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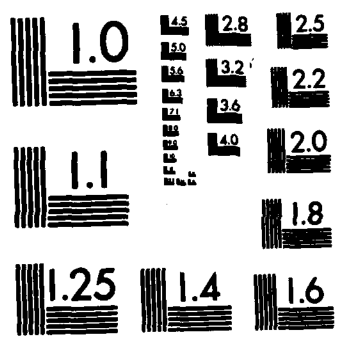

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THESIS

A STUDY OF THE APPLICATION OF THE LOGNORMAL AND GAMMA DISTRIBUTIONS TO CORRECTIVE MAINTENANCE REPAIR TIME DATA

by

Ergam Camozu

October 1982

Thesis Advisor: M. B. Kline

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A Study of the Application of the
Lognormal and Gamma Distributions to
Corrective Maintenance Repair Time Data

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Submitted in partial fulfillment of the
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I. INTRODUCTION

A. OVERVIEW

The effectiveness of a system depends not only on its ability to meet its specified performance requirements, but also on its ability to perform when needed, for the duration of its assigned missions, and for its operational lifetime. The technical disciplines concerned with these time-related system characteristics are reliability, maintainability and logistic support.

Commonly used methods for prediction and demonstration of corrective maintenance repair times assume the validity of the lognormal distribution.

In order to help in focusing attention on these matters, a statistical analysis on data sets of demonstrated and field repair times has been conducted as part of a study on the application of the lognormal and gamma distributions to corrective maintenance data. The results of the analysis are given in this thesis. A previous study of electronic systems and equipments concluded that the lognormal distribution was a suitable descriptor for repair times [Ref. 1].

B. PURPOSE AND APPROACH

1. Objectives

The objectives of this study have been;

(a) To test the exponential, lognormal, and gamma distributions as descriptors of repair time data on mechanical and electronic systems and equipments,

(b) to verify that the lognormal or gamma distribution is a suitable descriptor for repair times,

(c) to estimate the percentage error caused in assuming exponentiality for availability and maintainability calculations when in fact the distribution is lognormal.

2. Systems and Data Analyzed

Forthy-six sets of repair time data for electronic and mechanical systems / equipments were analyzed (Table 1). Data for the electronic systems (Sets 8-20) came from formal maintanability demonstration test reports furnished to us by the Rome Air Development Center, U.S. Air Force. Mechanical equipments included field repair time pumps used in French nuclear electric power stations (Sets 1-7), and additional 26 sets of field repair data from British Aircraft, provided by the United Kingdom Ministry of Defence, representing elapsed times (Sets 21-33), and man hours (Sets 34-46). No detailed reports are available for the mechanical items.

3. Analysis Approach

Three approaches were used for testing data.

Table 1

SYSTEMS / EQUIPMENTS ANALYZED

Set No	S Y S T E M N A M E
1	French Fessenheim Pumps Down Time
2	French Fessenheim Pumps Repair Time
3	Condenser Extraction Pump
4	Heater Drain Pump
5	Main Feedwater Pump
6	Reactor Coolant Pump
7	Condenser Vacuum Pump
8	AN/GRT-20
9	AN/ARC-162(V) Radio Set Intermediate
10	AN/ARC-163(V) UHF Modern Radio Set
11	AN/ALQ-125 TEREC , Direction Finder Set
12	CGT MAX. MPU and RSS Terminal Organize
13	AN/TPN-28 Radar Beacon Transponder
14	AN/MST-T1 Electronic Warfare Training Set
15	AN/UYK-14C
15a	Revised AN/UYK-14C
16	Integrated Microwave G/G Subsystem
17	CCG-NBOT Compass Ears
18	AN/GXS-2V
19	AN/ARN-101(V)
20	AN/FCC-98 Multiplexer
21,34	Fuselage Structure
22,35	Flying Controls
23,36	Landing Gear
24,37	Hydraulics
25,38	Ice and Rain Protection
26,39	Power Plant
27,40	Engine Starting
28,41	Main Rotor Head and Blades
29,42	Main Rotor Drive
30,43	Electrical Power Supply and Distribution
31,44	Communications
32,45	Surveillance and Search
33,46	Automatic Flight Control

Note: In sets 21-46, the two numbers represent two part data. The first part is ELAPSED TIME and second part is MAN HOURS.

(a) Rough check using a technique called Boxplot. This gives a rough idea that the data comes from symmetric (normal) or skewed distributions lognormal, exponential or gamma).

(b) Plotting the data using appropriate probability paper.

(c) Statistical tests of significance for assumed distributions.

The following procedure was used for analyzing the data,

i. Sketch boxplot and histogram of data and determine if data appears to come from a symmetric or skewed distribution. Obtain an idea about center and dispersion of data.

ii. Plot the data on appropriate probability paper for the exponential and lognormal distributions. In some cases, plots were made for the gamma distribution.

iii. A Chi-Square Goodness-Of-Fit test was performed for lognormal and exponential assumptions, [Ref. 2]

iv. A W test, due to Shapiro and Wilk, [Ref. 3] was used to test the lognormal assumption for sample sizes less than 51, (due to availability of the tables),

v. A Kolmogorov-Smirnov (K-S) test was used for lognormal and gamma assumptions [Ref. 4].

II. STATISTICAL CONSIDERATIONS

A. ROUGH CHECK METHODS

1. Boxplot [Refs. 5 and 6]

Purpose: Many batches of data pile up in the middle. To analyze the behavior of a batch, we need a picture of where the middle (median) lies and how the tails relate to it. The middle is generally better defined than the tails so we want to see more at the tails. Some values called outliers are so low or so high that they seem to stand apart from the rest of the batch.

One technique for displaying such batches of data which is often more convenient than a histogram is the boxplot. But the histogram is still good for showing the shape of the distribution. Boxplot will not show multiple modes. There may be several "modes", and we want to see them.

A boxplot is obtained by first calculating the lower and upper quartiles and the median of the batch (sample) of numbers and then plotting the numbers on a horizontal line.

The lower quartile is the value that divides the batch into two parts, with 1/4 of the numbers below this value and 3/4 above it. The upper quartile is the value with 3/4 of the observations below and 1/4 observations above it.

The next step is to draw a narrow rectangular box with ends corresponding to the lower and upper quartiles, and to display the median point by a plus sign. The length of this box is called the interquartile range or H-Spread (Figure 1).

To the above are added lines marking 1/2 H-Spread on each side of the box. Data values outside the lines (outliers) are marked with stars and rectangular circles.

```
                         ---------
***     ---------I   +    I-----------   *  *    0000
                         ---------

                 |-------|
                 Interquartile range
        |-----------------|
        1.5 interquartile range
 |-----------------------|
 2 interquartile range
```

-Figure 1 - Boxplot-

First, the data are ordered such that,

X(1) ≤ X(2)............... ≤ X(n)   where,

X(1) : smallest (minimum) value

X(n) : largest (maximum) value

median=M , center of batch

if n is an odd number, M=X((n+1)/2)

if n is an even number , M=(X((N/2)+1)+X((n/2)-1))/2 quartiles=H , center of half of batch

$d(H)=(|d(M)|+1)/2$ where $|A|$ means integer part of A

interquartile distance = upper quartile -lower quartile

Here are good boxplot examples for the exponential and lognormal model (data generated randomly).

BOXPLOT EXPONENTIAL

```
 --------
-I +    I--------                                 O
 --------
```

-Figure 2 - Boxplot Exponential-

BOXPLOT NORMAL (log is taken then boxplot)

```
          -----------
----------I    +    I---------------
          -----------
```

-Figure 3 - Boxplot Normal-

2. Histogram [Ref. 5 and 6]

Boxplot is good for comparison but a histogram is still good for showing shape. Sometimes histogram is more helpful than the boxplot, especially for detection of more bumps in the data. Histogram prints data in classes. Examples are given below in Figure 4.

HISTOGRAM EXPONENTIAL

```
MIDDLE OF    NUMBER OF
INTERVAL     OBSERVATIONS
    0.          9     *********
    1.         10     **********
    2.          2     **
    3.          4     ****
    4.          0
    5.          0
    6.          0
    7.          0
    8.          1     *
```

HISTOGRAM NORMAL (log is taken then histogram)

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
1.0	1	*
1.5	3	***
2.0	6	******
2.5	6	******
3.0	5	*****
3.5	2	**
4.0	1	*
4.5	2	**

-Figure 4 - Histograms-

B. STATISTICAL DISTRIBUTIONS

1. The Exponential Distribution

The probability density function of the exponential distribution is [Ref. 2],

$$f(x,\lambda) = \lambda e^{-\lambda x} , \qquad x \geq 0 \text{ and } \lambda > 0$$

Its cumulative distribution function is,

$$F(x,\lambda) = \int_0^x \lambda e^{-\lambda t} dt = 1 - e^{-\lambda x}$$

which can be easily evaluated.

The exponential distribution is used frequently as a time to failure model for a system when a constant failure rate is assumed [Ref. 4]. But for repair times, it can be shown that it is not an appropriate model. Because repair time includes diagnostic, correction, and verification tasks a repair time density must have a value of zero at time t=0. It should then increase to its maximum value rapidly and then gradually decrease towards zero as time increases.

The maximum likelihood estimator of the parameter of the exponential distribution from given data is,

$$\hat{\lambda}=\frac{1}{\bar{x}} \quad \text{where} \quad \bar{x}=\frac{\sum_{i=1}^{n} x(i)}{n}$$

2. Lognormal Distribution

This distribution has many different shapes for non-negative variates. It is skewed to the right, the degree of skewness increasing with increasing values of standard deviation. The mean and standard deviation are scale and shape parameters respectively.

A random variable is said to have a lognormal dis - tribution if the logarithm of the variable is normally distributed,

$$f(x)=\frac{1}{x\sigma\sqrt{2\pi}}\int_{0}^{\infty} e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2} dx$$

with mean and variance

μ = E (lnX) and σ^2= Variance(lnX)

If data X(1), X(2), ... , X(n) and if

Y(1)=lnX(1), Y(2)=lnX(2), ..., Y(n)=lnX(n) than, maximum likelihood estimators of the lognormal distribution are,

$$\hat{\mu}=\bar{y} \qquad \hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}(y(i)-\bar{y})^2$$

The p^{th} percentile is calculated by [Refs. 7 and 8]

Median $Z(.5)=0$ $X_{0.5} = e^{\mu}$

90^{th} per. $Z(.9)=1.282$ $X_{0.9} = e^{\mu+1.282\sigma}$

95^{th} per. $Z(.95)=1.645$ $X_{0.95} = e^{\mu+1.645\sigma}$

3. Gamma Distribution

A random variable x has a gamma distribution, and it is referred to as a gamma random variable, if and only if its probability density is given by, [Ref. 9]

$$f(x) = \begin{cases} \dfrac{1}{b^{a}\,\Gamma(a)} x^{a-1} e^{-x/b} & \text{for } x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

where $a \geq 0$ and $b \geq 0$

a is shape parameter

b is scale parameter

Γ is the gamma function

The mean and variance of the gamma distribution are given by,

$$\text{mean}=a.b \quad \text{and} \quad \text{variance}=a.b^{2}$$

Special cases of the gamma distribution play an important roles in statistics. For instance, for a=1 and b=Q, we get the exponential distribution with $\lambda=1/Q$.

For the gamma distribution the scale and shape parameters can be estimated from data in many ways; here are two

a. Method Of Moment (M.O.M) estimators

$$a=\frac{(\text{sample mean})^2}{\text{sample variance}} \qquad b=\frac{\text{sample variance}}{\text{sample mean}}$$

where mean=a.b and variance=a.b^2

b. Maximum Likelihood Method (M.L.E.) estimators

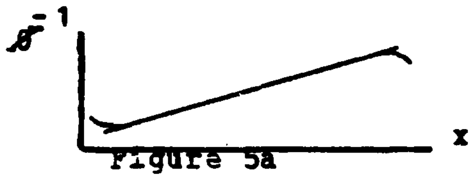
This method is more complex as compared to the M.O.M.; it requires numerical calculation. There is an APL function named ESTGAM that gives directly M.L.E. (App.C). It is used because it provides a better estimator of the parameters a and b in large samples, provided the data truely come from a gamma model.

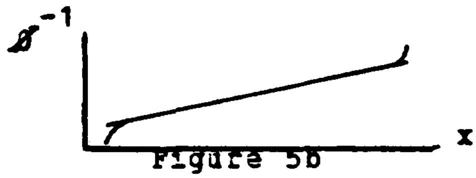
C. PROBABILITY PLOTTING

Plotting data points on probability paper is quite easy and does not require many calculations. It is a visual examination rather than statistical calculation (Figure 5).

What can we learn from a probability plot?

1. Visual indication of assumed distribution.
2. Examine departures from assumed distribution.





5a. Tails longer than normal
5b. Tails shorter than normal

-Figure 5-

1. Lognormal Distribution

Consider the order statistics of a sample

$x(1) \leq x(2) \leq \ldots\ldots\ldots\ldots\ldots\ldots \leq x(N)$ Then, $x(i)$ is an estimate of the (i/N) th percentile of X

If X is N(0,1) then we expect

$x(i) \approx \Phi^{-1}(i/N)$ for $i=1,2,3\ldots\ldots\ldots N$

Then a plot of $X(i)$ vs $\Phi^{-1}(i/N)$ should give a straight line. The only problem is

$\Phi^{-1}(i/N)$=infinity, so we use $\Phi^{-1}(i/(N+s))$, s=1.

PROCEDURES

a. Use normal probability paper

Plot (i/N+s) vs X(i)

b. Use normplot in APL workspace [Ref. 21]

Use MINITAB

__NSCORES of C1,put in C2

__PLOT C2 vs C1

NSCORES are (i/N+s)

Consider log of data $Y(1) \leq Y(2) \leq \ldots\ldots\ldots\ldots \leq Y(N)$ where $Y(i)=\mathrm{Ln}X(i)$ and then apply above procedure to get lognormal plot.

2. Exponential Distribution

Suppose X is exponential λ with sample size n, so $\Pr(X \leq x)=1-\exp(-\lambda X)$ where $X \geq 0$. Then $(i/n+s)$ is an estimate of $\Pr(X \leq X(i))=1-\exp(-\lambda x(i))$

So if the data really come from an exponential distribution $(i/N+s)$ vs. $1-\exp(-\lambda X(i))$, should yield a straight line.

PROCEDURES

a. $\ln(i/N+s)$ vs. $X(i)$ on rectangular paper,

b. EXPONPLOT in APL workspace, [Ref. 21]

c. MINITAB,

d. Use Gamma probability paper for ALPHA=1 BETA=Q where Q is exponential parameter so that,

$$f(x)=(1/Q)*\exp(-X/Q) \text{ for } X \geq 0$$

3. Gamma Distribution

There is an APL program function named GAM that uses estimated gamma parameters and gives test statistic D. The program also gives a matrix for probability plotting purposes. (see Appendix C)

D. STATISTICAL TESTS FOR DISTRIBUTIONAL ASSUMPTIONS

We need to know whether we are actually dealing with a sample from an exponential population or whether the data values of random variables have a lognormal or some other distribution. A statistical test of significance of a distributional assumption provides an objective technique for tentatively assessing whether or not an assumed model provides an adequate description of the observed data.

There are three basic steps involved in statistical test methods.

1. A test statistic is calculated from the data,
2. The probability of obtaining the calculated test statistic is determined,
3. Assessment is made of the adequacy of the assumed distribution.

    a. If the probability of obtaining the calculated test statistic is "LOW", one can say that the assumed distribution does not provide an adequate representation.

    b. If the probability of obtaining the calculated test statistic "HIGH" , then the data provides no evidence that the assumed distribution is not adequate, judged by the test.

The definition of "LOW" or "HIGH" depends on the user's preferences, and the consequences of rejecting the distribution. For example, the rejection statistic is typically be 0.01, 0.05, or 0.1. For the purpose of this study 0.05 was selected as the reject criterion.

It should be pointed out that a statistical test, although it allows one to reject an assumption as inadequate does not prove that the assumed distribution is correct.

1. The Chi-Squared Goodness-of-Fit Test

This test is one of the oldest and most commonly used for evaluating distributional assumptions. Basically, the given data are grouped into cells and compared to the expected number of observation in the cells based on the assumed distribution. Then if the test statistic, calculated from this comparison exceeds a critical chi-square value the assumed distribution is rejected.

The only problem is dividing the data into cells. If the number of observations is small it is usually suggested to use a number of cells as large as possible, subject to the restriction that it should not exceed N/5 (N is sample size).

The computations involved in the Chi-Squared Goodness-Of-Fit test were made by a computer program, in which are given as inputs the assumed cumulative distribution function, the number of observations, the number of equiprobable cells, and the number of parameters estimated from the sample. The outputs are the Chi-Square statistic and its probability of exceeding a chi-square value for a given number of degrees of freedom.

The procedure used is described as follows: [Ref. 2]

a. The cell boundaries are determined from the assumed cumulative distribution as the values such that the estimated probability of the observation value falling within a given class is 1/K. For each class,

$Pr(X \leq x(i)) = i/k$, where

x=the random observation to be assigned to the i-th cell

x(i)=the i-th cell boundary to be solved from the above formula

k = the number of cells, in this study k = n/5

The lower bound of the first cell and the upper bound of the last cell are the smallest and the largest values of the observations.

b. The expected number of observations for each cell E(i) is equal to n/k for each cell.

c. The number of observed values in each cell M(i) is counted based on the results of the above equation.

d. The test statistic is

$$\chi^2 = \frac{k}{n}\left[\sum_{i=1}^{k} M(i)^2\right] - n$$

e. The computed value χ^2 is used to compute the level of significance, or the probability of a Chi-Square value with degrees of freedom equal to K-M-1 where M is the number of estimated parameters from the sample to exceed calculated chi-square

$$\alpha = Pr(\chi^2_{1-\alpha}(\nu) \geq X^2) \qquad \text{with } \nu \text{ degrees of freedom}$$

If α is less than or equal to .05 the assumed distribution can be rejected as inadequate.

2. <u>The Kolmogorov-Smirnov Test</u> (<u>K-S Test</u>)

K-S test examines the cumulative distribution function rather than probability density function . It simply compares the sample cumulative distribution function S(.) to the hypothesized distribution function F(.). As a measure of comparison the test uses, $D = Max|F(.)-S(.)|$; this is the main difference between the Chi-Squared G.O.F. test and the K-S. test. [Ref. 4]

On the other hand here we are dealing with Type I error, that is the hypothesis is true and reject it. The probability of commiting a Type I error is called the significance level for the test alpha. Here alpha is taken to be 0.05. This means no more than a 5 % chance of rejecting the true hyphothesis.

X has CDF $F(x) = Pr(X \leq x)$,

Take a sample $X(1) \leq X(2) \leq \ldots\ldots\ldots \leq X(N)$,

We want to estimate $F(x) = Pr(X \leq x) = S_n(x)$

$S_n(x)$	= 0	for	$X < X(1)$
"	1/N	"	$X \geq X(1)$ .AND. $X < X(2)$
"	2/N	"	$X \geq X(2)$ .AND. $X < X(3)$
"	.		. . . .
"	.		. . . .
"	K/N	"	$X \geq X(K)$ .AND. $X < x(K+1)$
"	.		. . . .
"	.		. . . .
"	1		$X \geq X(N)$

Find $D = MAX| S(x)-F(x)|$

Using Table 9 of reference 4 for given significance level,

If D exceeds the tabulated value that comes from the table, reject the hypothetical distribution.

If D does not exceed the tabulated value that comes from table, do not reject the hypothetical distribution.

There are two kinds of tables for the K-S test. One of these is used if the parameters are estimated from data, and the other one is used if the parameters are given. Here parameters are estimated from data.

3. <u>The W test for Lognormal Assumption</u>

The W test is shown in Ref. 3 to be an effective procedure for evaluating the assumption of normality against nonnormal alternatives, even if only a relatively small number of observations are available. Hahn and Shapiro suggest that the W test may also be used to evaluate the assumptions of the lognormal distribution. This follows because of the property that if the logarithms of the observations follow a normal distribution, then the original values of the observations are lognormally distributed.

The following procedure was applied using tables from Reference 2.

a. The observations are ordered,

$$X(1) \leq X(2) \leq \ldots \leq X(i) \ldots\ldots \leq X(n-1) \leq X(n)$$

b. The following parameters are computed,

i.

$$S^2 = \sum_{i=1}^{n} \left[\ln x(i)\right]^2 - \frac{\left[\sum_{i=1}^{n} \ln x(i)\right]^2}{n}$$

ii. If n is even, k=n/2
n is odd, k=(n-1)/2

$$B=\sum_{i=1}^{k}[\ a(k-i+1)*[(\ln x(n-i+1))-(\ln x(i))]]$$

where the values of a(k-i+1) for i=1,2,..k are given in table IX [Ref. 2] for n=3 to n=50

iii. The test statistic W is

$$W= B^2 / S^2$$

c. The approximate probability of obtaining the calculated value of W can be obtained from the formula

$$Z=\lambda+\eta\ln\left|\frac{W-\epsilon}{1-W}\right|$$

using the values of λ, η and ϵ given in Table XI of Ref. 2 for the appropriate sample size and then using standardized normal distribution to determine the probability of obtaining a value less than or equal to Z, which is the significance level of the test.

$$q = Pr(Z_q \le Z)$$

If q is less than or equal to .05, the selected level of significance in this study, the lognormal is rejected as an inadequate assumption.

III. DATA ANALYSIS RESULTS

A. RESULT OF TESTS FOR DISTRIBUTIONAL ASSUMPTIONS

1. Summary of results

Tables 2,3 and 4 summarize the results of the statistical test analysis. In all cases the 5 % level was chosen as the reject criterion. The 46 sets of data used in this study can be divided into three parts as indicated in Section on IB2.

The following is an example of the computer program summary table which includes the results for the Chi-Square test and calculated parameters from the sample data.

SET NO 18 - AN/GXS-2(V)

SAMPLE SIZE N = 26		NO. OF CELLS K = 5		(a)
SAMPLE MEAN = 19.70		STANDARD DEV = 22.62		(b)
	EXPONENTIAL	LOGNORMAL	DIFFERENCE	
PARAM1	0.05	2.56		(c)
PARAM2		0.78		
MTTR	19.70	19.13	2.96 %	(d)
50-TH PERCNT	13.65	12.93	5.61 %	
90-TH PERCNT	45.35	40.22	12.77 %	
95-TH PERCNT	59.00	55.46	6.39 %	
CHI-SQR STAT	5.92	0.54		(e)
DEG OF FREED	3	2		(f)
SIGNIF LEVEL	0.115E+00	0.764E+00		(g)

Table 2

TOTAL DATA ANALYSIS RESULTS

Set No	Sample Size	EXP. TEST Pr. Pl.	EXP. TEST Ch. Sq.	Rej.	LOG NORMAL TEST Bo. Pl.	LOG NORMAL TEST Pr. Pl.	LOG NORMAL TEST Ch. Sq.	W T.	K-S T.	Rej.	GAMMA TEST M. OM	GAMMA TEST M. LE
1	43	B	R	+	G	G	A	A	R		R	R
2	43	B	R	+	G	G	A	R	A		R	R
3	30	B	R	+	G	F	A	A	R		R	R
4	43	B	R	+	B	B	R	R	R	+	R	R
5	33	B	R	+	B	P	R	R	A	*	R	R
6	21	B	A		B	B	R	R	R	+	R	A
7	59	B	R	+	P	F	R	--	R	+	R	R
8	20	G	R	+	P	F	R	R	R	+	A	A
9	49	P	R	+	G	G	A	A	R		A	A
10	50	P	R	+	F	F	R	A	R	*	R	R
11	50	P	R	+	G	G	A	A	A		A	A
12	43	B	R	+	F	F	R	A	R	*	A	A
13	30	B	R	+	P	P	R	R	R	+	A	A
14	26	P	R	+	G	G	A	A	A		A	A
15a	33	G	R	+	P	G	A	A	R		R	R
16	30	B	R	+	B	B	R	R	R	+	R	R
17	50	P	R	+	G	G	A	A	A		R	A
18	26	F	A		F	F	A	A	A		A	A
19	57	B	R	+	B	B	A		R	+	A	A
20	57	B	R	+	G	F	R	--	R	+	R	A
21	157	P	R	+	G	F	R	--	R		R	R
22	28	F	A		G	G	A	-A	A		R	A
23	36	B	R	+	P	B	R	R	A	*	R	R
24	58	B	R	+	B	G	R		R	+	R	R
25	20	F	R	+	G	B	R	-A	A		A	A
26	92	F	R	+	F	P	R		R	+	R	R
27	21	B	R	+	B	B	R	-A	R	*	A	A
28	33	B	R	+	G	G	A	A	A		R	A
29	47	P	R	+	G	F	A	A	A		R	A
30	35	B	R	+	F	G	A	A	R		A	A
31	81	P	R	+	B	B	R		R	+	A	A
32	22	B	R	+	G	G	A	-A	R		A	A
33	45	B	R	+	B	B	R	R	R	+	R	R
34	171	B	R	+	G	F	R		R	+	R	R
35	31	F	A		G	G	A	-A	A		A	A
36	37	B	R	+	P	G	A	A	A		R	R
37	60	F	R	+	B	B	R		R	+	R	R
38	20	G	R	+	G	F	A	-A	A		A	A
39	95	B	R	+	F	G	A		A		R	R
40	22	P	A		G	G	A	-A	A		A	A
41	35	F	R	+	G	G	A	A	A		R	A
42	51	F	A		G	G	A		A		R	A
43	38	B	R	+	F	F	A	-R	R	*	A	A
44	87	F	R	+	P	G	A		R		A	A
45	22	F	R	+	G	G	A	-A	R		A	A
46	47	B	R	+	B	B	R	R	R	+	R	R

--- Criteria for rejection $Pr(X^2)$ or Pr(W) less than .05 or D value for K-S test bigger than tabulated value.

* To be determined from probability plot or Boxplot

where B=Bad, P=Poor, F=Fair, G=Good

A=Do not reject, R=Reject

Pr.Pl.= Probability Plot, Bo.Pl.= Boxplot

Table 3

LOGNORMAL AND EXPONENTIAL RESULTS

Set No	Sample Size	E X P. $P(X^2)$	L O G N O R M A L $P(X^2)$	P(W)	K-S Test. Ts.St.	Tab.Va.
1	43	0.0219	0.659	0.335	.146	.135
2	43	0.00166	0.358	0.025	.079	.135
3	30	0.48E-4	0.0588	0.0745	.205	.161
4	43	0.59E-7	0.0005	0.0	.238	.135
5	33	0.018	0.0128	0.0328	.148	.154
6	21	0.245	0.0468	0.0002	.27	.193
7	59	0.238E-6	0.596E-7	-----	.172	.115
8	20	0.0101	0.0143	0.017	.235	.190
9	49	0.119E-6	0.459	0.31	.217	.126
10	50	0.596E-7	0.0445	0.099	.212	.125
11	50	0.00011	0.979	0.632	.086	.125
12	43	0.596E-6	0.0421	0.19	.143	.135
13	30	0.28E-5	0.0186	0.01	.3	.161
14	26	0.129E-4	0.292	0.238	.151	.174
15	50	0.208E-4	0.00408	0.001	.245	.125
15a	33	0.216E-4	0.484	0.06	.211	.154
16	30	0.811E-4	0.0321	0.0	.199	.161
17	50	0.0323	0.494	0.583	.125	.125
18	26	0.115	0.764	0.36	.084	.174
19	57	0.596E-7	0.360	-----	.23	.117
20	57	0.0	0.116E-4	-----	.441	.117
21	157	0.0	0.0	-----	.073	.07
22	28	0.381	0.526	0.624	.145	.167
23	36	0.808E-4	0.173E-3	0.0074	.132	.147
24	58	0.132E-3	0.0255	-----	.143	.116
25	20	0.0334	0.912E-2	0.35	.16	.190
26	92	0.656E-6	0.380E-4	-----	.102	.092
27	21	0.174E-2	0.231E-2	0.175	.301	.193
28	33	0.018	0.554	0.345	.143	.154
29	47	0.69E-2	0.711	0.524	.106	.130
30	35	0.254E-4	0.592	0.617	.263	.149
31	81	0.186E-3	0.861E-4	-----	.121	.098
32	22	0.468E-2	0.063	0.59	.219	.189
33	45	0.427E-3	0.315E-3	0.013	.147	.132
34	171	0.596E-7	0.596E-7	-----	.069	.068
35	31	0.456	0.724	0.97	.117	.159
36	37	0.0258	0.571	0.11	.083	.145
37	60	0.542E-5	0.430E-2	-----	.123	.114
38	20	0.0334	0.094	0.896	.099	.110
39	95	0.298E-6	0.22	-----	.091	.091
40	22	0.0716	0.201	0.424	.156	.189
41	35	0.0476	0.126	0.48	.125	.149
42	51	0.193	0.103	-----	.092	.124
43	38	0.470E-2	0.242	0.04	.265	.144
44	87	0.182E-2	0.193	-----	.106	.095
45	22	0.468E-2	0.340	0.57	.224	.189
46	47	0.757E-5	0.192E-3	0.004	.179	.13

Table 4

GAMMA DISTRIBUTION TEST RESULTS
(Kolmogorov Smirnov G.O.F.Test)

Set No	D value of M.O.M.Est.	A/R	D value of M.L.E.Est.	A/R	Tabulated Value
1	.362	R	.154	R	.135
2	.286	R	.182	R	.135
3	.22	R	.185	R	.161
4	.458	R	.344	R	.135
5	.37	R	.21	R	.154
6	.23	R	.164	A	.193
7	.299	R	.138	R	.115
8	.18	A	.154	A	.190
9	.093	A	.11	A	.126
10	.143	R	.147	R	.125
11	.081	A	.081	A	.125
12	.124	A	.107	A	.135
13	.148	A	.152	A	.161
14	.168	A	.153	A	.174
15	.158	R	.162	R	.125
15a	.17	R	.114	R	.154
16	.231	R	.162	R	.161
17	.136	R	.097	A	.125
18	.232	R	.143	A	.174
19	.073	A	.074	A	.117
20	.145	R	.109	A	.117
21	.192	R	.163	R	.07
22	.12	A	.112	A	.157
23	.285	R	.201	R	.147
24	.26	R	.164	R	.116
25	.172	A	.155	A	.190
26	.222	R	.14	R	.092
27	.186	A	.187	A	.193
28	.172	R	.09	A	.154
29	.187	R	.113	A	.130
30	.1	A	.098	A	.149
31	.086	A	.085	A	.098
32	.105	A	.114	A	.189
33	.185	R	.142	R	.132
34	.284	R	.13	R	.068
35	.153	A	.098	A	.159
36	.268	R	.158	R	.145
37	.286	R	.172	R	.114
38	.124	A	.107	A	.19
39	.336	R	.161	R	.091
40	.08	A	.085	A	.184
41	.154	R	.099	A	.149
42	.163	R	.099	A	.124
43	.131	A	.126	A	.144
44	.088	A	.075	A	.095
45	.12	A	.089	A	.189
46	.238	R	.175	R	.13

Notes:

a. The number of equiprobable cells, K, was chosen as N/5 where N the sample size.

b. The sample mean and standard deviation are calculated based on the maximum likelihood estimates.

c. PARAM1 for the exponential distribution is the reciprocal of the sample mean, for the lognormal distribution PARAM1 and PARAM2 are $\hat{\mu}$ and $\hat{\sigma}^2$, the parameters of the lognormal distribution.

d. The MTTR and the 50th, 90th and 95th percentiles are calculated from the sample and are based on the relationships between the calculated parameters and their distribution functions. The percentage difference is between the exponential and lognormal MTTR and percentiles.

e. The Chi-Square statistic.

f. The number of degrees of freedom (K-M-1) is K-2 for the exponential distribution and K-3 for the lognormal. This is because one parameter is estimated from the sample in the exponential case and two in the lognormal case.

g. The level of significance is the probability of a Chi-Square variate with the specified degrees of freedom exceeding the calculated Chi-square statistic.

For the exponential model, only the Chi-Squared test was used. In 6 out of 46 cases we would accept the exponential model. If we compare the exponential vs. lognormal model using the Chi-Square test, test prefers the lognormal model to the exponential model. In only one case would the exponential model be accepted and the lognormal rejected.

		exponential A	exponential R	
lognormal	A	5	19	24
	R	1	21	22
		6	40	

Therefore the lognormal model is a better descriptor with respect to the exponential model.

In most cases the lognormal assumption works reasonably well. In those cases in which the results of the chi-squared test, W test and K-S test do not agree, a plot on lognormal probability paper was used to determine the appropriateness of this assumption (* in Table 2). Comparison between test methods were made in three ways for the lognormal model and tabled below.

a. Chi-Sq. vs. K-S test (46 cases)

		K-S A	K-S R	
Chi-Sq.	A	15	10	25
	R	3	18	21
		18	28	

b. Chi-Sq. vs. W test (34 cases)

Chi-Sq. \ W	A	R	
A	19	2	21
R	4	9	13
	23	11	

c. W vs. K-S test (34 cases)

W \ K-S	A	R	
A	13	10	23
R	3	8	11
	16	18	

For the lognormal methods the test methods sometimes give different results. The K-S test and the chi-square test agree in 74 % of the cases. The chi-square agrees with the W test in 82 % of the cases. W and K-S test agree 62 %. Therefore, there is no significant difference among the three test methods.

A previous study has been done using 24 data sets of electronic systems and equipments, using only the W and chi-squared tests for the lognormal model [Ref. 1]. The table below gives a comparison between the exponential and lognormal for the previous study. For the exponential model do not reject (A) or reject (R) decision was made based on the chi-squared test. For the lognormal model accept by either the chi-square or W test was used as a criterion.

		exponential A	exponential R	
lognormal	A	7	14	21
	R	1	2	3
		8	16	

The tables below show same kind of results for the current study. Only difference between the two studies is there are different kind of systems and equipments (e.g. mechanic or electronic) in the current study.

		exponential A	exponential R	
lognormal	A	5	23	28
	R	1	17	18
		6	40	

For all cases (46 cases)

		exponential A	exponential R	
lognormal	A	1	7	8
	R	0	5	5
		1	12	

For electronic equipments only (13 cases)

These results indicate that the lognormal model is better than the exponential model for repair time data.

For the gamma model, the MLE estimator is more powerful than the MOM estimator, with the K-S test used for testing the data. The decision was made to accept the results of the MLE when the two estimators gave different results, since in no case did the MOM suggest acceptance when the MLE said to reject. In the eight cases where there was a difference, therefore, the MLE was used as the criterion. This gives the following results for the gamma model.

		MOM A	MOM R	
MLE	A	19	8	27
	R	0	19	19
		19	27	

Therefore using the MLE, we would accept the gamma model in 27 out of 46 cases. In 16 of these 27 cases, the lognormal model is also accepted and in 5 additional cases all three distribution assumptions would be accepted. Again only one case (case 6) would we accept the gamma and exponential model but reject the lognormal.

We can also compare lognormal and gamma model using the K-S test.

		gamma A	gamma R	
lognormal	A	13	5	18
	R	13	15	28
		26	20	

The above table shows that the K-S test prefers the gamma model. 27 out of 46 cases accepted the gamma model, but only 18 out of 46 cases accepted the lognormal model.

Tables 6-9, (Appendix B) summarize the results for all cases and for each of the three different types of data. Table 10 (Appendix B) gives total results in terms of sample sizes of 20-29, 30-39, 40-50 and over 50. For exponential assumption, increasing sample size increases rejection per - centages. For gamma assumption, increasing sample size also increases rejection. In lognormal, increasing the sample size decreases rejection up to sample sizes of forthy, then rejection increases rapidly.

Because of the large number of data sets, only some interesting cases of the data analysis and probability plots for those sets are discussed in the next section.

2. Discussion of specific cases

The following discussions illustrate some cases which are of special interest. The data analysis results and plots are given in Appendix A.

a. Set No. 18 - AN/GXS-2(V)

This case is an illustration where all three models are accepted (not rejected) for all tests. Data were collected from 26 maintenance tasks. In this case, as in most other of the formal maintainability demonstration (cases 8-20), the tests were performed in accordance with the

U.S. military standard for maintainability demonstration (MIL-STD-471) [Ref. 23] in which sample selection was generated from knowledge or estimates of component failure rates and tests were conducted using service technicians of the appropriate skill levels and given prior training on the equipments.

This case fits the lognormal, exponential and also gamma for both estimators. All three tests give high significance for lognormality.

b. Set No. 6 - Reactor Coolant Pump

This case is the only one the exponential model accepted and the lognormal model rejected. However the plots are not good. Further investigation is necessary in this case but since only the data was available, it was not possible for us to do so.

c. Sets No. 28 and 41 - Main Rotor Head and Blades

These two cases indicate a very good example of lognormality. All three tests signal "do not reject" for the lognormal assumption. The plots are also very good. The gamma family does not well represent the data sets.

d. Sets No. 1 and 2 - French Fessenheim Pumps

(Down / Repair) times

These two cases are discussed as unsuccessful for gamma and exponential but good for lognormal since at least 2 out of 3 tests give acceptance for the lognormal assumption. For repair times the W test rejects the assumption while the Chi-squared and K-S tests do not, but down times are rejected by only the K-S test. There are some outliers around 5%, but the rest of the data show a good lognormal fit in the probability plot.

The existence of outliers could be a reason for rejection by one of the 3 tests. For downtime data, after trimming the two high outliers the lognormal assumption was also accepted by the K-S test. For repair time, after trimming one outlier the data shows high significance (0.15) for the W test. Thus, in some cases it would be valuable to examine in more detail the reason for the high or low outliers. In this case, only the data was made available so that this was not possible.

e. Set No. 4 - Heater Drain Pump

Set No. 5 - Main Feedwater Pump

Set No. 4 illustrates the case where all three tests for the lognormal model are rejected. Here outliers are important factors. The effect of outliers can be seen easily from the boxplot, histogram and probability plot. No 5 is almost the same as no 4, the only difference between the two being that the K-S test barely accepts the lognormality in set no 5. For these cases, trimming the outliers did not change anything in so far as significance levels or acceptance or rejection are concerned.

f. Set No. 9 - AN/ARC-162 (Intermediate)

Set No. 10 - AN/ARC-163 (Intermediate)

These two cases are discussed together, because they give opposite results. Detailed reports exist in both cases. Test technicians were used for running the tests and removing the faults. [Refs. 12 and 13]

The exponential assumption is rejected in both cases. In set No 9 the lognormal assumption is rejected only by the K-S test while the other tests give high significance. The probability plot and boxplot also signal that "do not reject" is appropriate for this case. The gamma model fits this set for both estimators.

In Set No. 10 only the W test accepts, but the Chi-Squared test barely rejects (0.045) and the K-S test rejects the lognormal model. However the boxplot and probability plot are fairly good for this set. An examination of the report indicates that some bias exists by the use of "average" times for some repair time segments for much of the data instead of normal data. Therefore, a further examination of this data would be in order.

g. Set No. 15 - AN/UYK-14(C)

Set No. 15a - Revised AN/UYK-14(C)

Set No. 15 illustrates what happens when the data points are not selected at random and thus introduce bias. In this case, 33 tests were selected according to the sample selection procedure in MIL-STD-471A [Ref. 23]. However in order to have 50 data points according to the test plan 17 additional data points using "average" repair times for several components from a previous test were used. This therefore introduces a bias in the data, resulting in the initial rejection of the lognormal model (Table 3). After taking out the biased data points the analysis shows that the lognormal model is a good descriptor. The histogram does not look very good, but the probability does for the lognormal model.

This example then shows that if data does not fit an assumed distribution and if the reason is bias, then after finding a good reason for throwing out some bias points, the data frequently will agree with a lognormal model.

h. Sets No 21 and 34 - Fuselage Structure

These two cases are example of large sample sizes (157 and 171). For both data sets all distributional assumptions were rejected by all test methods. But for the lognormal case probability plots, boxplots and histograms are fairly good. These cases should be further investigated.

B. ERRORS IN CALCULATED PARAMETERS

1. Error in MTTR when Assuming an Exponential Distribution Instead of Lognormal

The percent error in the mean-time-to-repair (MTTR) is calculated as follows,

$$E = \frac{|M_{log} - M_{exp}|}{M_{log}} \times 100 \qquad \text{where,}$$

M_{log} = the lognormal mean = $e^{\hat{\mu}+0.5\hat{\sigma}^2}$

M_{exp} = the exponential mean = $\bar{X}$

The results of the percentage error in the mean which are summarized in Table 5. All cases have an error less than 18 %. Therefore since the inherent availability formula normally used is,

$$A_I = \frac{MTBF}{MTBF + MTTR} = \frac{1}{1 + \frac{MTTR}{MTBF}}$$

Table 5

PERCENTAGE ERROR IN MTTR WHEN ASSUMING AN EXPONENTIAL INSTEAD OF LOGNORMAL DISTRIBUTION

Set No -1-	Sample Mean (EXP) -2-	Sample Mean (LOG) -3-	Error (%) $\frac{\|3-2\|}{3}$
1	119.51	112.44	6.29
2	24.0	22.17	8.25
3	121.63	140.84	13.63
5	57.21	48.69	17.49
9	7.23	7.22	0.16
10	9.35	9.34	0.08
11	43.05	43.59	1.23
12	15.75	15.74	0.02
14	15.73	15.54	1.25
15	15.85	15.59	1.65
17	17.42	17.76	1.93
18	19.7	19.13	2.96
22	1.87	2.07	9.73
23	1.55	1.48	4.9
25	1.76	1.78	1.36
27	0.57	0.31	1.36
28	0.41	0.28	1.29
29	2.45	2.44	0.69
30	1.7	1.72	1.45
32	1.64	1.66	0.85
35	4.03	4.57	11.84
36	2.55	2.39	6.96
38	2.26	2.33	2.96
39	4.22	4.1	2.73
40	2.5	2.57	2.57
41	3.5	3.51	0.27
42	3.63	3.66	0.63
43	2.4	2.44	1.78
44	1.72	1.87	7.88
45	2.58	2.60	0.58

NOTE: Only cases included when lognormal model was accepted (at least one out of three test methods) and since for a reasonable availability MTTR << MTBF (e.g. MTTR is measured in hours, MTBF is 100's of hours), then it is shown that the exponential assumption for MTTR introduces negligible error in availability.

C. CONCLUSIONS

From the data analysis in this study as well as the previous study [Ref. 1], it is concluded that the lognormal model is a good descriptor for repair times. 28 out of 46 sets show that, within a 0.05 level of significance for at least one of the 3 test methods , this model can not be rejected. Similarly, the data analysis shows that the assumption of the exponential distribution should be rejected in 40 sets. The gamma assumption is not rejected in 27 cases.

The percent difference in the MTTR, when assuming exponential distribution when the true distribution is lognormal has been found to be negligible for calculated system availability.

When the results of the statistical tests indicate opposite conclusions, probability plots, boxplots and histograms are helpful in determining an acceptable model and to explore anomalies in the data. Probability plots are also useful in estimating percentiles, as well as density parameters.

APPENDIX A

SET NO 1 - FRENCH FESSENHEIM PUMPS (DOWN TIME)

SAMPLE SIZE N = 43   NO. OF CELLS K = 7

SAMPLE MEAN = 119.51   STANDARD DEV = 225.79

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.01	3.89	
PARAM2		1.66	
MTTR	119.51	112.44	6.29 %
50-TH PERCNT	82.84	49.10	68.70 %
90-TH PERCNT	275.19	255.75	7.60 %
95-TH PERCNT	358.02	408.08	12.27 %
CHI-SQR STAT	13.16	2.42	
DEG OF FREED	5	4	
SIGNIF LEVEL	0.219E-01	0.659E+00	

BOXPLOT SET1

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  I+  I---- *      O     O                     O
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BOXPLOT LNSET1

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          ----------------
  --------I         +    I----------------
          ----------------
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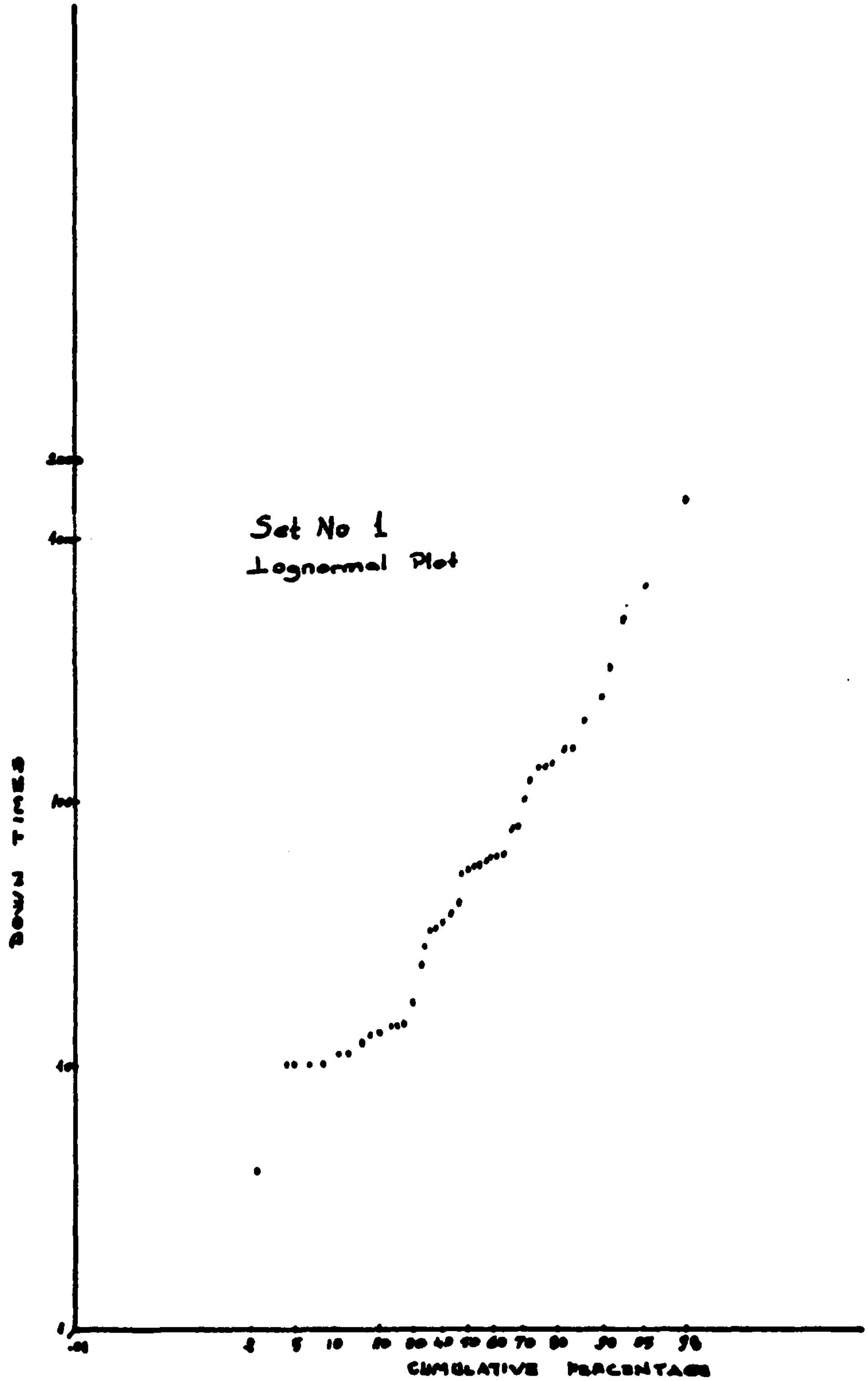
HISTOGRAM SET1

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
10.00	6	******
15.00	6	******
20.00	0	
25.00	1	*
30.00	3	***
35.00	2	**
40.00	1	*
45.00	0	
50.00	0	
55.00	4	****
60.00	4	****
65.00	0	
70.00	0	
75.00	0	
80.00	2	**

HISTOGRAM LNSET1

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
1.5	1	*
2.0	0	
2.5	11	***********
3.0	2	**
3.5	6	******
4.0	8	********
4.5	3	***
5.0	6	******
5.5	2	**
6.0	2	**
6.5	1	*
7.0	1	*


Set No 1
Lognormal Plot
DOWN TIMES
CUMULATIVE PERCENTAGE

SET NO 2 - FRENCH FESSENHEIM PUMPS (REPAIR TIME)

SAMPLE SIZE N = 43 NO. OF CELLS K = 7

SAMPLE MEAN = 24.00 STANDARD DEV = 31.95

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.04	2.73	
PARAM2		0.73	
MTTR	24.00	22.17	8.25 %
50-TH PERCNT	16.64	15.40	7.99 %
90-TH PERCNT	55.26	46.00	20.14 %
95-TH PERCNT	71.90	62.70	14.67 %
CHI-SQR STAT	19.35	4.37	
DEG OF FREED	5	4	
SIGNIF LEVEL	0.166E-02	0.358E+00	

BOXPLOT SET2

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  -----
-I + I----   *     O     O                 O   O
  -----
```

BOXPLOT LNSET2

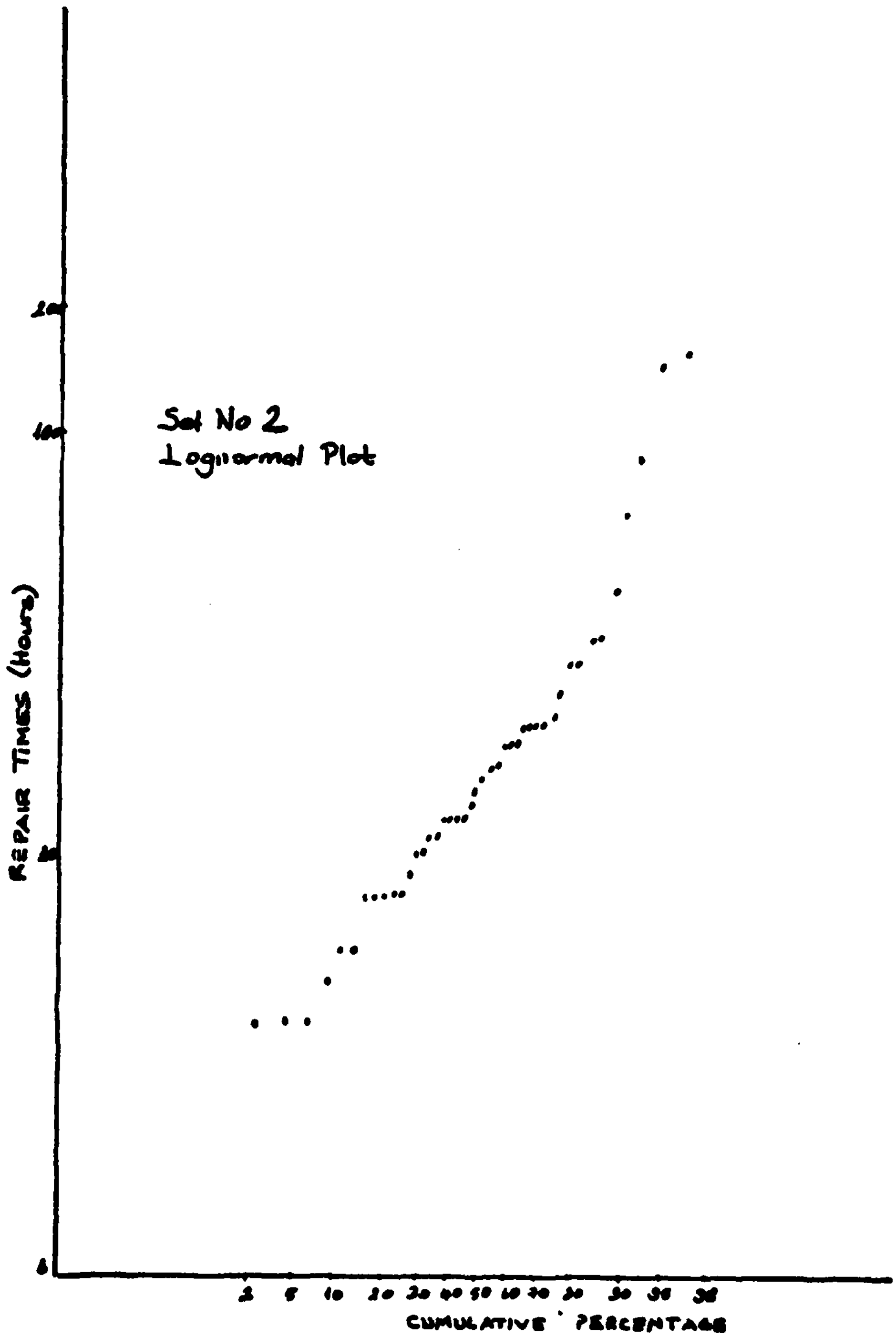
```
       -----------
-------I    +    I-----------      *     **
       -----------
```

HISTOGRAM SET2

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
0.	12	************
20.	24	************************
40.	3	***
60.	1	*
80.	1	*
100.	0	
120.	0	
140.	1	*
160.	1	*

HISTOGRAM LNSET2

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
1.5	4	****
2.0	8	********
2.5	11	***********
3.0	11	***********
3.5	5	*****
4.0	1	*
4.5	1	*
5.0	2	**



Set No 2
Lognormal Plot
REPAIR TIMES (Hours)
CUMULATIVE PERCENTAGE

SET NO 4 - HEATER DRAIN PUMP

SAMPLE SIZE N = 43   NO. OF CELLS K = 7

SAMPLE MEAN = 67.60   STANDARD DEV = 154.16

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.01	2.90	
PARAM2		1.81	
MTTR	67.60	44.72	51.18 %
50-TH PERCNT	46.86	18.09	> 100 %
90-TH PERCNT	155.67	101.51	53.36 %
95-TH PERCNT	202.53	165.42	22.43 %
CHI-SQR STAT	56.47	20.00	
DEG OF FREED	5	4	
SIGNIF LEVEL	0.596E-07	0.500E-03	

BOXPLOT SET4

```
          --
          +I  O  O  O                    O   O  O    O
          --
```

BOXPLOT LNSET4

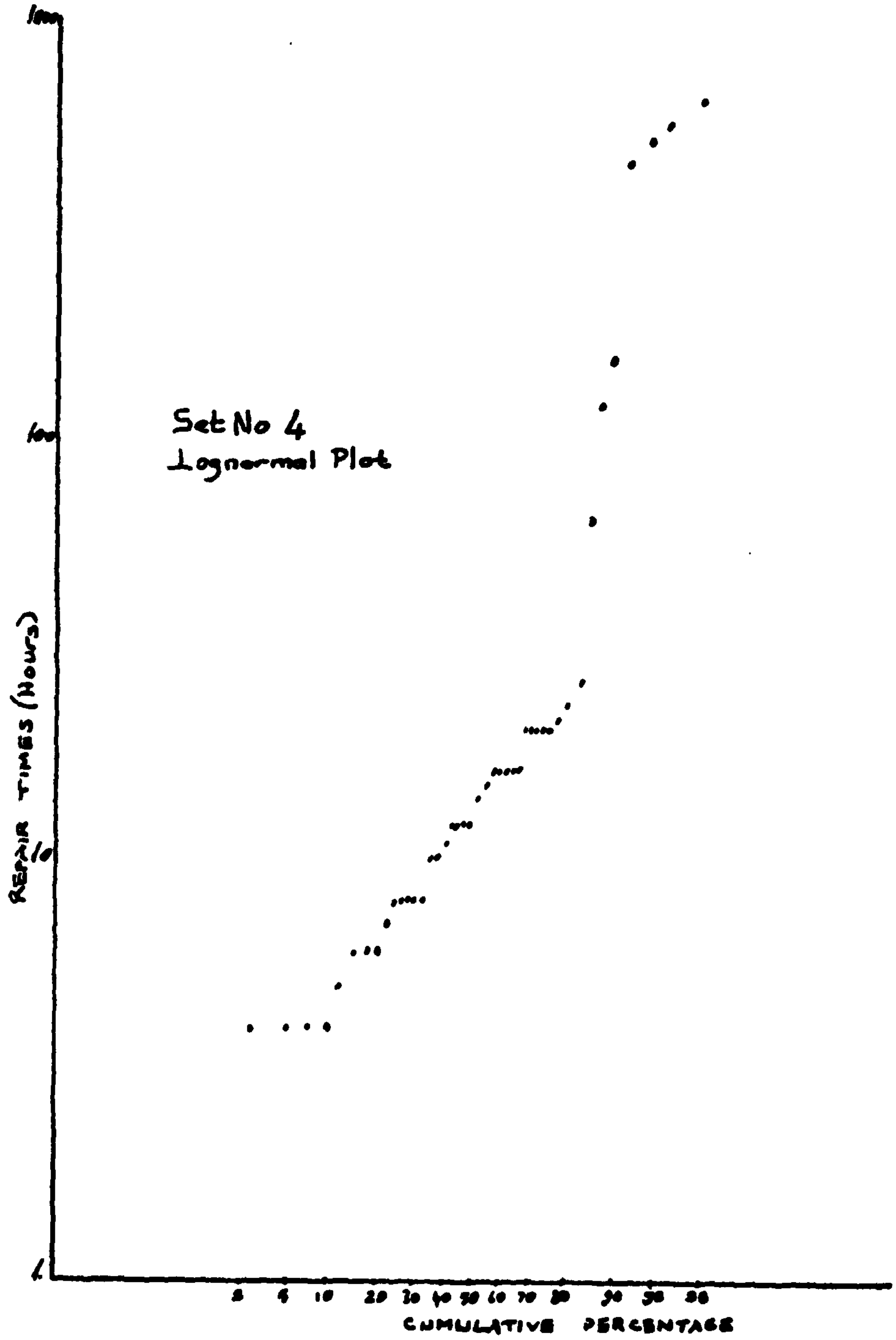
```
                 ----------
          ------I    +    I-----------        * *          0000
                 ----------
```

HISTOGRAM SET4

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
5.00	9	*********
10.00	12	************
15.00	7	*******
20.00	6	******
25.00	2	**

HISTOGRAM LNSET2

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
1.5	5	*****
2.0	9	*********
2.5	9	*********
3.0	12	************
3.5	1	*
4.0	1	*
4.5	0	
5.0	2	**
5.5	0	
6.0	2	**
6.5	2	**


Set No 4
Lognormal Plot
REPAIR TIMES (HOURS)
CUMULATIVE PERCENTAGE

SET NO 5 - MAIN FEEDWATER PUMP

SAMPLE SIZE N = 33 NO. OF CELLS K = 6

SAMPLE MEAN = 57.21 STANDARD DEV = 105.37

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.02	3.20	
PARAM2		1.38	
MTTR	57.21	48.69	17.49 %
50-TH PERCNT	39.66	24.47	62.03 %
90-TH PERCNT	131.74	110.10	19.65 %
95-TH PERCNT	171.39	168.54	1.69 %
CHI-SQR STAT	11.91	10.82	
DEG OF FREED	4	3	
SIGNIF LEVEL	0.180E-01	0.128E-01	

```
BOXPLOT SET5

          ----
          I+ I--             O  O      O                 O
          ----

BOXPLOT LNSET5

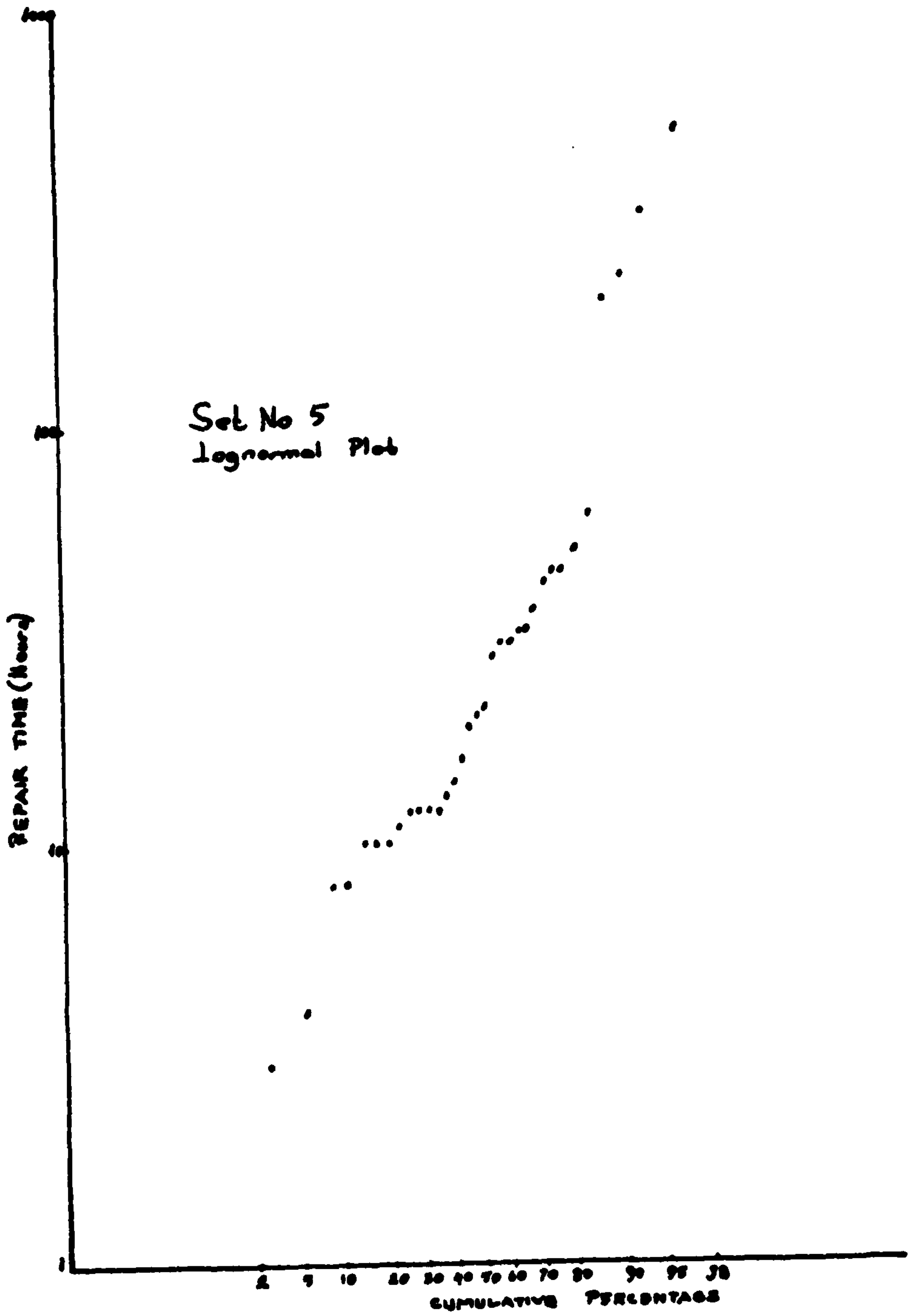
                    --------
          ---------I  +    I-----------   *  *
                    --------
```

HISTOGRAM SET5

```
   MIDDLE OF     NUMBER OF
   INTERVAL      OBSERVATIONS
       5.00         2      **
      10.00        10      **********
      15.00         3      ***
      20.00         3      ***
      25.00         0
      30.00         5      *****
      35.00         1      *
      40.00         1      *
      45.00         2      **
      50.00         1      *
      55.00         0
      60.00         1      *
```

HISTOGRAM LNSET5

```
   MIDDLE OF     NUMBER OF
   INTERVAL      OBSERVATIONS
        1.0         1      *
        1.5         1      *
        2.0         2      **
        2.5        10      **********
        3.0         4      ****
        3.5         7      *******
        4.0         4      ****
        4.5         0
        5.0         0
        5.5         2      **
        6.0         2      **
```


Set No 5
Lognormal Plot
REPAIR TIME (Hours)
CUMULATIVE PERCENTAGE

SET NO 6 - REACTOR COOLANT PUMP

SAMPLE SIZE N = 21 NO. OF CELLS K = 4

SAMPLE MEAN = 58.57 STANDARD DEV = 39.22

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.02	3.64	
PARAM2		1.49	
MTTR	58.57	80.08	26.86 %
50-TH PERCNT	40.60	38.05	6.69 %
90-TH PERCNT	134.87	181.78	25.81 %
95-TH PERCNT	175.46	283.05	38.01 %
CHI-SQR STAT	2.81	3.95	
DEG OF FREED	2	1	
SIGNIF LEVEL	0.245E+00	0.468E-01	

BOXPLOT SET6

```
               --------------
       -------I      +      I------------------
               --------------
```

BOXPLOT LNSET6

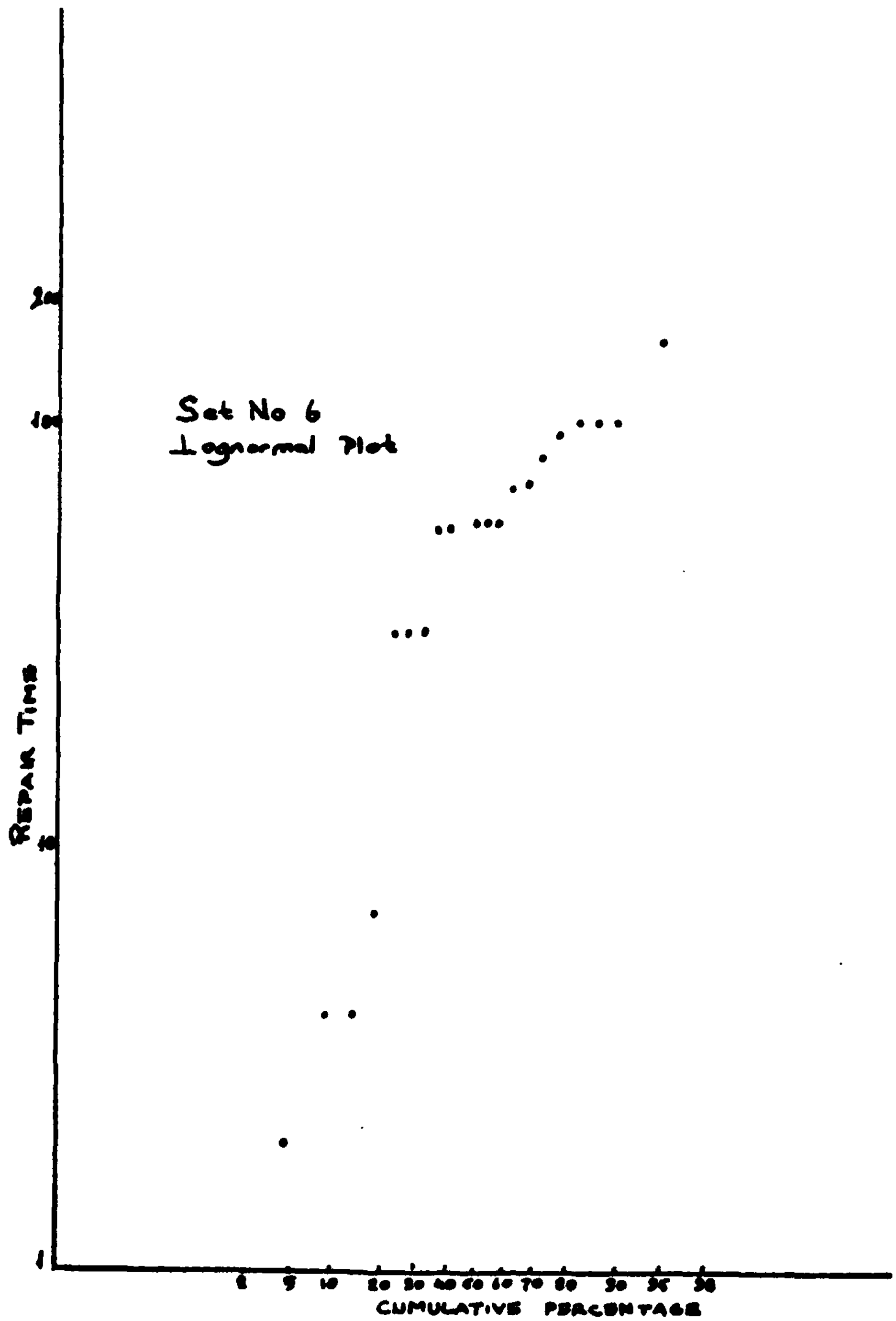
```
                                     -----------
  *     *     *                      I     +    I------
                                     -----------
```

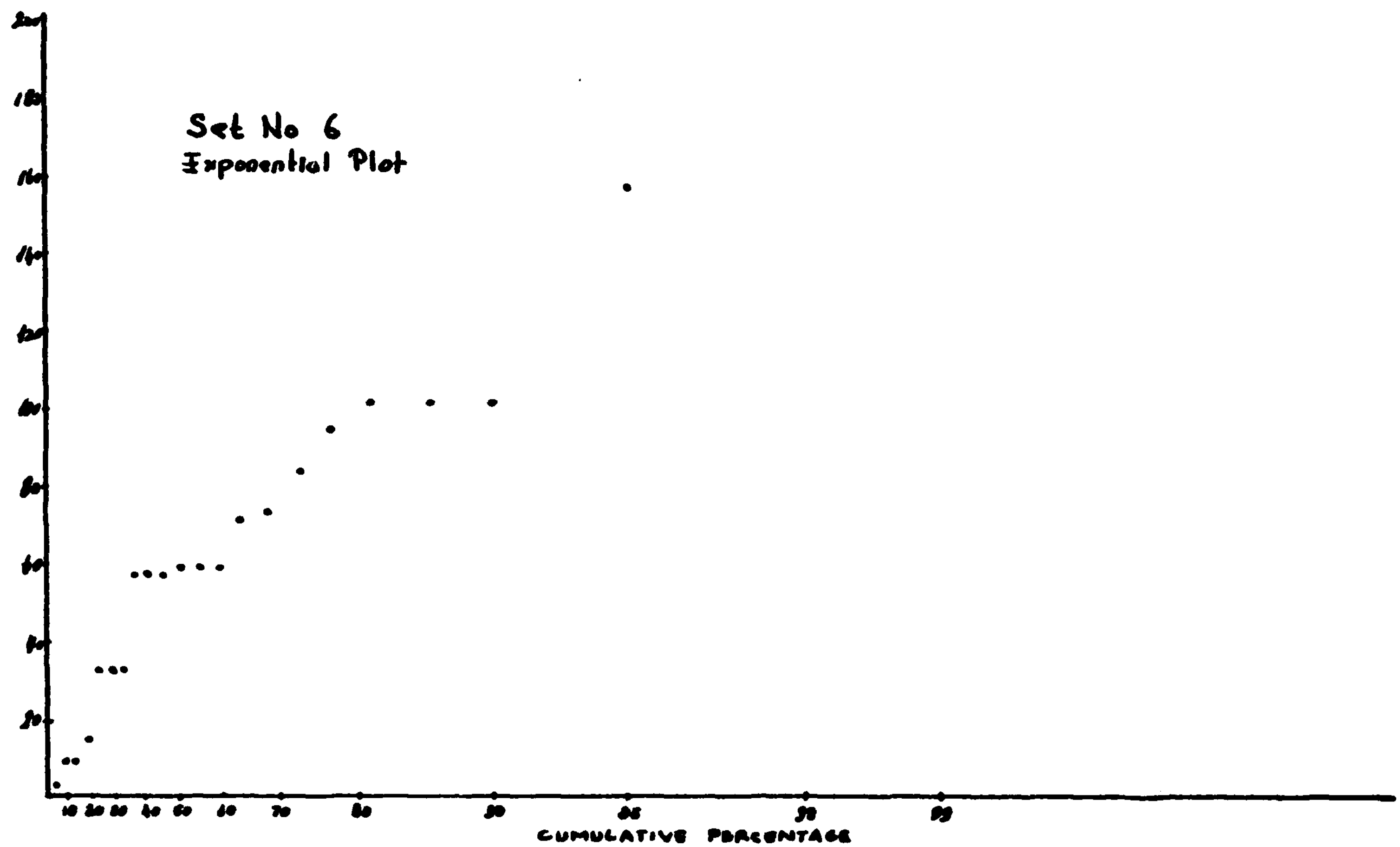
HISTOGRAM SET6

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
0.	4	****
20.	0	
40.	3	***
60.	6	******
80.	3	***
100.	4	****
120.	0	
140.	0	
160.	1	*

HISTOGRAM LNSET6

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
0.5	1	*
1.0	0	
1.5	2	**
2.0	1	*
2.5	0	
3.0	0	
3.5	3	***
4.0	7	*******
4.5	6	******
5.0	1	*


Set No 6
Lognormal Plot
REPAIR TIME
CUMULATIVE PERCENTAGE


Set No 6
Exponential Plot
CUMULATIVE PERCENTAGE

SET NO 9 - AN/ARC-162 (V) (INTERMEDIATE LEVEL)

SAMPLE SIZE N = 49   NO. OF CELLS K = 10

SAMPLE MEAN = 7.23   STANDARD DEV = 3.44

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.14	1.88	
PARAM2		0.19	
MTTR	7.23	7.22	0.16 %
50-TH PERCNT	5.01	6.57	23.64 %
90-TH PERCNT	16.65	11.49	44.98 %
95-TH PERCNT	21.67	13.46	60.99 %
CHI-SQR STAT	47.94	6.71	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.119E-06	0.459E+00	

BOXPLOT SET9

```
           ---------------
     ------I  +          I-----------------     **        *
           ---------------
```

BOXPLOT LNSET9

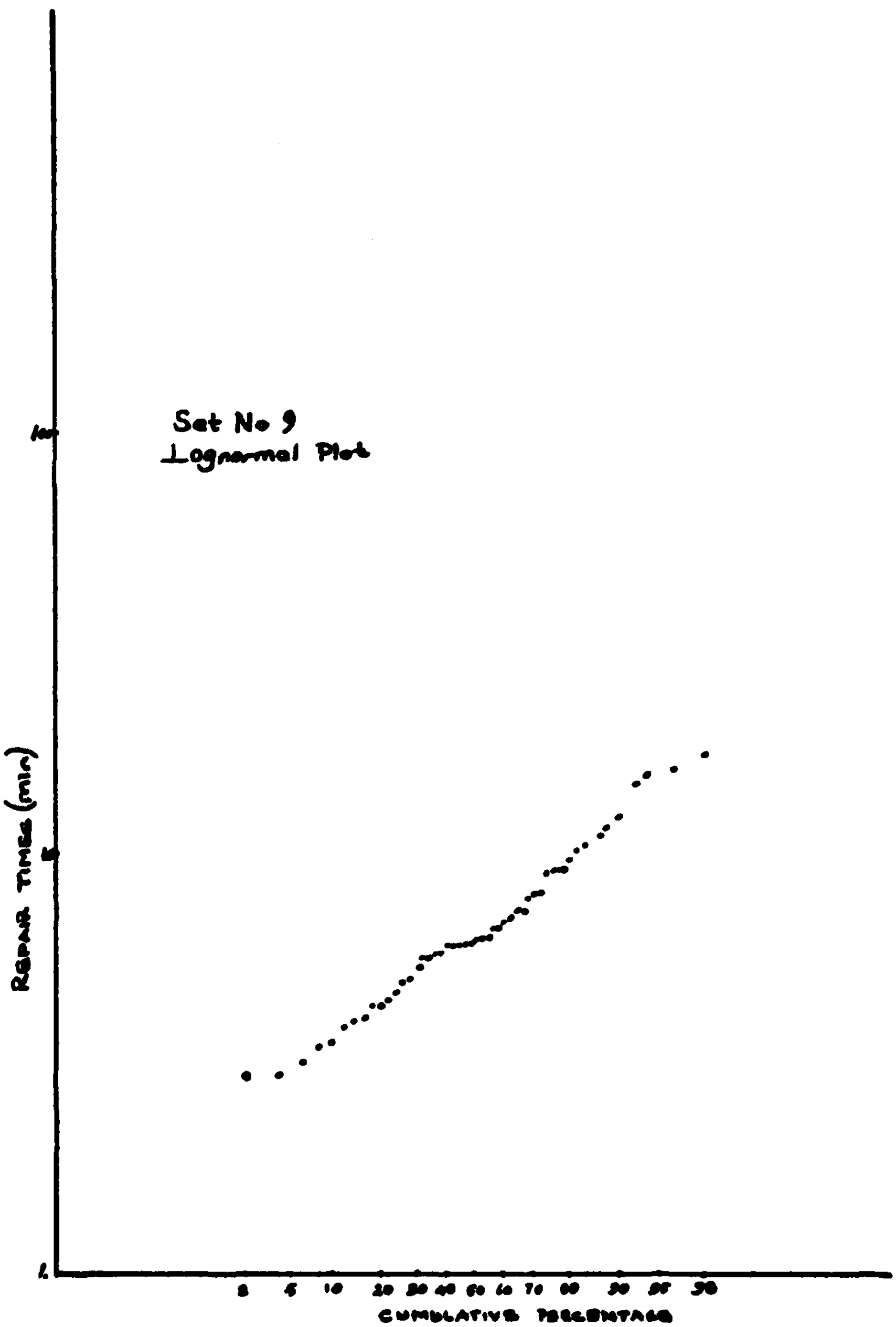
```
                 ----------------
     ------------I     +        I-----------------
                 ----------------
```

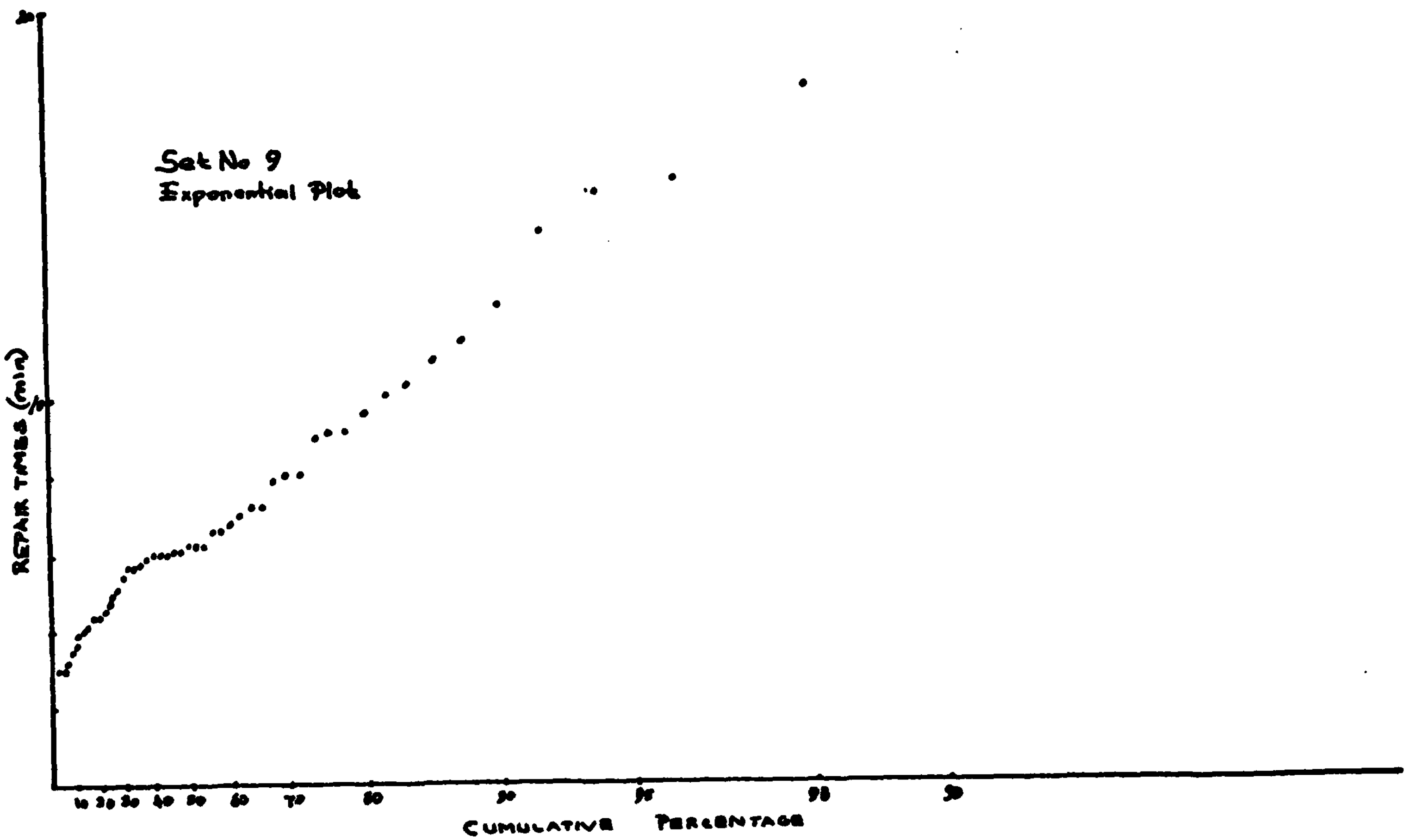
HISTOGRAM SET9

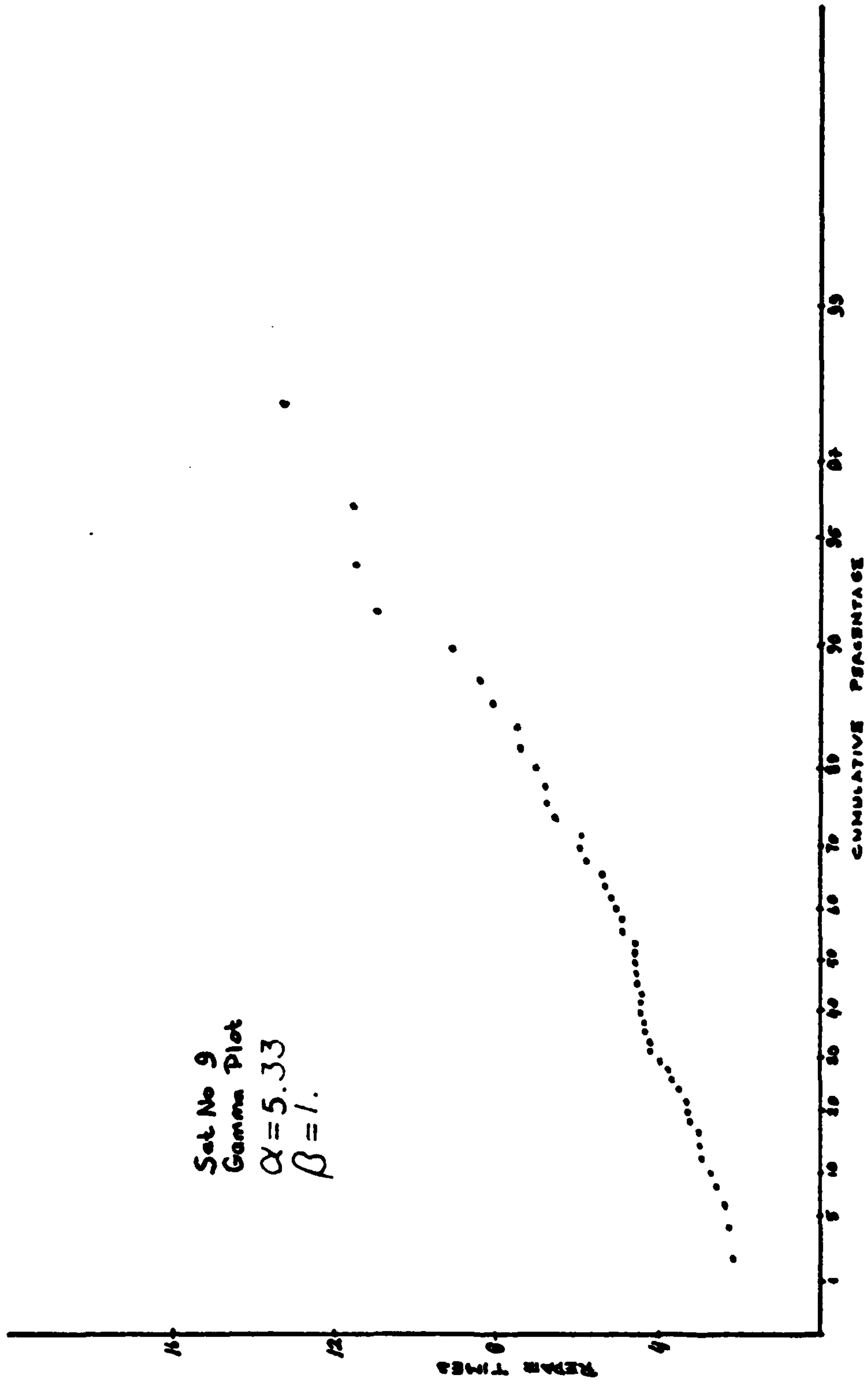
```
MIDDLE OF    NUMBER OF
INTERVAL     OBSERVATIONS
    2.          0
    4.         12     ************
    6.         18     ******************
    8.          6     ******
   10.          6     ******
   12.          3     ***
   14.          1     *
   16.          2     **
   18.          1     *
```

HISTOGRAM LNSET9

```
MIDDLE OF    NUMBER OF
INTERVAL     OBSERVATIONS
   1.0          2     **
   1.2          3     ***
   1.4          5     *****
   1.6          5     *****
   1.8         14     **************
   2.0          7     *******
   2.2          4     ****
   2.4          4     ****
   2.6          2     **
   2.8          3     ***
```


Set No 9
Lognormal Plot
REPAIR TIMES (min)
CUMULATIVE PERCENTAGE


Set No 9
Exponential Plot
REPAIR TIMES (min)
10 20 30 40 50 60 70 80 90 95 98 99
CUMULATIVE PERCENTAGE


Set No 9
Gamma Plot
$\alpha = 5.33$
$\beta = 1.$
CUMULATIVE PERCENTAGE
REPAIR TIMES

SET NO 10 - AN/ARC-163 (V) (INTERMEDIATE LEVEL)

SAMPLE SIZE N = 50 NO. OF CELLS K = 10

SAMPLE MEAN = 9.35 STANDARD DEV = 4.27

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.11	2.14	
PARAM2		0.18	
MTTR	9.35	9.34	0.08 %
50-TH PERCNT	6.48	8.53	24.03 %
90-TH PERCNT	21.52	14.73	46.09 %
95-TH PERCNT	28.00	17.20	62.82 %
CHI-SQR STAT	53.20	14.40	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.596E-07	0.445E-01	

BOXPLOT SET10

```
              ----------
-------I    +    I-------------     ***   *    **
              ----------
```

BOXPLOT LNSET10

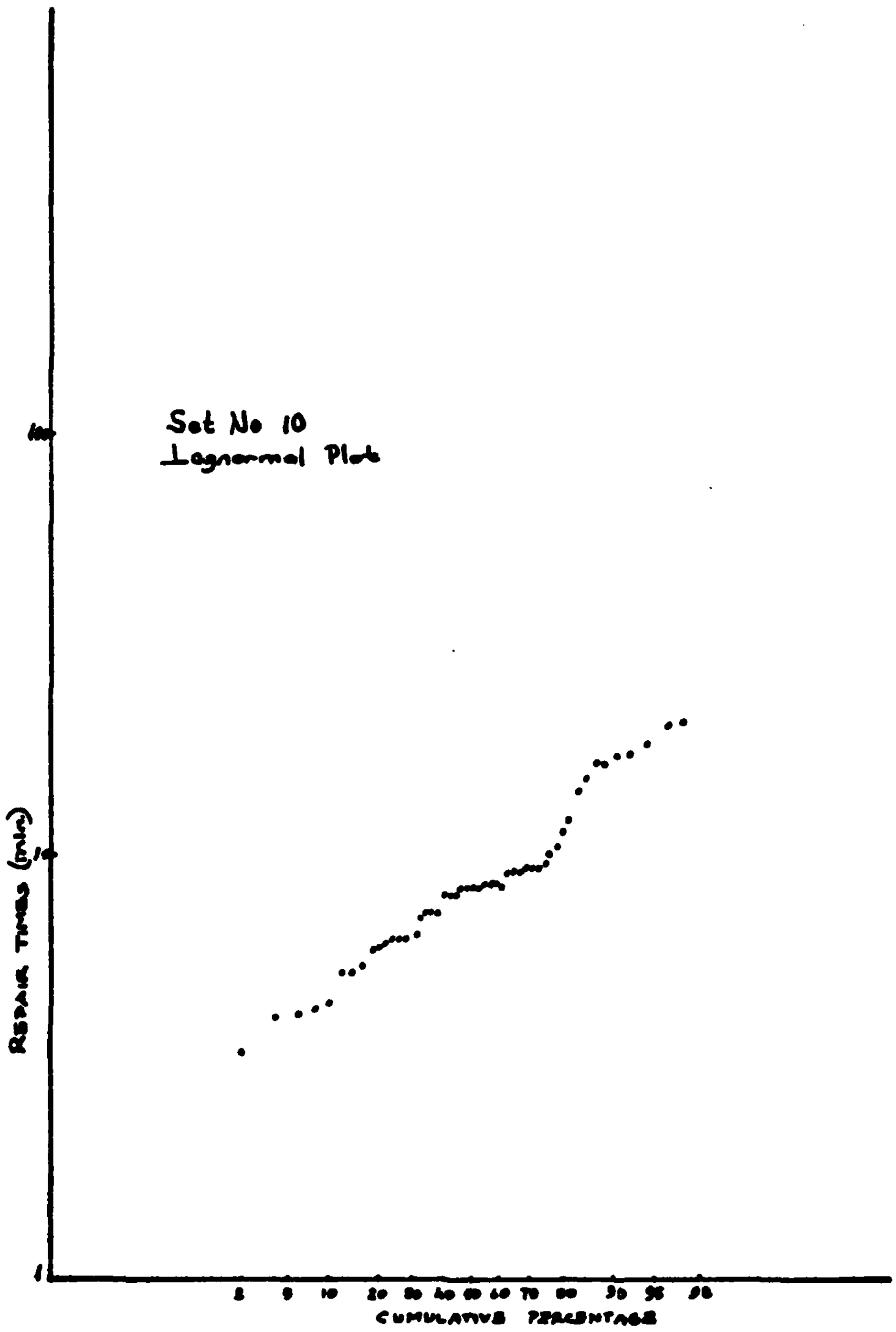
```
                      -------------
--------------I        +        I----------------
                      -------------
```

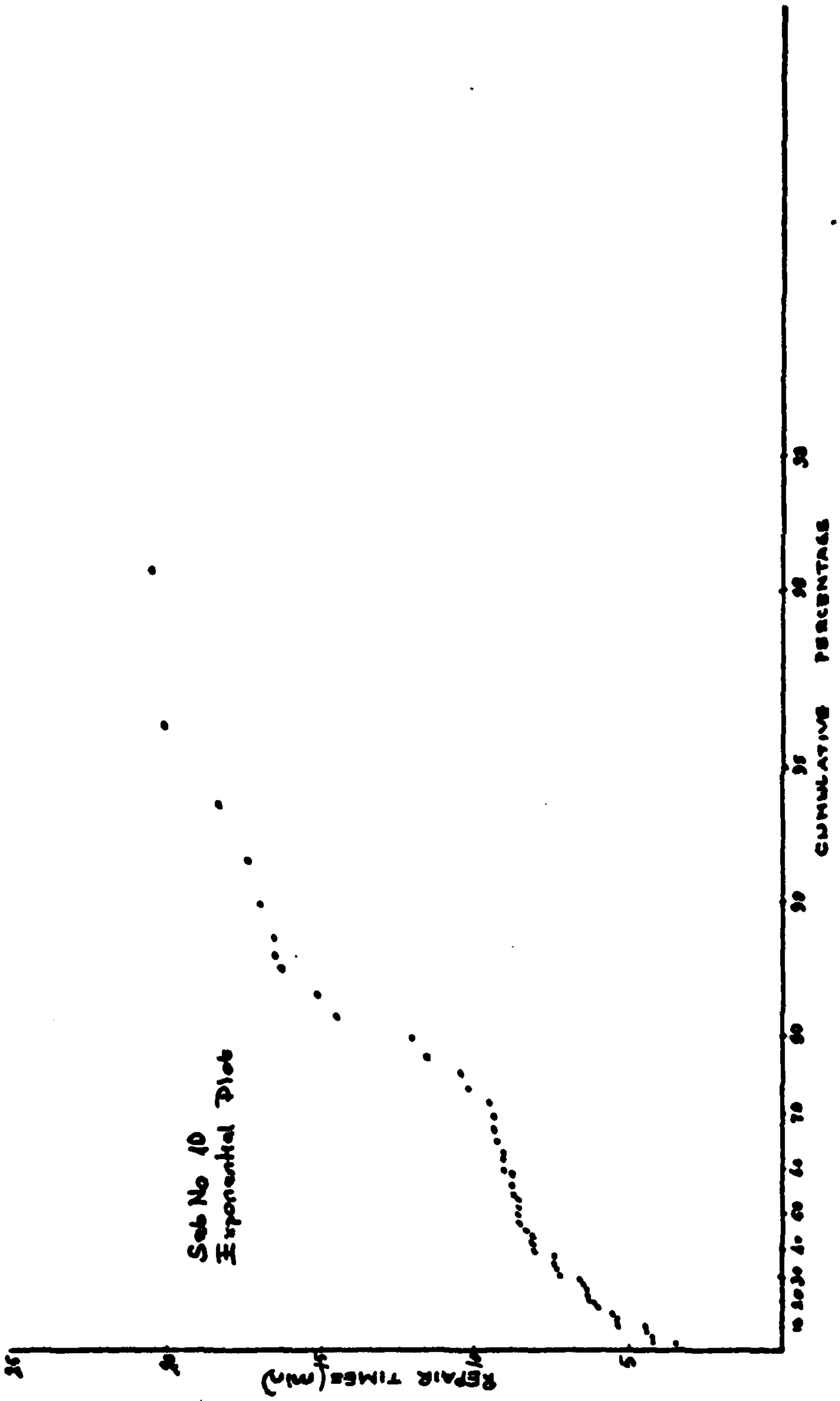
HISTOGRAM SET10

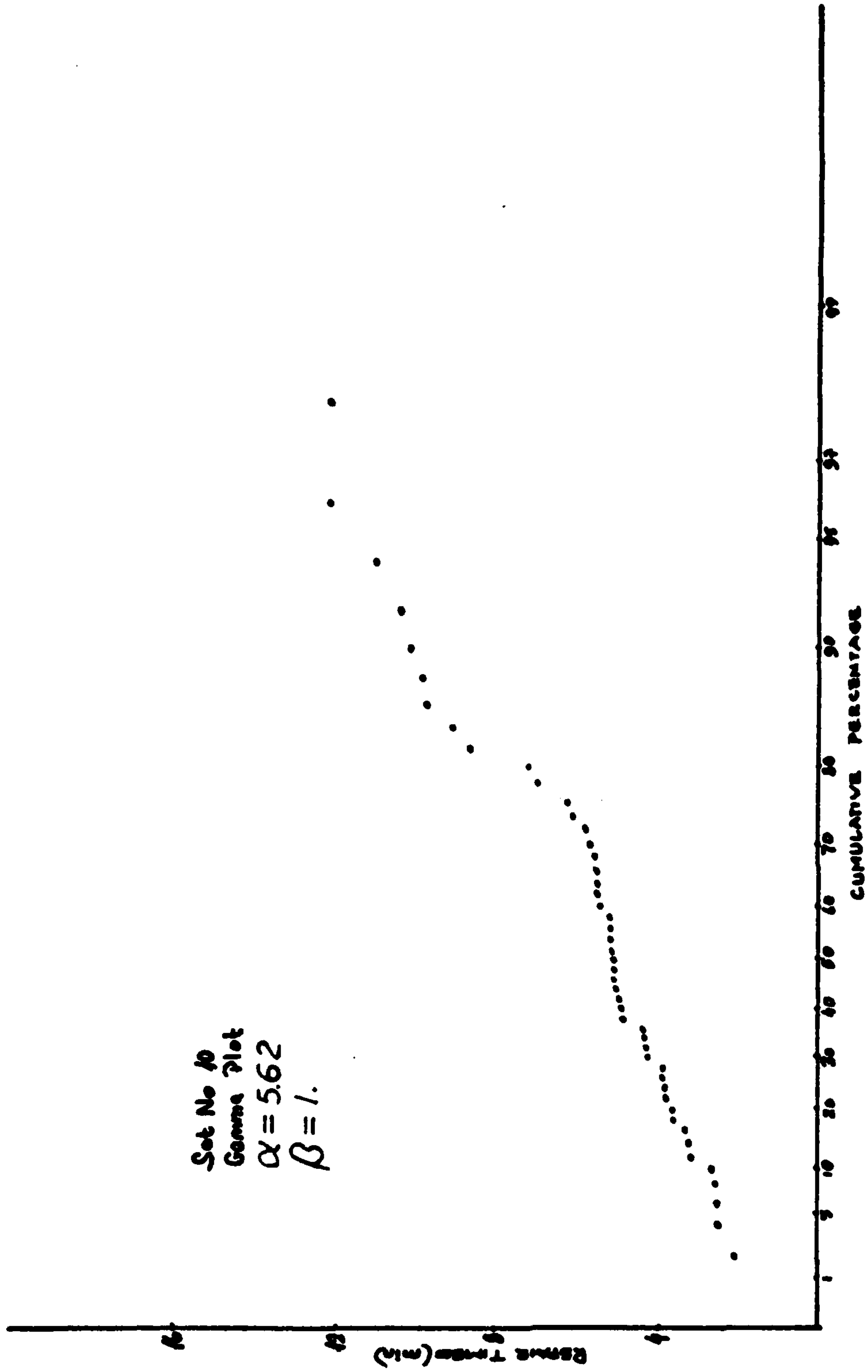
MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
4.	5	*****
6.	10	**********
8.	15	***************
10.	9	*********
12.	2	**
14.	1	*
16.	3	***
18.	3	***
20.	2	**

HISTOGRAM LNSET10

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
1.2	1	*
1.4	3	***
1.6	3	***
1.8	8	********
2.0	7	*******
2.2	15	***************
2.4	4	****
2.6	1	*
2.8	5	*****
3.0	3	***


Set No 10
Lognormal Plot
100
10
1
REPAIR TIMES (min)
CUMULATIVE PERCENTAGE


Sub No 10
Exponential Plot
CUMULATIVE PERCENTAGE
REPAIR TIMES (min)


Set No 10
Gamma Plot
$\alpha = 5.62$
$\beta = 1.$
CUMULATIVE PERCENTAGE

SET NO 15 - AN/UYK-14(C)

SAMPLE SIZE N = 50 NO. OF CELLS K = 10

SAMPLE MEAN = 15.85 STANDARD DEV = 11.06

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.06	2.58	
PARAM2		0.33	
MTTR	15.85	15.59	1.65 %
50-TH PERCNT	10.99	13.21	16.85 %
90-TH PERCNT	36.50	27.63	32.09 %
95-TH PERCNT	47.48	34.05	39.45 %
CHI-SQR STAT	35.60	20.80	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.208E-04	0.408E-02	

BOXPLOT SET15

```
       -----------
---I +         I----------         *      *
       -----------
```

BOXPLOT LNSET15

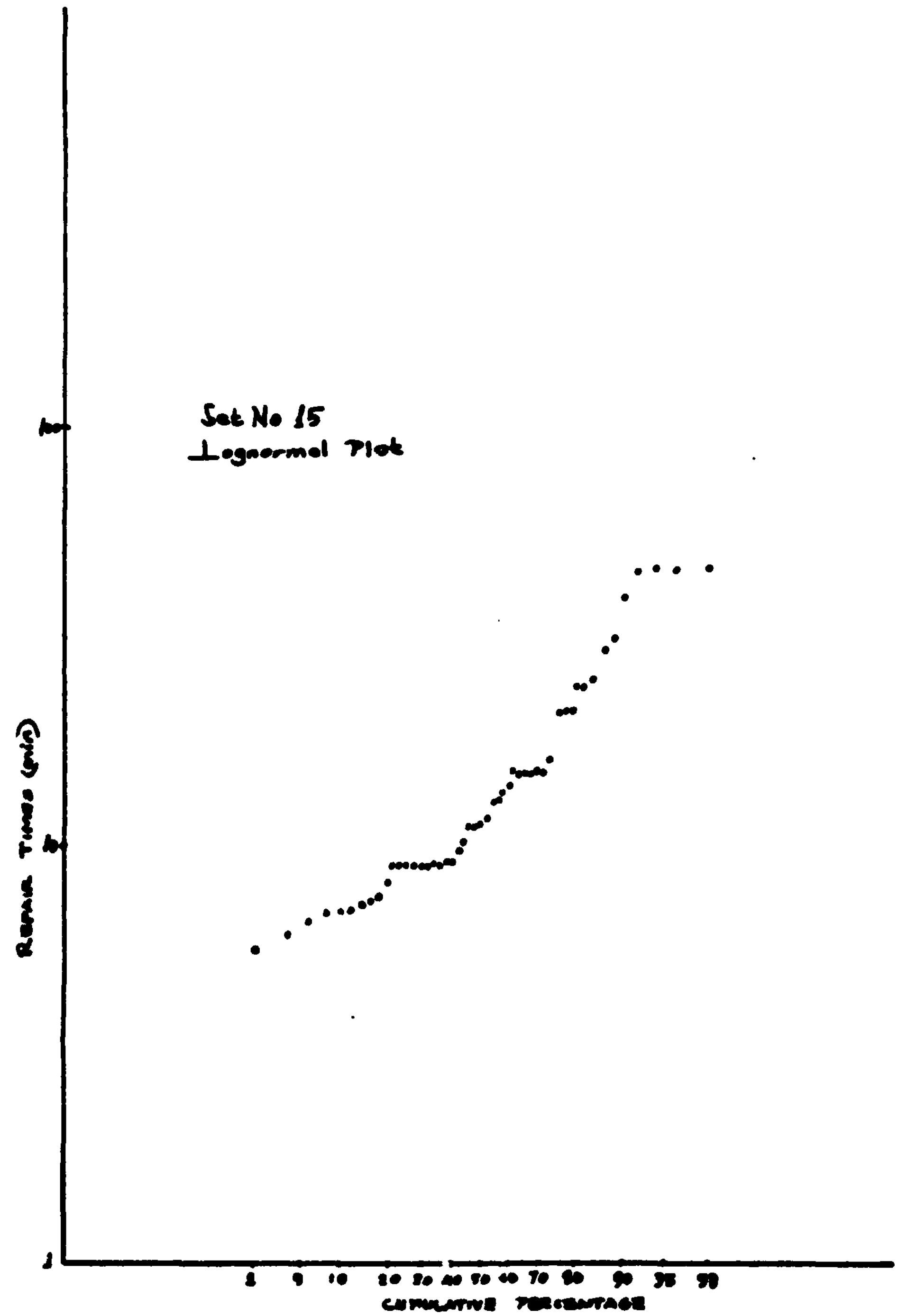
```
        ---------------
---------I   +             I---------------
        ---------------
```

HISTOGRAM SET15

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
5.	8	********
10.	18	******************
15.	11	***********
20.	3	***
25.	3	***
30.	2	**
35.	0	
40.	1	*
45.	4	****

HISTOGRAM LNSET15

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
1.8	3	***
2.0	6	******
2.2	12	************
2.4	5	*****
2.6	4	****
2.8	7	*******
3.0	3	***
3.2	3	***
3.4	2	**
3.6	1	*
3.8	4	****



Set No 15
Lognormal Plot
Repair Times (min)
CUMULATIVE PERCENTAGE

SET NO 15A - REVISED AN/UYK-14(C)

SAMPLE SIZE N = 33 NO. OF CELLS K = 6

SAMPLE MEAN = 14.37 STANDARD DEV = 9.66

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.07	2.50	
PARAM2		0.31	
MTTR	14.37	14.19	1.29 %
50-TH PERCNT	9.96	12.15	17.98 %
90-TH PERCNT	33.10	24.83	33.30 %
95-TH PERCNT	43.06	30.40	41.65 %
CHI-SQR STAT	26.82	2.45	
DEG OF FREED	4	3	
SIGNIF LEVEL	0.216E-04	0.484E+00	

BOXPLOT SET15A

```
     ----------
    -I    +    I---------   * *       *      O
     ----------
```

BOXPLOT LNSET15A

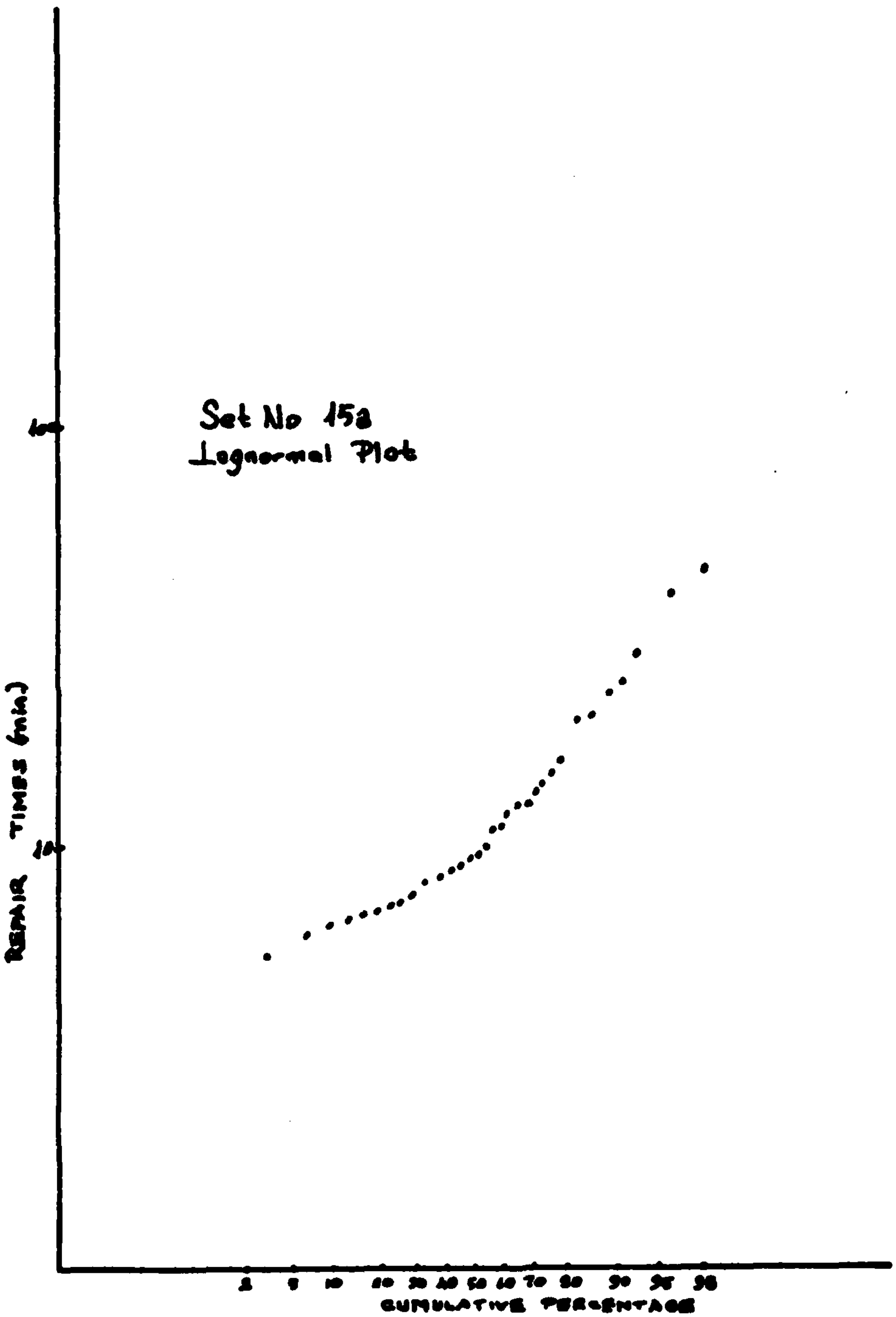
```
         ----------------
    -----I       +      I--------------------
         ----------------
```

HISTOGRAM SET15A

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
5.	8	********
10.	11	***********
15.	6	******
20.	2	**
25.	2	**
30.	2	**
35.	0	
40.	1	*
45.	1	*

HISTOGRAM LNSET15A

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
1.8	3	***
2.0	6	******
2.2	6	******
2.4	4	****
2.6	4	****
2.8	2	**
3.0	2	**
3.2	2	**
3.4	2	**
3.6	1	*
3.8	1	*



Set No 15a
Lognormal Plot
REPAIR TIMES (min)
CUMULATIVE PERCENTAGE

SET NO 18 - AN/GXS-2 (V)

SAMPLE SIZE N = 26   NO. OF CELLS K = 5

SAMPLE MEAN = 19.70   STANDARD DEV = 22.62

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.05	2.56	
PARAM2		0.78	
MTTR	19.70	19.13	2.96 %
50-TH PERCNT	13.65	12.93	5.61 %
90-TH PERCNT	45.35	40.22	12.77 %
95-TH PERCNT	59.00	55.46	6.39 %
CHI-SQR STAT	5.92	0.54	
DEG OF FREED	3	2	
SIGNIF LEVEL	0.115E+00	0.764E+00	

BOXPLOT SET18

```
        -------
      --I  +   I------            O                 O
        -------
```

BOXPLOT LNSET18

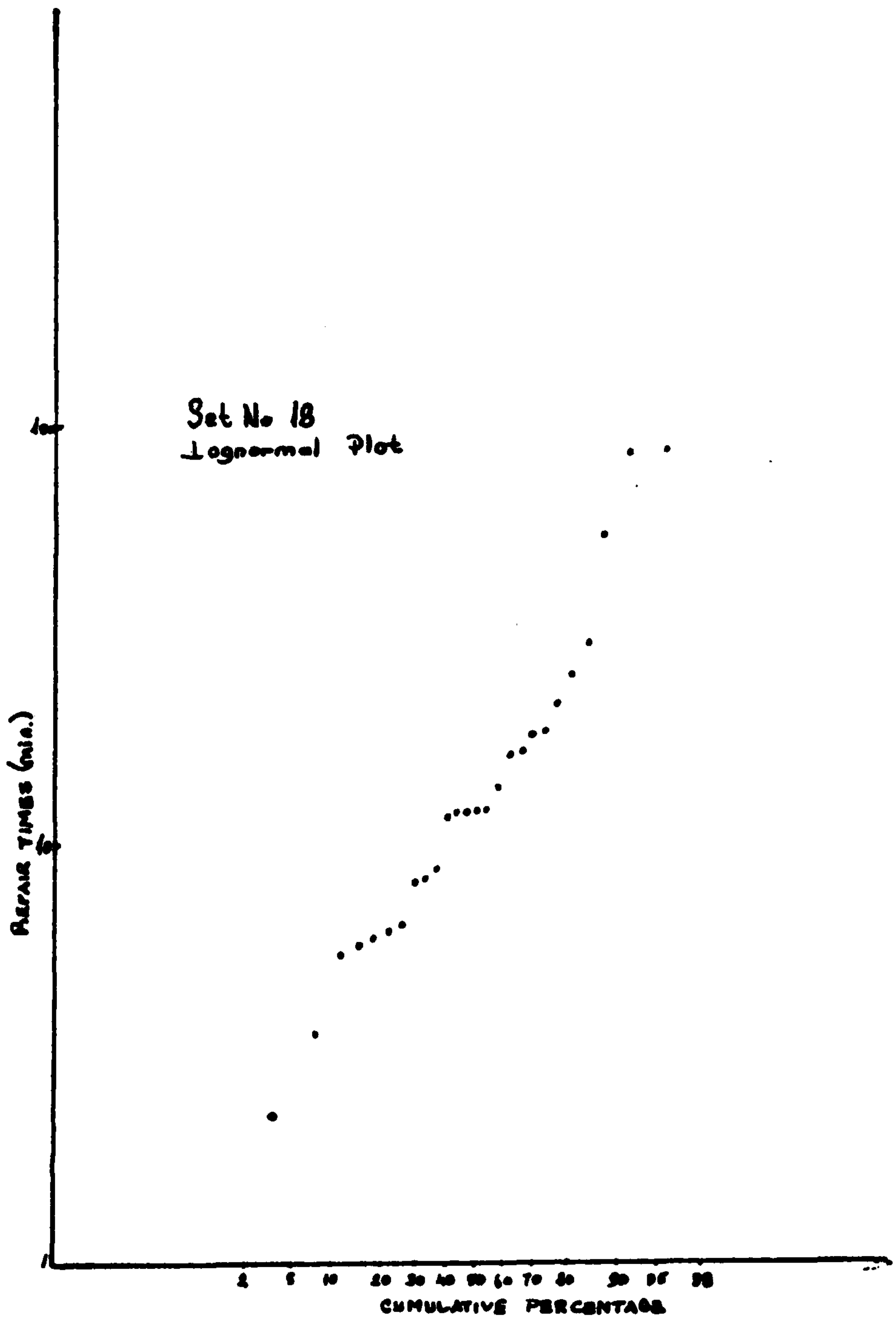
```
                ------------
      ----------I     +     I---------------
                ------------
```

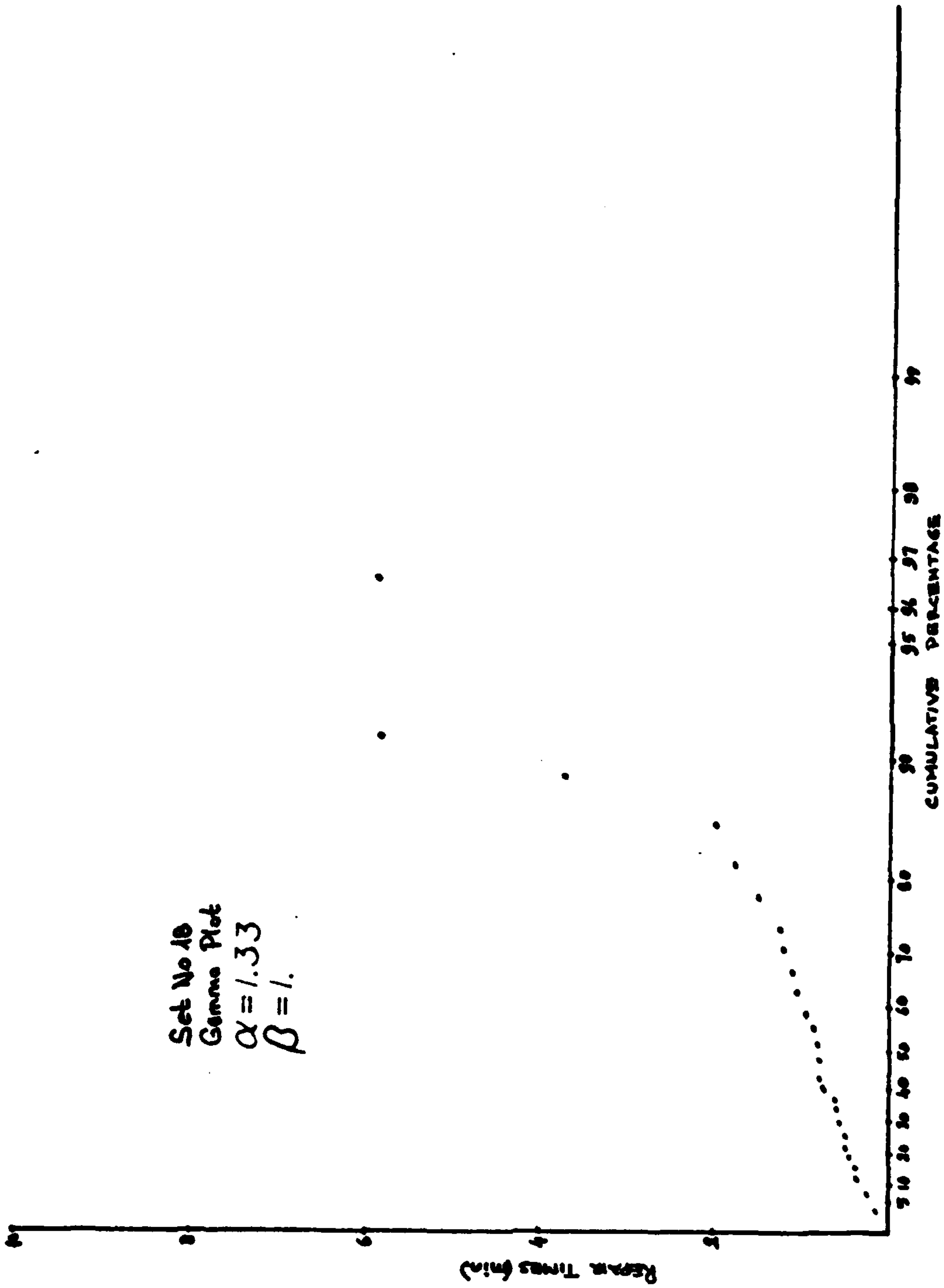
HISTOGRAM SET18

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
0.	2	**
10.	14	**************
20.	5	*****
30.	2	**
40.	0	
50.	0	
60.	1	*
70.	0	
80.	0	
90.	2	**

HISTOGRAM LNSET18

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
1.0	1	*
1.5	3	***
2.0	6	******
2.5	6	******
3.0	5	*****
3.5	2	**
4.0	1	*
4.5	2	**


Set No 18
Lognormal Plot
REPAIR TIMES (min.)
CUMULATIVE PERCENTAGE
2 5 10 20 30 40 50 60 70 80 90 95 98


Set No 18
Gamma Plot
α = 1.33
β = 1.
CUMULATIVE PERCENTAGE
REPAIR TIMES (min)

SET NO 21 - FUSELAGE STRUCTURE

SAMPLE SIZE N = 157 NO. OF CELLS K = 24

SAMPLE MEAN = 1.54 STANDARD DEV = 1.83

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.65	-0.06	
PARAM2		0.97	
MTTR	1.54	1.53	1.01 %
50-TH PERCNT	1.07	0.94	13.52 %
90-TH PERCNT	3.56	3.33	6.93 %
95-TH PERCNT	4.63	4.75	2.64 %
CHI-SQR STAT	169.06	155.0	
DEG OF FREED	22	21	
SIGNIF LEVEL	0.0	0.0	

BOXPLOT SET21

```
    --------
--I  +    I------      **   **     000 0 0      0        00
    --------
```

BOXPLOT LNSET21

```
                 -------------
-----------------I     +     I-----------------
                 -------------
```

```
HISTOGRAM SET 21
   EACH * REPRESENTS   2 OBSERVATIONS

   MIDDLE OF    NUMBER OF
   INTERVAL     OBSERVATIONS
       0.          33      *****************
       1.          73      *************************************
       2.          27      **************
       3.           7      ****
       4.           4      **
       5.           2      *
       6.           5      ***
       7.           3      **
       8.           1      *
       9.           0
      10.           2      *

HISTOGRAM LNSET21

   MIDDLE OF    NUMBER OF
   INTERVAL     OBSERVATIONS
     -2.5           3      ***
     -2.0           2      **
     -1.5          12      ************
     -1.0          16      ****************
     -0.5          34      **********************************
      0.0          36      ************************************
      0.5          27      ***************************
      1.0          10      **********
      1.5           7      *******
      2.0           8      ********
      2.5           2      **
```

SET NO 28 - MAIN ROTOR HEAD AND BLADES (ELAPSE TIME)

SAMPLE SIZE N = 33 NO. OF CELLS K = 6

SAMPLE MEAN = 2.07 STANDARD DEV = 1.87

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.48	0.46	
PARAM2		0.52	
MTTR	2.07	2.05	0.89 %
50-TH PERCNT	1.44	1.58	9.09 %
90-TH PERCNT	4.77	4.00	19.31 %
95-TH PERCNT	6.21	5.20	19.33 %
CHI-SQR STAT	11.91	2.09	
DEG OF FREED	4	3	
SIGNIF LEVEL	0.180E-01	0.554E+00	

BOXPLOT SET28

```
      ---------
    --I  +     I----------                                    O
      ---------
```

BOXPLOT LNSET28

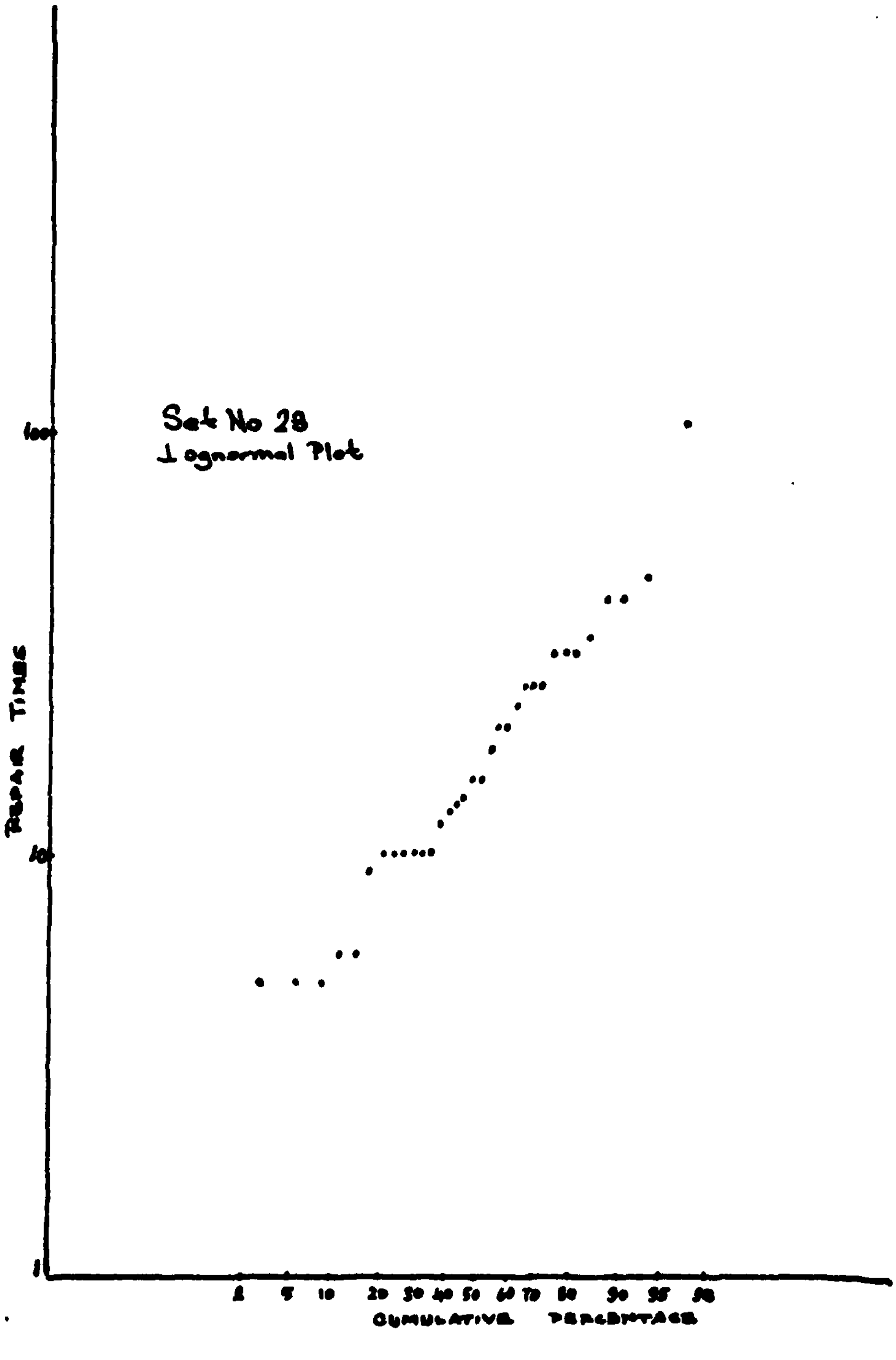
```
             --------------
    ---------I     +      I---------            *
             --------------
```

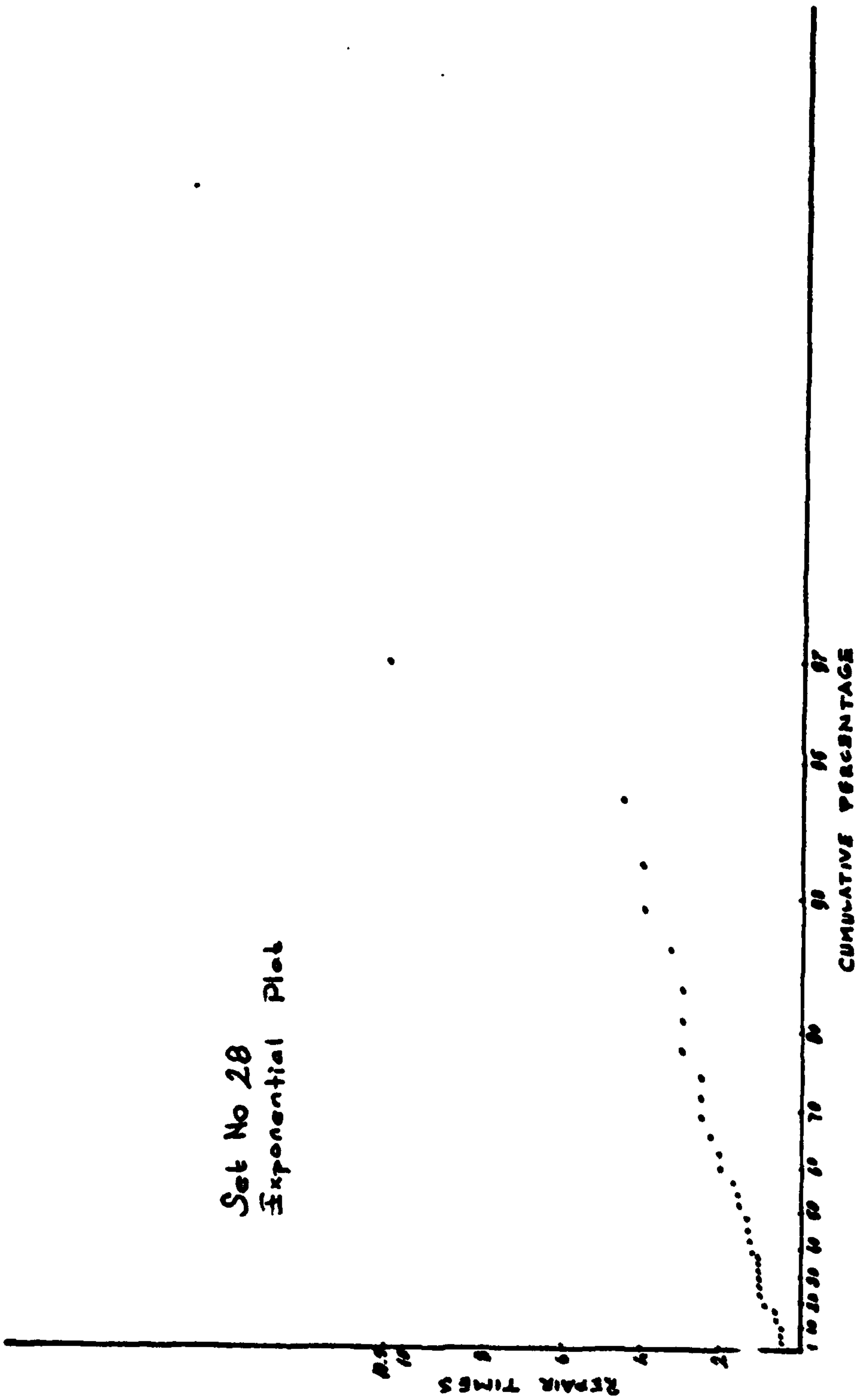
```
HISTOGRAM SET28

   MIDDLE OF    NUMBER OF
   INTERVAL     OBSERVATIONS
        0.          0
        1.         16      ****************
        2.          6      ******
        3.          7      *******
        4.          2      **
        5.          1      *
        6.          0
        7.          0
        8.          0
        9.          0
       10.          0
       11.          1      *

HISTOGRAM LNSET28

   MIDDLE OF    NUMBER OF
   INTERVAL     OBSERVATIONS
      -0.8          3      ***
      -0.4          2      **
       0.0          8      ********
       0.4          6      ******
       0.8          6      ******
       1.2          6      ******
       1.6          1      *
       2.0          0
       2.4          1      *
```


Set No 28
Lognormal Plot
100
10
1
REPAIR TIMES
2
5
10
20
30
40
50
60
70
80
90
95
98
CUMULATIVE PERCENTAGE


Set No 28
Exponential Plot
REPAIR TIMES
CUMULATIVE PERCENTAGE

SET NO 34 - FUSELAGE STRUCTURE

SAMPLE SIZE N = 171 NO. OF CELLS K = 34

SAMPLE MEAN = 2.83 STANDARD DEV = 4.60

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.35	0.36	
PARAM2		1.26	
MTTR	2.83	2.68	5.26 %
50-TH PERCNT	1.96	1.43	36.77 %
90-TH PERCNT	6.51	6.03	7.95 %
95-TH PERCNT	8.46	9.05	6.51 %
CHI-SQR STAT	126.65	92.85	
DEG OF FREED	32	31	
SIGNIF LEVEL	0.596E-07	0.596E-07	

BOXPLOT SET34

```
-----
I+  I----* **OO     O OOO           O    O          O
-----
```

BOXPLOT LNSET34

```
                      -----------
*    ---------I   +     I--------------**
                      -----------
```

```
HISTOGRAM SET34

   5 OBSERVATIONS ARE BELOW THE FIRST CLASS

   MIDDLE OF    NUMBER OF
   INTERVAL     OBSERVATIONS
     0.500         38      **************************************
     1.000         33      *********************************
     1.500         25      *************************
     2.000         15      ***************
     2.500          9      *********
     3.000         13      *************
     3.500          2      **
     4.000          3      ***
     4.500          3      ***
     5.000          3      ***
     5.500          0
     6.000          5      *****
     6.500          1      *
     7.000          1      *

HISTOGRAM LNSET34

   MIDDLE OF    NUMBER OF
   INTERVAL     OBSERVATIONS
    -2.500          2      **
    -2.250          0
    -2.000          0
    -1.750          3      ***
    -1.500          5      *****
    -1.250          0
    -1.000         11      ***********
    -0.750         15      ***************
    -0.500          6      ******
    -0.250         13      *************
     0.0           19      *******************
     0.250         15      ***************
     0.500         19      *******************
     0.750         12      ************
     1.000         18      ******************
     1.250          3      ***
     1.500          8      ********
     1.750          6      ******
     2.000          2      **
     2.250          7      *******
     2.500          1      *
     2.750          3      ***
     3.000          0
     3.250          2      **
     3.500          1      *
```

SET NO 41 - MAIN ROTOR HEAD AND BLADES (MAN HOURS)

SAMPLE SIZE N = 35 NO. OF CELLS K = 7

SAMPLE MEAN = 3.50 STANDARD DEV = 3.42

	EXPONENTIAL	LOGNORMAL	DIFFERENCE
PARAM1	0.29	0.90	
PARAM2		0.71	
MTTR	3.50	3.51	0.27 %
50-TH PERCNT	2.43	2.47	1.57 %
90-TH PERCNT	8.06	7.25	11.29 %
95-TH PERCNT	10.49	9.83	6.72 %
CHI-SQR STAT	11.20	7.20	
DEG OF FREED	5	4	
SIGNIF LEVEL	0.476E-01	0.126E+00	

BOXPLOT SET41

```
         ---------
        -I  +    I---------- *                          O
         ---------
```

BOXPLOT LNSET41

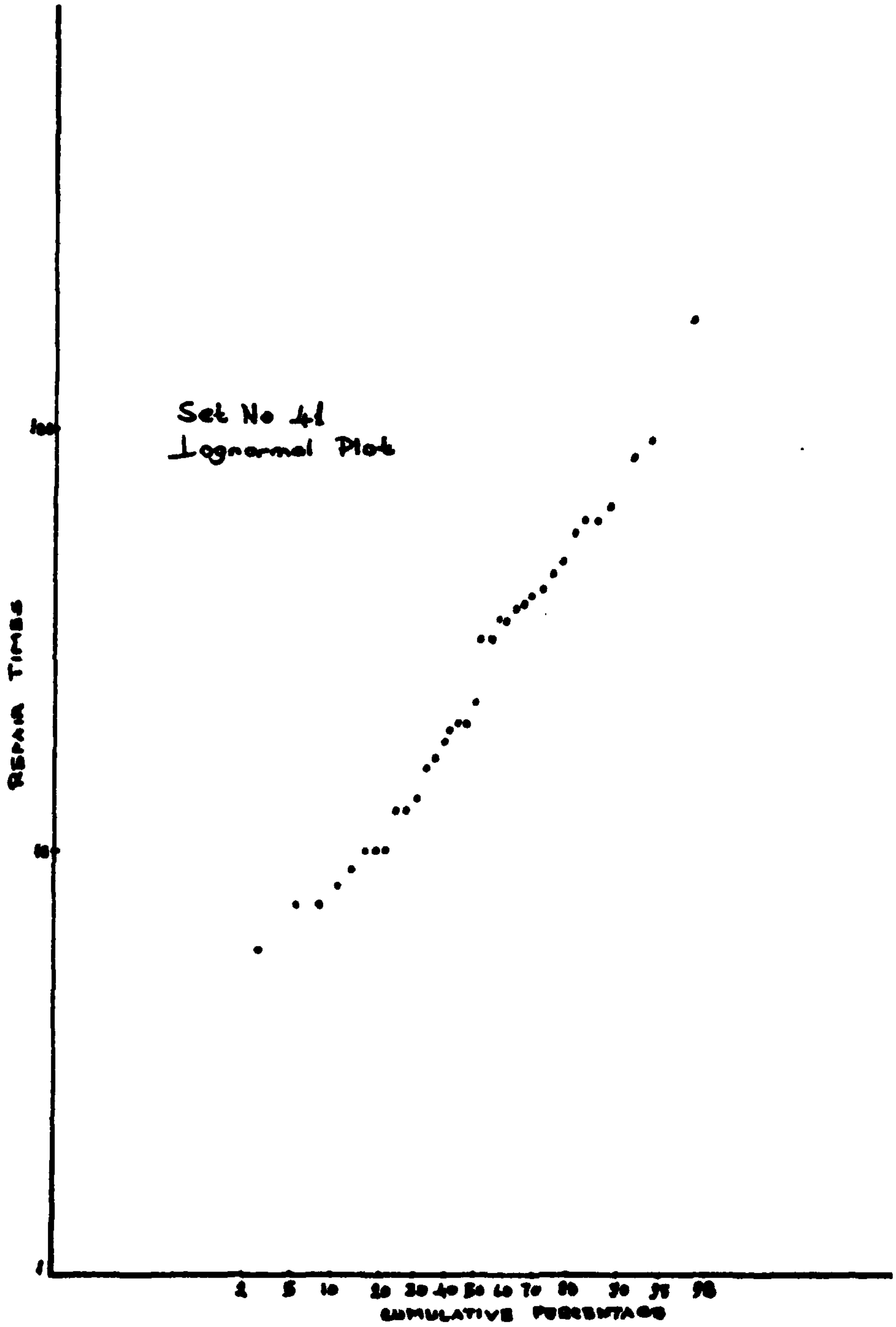
```
                   -------------------
        ----------I         +         I---------------------
                   -------------------
```

HISTOGRAM SET41

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
0.	5	*****
2.	13	*************
4.	10	**********
6.	4	****
8.	1	*
10.	1	*
12.	0	
14.	0	
16.	0	
18.	1	*

HISTOGRAM LNSET41

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
-0.5	3	***
0.0	7	*******
0.5	7	*******
1.0	3	***
1.5	9	*********
2.0	5	*****
2.5	0	
3.0	1	*



Set No 41
Lognormal Plot
REPAIR TIMES
CUMULATIVE PERCENTAGE

APPENDIX B

Table 6

TOTAL RESULTS (46 cases)

Exponential Distribution : Only Chi Square test used.

Accept	Reject
6	40

Lognormal Distribution : Chi Square, Kolmogorov Smirnov and W test used.

K-S test (A = accept, R = reject):

		A: W test A	A: W test R	R: W test A	R: W test R
Chi Sq. test	A	12	1	7	1
	R	1	2	3	7

(34 cases)

No W test

		K-S test A	K-S test R
Ch.Sq. test	A	2	2
	R	0	8

(12 cases)

Gamma Distribution : Kolmogorov Smirnov test used.

		M.O.M. estimators A	M.O.M. estimators R
M.L.E. estimators	A	19	8
	R	0	19

Table 7

PUMP DATA RESULTS (7 sets)

Exponential Distribution : Only Chi Square test used.

Accept	Reject
1	6

Lognormal Distribution : Chi Square, Kolmogorov Smirnov and W test used.

		K-S test A		K-S test R	
		W test A	W test R	W test A	W test R
Chi Sq. test	A	0	1	2	0
	R	0	1	0	2

(6 cases)

No W test

Ch.Sq. test	K-S test A	K-S test R
A	0	0
R	0	1

(1 case)

Gamma Distribution : Kolmogorov Smirnov test used.

		M.O.M. estimators A	M.O.M. estimators R
M.L.E. estimators	A	0	1
	R	0	6

AD-A124 632 A STUDY OF THE APPLICATION OF THE LOGNORMAL AND GAMMA DISTRIBUTIONS TO CORRECTIVE MAINTENANCE REPAIR TIME DATA(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA E CAMOZU 2/2

UNCLASSIFIED OCT 82 F/G 15/5 NL

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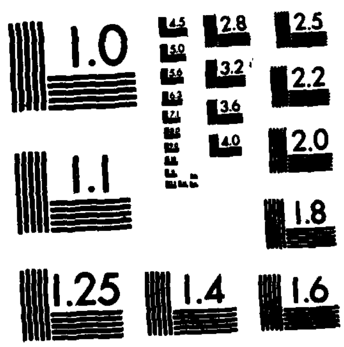

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Table 8

REPORT DATA RESULTS (13 sets)

Exponential Distribution : Only Chi Square test used.

Accept	Reject
1	12

Lognormal Distribution : Chi Square, Kolmogorov Smirnov and W test used.

		K-S test A		K-S test R	
		W test		W test	
		A	R	A	R
Chi Sq. test	A	4	0	2	0
	R	0	0	2	3

(11 cases)

No W test

		K-S test A	K-S test R
Ch.Sq. test	A	0	1
	R	0	1

(2 cases)

Gamma Distribution : Kolmogorov Smirnov test used.

		M.O.M. estimators A	M.O.M. estimators R
M.L.E. estimators	A	7	3
	R	0	3

Table 9

FIELD DATA RESULTS (26 sets)

Exponential Distribution : Only Chi Square test used.

Accept	Reject
4	22

Lognormal Distribution : Chi Square, Kolmogorov Smirnov and W test used.

K-S test

		A: W test A	A: W test R	R: W test A	R: W test R
Chi Sq. test	A	8	0	3	1
	R	1	1	1	2

(17 cases)

No W test

		K-S test A	K-S test R
Ch.Sq. test	A	2	1
	R	0	6

(9 cases)

Gamma Distribution : Kolmogorov Smirnov test used.

		M.O.M. estimators A	M.O.M. estimators R
M.L.E. estimators	A	11	5
	R	0	10

Table 10

TOTAL % RESULTS (w.r.t. sample sizes)

For exponential distribution :

Sample Size	A	R
20-29	30.8	59.2
30-39	8.3	91.7
40-50	0.	100.
GT.50	8.3	91.7

For lognormal distribution :

Sample Size	Chi-Sq.test A	Chi-Sq.test R	W test A	W test R	K-S test A	K-S test R
20-29	53.8	46.2	71.4	28.6	46.2	53.8
30-39	66.7	33.3	58.3	41.7	50.	50.
40-50	58.3	41.7	58.3	41.7	33.3	66.7
GT.50	25.	75.	---	---	16.7	83.3

For gamma distribution :

Sample Size	A in both	R in both	A in MLE R in MOM
20-29	84.6	7.7	7.7
30-39	33.3	50.	16.7
40-50	25.	62.5	12.5
GT.50	16.7	16.7	66.6

APPENDIX C

COMPUTER PROGRAMS FOR DATA ANALYSIS

1. LGNRML (FORTRAN) [Ref. 1]

The program makes a Chi-Squared Goodness-Of-Fit test for the exponential and lognormal assumptions. The main program computes expected values of ordered observations, sample mean and standard deviation based on the maximum likelihood estimates of the parameters of the lognormal distribution, calculates the 50th, 90th, 95th percentiles of the exponential and lognormal distribution assumptions, percentage differences between the calculated parameters of the exponential and lognormal distributions.

2. BOXPLOT (MINITAB) [Ref. 5]

Syntax: Boxplot Vector

Description: Generates a boxplot display for a vector of data. A rectangular box with ends corresponding to lower and upper quartiles is presented with the median marked with an plus sign, and lines are drawn on each side of the box with crosses marking the lowest and highest data values within half quartile distance in each side. Outliers are marked with stars and circles.

3. NORMPLOT (APL) [Ref. 21]

Syntax: Normplot vector

Parameters: Wid- controls the horizontal size of display
Dep- controls the vertical size of display

Subprograms: NSCORES, PLOT

Description: Generates a normal probability plot for a vector of data. If data fits normal then plot is straight line.

4. EXPONPLOT (APL) [Ref. 21]

Syntax: Exponplot vector

Parameters: Wid- controls the horizontal size of display
Dep- controls the vertical size of display

Subprogram: PLOT

Description: Generates a exponential probability plot for a vector of data. If data fits exponential then plot is straight line.

5. GAM (APL)

Syntax: GAM vector

Subprograms: ESTGAM, GAMMA

Description: Perform Kolmogorov-Smirnov Goodness-of-Fit test and gives test statistic and "reject" / "do not reject" idea for gamma assumption for both MOM and MLE estimators.

6. KOL (APL)

Syntax: Kol vector

Subprogram: MOM, INVV

Description: Performs a Kolmogorov Smirnov Goodnees-Of-Fit test for a data vector for lognormal assumption and print test statistic. If test statistic is bigger than the tabulated value "reject" the assumed distribution, otherwise "do not reject".

7. W (APL)

Syntax: A W Vector

Description: Performs a W test for the lognormal as - sumption. A must be a data vector (from Table IX, Ref. 2) and X vector that is testing against lognormality. A and X must have the same dimension, output will be W test statistic. Then using the value of λ, μ and ϵ (Table XI, Ref. 2) for the appropriate sample size and than using standardized normal distribution determine the probability of obtaining value less than or equal Z, which is the significance level of the test.

```
      ∇ GAM [⎕]∇
     ∇ GAM Y
     ⍝ PROGRAM PERFORMS KOLMOGOROV SMIRNOF TEST FOR
     ⍝ GAMMA ASSUMPTION FOR MOM AND MLE ESTIMATORS
      Y←Y[⍋Y]
      ESTGAM Y
      X1←Y÷E[1;2]
      X2←Y÷E[2;2]
      N←⍴Y
      Z←0.886÷N*0.5
      A1←E[1;1]
      A2←E[2;1]
      K←0,KK←(⍳(N−1))÷N
      I←II←0
      J←JJ←0/0
    L2:I←I+1
      →L1×⍳(I>N)
      A1 GAMMA X1[I]
      J←J,P
      →L2
    L1:R←|(J−K)
      RR←R[⍒R]
      ' TEST STATISTIC FOR MOM ESTIMATORS IS ',⍕RR[1]
      →L3×⍳(RR[1]<Z)
      ' REJECT FOR ALPHA=0.05 FOR MOM ESTIMATORS '
      →L5
    L3:' DO NOT REJECT FOR ALPHAx0.05 FOR MOM ESTIMATORS '
    L5:II←II+1
      →L6×⍳(II>N)
      A2 GAMMA X2[II]
      JJ←JJ,P
      →L5
    L6:S←|(JJ−K)
      SS←S[⍒S]
      ' TEST STATISTIC FOR MLE ESTIMATOR IS ',⍕SS[1]
      →L7×⍳(SS[1]<Z)
      ' REJECT FOR ALPHAx0.05 FOR MLE ESTIMATORS '
      →99
    L7:' DO NOT REJECT FOR ALPHAx0.05 FOR MLE ESTIMATORS '
      →99
      ∇
      ⋄

       GAM SET1
 TEST STATISTIC FOR MOM ESTIMATORS IS 0.363
 REJECT FOR ALPHA=0.05 FOR MOM ESTIMATORS
 TEST STATISTIC FOR MLE ESTIMATOR IS 0.154
 REJECT FOR ALPHAx0.05 FOR MLE ESTIMATORS
      ⋄
```

```
      ∇ KOL [⎕]∇
    ∇ KOL X
[1]   ⍝ PROGRAM PERFORMS KOLMOGOROV-SMIRNOV
[2]   ⍝ GOODNESS-OF-FIT TEST FOR LOGNORMAL
[3]   ⍝ ASSUMPTION , X MUST BE DATA ARRAY
[4]   ⍝ ORDER DATA AND FIND SAMPLE MEAN AND VARIANCE OF LOG DATA
[5]    X←X[⍋X]
[6]    MOM⍟X
[7]   ⍝ PUT DATA IN STANDARD NORMAL CASE
[8]    Z←((⍟X)-XB)÷S
[9]    N←⍴X
[10]   PPP←0/0
[11]   I←II←0
[12]  L3:II←II+1
[13]   →L4×⍳(II>N)
[14]  ⍝ FIND P VALUE FOR EVERY DATA POINTS AND PUT IN ARRAY
[15]  L1:I←I+1
[16]   →L4×⍳(I>999)
[17]   INVV P←I×0.001
[18]   →L2×⍳(ZZ>Z[II])
[19]   →L1
[20]  L2:PPP←PPP,P
[21]   →L3
[22]  L4:K←(⍳(N-1))÷N
[23]   K←0,K
[24]   CC←(⍴K)-(⍴PPP)
[25]   CD←(1,CC)⍴1
[26]   DC←(,CD)
[27]   PPP←PPP,DC
[28]  ⍝ FIND MAXIMUM ABSOLUTE DEVIATION AND PRINT AS TEST STATISTIC
[29]   KK←|(PPP-K)
[30]   L←KK[⍒KK]
[31]   'TEST STATISTIC =',⍕L[1]
[32]   TV←0.886÷(N*0.5)
[33]   'TABULATED VALUE =',⍕TV
     ∇

      KOL SET1
TEST STATISTIC =0.146
TABULATED VALUE =0.135
```

```
      ∇ W [⎕]∇
     ∇ Y W X;A1;A2;A3;S2;K;N;NN
     ⍝ PRODUCES W TEST STATISTIC FOR W TEST
     ⍝ Y=VECTOR FOR W TEST LENGTH OF ARGUMENT
     ⍝ X= VECTOR THAT IS TESTING
     ⍝ X AND Y MUST BE SAME DIMENSIONS
      N←⍴X
      I←B←0
     ⍝ ORDER DATA
      X←X[⍋X]
     ⍝ CALCULATE S*2 TERM
      A1←+/⍟X
      A2←+/(⍟X)*2
      A3←(A1*2)÷N
      S2←A2−A3
     ⍝ CALCULATE B TERM
      K←⌊N÷2
      LX←⍟X
      LLX←LX[⍒LX]
      B←+/Y×((K↑LLX)−(K↑LX))
     ⍝ FIND W TEST STATISTIC AND PRINT AS W
      Z←(B*2)÷S2
      ' W TEST STATISTIC = ',⍕Z
     ∇
      ⋄

      A43 W SET1
 W TEST STATISTIC = 0.967
      ⋄
```

```
    ∇ GAMMA [⎕]∇
   ∇ A GAMMA X;V;Y;Z;R;A̲;N̲
[1]  ⍝ INC. GAMMA FNC.-SHAPE=A>0, VECTOR ARG. X≥0, SCALE PARAM.=1
[2]  ⍝ GIVES THE LEFT TAIL AREA FOR GIVEN A AND X
[3]  ⍝ A IS SHAPE PARAMETER, X IS DATA VALUE
[4]   A̲←⌊A
[5]   P←1-*-X
[6]   →0×⍳1=A
[7]   P←(⍴X←,X)⍴0
[8]   V←(X<A)∨X≤1
[9]   →L×⍳0=+/V
[10]  Y←V/X
[11]  N̲←⌈¯7÷⍟(⌈/Y)÷A+1
[12]  Z←1++/×\Y∘.÷A+⍳N̲
[13]  →(1↑3+⎕LC)×⍳1<A̲
[14]  P[V/⍳⍴X]←Z×(Y*A)÷(!A)×*Y
[15]  →L
[16]  P[V/⍳⍴X]←(Y÷A)×Z×(×/(Y×*-Y÷A̲-1)∘.÷A-⍳A̲-1)×(Y*A-A̲)÷!A-A̲
[17] L:→0×⍳0=+/~V
[18]  Y←(~V)/X
[19]  R̲←20⌈2×A
[20]  Z←A CF Y
[21]  →(1↑3+⎕LC)×⍳1<A̲
[22]  P[(~V)/⍳⍴X]←1-Z×(Y*A)×(*-Y)÷!A-1
[23]  →0
[24]  P[(~V)/⍳⍴X]←1-Y×Z×(×/(Y×*-Y÷A̲-1)∘.÷A-⍳A̲-1)×(Y*A-A̲)÷!A-A̲
     ∇

    ∇ INVV [⎕]∇
   ∇ INVV A
[1]  ⍝ 'INVV A' RETURNS THE VALUE OF STANDARD
[2]  ⍝ NORMAL DISTRIBUTION AS LEFT TAIL AREA A
[3]   B←|(1×A>0.5)-A
[4]   X←(-⍟B×B)*0.5
[5]   X1←2.515517+X×0.802853+0.010328×X
[6]   X2←1+X×1.432788+X×0.189269+0.001308×X
[7]   ZZ←(X1÷X2)-X
[8]   ZZ←ZZ-2×ZZ×A>0.5
     ∇

    ∇ MOM [⎕]∇
   ∇ MOM X
[1]  ⍝ RETURNS SAMPLE MEAN AND VARIANCE
[2]   XB←(+/X)÷⍴X
[3]   S←(+/(X-XB)*2)÷¯1+⍴X
     ∇
```

```
      ∇ ESTGAM [⎕]∇
     ∇ ESTGAM X;A;B;F;FP
[1]   ⍝ RETURNS METH. OF MOMENTS EST. THEN MAX LIK EST. OF GAMMA DATA
[2]   ⍝ OUTPUT AS 2x2 MATRIX ITS FIRST ROW MOM AND SECOND ROW MLE
[3]   ⍝ ESTIMATORS SHAPE AND SCALE PARAMETERS RESPECTIVELY
[4]    MOM X
[5]    A←XB÷B←S÷XB
[6]    E←A,B
[7]    B←(+/⍟X)÷⍴X
[8]   L:A0←A
[9]    F←B+(⍟A÷XB)-0 PSI A
[10]   FP←(÷A)-1 PSI A
[11]   A←A-F÷FP
[12]   →L×⍳(0.0001)<|A-A0
[13]   E← 2 2 ⍴E,A,XB÷A
     ∇
     ⋄

      ∇ PSI [⎕]∇
     ∇ P←N PSI Y;C;IV;JIV;K;KK;YY;V;Z;T;I
[1]   ⍝ N IS THE ORDER OF THE DERIVATIVE OF THE PSI FUNCTION
[2]   ⍝ Y (>0) IS THE ARGUMENT,SCALAR OR VECTOR
[3]    C←10
[4]    IV←⍳⍴Y←,Y
[5]    P←Z←K←(⍴Y)⍴0
[6]    KK←⌈C-Y[JIV←(V←Y<C)/IV]
[7]    →L1×⍳(⍴IV)=⍴(~V)/IV
[8]    T←Y[JIV]
[9]    I←0
[10]  L2:I←I+1
[11]   YY←KK[I]⍴T[I]
[12]   Z[I]←(!N)×+/((YY-1)+⍳KK[I])*-1+N
[13]   →L2×⍳I<⍴JIV
[14]   Z←V\Z[⍳⍴JIV]
[15]   K←V\KK
[16]  L1:→S1×⍳N>0
[17]   P←-(⍟K+Y)-(2×K+Y)*-1
[18]   →2+⎕LC
[19]  S1:P←((!N-1)×(Y+K)*-N)+(!N)×0.5×(Y+K)*-N+1
[20]   P←((¯1)*N+1)×P+Z+N JEX Y+K
     ∇
     ⋄
```

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