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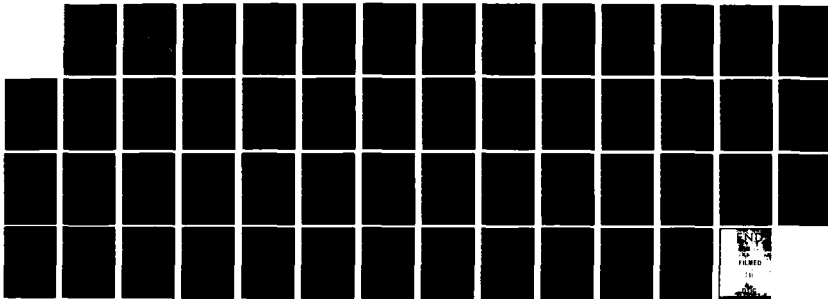
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ACCEPTANCE CONTROL CHARTS
BASED ON THE EXACT AND NORMAL APPROXIMATIONS TO
THE BINOMIAL DISTRIBUTION

Research Report No. 82-6

by

Carlos Amado
Richard S. Leavenworth
Richard L. Scheaffer

RESEARCH REPORT

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Engineering Department
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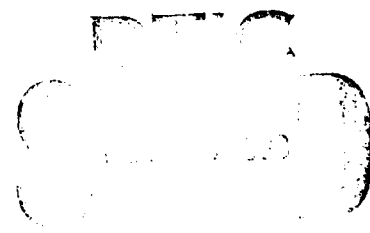
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December 1982

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ABSTRACT

Procedures are developed for finding the sample size and control limit for Acceptance Control Charts for proportion of nonconforming units using the exact binomial distribution, the standard normal approximation to the binomial, and a normalized arcsin transformation of the data. The user must select an Acceptable Process Level and a Rejectable Process Level and the associated risks for each. The approximation methods are compared to the exact method over a wide range of design specifications. It was found that the arcsin transformation is considerably more accurate than the standard normal approximation and, although more complex to figure, is preferable if the user is familiar with small scientific pocket calculators. If a microprocessor or minicomputer is available, the exact binomial may be used with ease to achieve at least the stipulated risk protection desired. FORTRAN programs for the three formulations are included.

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Procedures are developed for finding the sample size and control limit for Acceptance Control Charts for proportion of nonconforming units using the exact binomial distribution, the standard normal approximation to the binomial, and a normalized arcsin transformation of the data. The user must select an Acceptable Process Level and a Rejectable Process Level and the associated risks for each. The approximation methods are compared to the exact method over a wide range of design specifications. It was found that the arcsin transformation is considerably more accurate than the standard normal approximation and, although more complex to figure, is preferable if the user is familiar with small scientific pocket calculators. If a micro-processor or minicomputer is available, the exact binomial may be used with ease to achieve at least the stipulated risk protection desired. FORTRAN programs for the three formulations are included.

ACCEPTANCE CONTROL CHARTS
BASED ON THE EXACT AND NORMAL APPROXIMATIONS TO
THE BINOMIAL DISTRIBUTION

INTRODUCTION

The control chart for fraction rejected, or p-chart, probably is the most widely used of all control chart procedures. It may be applied to one or more than one quality characteristic whether measured on a go, not-go, basis or as variables measurements. So long as the result of an inspection is to classify an item as meeting specifications (acceptable) or not meeting specifications (unacceptable), a single p-chart may be used.

When the sample subgroup size is constant, the chart for $\underline{np}$ may be used conveniently since it records the actual count of rejected units in a subgroup of size $\underline{n}$ rather than the proportion rejected. In either case, the binomial probability density function may be used to model the process.

The standard Shewhart control chart for $\underline{np}$ places the upper and lower control limits at:

$$\underline{n\bar{p}} \pm 3 \sqrt{\underline{n\bar{p}}(1-\bar{p})}$$

where:

$\underline{n\bar{p}}$ is the average count of rejected units for a series of subgroups of constant size, $\underline{n}$.

$\sqrt{\underline{n\bar{p}}(1-\bar{p})}$ is the standard deviation of the binomial count ($\sigma_{\underline{np}}$).

Thus these limits are standard 3-sigma limits of the Shewhart control chart. While variation beyond these limits at random should occur very rarely indeed, the same probabilities often associated with random variation beyond 3 sigma limits on an $\bar{X}$ chart should not be applied. Nevertheless, exact probabilities

calculated from the binomial distribution may be found for $\underline{np}$ control charts with 3-sigma limits or any other set of limits which are a multiple of $\sigma_{\underline{np}}$.

In the case of the Acceptance Control Chart, at each point a sample is selected a decision is to be made either to accept the hypothesis that the process is operating as specified, at the acceptable quality level or better, or that the actual quality level is beyond an acceptable level (a higher value of $\underline{p}$). The decision criterion is the upper control limit on $\underline{np}$, $\underline{np} + \underline{Z}_\gamma \sigma_{\underline{np}}$, where γ is the probability of acceptance of the hypothesis with respect to $\underline{p}$ using a normal approximation to the binomial distribution. ($\underline{Z}_\gamma$ equals 3 for the standard Shewhart control chart.)

This paper explores the development of Acceptance Control Charts for binomial counts of the number of units rejected. Two parameters are to be found, one the value of the control limit, and the second, the appropriate (constant) sample size, $\underline{n}$.

The problem is formulated in three ways, using the exact binomial distribution, using a standard normal approximation to the binomial, and using a normalized arcsin transformation. Analytical results of application of the three methods are compared and the results of a simulation study using computer-generated synthetic data are presented.

PROBLEM FORMULATION: EXACT BINOMIAL

If samples are being drawn from a continuous process generating nonconforming items at a constant rate, $\underline{p}$, the binomial distribution describes this process accurately. The probability density function is

$$f(r|\underline{n}, \underline{p}) = \binom{\underline{n}}{r} \underline{p}^r (1-\underline{p})^{\underline{n}-r} \quad (1)$$

where r = no. of units rejected
 $\underline{n}$ = sample (or subgroup) size.

The probability that the number of units rejected is less than or equal to some fixed value, c , is:

$$P[r \leq c | n, p] = \sum_{r=0}^c \binom{n}{r} p^r (1-p)^{n-r} = \gamma \quad (2)$$

Limits for acceptance control charts, like acceptance numbers for sampling plans, usually are designed by selecting two points on an Operating Characteristic (OC) curve, as illustrated in Figure 1. One point assures that an Acceptable Process Level (APL), p_1 , has a high probability of being accepted of at least $1-\alpha$; the other assures that a Rejectable Process Level (RPL), p_2 , will have a suitable low probability of acceptance of at most β . α and β are the design risk levels associated with the process quality levels p_1 and p_2 , respectively, where p_2 is greater than p_1 .

The two equations needed should express the fact that it is desired that the OC curve pass through, or pass as close as possible to, the two points specified as the design criteria, namely, $(p_1, 1-\alpha)$ and (p_2, β) . These equations are:

$$P[r \leq c | n, p_1] = \sum_{r=0}^c \binom{n}{r} p_1^r (1-p_1)^{n-r} > 1-\alpha \quad (3)$$

$$P[r \leq c | n, p_2] = \sum_{r=0}^c \binom{n}{r} p_2^r (1-p_2)^{n-r} < \beta \quad (4)$$

They are expressed in inequality form because, for any integer values of n and c , it is unlikely that the cumulative probability can exactly satisfy $1-\alpha$ and β . As the result there are an infinite number of (n, c) pairs satisfying (3) and (4) above some minimum combination. The complete formulation thus requires the further stipulations:

$$\left. \begin{array}{l} \text{minimize } n \text{ and } c \\ \text{where: } p_2 > p_1 \text{ and } 1-\alpha > \beta \end{array} \right\} \quad (5)$$

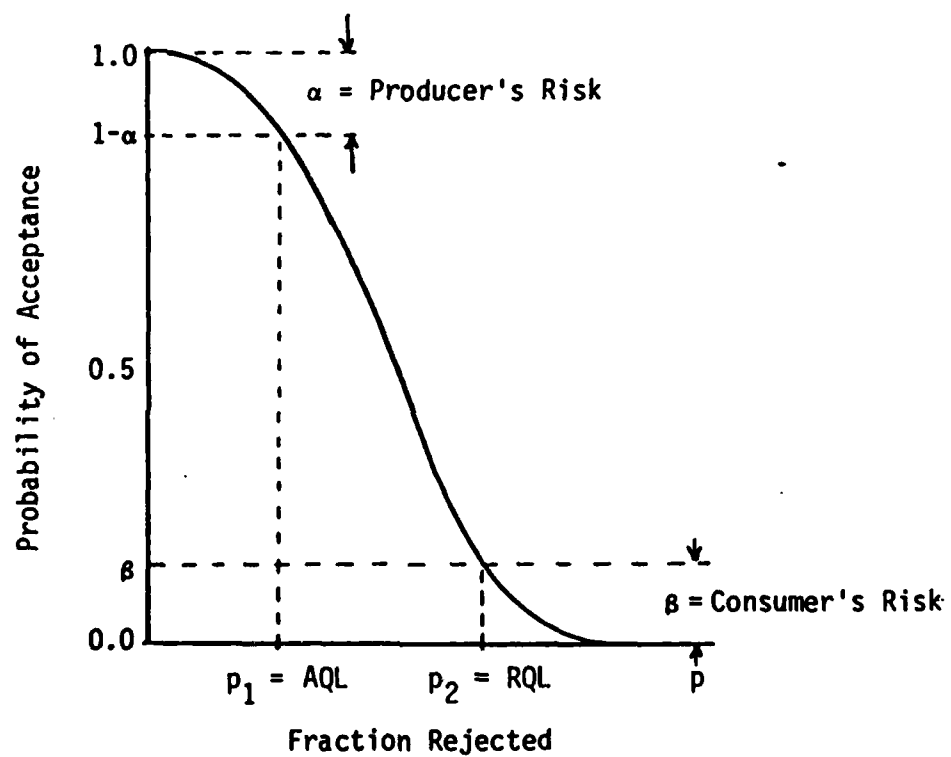


Figure 1. Design Operating Characteristic (OC) Curve for np Chart.

Referring to Fig. 1, equation (3) assures that the resulting OC curve will pass above and to the right of the design curve at the point $(p_1, 1-\alpha)$. Equation (4) assures that the resulting OC curve will pass below and to the left of the point (p_2, β) . Stipulating the choice of the minimum (n, c) pair assures that the resulting OC curve passes as closely as possible to the design curve.

The value of n , of course, yields the constant subgroup size to be used in sampling. Thus the np chart becomes a reasonable and easily understandable alternative to the p chart. The value of c is the maximum count of rejected units that should lead to no corrective action on the process. Only if $(c+1)$ or more units are rejected should action be taken. However, if the Acceptance Control Limit (ACL) is plotted exactly at the value of c , the user may become confused as to whether or not to take action. In accordance with the rules of control chart interpretation, this is not a coin-flip situation. A reasonable procedure is to plot the Acceptance Control Limit at:

$$ACL = c + 0.5 \quad (6)$$

thus avoiding any confusion in chart interpretation.

PROBLEM FORMULATION: NORMAL APPROXIMATIONS

Working with the binomial formula presents a number of mechanical problems some of which are discussed subsequently. Suffice it to say at this point in the discussion that no closed form solution for the values of n and c exists. Its application involves repetitive use of some form of search algorithm. Therefore no one should be surprised that considerable attention and ingenuity have been applied in the development of useful approximations to the cumulative binomial.

Johnson and Kotz (1969) provide a rather extensive survey of binomial approximation techniques and Raff (1956) has compared the accuracy of several of them. The two presented and compared in this study are the standard normal approximation and a normalized arcsin transformation. The standard normal approximation is the most familiar and used as alluded to in the Introduction. It is easy to apply since $\underline{n}$ and $\underline{c}$ may be obtained directly with the use of a slide rule or pocket calculator and a table of the standard normal curve. The arcsin transformation is somewhat more complicated but easily adaptable for use on a programmable pocket calculator.

Standard Normal Approximation

The mean and standard deviation of the binomial distribution are:

$$\begin{aligned} E(r) &= np \\ \sigma_r &= \sqrt{np(1-p)}. \end{aligned}$$

The distribution of the standardized binomial variable

$$Z = (r - np) / \sqrt{np(1-p)}$$

tends to the standard normal distribution as $\underline{n}$ becomes large. (See Johnson and Kotz, 1969.) That is, for any real number, X :

$$\lim_{n \rightarrow \infty} P[Z < X] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^X \exp(-u^2/2) du$$

which values may be found in a table of the normal curve or solved for on many programmable pocket calculators. Thus, if given the value of an ACL (say, derived from the exact binomial, non-integer, and equal to $\underline{c} + 0.5$), the probability of $\underline{c}$ or less occurrences with $\underline{n}$ and $\underline{p}$ known, $\Phi(Z)$, requires only the calculation of:

$$Z_Y = (ACL - np) / \sqrt{np(1-p)} \quad (7)$$

and $\Phi(Z_Y) = \gamma$ from a cumulative (left-hand) normal curve table.

Arcsin Transformation

The arcsin transformation

$$y = \sin^{-1} \sqrt{(r + 3/8)/(n + 3/4)}$$

produces a random variable, y , which is approximately normally distributed. (See Johnson and Kotz, 1969.) Thus a normalized random variable, Z , produces a statistic, the asymptotic distribution of which is normal, where:

$$Z_y = 2\sqrt{n} \left[\sin^{-1} \sqrt{\frac{c + 3/8}{n + 3/4}} - \sin^{-1} \sqrt{p} \right] \quad (8)$$

and $\Phi(Z_y) = \gamma$ is then found on a cumulative normal curve table.

PROBLEM SOLUTION - EXACT BINOMIAL

Restating the problem, the objective is to find an (n, c) pair such that:

minimize: n, c

subject to:

$$P[r < c | n, p_1] > 1 - \alpha \quad (3)$$

$$P[r < c | n, p_2] < \beta \quad (4)$$

where:

$$p_1 < p_2 \text{ and } \beta < 1 - \alpha. \quad (5)$$

Guenther (1969) develops a search procedure for finding (n, c) pairs that satisfy equations 3, 4, and 5*. While Guenther's algorithm is aimed at

*Actually Guenther's paper is devoted to finding sample sizes (n) and acceptance numbers (c) for single sampling acceptance plans. The procedure, however, is the same. In his paper, the hypergeometric, binomial, and Poisson distributions are used.

deriving plans by hand calculation and the use of tables, it is an iterative, brute-force technique more amenable to computerization than to hand calculation.

Hailey (1980) programmed Guenther's algorithm to find the minimum single sampling acceptance plan satisfying equations (3) and (4). His paper contains the FORTRAN IV computer code for deriving plans using either the binomial distribution or the Poisson.

The algorithm operates basically as follows. For any stipulated value of $\underline{c}$, there is a minimal sample size, $\underline{n}$, satisfying equation (4). That value is designated $\underline{n}_s$, a minimal value of $\underline{n}$. For the same value of $\underline{c}$, there also exists a maximum value of $\underline{n}$ satisfying equation (3). That value is designated $\underline{n}_\ell$, a maximum value for $\underline{n}$. If the solved value of $\underline{n}_\ell$ is less than the solved value of $\underline{n}_s$ for fixed $\underline{c}$, no feasible solution exists for that value of $\underline{c}$ or any lesser value. If $\underline{n}_\ell$ is greater than (or equal to) $\underline{n}_s$, then any value of $\underline{n}$ (a unique solution) is the range $\underline{n}_s < \underline{n} < \underline{n}_\ell$ is feasible for that value of $\underline{c}$. In fact, feasible solutions exist for any value of $\underline{c}$ greater than the designated value, as well. That is, an infinite number of plans exist satisfying equations (3) and (4).

The search procedure begins by setting $\underline{c}$ equal to zero and solving for $\underline{n}_s$ and $\underline{n}_\ell$. The value of $\underline{c}$ is increased by one and the process repeated until a feasible range of $\underline{n}$ is found. Hailey's program immediately selects the minimum $\underline{n}$, $\underline{n}_s$, and terminates with a series of output options. A variation of this program was used to derive sample sizes and ACL's based on the binomial distribution. The sample size, $\underline{n}$, was set equal to $\underline{n}_s$ and the Acceptance Control Limit

$$ACL = c + 0.5$$

in order to avoid any confusion in the interpretation of points falling on the control limit.

PROBLEM SOLUTION: NORMAL APPROXIMATIONS

Hand calculation using the algorithm stated in the previous section would become most tedious and time-consuming. Available tables of the binomial may not cover the ranges of $\underline{n}$ or $\underline{p}$ required. To evaluate the binomial where $\underline{c}$ equal 50 requires the calculation and summing of 51 terms, which is a large task even with the aid of a sophisticated pocket calculator or small computer. This procedure would have to be repeated many times before the minimum $(\underline{n}, \underline{c})$ pair are found. However, approximations usually require the evaluation of only two equations: one for $\underline{n}$ and one for $\underline{c}$.

Standard Normal Approximation

If Z_{α} denotes the value that cuts off an upper tail area of α under the standard normal curve, as illustrated in Figure 2, then the acceptance plan $(\underline{n}, \underline{c})$ pair can be found from the following equations:

$$Z_{1-\alpha} = \frac{c - n p_1}{\sqrt{n p_1 (1-p_1)}}$$

$$-Z_{1-\beta} = \frac{c - n p_2}{\sqrt{n p_2 (1-p_2)}}$$

Solving these equations simultaneously yields:

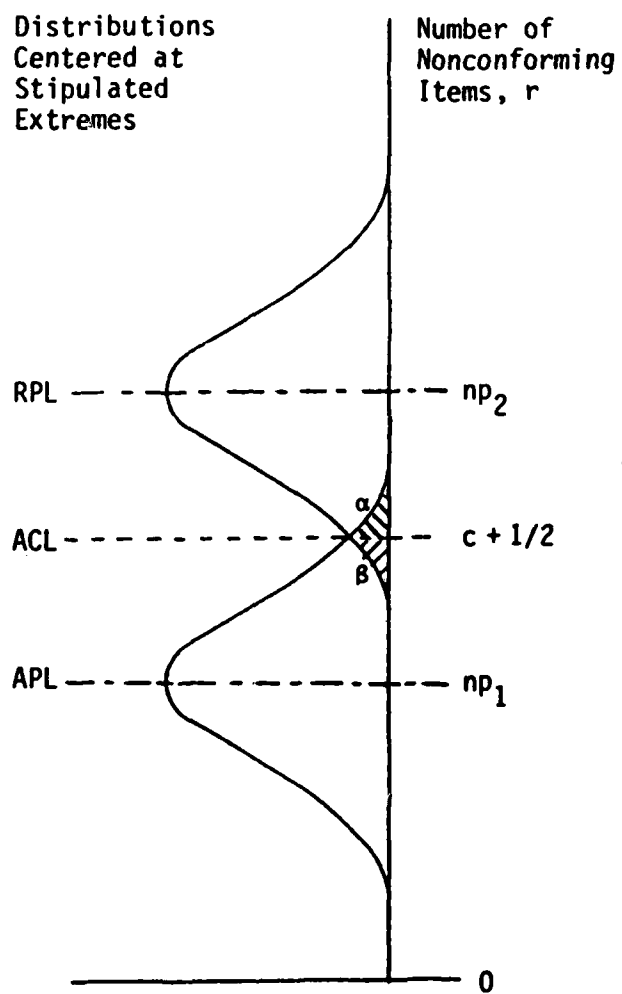


Figure 2. Acceptance Control Charting Scheme for Binomial Counts.

$$n = [(Z_{1-\alpha} \sqrt{p_1(1-p_1)} + Z_{1-\beta} \sqrt{p_2(1-p_2)}) / (p_2 - p_1)]^2 \quad (9)$$

$$c = Z_{1-\alpha} \sqrt{n p_1(1-p_1)} + n p_1 \quad (10)$$

$$= -Z_{1-\beta} \sqrt{n p_2(1-p_2)} + n p_2$$

The values of $\underline{n}$ and $\underline{c}$ may be calculated rather quickly by hand or by the use of a programmable pocket calculator. This is the simplest formulation of the problem. The value of $\underline{n}$, of course, is rounded to the nearest integer.

Arcsin Transformation

The acceptance control chart plan $(\underline{n}, \underline{c})$ pair can be found by solving the following equations:

$$Z_{1-\alpha} = 2 \sqrt{n} [\sin^{-1} \sqrt{\frac{c + 3/8}{n + 3/4}} - \sin^{-1} \sqrt{p_1}]$$

$$-Z_{1-\beta} = 2 \sqrt{n} [\sin^{-1} \sqrt{\frac{c + 3/8}{n + 3/4}} - \sin^{-1} \sqrt{p_2}]$$

Solving these two equations simultaneously yields:

$$n = \{[(Z_{1-\alpha} + Z_{1-\beta}) / [2 (\sin^{-1} \sqrt{p_2} - \sin^{-1} \sqrt{p_1})]]\}^2 \quad (11)$$

$$c = (n + 3/4) \{\sin [Z_{1-\alpha} / (2 \sqrt{n}) + \sin^{-1} \sqrt{p_1}]\}^2 - 3/8 \quad (12)$$

Again, the value of $\underline{n}$ is rounded to the nearest integer. The derived function $\sin^{-1}(x) = \tan^{-1}(x / \sqrt{1 - x^2})$ may be used to obtain the inverse sin of x , where $-1 < x < 1$, in computers which do not have an inverse sin function, (Standard Mathematical Tables, 1973).

ANALYSIS

The three methods of calculating $(\underline{n}, \underline{c})$ pairs for Acceptance Control Charts were investigated using the popular risk levels of α equals 0.05 and β equals 0.10. The value of $\underline{p}_1$ ranges from 0.005 to 0.06 in increments of 0.005. The value of $\underline{p}_2$ changes iteratively in order to maintain specified levels of the discrimination ratio, $\underline{D} = \underline{p}_2/\underline{p}_1$. The value of $\underline{D}$ ranges from 1.5 to 5.0 in increments of 0.5.

Acceptance Control Chart plans in terms of $(\underline{n}, \underline{c})$ pairs are presented in Table 1. The three methods are labeled BINOMIAL, NORMAL, and ARCSIN to identify, respectively the cumulative binomial distribution, standard normal approximation, and normalized arcsin transformation.

Subgroup sizes, the $\underline{n}$'s, obtained from the two approximations, ARCSIN and NORMAL, have been rounded to the nearest integer. The acceptance control limits, the $\underline{c}$'s, were not converted to integers because values from these approximations yield different risk protections depending on whether a continuity correction is added or subtracted from $\underline{c}$ before it is converted. For example:

<u>Acceptance Control Limit (Integerized)</u>		<u>Truncate Value of</u>	<u>Risk Protection Favored</u>
c'	=	$c + 0.5$	Type I error
c	=	c	Inconsistent
c^*	=	$c - 0.5$	Type II error

where 0.5 is a continuity correction implicit in Laplace's Theorem. (See Johnson and Kotz, 1969, p.53.) Thus, the conversion of $\underline{c}$ to integer is performed after determining the continuity correction which will yield the type of risk protection most favorable to the quality control program.

METHOD	D	0.0050			0.0100			0.0150		
		P1	P2	n	c	P2	n	c	P2	n
BINOMIAL ARCSINE NORMAL	1.5		0.0075	8474.0	53.00	0.0150	4163.0	52.00	0.0225	2774.0
				8428.0	53.09		4188.0	52.77		2774.0
				8218.0	51.61		4083.0	51.29		2705.0
BINOMIAL ARCSINE NORMAL	2.0		0.0100	2473.0	18.00	0.0200	1235.0	18.00	0.0300	822.0
				2478.0	18.47		1230.0	18.34		814.0
				2373.0	17.52		1178.0	17.39		779.0
BINOMIAL ARCSINE NORMAL	2.5		0.0125	1230.0	10.00	0.0250	614.0	10.00	0.0375	409.0
				1258.0	10.70		623.0	10.62		412.0
				1188.0	9.94		588.0	9.85		389.0
BINOMIAL ARCSINE NORMAL	3.0		0.0150	783.0	7.00	0.0300	390.0	7.00	0.0450	260.0
				792.0	7.52		392.0	7.46		259.0
				739.0	6.85		366.0	6.78		241.0
BINOMIAL ARCSINE NORMAL	3.5		0.0175	600.0	6.00	0.0350	299.0	6.00	0.0525	175.0
				559.0	5.84		276.0	5.79		182.0
				517.0	5.22		255.0	5.16		168.0
BINOMIAL ARCSINE NORMAL	4.0		0.0200	442.0	5.00	0.0400	198.0	4.00	0.0600	132.0
				423.0	4.81		209.0	4.76		138.0
				388.0	4.23		191.0	4.18		126.0
BINOMIAL ARCSINE NORMAL	4.5		0.0225	354.0	4.00	0.0450	176.0	4.00	0.0675	117.0
				336.0	4.11		166.0	4.07		109.0
				306.0	3.56		151.0	3.51		99.0
BINOMIAL ARCSINE NORMAL	5.0		0.0250	266.0	3.00	0.0500	132.0	3.00	0.0750	88.0
				277.0	3.61		136.0	3.58		90.0
				250.0	3.08		123.0	3.04		80.0

Table 1 (a). Acceptance Control Chart Parameters Obtained by Three Methods.

METHOD	P1 ^m	0.0200			0.0250			0.0300		
		D	r ²	n	c	r ²	n	c	r ²	n
BINOMIAL	1.5	1.5	0.0300	2043.0	51.00	0.0375	1633.0	51.00	0.0450	1360.0
ARCSINE				2068.0	52.11		1643.0	51.79		1361.0
NORMAL				2014.0	50.66		1602.0	50.34		1327.0
BINOMIAL	2.0	2.0	0.0400	614.0	18.00	0.0500	492.0	18.00	0.0600	410.0
ARCSINE				606.0	18.08		481.0	17.95		398.0
NORMAL				580.0	17.14		460.0	17.01		380.0
BINOMIAL	2.5	2.5	0.0500	306.0	10.00	0.0625	244.0	10.00	0.0750	203.0
ARCSINE				306.0	10.45		243.0	10.37		201.0
NORMAL				289.0	9.69		229.0	9.60		189.0
BINOMIAL	3.0	3.0	0.0600	194.0	7.00	0.0750	155.0	7.00	0.0900	129.0
ARCSINE				192.0	7.33		152.0	7.27		126.0
NORMAL				179.0	6.65		141.0	6.59		116.0
BINOMIAL	3.5	3.5	0.0700	131.0	5.00	0.0875	104.0	5.00	0.1050	87.0
ARCSINE				135.0	5.68		107.0	5.63		88.0
NORMAL				124.0	5.05		98.0	5.00		81.0
BINOMIAL	4.0	4.0	0.0800	98.0	4.00	0.1000	78.0	4.00	0.1200	65.0
ARCSINE				102.0	4.67		81.0	4.62		66.0
NORMAL				93.0	4.08		73.0	4.02		60.0
BINOMIAL	4.5	4.5	0.0900	87.0	4.00	0.1125	70.0	4.00	0.1350	58.0
ARCSINE				81.0	3.99		64.0	3.95		52.0
NORMAL				73.0	3.42		57.0	3.37		47.0
BINOMIAL	5.0	5.0	0.1000	65.0	3.00	0.1250	52.0	3.00	0.1500	43.0
ARCSINE				66.0	3.50		52.0	3.46		43.0
NORMAL				59.0	2.95		46.0	2.91		38.0

Table 1 (b).

METHOD	P1=	0.0350			0.0400			0.0450		
		D	P2	n	c	P2	n	c	P2	n
BINOMIAL	1.5	1.5	0.0525	1145.0	51.00	0.0600	1001.0	50.00	0.0675	889.0
ARCSINE				1159.0	51.14		1007.0	50.81		890.0
NORMAL				1130.0	49.71		982.0	49.39		867.0
BINOMIAL	2.0	2.0	0.0700	334.0	17.00	0.0800	292.0	17.00	0.0900	259.0
ARCSINE				338.0	17.69		294.0	17.56		259.0
NORMAL				323.0	16.76		281.0	16.63		247.0
BINOMIAL	2.5	2.5	0.0875	174.0	10.00	0.1000	152.0	10.00	0.1125	135.0
ARCSINE				170.0	10.20		148.0	10.12		130.0
NORMAL				160.0	9.44		139.0	9.35		122.0
BINOMIAL	3.0	3.0	0.1050	110.0	7.00	0.1200	96.0	7.00	0.1350	85.0
ARCSINE				107.0	7.14		92.0	7.08		81.0
NORMAL				99.0	6.46		85.0	6.39		75.0
BINOMIAL	3.5	3.5	0.1225	74.0	5.00	0.1400	65.0	5.00	0.1575	57.0
ARCSINE				75.0	5.52		65.0	5.47		57.0
NORMAL				68.0	4.88		59.0	4.83		52.0
BINOMIAL	4.0	4.0	0.1400	56.0	4.00	0.1600	48.0	4.00	0.1800	43.0
ARCSINE				56.0	4.53		48.0	4.48		43.0
NORMAL				51.0	3.92		44.0	3.87		38.0
BINOMIAL	4.5	4.5	0.1575	49.0	4.00	0.1800	43.0	4.00	0.2025	38.0
ARCSINE				44.0	3.86		38.0	3.82		33.0
NORMAL				39.0	3.28		34.0	3.23		30.0
BINOMIAL	5.0	5.0	0.1750	37.0	3.00	0.2000	32.0	3.00	0.2250	28.0
ARCSINE				36.0	3.39		31.0	3.35		27.0
NORMAL				32.0	2.82		27.0	2.77		24.0

Table 1 (c).

METHOD	D	P1= 0.0500			0.0550			0.0600		
		r2	n	c	r2	n	c	r2	n	c
BINOMIAL ARCSINE NORMAL	1.5	0.0750	800.0 795.0 775.0	50.00 50.16 48.76	0.0825	713.0 718.0 700.0	49.00 49.83 48.44	0.0900	654.0 654.0 638.0	49.00 49.50 48.12
BINOMIAL ARCSINE NORMAL	2.0	0.1000	233.0 231.0 221.0	17.00 17.29 16.37	0.1100	212.0 209.0 199.0	17.00 17.16 16.25	0.1200	194.0 190.0 181.0	17.00 17.03 16.12
BINOMIAL ARCSINE NORMAL	2.5	0.1250	121.0 116.0 109.0	10.00 9.95 9.18	0.1375	110.0 105.0 98.0	10.00 9.86 9.10	0.1500	100.0 95.0 89.0	10.00 9.78 9.01
BINOMIAL ARCSINE NORMAL	3.0	0.1500	77.0 72.0 67.0	7.00 6.95 6.26	0.1650	69.0 65.0 60.0	7.00 6.88 6.19	0.1800	63.0 59.0 54.0	7.00 6.82 6.13
BINOMIAL ARCSINE NORMAL	3.5	0.1750	51.0 50.0 46.0	5.00 5.36 4.71	0.1925	46.0 45.0 41.0	5.00 5.31 4.66	0.2100	42.0 41.0 37.0	5.00 5.25 4.60
BINOMIAL ARCSINE NORMAL	4.0	0.2000	38.0 38.0 34.0	4.00 4.39 3.77	0.2200	35.0 34.0 30.0	4.00 4.34 3.72	0.2400	32.0 31.0 27.0	4.00 4.30 3.67
BINOMIAL ARCSINE NORMAL	4.5	0.2250	28.0 30.0 26.0	3.00 3.74 3.14	0.2475	31.0 27.0 23.0	4.00 3.69 3.09	0.2700	23.0 24.0 21.0	3.00 3.65 3.04
BINOMIAL ARCSINE NORMAL	5.0	0.2500	25.0 24.0 21.0	3.00 3.27 2.68	0.2750	23.0 22.0 19.0	3.00 3.23 2.64	0.3000	21.0 19.0 17.0	3.00 3.19 2.59

Table 1 (d).

For all three methods, $\underline{n}$ and $\underline{c}$ decrease as the difference between the AQL ($\underline{p}_1$) and RQL ($\underline{p}_2$) increases. This can be observed by either maintaining a constant discrimination ratio ($\underline{D} = \underline{p}_2/\underline{p}_1$) and increasing $\underline{p}_1$, or by maintaining a constant $\underline{p}_1$ and increasing $\underline{D}$.

Tables 2, 3, and 4 indicate the risk over-protection, or under-protection (negative values), which each plan provides. The table values are the differences between actual ($P[k \leq c | p]$) values and stipulated ($1-\alpha$ or β) risk protection probabilities. The actual probabilities were obtained using a FORTRAN program, in single precision mode, on a PDP 11/34 computer. The probabilities obtained from this program compared with published binomial probability tables [N.B.S. (1950), Romig (1953), and Harvard (1955)] with an 0.00001 accuracy.

Binomial Distribution

As evidenced in Table 2, ACC plans obtained from the binomial distribution over-protect both the producer and consumer by having low probabilities of committing Type I or Type II errors; i.e., rejecting good quality products or accepting bad quality products. The largest difference between actual and stipulated probabilities is 2.21%. The over-protection is due to the fact that since $\underline{n}$ and $\underline{c}$ are both integers, an OC curve usually cannot pass exactly through points $(\underline{p}_1, 1-\alpha)$ and $(\underline{p}_2, \beta)$, thus the search procedure finds the minimum $\underline{n}$ and $\underline{c}$ which satisfy inequalities (3) and (4).

Standard Normal Approximation

The acceptance control plans obtained from the standard normal approximation generally have smaller subgroup sizes, $\underline{n}$, than the plans obtained from the other two models. As evidenced in Table 4, this method consistently protects against Type II error when $\underline{c}^*$ is used, but the underprotection

$\frac{p_1}{D}$	0.0050	0.0100	0.0150	0.0200	0.0250	0.0300	0.0350	0.0400	0.0450	0.0500	0.0550	0.0600
$R = P(k \leq c p_1) - (1 - \alpha)$												
1.5	0.0004	0.0008	0.0013	0.0004	0.0001	0.0022	0.0029	0.0006	0.0023	0.0018	0.0005	0.0003
2.0	0.0029	0.0035	0.0048	0.0053	0.0047	0.0069	0.0014	0.0020	0.0035	0.0039	0.0041	0.0052
2.5	0.0011	0.0023	0.0029	0.0039	0.0053	0.0062	0.0065	0.0075	0.0081	0.0093	0.0100	0.0124
3.0	0.0043	0.0055	0.0060	0.0074	0.0081	0.0088	0.0102	0.0112	0.0123	0.0115	0.0147	0.0158
3.5	0.0171	0.0178	0.0004	0.0013	0.0032	0.0029	0.0048	0.0046	0.0075	0.0089	0.0107	0.0118
4.0	0.0199	0.0000	0.0005	0.0027	0.0040	0.0044	0.0040	0.0078	0.0070	0.0103	0.0087	0.0096
4.5	0.0162	0.0172	0.0179	0.0193	0.0190	0.0201	0.0220	0.0221	0.0231	0.0009	0.0242	0.0041
5.0	0.0042	0.0058	0.0062	0.0086	0.0091	0.0105	0.0104	0.0123	0.0146	0.0159	0.0150	0.0159
$R = \beta - P(k \leq c p_2)$												
1.5	0.0001	0.0001	0.0004	0.0003	0.0002	0.0001	0.0002	0.0008	0.0004	0.0012	0.0001	0.0019
2.0	0.0002	0.0004	0.0002	0.0007	0.0005	0.0017	0.0003	0.0008	0.0002	0.0011	0.0031	0.0029
2.5	0.0001	0.0005	0.0014	0.0013	0.0004	0.0011	0.0026	0.0031	0.0041	0.0029	0.0045	0.0006
3.0	0.0005	0.0000	0.0018	0.0007	0.0018	0.0029	0.0018	0.0022	0.0018	0.0075	0.0011	0.0008
3.5	0.0004	0.0006	0.0014	0.0026	0.0009	0.0050	0.0033	0.0074	0.0029	0.0022	0.0005	0.0009
4.0	0.0004	0.0004	0.0028	0.0005	0.0006	0.0031	0.0080	0.0010	0.0073	0.0014	0.0103	0.0118
4.5	0.0009	0.0010	0.0024	0.0012	0.0066	0.0068	0.0030	0.0072	0.0074	0.0037	0.0124	0.0039
5.0	0.0010	0.0008	0.0038	0.0004	0.0037	0.0038	-0.0084	0.0069	0.0037	0.0038	0.0124	0.0144

Table 2. BINOMIAL distribution risk protection difference (R) between actual ($P[k \leq c | p]$) and stipulated ($1 - \alpha$ or β) probabilities of acceptance plans in Table 1.

p1	0.0050	0.0100	0.0150	0.0200	0.0250	0.0300	0.0350	0.0400	0.0450	0.0500	0.0550	0.0600
D	R = P[k<c p1] - (1-α) c = (c - 0.5) truncated.											
1.5	0.0037	-0.0017	-0.0017	0.0089	-0.0013	0.0033	0.0077	-0.0025	0.0032	0.0081	-0.0022	0.0020
2.0	0.0159	-0.0045	-0.0003	0.0030	0.0043	0.0098	0.0127	0.0149	0.0184	-0.0031	0.0011	0.0045
2.5	0.0075	0.0131	0.0150	0.0173	0.0195	0.0217	-0.0058	-0.0032	0.0009	0.0033	0.0044	0.0093
3.0	0.0153	0.0174	0.0197	0.0215	0.0240	0.0240	-0.0092	-0.0041	-0.0019	0.0003	0.0040	0.0082
3.5	0.0025	0.0055	0.0079	0.0110	0.0133	0.0149	0.0182	0.0198	0.0217	0.0237	0.0243	0.0282
4.0	0.0030	0.0040	0.0080	0.0108	0.0138	0.0140	0.0175	0.0195	0.0231	0.0241	0.0273	0.0293
4.5	0.0303	0.0314	-0.0124	-0.0086	-0.0043	-0.0021	0.0033	0.0043	0.0058	0.0113	0.0150	0.0159
5.0	0.0122	0.0144	0.0174	0.0195	0.0225	0.0234	0.0253	0.0285	0.0288	0.0311	0.0318	0.0334
R = β - P[k<c p2] c = (c) truncated.												
1.5	-0.0182	-0.0021	-0.0075	-0.0188	-0.0014	-0.0091	-0.0178	-0.0003	-0.0089	-0.0181	0.0004	-0.0044
2.0	-0.0384	0.0007	-0.0049	-0.0100	-0.0144	-0.0235	-0.0304	-0.0349	-0.0453	-0.0008	-0.0077	-0.0133
2.5	-0.0243	-0.0304	-0.0331	-0.0412	-0.0475	-0.0542	0.0012	-0.0019	-0.0088	-0.0122	-0.0184	-0.0234
3.0	-0.0339	-0.0404	-0.0475	-0.0527	-0.0625	-0.0704	0.0035	-0.0029	-0.0051	-0.0074	-0.0144	-0.0238
3.5	-0.0107	-0.0140	-0.0204	-0.0274	-0.0323	-0.0349	-0.0457	-0.0484	-0.0517	-0.0409	-0.0724	-0.0813
4.0	-0.0118	-0.0172	-0.0201	-0.0259	-0.0337	-0.0388	-0.0408	-0.0463	-0.0413	-0.0419	-0.0788	-0.0885
4.5	-0.0004	-0.0042	0.0079	0.0037	-0.0038	-0.0055	-0.0175	-0.0161	-0.0145	-0.0318	-0.0429	-0.0413
5.0	-0.0270	-0.0317	-0.0409	-0.0443	-0.0544	-0.0574	-0.0443	-0.0823	-0.0780	-0.0917	-0.0903	-0.1019
R = P[k<c p1] - (1-α) c = (c) truncated.												
1.5	-0.0104	-0.0017	-0.0174	-0.0040	-0.0013	0.0033	-0.0077	-0.0025	0.0032	-0.0074	-0.0022	0.0020
2.0	-0.0079	-0.0045	-0.0003	0.0030	0.0043	0.0098	-0.0144	-0.0112	-0.0039	-0.0031	0.0011	0.0045
2.5	-0.0300	-0.0247	-0.0215	-0.0174	-0.0139	-0.0101	-0.0058	-0.0032	0.0009	0.0033	0.0044	0.0093
3.0	-0.0310	-0.0273	-0.0230	-0.0194	-0.0148	-0.0108	-0.0072	-0.0041	-0.0019	0.0003	0.0040	0.0082
3.5	0.0025	0.0055	0.0079	0.0110	-0.0497	-0.0444	-0.0398	-0.0348	-0.0339	-0.0284	-0.0230	-0.0184
4.0	0.0030	0.0040	0.0080	0.0108	0.0138	-0.0357	-0.0378	-0.0484	-0.0402	-0.0381	-0.0303	-0.0253
4.5	-0.0190	-0.0159	-0.0124	-0.0088	-0.0043	-0.0021	0.0033	0.0043	0.0058	0.0113	0.0150	0.0159
5.0	0.0122	0.0144	-0.0492	-0.0444	-0.0574	-0.0550	-0.0504	-0.0418	-0.0413	-0.0349	-0.0334	-0.0282
R = β - P[k<c p2] c = (c) truncated.												
1.5	0.0043	0.0021	0.0141	0.0044	-0.0014	-0.0091	0.0077	-0.0003	-0.0089	0.0080	0.0004	-0.0044
2.0	0.0054	0.0007	-0.0049	-0.0100	-0.0144	0.0183	0.0134	0.0103	0.0027	-0.0008	-0.0077	-0.0133
2.5	0.0243	0.0224	0.0194	0.0138	0.0117	0.0074	0.0012	-0.0019	-0.0088	-0.0122	-0.0184	-0.0234
3.0	0.0243	0.0237	0.0194	0.0147	0.0108	0.0040	0.0035	-0.0029	-0.0051	-0.0074	-0.0144	-0.0238
3.5	-0.0107	-0.0140	-0.0204	-0.0274	0.0375	0.0344	0.0308	0.0275	0.0283	0.0234	0.0175	0.0130
4.0	-0.0118	-0.0172	-0.0201	-0.0259	-0.0337	0.0397	0.0395	0.0370	0.0277	0.0300	0.0217	0.0172
4.5	0.0143	0.0119	0.0079	0.0037	-0.0038	-0.0055	-0.0175	-0.0161	-0.0145	-0.0318	-0.0429	-0.0413
5.0	-0.0270	-0.0317	0.0447	0.0427	0.0384	0.0384	0.0341	0.0282	0.0312	0.0253	0.0272	0.0224
R = P[k<c p1] - (1-α) c = (c - 0.5) truncated.												
1.5	-0.0104	-0.0188	-0.0174	-0.0040	-0.0190	-0.0132	-0.0077	-0.0207	-0.0137	-0.0074	-0.0210	-0.0154
2.0	-0.0079	-0.0392	-0.0333	-0.0288	-0.0239	-0.0188	-0.0144	-0.0112	-0.0039	-0.0398	-0.0337	-0.0288
2.5	-0.0300	-0.0247	-0.0215	-0.0174	-0.0139	-0.0101	-0.0058	-0.0544	-0.0499	-0.0440	-0.0404	-0.0341
3.0	-0.0310	-0.0273	-0.0230	-0.0194	-0.0148	-0.0108	-0.0042	-0.0753	-0.0714	-0.0679	-0.0613	-0.0534
3.5	-0.0700	-0.0444	-0.0400	-0.0541	-0.0497	-0.0444	-0.0398	-0.0348	-0.0339	-0.0284	-0.0230	-0.0184
4.0	-0.0018	-0.0761	-0.0722	-0.0444	-0.0403	-0.0337	-0.0378	-0.0484	-0.0402	-0.0381	-0.0303	-0.0253
4.5	-0.0190	-0.0159	-0.1344	-0.1298	-0.1208	-0.1147	-0.1054	-0.1037	-0.1009	-0.0884	-0.0801	-0.0784
5.0	-0.0011	-0.0742	-0.0492	-0.0444	-0.0574	-0.0550	-0.0504	-0.0418	-0.0413	-0.0349	-0.0334	-0.0282
R = β - P[k<c p2] c = (c) truncated.												
1.5	0.0043	0.0201	0.0141	0.0044	0.0210	0.0149	0.0077	0.0223	0.0153	0.0080	0.0233	0.0179
2.0	0.0054	0.0333	0.0314	0.0278	0.0232	0.0183	0.0134	0.0103	0.0027	0.0342	0.0315	0.0277
2.5	0.0243	0.0224	0.0194	0.0138	0.0117	0.0074	0.0440	0.0443	0.0402	0.0383	0.0344	0.0314
3.0	0.0243	0.0237	0.0194	0.0147	0.0108	0.0040	0.0532	0.0508	0.0498	0.0488	0.0453	0.0403
3.5	0.0481	0.0435	0.0434	0.0398	0.0375	0.0344	0.0308	0.0275	0.0283	0.0234	0.0175	0.0130
4.0	0.0518	0.0494	0.0483	0.0454	0.0421	0.0399	0.0395	0.0370	0.0277	0.0300	0.0217	0.0172
4.5	0.0143	0.0119	0.0448	0.0453	0.0424	0.0420	0.0372	0.0383	0.0384	0.0324	0.0481	0.0494
5.0	0.0503	0.0484	0.0447	0.0427	0.0384	0.0384	0.0341	0.0282	0.0312	0.0253	0.0272	0.0224

Table 3. Standard NORMAL approximation risk protection difference (R) between actual $P[k < c | p]$ and stipulated $(1-\alpha$ or $\beta)$ probabilities of acceptance. Plans in Table 1.

p1	0.0050	0.0100	0.0150	0.0200	0.0250	0.0300	0.0350	0.0400	0.0450	0.0500	0.0550	0.0600
D	R = $P(k \leq c p) - (1 - \alpha)$ $c = (c - 0.5)$ truncated.											
1.5	0.0077	0.0118	0.0017	0.0067	0.0104	0.0015	0.0059	0.0109	0.0017	0.0060	0.0097	0.0140
2.0	0.0022	0.0050	0.0082	0.0112	0.0135	0.0165	0.0194	0.0210	0.0035	0.0070	0.0092	0.0123
2.5	0.0226	0.0248	0.0264	0.0039	0.0064	0.0087	0.0122	0.0138	0.0148	0.0189	0.0201	0.0229
3.0	0.0299	0.0043	0.0048	0.0095	0.0120	0.0133	0.0154	0.0187	0.0204	0.0228	0.0242	0.0259
3.5	0.0261	0.0278	0.0293	0.0304	0.0318	0.0332	0.0339	0.0044	0.0075	0.0122	0.0142	0.0154
4.0	0.0292	0.0306	0.0317	0.0331	0.0339	0.0357	0.0364	0.0078	0.0070	0.0103	0.0130	0.0142
4.5	0.0220	0.0236	0.0254	0.0264	0.0280	0.0304	0.0314	0.0329	0.0347	0.0344	0.0353	0.0373
5.0	0.0366	0.0378	0.0383	0.0396	0.0091	0.0105	0.0137	0.0140	0.0185	0.0202	0.0198	0.0257
R = $\beta - P(k \leq c p) - (1 - \alpha)$ $c = (c - 0.5)$ truncated.												
1.5	-0.0089	-0.0143	0.0004	-0.0073	-0.0145	0.0011	-0.0070	-0.0146	0.0019	-0.0075	-0.0142	-0.0253
2.0	0.0019	-0.0030	-0.0083	-0.0139	-0.0199	-0.0257	-0.0338	-0.0387	0.0002	-0.0043	-0.0093	-0.0156
2.5	-0.0396	-0.0442	-0.0517	-0.0013	-0.0021	-0.0057	-0.0136	-0.0157	-0.0230	-0.0281	-0.0298	-0.0389
3.0	-0.0610	0.0029	-0.0004	-0.0054	-0.0098	-0.0112	-0.0151	-0.0244	-0.0290	-0.0357	-0.0384	-0.0436
3.5	-0.0422	-0.0484	-0.0537	-0.0592	-0.0635	-0.0709	-0.0727	0.0074	0.0029	-0.0084	-0.0115	-0.0124
4.0	-0.0500	-0.0550	-0.0584	-0.0638	-0.0680	-0.0812	-0.0858	0.0010	0.0073	0.0014	-0.0036	-0.0034
4.5	-0.0249	-0.0284	-0.0338	-0.0359	-0.0398	-0.0517	-0.0564	-0.0615	-0.0735	-0.0626	-0.0655	-0.0830
5.0	-0.0765	-0.0850	-0.0863	-0.0981	-0.0037	0.0038	-0.0033	-0.0070	-0.0129	-0.0150	-0.0071	-0.0332
R = $P(k \leq c p) - (1 - \alpha)$ $c = (c - 0.5)$ truncated.												
1.5	0.0077	-0.0021	0.0017	0.0047	-0.0038	0.0015	0.0059	-0.0034	0.0017	0.0040	-0.0051	0.0003
2.0	0.0022	0.0050	0.0082	0.0112	-0.0119	-0.0077	-0.0033	-0.0007	0.0035	0.0070	0.0092	0.0123
2.5	-0.0054	-0.0020	0.0007	0.0039	0.0044	0.0087	0.0122	0.0138	0.0148	-0.0158	-0.0138	-0.0089
3.0	0.0013	0.0043	0.0048	0.0095	0.0120	0.0133	0.0154	0.0187	0.0204	-0.0182	-0.0157	-0.0124
3.5	-0.0142	-0.0107	-0.0077	-0.0048	-0.0024	0.0007	0.0023	0.0044	0.0075	0.0122	0.0142	0.0154
4.0	-0.0132	-0.0102	-0.0078	-0.0045	-0.0024	0.0019	0.0040	0.0078	0.0070	0.0103	0.0130	0.0142
4.5	0.0220	0.0236	0.0254	-0.0296	-0.0264	-0.0205	-0.0173	-0.0143	-0.0094	-0.0108	-0.0084	-0.0026
5.0	-0.0016	0.0014	0.0031	0.0044	0.0091	0.0105	0.0137	0.0140	0.0185	0.0202	0.0198	0.0257
R = $\beta - P(k \leq c p) - (1 - \alpha)$ $c = (c - 0.5)$ truncated.												
1.5	-0.0089	0.0079	0.0004	-0.0073	0.0087	0.0011	-0.0070	0.0084	0.0019	-0.0075	0.0093	0.0019
2.0	0.0019	-0.0030	-0.0083	-0.0139	0.0199	0.0158	0.0099	0.0044	0.0002	-0.0043	-0.0093	-0.0156
2.5	0.0141	0.0097	0.0061	0.0013	-0.0021	-0.0057	-0.0136	-0.0157	-0.0230	0.0261	0.0252	0.0195
3.0	0.0048	0.0029	-0.0004	-0.0054	-0.0098	-0.0112	-0.0151	-0.0244	-0.0290	0.0273	0.0280	0.0253
3.5	0.0260	0.0226	0.0198	0.0168	0.0146	0.0105	0.0099	0.0074	0.0029	-0.0084	-0.0115	-0.0124
4.0	0.0259	0.0234	0.0217	0.0181	0.0173	0.0101	0.0080	0.0010	0.0073	0.0014	-0.0036	-0.0034
4.5	-0.0249	-0.0284	-0.0338	0.0407	0.0392	0.0335	0.0314	0.0294	0.0239	0.0304	0.0294	0.0213
5.0	0.0169	0.0128	0.0128	0.0048	0.0037	0.0038	-0.0033	-0.0070	-0.0129	-0.0150	-0.0071	-0.0332
R = $P(k \leq c p) - (1 - \alpha)$ $c = (c - 0.5)$ truncated.												
1.5	-0.0074	-0.0021	-0.0151	-0.0087	-0.0038	-0.0134	-0.0079	-0.0034	-0.0158	-0.0100	-0.0051	0.0003
2.0	-0.0283	-0.0243	-0.0198	-0.0153	-0.0119	-0.0077	-0.0033	-0.0007	-0.0287	-0.0235	-0.0204	-0.0157
2.5	-0.0054	-0.0020	0.0007	-0.0044	-0.0036	-0.0038	-0.0249	-0.0244	-0.0194	-0.0138	-0.0138	-0.0089
3.0	0.0013	-0.0515	-0.0474	-0.0426	-0.0382	-0.0359	-0.0322	-0.0259	-0.0224	-0.0182	-0.0157	-0.0124
3.5	-0.0142	-0.0107	-0.0077	-0.0048	-0.0024	0.0007	0.0023	-0.0485	-0.0430	-0.0334	-0.0498	-0.0472
4.0	-0.0132	-0.0102	-0.0078	-0.0045	-0.0024	0.0019	0.0040	-0.0749	-0.0770	-0.0704	-0.0647	-0.0625
4.5	-0.0397	-0.0363	-0.0322	-0.0294	-0.0264	-0.0205	-0.0173	-0.0143	-0.0094	-0.0108	-0.0084	-0.0026
5.0	-0.0016	0.0014	0.0031	0.0044	-0.0708	-0.0881	-0.0808	-0.0758	-0.0697	-0.0659	-0.0677	-0.0521
R = $\beta - P(k \leq c p) - (1 - \alpha)$ $c = (c - 0.5)$ truncated.												
1.5	0.0142	0.0079	0.0201	0.0158	0.0087	0.0231	0.0145	0.0084	0.0242	0.0145	0.0093	0.0019
2.0	0.0353	0.0318	0.0281	0.0242	0.0199	0.0158	0.0099	0.0044	0.0359	0.0314	0.0274	0.0251
2.5	0.0141	0.0097	0.0061	0.0038	0.0018	0.0397	0.0348	0.0337	0.0291	0.0261	0.0252	0.0195
3.0	0.0048	0.0302	0.0484	0.0457	0.0433	0.0428	0.0408	0.0354	0.0330	0.0293	0.0280	0.0253
3.5	0.0260	0.0226	0.0198	0.0168	0.0146	0.0105	0.0099	0.0394	0.0374	0.0321	0.0309	0.0257
4.0	0.0259	0.0234	0.0217	0.0181	0.0173	0.0101	0.0080	0.0604	0.0637	0.0613	0.0594	0.0598
4.5	0.0451	0.0437	0.0413	0.0407	0.0392	0.0335	0.0314	0.0294	0.0239	0.0304	0.0294	0.0213
5.0	0.0169	0.0128	0.0128	0.0048	0.0037	0.0038	0.0662	0.0637	0.0626	0.0606	0.0640	0.0538

Table 4. ARCSINE transformation risk protection difference (R) between actual ($P(k \leq c | p)$) and stipulated ($1 - \alpha$ or β) probabilities of acceptance. Plans in Table 1.

against Type I error may be as large as 14%. When $\underline{c'}$ is adopted, protection against Type I error is usually attained, or is at most 1.25% below the stipulated producer's risk probability, but under-protection against Type II error may be as large as 10%. When no continuity correction is used with $\underline{c}$, there is no consistent risk protection, and under-protection against either type of error may be as large as 6%.

Arcsin Transformation

As evidenced in Table 5, acceptance control plans obtained from the arcsin transformation consistently provide one-tail protection. When $\underline{c^*}$ is used, protection against Type II error is attained, but not against Type I error. Under-protection against Type I error may be as large as 9%. When $\underline{c'}$ is adopted, protection against Type I error is attained, but under-protection against Type II error may be as large as 10%. When no continuity correction is used with $\underline{c}$, no consistent protection is attained, and under-protection against either type of error may be as large as 3%.

SIMULATION STUDY

A computer program simulating item manufacture and control charts, written by Davis (1977), was used to analyze the performance of ACC's derived from the binomial distribution, standard normal approximation, and arcsin transformation. Twenty replications of this simulated process were made, using common random numbers for variance reduction.

Process Description

It is desired to have producer and consumer risks of 5% and 10%, respectively. The cost of a Type I error is considered to be greater than that of a Type II error in this simulation; thus, $\underline{c'}$ is used with the

Table 5. BINOMIAL ACC Plan Simulation Results.

SUB GROUP	NONCON- FORMANCES	SUB GROUP	NONCON- FORMANCES	SUB GROUP	NONCON- FORMANCES
1	3	21	3	41	4
2	2	22	2	42	3
3	0	23	2	43	2
4	4	24	4	44	3
5	0	25	3	45	1
6	5	26	3	46	3
7	1	27	3	47	3
8	2	28	2	48	5
9	3	29	3	49	3
10	4	30	2	50	1
11	5	31	3	51	12
12	2	32	5	52	8
13	2	33	0	53	8
14	4	34	1	54	6
15	4	35	1	55	13
16	5	36	1	56	8
17	5	37	3	57	7
18	2	38	2	58	12
19	3	39	2	59	10
20	7	40	0	60	14

approximations because of its increased Type I error protection. In Table 1, when an APL of 1.5% and a discrimination ratio, $\underline{D}$, of 3.5 are chosen (i.e., $RPL = 5.25\%$), the acceptance control plans are as follows:

<u>Method</u>	<u>n</u>	<u>c</u>	<u>c'</u>	<u>c*</u>	<u>Plan Chosen</u>
BINOMIAL	175.0	5.00			(175, 5)
ARCSIN	182.0	6.73	7	6	(182, 7)
NORMAL	168.0	5.11	5	4	(168, 5)

Thus the Acceptance Control Limits were positioned at 5.5 for the binomial and standard normal approximation and at 7.5 for the normalized arcsin transformation. The simulation "manufactured" 10,500 items. Initially the manufacturing process had a nonconforming percentage of 1.5, the AQL. After the 8,750th item produced, the process shifted to 5.25% nonconforming, the RQL level. Thus there was a shift from the expected acceptable process level to the rejectable process level to test the plans at the stipulated extremes.

Control Chart Analysis

The control charts in Figures 3 and 4 show the number of nonconforming units found in each subgroup of the three acceptance plans. Tables 5, 6, and 7 contain the simulated data used to plot these Figures. The process shifted to the RPL after the 50th subgroup for the exact binomial plan, and in the middle of the 49th and 53rd subgroups for the arcsin transformation and normal approximation plans, respectively.

The binomial plan made one Type I error (subgroup 20) in 50 subgroups of good quality and no Type II errors were made in the final 10 subgroups. The arcsin transformation plan made one Type I error (subgroup 32) out of 48 good quality subgroups and no Type II errors in the last ten subgroups. The standard normal approximation plan made no Type I errors in the first 52

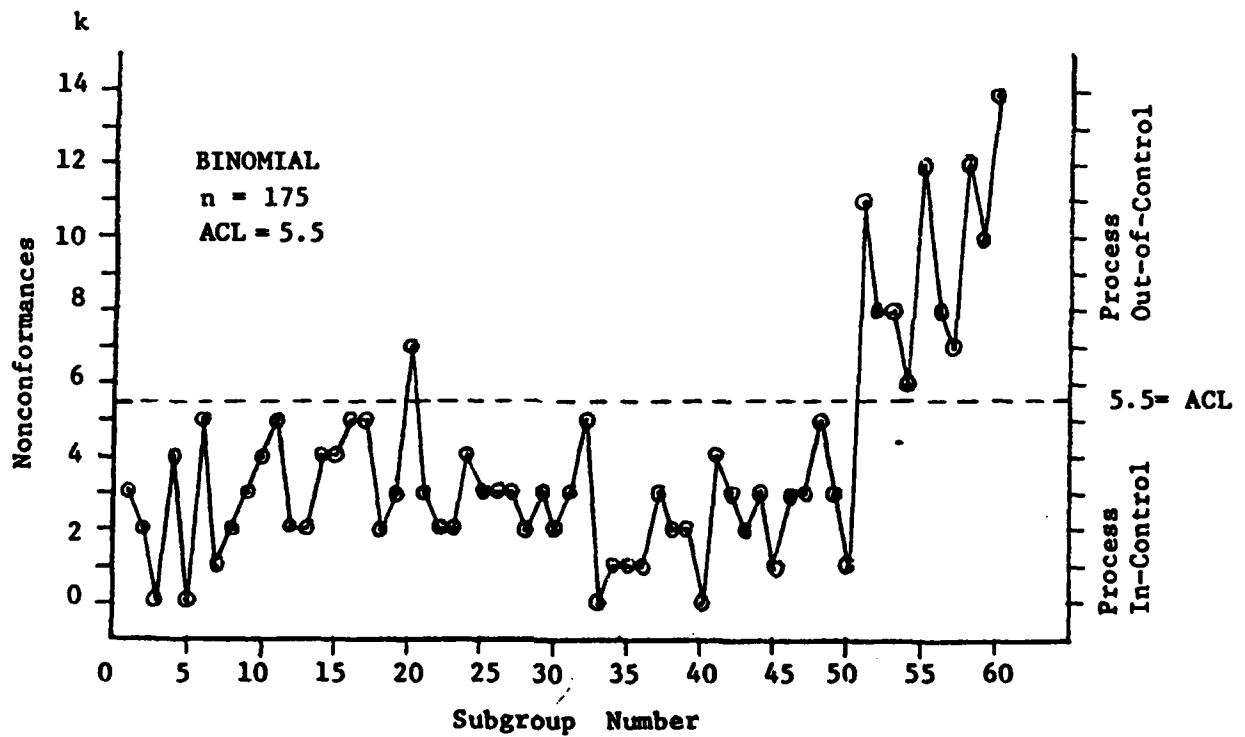


Figure 3. Acceptance Control Chart, Exact Binomial

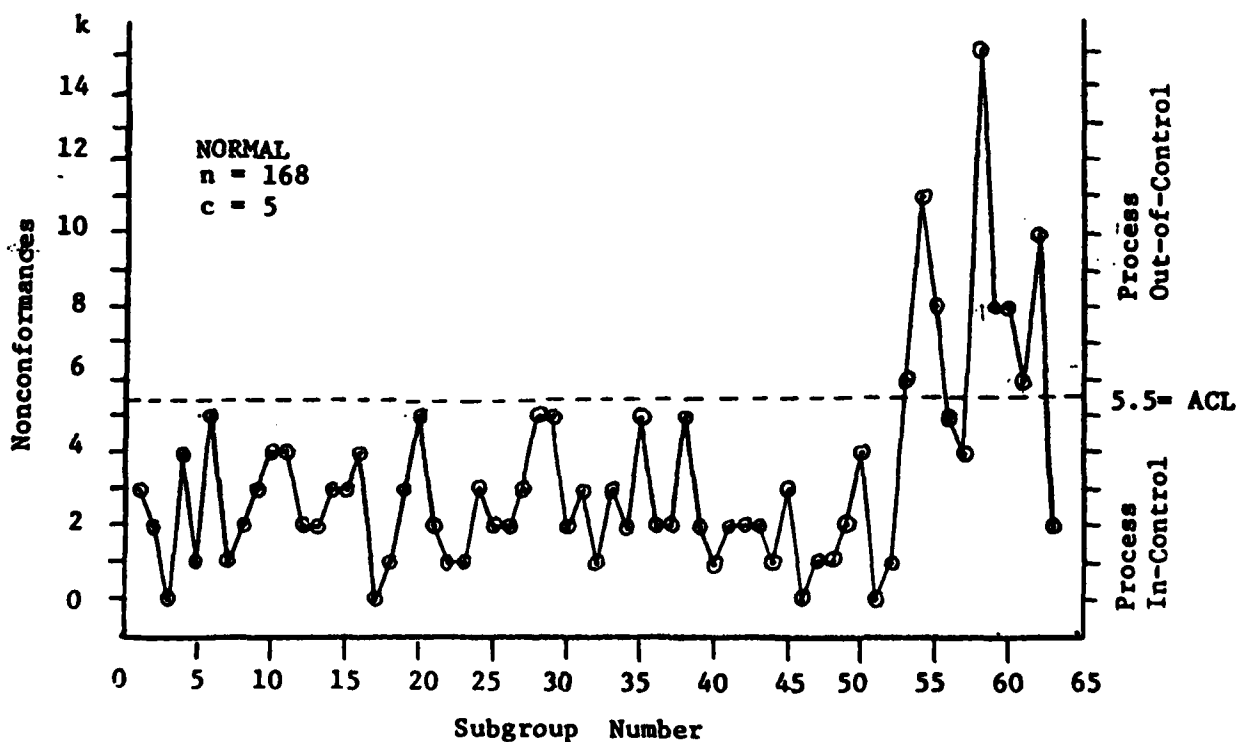
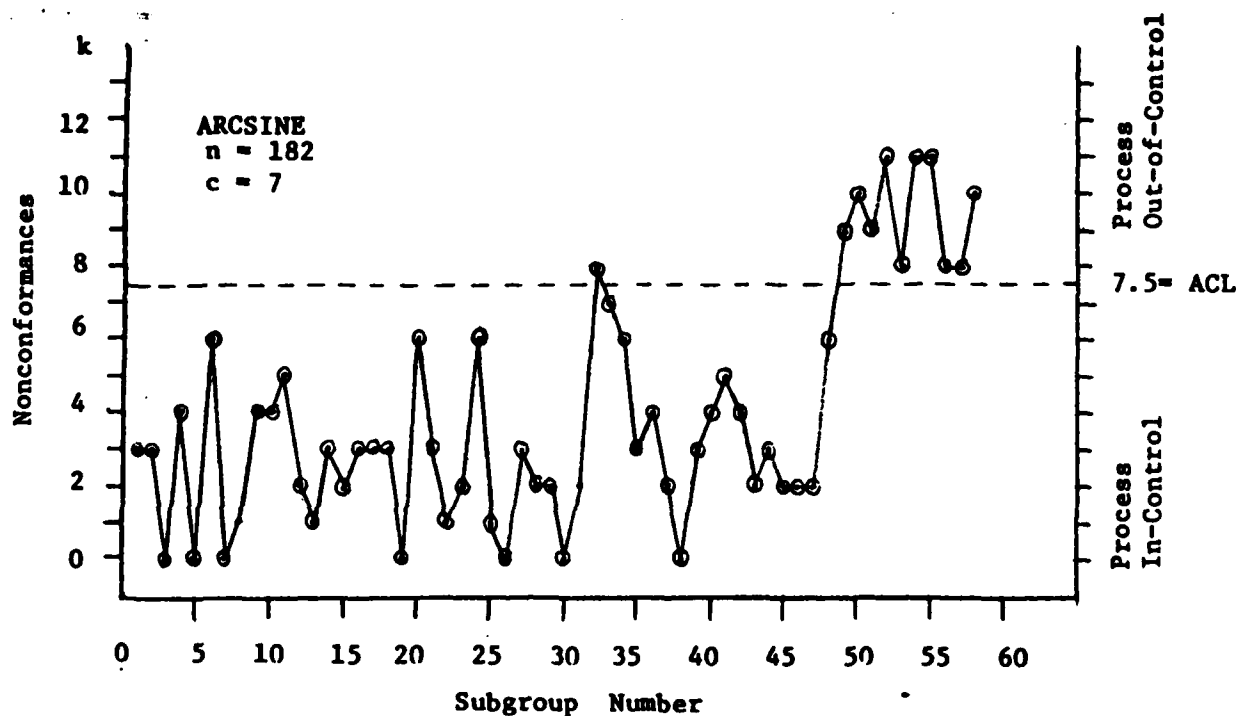


Figure 4. Acceptance Control Charts for ARCSIN and NORMAL approximations.

Table 6. ARCSINE ACC Plan Simulation Results.

SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES
1	3	21	3	41	5
2	3	22	1	42	4
3	0	23	2	43	2
4	3	24	6	44	3
5	0	25	1	45	2
6	6	26	0	46	2
7	0	27	3	47	2
8	1	28	2	48	6
9	4	29	2	49	9
10	4	30	0	50	10
11	5	31	2	51	9
12	2	32	8	52	11
13	1	33	7	53	8
14	3	34	6	54	12
15	2	35	3	55	12
16	3	36	4	56	8
17	3	37	2	57	8
18	3	38	0	58	10
19	0	39	3		
20	6	40	4		

Table 7. NORMAL ACC Plan Simulation Results.

SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES
1	3	21	2	41	2
2	2	22	1	42	2
3	0	23	1	43	2
4	4	24	3	44	1
5	1	25	2	45	3
6	5	26	2	46	0
7	1	27	3	47	1
8	2	28	5	48	1
9	3	29	5	49	2
10	4	30	2	50	4
11	4	31	3	51	0
12	2	32	1	52	1
13	2	33	3	53	6
14	3	34	2	54	11
15	3	35	5	55	8
16	4	36	2	56	15
17	0	37	2	57	5
18	1	38	5	58	4
19	3	39	2	59	8
20	5	40	1	60	8
				61	6
				62	10
				63	3

subgroups, but had three Type II errors (subgroups 57, 58 and 60) in the last eleven subgroups, when the process had shifted to the RPL.

The total number rejected in a subgroup may be expected to exceed the ACL for either of two reasons, (1) the existence of assignable causes, or (2) the existence of a quality level which exceeds the APL. In either case, the only clue given by the acceptance control chart as to the cause of lack of control is the time at which lack of control at the desired level was observed. For this reason, immediate corrective action should be taken whenever a point exceeds the ACL. This simulation did not include such corrective action because it was desired to observe the consistency of each control chart in providing the desired risk protections.

CONCLUSIONS

Three methods for obtaining subgroup sizes and acceptance control limits were compared. In addition to utilizing the exact binomial distribution, the standard normal approximation and a normalized arcsin transformation were used. Acceptance plans obtained by using the binomial distribution provide the stipulated risk protections (guaranteed over protection) for both producer and consumer. This results from the strict application of the inequality constraints of equations (3) and (4). When a continuity correction of 0.5 is added to (subtracted from) the $\underline{c}$ derived from either of the latter two approximation methods, strict risk protection against Type I (Type II) error is attained, but not against both error types. If no continuity correction is used, the risk protection from these approximations is inconsistent; i.e., risk protection alternates against both types of errors, with no discernable pattern.

For the risk levels studied ($1-\alpha = 0.95$ and $\beta = 0.10$) and the wide range of values of p_1 and D studied, the normalized arcsin transformation yielded results closer to design than did the standard normal approximation. Risk protection losses ranged as high as 2.96% for the producer ($1-\alpha$) and 3.38% for the consumer (β) with no continuity correction factor applied. These are the maximum (underlined) negative numbers in Table 4. With a continuity correction factor of 0.5 added to the solving value of $\underline{c}$, the producer received at least the required protection but the consumer loss of protection reached as high as 9.81% (nearly doubled). With 0.5 subtracted from the solving value of $\underline{c}$, consumer protection at the specified level was assured but loss of producer protection increased to 9.08%, i.e., from a design level of 0.05 to as high as 0.1408. Unless protection at one level is vital, as opposed to protection at the other level, no continuity correction is recommended when the arcsin transformation is to be used.

Results from the standard normal approximation were not as good, in general, as those achieved by applying the arcsin transformation. Without adjustment by a continuity correction factor, loss of producer protection ranged as high as 6.92% and loss of consumer protection as high as 4.29% (negative underlined values in Table 3). With 0.5 added to the solving value of $\underline{c}$, loss of producer protection was reduced to 1.2% but loss of consumer protection was increased markedly to 10.19%. With 0.5 subtracted from the solving value of $\underline{c}$, loss of producer protection increased to 13.64% but consumer protection at the design level was assured. As was the case with the normalized arcsin transformation, no continuity correction can be recommended unless it is imperative to meet (or nearly meet) the design level of protection at either the producer or the consumer quality protection level.

Both the standard normal and arcsin transformation require the evaluation of only two equations to obtain an ACC plan $(\underline{n}, \underline{c})$ pair. These equations easily may be evaluated using Standard Mathematical Tables or a scientific calculator, since they only require the use of square root, sine, and inverse sine functions. The binomial distribution requires at least $\underline{c}^2$ evaluations of equation 2, which contains factorial and exponential terms, and a search for the minimum $\underline{n}$ among various $(\underline{n}, \underline{c})$ pairs that satisfy the inequalities in equations 3 and 4. However, even most home computers have the capability of performing these evaluation and search tasks quickly; they would be tedious and time consuming if performed using tables and/or pocket calculators.

Finally, to assure the desired protection for both producer and consumer, the exact binomial should be used to obtain ACC plans, provided computer facilities are available. The normalized arcsin transformation is preferable to the standard normal approximation because its likely degree of under-protection is about half that of the standard normal. Possible under-protection afforded by the standard normal is about double that of the arcsin in absolute terms.

A complete listing of the computer programs used to develop and evaluate the various plans is provided in the Appendix.

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APPENDIX

```

10 REM*****
20 REM* PROGRAM: BINOMIAL ACCEPTANCE CONTROL PLAN (N,C) *
30 REM*****
40 DIM SUMLOG(1000)
50 NMAX = 1
60 SUMLOG(NMAX) = 0
70 INPUT "ENTER PRODUCER & CONSUMER'S RISKS ", A,B
80 INPUT "ENTER ACCEPTABLE & REJECTABLE PROCESS LEVELS ", P1,P2
90 GOSUB 130
100 PRINT "SUBGROUP SIZE N = ";N
110 PRINT "CONTROL LIMIT C = ";C
120 END
130 REM*****
140 REM* SUBROUTINE: SINGLE SAMPLING PLAN SEARCH *
150 REM* *
160 REM* REFERENCE: GUENTHER (1969) & HAILEY (1980) *
170 REM* *
180 REM* GIVEN: A = PRODUCER'S RISK *
190 REM* B = CONSUMER'S RISK *
200 REM* P1 = ACCEPTABLE QUALITY LEVEL (AQL) *
210 REM* P2 = REJECTABLE QUALITY LEVEL (RQL) *
220 REM* *
230 REM* FINDS: N = MINIMUM SUBGROUP SIZE *
240 REM* C = NONCONFORMANCE CONTROL LIMIT *
250 REM*****
260 N = 1
270 C = -1
280 C = C + 1
290 P = P2
300 N = N + 1
310 GOSUB 370 \ REM*** CALL BINOMIAL (P2, N, C, PACC)
320 IF PACC > B THEN 300
330 P = P1
340 GOSUB 370 \ REM*** CALL BINOMIAL (P1, N, C, PACC)
350 IF PACC < (1-A) THEN 280
360 RETURN
370 REM*****
380 REM* SUBROUTINE: CUMULATIVE BINOMIAL PROBABILITY *
390 REM* *
400 REM* GIVEN: N = SUBGROUP SIZE *
410 REM* C = NONCONFORMANCE CONTROL LIMIT *
420 REM* P = PROBABILITY OF NONCONFORMANCE *
430 REM* *
440 REM* FINDS: PACC = PROBABILITY OF ACCEPTANCE (CUM. BINOMIAL) *
450 REM*****
460 Q = 1-P
470 CUMULA = Q^N
480 IF C = 0 THEN 600
490 IF N <= NMAX THEN 540 \ REM*** LOG SUMS ALREADY IN MEMORY
500 FOR K = (NMAX+1) TO N \ REM*** COMPUTE ONLY NEW LOG SUMS
510 SUMLOG(K) = LOG10(K) + SUMLOG(K-1)
520 NEXT K
530 NMAX = N \ REM*** LARGEST LOG SUM (FACTORIAL) IN MEMORY
540 PLOG = LOG10(P)
550 QLOG = LOG10(Q)
560 FOR K = 1 TO C \ REM*** COMPUTE CUMULATIVE PROBABILITY
570 FACTOR = SUMLOG(N) - SUMLOG(N-K) - SUMLOG(K)
580 CUMULA = 10^(FACTOR + K*PLOG + (N-K)*QLOG) + CUMULA
590 NEXT K
600 PACC = CUMULA
610 RETURN

```

SINGLE,SINGLE/LI:1=SINGLE

```
CC-----
CC  PROGRAM:  COMPARES SINGLE SAMPLING PLANS FOR A PROCESS
CC              WHICH HAS BINOMIALLY DISTRIBUTED DEFECTIVES.
CC
CC  PROGRAMMER:  CARLOS AMADO                      FALL '81
CC
CC  RESEARCH FOR: DR. R.L. LEAVENWORTH, ISE DEPT, UNIV. OF FLORIDA
CC-----
0001  DIMENSION CLU(2), CLI(2)
0002  CALL ASSIGN (2,'SINGLE.DAT') ! OUTPUT FILE
0003  130  CALL PUTCHR (26,5) ! CLEAR CRT SCREEN
0004  WRITE (5,131)
0005  131  FORMAT ('ENTER DATA (START IN 1ST LETTER OF TITLE)')//
      I      ' P0      ALFA      P1      BETA      ITER8')
0006  READ (5,133,ERR=130) P0, ALFA, P1, BETA, ETER8
0007  133  FORMAT (5(F7.4,1X))
0008  IF (P0.LE.0) GO TO 199
0010  IF (P0.EQ.P1) GO TO 127
0012  IF (ETER8.NE.0) GO TO 126
0014  GO TO 129
C
0015  127  WRITE (5,*)
0016  WRITE (5,*) 'P0 CAN NOT EQUAL P1 (ZERO DIVIDE ERROR)'
0017  GO TO 194
C
0018  126  CALL ITER8 (ALFA, P0, BETA, P1, A, ETER8)
0019  IF (A.EQ.'Y') GO TO 196
0021  IF (A.EQ.'R') GO TO 194
0023  GO TO 130
C
0024  129  WRITE (2,128) P0, ALFA, P1, BETA
0025  128  FORMAT (////' GIVEN (P0, A, P1, B):',4(F7.4,2X))
0026  CALL ARCSIN (ALFA, P0, BETA, P1, SN, C, CLU, CLI)
0027  WRITE (5,*) SN, C, ' FROM ARCSINE'
0028  WRITE (2,*) ' '
0029  WRITE (2,*) 'ARCSINE TRANSFORMATION'
0030  M= 1
0031  GO TO 134
C
0032  132  CALL NORMBI (ALFA, P0, BETA, P1, SN, C, CLU, CLI)
0033  WRITE (2,*) ' '
0034  WRITE (2,*) 'NORMAL APPROXIMATION'
0035  M= 2
0036  GO TO 134
C
0037  136  CALL GUNTHR (ALFA, P0, BETA, P1, SN, C)
0038  WRITE (2,*) ' '
0039  WRITE (2,*) 'EXACT BINOMIAL'
0040  M= 3
C
0041  134  IF (C.EQ.999.) GO TO 199
0043  CALL EXACT (SN,P0,C,BXLECN)
0044  CALL EXACT (SN,P1,C,P,N)
C
```

SINGLE,SINGLE/LI:1=SINGLE

```
0045      WRITE (2,135) SN, BXLEC, C, P
0046  135  FORMAT (' SINGLE SAMPLING PLAN: '//
              I      ' SAMPLE SIZE  =',F9.2,5X,
              I      ' P > 1-A =',F9.4/
              I      ' MAX DEFECTS  =',F9.2,5X,
              I      ' P < B   =',F9.4)
0047      GO TO (132,136,130), M
0048  194  WRITE (5,*)
0049      WRITE (5,*)' INPUT ERROR!  TYPE "R" TO RESTART'
0050  196  READ (5,197) A
0051  197  FORMAT (A1)
0052      IF (A.EQ.'R'.OR.A.EQ.'N') GO TO 130
0054  199  WRITE (5,*)
0055      STOP '      HAVE A GOOD LIFE'
0056      END
```

SINGLE, SINGLE/LI:1=SINGLE

0001 SUBROUTINE GUNTHR (ALFA, P0, BETA, P1, SN, C)

CC

CC PROGRAM: OBTAINS EXACT BINOMIAL SINGLE SAMPLING PLAN

CC

CC REF: GUENTHER, W.C., "USE OF BINOMIAL, HYPERGEOMETRIC, AND
CC POISSON TABLES TO OBTAIN SAMPLING PLANS", JOURNAL OF
CC QUALITY TECH, VOL 1, NO. 2, APRIL 1969, PP 105-109.

CC

0002 1 C= -1. ! START AT C=0
0003 SNS= 1. ! START AT SNS=2

C

0004 2 C= C+ 1.

C

0005 3 SNS= SNS+ 1.
0006 CALL EXACT (SNS, P1, C, BXLEC, N)
0007 IF (BXLEC.GT.BETA) GO TO 3

C

0009 SNL= SNS- 1. ! SMALLER SNL NOT NEEDED
0010 4 SNL= SNL+ 1.

C

0011 WRITE (5,*) SNS, C, SNL, BXLEC
0012 CALL EXACT (SNL, P0, C, BXLEC, N)
0013 IF (BXLEC.GE.(1.-ALFA)) GO TO 4
0015 SNL= SNL- 1. ! PREVIOUS SNL IS THE ONE WANTED
0016 IF (SNS.GT.SNL) GO TO 2
0018 SN= SNS
0019 RETURN
0020 END

C

SINGLE, SINGLE/LI:1-SINGLE

```
0001      SUBROUTINE EXACT (SN, P, C, BXLEC, N)
CC-----
CC      PROGRAM:  COMPUTES EXACT CUMM. BINOMIAL PROBABILITY OF X.LE.C
CC
CC      METHOD:
CC          A. "N" NUMBER OF LOG(10) SUMS ARE COMPUTED ONCE, ONLY,
CC              AND STORED IN "SUMLOG(I)" VECTOR.
CC          B. 10*( SUMLOG(I) ) REPRESENTS I-FACTORIAL; THEREFORE,
CC              FACTORIAL MULTIPLICATIONS ARE REDUCED TO SUMMATIONS,
CC              ALL FACTORIALS NEEDED ARE IN STORAGE UP TO SUMLOG(N).
CC          C. WHENEVER ITERATIONS ARE RUN, THE SUMLOG(I)'S IN MEMOR
CC              ARE NOT RECOMPUTED.
CC          D.  $N > C > 0$ ;  $N-FAC > C-FAC$ ; AND  $SUMLOG(N) > SUMLOG(C)$ .
CC          E.  $SUMLOG(I)$  SUBTRACTION = I-FACTORIAL DIVISION.
CC          F. LOG(10) ALLOWS EFFICIENT HANDLING OF LARGE NUMBERS.
CC
CC      PROGRAMMER:  CARLOS H. AMADO, ISE DEPT., UNIV. OF FLORIDA
CC-----
CC      GIVEN:      SN      = SAMPLE NUMBER (SIZE)
CC                  P      = PROB. OF DEFECTIVES
CC                  C      = # OF DEFECTIVES IN SAMPLE
CC
CC      COMPUTES:  BXLEC = BINOMIAL PROB (X.LE.C)
CC                  SUMLOG(I) = VECTOR CONTAINING SUM OF LOG(1) THRU LOG
CC-----
0002      VIRTUAL SUMLOG(9000)
0003      R = 1.-P
0004      NN = SN
C
C      >>> BINOMIAL PROB. WHEN C=0 <<<
0005      CSUMS = R **NN
0006      IF (C.EQ.0.) GO TO 333
C
C      >>> AVOID RECOMPUTING SUMLOG(I)'S ALREADY IN MEMORY <<<
C
0008      IF (NN.GT.9000) GO TO 998
0010      IF (N-NN) 100,211,211
0011      100  M= N+ 1
C
C      >>> COMPUTE N SUMLOGS --EQUIVALENT TO N-FACTORIAL <<<
C
0012      IF (M.GT.1) GO TO 110
0014      SUMLOG(1) = 0.
0015      M = 2
0016      110  DO 111 I= M, NN
0017          SUMLOG(I) = ALOG10(FLOAT(I)) +SUMLOG(I-1)
0018      111  CONTINUE
C
C      >>> COMPUTE C CSUMS --EQUIVALENT TO SUM OF PROB COMBINATIONS
C          I.E., CUMMULATIVE BINOMIAL DISTRIBUTION COMPUTATION
C
0019      211  KC = C
0020      N= NN
C
```

SINGLE, SINGLE/LI:1=SINGLE

```

      C      >>> SUM (ACCUMULATE) PROBABILITIES <<<
      C
0021      PLOG= ALOG10(P)
0022      RLOG= ALOG10(Q)
0023      DO 322 K= 1, KC
0024      CSUMS = 10.**( SUMLOG(N) -SUMLOG(N-K) -SUMLOG(K)
      Z      +K*PLOG +(N-K)*RLOG ) +CSUMS
0025      322 CONTINUE
      C
0026      333 BXIEC= CSUMS
0027      RETURN
0028      998 WRITE (5,999) SN, P, C
0029      999 FORMAT ('OWHOOPS!!! SAMPLE SIZE =',F10.2//
      Z      ' AT P =',F10.4,' AND C =', F10.4)
0030      STOP
0031      END

```

C-----

SINGLE, SINGLE/LI:1=SINGLE

```
0001      SUBROUTINE NORMBI (ALFA, P0, BETA, P1, SN, C, CLU, CLI)
CC-----
CC      PROGRAM:  COMPUTES A SINGLE SAMPLING PLAN USING THE
CC                NORMAL APPROXIMATION FOR A BINOMIAL DISTRIBUTION
CC-----
0002      DIMENSION CLU(2), CLI(2), ZV(12), ZT(12), Z(2)
0003      DATA ZV(1)/2.326/, ZV(2)/2.054/, ZV(3)/1.881/, ZV(4)/1.751/,
I      ZV(5)/1.645/, ZV(6)/1.555/, ZV(7)/1.474/, ZV(8)/1.405/,
I      ZV(9)/1.34/, ZV(10)/1.282/, ZV(11)/1.036/, ZV(12)/.842/
0004      DO 29 I= 1, 10
0005      29      ZT(I)= 1.- I/100.
0006      ZT(11)= .85
0007      ZT(12)= .80
0008      PZ= 1.-ALFA
0009      K= 1
0010      30      DO 31 I= 1, 12
0011      IF (PZ.NE.ZT(I)) GO TO 31
0013      Z(K)= ZV(I)
0014      GO TO (32,33), K
0015      31      CONTINUE
0016      WRITE (5,*) 'ERROR: ALFA OR BETA NOT IN DATA STATEMENT'
0017      C= 999.
0018      RETURN
C
0019      32      PZ= 1.-BETA
0020      K= 2
0021      GO TO 30
C
0022      33      PR0= (1- P0)* P0
0023      PR1= (1- P1)* P1
0024      SN= ((Z(1)* SQRT(PR0) + Z(2)* SQRT(PR1))/ (P1- P0))**2
0025      C= Z(1)* SQRT(SN*PR0)+ SN* P0
C
0026      RETURN
0027      END
C-----
```

SINGLE,SINGLE/LI:1=SINGLE

0001 SUBROUTINE ARCSIN (ALFA, P0, BETA, P1, SN, C)

```

CC-----
CC  PROGRAM:  SINGLE SAMPLING PLAN USING AN ARCSINE NORMALIZING
CC             TRANSFORMATION* TO APPROXIMATE A BINOMIAL PROCESS.
CC
CC             1.  NORMAL                      ARCSINE TRANSFORMATION
CC             2.  FREEMAN & TUCKEY (F & T)
CC
CC  REF:  JOHNSON, N. S. KOTZ, 'DIST. IN STATISTICS -DISCRETE DIST.'
CC             HOUGHTON MIFFLIN CO, BOSTON, 1969, P 65.
CC
CC  GIVEN:  ALFA  = PRODUCER'S RISK PROBABILITY OF REJECTION
CC           BETA  = CONSUMER'S " " " " ACCEPTANCE
CC           P0    = ACCEPTABLE PROCESS LEVEL
CC           P1    = REJECTABLE " "
CC
CC  COMPUTES: SN  = SAMPLE SIZE
CC              C  = MAX NUMBER OF DEFECTIVES IN ACCEPTANCE
CC
CC  VARIABLES:
CC              Z(1)  = Z(1-ALFA)
CC              Z(2)  = Z(1-BETA)
CC              SINV(1) = ARCSINE OF SQRT(1-ALFA)
CC              SINV(2) = " " " (1-BETA)
CC              SS(1)  = SAMPLE SIZE OF NORMAL ARCSINE TRANSFORMATION
CC              SS(2)  = " " " F & T "
CC              D(1)   = ACCEPTABLE # DEFECTIVES IN NORMAL "
CC              D(2)   = ACCEPTABLE # DEFECTIVES IN F & T "
CC-----

```

```

0002  DIMENSION Z(2), SINV(2), SS(2), D(2), ZT(12), ZV(12), P(2)
0003  DATA ZV(1)/2.326/, ZV(2)/2.054/, ZV(3)/1.881/, ZV(4)/1.751/,
I      ZV(5)/1.645/, ZV(6)/1.555/, ZV(7)/1.474/, ZV(8)/1.405/,
I      ZV(9)/1.34/, ZV(10)/1.282/, ZV(11)/1.036/, ZV(12)/.842/
0004  DO 29 I= 1, 10
0005      29  ZT(I)= 1.- I/100.
0006      ZT(11)= .85
0007      ZT(12)= .80
0008      P(1)= P0
0009      P(2)= P1
C
0010      PZ= 1.- ALFA
0011      K= 1
0012      30  DO 31 I= 1, 12
0013          IF (PZ.NE.ZT(I)) GO TO 31
0015          Z(K)= ZV(I)
0016          SQRPF= SQRT(P(K))
0017          SINV(K)= ATAN( SQRPF/ SQRT(-SQRPF*SQRPF+1) ) ! ARCSIN
0018          GO TO (32,33), K
0019      31  CONTINUE
0020      WRITE (5,*) 'ERROR: ALFA OR BETA NOT IN DATA STATEMENT'
0021      C= 999.
0022      RETURN
C
0023      32  PZ= 1.-BETA

```

CONTINUED
SINGLE, SINGLE/LI:1-SINGLE

```
0024      K= 2
0025      GO TO 30

C
0026 33    SS(2)= ( (Z(1)+Z(2))/ (SINV(2)-SINV(1)) )**2
0027      SS(1)= SS(2) /4
0028      X= SINV(1)+ Z(1)/ (2*SQRT(SS(1)))
0029      D(1)= (SS(1)+0.75) * (SIN(X)**2) -(3./8.)

C
0030      SN= SS(1)
0031      C= D(1)
0032      RETURN
0033      END
```

SINGLE,SINGLE/LI:1=SINGLE

```
0001      SUBROUTINE ITER8 (ALFA, P0, BETA, P1, A, ETER8)
CC-----
CC      PROGRAM: DETERMINES SEVERAL SINGLE SAMPLING PLANS
CC              BY CHANGING PARAMETER VALUES IN EACH ITERATION.
CC-----
0002      DIMENSION SS(3), CC(3), R0(3), R1(3)
0003      COMMON /ETER/ AL,AS,AH,BL,BS,BH,POL,POS,POH,P1L,P1S,P1H
0004      WRITE (5,101)
0005      101  FORMAT ('ENTER ITERATION (STEP) AND HIGHEST VALUES'//
Z          ' (ALFA) ALFA (P0) P0 (P1) P1 (BETA) BETA'
0006      READ (5,103) AS, AH, POS, POH, P1S, P1H, BS, BH
0007      103  FORMAT (BF7.4)
C
0008      IF (AS.LT.0.OR.AH.LT.ALFA) GO TO 194
0010      IF (BS.GT.0) GO TO 104
0012      BS= AS
0013      BH= AH
C
0014      104  IF (POS.LT.0.OR.POH.LT.P0.OR.P1S.LT.0.OR.P1H.LT.P1.OR.
Z          P1S.LT.POS.OR.BH.LT.BETA) GO TO 194
0016      AL= ALFA
0017      BL= BETA
0018      POL= P0
0019      P1L= P1
0020      IF (ETER8.EQ.8.) GO TO 114
0022      GO TO 110 ! BEGIN ITERATIONS
C
0023      106  P1= P1+ P1S
0024      IF (P1.LE.P1H) GO TO 110
0026      P1= P1L
0027      P0= P0+ POS
0028      IF (P0.LE.POH) GO TO 110
0030      P0= POL
0031      ALFA= ALFA+ AS
0032      IF (ALFA.LE.AH) GO TO 110
0034      ALFA= AL
0035      BETA= BETA+ BS
0036      IF (BETA.LE.BH) GO TO 110
0038      WRITE (5,*)
0039      WRITE (5,*)' END OF ITERATIONS! END SESSION? (Y/N)'
0040      A= 'Y'
0041      RETURN
0042      194  A='R'
0043      RETURN
C
0044      110  CALL ARCSIN (ALFA, P0, BETA, P1, SS(1), CC(1))
0045      CALL NORMBI (ALFA, P0, BETA, P1, SS(2), CC(2))
0046      CALL GUNTHR (ALFA, P0, BETA, P1, SS(3), CC(3))
0047      DO 112 I= 1, 3
0048          CALL EXACT (SS(I), P0, CC(I), R0(I), N)
0049      112  CALL EXACT (SS(I), P1, CC(I), R1(I), N)
0050      A1= 1.-ALFA
0051      WRITE (2,113) A1, BETA, P0, P1, SS(3), CC(3), R0(3), R1(3),
Z          SS(1), CC(1), R0(1), R1(1), SS(2), CC(2), R0(2), R1(2)
```

SINGLE,SINGLE/LI:1=SINGLE

```
0052 113 FORMAT ('O GIVEN (1-A, B, P0, P1):',4(F7.4,',')//  
      Z      ' SINGLE SAMPLING PLAN:      N',7X,'C      P>1-A    P<B'//  
      Z      ' EXACT BINOMIAL',8X,2F8.2,2F8.4/  
      Z      ' ARCSINE TRANSFORMATION',2F8.2,2F8.4/  
      Z      ' NORMAL APPROXIMATION  ',2F8.2,2F8.4)  
0053      GO TO 106  
0054 114 CALL ITER88 (ALFA, P0, BETA, P1, A)  
0055      RETURN  
0056      END
```

C-----

SINGLE, SINGLE/LI:1=SINGLE

```

0001      SUBROUTINE ITER88 (ALFA, P0, BETA, P1, A)
C=====
C      ROUTINE: COMPARES DESCRIMINATION RATIOS (P1/P0)
C=====
0002      DIMENSION SS(9), CC(9), R0(9), R1(9), PA(3), PB(3)
0003      COMMON /ETER/ AL, AS, AH, BL, BS, BH, POL, POS, POH, P1L, P1S, P1H
0004      112  WRITE (5,*)
0005      WRITE (5,*) 'ENTER D-STEP FOR P1'
0006      READ (5,*) DS
0007      IF (DS.LT.0) GO TO 112
C
0009      114  A1= 1.-ALFA
0010      WRITE (2,115) A1,ALFA,BETA
0011      WRITE (1,115) A1,ALFA,BETA
0012      115  FORMAT ('O GIVEN (1-A, A, B):',3(F7.4,', '))
0013      PA(1)= POL
0014      PB(1)= P1L
0015      PH= P1H+ 0.001
0016      SP1= P1S
C
0017      116  D= SP1/POS
0018      DO 111 M= 1, 3
0019          DD= PB(M)/PA(M)
0020          IF (DD+0.01.LE.D.OR.DD-0.01.GE.D) GO TO 128
0022          IF (PB(M).GT.PH) GO TO 126
C
0024          CALL GUNTHR (ALFA, PA(M), BETA, PB(M), SS(M), CC(M))
0025          CALL ARCSIN (ALFA, PA(M), BETA, PB(M), SS(M+3), CC(M+3))
0026          CALL NORMBI (ALFA, PA(M), BETA, PB(M), SS(M+6), CC(M+6))
C
0027          IF (M.GE.3) GO TO 111
0029          PA(M+1)= PA(M)+ POS
0030          PB(M+1)= PB(M)+ SP1
0031      111  CONTINUE
C
0032      DO 122 M= 1, 9
0033          I= SS(M)+ 0.5
0034      122  SS(M)= I
0035      117  DO 121 M= 1, 3
0036          DO 119 I= M, 9, 3
0037              CALL EXACT (SS(I),PA(M),CC(I),R0(I),N)
0038      119  CALL EXACT (SS(I),PB(M),CC(I),R1(I),N)
0039      121  CONTINUE
0040      WRITE (2,123) (PA(I),I=1,3),D,PB(1),SS(1),CC(1),PB(2),SS(2),CC(
H          PB(3),SS(3),CC(3),D,PB(1),SS(4),CC(
I          PB(2),SS(5),CC(5),PB(3),SS(6),CC(6)
T          D,PB(1),SS(7),CC(7),PB(2),SS(8),CC(
S          PB(3),SS(9),CC(9)
0041      123  FORMAT ('OMETHOD  D\N P1=',3(F8.4,'      N',7X,'C  ')/
B          ' BINOMIAL',F6.1,3(F8.4,F8.1,F8.2)/
A          ' ARCSINE ',F6.1,3(F8.4,F8.1,F8.2)/
R          ' NORMAL ',F6.1,3(F8.4,F8.1,F8.2))
0042      WRITE (1,125) (PA(I),I=1,3),D,PB(1),R0(1),R1(1),PB(2),R0(2),R1(
P          PB(3),R0(3),R1(3),D,PB(1),R0(4),R1(

```

SINGLE,SINGLE/LI:1=SINGLE

```

      E                                PB(2),B0(5),B1(5),PB(3),B0(6),B1(6)
      T                                D,PB(1),B0(7),B1(7),PB(2),B0(8),B1
      S                                PB(3),B0(9),B1(9)
0043  125  FORMAT ('OMETHOD  D\PI=',3(F8.4' P>1-A  P<B  ')/
      M      ' BINOMIAL',F7.1,9F8.4/' ARCSINF ',F7.1,9F8.4/
      E      ' NORMAL ',F7.1,9F8.4)
0044      PB(1)= PB(1)+ DS
0045      SP1= (D+ 0.5) *PA(2) -PB(1)
0046      GO TO 116
C
0047  126  BETA= BETA+ BS
0048      IF (BETA.LE.BH) GO TO 114
0050      BETA= BL
0051      ALFA= ALFA+ AS
0052      IF (ALFA.LE.AS) GO TO 114
0054      WRITE (5,*)
0055      WRITE (5,*)'END OF ITERATIONS!  END? (Y/N)'
0056      A='Y'
0057      RETURN
C
0058  128  WRITE (5,*)
0059      WRITE (5,*)'DESCRIMINATION RATIO ERROR; D=',D,'  DD=',DD
0060      A= 'R'
0061      RETURN
0062      END

```

SINGLE,SINGLE/LI:1=SINGLE

```
0001      SUBROUTINE NORMAL (X, U, SD, ZTAB, TABZ)
CC-----
CC      PROGRAM:  NORMAL PROBABILITY DISTRIBUTION APPROXIMATION
CC
CC              CUM. NORMAL= 1 - F(X) * (B1 * T + B2 * T**2 + B3 * T*
CC                                     + B4 * T**4 + B5 * T*
CC              MIN. ACCURACY: +- 0.000 000 075
CC
CC      REF:  ABRAMOWITZ, M., I. STEGUN, EDS. "HANDBOOK OF MATH.
CC            FUNCTS. WITH FORMULAS, GRAPHS, AND MATH TABLES", APPL
CC            MATH SERIES #55, WASH. DC, NAT'L BUREAU OF STDS., 196
CC
CC      GIVEN:   X      = SAMPLE STATISTIC
CC              U      = DISTRIBUTION MEAN
CC              SD     = STANDARD DEVIATION
CC
CC      COMPUTES: ZTAB  = NORMAL CUMMULATIVE PROBABILITY
CC              TABZ   = (1 - ZTAB)
CC-----
0002      DATA B1/0.31938153/, B2/-0.356563782/, B3/1.781477937/, DUMY/0.
Z      B4/-1.821255978/, B5/1.330274429/, CONST/0.39894228/
0003      Z= (X - U)/SD
0004      IF (Z.GE.0.) GO TO 21
0006      Z = -Z
0007      DUMY = 1.
0008      21  T = 1./(1.+Z *.2316419)
C      C = 1/SQRT(2* 3.141526536); SEE DATA
0009      F = C* EXP( -(Z**2)/2.)
0010      TABZ = F* (B1* T +B2* T**2 +B3* T**3 +B4* T**4 +B5* T**5)
0011      IF (DUMY.EQ.0.) GO TO 23
0013      TABZ = 1.-TABZ
0014      23  ZTAB = 1.-TABZ
0015      RETURN
0016      END
```

SINGLE,SINGLE/LI:1=SINGLE

0001 SUBROUTINE REVNDR (X, U, SD, ZTAB, TABZ)

CC-----
CC PROGRAM: REVERSE NORMAL PROBABILITY DISTRIBUTION
CC
CC $X = T - (C0 + C1 * T + C2 * T ** 2) / (1 + D1 * T + D2 * T ** 2 + D3 * T ** 3)$
CC
CC MIN. ACCURACY: ± 0.00045
CC
CC REF: HAISTINGS, CECIL JR, 'APPROXS FOR DIGITAL COMPS', PRINCETON
CC NJ, PRINCETON UNIV. PRESS, 1955.
CC
CC GIVEN: U = DISTRIBUTION MEAN
CC SD = STANDARD DEVIATION
CC ZTAB = NORMAL CUMULATIVE PROBABILITY
CC TABZ = (1 - ZTAB)
CC
CC COMPUTES: X = POINT STATISTIC
CC-----

0002 DATA C0/2.515517/, C1/0.802853/, C2/0.010328/, DUMY/1./,
I D1/1.432788/, D2/0.189269/, D3/0.001308/
0003 F= ZTAB
0004 TABZ= 1- ZTAB
0005 IF (ZTAB.GE.1) GO TO 35
0007 IF (ZTAB.LT.0.5) GO TO 31
0009 DUMY= 0.
0010 F= 1.- ZTAB
0011 31 TT= ALOG (1./F**2)
0012 T= SQRT (TT)
0013 XN= C0+ C1* T+ C2* T**2
0014 XD= 1.+ D1* T+ D2* T**2 +D3* T**3
0015 X= T- XN/ XD
0016 IF (DUMY.EQ.0.) GO TO 33
0018 X= -X
0019 33 X= X* SD+ U
0020 GO TO 37
0021 35 X= 999.999
0022 37 RETURN
0023 END

C-----

END

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