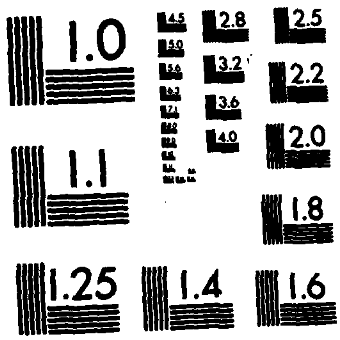

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ANALYTICAL DESIGN FOR INTERNAL BURNING STAR
GRAINS OF SOLID ROCKETS

by

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ANALYTICAL DESIGN FOR INTERNAL BURNING STAR GRAINS OF SOLID ROCKETS*

Lü Chang-tang

With regard to the design of internal burning star grains of solid rocket engines, they have been frequently designed based on pure geometrical studies and their corresponding trial methods. The formula and computation curves shown in [1] are representative which are widely used in engineering design and scientific studies both in our country and abroad. Actual practice showed that the pure geometrical formula and curves given in [1] are not only too complex, but also incomplete. The corresponding trial method not only involves a huge computational load, but also is rather blind. It is very difficult to ensure various technical objectives. Especially, it is extremely difficult to reach the optimum design. In this paper, we will attempt to use analytical design to replace trial design. Different from the traditional pure geometrical studies, we combined the various geometrical parameters of the internal star, the various characteristic parameters of the solved propellent and the technical objectives of the engine to establish the grain design equation series according to the optimization principle of grain design. It was matched with simple computational formulas and curves to quickly solve the equation series. Compared to [1-3], not only the computational load is less, but also the required technical objectives are assured. In addition, it approaches the optimum design in improving the performance of the engine so that significant savings in materials and ease of computer calculation are realized.

1. Geometrical functional relationship of the internal star. As shown in Figure 1, based on the Piobert combustion law, it is not difficult to obtain the functional relations between the geometrical parameters of internal star grains presented in Figure 1. To simplify the calculation, only functions k (Figure 2) and s_{σ} (the definitions

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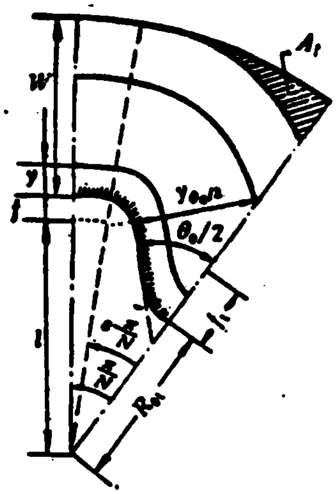

Figure 1. Diagram of design unit of the internal star

are shown in Figure 1), which are related to the star corner number N and the half angle of the star tip $\theta_o/2$ are given:

$$k = 2N\left(\frac{\pi}{2} + \frac{\pi}{N} - \frac{\theta_0}{2} - \mathrm{ctg}\,\frac{\theta_0}{2}\right) \tag{1}$$

$$s_0 = 2Nl\,\frac{\sin \varepsilon \frac{\pi}{N}}{\sin \frac{\theta_0}{2}} + 2\pi l(1 - \varepsilon) + kf \tag{2}$$

Therefore, the first stage combustion area A_{bl} and the initial gas passage area A_{pol} can be simplified into the two following equations:

$$A_{bl} = L_p[s_0 + kf_1 + 2\pi(y - f_1)] \tag{3}$$

$$A_{p0l} = s_0 f + l^2 c + \pi f_1^2 - 0.5k(f^2 + f_1^2) \tag{4}$$

where L_p is the length of the grain and other symbols are as presented in Figure 1. The value of c is given in the following equation:

$$c = (1 - \varepsilon)\pi + N \sin \varepsilon \frac{\pi}{N} \cos \varepsilon \frac{\pi}{N} - N \sin^2 \varepsilon \frac{\pi}{N}\, \mathrm{ctg}\, \frac{\theta_0}{2} \tag{5}$$

Based on analysis, we know that the angle $\theta/2$ varies in a monotonically increasing manner from the star side disappearing point $y\theta_o/2$ with combustion time in the region of $[\theta_o/2, \theta_w/2]$. It is called the dynamic angular variable. $\theta_w/2$ is the star tip half angle when combustion stops. $\theta/2$ has the following relation with y which varies with time:

$$\frac{\theta}{2} = \arccos \frac{\sin \varepsilon \frac{\pi}{N}}{(y + f)/l} \tag{6}$$

Using this dynamic angular variable $\theta/2$, the function of the second stage combustion area A_{bH} and residual grain area A_f are simplified into the following forms:

$$A_{bH} = L_p\left[2\pi + 2N\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right] \frac{l \cdot \sin \varepsilon \frac{\pi}{N}}{\cos(\theta/2)} + 2\pi l L_p(1 - \varepsilon) \tag{7}$$

$$A_f = \pi l(D_p - l)\varepsilon + N(W+f)^2\left(\frac{\theta_W}{2} - \frac{\pi}{2} - \sin\frac{\theta_W}{2}\cos\frac{\theta_W}{2}\right) - Nl^2\sin\varepsilon\frac{\pi}{N}\cos\varepsilon\frac{\pi}{N} \tag{8}$$

where the residual grain area star tip half angle $\theta_W/2$ when combustion stops is

$$\frac{\theta_W}{2} = \arccos\frac{\sin\varepsilon\frac{\pi}{N}}{(W+f)/l} \tag{9}$$

From mathematical analysis, we know that A_{bl1} has a minimum A_{bmin} at $\bar{\theta}/2$ and its dimensionless form is (Figure 3 is the dimensionless functional curve)

$$\frac{f_{min}}{l} = 2N\frac{\sin\varepsilon\frac{\pi}{N}}{\sin(\bar{\theta}/2)} + 2\pi(1-\varepsilon) \tag{10}$$

Similarly it can be proven that A_f has a minimum A_{fmin} at $\frac{\theta_W}{2} = \frac{\pi}{2} - \varepsilon\frac{\pi}{N}$, i.e., $\frac{W+f}{l} = 1$ 18

$$A_{fmin} = 2l^2\left(\pi\varepsilon - 2N\sin\varepsilon\frac{\pi}{N}\cos\varepsilon\frac{\pi}{N}\right) \tag{11}$$

The existence of A_{bmin} and A_{fmin} is an important characteristic of internal star grain cylinders.

The effective grain area A_{eo} without star tip angle, i.e., $f_1=0$, is very important to the establishment and solution of the design equation. It is given by the following equation:

$$A_{eo} = \frac{\pi}{4}D_p^2 - \pi l(2f+l) - 0.5f^2k - 2\pi W'l\varepsilon - \frac{2Nfl}{\sin(\theta_b/2)}\sin\varepsilon\frac{\pi}{N} + Nl^2\operatorname{ctg}\frac{\theta_b}{2}\sin^2\varepsilon\frac{\pi}{N} - N(W+f)^2\cdot\left(\frac{\theta_W}{2} - \frac{\pi}{2} - \sin\frac{\theta_W}{2}\cos\frac{\theta_W}{2}\right) \tag{12}$$

Therefore, when there are star tip angles, i.e., $f_1 \neq 0$, the effective grain area A_{eo1} is

$$A_{eo1} = A_{eo} - 0.5(2\pi - k)f_1^2 \tag{13}$$

The minimum radius R_o of the internal star at $f_1=0$ provides the constraint relation between the star shaped geometrical parameters. Furthermore, because it is always true that $\frac{d(R_o/l)}{d\varepsilon} < 0$, R_o is a decreasing function of the angular coefficient ε. Let $R_o>0$, then the upper limit of ε which is $\varepsilon_{N\ upper}$ is:

$$\varepsilon_{N\ upper} = \frac{N}{\pi}\left(\arcsin\frac{f}{l}+\frac{\theta_0}{2}\right) \tag{14}$$

As long as $\varepsilon<\varepsilon_{N\ upper}$, then $R_o>0$. Furthermore, $R_{oi}>0$ too (when $f_i\neq 0$).

2. <u>The establishment of the grain design equation series</u>

The known conditions are:

1) the technical objectives required by the engine: the total impulse value I_r or in the required range, the thrust law $F_{min}-F_{max}$, the working time $t_{min}-t_{max}$, the working temperature $T_{min}-T_{max}$ °C, the external diameter of the engine D and other limitations.

2) the characteristics of the solid propellent: the specific impulse I_{sp}, the combustion speed $r=ap^n$, the temperature coefficient σ_T, the flow sensitivity coefficient limit K_{kp}, the thrust coefficient C_1, the characteristic speed C^*, the critical pressure P_{kp}, and the density ρ.

The design equation series obeys the grain design optimization principle: to ensure the realization of the total impulse and thrust requirements as well as the working time, to carry out stable and normal combustion, to have a large effective grain coefficient η_e and a small residual grain coefficient η_f. Stress is concentrated in the allowable range.

In order to establish the design functions, we must carry out the following transformation:

Transform the total impuse I_t into the volume V_e of the affective grain quantity G_e

$$V_e = \frac{I_t}{I_{sp}\rho}$$

The thrusts are transformed into grain combustion areas

$$A_{bmin} = \frac{A_t}{c^*\rho}\left(\frac{P}{aP^n}\right)_{-\tau c}$$

$$A_{bmax} = \frac{A_t}{c^*\rho}\left(\frac{P}{aP^n}\right)_{+\tau c}$$

where A_t is the critical area of the nozzle, $A_t = F_{min}/P_{min}C_{F-\tau c}$, with $P_{min} \geq P_{xp}$.

The working time of the engine is transformed into the combustion arch thickness W which is calculated from $W = rt$ or $W = 2V_e/(A_{bmin} + A_{bmax})$ and also verified experimentally.

The grain cylinder diameter D_f is D minus the thicknesses of the shell, gap and packaging.

After W is determined, in order to make A_f approach $A_{f\ min}$, the star angle transition arc radius f is calculated according to the reference value f_o and the adjustment formula α_f:

$$f_0 = 0.25D_p - W \tag{15}$$

$$\alpha_f = \frac{2(f - f_0)}{0.25D_p - (f - f_0)} \tag{16}$$

Adjust f to make $|\alpha_f|$ small to the extent possible. Simultaneously, under conditions allowable by grain stress, small f should be chosen in order to obtain better design performance.

Therefore, the design equation to ensure the engine total impulse requirement is

$$L_p[A_b - 0.9(2\pi - k)f^2] = V_e \tag{17}$$

The design function to ensure the lower limit of thrust is

$$2N\left[\frac{\sin\varepsilon\frac{\pi}{N}}{\sin\frac{\theta}{2}} + (1-\varepsilon)\frac{\pi}{N}\right] = \frac{A_{bmin}}{lL_p} \tag{18}$$

The initial pressure peak limit equation to ensure stable combustion is

$$\frac{[s_0 - (2\pi - k)f_1]L_p}{A_{p0} + 0.5(2\pi - k)f_1} = \varkappa_{np} \tag{19}$$

Eliminate the grain cylinder length L_p and f_1 from the above series of equations and then substitute equations (1), (2), (8), (12) into them. We can obtain the equation related to the angular coefficient ε as:

$$\begin{aligned}
&\frac{\pi}{4}\varkappa_{np}D_p^2 - \pi\varkappa_{np}l\varepsilon(D_p - l) - N\varkappa_{np}(W+f)^2\left[\arccos\frac{l\sin\varepsilon\frac{\pi}{N}}{W+f} - \frac{\pi}{2}\right.\\
&\left.\quad - \frac{l\sin\varepsilon\frac{\pi}{N}}{W+f}\sin\left(\arccos\frac{l\sin\varepsilon\frac{\pi}{N}}{W+f}\right)\right] + N\varkappa_{np}l^2\sin\varepsilon\frac{\pi}{N}\cos\varepsilon\frac{\pi}{N} - \varkappa_{np}V_eA_{bmin}^{-1}2\pi l(1-\varepsilon)\\
&\quad - 2NA_{bmin}^{-1}lV_e\varkappa_{np}\frac{\sin\varepsilon\frac{\pi}{N}}{\sin\frac{\theta}{2}} - \left[2Nl\frac{\sin\varepsilon\frac{\pi}{N}}{\sin\frac{\theta}{2}} + 2\pi l(1-\varepsilon)\right]^{-1}\cdot\left[2NlA_{bmin}\frac{\sin\varepsilon\frac{\pi}{N}}{\sin\frac{\theta_0}{2}}\right.\\
&\left.\quad + 2\pi lA_{bmin}(1-\varepsilon) + 2NfA_{bmin}\left(\frac{\pi}{2}+\frac{\pi}{N}-\frac{\theta_0}{2}-\mathrm{ctg}\frac{\theta_0}{2}\right)\right]\\
&\quad + A_{bmin}\left[2Nl\frac{\sin\varepsilon\frac{\pi}{N}}{\sin\frac{\theta}{2}} + 2\pi l(1-\varepsilon)\right]^{-1}\cdot\Big\{\pi^2D_p^2 - 4\pi^2l(2f+l)\\
&\quad - 2\pi f^2\left(N\pi + 2\pi - 2N\frac{\theta_0}{2} - 2N\,\mathrm{ctg}\frac{\theta_0}{2}\right) - 8\pi^2lW\varepsilon - \frac{8\pi Nfl}{\sin\frac{\theta_0}{2}}\sin\varepsilon\frac{\pi}{N} + 4\pi Nl^2\,\mathrm{ctg}\frac{\theta_0}{2}\\
&\quad \times\sin^2\varepsilon\frac{\pi}{N} - 4\pi N(W+f)^2\left[\arccos\frac{l\sin\varepsilon\frac{\pi}{N}}{W+f} - \frac{\pi}{2} - \sin\left(\arccos\frac{l\sin\varepsilon\frac{\pi}{N}}{W+f}\right)\frac{l\sin\varepsilon\frac{\pi}{N}}{W+f}\right]\\
&\quad - 4\pi V_eA_{bmin}\left[2Nl\frac{\sin\varepsilon\frac{\pi}{N}}{\sin\frac{\theta}{2}} + 2\pi l(1-\varepsilon)\right]^{-1} - \frac{\pi}{2}ND_p^2\cdot\left(\frac{\pi}{2}+\frac{\pi}{N}-\frac{\theta_0}{2}-\mathrm{ctg}\frac{\theta_0}{2}\right)\\
&\quad + 2\pi Nl(2f+l)\cdot\left(\frac{\pi}{2}+\frac{\pi}{N}-\frac{\theta_0}{2}-\mathrm{ctg}\frac{\theta_0}{2}\right) + 2Nf^2\left(\frac{\pi}{2}+\frac{\pi}{N}-\frac{\theta_0}{2}-\mathrm{ctg}\frac{\theta_0}{2}\right)^2
\end{aligned}$$

$$+ 4\pi N l W \varepsilon \left(\frac{\pi}{2} + \frac{\pi}{N} - \frac{\theta}{2} - \operatorname{ctg}\frac{\theta}{2}\right) + 4N^2 l^2 \left(\frac{\pi}{2} + \frac{\pi}{N} - \frac{\theta_0}{2} - \operatorname{ctg}\frac{\theta_0}{2}\right)$$

$$\times \frac{\sin \varepsilon \frac{\pi}{N}}{\sin \frac{\theta_0}{2}} - 2N^2 l^2 \left(\frac{\pi}{2} + \frac{\pi}{N} - \frac{\theta_0}{2} - \operatorname{ctg}\frac{\theta_0}{2}\right) \operatorname{ctg}\frac{\theta_0}{2} \cdot \sin^2 \varepsilon \frac{\pi}{N} + 2N^2 (W + f)^2$$

$$\times \left[\arccos \frac{l \sin \varepsilon \frac{\pi}{N}}{W + f} - \frac{\pi}{2} - \sin\left(\arccos \frac{l \sin \varepsilon \frac{\pi}{N}}{W + f}\right) \frac{l \sin \varepsilon \frac{\pi}{N}}{W + f}\right] \left(\frac{\pi}{2} + \frac{\pi}{N} - \frac{\theta_0}{2} - \operatorname{ctg}\frac{\theta_0}{2}\right)$$

$$+ 2N V_e A_{bmin}^{-1} \cdot \left(\frac{\pi}{2} + \frac{\pi}{N} - \frac{\theta_0}{2} - \operatorname{ctg}\frac{\theta_0}{2}\right) \left[2\pi l (1 - \varepsilon) + 2Nl \cdot \frac{\sin \varepsilon \frac{\pi}{N}}{\sin \frac{\theta}{2}}\right]\Bigg\}^{\frac{1}{2}} = 0 \qquad (20)$$

We can see that this equation simultaneously includes the required technical objectives, the propellent characteristics and the internal star geometrical parameters.

3. <u>The solution of the design equation and the acquisition of the grain geometrical parameters.</u> Obviously, it is difficult to directly solve for ε from equation (20). For this purpose, let us change ε into two functional curves Y_1 and Y_2.

$$Y_1 = \varepsilon_{up}\left(\frac{\pi}{4} D_p^2 - A_f - \frac{V_c}{L_p}\right), \quad Y_2 = L_p [s_0 - \sqrt{2\Delta(2\pi - k)}\,] \qquad (21)$$

where

$$\Delta = A_\omega - (V_e / L_p) \qquad (22)$$

Δ is the necessary formula to judge the solution. Only where $\Delta \geq 0$ there is a solution.

For each positive integer N, there is an upper limit $\theta_{N\ upper}/2$ for the angle $\theta_o/2$ which makes $\Delta = 0$ and $f_1 = 0$. $\theta_{N\ upper}/2$ is an increasing function of N; therefore $\theta_o/2$ must be determined in the range of $(0, \theta_{N\ upper}/2)$ in combination with the initial thrust requirement. N and $\theta_o/2$ are the variable parameters of equation (20). In order to have better overall performance for the grains, N and $\theta_o/2$ should be choosen to be small (e.g. N=2.3, $\theta_o/2$=10°). After N and $\theta_o/2$ are

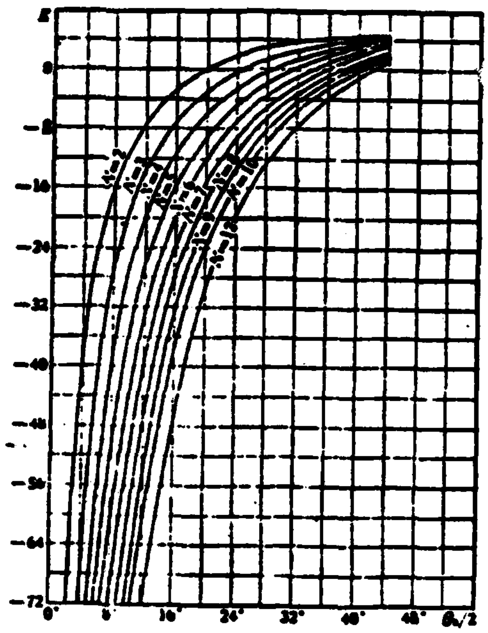

Figure 2. The k function curves used in calculation

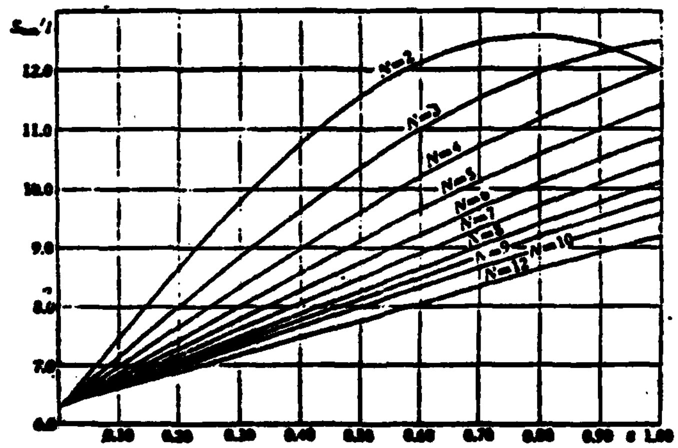

Figure 3. The curves of the S_{min}/l function used in calculation

selected, use $(N_1\theta_o/2)$ to check the k value in Figure 2, and then calculate the functional curves Y_1 and Y_2 of ε. For each chosen ε, use (N,ε) to look up the $S_{min}/1$ value in Figure 3. Substitute into the following equation to obtain L_p:

$$L_p = \frac{A_{bmin}}{l \cdot \frac{s_{min}}{l}} \tag{23}$$

Consequently, V_e/L_p can be obtained. Substitute ε and N, $\theta_o/2$ and k into equation (12) to get A_{eo} in order to obtain Δ. When $\Delta \geq 0$, from equations (2) and (8), we can obtain s_o and A_t. Furthermore, from equation (21) we can get the functions Y_1 and Y_2 corresponding to ε. Similarly, a series Y_1 and Y_2 functions of ε are calculated. The abscissa of the intersect point of the decreasing function Y_1 and increasing function Y_2 of ε gives the solution $\varepsilon_{solution}$ of equation (20) Therefore, the angular coefficient of the grain is obtained. The ordinate is the A_{b01} of the grain. From (N, $\varepsilon_{solution}$), we can obtain $S_{min}/1$ by checking Figure 3. Similarly, to the procedures described before, the grain length L_p, V_e/L_p, A_{eo}, Δ, A_f, s_o, etc., can be calculated. V_e/L_p is the value of A_{eo1}.

Use the following two equations to calculate f_1 and R_{01}

$$f_1 = \sqrt{\frac{2\Delta}{2\pi - k}} \tag{24}$$

$$R_{01} = \csc\frac{\theta_0}{2}\left[l \sin\left(\frac{\theta_0}{2} - \varepsilon\frac{\pi}{N}\right) + f + f_1\right] - f_1 \tag{25}$$

When $f_1 = 0$, from equation (14) we know that, as long as $\varepsilon < \varepsilon_{N\ upp}$ then it is necessary to have $R_o > 0$. When $f_1 \neq 0$, as long as the equation has a solution, $R_{01} > 0$ is valid. Furthermore, the smaller $\theta_o/2$ is, the larger R_{o1} becomes.

Discussion on the solution: When varying ε and Δ<0 still exists, i.e., the equation has no solution, it is necessary to choose a larger N and a smaller $\theta_o/2$. It is also possible to vary K_{kp} and other known parameters in the allowable range (Note: larger N means larger R_{o1}, larger $\theta_2/2$ means smaller A_1. When necessary, we must adjust N and $\theta_o/2$ simultaneously).

Up to this point, the engire geometrical parameters of internal star grains are solved. Finally, let us calculate the variation of the combustion surface:

$[0, y\theta_0/2]$ is the first stage of combustion which is the linear stage, $y_{\theta_0/2} = \dfrac{l \sin \epsilon \frac{\pi}{N}}{\cos \frac{\theta_0}{2}} - f$ 187

In the interval $[0, f_1]$, we have

$$A_{bl} = [s_0 + kf_1 + 2\pi(y - f_1)]L_p \tag{26}$$

In the interval $[f_1, y\theta_0/2]$, we have

$$A_{bl} = (s_0 + ky)L_p \tag{27}$$

$[y_{\theta_0/2}, W']$ is the second stage of combustion which is the non-linear combustion stage. Using the dynamic angular variable, $\theta/2$, we can quickly obtain the variation of A_{bll} from equation (7).

Incidentally, F. A. Williams et al stated in [2] that "in the second stâge of combustion...the grains are the increasing type, i.e., A_b increases linearly according to y=rt". This is not accurate. In the second stage, A_{bll} is nonlinear; furthermore, when $\theta_0/2 < \bar{\theta}/2$, it decreases before it begins to increase.

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