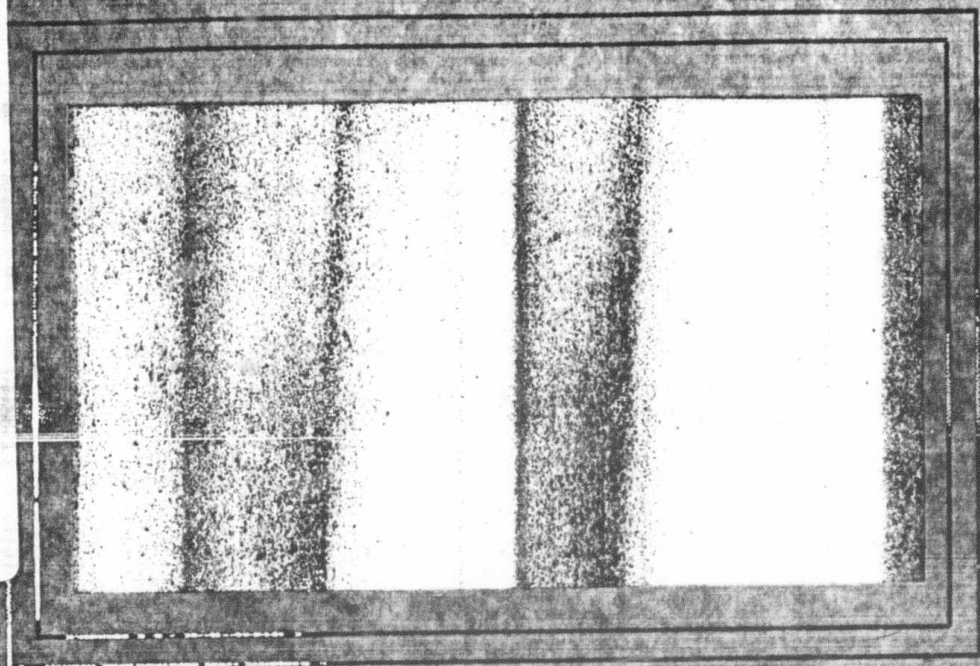


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**BIMEAN CLUSTERING**

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**ABSTRACT**

An algorithm is presented which finds the best-fitting pair of constants, in the least squares sense, to a set of scalar data; we call this pair of constants the "bimean" of the data. The relationship of the bimean clustering to the ISODATA clustering algorithm, and its application to image thresholding, are also discussed.

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## 1. Introduction

Let us first consider the following minimization problem:

$$\text{minimize } f = \sum_{i=1}^n (x_i - \mu)^2 \quad (1)$$

where we may assume without loss of generality that  $x_1 \leq x_2 \leq \dots \leq x_n$ . It is well known that  $f$  is minimized when  $\mu$  is the average of the  $x_i$ , that is,

$$\mu = \frac{1}{n} \sum_{i=1}^n x_i \quad (2)$$

We now extend this to the following minimization problem:

$$\text{minimize } f = \sum_{i=1}^n (\min((x_i - \mu)^2, (x_i - v)^2)) \quad (3)$$

That is to say, we are interested in the best fitting pair of constants to the set  $x_1, \dots, x_n$  of data. We refer to this pair of constants  $(\mu, v)$  as the bimean. This is because, as we shall see,

$$\begin{aligned} \mu &= \frac{1}{k} \sum_{i=1}^k x_i \equiv \mu_k \\ v &= \frac{1}{(n-k)} \sum_{i=k+1}^n x_i \equiv v_k \end{aligned} \quad (4)$$

for some  $k$ ,  $1 \leq k \leq n$ . That is, the constants  $\mu$  and  $v$  are such that they are averages or means of subsets of the  $n$  data points.

We are interested in the bimean because it defines a natural clustering of the  $x$ 's into two subsets. For example, if the  $x$ 's are the gray levels of pixels in an image, clustering can be used to segment the image, e.g., into objects and background. Velasco [1] recently showed that the segmentation can be done with the ISODATA clustering algorithm. We will

show that the ISODATA algorithm should ideally converge to the bimean clustering result, but that there are cases where even this ideal clustering approach does not select the appropriate threshold.

This minimization problem has attracted much recent interest. Hartigan and Wong [2] present an algorithm for k-mean clustering which produces a global minimum only for the two-mean case. Pollard [3] discusses the convergence of the k-mean clustering. We consider this question briefly in Section 4 and show that the two-class case of the ISODATA clustering algorithm is relevant to the bimean. Fisher [4] discusses algorithms for clustering, but does not consider the special case of two means.

In Section 2, we develop the bimean clustering algorithm, and in Section 3, we show how the algorithm is applied to image segmentation. Concluding comments are presented in Section 4.



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## 2. The Bimean Clustering Algorithm

In this section we present an algorithm to compute the values of  $\mu$  and  $\nu$  such that

$$f = \sum_{i=1}^n \min((x_i - \mu)^2, (x_i - \nu)^2) \quad (5)$$

is minimized where  $x_1 \leq x_2 \leq \dots \leq x_n$ . First note that since  $f$  is continuous and  $f \geq 0$ , a minimum exists. We shall show that if among the  $n > 1$  values  $x_1, \dots, x_n$  there are at least two distinct values, the minimum is attained for some  $\mu < \nu$ .

Theorem 1: Let  $f(\mu) = \sum_{i=1}^k (x_i - \mu)^2$  where  $x_1 \leq x_2 \leq \dots \leq x_k$ . Then there exists one value of  $\mu$  at which the minimum is attained, given by

$$\mu^* = \frac{1}{k} \sum_{i=1}^k x_i \quad (6)$$

and for  $\mu \neq \mu^*$ ,  $f(\mu) > f(\mu^*)$ .

Proof: With  $\mu^*$  as defined above, we may write

$$f(\mu) = \sum_{i=1}^k (x_i - \mu)^2 \quad (7)$$

$$\text{as} \quad f(\mu) = \sum_{i=1}^k (x_i - \mu^* + \mu^* - \mu)^2 \quad (8)$$

which can be expanded as

$$\begin{aligned} f(\mu) &= \sum_{i=1}^k (x_i - \mu^*)^2 + \sum_{i=1}^k (\mu - \mu^*)^2 + 2(\mu - \mu^*) \sum_{i=1}^k (\mu^* - x_i) \\ &= \sum_{i=1}^k (x_i - \mu^*)^2 + k(\mu - \mu^*)^2 \end{aligned} \quad (9)$$

since the last term is zero with  $\mu^*$  as defined above. The minimum is obtained when  $\mu = \mu^*$ , and for  $\mu \neq \mu^*$ ,  $f(\mu) > f(\mu^*)$ . ||

We now consider the function

$$f(\mu, \nu) = \sum_{i=1}^n \min((x_i - \mu)^2, (x_i - \nu)^2) \quad (10)$$

As a corollary to Theorem 1, we now show that the minimum of  $f(\mu, \nu)$  cannot occur for  $\mu = \nu$ .

Theorem 2: If the  $n > 1$  values  $x_1 \leq x_2 \leq \dots \leq x_n$  contain at least two distinct values, the minimum is attained for some  $\mu < \nu$ .

Proof: We assume to the contrary that the minimum is attained for  $\mu = \nu$ . Theorem 1 implies a minimum at

$$\bar{\mu} = \frac{1}{n} \sum_{i=1}^n x_i \quad (11)$$

Therefore  $\bar{\mu} < x_n$ , because  $x_1, \dots, x_n$  contain at least two distinct values. We now consider

$$f(\bar{\mu}, x_n) = \sum_{i=1}^n \min((x_i - \bar{\mu})^2, (x_i - x_n)^2) \quad (12)$$

and we see that

$$f(\bar{\mu}, x_n) \leq \sum_{i=1}^{n-1} (x_i - \bar{\mu})^2$$

and also that

$$\sum_{i=1}^{n-1} (x_i - \bar{\mu})^2 < \sum_{i=1}^n (x_i - \bar{\mu})^2$$

since  $x_n - \bar{\mu} > 0$ .

Since  $f(\bar{\mu}, \bar{\mu}) = \sum_{i=1}^n (x_i - \bar{\mu})^2$  and from Theorem 1 follows that

this expression attains the unique minimum at  $(\bar{\mu}, \bar{\mu})$ .

We see that

$$f(\bar{\mu}, x_n) < f(\bar{\mu}, \bar{\mu})$$

which is a contradiction to our assumption that  $f(\mu, \nu)$  is

minimized when  $\mu = v$ . We conclude that  $\mu \neq v$  if the  $n > 1$  values  $x_1 \leq x_2 \leq \dots \leq x_n$  contain at least two distinct values. ||

From the very definition, it follows that  $f(\mu, v) = f(v, \mu)$ . With this fact and Theorem 2, we can restrict the domain of definition of the function  $f(\mu, v)$  to be  $\{(\mu, v) : \mu, v \in \mathbb{R} \text{ and } \mu < v\}$ .

We now define the numbers  $\mu_k$  and  $v_k$  to be

$$\begin{aligned}\mu_k &= \frac{1}{k} \sum_{i=1}^k x_i \\ v_k &= \frac{1}{n-k} \sum_{i=k+1}^n x_i\end{aligned}\tag{13}$$

for all  $k$ ,  $1 \leq k \leq n$ . We prove our final theorem from which the bimean clustering algorithm follows.

Theorem 3: If  $(\mu^*, v^*)$  is the minimum of  $f(\mu, v)$  for  $\mu < v$  then there exists an index  $k$  such that

$$\begin{aligned}\mu^* &= \mu_k, \quad v^* = v_k \\ x_k &\leq \frac{1}{2}(\mu^* + v^*) \\ x_{k+1} &> \frac{1}{2}(\mu^* + v^*)\end{aligned}\tag{14}$$

Proof: Let  $k$  be the largest index such that

$$x_k \leq \frac{1}{2}(\mu^* + v^*)$$

is true. We can write  $f(\mu^*, v^*)$  as

$$f(\mu^*, v^*) = \sum_{i=1}^k (x_i - \mu^*)^2 + \sum_{i=k+1}^n (x_i - v^*)^2\tag{15}$$

Assume to the contrary that  $\mu^* \neq \mu_k$  or  $v^* \neq v_k$

Case 1:  $\mu^* \neq \mu_k$ .

Consider  $f(\mu_k, v^*) = \sum_{i=1}^n \min((x_i - \mu_k)^2, (x_i - v^*)^2)$

$$\leq \sum_{i=1}^k (x_i - \mu_k)^2 + \sum_{i=k+1}^n (x_i - v^*)^2 \quad (16)$$

By Theorem 1, the first term of equation (16) is minimized, thus

$$f(\mu^*, v^*) > \sum_{i=1}^k (x_i - \mu_k)^2 + \sum_{i=k+1}^n (x_i - v^*)^2 \quad (17)$$

From equations (16) and (17)

$$f(\mu_k, v^*) < f(\mu^*, v^*)$$

and we have achieved a contradiction to the assumption that the minimum of  $f(\mu, v)$  is attained at  $(\mu^*, v^*)$ .

Case 2:  $v^* \neq v_k$ .

The proof for this case is analogous to that for the first case and shall be omitted. Thus we have shown that  $\mu^* = \mu_k$  and  $v^* = v_k$  for some  $k$ ,  $1 \leq k \leq n$ . ||

With Theorem 3 in hand, we present the bimean clustering algorithm:

Step 1) Find the set  $K$  of indices which satisfy

$$\begin{aligned} x_k &\leq \frac{1}{2}(\mu_k + v_k) \\ x_{k+1} &> \frac{1}{2}(\mu_k + v_k) \end{aligned}$$

Step 2) For all  $k \in K$  evaluate  $f(\mu_k, v_k)$

Step 3) Find the minimum of  $f(\mu_k, v_k)$  for all  $k \in K$ . Set  $\mu^* = \mu_j$  and  $v^* = v_j$  where  $j$  is the largest element of  $K$  for which the minimum is attained.

### 3. Application to thresholding

A possible application of the bimean is to segment an image so as to separate objects from their background, by clustering the gray levels of the image's histogram into two clusters. Thus, the index  $k$  in the bimean clustering algorithm is the gray level above which the gray levels belong to the objects, and the cluster of gray levels below the index  $k$  belong to the background.

Figure 1 shows results of applying the bimean clustering algorithm to a set of infrared images of tanks. The original images are shown in the first column, and the bimean results in the third column. The second column shows ISODATA results (see below), with the initial threshold taken at the mean gray level of the image. We see that the first five images are reasonably segmented by the bimean algorithm, but the last three are not; and the ISODATA results are not as good (e.g., the fourth image is poorly segmented).

Velasco [1] showed that a two-class, one-dimensional ISODATA clustering algorithm (e.g., [5]) could be used to segment images into two gray level classes. Our experiments show that this is not always the case. ISODATA is an iterative process based on the same distance measure that we are minimizing in the bimean algorithm; but ISODATA may converge only to a local minimum of this measure, whereas our algorithm finds the global minimum. In spite of this, we do not always obtain good thresholds, and neither does ISODATA.

#### 4. Discussion and concluding remarks

We have presented a new algorithm for clustering single dimensional data into two clusters. The computation is relatively quick, and the equations can be rewritten so as to only perform a single pass through the data.

The ISODATA algorithm should ideally converge to the Bimean, but this requires a suitable initial choice of means for ISODATA. For example, if the second mean is set equal to one outlier, the ISODATA algorithm converges but possibly not to the true bimean. Thus, the Bimean algorithm is a more reliable method of obtaining a globally optimal threshold than iterative algorithms such as ISODATA. Figure 1 shows, however, that this method does not always perform well in practice.

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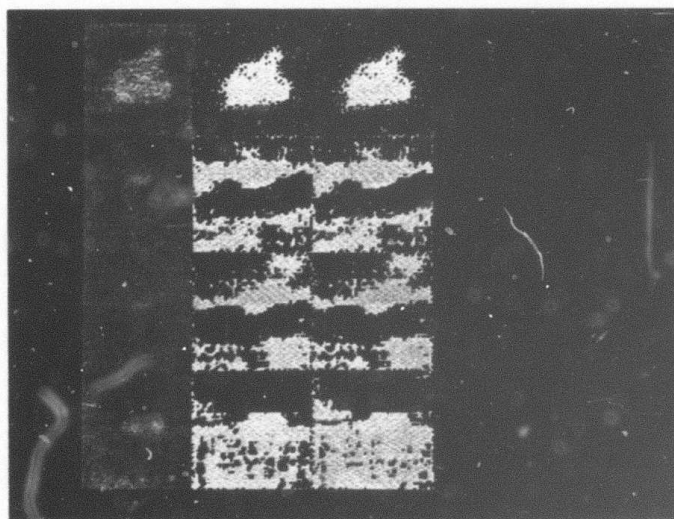
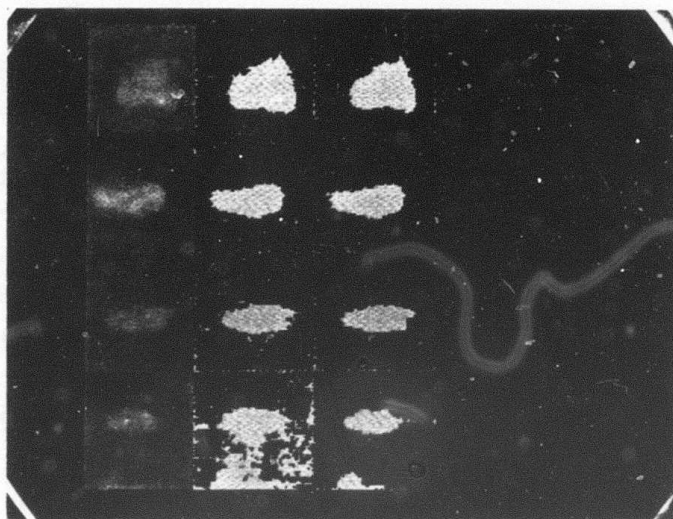


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