1.10
Skewness	.80	.60	.65	1.03	1.55
Kurtosis	3.96	3.64	3.69	4.86	7.10

<u>Percentage Points</u>	<u>36</u>		<u>37</u>		<u>38</u>		<u>39</u>		<u>40</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	15	16	15	15	14	14	12	13	10	11
.60	17	18	17	18	17	18	16	16	14	15
.70	19	20	19	20	20	20	19	20		
.80	22	22	23	23	24	25	26	23	27	27
.85	24	25	25	26	27	27	30	26	32	33
.90	26	27	27	28	31	31	35	35	39	40
.95	30	29	32	32	37	37	44	44	51	51
.97	32	32	35	34	41	41	50	50	60	61
.99	38	36	41	40	50	51	64	63	80	80

Table II-9. Lead Time Variability Sensitivity for HI Base Case

Moments of the LPG Distribution

As noted above, we computed the first four moments of the LPG distribution. Figures II-8 and II-9 present the means and standard deviations associated with each of the forty data sets studied, while Figure II-10 presents a plot of $(\text{skewness})^2$ value versus the kurtosis for the forty data set calculations. Figure II-10 is particularly interesting. As shown in Johnson and Kotz (1969, Pg 39), the skewness²-kurtosis relationships of many well known probability distributions are described by specific lines, points, or regions when plotted on a chart similar to Figure II-10. For example, the lines and regions associated with Normal, Gamma, Log-normal, Poisson, Binomial, and Negative Binomial Distributions are shown in Figure II-10. Hence, the plot of skewness² versus kurtosis for a specific probability distribution may be used to gain insights into appropriate approximations for that distribution. As shown in Figures II-10 thru II-13, all of the sample skewness²-kurtosis points associated with our LPG calculations fall on a line which is almost identical with that associated with the gamma probability distribution. Hence, it appears that the gamma may be a very useful approximation for LPG calculations.

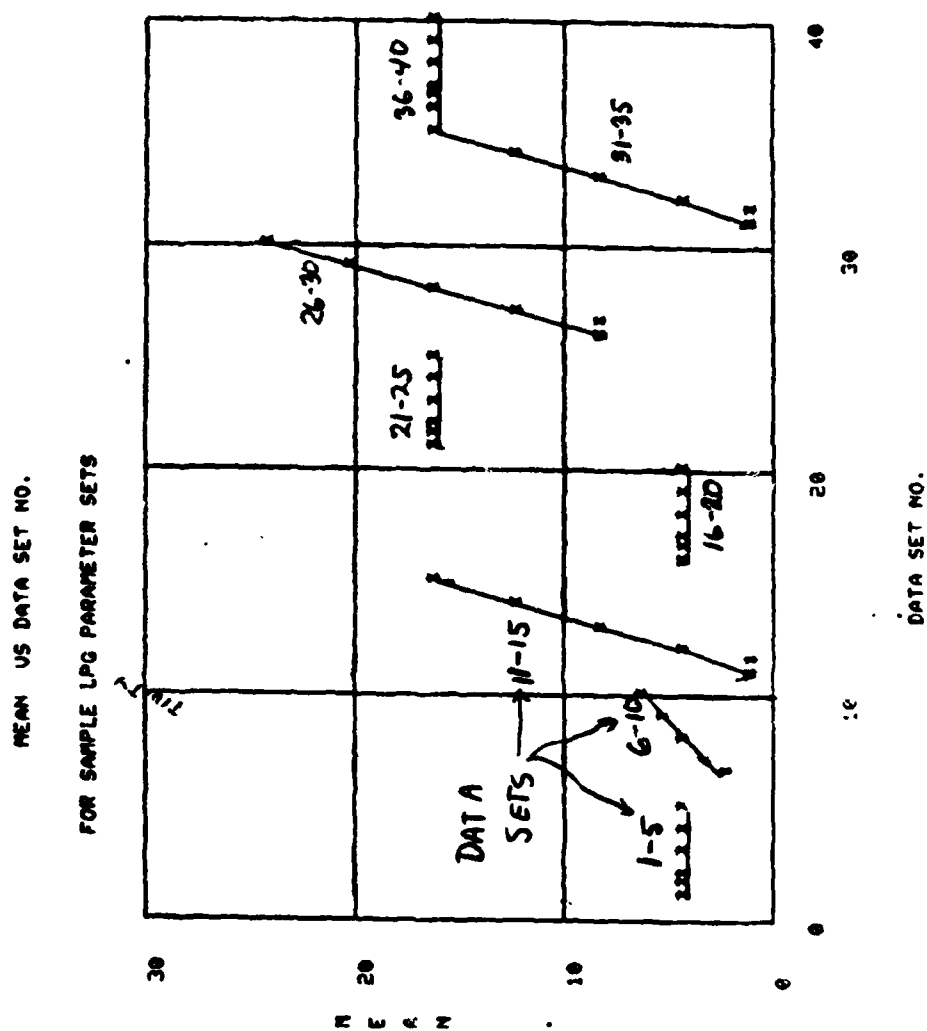


Figure II-8. Sample LPG Means.

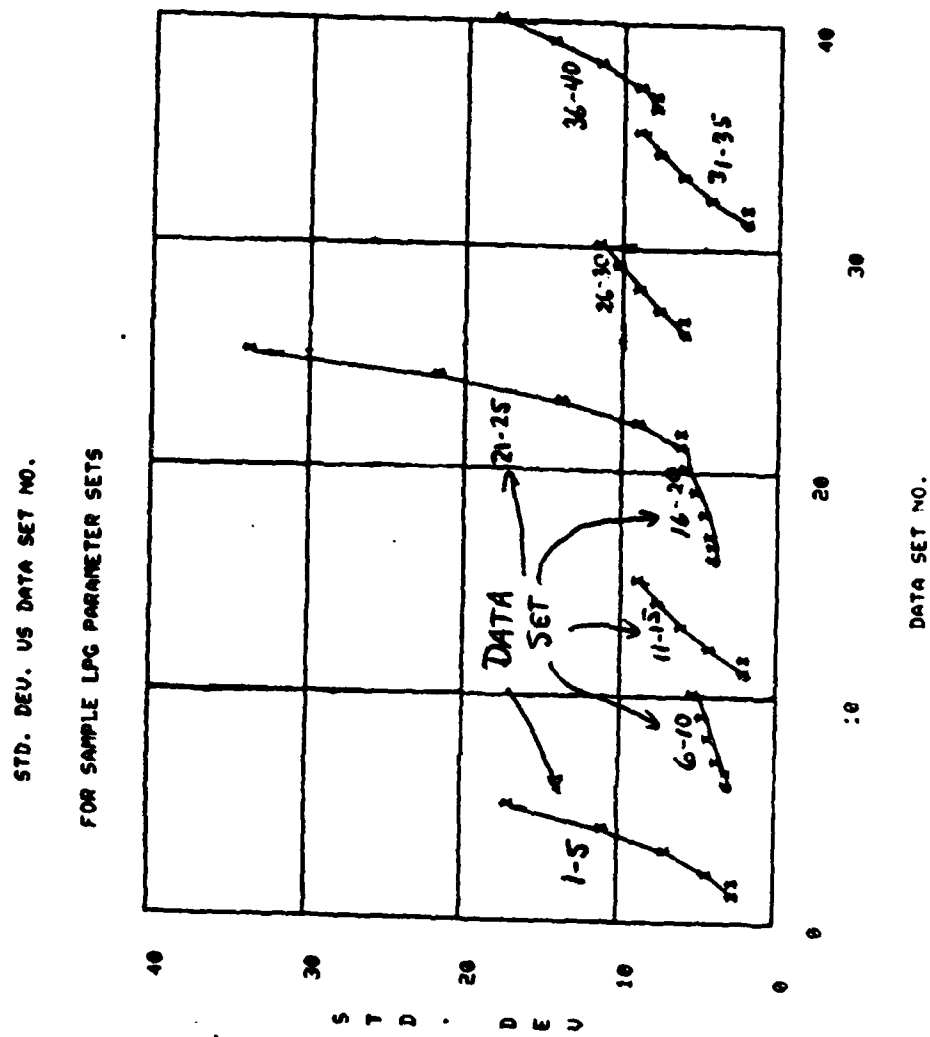


Figure II-9. Sample LPG Standard Deviation.

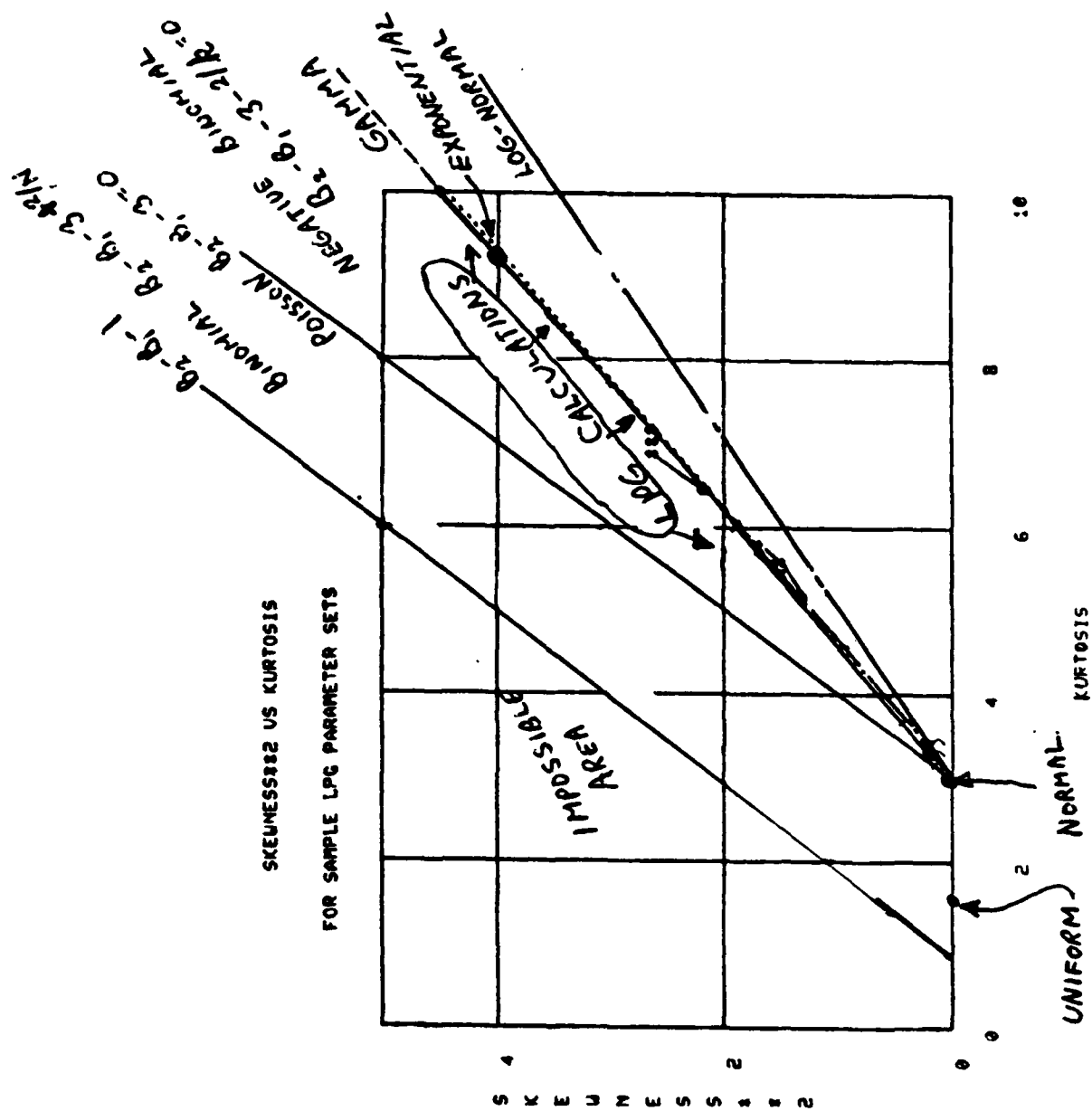


Figure II-10. Skewness² vs Kurtosis for Sample LPG Distribution.

SKEWNESS² VS KURTOSIS

FOR SAMPLE LPG PARAMETER SETS

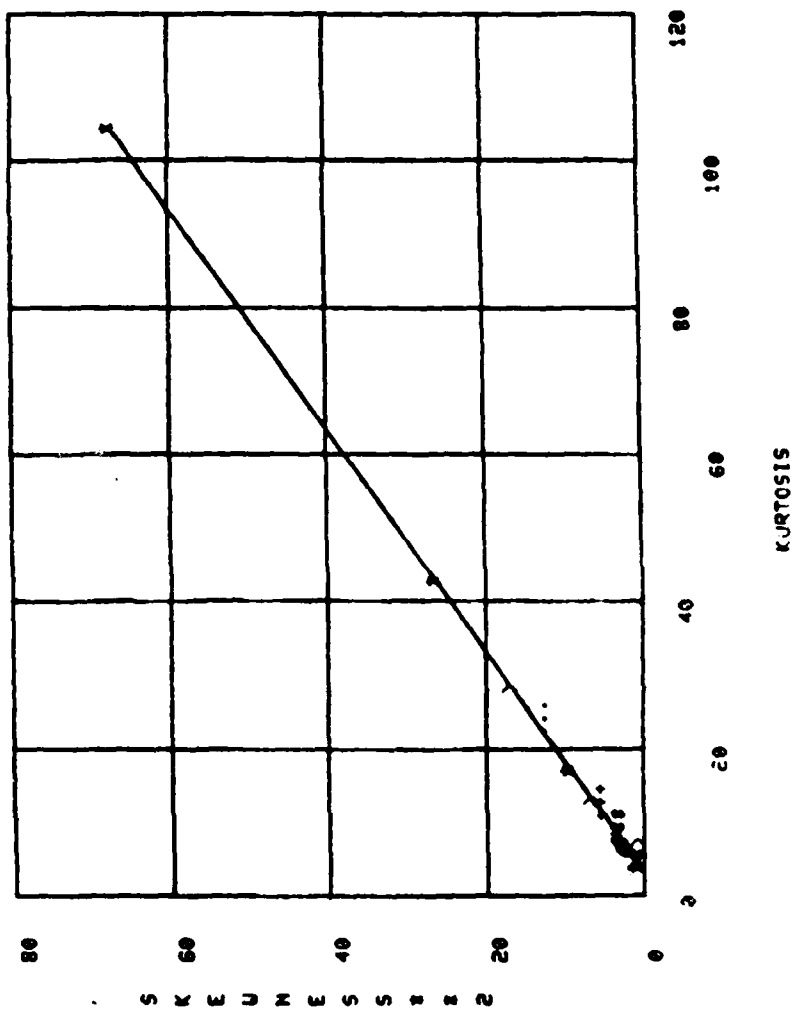
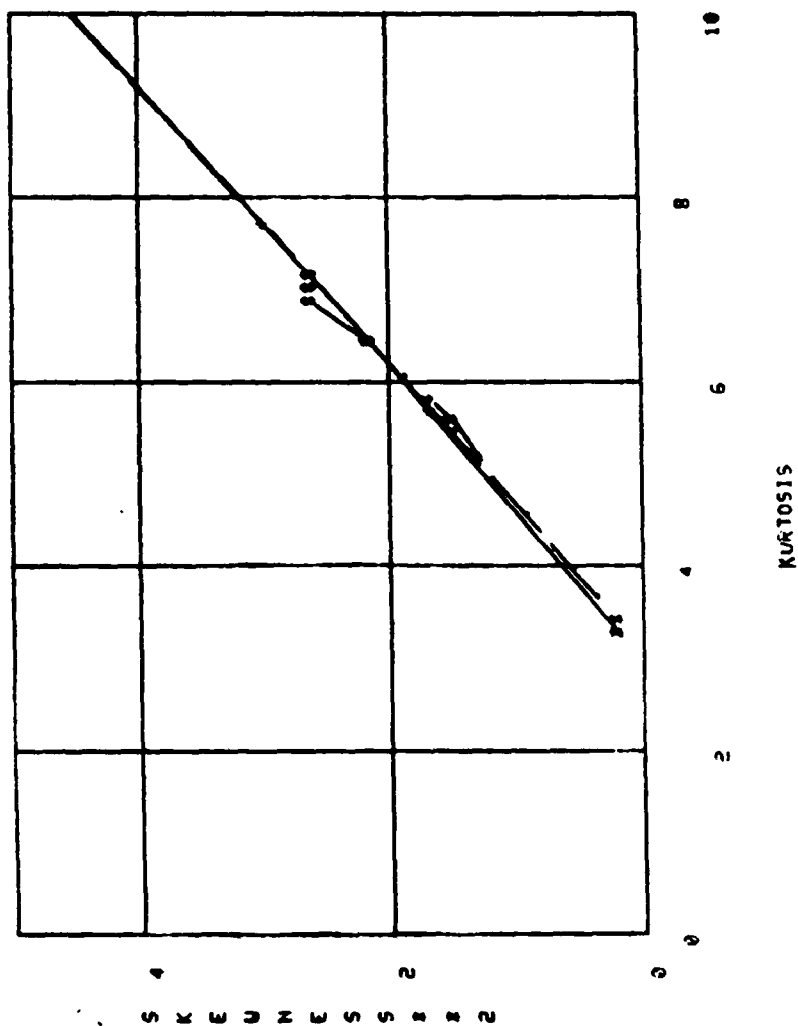


Figure II-11. Skewness² vs Kurtosis for Sample LPG Distribution with Unrestricted Scales.

SKEWNESS² VS KURTOSIS

LPG VALUES FOR E(D)=0.5

Figure II-12. Skewness² vs Kurtosis for LO Base Case.

SKENESS82 VS KURTOSIS

LPG VALUES FOR E(D)-2.0

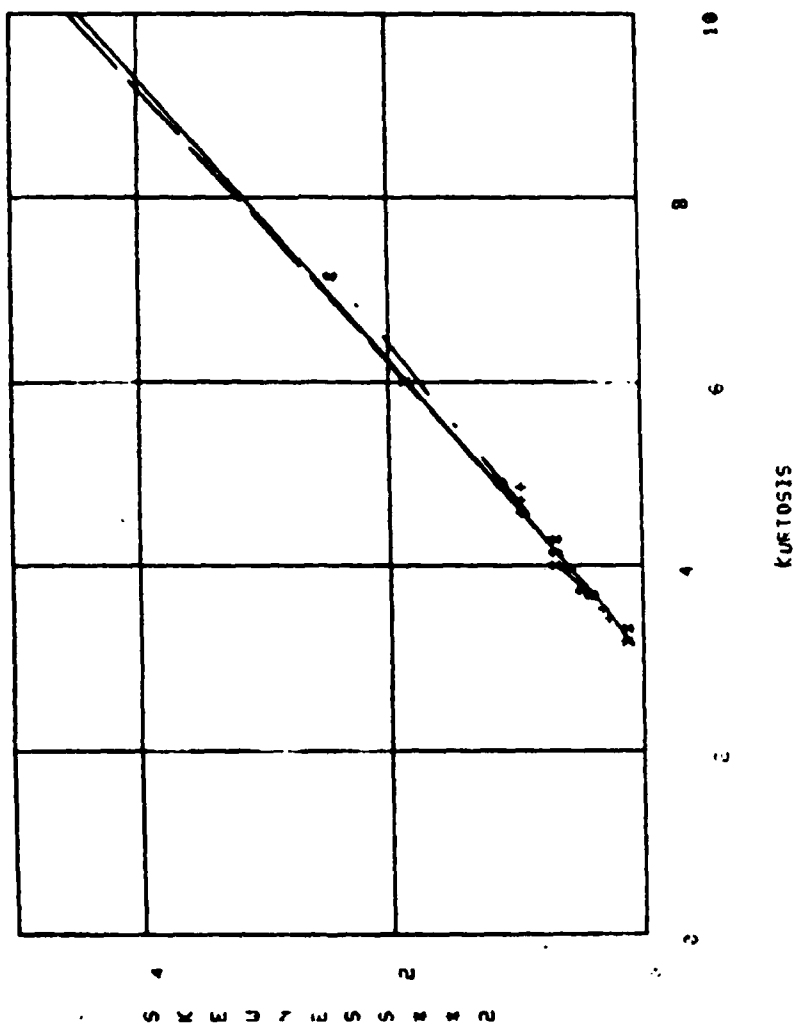


Figure II-13. Skewness² vs Kurtosis for HI Base Case.

Section III

Summary and Conclusions

In this paper, we have presented sample distribution functions for the Logarithmic-Poisson-Gamma (LPG) distributed and for the Scaled Negative Binomial (SNB) approximation to the LPG. Two base cases were considered; these cases have the following characteristics.

	<u>LO Base Case</u>	<u>HI Base Case</u>
Average Requisition Size (units)	2	2
Average Units Demanded per Month	.5	2
Average Lead Time (Months)	8	8
Coefficient of Variation of Lead Time	.25	.25

The effects of varying each of these parameters while holding the other three constant were investigated for base cases. We found that the SNB approximation provides percentage points very similar to the LPG except when average requisition sizes exceed 4 units or when demand per month is very small. Hence, use of the SNB approximation appears to be a promising way to significantly reduce the computer requirements associated with the LPG distribution.

We also analyzed the skewness² versus kurtosis relationships for the sample data sets. These relationships indicate that the continuous Gamma distribution may be an excellent approximation to the Logarithmic-Poisson-Gamma.

Section IV

References

1. Nahmias, Steven and W. Steven Demmy, The Logarithmic-Poisson-Gamma Distribution: A Model for the Distribution of Demand in a Lead Time, Working Paper WP-81-04, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Oh 45431, August 1981, 22 pp.
2. Johnson, N.L. and S. Kotz, Discrete Distributions, Wiley, 1969.

Appendix A

Detailed Item Plots

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
Ave. Req. Size	1.01	2	4	8	16
Ave. Demand/Mo.	.5	.5	.5	.5	.5
Mean Lead Time	8.0	8.0	8.0	8	8
CV of Lead Time	.25	.25	.25	.25	.25
<u>LPG Parameter</u>					
Theta	.02	.72	.90	.96	.99
Lambda	.49	.25	.12	.06	.03
Alpha	16	16	16	16	16
Beta	2	2	2	2	2
<u>LPG Moments</u>					
Mean	4	4	4	4	4
CV	.56	.97	1.63	2.63	4.15
Skewness	.41	1.46	2.96	5.08	8.18
Kurtosis	3.23	6.40	16.45	42.13	103.79
<u>Percentage Points</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
	<u>LPG</u> <u>NB</u>	<u>LPG</u> <u>NB</u>	<u>LPG</u> <u>NB</u>	<u>LPG</u> <u>NB</u>	<u>LPG</u> <u>NB</u>
.50	4 5	3 4	1 4	0 8	0 18
.60	4 5	4 5	2 5	0 10	0 22
.70	5 6	5 6	4 6	2 11	0 26
.80	6 7	7 7	7 10	4 13	1 29
.85	6 7	8 8	9 12	7 14	3 31
.90	7 8	9 9	12 14	12 22	7 33
.95	8 9	12 12	17 16	22 33	22 43
.97	9 10	13 13	21 22	31 38	38 68
.99	10 11	17 16	31 26	52 42	82 93

Table II-2. Requisition Size Sensitivity for LO Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 1.01
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

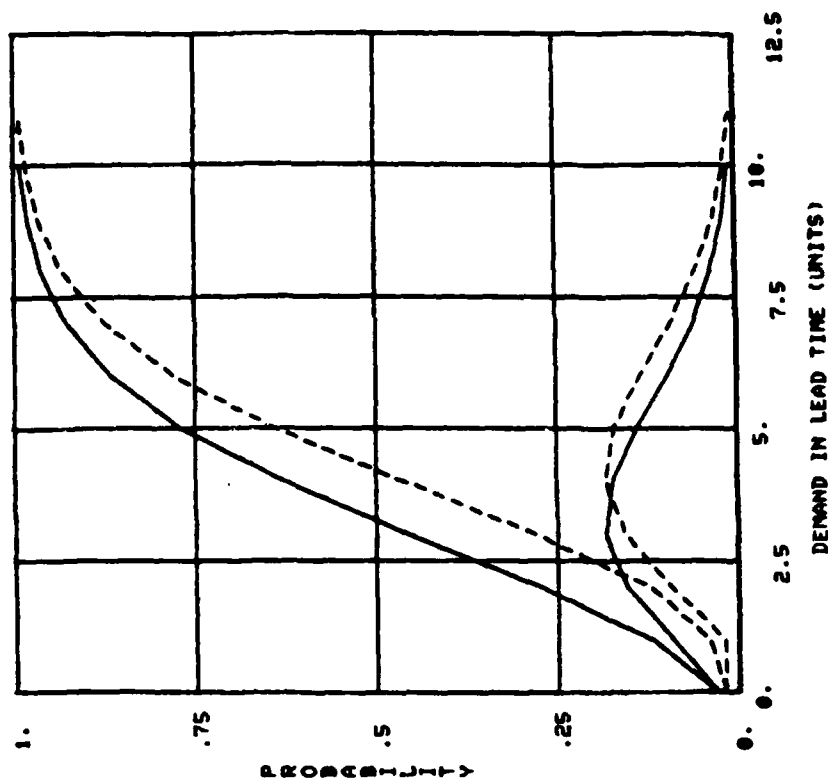
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF V OF DLT 0.56
 SKEWNESS 0.41
 KURTOSIS 3.23

LPG PARAMETERS

THETA 0.02
 LAMBDA 0.49
 ALPHA 16.00
 BETA 2.00

X	FX	FNBX	DIFF
0.	0.03	0.02	0.01
1.	0.12	0.04	0.08
2.	0.27	0.12	0.15
3.	0.46	0.28	0.18
4.	0.63	0.47	0.16
5.	0.77	0.64	0.13
6.	0.87	0.78	0.09
7.	0.93	0.87	0.05
8.	0.96	0.93	0.03
9.	0.98	0.97	0.02
10.	0.99	0.98	0.01



DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

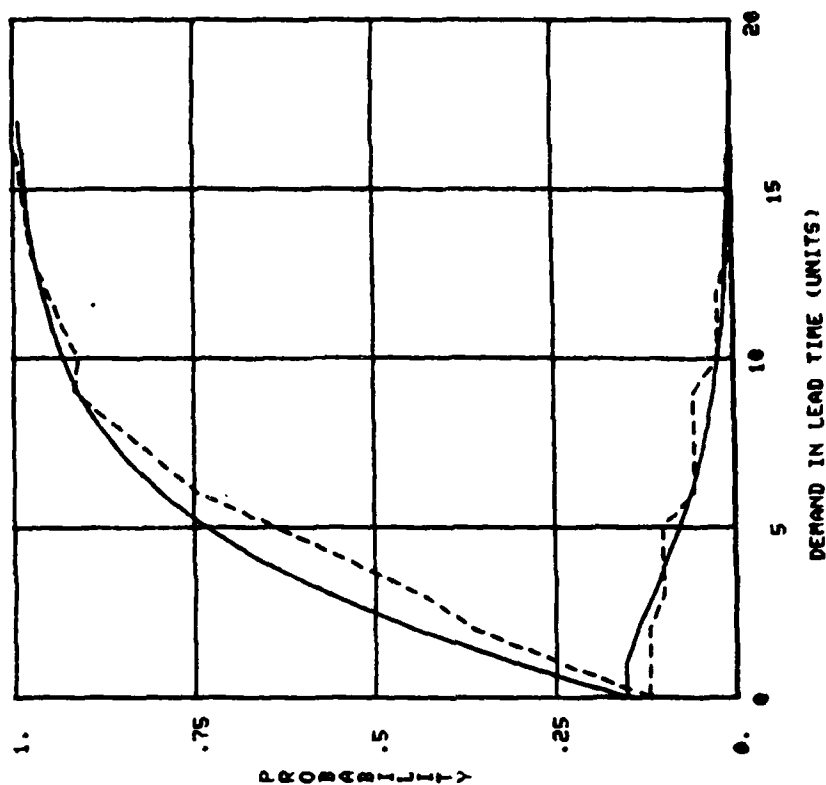
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF V OF DLT 0.97
 SKEWNESS 1.46
 KURTOSIS 6.40

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 2.00

X	FX	FNBX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.93	0.02
11.	0.95	0.95	0.01
12.	0.96	0.97	-0.00
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.01
16.	0.99	0.99	-0.01



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE- 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 4.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF U OF LT 0.25

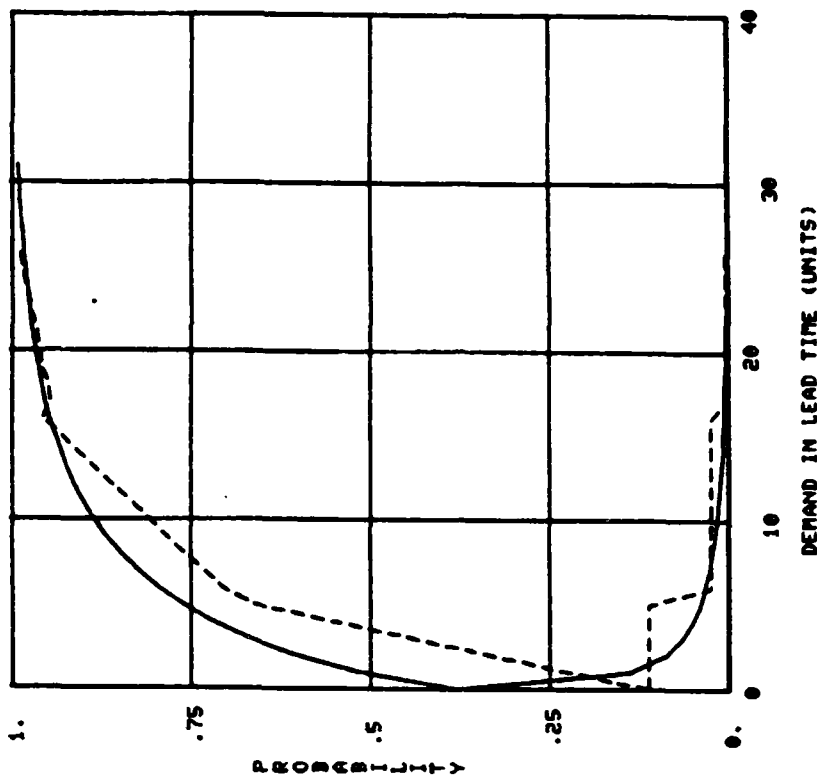
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF U OF DLT 1.63
 SKEWNESS 2.96
 KURTOSIS 16.45

LPG PARAMETERS

THETA 0.90
 LAMBDA 0.12
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.38	0.11	0.27
1.	0.52	0.22	0.30
2.	0.61	0.33	0.27
3.	0.67	0.44	0.23
4.	0.72	0.55	0.17
5.	0.76	0.66	0.10
6.	0.80	0.71	0.09
7.	0.82	0.73	0.09
8.	0.85	0.76	0.09
9.	0.87	0.78	0.08
10.	0.88	0.81	0.08
11.	0.90	0.83	0.07
12.	0.91	0.86	0.05
13.	0.92	0.88	0.04
14.	0.93	0.91	0.02
15.	0.94	0.93	0.01
16.	0.95	0.96	-0.01
17.	0.95	0.95	0.01
18.	0.96	0.95	0.01
19.	0.96	0.96	0.01
20.	0.97	0.96	0.01



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.296 FOR X = 1

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 8.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

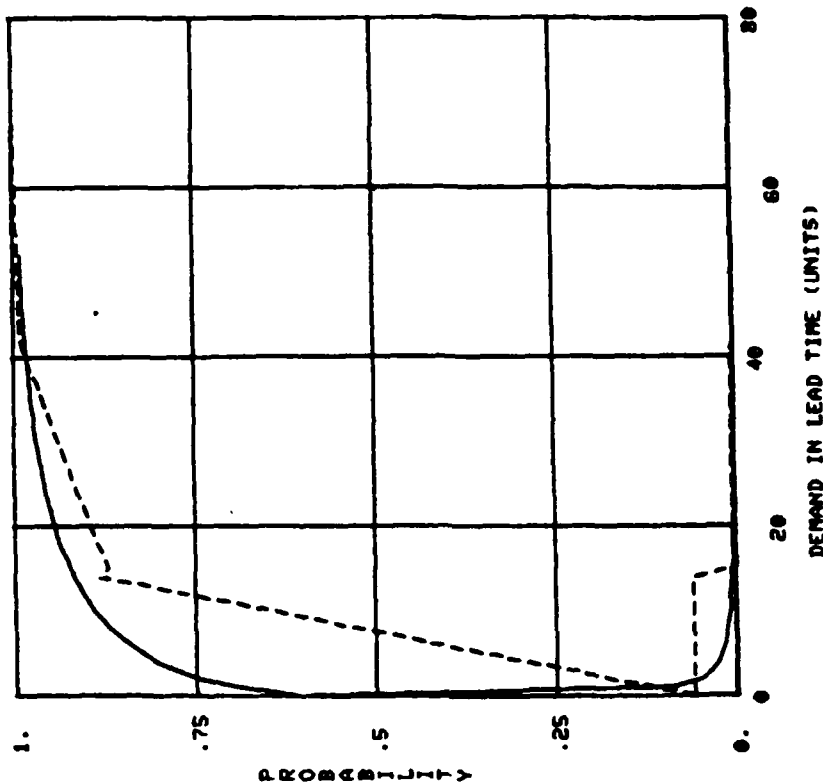
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF V OF DLT 2.63
 SKEWNESS 5.08
 KURTOSIS 42.13

LPG PARAMETERS

THETA 0.96
 LAMBDA 0.06
 ALPHA 16.00
 BETA 2.00

X	FX	FNBX	DIFF
0.	0.61	0.06	0.55
1.	0.70	0.18	0.58
2.	0.75	0.23	0.57
3.	0.78	0.29	0.54
4.	0.80	0.35	0.51
5.	0.82	0.41	0.47
6.	0.84	0.47	0.43
7.	0.85	0.53	0.39
8.	0.87	0.59	0.34
9.	0.88	0.64	0.29
10.	0.89	0.70	0.24
11.	0.90	0.76	0.19
12.	0.90	0.82	0.14
13.	0.91	0.88	0.09
14.	0.92	0.87	0.04
15.	0.92	0.87	0.05
16.	0.93	0.88	0.05
17.	0.93	0.88	0.05
18.	0.94	0.89	0.05
19.	0.94	0.89	0.05
20.	0.94	0.89	0.05



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE - 0.580 FOR X - 1

DEMAND IN LEAD TIME PARAMETERS

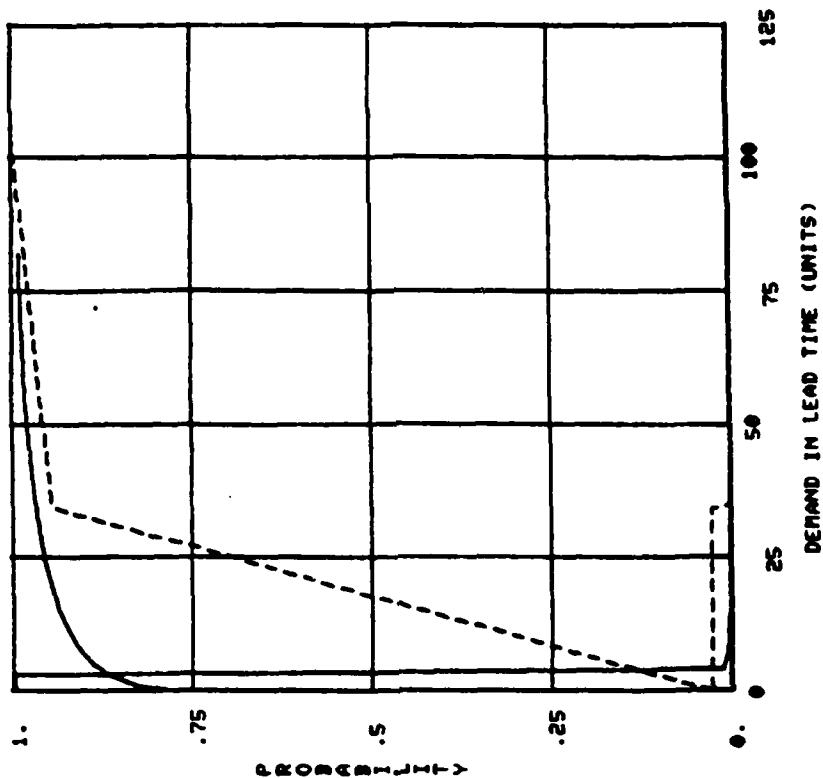
AVE REQ SIZE 16.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF U OF LT 0.25

LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF U OF DLT 4.15
 SKEWNESS 8.18
 KURTOSIS 103.70

LPG PARAMETERS

THETA 0.99
 LAMBDA 0.03
 ALPHA 16.00
 BETA 2.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE- 0.772 FOR X = 1

X	FX	FMBX	DIFF
0.	0.78	0.03	0.75
1.	0.83	0.05	0.77
2.	0.85	0.08	0.77
3.	0.86	0.11	0.76
4.	0.88	0.13	0.74
5.	0.89	0.16	0.73
6.	0.89	0.19	0.71
7.	0.90	0.21	0.69
8.	0.91	0.24	0.67
9.	0.91	0.27	0.64
10.	0.92	0.29	0.62
11.	0.92	0.32	0.60
12.	0.92	0.35	0.58
13.	0.93	0.37	0.55
14.	0.93	0.40	0.53
15.	0.93	0.43	0.51
16.	0.94	0.45	0.48
17.	0.94	0.48	0.46
18.	0.94	0.51	0.43
19.	0.94	0.53	0.41
20.	0.95	0.55	0.39

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo	.50	.50	.50	.50	.50
Mean Lead Time	4.00	6.00	8.00	10.00	12.00
CV of Lead Time	.25	.25	.25	.25	.25

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	.25	.25	.25	.25	.25
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	4.00	2.67	2.00	1.60	1.33

LPG Moments

Mean	2.00	3.00	4.00	5.00	6.00
Skewness	2.16	1.72	1.46	1.28	1.14
Kurtosis	10.22	7.67	6.40	5.65	5.15

<u>Percentage Points</u>	<u>6</u>		<u>7</u>		<u>8</u>		<u>9</u>		<u>10</u>	
	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	1	2	2	3	3	4	4	5	5	6
.60	2	2	3	4	4	5	5	6	6	7
.70	2	4	4	5	5	6	6	7	8	8
.80	3	5	5	6	7	7	8	9	9	11
.85	4	5	6	7	8	8	9	9	11	12
.90	5	6	7	8	9	9	11	12	12	13
.95	7	8	10	9	12	12	14	14	15	15
.97	9	9	11	12	13	13	15	15	17	16
.99	12	9	15	14	17	16	19	19	22	20

Table II-3. Lead Time Sensitivity for L0 Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 4.00
 C OF V OF LT 0.25

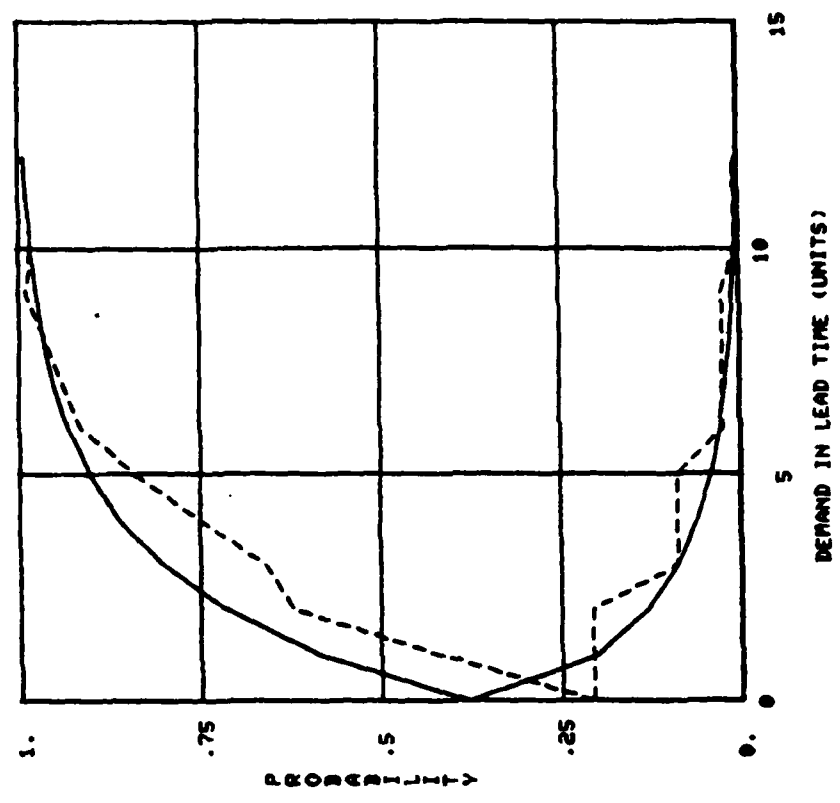
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 2.00
 C OF V OF DLT 1.35
 SKEWNESS 2.16
 KURTOSIS 10.22

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 4.00

X	FX	FNDX	DIFF
0.	0.38	0.21	0.17
1.	0.58	0.41	0.17
2.	0.71	0.62	0.09
3.	0.80	0.66	0.14
4.	0.86	0.75	0.11
5.	0.89	0.84	0.06
6.	0.93	0.91	0.02
7.	0.95	0.94	0.01
8.	0.97	0.96	0.00
9.	0.98	0.99	-0.01
10.	0.98	0.98	-0.00
11.	0.99	0.99	-0.00
12.	0.99	0.99	-0.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE 0.172 FOR X - 0

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 6.00
 C OF V OF LT 0.25

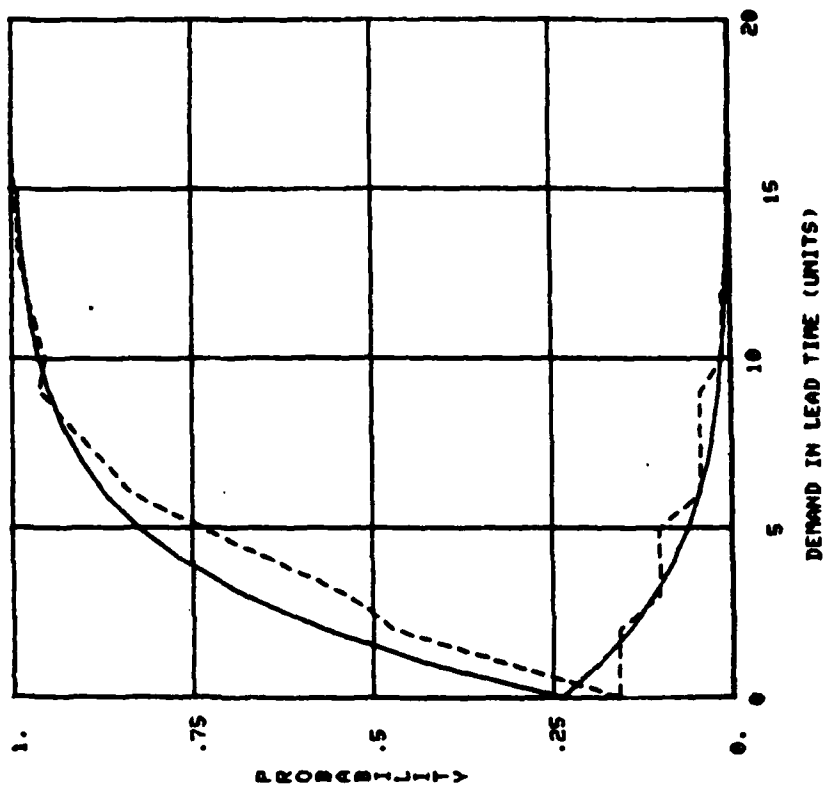
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 3.00
 C OF V OF DLT 1.11
 SKEWNESS 1.72
 KURTOSIS 7.67

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 2.67

X	FX	FNDX	DIFF
0.	0.24	0.16	0.08
1.	0.42	0.32	0.11
2.	0.57	0.47	0.09
3.	0.68	0.54	0.14
4.	0.76	0.54	0.12
5.	0.82	0.74	0.09
6.	0.87	0.83	0.04
7.	0.90	0.87	0.03
8.	0.93	0.92	0.01
9.	0.95	0.96	-0.01
10.	0.96	0.95	0.01
11.	0.97	0.97	0.00
12.	0.98	0.98	0.00
13.	0.99	0.99	-0.00
14.	0.99	0.99	-0.00
15.	0.99	1.00	-0.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE- 0.142 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

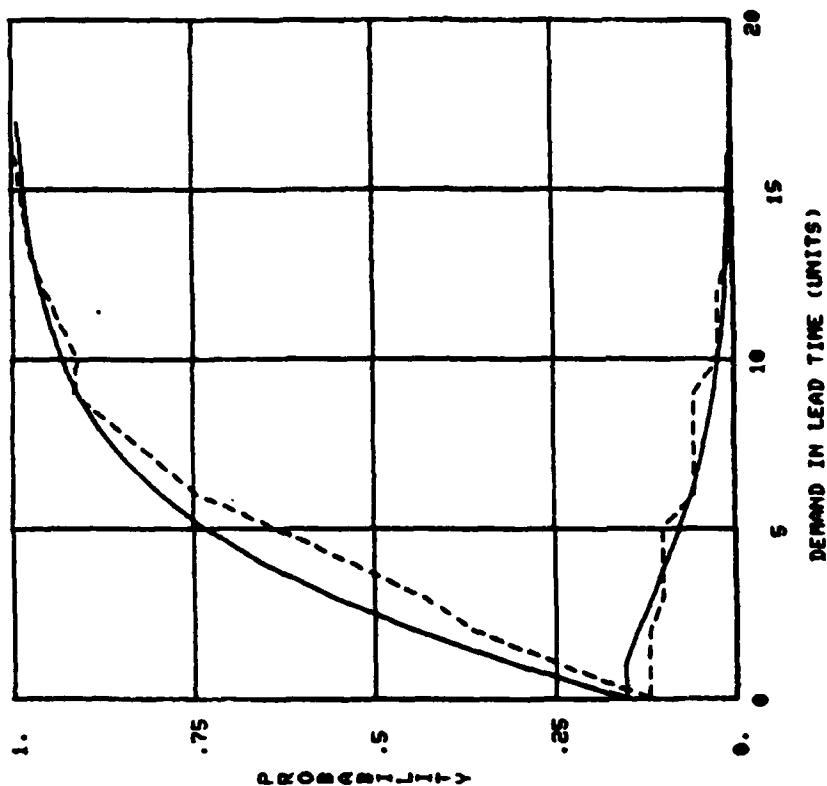
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF V OF DLT 0.97
 SKEWNESS 1.46
 KURTOSIS 6.40

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 2.00

X	FX	FNBX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.78	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.93	0.02
11.	0.95	0.95	0.01
12.	0.96	0.97	-0.00
13.	0.97	0.98	-0.00
14.	0.98	0.99	-0.00
15.	0.99	0.99	-0.01
16.	0.99	0.99	-0.01



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 10.00
 C OF V OF LT 0.25

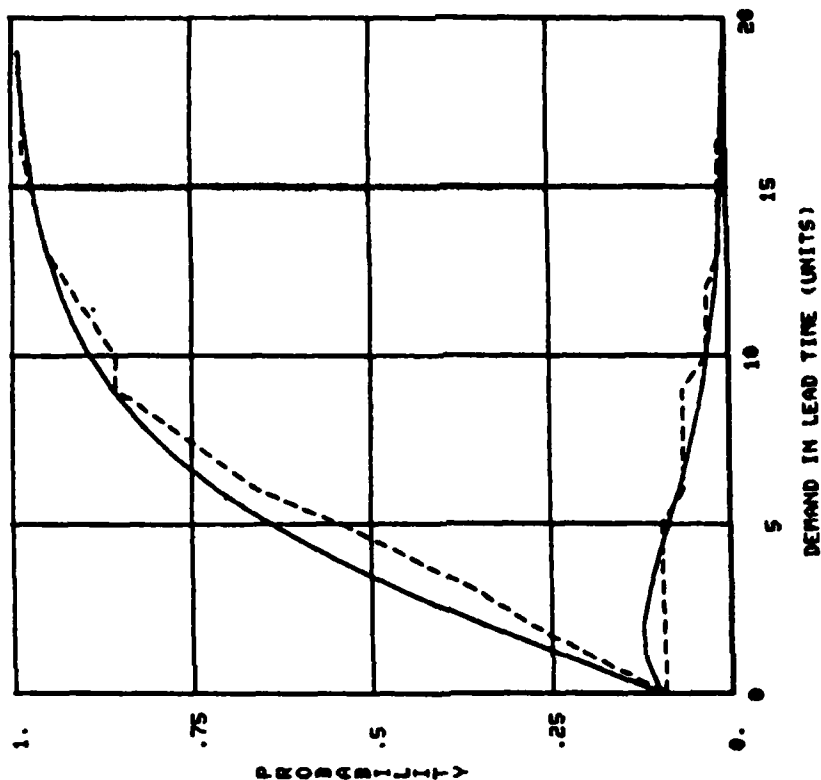
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 5.00
 C OF V OF DLT 0.87
 SKEWNESS 1.28
 KURTOSIS 5.65

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 1.60

X	FX	FMDX	DIFF
0.	0.10	0.09	0.01
1.	0.22	0.19	0.03
2.	0.34	0.28	0.06
3.	0.45	0.35	0.10
4.	0.55	0.45	0.11
5.	0.64	0.54	0.10
6.	0.71	0.66	0.06
7.	0.77	0.72	0.05
8.	0.82	0.79	0.03
9.	0.86	0.85	0.01
10.	0.89	0.89	0.04
11.	0.92	0.89	0.03
12.	0.93	0.92	0.02
13.	0.96	0.95	0.00
14.	0.96	0.96	0.00
15.	0.97	0.97	-0.00
16.	0.98	0.99	-0.01
17.	0.98	0.99	-0.00
18.	0.99	0.99	-0.00
19.	0.99	0.99	-0.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.108 FOR X = 4

DEMAND IN LEAD TIME PARAMETERS

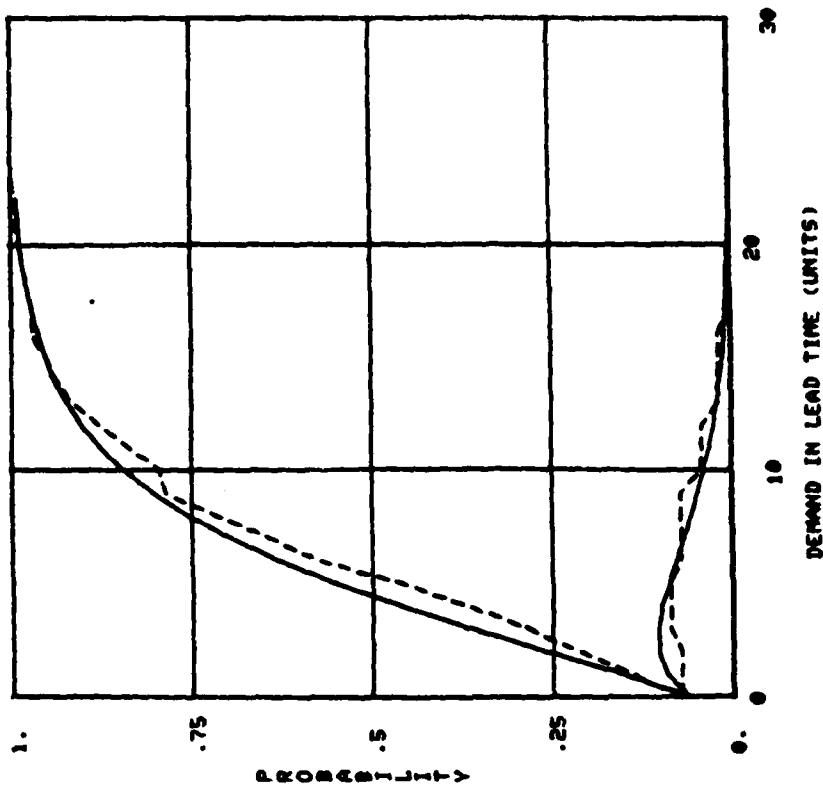
AUE REQ SIZE 2.00
 AUE MONTHLY DEMAND 0.50
 AUE LEAD TIME 12.00
 C OF U OF LT 0.25

LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 6.00
 C OF U OF DLT 0.81
 SKEWNESS 1.14
 KURTOSIS 5.15

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 1.33



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.093 FOR X = 5

X	FX	FNBX	DIFF
0.	0.06	0.07	-0.01
1.	0.16	0.14	0.01
2.	0.26	0.21	0.04
3.	0.36	0.28	0.08
4.	0.46	0.37	0.09
5.	0.55	0.46	0.09
6.	0.63	0.57	0.06
7.	0.70	0.64	0.05
8.	0.75	0.72	0.04
9.	0.80	0.79	0.02
10.	0.84	0.83	0.04
11.	0.87	0.87	0.03
12.	0.90	0.91	0.01
13.	0.92	0.93	0.00
14.	0.95	0.95	-0.00
15.	0.96	0.97	-0.01
16.	0.98	0.97	0.00
17.	0.97	0.97	0.00
18.	0.98	0.98	-0.00
19.	0.98	0.98	-0.00
20.	0.99	0.99	-0.00

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo	.10	.50	1.00	1.50	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	.05	.25	.50	.75	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00

LPG Moments

Mean	.80	4.00	8.00	12.00	16.00
CV	2.11	.97	.71	.60	.53
Skewness	3.52	1.46	.95	.73	.60
Kurtosis	21.73	6.40	4.53	3.93	3.64

<u>Percentage Points</u>	<u>11</u>		<u>12</u>		<u>13</u>		<u>14</u>		<u>15</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	0	1	3	4	7	8	11	12	15	15
.60	0	2	4	5	8	9	13	13	17	18
.70	1	2	5	6	10	11	15	15	19	20
.80	1	2	7	7	12	13	17	19	23	23
.85	2	2	8	8	14	14	19	20	26	26
.90	3	5	9	9	16	16	22	22	27	28
.95	4	5	12	12	19	19	25	26	32	32
.97	5	6	13	13	21	21	28	28	35	35
.99	8	8	17	16	26	25	34	33	41	40

Table II-4. Demand Rate Sensitivity for LO Base Case

END IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.10
 AVE LEAD TIME 0.00
 C OF V OF LT 0.25

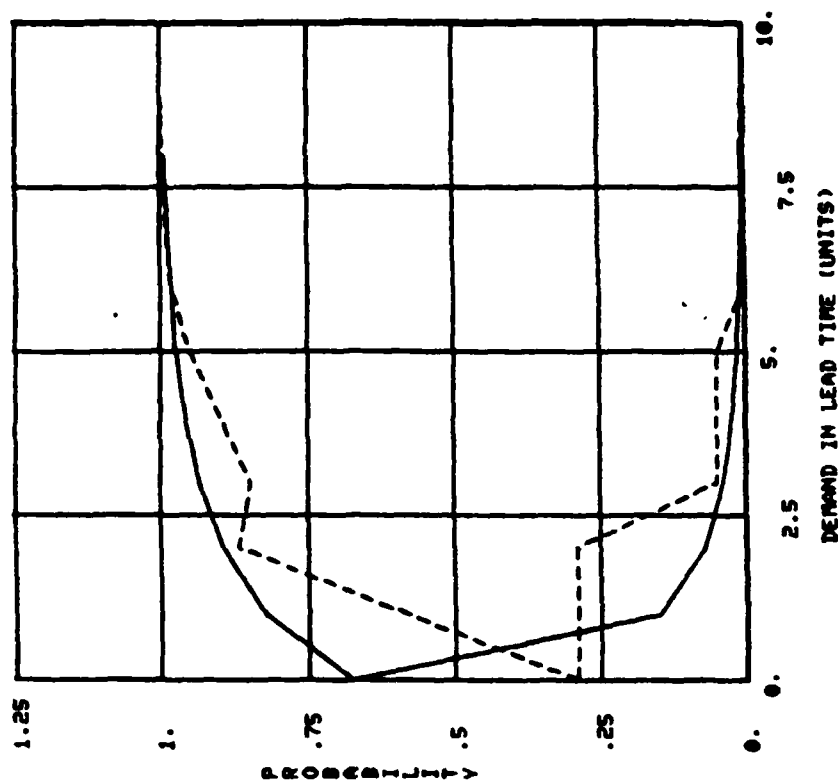
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 0.80
 C OF V OF DLT 2.11
 SKEWNESS 3.52
 KURTOSIS 21.73

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.05
 ALPHA 16.00
 BETA 2.00

X	FX	FMBX	DIFF
0.	0.67	0.29	0.38
1.	0.82	0.52	0.24
2.	0.89	0.87	0.03
3.	0.93	0.85	0.09
4.	0.96	0.90	0.06
5.	0.97	0.95	0.02
6.	0.98	0.98	-0.00
7.	0.99	0.99	-0.00
8.	0.99	0.99	-0.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE 0.384 FOR X = 0

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF U OF LT 0.25

LPG DISTRIBUTION PARAMETERS

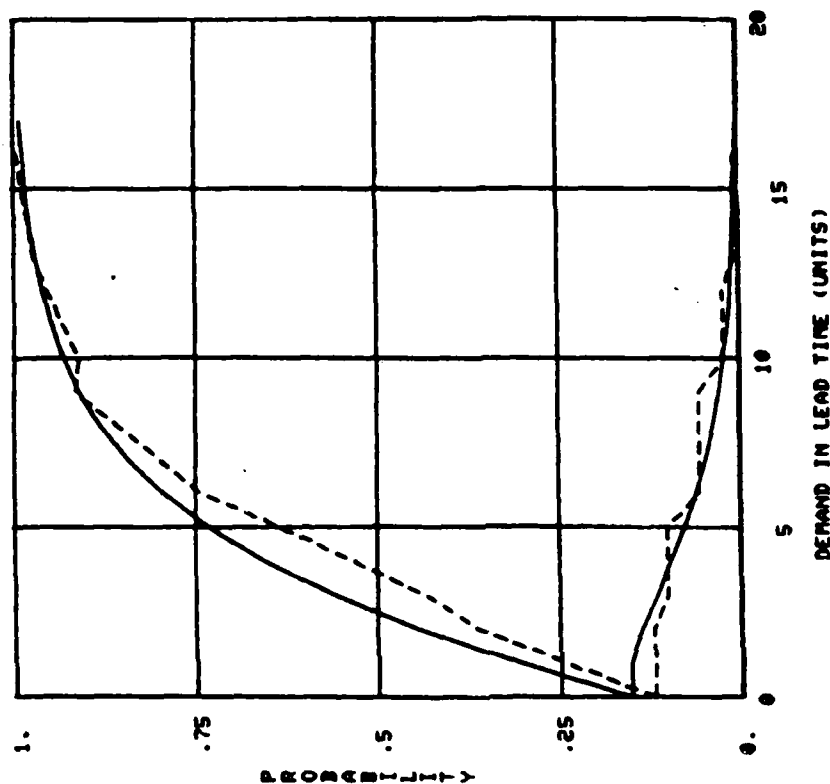
MEAN DEMAND 4.00
 C OF U OF DLT 0.97

SKEWNESS 1.46
 KURTOSIS 6.40

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.91	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.99	0.99	-0.00
16.	0.99	0.99	-0.01



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 1.00
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

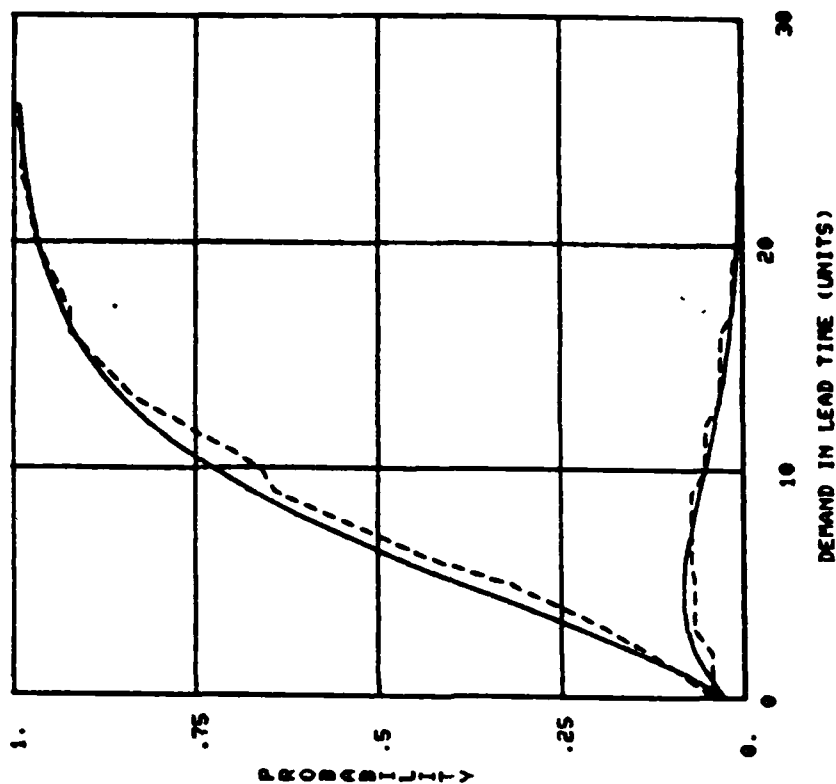
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 8.00
 C OF V OF DLT 0.71
 SKEWNESS 0.95
 KURTOSIS 4.53

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.50
 ALPHA 16.00
 BETA 2.00

X	FX	FNBX	DIFF
0.	0.03	0.04	-0.02
1.	0.08	0.09	-0.01
2.	0.15	0.13	0.02
3.	0.23	0.19	0.04
4.	0.31	0.25	0.05
5.	0.39	0.32	0.07
6.	0.47	0.43	0.04
7.	0.54	0.50	0.05
8.	0.61	0.57	0.04
9.	0.67	0.64	0.03
10.	0.73	0.66	0.07
11.	0.77	0.71	0.06
12.	0.81	0.77	0.04
13.	0.85	0.83	0.02
14.	0.87	0.86	0.02
15.	0.90	0.89	0.01
16.	0.92	0.92	-0.00
17.	0.93	0.92	0.01
18.	0.95	0.94	0.01
19.	0.96	0.95	0.00
20.	0.97	0.97	-0.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.070 FOR X = 5

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 1.50
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

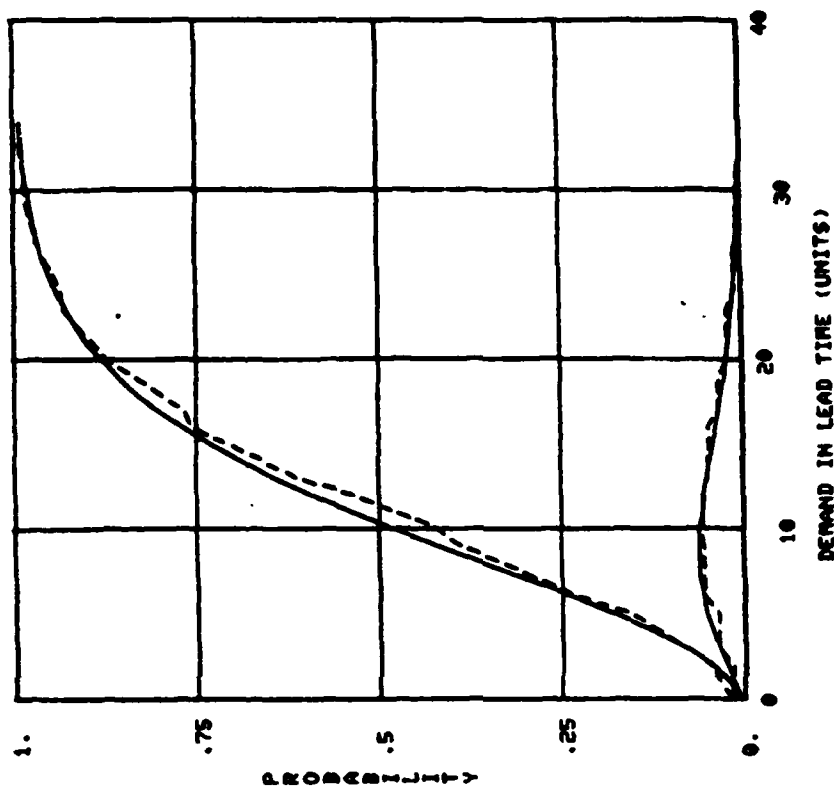
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 12.00
 C OF V OF DLT 0.60
 SKEWNESS 0.73
 KURTOSIS 3.92

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.75
 ALPHA 16.00
 BETA 2.00

X	FX	FMAX	DIFF
0.	0.01	0.02	-0.01
1.	0.02	0.03	-0.01
2.	0.05	0.05	-0.00
3.	0.08	0.08	0.00
4.	0.13	0.12	0.01
5.	0.18	0.15	0.03
6.	0.24	0.23	0.01
7.	0.30	0.28	0.02
8.	0.36	0.34	0.02
9.	0.42	0.39	0.03
10.	0.48	0.42	0.06
11.	0.54	0.48	0.06
12.	0.59	0.53	0.06
13.	0.64	0.61	0.03
14.	0.69	0.66	0.03
15.	0.73	0.71	0.02
16.	0.77	0.76	0.01
17.	0.80	0.77	0.04
18.	0.83	0.80	0.03
19.	0.86	0.83	0.02
20.	0.88	0.87	0.01



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.062 FOR X = 11

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

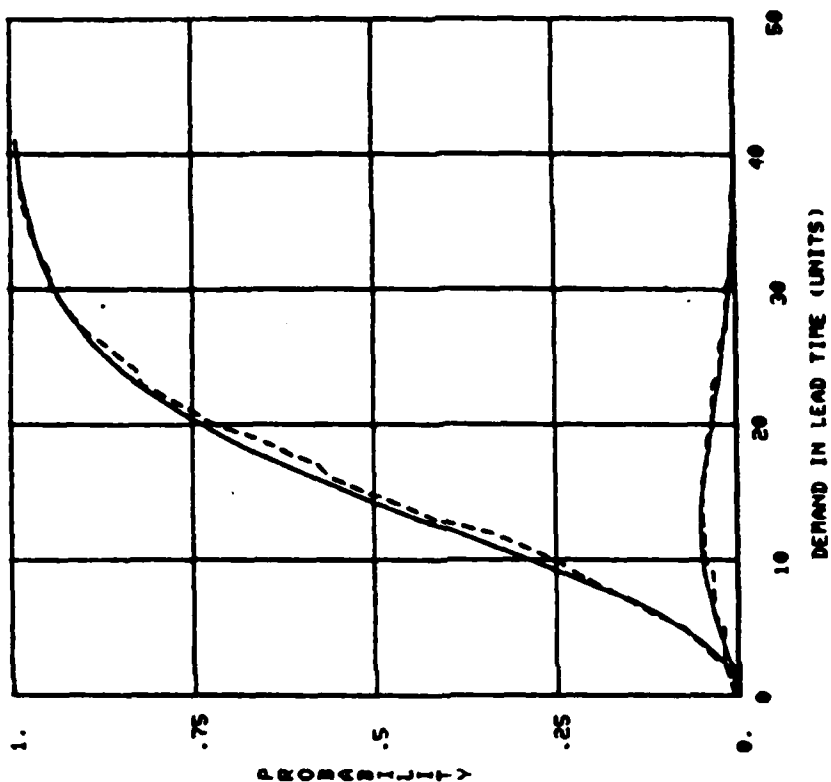
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 0.53
 SKEWNESS 0.60
 KURTOSIS 3.64

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.03
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.58	0.56	0.02
17.	0.62	0.58	0.05
18.	0.67	0.62	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE 0.047 FOR X = 17

SAMPLE LPG CALCULATION

Data Set No.	<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	0.5	0.5	0.5	0.5	0.5
Mean Lead Time	8	8.0	8.0	8.0	8.0
CV of Lead Time	.01	.25	.50	.75	1.0

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	.25	.25	.25	.25	.25
Alpha	1,000	16.00	4.00	1.78	1.00
Beta	1,250	2.00	.50	.22	.13

LPG Moments

Mean	4.00	4.00	4.00	4.00	4.00
CV	.94	.97	1.06	1.20	1.37
Skewness	1.61	1.46	1.21	1.13	1.29
Kurtosis	6.84	6.40	5.55	5.15	5.76

<u>Percentage Points</u>	<u>16</u>		<u>17</u>		<u>18</u>		<u>19</u>		<u>20</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	3	4	3	4	3	4	2	3	2	2
.60	4	5	4	5	4	5	3	5	3	4
.70	5	6	5	6	5	6	5	6	5	6
.80	7	7	7	7	7	8	7	8	7	8
.85	8	8	8	8	8	9	8	9	9	9
.90	9	9	9	9	10	11	10	11	11	12
.95	11	12	12	12	12	13	14	14	15	15
.97	13	13	13	13	14	14	16	17	18	19
.99	17	15	17	16	19	19	21	21	25	25

Table 11-5. Lead Time Variability Sensitivity for LO Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF V OF LT 0.01

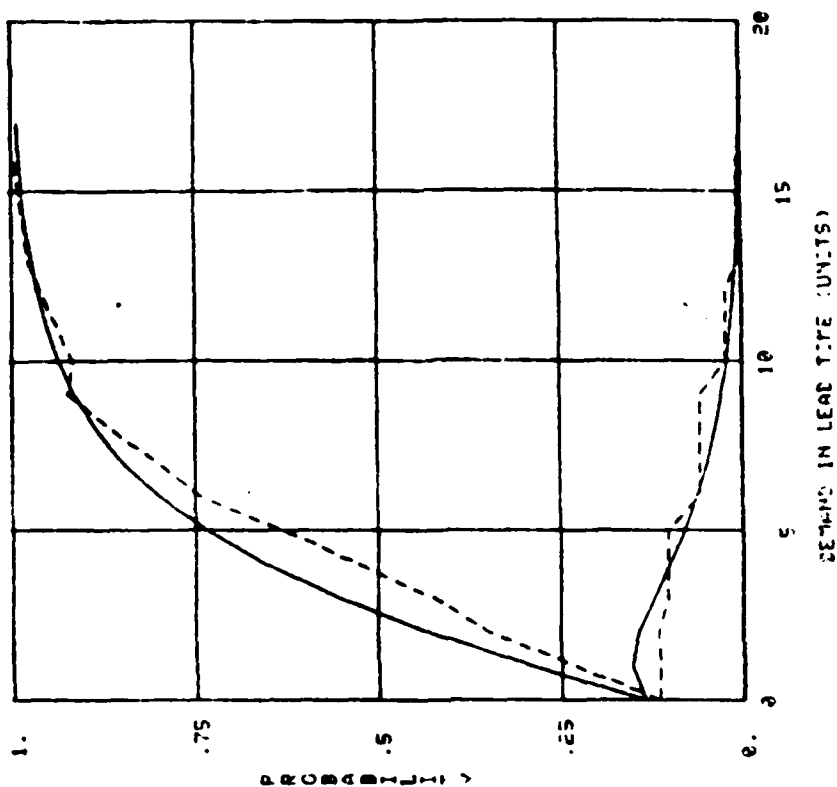
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF V OF DL 0.54
 SKEWNESS 1.61
 KURTOSIS 6.84

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 1000.00
 BETA 1250.00

X	FX	FNBX	DIFF
0.	0.14	0.12	0.02
1.	0.29	0.23	0.06
2.	0.43	0.35	0.08
3.	0.55	0.42	0.13
4.	0.66	0.53	0.13
5.	0.74	0.63	0.09
6.	0.80	0.74	0.05
7.	0.85	0.80	0.04
8.	0.89	0.86	0.02
9.	0.91	0.92	-0.01
10.	0.94	0.97	0.02
11.	0.95	0.99	0.01
12.	0.97	0.99	0.00
13.	0.97	0.98	-0.00
14.	0.98	0.97	-0.00
15.	0.99	0.95	-0.00
16.	0.99	0.90	-0.01



----- LPG PROBABILITY
 - - - - - NORMAL NEGATIVE SERIAL

ANY CDF DIFFERENCE 0.13: FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

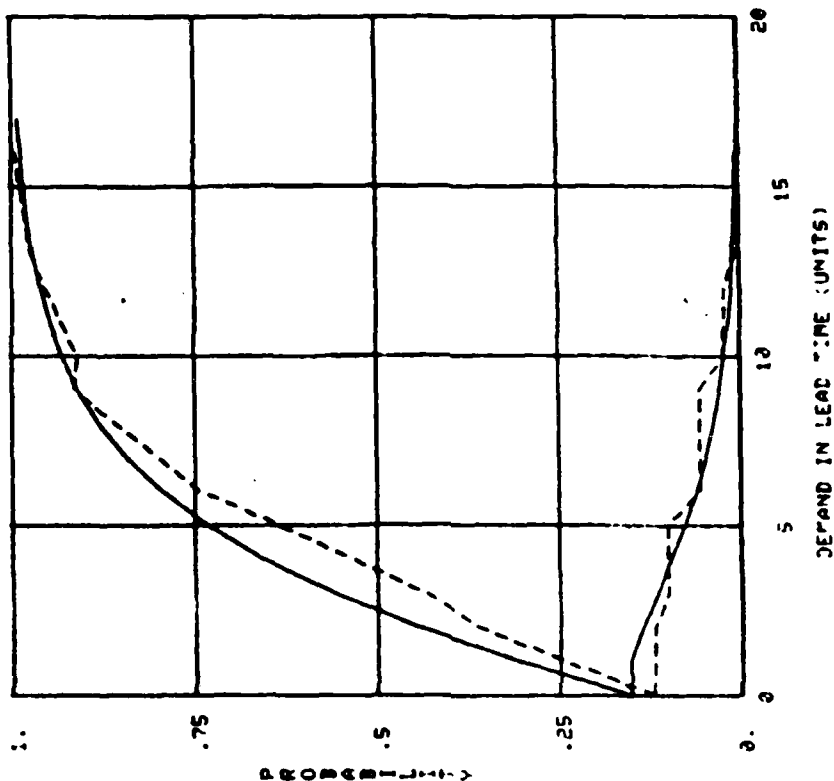
LPC DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF V OF DLT 0.97
 SKEWNESS 1.46
 KURTOSIS 6.40

LPC PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.93	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.01
16.	0.99	0.99	-0.01



----- EXACT LPC PROBABILITIES
 - - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF U OF LT 0.50

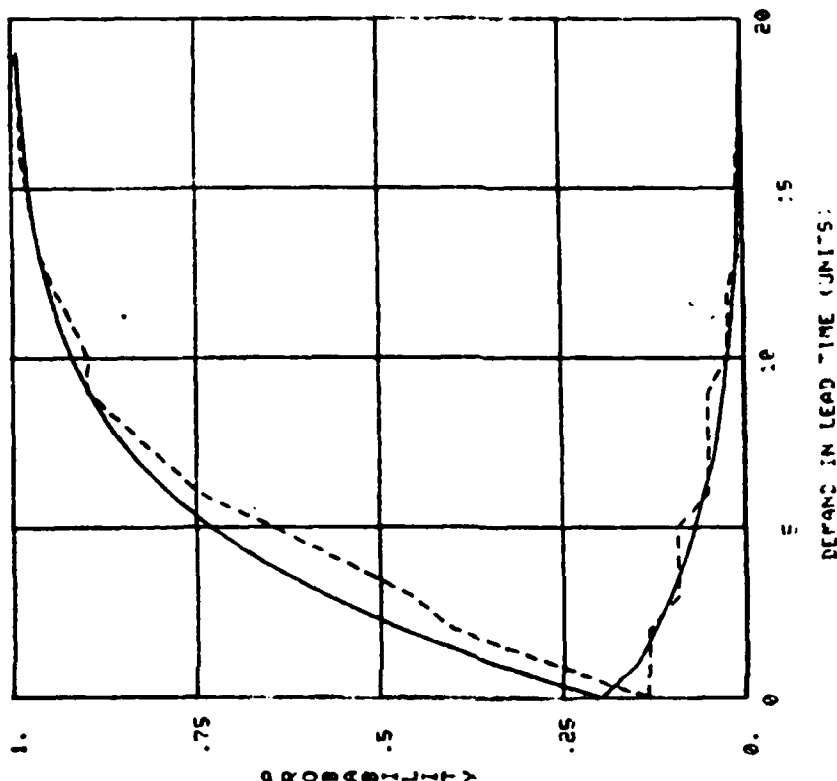
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF U OF DLT 1.06
 SKEWNESS 1.21
 KURTOSIS 5.55

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 4.00
 BETA 0.50

X	FX	FNDX	DIFF
0.	0.20	0.13	0.06
1.	0.35	0.27	0.08
2.	0.47	0.40	0.07
3.	0.58	0.46	0.12
4.	0.66	0.55	0.11
5.	0.73	0.65	0.08
6.	0.79	0.74	0.04
7.	0.83	0.80	0.04
8.	0.87	0.85	0.02
9.	0.90	0.90	-0.00
10.	0.92	0.90	0.02
11.	0.94	0.92	0.02
12.	0.95	0.94	0.01
13.	0.96	0.96	-0.00
14.	0.97	0.97	-0.00
15.	0.98	0.98	-0.00
16.	0.98	0.99	-0.01
17.	0.99	0.99	-0.00
18.	0.99	0.99	-0.00
19.	1.00	1.00	-0.00



----- EXACT LPG PROBABILITIES
 - - - - - SCALED NEGATIVE BINOMIAL

TAX CDF DIFFERENCE = 0.116 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF V OF LT 0.75

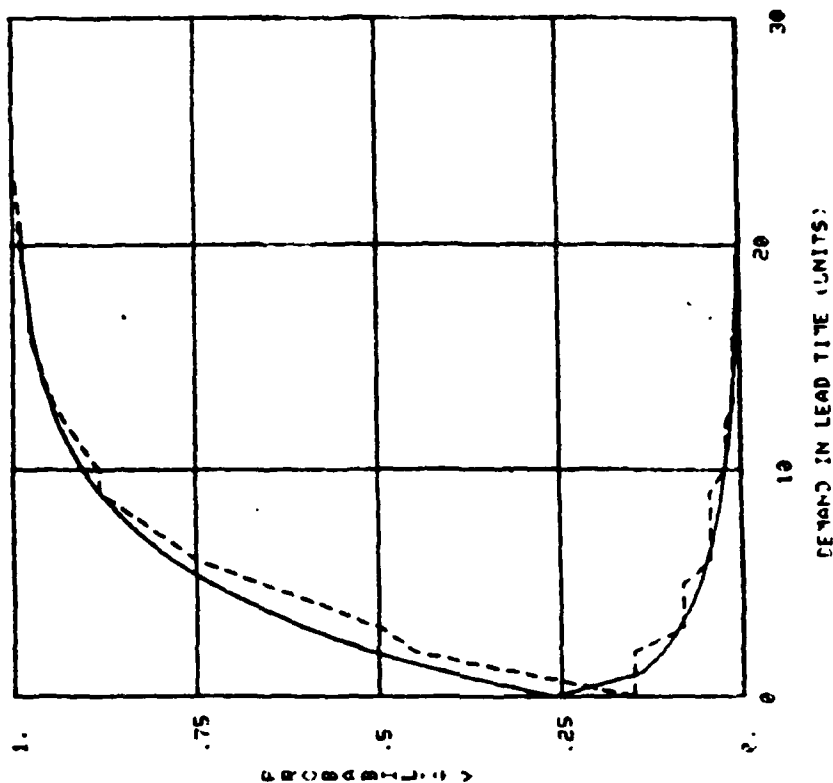
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF V OF DLT 1.20
 SKEWNESS 1.13
 KURTOSIS 5.15

LPG PARAMETERS

T-META 0.72
 LAMBDA 0.25
 ALPHA 1.78
 BETA 0.22

X	FX	FNBX	DIFF
0.	0.26	0.15	0.11
1.	0.40	0.30	0.10
2.	0.51	0.45	0.06
3.	0.60	0.50	0.10
4.	0.67	0.58	0.09
5.	0.73	0.66	0.07
6.	0.78	0.75	0.03
7.	0.82	0.79	0.03
8.	0.85	0.84	0.02
9.	0.88	0.88	0.00
10.	0.92	0.90	0.02
11.	0.94	0.92	0.01
12.	0.95	0.93	0.00
13.	0.96	0.95	0.00
14.	0.97	0.97	-0.00
15.	0.97	0.97	-0.00
16.	0.98	0.98	0.00
17.	0.98	0.98	0.00
18.	0.99	0.99	0.00
19.	0.99	0.99	0.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.111 FOR X = 0

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF V OF LT 1.00

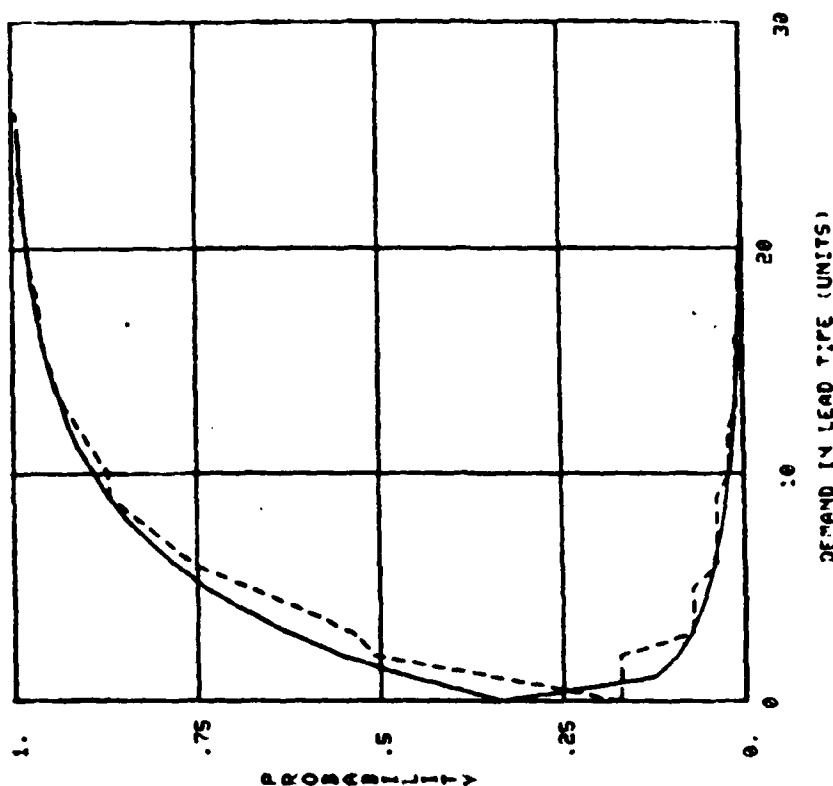
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF V OF DLT 1.37
 SKEWNESS 1.29
 KURTOSIS 5.76

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 1.00
 BETA 0.13

X	FX	FNDX	DIFF
0.	0.33	0.17	0.16
1.	0.46	0.34	0.12
2.	0.55	0.51	0.09
3.	0.63	0.54	0.09
4.	0.69	0.61	0.08
5.	0.74	0.68	0.06
6.	0.78	0.75	0.03
7.	0.81	0.79	0.02
8.	0.84	0.83	0.01
9.	0.87	0.87	0.00
10.	0.91	0.89	0.02
11.	0.92	0.91	0.21
12.	0.93	0.93	0.00
13.	0.94	0.94	0.00
14.	0.95	0.95	0.00
15.	0.96	0.96	-0.00
16.	0.97	0.96	0.00
17.	0.97	0.97	0.00
18.	0.98	0.97	0.00
19.	0.98	0.98	-0.00



----- EXACT LPG PROBABILITIES
 - - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.164 FOR X = 0

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>					
Ave. Req. Size	1.01	2.00	4.00	8.00	16.00					
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00					
Mean Lead Time	8.00	8.00	8.00	8.00	8.00					
CV of Lead Time	.25	.25	.25	.25	.25					
<u>LPG Parameter</u>										
Theta	.02	.72	.90	.96	.99					
Lambda	1.98	1.00	.50	.25	.13					
Alpha	16.00	16.00	16.00	16.00	16.00					
Beta	2.00	2.00	2.00	2.00	2.00					
<u>LPG Moments</u>										
Mean	16.00	16.00	16.00	16.00	16.00					
CV	.36	.53	.84	1.33	2.09					
Skewness	.27	.60	1.35	2.44	4.02					
Kurtosis	3.13	3.64	5.97	12.32	27.79					
<u>Percentage</u>	<u>21</u>		<u>22</u>		<u>23</u>		<u>24</u>		<u>25</u>	
<u>Points</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	15	16	15	15	13	14	8	13	2	22
.60	17	18	17	18	16	19	12	18	5	26
.70	19	19	19	20	20	23	18	26	11	31
.80	21	21	23	23	25	27	27	35	23	37
.85	22	22	25	26	29	31	33	39	33	56
.90	24	24	27	28	34	35	43	47	50	75
.95	26	27	32	32	42	42	59	61	80	93
.97	28	28	35	34	49	46	72	67	105	101
.99	31	32	41	40	61	57	99	88	163	148

Table II-6. Requisition Size Sensitivity for HI Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 1.01
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

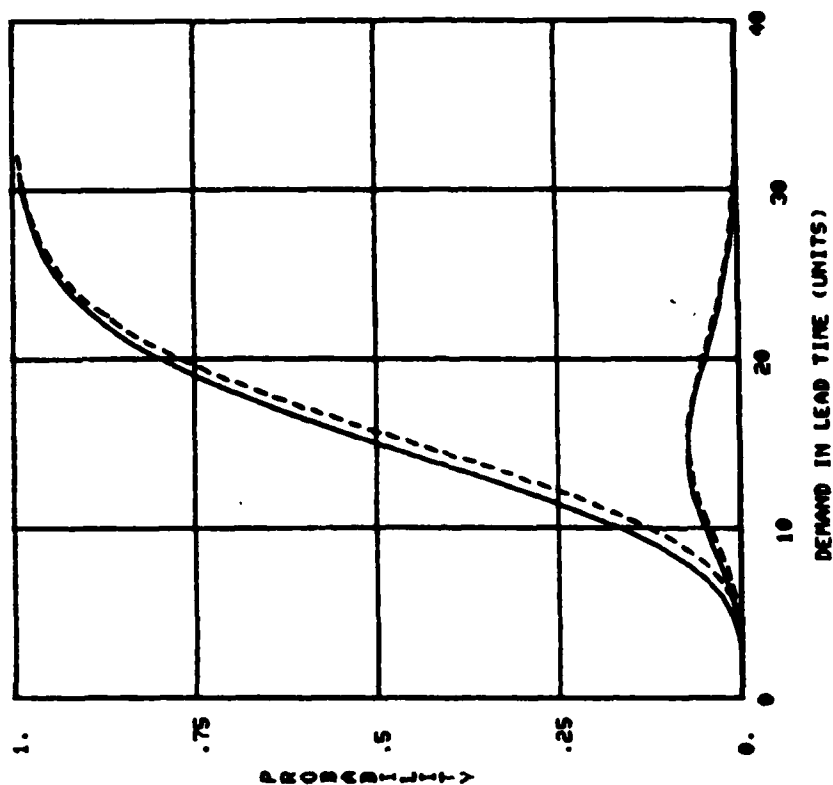
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 15.00
 C OF V OF DLT 0.36
 SKEWNESS 0.27
 KURTOSIS 3.13

LPG PARAMETERS

THETA 0.02
 LAMBDA 1.98
 ALPHA 15.00
 BETA 2.00

X	FX	FMDX	DIFF
0.	0.00	0.00	0.00
1.	0.00	0.00	0.00
2.	0.00	0.00	0.00
3.	0.00	0.00	0.00
4.	0.01	0.00	0.00
5.	0.01	0.01	0.01
6.	0.03	0.01	0.01
7.	0.05	0.03	0.02
8.	0.08	0.05	0.03
9.	0.12	0.08	0.04
10.	0.16	0.12	0.04
11.	0.22	0.18	0.05
12.	0.29	0.24	0.05
13.	0.36	0.30	0.05
14.	0.43	0.38	0.05
15.	0.50	0.45	0.05
16.	0.57	0.52	0.04
17.	0.64	0.59	0.04
18.	0.70	0.66	0.03
19.	0.75	0.72	0.03
20.	0.80	0.77	0.03



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.053 FOR X = 14

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 2.00
 C OF V OF LT 0.25

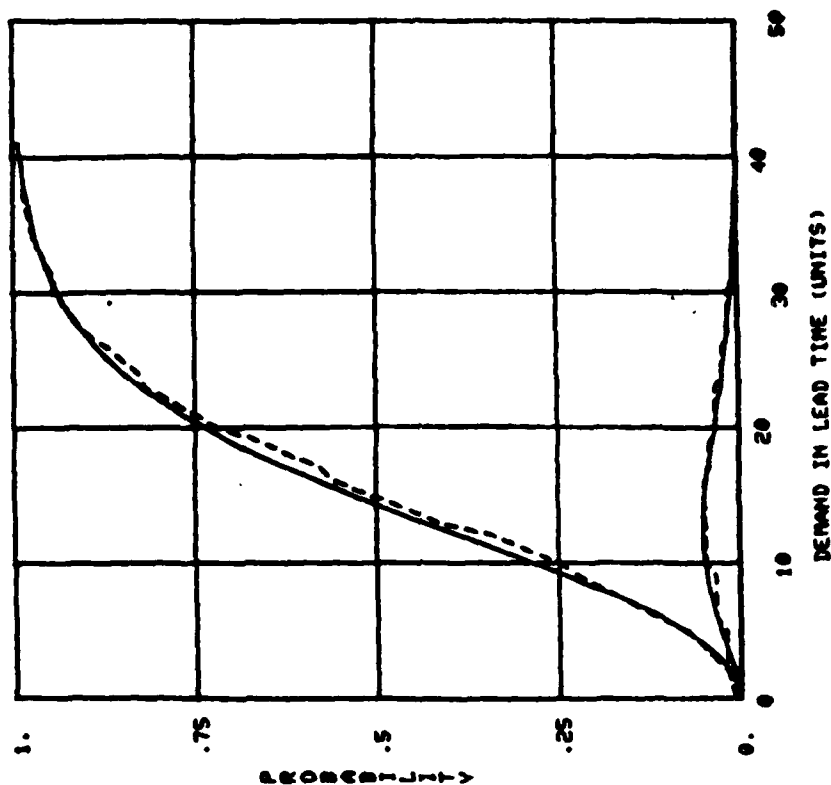
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 0.53
 SKEWNESS 0.60
 KURTOSIS 3.64

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	-0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.03
14.	0.49	0.46	0.03
15.	0.54	0.51	0.02
16.	0.58	0.56	0.02
17.	0.62	0.59	0.05
18.	0.67	0.62	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE: 0.047 FOR X = 17

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 4.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

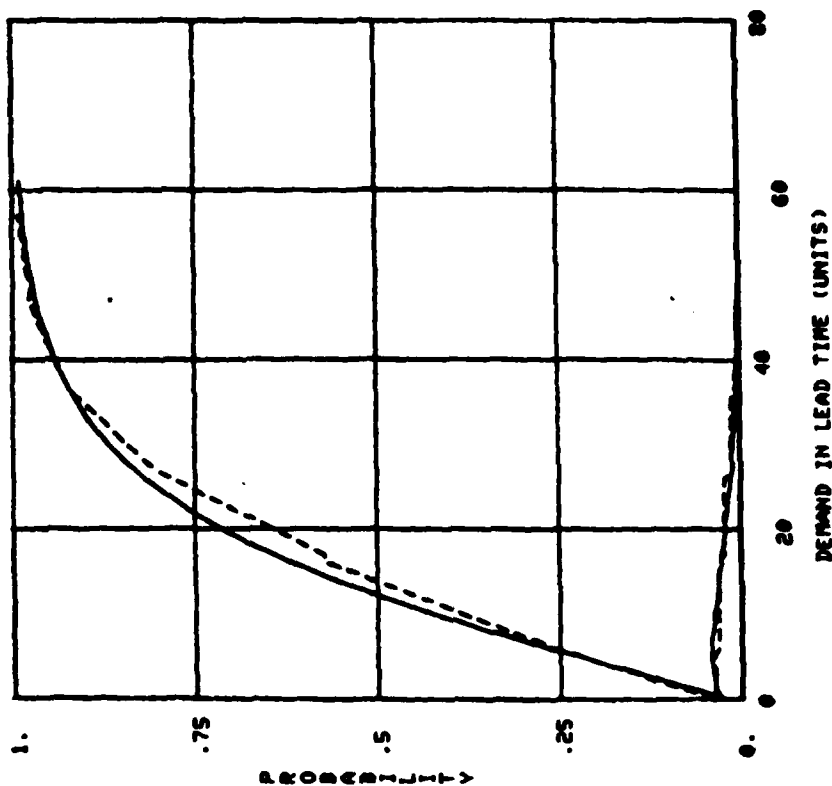
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 0.84
 SKEWNESS 1.35
 KURTOSIS 5.97

LPG PARAMETERS

THETA 0.90
 LAMBDA 0.50
 ALPHA 16.00
 BETA 2.00

X	FX	FNBX	DIFF
0.	0.03	0.04	-0.01
1.	0.06	0.07	-0.01
2.	0.10	0.11	-0.01
3.	0.14	0.15	-0.01
4.	0.18	0.19	-0.01
5.	0.23	0.22	0.00
6.	0.27	0.26	0.01
7.	0.31	0.29	0.02
8.	0.35	0.32	0.03
9.	0.39	0.35	0.04
10.	0.43	0.38	0.04
11.	0.46	0.42	0.05
12.	0.50	0.46	0.05
13.	0.53	0.48	0.05
14.	0.56	0.51	0.05
15.	0.59	0.54	0.05
16.	0.62	0.57	0.05
17.	0.64	0.57	0.07
18.	0.67	0.60	0.07
19.	0.69	0.62	0.07
20.	0.71	0.64	0.07



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE- 0.071 FOR X - 18

DEMAND IN LEAD TIME PARAMETERS

AUE REQ SIZE 8.00
 AUE MONTHLY DEMAND 2.00
 AUE LEAD TIME 8.00
 C OF V OF LT 0.25

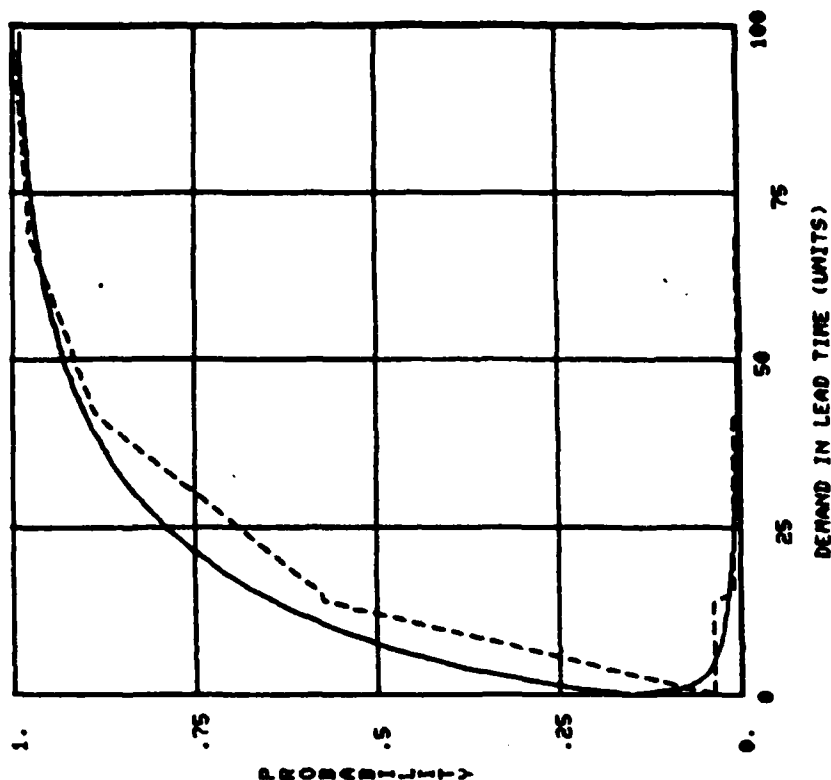
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 1.33
 SKEWNESS 2.44
 KURTOSIS 12.32

LPG PARAMETERS

THETA 0.96
 LATBDA 0.25
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.15	0.04	0.11
1.	0.23	0.08	0.15
2.	0.29	0.11	0.17
3.	0.34	0.15	0.19
4.	0.38	0.19	0.19
5.	0.42	0.23	0.19
6.	0.45	0.27	0.19
7.	0.48	0.31	0.18
8.	0.51	0.34	0.17
9.	0.54	0.38	0.16
10.	0.56	0.42	0.14
11.	0.58	0.46	0.13
12.	0.61	0.50	0.11
13.	0.63	0.54	0.09
14.	0.64	0.57	0.07
15.	0.66	0.58	0.08
16.	0.68	0.59	0.09
17.	0.69	0.60	0.09
18.	0.71	0.61	0.10
19.	0.72	0.62	0.10
20.	0.73	0.63	0.10



DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 16.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

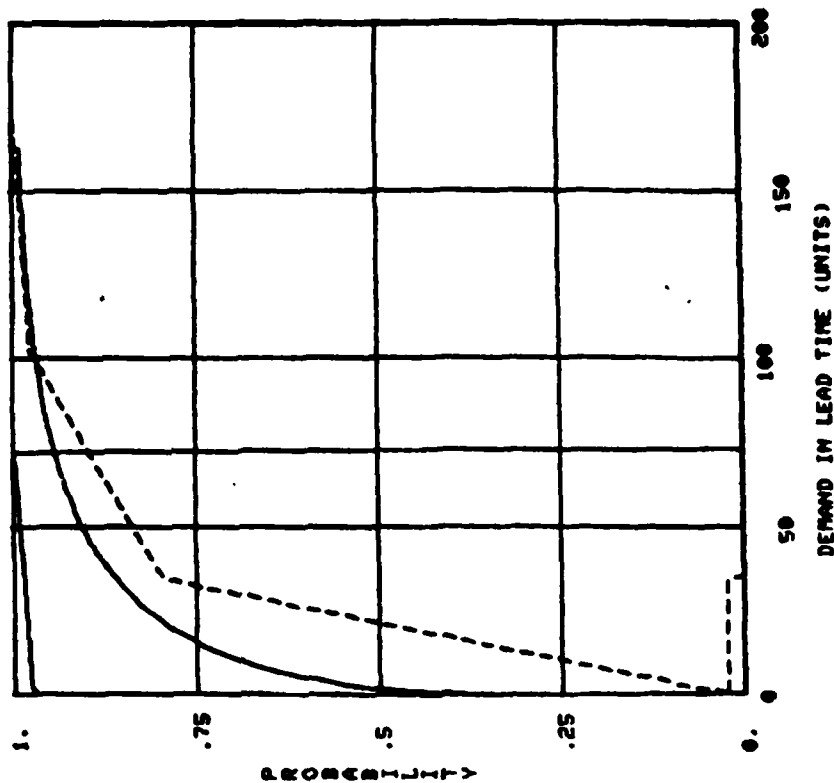
LPC DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 2.00
 SKEWNESS 4.02
 KURTOSIS 27.70

LPC PARAMETERS

THETA 0.99
 LAMBDA 0.13
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.38	0.02	0.36
1.	0.46	0.04	0.42
2.	0.51	0.07	0.45
3.	0.55	0.09	0.46
4.	0.58	0.11	0.47
5.	0.60	0.13	0.47
6.	0.63	0.16	0.47
7.	0.64	0.18	0.47
8.	0.66	0.20	0.46
9.	0.68	0.22	0.45
10.	0.69	0.25	0.44
11.	0.70	0.27	0.43
12.	0.71	0.29	0.42
13.	0.73	0.31	0.41
14.	0.74	0.34	0.40
15.	0.75	0.36	0.39
16.	0.75	0.38	0.37
17.	0.76	0.40	0.36
18.	0.77	0.43	0.34
19.	0.78	0.45	0.33
20.	0.79	0.47	0.31



----- EXACT LPC PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE- 0.470 FOR X - 5

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>26</u>	<u>27</u>	<u>28</u>	<u>29</u>	<u>30</u>
Ave. Req Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	4.00	6.00	8.00	10.00	12.00
CV of Lead Time	.25	.25	.25	.25	.25

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	1.00	1.00	1.00	1.00	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	4.00	2.67	2.00	1.60	1.33

LPG Moments

Mean	8.00	12.00	16.00	20.00	24.00
CV	.71	.60	.53	.49	.46
Skewness	.95	.73	.60	.52	.47
Kurtosis	4.53	3.92	3.64	3.48	3.38

<u>Percentage Points</u>	<u>26</u>		<u>27</u>		<u>28</u>		<u>29</u>		<u>30</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	7	8	11	12	15	15	19	20	23	23
.60	8	9	13	13	17	18	21	22	25	26
.70	10	11	15	15	19	20	24	25	29	29
.80	12	13	17	19	23	23	28	28	33	33
.85	14	14	19	20	25	26	30	30	35	36
.90	16	16	22	22	27	28	33	34	39	39
.95	19	19	25	26	32	32	38	37	44	44
.97	21	21	28	28	35	34	41	41	48	48
.99	26	25	34	33	41	40	48	48	55	55

Table II-7. Lead Time Sensitivity for HI Base Case

DEMAND IN LEAD TIME PARAMETERS

AUE REQ SIZE 2.00
 AUE MONTHLY DEMAND 2.00
 AUE LEAD TIME 4.00
 C OF U OF LT 0.25

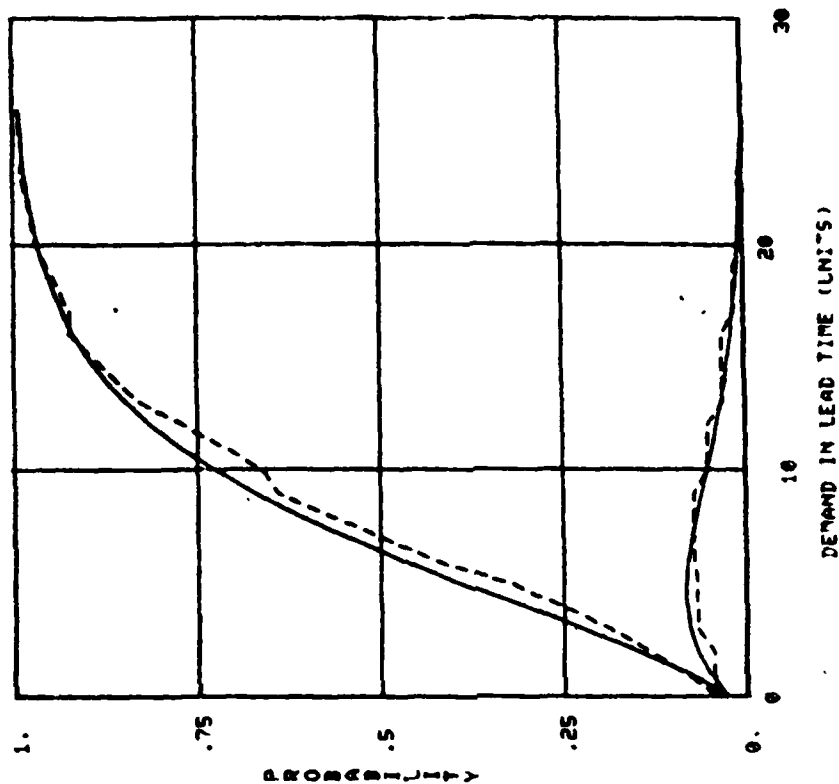
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 8.00
 C OF U OF DLT 0.71
 SKEWNESS 0.95
 KURTOSIS 4.53

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 16.00
 BETA 4.00

X	FX	FNBX	DIFF
0.	0.03	0.04	-0.02
1.	0.08	0.09	-0.01
2.	0.15	0.13	0.02
3.	0.23	0.19	0.04
4.	0.31	0.25	0.05
5.	0.39	0.32	0.07
6.	0.47	0.43	0.04
7.	0.54	0.50	0.05
8.	0.61	0.57	0.04
9.	0.67	0.64	0.03
10.	0.72	0.66	0.07
11.	0.77	0.71	0.06
12.	0.81	0.77	0.04
13.	0.85	0.83	0.02
14.	0.87	0.86	0.02
15.	0.90	0.89	0.01
16.	0.92	0.92	-0.00
17.	0.93	0.92	0.01
18.	0.95	0.94	0.01
19.	0.96	0.95	0.00
20.	0.97	0.97	-0.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.070 FOR X = 5

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 6.00
 C OF U OF LT 3.25

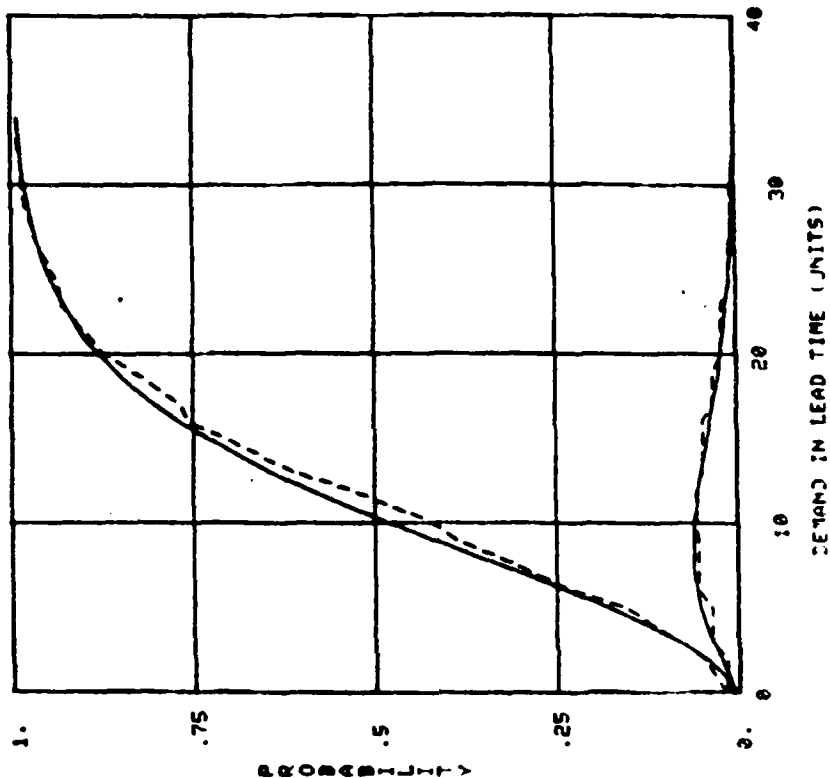
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 12.00
 C OF U OF DLT 0.60
 SKEWNESS 0.73
 KURTOSIS 3.92

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 16.00
 BETA 2.67

X	FX	FNBX	DIFF
0.	0.01	0.02	-0.01
1.	0.02	0.03	-0.01
2.	0.05	0.05	-0.00
3.	0.08	0.08	0.00
4.	0.13	0.12	0.01
5.	0.18	0.15	0.03
6.	0.24	0.23	0.01
7.	0.30	0.28	0.02
8.	0.36	0.34	0.02
9.	0.42	0.39	0.03
10.	0.48	0.42	0.06
11.	0.54	0.48	0.36
12.	0.59	0.53	0.06
13.	0.64	0.61	0.03
14.	0.69	0.66	0.03
15.	0.73	0.71	0.02
16.	0.77	0.76	0.01
17.	0.80	0.77	0.04
18.	0.83	0.80	0.03
19.	0.86	0.83	0.02
20.	0.88	0.87	0.01



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.062 FOR X = 1:

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

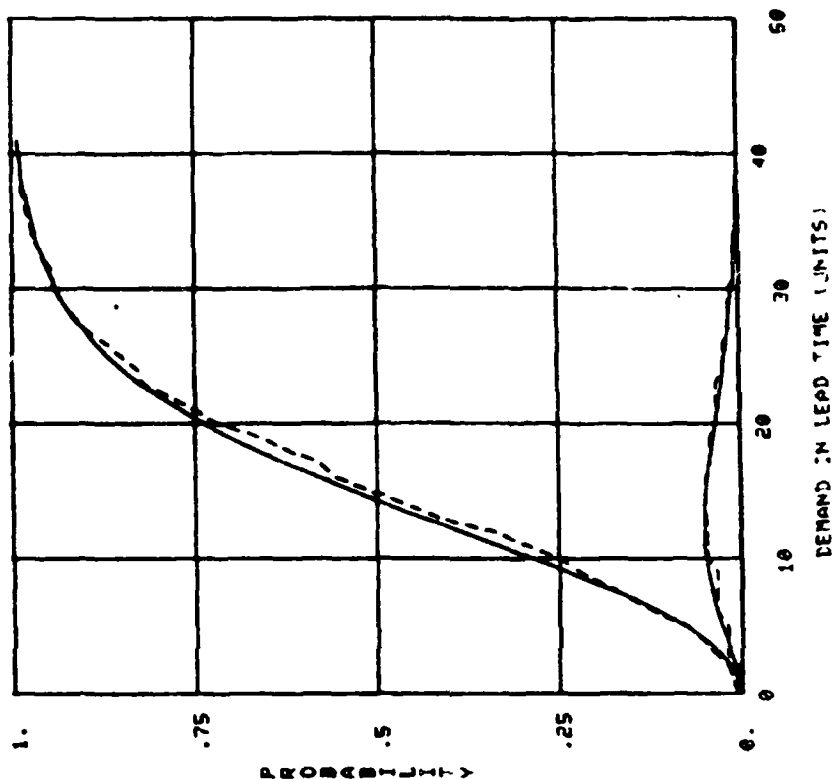
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 0.53
 SKEWNESS 0.60
 KURTOSIS 3.64

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.03
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.59	0.55	0.02
17.	0.62	0.58	0.05
18.	0.65	0.61	0.04
19.	0.67	0.63	0.02
20.	0.70	0.66	0.02
21.	0.72	0.68	0.02



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 2.047 FOR X = 17

DEMAND IN LEAD TIME PARAMETERS

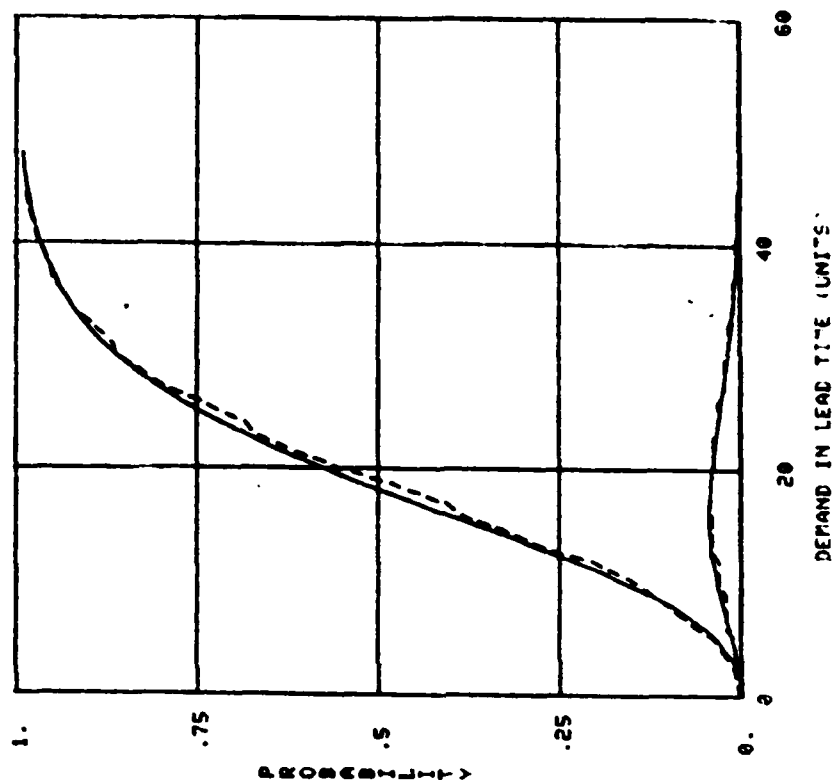
AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 10.00
 C OF V OF LT 0.25

LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 20.00
 C OF V OF DLT 0.49
 SKEWNESS 0.52
 KURTOSIS 3.48

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 16.00
 BETA 1.60



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.042 FOR X = 18

X	FY	FNBX	DIFF
0.	0.00	0.00	-0.00
1.	0.00	0.01	-0.00
2.	0.01	0.01	-0.00
3.	0.01	0.02	-0.01
4.	0.02	0.03	-0.01
5.	0.03	0.04	-0.00
6.	0.05	0.06	-0.01
7.	0.07	0.08	-0.01
8.	0.10	0.10	-0.00
9.	0.13	0.12	0.00
10.	0.16	0.14	0.02
11.	0.20	0.18	0.02
12.	0.24	0.21	0.03
13.	0.28	0.27	0.01
14.	0.32	0.31	0.01
15.	0.36	0.35	0.02
16.	0.41	0.39	0.02
17.	0.45	0.41	0.04
18.	0.49	0.45	0.04
19.	0.53	0.49	0.04
20.	0.57	0.55	0.02

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 12.00
 C OF U OF LT 0.25

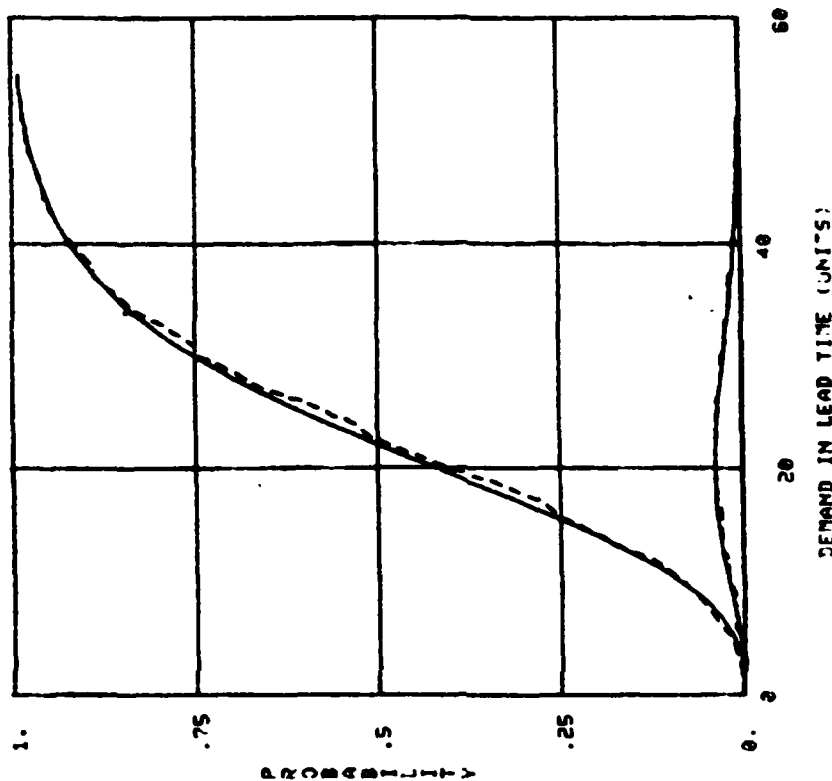
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 24.00
 C OF U OF DLT 0.46
 SKEWNESS 0.47
 KURTOSIS 3.38

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 16.00
 BETA 1.33

X	FX	FNBX	DIFF
0.	0.00	0.00	-0.00
1.	0.00	0.00	-0.00
2.	0.00	0.00	-0.00
3.	0.00	0.01	-0.00
4.	0.01	0.01	-0.00
5.	0.01	0.02	-0.00
6.	0.02	0.03	-0.01
7.	0.03	0.24	-0.01
8.	0.05	0.05	-0.01
9.	0.07	0.07	-0.00
10.	0.09	0.08	0.00
11.	0.11	0.10	0.01
12.	0.14	0.12	0.01
13.	0.17	0.15	0.03
14.	0.20	0.19	0.00
15.	0.23	0.22	0.01
16.	0.27	0.25	0.01
17.	0.30	0.28	0.03
18.	0.34	0.31	0.03
19.	0.38	0.35	0.03
20.	0.42	0.40	0.02



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.035 FOR X = 24

SAMPLE LPG CALCULATIONS

<u>Data Set No.</u>	<u>31</u>	<u>32</u>	<u>33</u>	<u>34</u>	<u>35</u>
Ave. Req Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	.10	.50	1.00	1.50	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda ..	.05	.25	.50	.75	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00

LPG Moments

Mean	.80	4.00	8.00	12.00	16.00
CV	2.11	.97	.71	.60	.53
Skewness	3.52	1.46	.95	.73	.60
Kurtosis	21.73	6.40	4.53	3.92	3.64

<u>Percentage Points</u>	<u>31</u>		<u>32</u>		<u>33</u>		<u>34</u>		<u>35</u>		
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	
.50	0	1	3	4	7	8	11	12	15	15	
.60	0	2	4	5	8	9	13	13	17	18	
.70	1	2	5	6	10	11	15	15	19	20	
.80	1	2	7	7	12	13	17	19	23	23	
.85	2	2	8	8	14	14	19	20	25	26	
.90	3	5	1	9	9	16	16	22	22	27	28
.95	4	5	12	12	19	19	25	26	32	32	
.97	5	6	13	3	21	21	28	28	35	34	
.99	8	8	17	16	26	25	34	33	41	40	

Table II-8. Demand Rate Sensitivity for HI Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.10
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

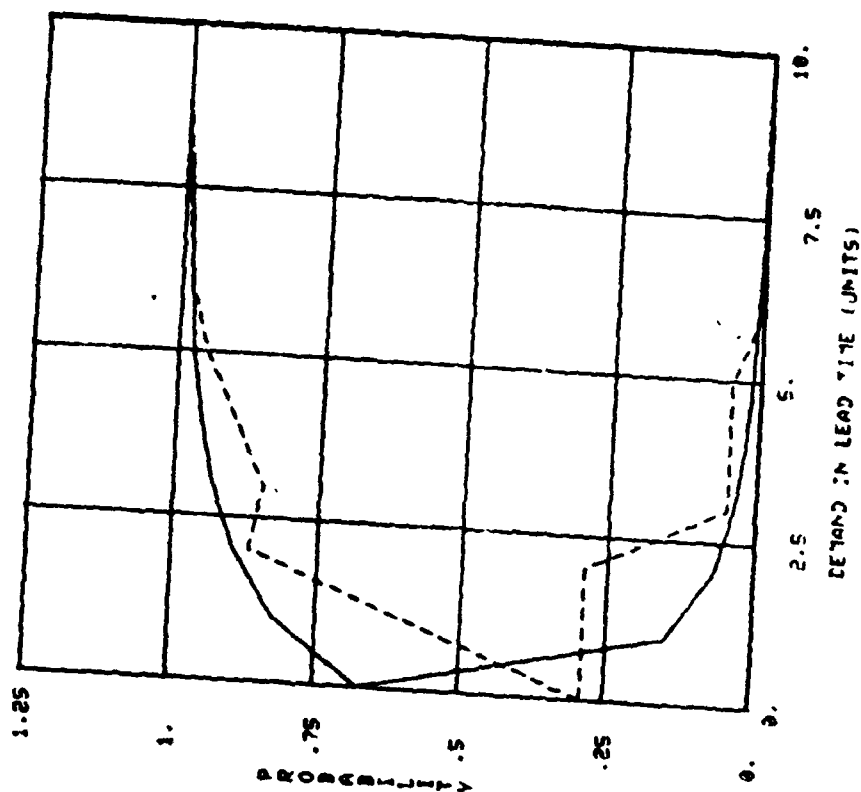
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 0.80
 C OF V OF DLT 2.11
 SKEWNESS 3.52
 KURTOSIS 21.73

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.05
 ALPHA 16.00
 BETA 2.00

X	FY	FNBX	DIFF
0.	0.67	0.29	0.38
1.	0.82	0.58	0.24
2.	0.89	0.87	0.03
3.	0.93	0.85	0.09
4.	0.96	0.90	0.06
5.	0.97	0.95	0.02
6.	0.98	0.98	-0.00
7.	0.99	0.99	-0.00
8.	0.99	0.99	-0.00



----- EXACT LPG PROBABILITIES
 - - - - - SCHEMATIC NEGATIVE BINOMIAL

MAX CDF DIFFERENCE = 0.284 FOR X = 0

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 0.50
 AVE LEAD TIME 8.00
 C OF U OF LT 0.25

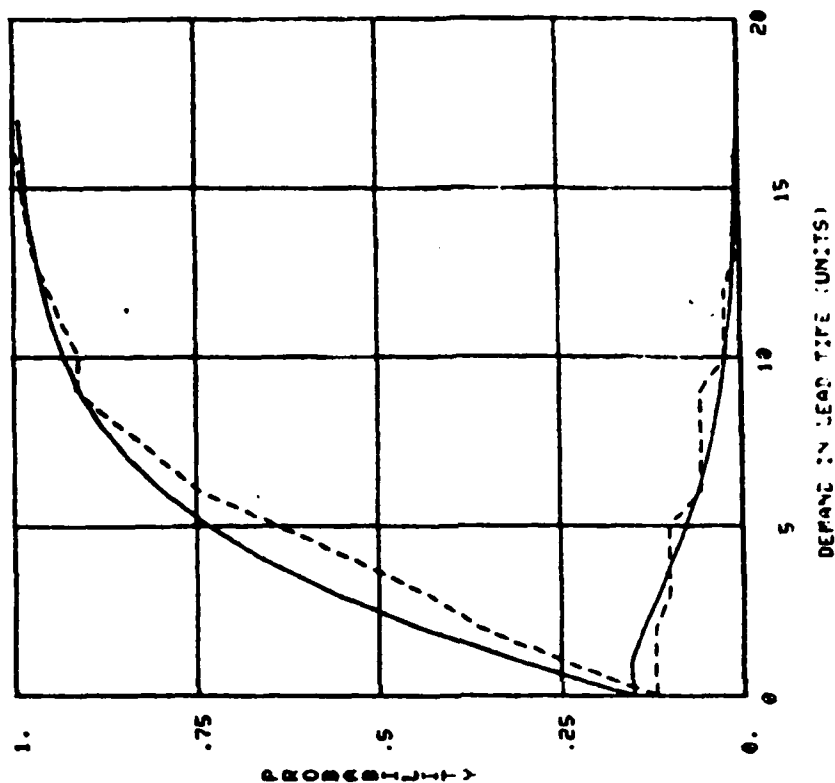
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 4.00
 C OF U OF DLT 0.87
 SKEWNESS 1.46
 KURTOSIS 6.40

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.25
 ALPHA 16.00
 BETA 2.00

X	FX	FNBX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.93	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.01
16.	0.99	0.99	-0.01



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 1.00
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

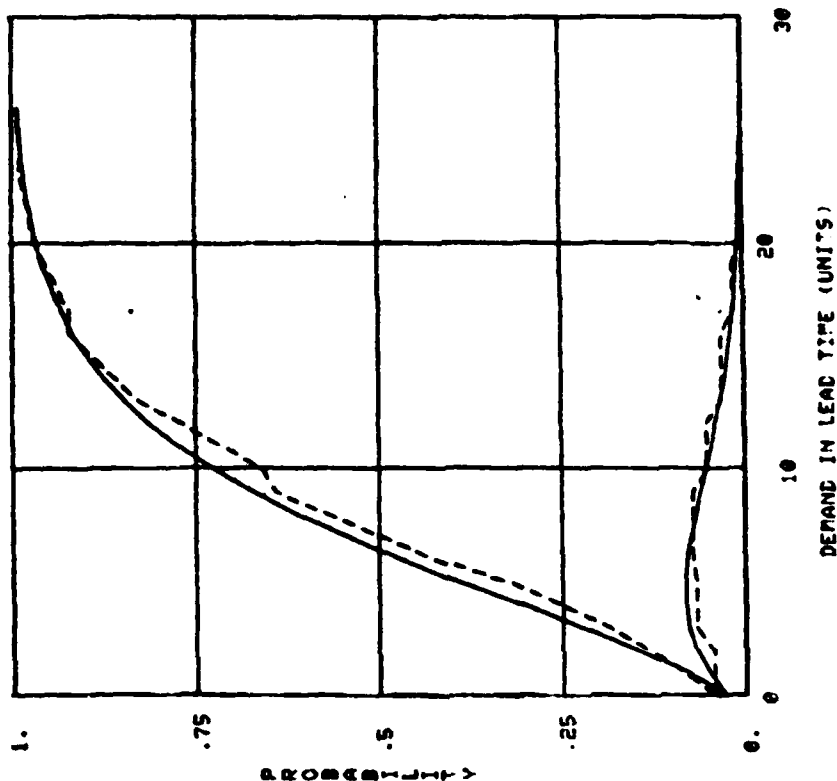
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 8.00
 C OF V OF DLT 0.71
 SKEWNESS 0.95
 KURTOSIS 4.53

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.50
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.03	0.04	-0.02
1.	0.08	0.09	-0.01
2.	0.15	0.13	0.02
3.	0.23	0.19	0.04
4.	0.31	0.25	0.05
5.	0.39	0.32	0.07
6.	0.47	0.43	0.04
7.	0.54	0.50	0.05
8.	0.61	0.57	0.04
9.	0.67	0.64	0.03
10.	0.73	0.66	0.07
11.	0.77	0.71	0.06
12.	0.81	0.77	0.04
13.	0.85	0.83	0.02
14.	0.87	0.86	0.02
15.	0.90	0.83	0.01
16.	0.92	0.92	-0.00
17.	0.93	0.92	0.01
18.	0.95	0.94	0.01
19.	0.95	0.95	0.00
20.	0.97	0.97	-0.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.070 FOR X = 5

DEMAND IN LEAD TIME PARAMETERS

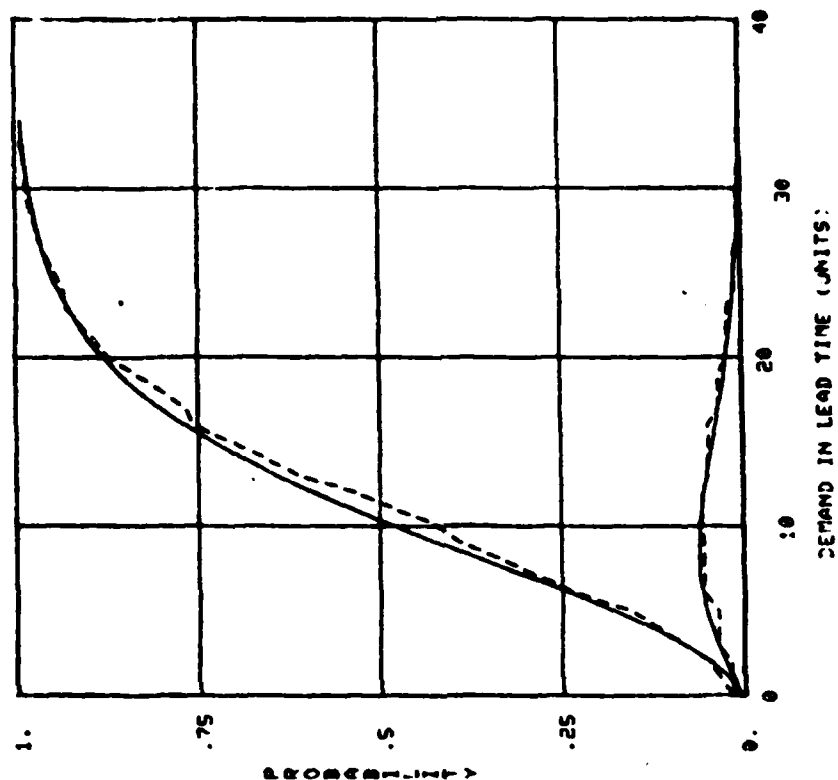
AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 1.50
 AVE LEAD TIME 3.00
 C OF V OF LT 0.25

LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 12.00
 C OF V OF DLT 0.60
 SKEWNESS 0.73
 KURTOSIS 3.92

LPG PARAMETERS

THETA 0.72
 LAMBDA 0.75
 ALPHA 16.00
 BETA 2.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

PAY CDF DIFFERENCE 0.062 FOR $\lambda = 11$

X	FX	FMBX	DIFF
0	0.01	0.02	-0.01
1	0.02	0.03	-0.01
2	0.05	0.05	-0.00
3	0.08	0.08	-0.00
4	0.13	0.12	0.01
5	0.18	0.15	0.03
6	0.24	0.23	0.01
7	0.30	0.28	0.02
8	0.36	0.34	0.02
9	0.42	0.39	0.03
10	0.48	0.42	0.06
11	0.54	0.48	0.06
12	0.59	0.53	0.06
13	0.64	0.61	0.03
14	0.69	0.66	0.03
15	0.73	0.71	0.02
16	0.77	0.76	0.01
17	0.80	0.77	0.03
18	0.83	0.82	0.01
19	0.86	0.87	0.00
20	0.88	0.93	0.05

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 8.00
 C OF V OF LT 0.25

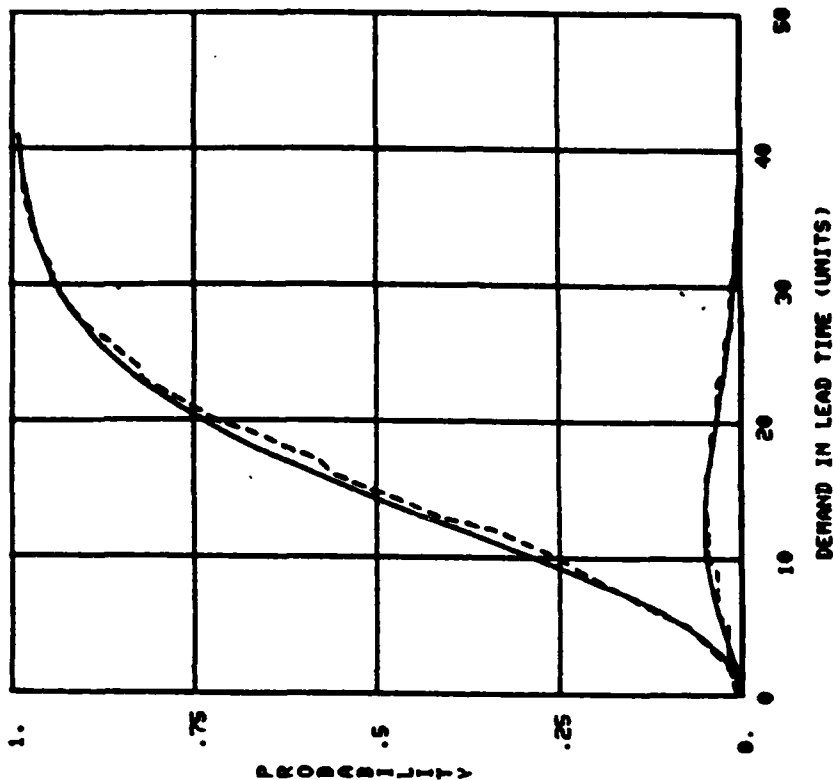
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 0.53
 SKEWNESS 0.60
 KURTOSIS 3.64

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 16.00
 BETA 2.00

X	FX	FNDX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.06	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.20	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.02
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.58	0.56	0.02
17.	0.62	0.58	0.05
18.	0.67	0.62	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.047 FOR X = 17

SAMPLE LPG CALCULATIONS

<u>Date Set No.</u>	<u>36</u>	<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.01	.25	.50	.75	1.00

LPG Parameter

Theta	.72	.72	.72	.72	.72
Lambda	1.00	1.00	1.00	1.00	1.00
Alpha	10,000	16.00	4.00	1.78	1.00
Beta	1,250	2.00	.50	.22	.13

LPG Moments

Mean	16.00	16.00	16.00	16.00	16.00
CV	.47	.53	.69	.88	1.10
Skewness	.80	.60	.65	1.03	1.55
Kurtosis	3.96	3.64	3.69	4.86	7.10

<u>Percentage Points</u>	<u>36</u>		<u>37</u>		<u>38</u>		<u>39</u>		<u>40</u>	
	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>	<u>LPG</u>	<u>NB</u>
.50	15	16	15	15	14	14	12	13	10	11
.60	17	18	17	18	17	18	16	16	14	15
.70	19	20	19	20	20	20	19	20		
.80	22	22	23	23	24	25	26	23	27	27
.85	24	25	25	26	27	27	30	26	32	33
.90	26	27	27	28	31	31	35	35	39	40
.95	30	29	32	32	37	37	44	44	51	51
.97	32	32	35	34	41	41	50	50	60	61
.99	38	36	41	40	50	51	64	63	80	80

Table II-9. Lead Time Variability Sensitivity for HI Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 8.00
 C OF U OF LT 0.01

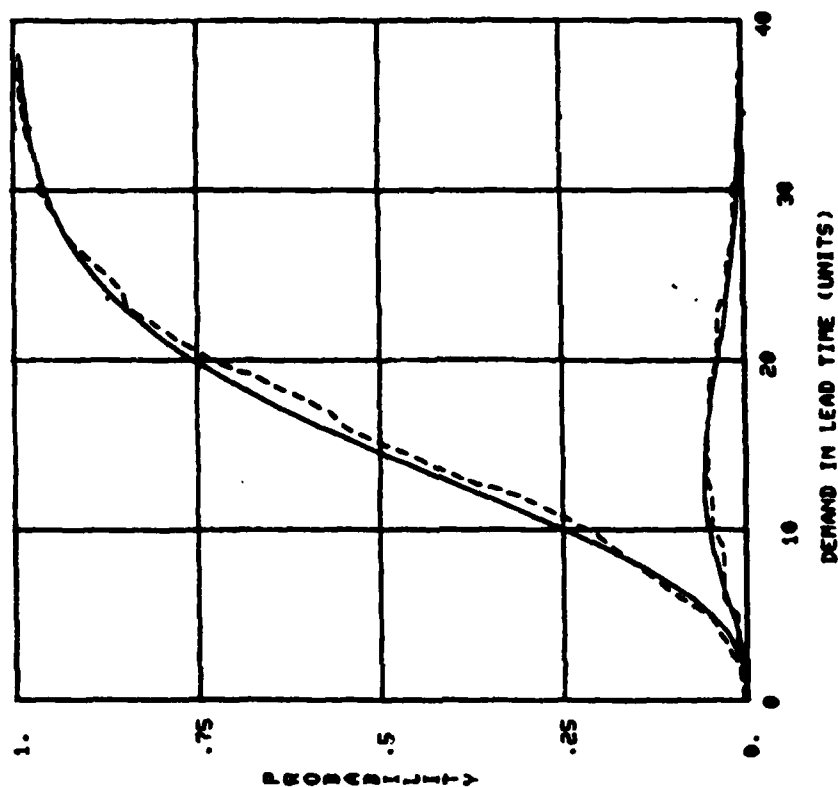
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF U OF DLT 0.47
 SKEWNESS 0.80
 KURTOSIS 3.96

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 10000.00
 BETA 1250.00

X	FX	FNBX	DIFF
0.	0.00	0.00	-0.00
1.	0.00	0.01	-0.01
2.	0.01	0.01	-0.01
3.	0.01	0.02	-0.01
4.	0.03	0.04	-0.01
5.	0.05	0.05	-0.00
6.	0.07	0.09	-0.02
7.	0.11	0.12	-0.01
8.	0.15	0.15	-0.00
9.	0.20	0.18	0.01
10.	0.25	0.21	0.03
11.	0.30	0.26	0.04
12.	0.36	0.31	0.05
13.	0.42	0.30	0.03
14.	0.47	0.44	0.03
15.	0.53	0.49	0.03
16.	0.58	0.55	0.03
17.	0.63	0.57	0.06
18.	0.67	0.62	0.05
19.	0.72	0.67	0.05
20.	0.75	0.73	0.02



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.058 FOR X = 17

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 ONE MONTHLY DEMAND 2.00
 AVE LEAD TIME 2.00
 C OF V OF LT 0.25

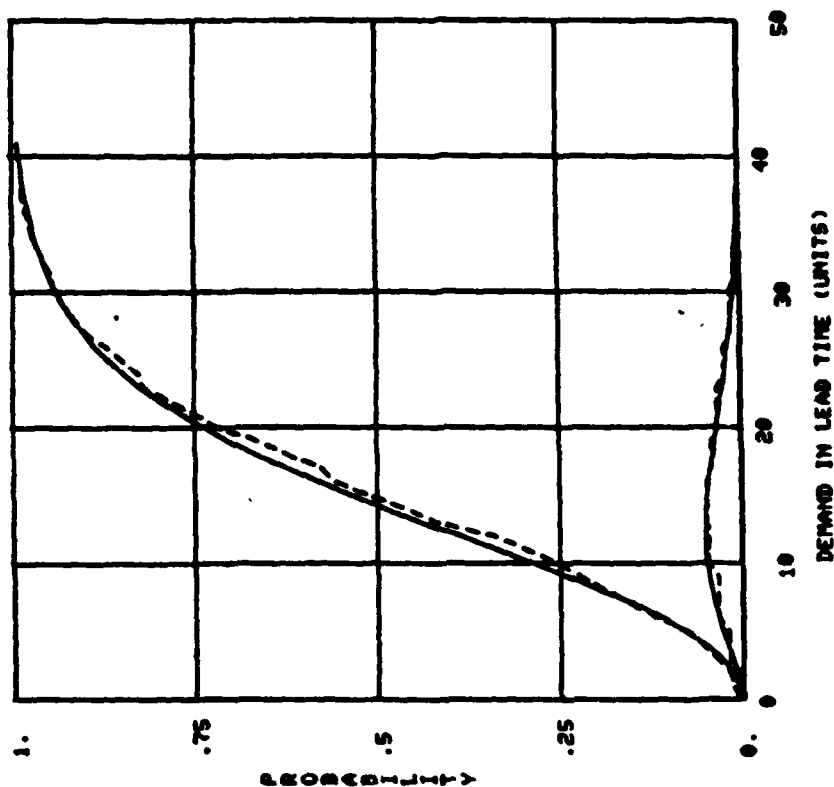
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 0.53
 SKEWNESS 0.60
 KURTOSIS 3.64

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 16.00
 BETA 2.00

X	FX	FMBX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.02
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.58	0.55	0.02
17.	0.62	0.58	0.05
18.	0.67	0.62	0.04
19.	0.70	0.65	0.04
20.	0.74	0.72	0.02



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.047 FOR X = 17

DEMAND IN LEAD TIME PARAMETERS

Avg REQ SIZE 2.00
 Avg MONTHLY DEMAND 2.00
 Avg LEAD TIME 8.00
 C OF V OF LT 0.50

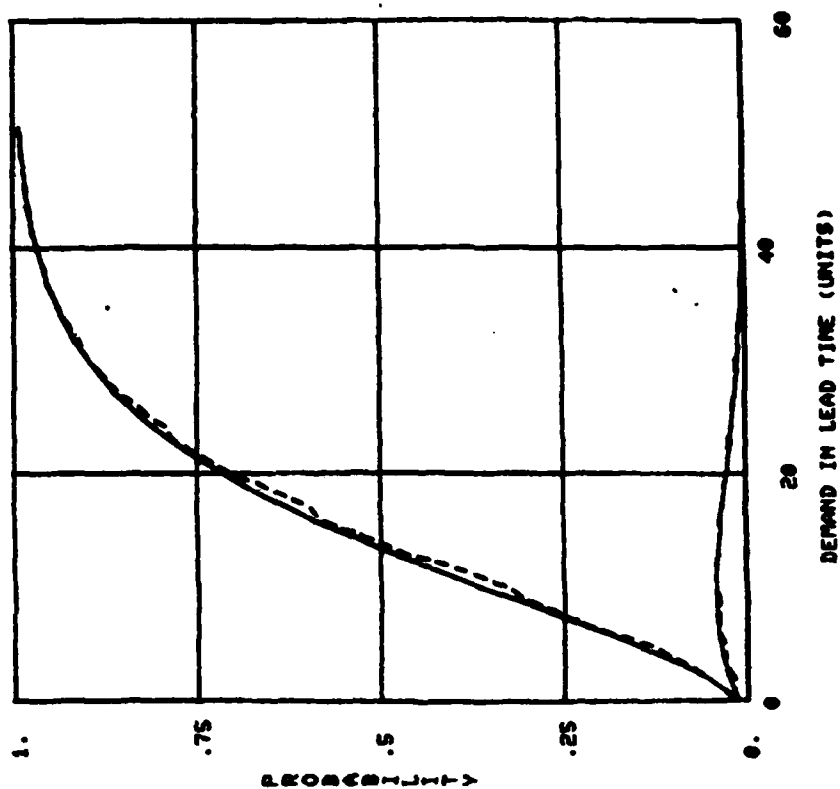
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 0.60
 SKEWNESS 0.65
 KURTOSIS 3.60

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 4.00
 BETA 0.50

X	FX	FNBX	DIFF
0.	0.01	0.02	-0.00
1.	0.03	0.03	-0.00
2.	0.06	0.05	0.00
3.	0.09	0.08	0.01
4.	0.12	0.11	0.01
5.	0.16	0.13	0.02
6.	0.19	0.19	0.01
7.	0.24	0.23	0.01
8.	0.28	0.27	0.01
9.	0.32	0.30	0.02
10.	0.36	0.33	0.04
11.	0.40	0.37	0.04
12.	0.45	0.41	0.04
13.	0.48	0.47	0.02
14.	0.52	0.51	0.02
15.	0.56	0.54	0.02
16.	0.59	0.58	0.01
17.	0.63	0.60	0.03
18.	0.66	0.63	0.03
19.	0.69	0.66	0.03
20.	0.72	0.70	0.01



DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 8.00
 C OF V OF LT 0.75

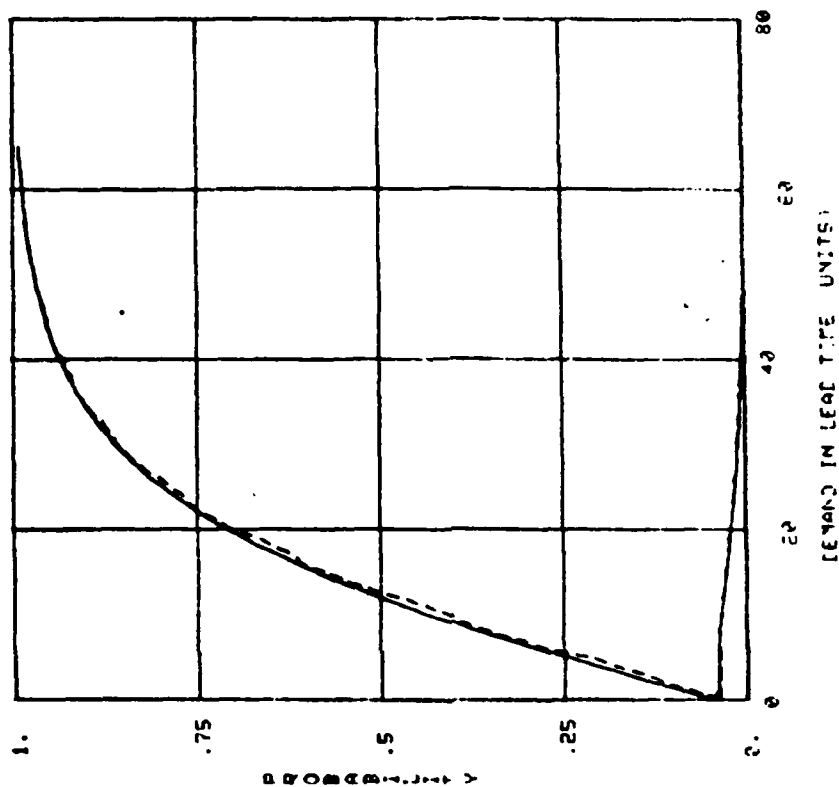
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF V OF DLT 0.88
 SKEWNESS 1.03
 KURTOSIS 4.87

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 1.78
 BETA 0.22

X	FX	FNE	DIF
0.	0.05	0.34	0.01
1.	0.09	0.28	0.01
2.	0.13	0.21	0.01
3.	0.17	0.14	0.03
4.	0.21	0.08	0.03
5.	0.25	0.02	0.03
6.	0.29	0.00	0.01
7.	0.33	0.00	0.01
8.	0.37	0.00	0.01
9.	0.40	0.00	0.01
10.	0.44	0.00	0.03
11.	0.47	0.00	0.03
12.	0.50	0.00	0.03
13.	0.53	0.00	0.01
14.	0.56	0.00	0.01
15.	0.59	0.00	0.01
16.	0.62	0.00	0.01
17.	0.64	0.00	0.02
18.	0.67	0.00	0.02
19.	0.69	0.00	0.01
20.	0.71	0.00	0.01



----- EXACT LPG PROBABILITIES
 - - - - - SCALED NEGATIVE BINOMIAL

PA = CDF DIFFERENCE = 0.031 FOR X = 10

Best available

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE 2.00
 AVE MONTHLY DEMAND 2.00
 AVE LEAD TIME 8.00
 C OF U OF LT 1.00

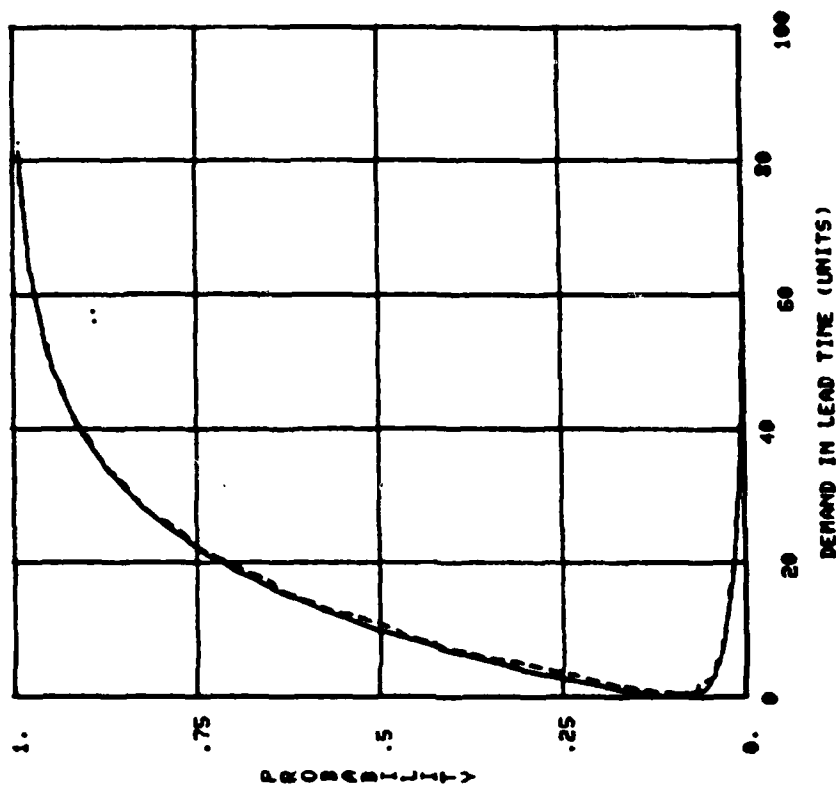
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND 16.00
 C OF U OF DLT 1.10
 SKEWNESS 1.55
 KURTOSIS 7.10

LPG PARAMETERS

THETA 0.72
 LAMBDA 1.00
 ALPHA 1.00
 BETA 0.13

X	FX	FNBX	DIFF
0.	0.11	0.07	0.05
1.	0.17	0.13	0.04
2.	0.22	0.20	0.02
3.	0.26	0.22	0.04
4.	0.30	0.26	0.04
5.	0.34	0.31	0.03
6.	0.38	0.36	0.01
7.	0.41	0.40	0.01
8.	0.44	0.43	0.01
9.	0.47	0.47	0.01
10.	0.50	0.48	0.03
11.	0.53	0.51	0.02
12.	0.56	0.53	0.02
13.	0.58	0.57	0.01
14.	0.60	0.59	0.01
15.	0.62	0.62	0.01
16.	0.64	0.64	0.00
17.	0.65	0.65	0.02
18.	0.68	0.67	0.02
19.	0.70	0.69	0.01
20.	0.72	0.71	0.00



----- EXACT LPG PROBABILITIES
 - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE- 0.046 FOR X = 0

Appendix B

LPG Program Source Listings

LIST

```

10 REM LP6.S--COMPUTE EXACT LPG PROB USING 6/19/81 RECURSION
20 REM
30 DIM T(200),T2(200)
40 REM
50 REM   T(K) = T(X,K) FOR THE CURRENT X
60 REM   T2(K) = T(X-1,K) FROM PREVIOUS X CALCULATION
70   FILES LP6.D
80   SCRATCH #1
90 REM       INPUT PARAMETERS
91   PRINT "LP6.S----  DEBUG ?  PRINT F(X)? (1 OR 0)"
93   INPUT D8, D7
94 REM
100       GOSUB 180
110 REM       DO LPG RECURSION
120   GOSUB 840
130 REM       DO SCALED BINOMIAL CALCULATIONS
140   GOSUB 3180
150   PRINT "CONTINUE? (Y OR N)"
155   INPUT A$
160   IF A$="Y" THEN 90
170   STOP
180 REM
181   PRINT
182   PRINT
190 PRINT "LP60.S--EXACT LPG PROB CALCULATIONS USING RECURSION"
191   PRINT
192   PRINT
200 REM
210 PRINT "INPUT AVE REQ, E(D), E(LT), C.OF.V OF LT "
220 INPUT R0,D1,E1,C
230   WRITE #1,R0,D1,E1,C
240 REM
250 REM       SOLVE FOR THETA = T1
260   GOSUB 1780
270   T1=0
280 REM
290 REM       ESTIMATE PARAMETERS FOR LPG
300 REM       S = STD DEV,  B = VAR TO MEAN RATIO
310   S = C*E1
320   B = S*S/E1
330   U1= D1*E1
340 REM
350   D1=1/B
360   A1 =D1*E1
370   R1=-T1/( (1-T1) *LOG(1-T1) )
380   L1 = U1/(E1*R1)
390 REM
400   PRINT "AVE REQ SIZE =",R1
410 PRINT
420   PRINT "THETA =",T1,"LAMBDA =",L1
430   PRINT "ALPHA=",  A1,"BETA =",D1
440 REM

```

55

```

450 REM
460  M1=-T1/((1-T1)*LOG(1-T1)*L1*A1/B1)
470 PRINT
480 PRINT "MEAN =",M1
490 PRINT
500REM  ESTIMATE THE FIRST FOUR MOMENTS M1,M2,M3,M4 OF THELP6 DISTRIBUTION
510  B1 = B1*(1-T1)
520  C4 = -L1*T1/(LOG(1.-T1))
530 REM
540REM
550  M1 = A1*C4/D1
560REM
570  M2 = A1*C4*(B1+C4)/D1^2
580REM
590  M3 = A1*C4*((B1^2)*(1+T1) +2*C4^2)/B1^3
600REM
610  M4 = B1^3*(1+4*T1+T1^2) + B1^2*C4*(3.*A1 + 1.)
620  M4 = M4 + 6*B1*C4*C4*A1 +C4^3*(3*A1+6.)
630  M4 = A1*C4*M4/D1^4
640REM
650  PRINT "MOMENTS"
660  PRINT "M1      M2      M3      M4"
670  PRINT M1,M2,M3,M4
680REM
690REM  COMPUTE STANDARDIZED MOMENTS
700  PRINT
710  S = SQR(M2)
720  PRINT "COEF OF VAR =", S/M1
730  PRINT "M3/S^3= ", M3/S^3
740  PRINT "M4/S^4= ", M4/S^4
750REM
760REM  OUTPUT LP6 PARAMETERS TO FILE
770 REM
780  WRITE B1,M1,S/M1,M3/S^3,M4/S^4
790  WRITE B1,T1,L1,A1,B1
800  RETURN
810REM *****
820REM  LP6 RECURSION CALCULATION
830 REM*****
840 REM
850 REM  REDEFINE C FOR PROBABILITY CALCULATIONS
860 REM
870  C= -L1/LOG(1.-T1)
880  C1 = C/(L1 + B1)
890 REM
900 REM  SET LIMIT ON X = 200
910 REM
920  L2=200
930 REM
940  C2=0
950 REM
960 REM  SET CONSTANTS FOR USE IN RECURSION

```

```

970 REM
980     S7=1
990     H1=(B1/(L1+B1))^A1
1000    H1=S7*H1
1010 REM           S7 = SCALE FACTOR
1020    H2=(C/(L1+B1))
1030 REM
1040    PRINT"C,C1,H1,H2",C;C1;H1;H2
1050 REM
1060 REM           EVALUATE H(X=0)
1070 REM
1080    T(0) = 0
1090    T2(0) = H1
1100    S1 = H1
1110    C2 = C2 + S1/S7
1120 REM
1130    PRINT
1131    WRITEN1,0,S1,C2
1140    IF D7<= 0 THEN 1200
1141    PRINT
1150    PRINT "           X           H(X)           F(X)"
1151    PRINT
1160    PRINT 0;S1;C2
1170 REM
1180 REM
1190 REM           EVALUATE H(X) FOR X > 0
1200 REM
1210    FOR X = 1 TO L2
1220        S1 = 0
1230        T(0)=0
1240 REM
1250        T(X)=(T1*H2*(A1+X-1)/X)*T2(X-1)
1260 REM
1270        IF X < 2 GOTO 1380
1280 REM
1290 REM
1300 REM
1310        FOR K=1 TO X-1
1320            T(K)=(T1/X) * (H2*(A1+K-1)*T2(K-1) +(X-1)*T2(K) )
1330            S1=S1+T(K)
1340            IF D8 <= 0 THEN 1360
1350            PRINT"X,K,T(X,K),S1=>",X;K;T(K);S1
1360        NEXT K
1370 REM           PICK UP T(X,X) TERM IN SUM
1380        S1=S1 + T(X)
1390 REM
1400 REM           PRINT TOTALS FOR H(X)
1410 REM

```

```

1420      C2 = C2 + S1/S7
1430      PRINT X;S1;C2
1440      WRITE #1,X,S1,C2
1450 REN
1460      IF CUM PROB EXCEEDS .99, STOP
1470 REN
1480      IF C2 > .99 GOTO 1670
1490 REN
1500      PRINT T(K) TERMS FOR DEBUGGING
1510 REN
1520      IF DB <= 0 THEN 1560
1530      FOR K=0 TO X
1540          PRINT "X,K,T(X,K) =>",X,K;T(K)
1550      NEXT K
1560 REN
1570 REN
1580      RECORD T(K) VALUES FOR USE IN NEXT PASS
1590 REN
1600      FOR K=0 TO X
1610          T2(K)=T(K)
1620      NEXT K
1630 REN
1640 REN-----END OF X LOOP
1650      NEXT X
1660 REN
1670      WRITE #1,-99,-99,-99 "    END OF LPG"
1680      RETURN
1690 REN*****END OF LPG RECURSION
1700 REN
1710 REN
1720 REN
1730 REN-----AVE REQ SIZE EQUATION
1740      DEF FNR(Q) = -Q/((1-Q)*LOG(1-Q) )
1750 REN
1760 REN-----
1770 REN      BINARY SEARCH ROUTINE
1780 REN-----
1790 REN
1800      SOLVE FOR THE VALUE OF Q WHICH GIVES AND
1810      AVE. REQUISITION SIZE OF R0
1820 REN
1830      SET UP END POINTS FOR SEARCH
1840 REN
1850      K= 0
1860      Q9=.999
1870      R9=FNR(Q9)
1880      Q1=.001
1890      R1=FNR(Q1)
1900      GOTO 2080
1910 REN

```

```

1920 REM      CHECK IF R0 <= R
1930 REM
1940      IF R0 > R THEN 2000
1950 REM
1960 REM      RE-SET TOP OF INTERVAL
1970      R9=R
1980      Q9=Q
1990      GOTO 2080
2000 REM
2010 REM      RE-SET BOTTON OF INTERVAL
2020      R1=R
2030      Q1=Q
2040      GOTO 2080
2050 REM
2060 REM      HALVE THE INTERVAL, AND RE-EVALUATE THE FUNCTION
2070 REM
2080      K=K+1
2090      Q=(Q1+Q9)/2
2100      R=FNR(Q)
2120 REM
2130 REM      IF R IS WITHIN .001 OF R0, THEN RETURN
2140 REM
2150      IF ABS(R-R0) < .001 THEN 2180
2160      IF K > 15 THEN 2180
2170      GOTO 1940
2180      RETURN
2190 REM
2200 REM      LPG.S---LP PROBABILITY CALCULATIONS
2210 REM
2220 REM      USE LPG.D AS THE OUTPUT FILE
2230 REM
2240 REM
2250      SCRATCH #1
2260      PRINT "THETA, LAMBDA, ALPHA, BETA, DEBUG?"
2270      INPUT T1,L1,A1,B1,B8
2280      PRINT "EXACT LPG PROB"
2290      GOSUB 2420
2300      PRINT "SCALED NEG. BIN"
2310      GOSUB 3230
2320      PRINT
2330      PRINT "CONTINUE?(Y OR N)"
2340      INPUT A6
2350      IF A6="Y" GOTO 2260
2360      STOP

```

```

2370 REM LP6.S--MASTER LPG ROUTINE LIBRARY
2380 REM LP61.S
2390 PRINT "THIS PROGRAM COMPUTES EXACT PROB FOR THE LPG DIST"
2400 DIM Y(100,100)
2410 INPUT T1,L1,A1,B1
2420 PRINT "THETA =" ; T1, "LAMBDA =" ; L1, "ALPHA =" ; A1, "BETA =" ; B1
2430   WRITE #1,1,2,3,4
2440   WRITE #1,T1,L1,A1,B1
2450 C = - L1 / LOG(1 - T1)
2460 M1 = -T1 / ((1-T1)*LOG(1-T1)) * L1 * A1 / B1
2470 C1 = C / (L1 + B1)
2480 L2 = INT(100 * M1)
2490 C2 = 0
2500 FOR X = 0 TO L2
2510 Y(X,0) = 0
2520 F1 = 1
2530 IF X < 2 THEN 2570
2540 FOR K = 1 TO X-1
2550 F1 = F1 * K
2560 NEXT K
2570 Y(X,X) = 1. / F1
2580 F1 = 1
2590 IF X <> 0 THEN 2630
2600 S1 = (B1 / (L1 + B1)) ^ A1
2610 GO TO 2740
2620 REM
2630 S1 = 0
2640 FOR K = 1 TO X
2650 IF X = 1 THEN 2670
2660 Y(X,K) = Y(X-1,K-1) / (X-1) + Y(X-1,K)
2670 F1 = 1
2680 FOR J = 0 TO K-1
2690 F1 = F1 * (A1 + J) * C1
2700 NEXT J
2710 S1 = S1 + Y(X,K) * F1
2720 NEXT K
2730 S1 = S1 * (B1 / (L1 + B1)) ^ A1 * T1 ^ X / X
2740 C2 = C2 + S1
2750 IF DB <= 0 THEN 2780
2760 PRINT X,S1,C2
2770 WRITE #1,X,S1,C2
2780 IF C2 > .99 THEN 2800
2790 NEXT X
2800 WRITE #1,-99,-99,-99
2810 RETURN

```

```

2820 REM
2830 REM *****
2840 REM LP62.S
2850 PRINT "SCALED POISSON"
2860 INPUT T1,L1,A1,B1
2870 PRINT "THETA =";T1,"LAMBDA =";L1,"ALPHA =";A1,"BETA =";B1
2880 C = -L1/LOG(1-T1)
2890 C3= 1./(1.-T1)
2900 K1 = T1*C
2910 C4 = (B1/(K1+B1))^A1
2920 U = K1/(K1+B1)
2930 C2 = 0
2940 FOR N = 0 TO 1000
2950 P1 = 1
2960 IF N<>0 THEN 3000
2970     P1 = C4
2980     GOTO 3040
2990 REM
3000 FOR I = 0 TO N-1
3010 P1 = P1*((A1+N-I-1)/(N-I)*U)
3020 NEXT I
3030 P1 = P1*C4
3040 C2 = C2 + P1
3050 IF N<=0 THEN 3080
3060 PRINT N;C3*N;P1;C2
3070 WRITE #1,C3*N,P1,C2
3080 IF C2>.99 THEN 3100
3090 NEXT N
3100 RETURN
3110 RETURN
3120 REM *****
3130 REM
3140 REM LP63.S
3150 PRINT "THIS PROGRAM COMPUTES APP PROB FOR LP6 DIST USING SCALED BIN"
3160 INPUT T1,L1,A1,B1
3161 PRINT
3170 PRINT "THETA =";T1,"LAMBDA =";L1,"ALPHA =";A1,"BETA =";B1
3180 PRINT
3190 PRINT "NEG BINOMIAL PROBABILITIES"
3200 PRINT
3210 PRINT "          X          P          F"
3220 PRINT
3230 C = 1/(1.-T1)
3240 K1 = -T1*L1/LOG(1-T1)
3250 C4 = (B1/(K1+B1))^A1
3260 U = K1/(K1+B1)
3270 C2 = 0
3280 FOR N = 0 TO 1000
3290 P1 = 1
3300 IF N<>0 THEN 3370
3310 P1 = C4
3320 A = -1
3330 B = INT(C/2 +0.5)
3340 S2 = P1/(C/2+1)
3350 GOTO 3440

```

```

3360 REM
3370 FOR I = 0 TO M-1
3380 P1 = P1*((A1+M-I-1)/(M-I)*U)
3390 NEXT I
3400 P1 = P1*C4
3410 A = INT((2*M-1)*C/2+0.5)
3420 B = INT((2*M+1)*C/2+0.5)
3430 S2 = P1/C
3440 L3 = C2
3450 FOR X = A+1 TO B
3460 F3 = C2 + S2*(X-A)
3470 F2 = F3 - L3
3480 L3 = F3
3490 IF B7 <= 0 THEN 3520
3500 PRINT X,F2,F3
3510 WRITEN1,X,F2,F3
3520 NEXT X
3530 C2 = C2 + P1
3540 IF C2 > .99 THEN 3560
3550 NEXT M
3560 WRITEN1,-99,-99,-99,"END OF NB"
3570 RETURN
3580 REM
3590 REM *****
3600 REM LP64.S
3610 REM COMPUTE REQUISITION SIZE R VS THETA TABLE
3620 REM
3630 FOR T1 = .01 TO .99 STEP .01
3640 B = -(1-T1)*LOG(1-T1)
3650 R = T1/B
3660 PRINT T1,R
3670 NEXT T1
3680 STOP
3690 REM *****
3700 REM DEFINITIONS IN NAHNIAS' PROGRAM LP61.S
3710 REM DEFINITIONS IN NAHNIAS' PROGRAM LP61.S
3720 REM
3730 REM LAMB L1
3740 REM ALPH A1
3750 REM BET B1
3760 REM MEAN M1
3770 REM CNST C1
3780 REM LIN L2
3790 REM CUM C2
3800 REM FACT F1
3810 REM SUM S1
3820 REM KN K1
3830 REM PROD P1
3840 REM CH C3
3850 REM CONST C4
3860 REM LAST L3
3870 REM FX F2
3880 REM FFX F3
3890 REM SL S2
3900 END

```

OLD 11	3	3	4	5	6	7	8
	MEAN	MEAN	Skew	Kurtosis	Std Dev		Skew
1	3.000	25.000	0.000	5.000	12.000	10	5.000
10	4.000	0.100	0.100	3.230	2.240	10	0.100
20	4.000	0.100	0.100	3.230	2.240	10	0.100
30	4.000	0.100	0.100	3.230	2.240	10	0.100
40	4.000	0.100	0.100	3.230	2.240	10	0.100
50	4.000	0.100	0.100	3.230	2.240	10	0.100
60	4.000	0.100	0.100	3.230	2.240	10	0.100
70	4.000	0.100	0.100	3.230	2.240	10	0.100
80	4.000	0.100	0.100	3.230	2.240	10	0.100
90	4.000	0.100	0.100	3.230	2.240	10	0.100
100	4.000	0.100	0.100	3.230	2.240	10	0.100
110	4.000	0.100	0.100	3.230	2.240	10	0.100
120	4.000	0.100	0.100	3.230	2.240	10	0.100
130	4.000	0.100	0.100	3.230	2.240	10	0.100
140	4.000	0.100	0.100	3.230	2.240	10	0.100
150	4.000	0.100	0.100	3.230	2.240	10	0.100
160	4.000	0.100	0.100	3.230	2.240	10	0.100
170	4.000	0.100	0.100	3.230	2.240	10	0.100
180	4.000	0.100	0.100	3.230	2.240	10	0.100
190	4.000	0.100	0.100	3.230	2.240	10	0.100
200	4.000	0.100	0.100	3.230	2.240	10	0.100
210	4.000	0.100	0.100	3.230	2.240	10	0.100
220	4.000	0.100	0.100	3.230	2.240	10	0.100
230	4.000	0.100	0.100	3.230	2.240	10	0.100
240	4.000	0.100	0.100	3.230	2.240	10	0.100
250	4.000	0.100	0.100	3.230	2.240	10	0.100
260	4.000	0.100	0.100	3.230	2.240	10	0.100
270	4.000	0.100	0.100	3.230	2.240	10	0.100
280	4.000	0.100	0.100	3.230	2.240	10	0.100
290	4.000	0.100	0.100	3.230	2.240	10	0.100
300	4.000	0.100	0.100	3.230	2.240	10	0.100
310	4.000	0.100	0.100	3.230	2.240	10	0.100
320	4.000	0.100	0.100	3.230	2.240	10	0.100
330	4.000	0.100	0.100	3.230	2.240	10	0.100
340	4.000	0.100	0.100	3.230	2.240	10	0.100
350	4.000	0.100	0.100	3.230	2.240	10	0.100
360	4.000	0.100	0.100	3.230	2.240	10	0.100
370	4.000	0.100	0.100	3.230	2.240	10	0.100
380	4.000	0.100	0.100	3.230	2.240	10	0.100
390	4.000	0.100	0.100	3.230	2.240	10	0.100
400	4.000	0.100	0.100	3.230	2.240	10	0.100
410	4.000	0.100	0.100	3.230	2.240	10	0.100
420	4.000	0.100	0.100	3.230	2.240	10	0.100
430	4.000	0.100	0.100	3.230	2.240	10	0.100
440	4.000	0.100	0.100	3.230	2.240	10	0.100
450	4.000	0.100	0.100	3.230	2.240	10	0.100
460	4.000	0.100	0.100	3.230	2.240	10	0.100
470	4.000	0.100	0.100	3.230	2.240	10	0.100
480	4.000	0.100	0.100	3.230	2.240	10	0.100
490	4.000	0.100	0.100	3.230	2.240	10	0.100

```

IDEN LPOSKU.P
FILE 11
DASH 1
LSPM 1
GRID 1
TLAB MEAN US DATA SET NO.
BLAB FOR SAMPLE LPG PARAMETER SETS
VLAB MEAN
XLAB DATA SET NO.
XCOL 2.2,49 2.2,49 2.2,49 2.2,49
YCOL 3.2,49
PLOT
PLOT
TLAB STD. DEV. US DATA SET NO.
YLAB STD. DEV
XCOL 6.2,49
PLOT
PLOT
TLAB SKEWNESS12 VS KURTOSIS
VLAB KURTOSIS
XLAB SKEWNESS12
XCOL 5.2,7 5.8,13 5.14,19 5.20,25 5.26,31
YCOL 4.2,7 4.8,13 4.14,19 4.20,25 4.26,31
SCALE 0.10 0.5
PLOT
CONT 1
XCOL 5.32,37 5.38,43 5.44,49
YCOL 4.32,37 4.38,43 4.44,49
PLOT
CONT 0
LPG VALUES FOR E(0)=0.5
XCOL 5.2,7 5.8,13 5.14,19 5.20,25
YCOL 4.2,7 4.8,13 4.14,19 4.20,25
PLOT
LPG VALUES FOR E(0)=2.0
XCOL 5.26,31 5.32,37 5.38,43 5.44,49
YCOL 4.26,31 4.32,37 4.38,43 4.44,49
PLOT
STOP

```

Plot Program and Input Data For LPG Skewness vs Kurtosis Plots.