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ESTIMATING SURVIVAL PROBABILITY OR RELIABILITY: SIMULATION ASSESSMENTS OF THE DELTA METHOD, JACKKNIFE, AND BOOTSTRAP

by

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October 1982

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Estimating Survival Probability or Reliability: Simulation Assessments of the Delta Method, Jackknife, and Bootstrap

by

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I. INTRODUCTION

A. OVERVIEW

A common problem in various areas of operations research and applied statistics, e.g. in reliability and maintainability studies, is that of predicting from available data the probability that a future observation exceeds a given value. An example arising in nuclear plant reliability is that a crucial repair or down time exceeds h (= 4.) hours. Another problem is to predict the "100-year flood", or earthquake, etc. The latter problem is difficult because there will usually be far less than 100-years worth of data to work with. Still another problem is that of predicting the probability of survival for h = 5 years for a cancer victim receiving a particular treatment. 1473

The simplest formulation is to assume that the data is a random sample from a probability distribution $F_x(x)$ (continuous, i.e. having a density), with density $f_x(x)$. That is, observed values are $x_1, x_2, \ldots, x_n$, being independent realizations of independent identically distributed random variables generically denoted by X. If one is willing to assume also that the mathematical form of $F_x(x) = F_x(x;\theta)$ is known (e.g. is Log-normal, or Gamma, or another candidate) then what one can do is:

(a) Estimate the possibly multidimensional parameter θ (e.g. θ could be μ, σ^2, the population mean and variance for a log-normal model, estimated by $\overline{\ln x} = \hat{\mu}$ and

$$S_{\ln x}{}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\ln x_i - \overline{\ln x})^2 \text{ classically}).$$

(b) Quote the point estimate $F_x(x;\theta)$, or, in the present case $1 - F_x(h;\hat{\theta})$ for probability of survival beyond h.

(c) Utilize facts about the sampling distribution of $\hat{\theta}$ to find a standard error or confidence limits on $\overline{F}_x(h;\theta)$, the survival probability.

The basic assumption, then, is that data can reasonably be assumed to be a random sample from a fixed distribution, the form of which is known. There are various ways in which such convenient assumptions can be violated, one obvious one being that the fixed distribution idea is not justifiable (perhaps because of important detectable variation in the distribution from location to location, or plant to plant, from repair crew to repair crew, etc.). Another might be that some data points are missing: too-short ones (down times) are not written down or else are recorded incorrectly, and too-long ones are regarded as being so exceptional as never to recur, and hence are removed. Possible or likely departures from the basic assumption should be investigated. The raw data should be carefully examined in an exploratory spirit (see J. W. Tukey [Ref. 1]), e.g. by graphics to check for departures from the basic "stationary" assumption. In this discussion we rule out such variations.

This paper gives an account and some evaluation of several different ways of accomplishing step (c) above (confidence limits for the probability of survival or excedance of time h and related topics). It will discuss four different methods for attacking the estimation and confidence limits problem.

1. Mathematical Formulation

We shall assume that $(x_1, x_2, \ldots, x_n)$ are the complete times of repair (or down times), and that they are independent realizations of the generic random variable X, where $Y = \ln X$ is normally distributed with mean μ and variance σ^2, both unknown. This kind of assumption is often made in practice. This implies that the probability that a randomly selected, future, down or repair time exceeds h is given by the formula

$$\bar{F}_X(x;\mu,\sigma^2) = \int_{\frac{\ln h-\mu}{\sqrt{\sigma^2}}}^{\infty} e^{-\frac{1}{2}z^2}\frac{dz}{\sqrt{2\pi}} \equiv \int_{\underline{q}}^{\infty} e^{-\frac{1}{2}z^2}\frac{dz}{\sqrt{2\pi}} \qquad (1.1)$$

In practice, this formula is not immediately applicable when μ and σ^2 are unknown but if we estimate

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\ln x_i = \overline{\ln x} \qquad (1.2)$$

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\ln x_i - \overline{\ln x})^2 , \tag{1.3}$$

we can go ahead and quote a point estimate; the latter depends on

$$\frac{\ln h - \hat{\mu}}{\sqrt{\hat{\sigma}^2}} . \tag{1.4}$$

If we examine this quantity (integration limit) it will be seen to be a single realization of a random variable written as

$$\theta = \frac{H - \overline{Y}}{\sqrt{S_y^{\,2}}} \tag{1.5}$$

where $\overline{Y}$ is $N(\mu,\sigma^2)$ and $S_y^{\,2}$ is $\sigma^2\chi^2_{(n-1)}$ proportional to a Chi-squared r.v., the latter being independent of $\overline{Y}$ by the convenient (log) normal assumption. Now re-write θ as

$$\theta = \frac{(H-\mu) - (\overline{Y}-\mu)}{\sqrt{n} \cdot \sqrt{\frac{S_y^{\,2}}{n}}} \tag{1.6}$$

If $(H-\mu) = 0$ then $(-\theta\sqrt{n})$ would be precisely distributed as a Student's t. On the other hand

$$-\theta\sqrt{n} = \frac{(\overline{Y}-\mu+\mu-H)\sqrt{n}}{S_y} \tag{1.7}$$

If we write $\delta = \mu-H$ then

$$-\theta\sqrt{n} = \frac{(\overline{Y}-\mu+\delta)\sqrt{n}}{S_y} \tag{1.8}$$

has a known density function, that of the Non-central t which is conveniently expressed in terms of the non-centrality parameter

$$\gamma = \frac{\sqrt{n}\,\delta}{\sigma}$$

Classical methods exist for utilizing this to establish tolerance limits. In this paper a different approach is followed. We examine the performance of several convenient approximate methods for assessing the uncertainty in the simple point estimate (1.1), where estimate (1.2) and (1.3) are used for the parameter values. These methods are the Delta method (linearization), the Jackknife, and the Bootstrap, as well as a completely distribution-free (Bernoulli trials) method. Details now follow.

B. PURPOSE AND APPROACH

1. Distribution-Free Approach

In general, suppose we want to solve the problem of estimating the survival probability without any distributional assumption, other than that observations are iid. The simplest way is to use the binomial approach. If $(x_1, x_2, \ldots, x_n)$ indicates the iid. sample of down or repair times, we can estimate $P(x>h)$, survival probability, by means of

$$\hat{\underline{P}}[X>h] = \frac{\#(X\text{'s} > h)}{n} = \hat{p} \qquad (1.9)$$

Then we can set up a confidence interval for the survival probability (1.9) by making use of the fact that for rge n the binomial distribution can be approximated by a rmal distribution. An approximate $(1-\alpha)\cdot 100\%$ confidence terval for the binomial parameter p is given by

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad (1.10)$$

ere $z_{\alpha/2}$ $(1-\alpha/2)\cdot 100\%$ point of the tabled unit normal.

2. Maximum Likelihood Approach

We can assume along with others, that repair time ta comes as a random sample from a log-normal population: = lnX where X is Normal(μ,σ^2). This assumption will be ucial in all three methods. Then the maximum likelihood timates (M.L.E.) of the parameter are as stated before:

$$\hat{\mu} = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad (1.11)$$

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 = s_y^{\,2} \qquad (1.12)$$

Strictly speaking, $\hat{\sigma}^2$ (1.12) is the M.L.E. multiplied (n/n-1) and is unbiased for σ^2. Furthermore, in repeated mples of size n we have that "exactly" (assuming the model correct):

(i) $\hat{\mu} = \bar{Y}$ is Normal$(\mu,\frac{\sigma^2}{n})$ (1.13)

(ii) $\hat{\sigma}^2 = s_y^{\,2}$ is $\frac{\sigma^2\chi^2(n-1)}{n-1}$, (1.14)

where $E[\hat{\sigma}^2] = \sigma^2$, and $\mathrm{Var}[\hat{\sigma}^2] = 2\sigma^4/(n-1)$

(iii) $\hat{\mu}$ and $\hat{\sigma}^2$ are statistically independent.

Thus for large n both $\hat{\mu}$ and $\hat{\sigma}^2$ tend to be close to their respective population values, guaranteeing a good approximation to the survival probability if model (1) is correct. Now according to the assumed model, the probability of exceeding h hours is

$$P(x>h) = \int_{\frac{\ln h-\mu}{\sigma}}^{\infty} e^{-\frac{1}{2}z^2} \frac{dz}{\sqrt{2\pi}} \tag{1.15}$$

The maximum likelihood estimate of this probability is obtained by replacing μ by $\hat{\mu}$, σ^2 by $\hat{\sigma}^2$.

$$\hat{P}(x>h) = \int_{\frac{\ln h-\hat{\mu}}{\hat{\sigma}^2}}^{\infty} e^{-\frac{1}{2}z^2} \frac{dz}{\sqrt{2\pi}} \tag{1.16}$$

Now find upper and lower limits for the parameter:

$$q = \frac{\ln h - \mu}{\sigma} \tag{1.17}$$

i.e. $\bar{q}$ and $\underline{q}$ are functions of the observations such that $\underline{q} \leq q \leq \bar{q}$ with prescribed probability $(1-\alpha)\cdot 100\%$, say 95%. These then translate into upper and lower limits on the probability of exceeding h

$$\int_{\bar{q}}^{\infty} e^{-\frac{1}{2}z^2} \frac{dz}{\sqrt{2\pi}} \leq P(x>h) \leq \int_{\underline{q}}^{\infty} e^{-\frac{1}{2}z^2} \frac{dz}{\sqrt{2\pi}} \tag{1.18}$$

If we compute $\underset{\sim}{q}$ and $\overline{q}$ from a sample, then, under the initial assumptions, we have the desired confidence limits for the probability of exceeding h.

3. Delta Method (DL)

The delta method is an approximate way of finding the distribution of q. It is known that functions such as q are approximately normally distributed for "sufficiently large" n (see Cramer [Ref. 2]). We estimate q by

$$\hat{q} = \frac{\ln h - \hat{\mu}}{\hat{\sigma}} \tag{1.19}$$

and use the "delta method", or method of linearization, or small errors, to estimate the variance:

$$\mathrm{Var}[\hat{q}] \cong \left(\frac{\partial q}{\partial \hat{\mu}}\right)^2 \mathrm{Var}[\hat{\mu}] + \left(\frac{\partial q}{\partial \hat{\sigma}^2}\right)^2 \mathrm{Var}[\hat{\sigma}^2] \tag{1.20}$$

There is no covariance term because of the (theoretical) independence of $\hat{\mu}$ and $\hat{\sigma}^2$, see (iii) above in section 2. This formula yields

$$\frac{\partial q}{\partial \hat{\mu}} = -\frac{1}{\hat{\sigma}},\ \mathrm{Var}[\hat{\mu}] = \frac{\sigma^2}{n},\ \mathrm{Var}[\hat{\sigma}^2] = \frac{2\sigma^4}{n-1},\ \frac{\partial q}{\partial \hat{\sigma}^2} = \frac{-(\ln h - \hat{\mu})}{2\cdot(\hat{\sigma}^2)^{3/2}}, \tag{1.21}$$

so

$$\mathrm{Var}[\hat{q}] \cong \frac{1}{\hat{\sigma}^2}\cdot\frac{\sigma^2}{n} + \frac{1}{4}\cdot\frac{(\ln h - \hat{\mu})^2}{(\sigma^2)^3}\cdot\frac{2\sigma^4}{(n-1)} \tag{1.22}$$

$$\mathrm{Var}[\hat{q}] \cong \frac{1}{n} + \frac{1}{2}\cdot\frac{(\ln h-\hat{\mu})^2}{\hat{\sigma}^2\cdot(n-1)} \cong \frac{1}{n}\left\{1 + \frac{1}{2}\cdot\frac{(\ln h-\hat{\mu})^2}{\hat{\sigma}^2}\right\} \equiv \hat{\sigma}_q{}^2 \tag{1.23}$$

Assume $\hat{q}$ can be taken to be normal with mean q and variance $\hat{\sigma}_q^{\,2}$ and quote these approximate confidence limits:

$$\bar{q}_{DL} = \hat{q} + z_{1-\alpha/2} \cdot \sqrt{\hat{\sigma}_q^{\,2}} \tag{1.24}$$

$$\underline{q}_{DL} = \hat{q} - z_{1-\alpha/2} \cdot \sqrt{\hat{\sigma}_q^{\,2}} \tag{1.25}$$

This translates into the desired (but approximate) confidence limits for the probability of exceeding h:

$$\int_{\bar{q}_{DL}}^{\infty} e^{-\frac{1}{2}z^2} \frac{dz}{\sqrt{2\pi}} \leq P(x>h) \leq \int_{\underline{q}_{DL}}^{\infty} e^{-\frac{1}{2}z^2} \frac{dz}{\sqrt{2\pi}} \tag{1.26}$$

Several approximations have been made in the process described and the validity, for moderate n, of such a relatively simple process, must be checked. Notice that the exact distribution of q is non-central t under the basic model assumption. This approach replaces the n-c.t by a convenient normal approximation.

4. Jackknife Method (JK)

The jackknife is an alternative way of putting confidence limits on the parameter

$$q = \frac{\ln h - \mu}{\sigma}$$

For further discussion see Mosteller and Tukey [Ref. 3] and Efron [Ref. 4]. In brief, the jackknife method has the capacity to reduce the bias of estimates of such quantities and, more importantly, to furnish confidence limits that behave in a satisfactory manner.

Jackknife estimates and confidence limits are constructed by successively leaving out parts of the available data to construct pseudovalues. These are then averaged, and the stability of the average assessed by use of Student's t or the Normal in order to obtain confidence limits. The procedure is given below for our case:

(1) Form the estimate

$$q_n(y_1, y_2, y_3, \ldots, y_n) = \frac{\ln h - \bar{y}}{s_y} . \tag{1.27}$$

This is the m.l.e. using all the data, just as before.

(2) Form the estimates $q_{(n-1),i}(y_1, y_2, \ldots, y_{i-1}, y_i, \ldots, y_n)$ $i = 1, 2, \ldots, n$; these are similar to q_n, but omit successively each single observation $y_1, y_2, \ldots, y_n$; at the next stage each observation is then restored and the following taken out, as i runs from 1 to n and thus there are n values $q_{(n-1),i}$.

(3) Compute the pseudovalues as follows:

$$u_i = nq_n - (n-1)q_{(n-1),i} \quad i = 1, 2, \ldots, n \tag{1.28}$$

(4) Compute the mean and variance of the pseudovalues:

$$\bar{u} = \frac{1}{n} \sum_{i=1}^{n} u_i \tag{1.29}$$

$$s_u{}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (u_i - \bar{u})^2 \tag{1.30}$$

(5) Approximate (accuracy increasing with n increasing) $(1-\alpha)\cdot 100\%$ confidence limits for q are given by:

$$\underline{q}_{JK} \equiv \bar{u} - \frac{S_u}{\sqrt{n}} t_{\alpha/2}(n-1) \le q \le \bar{u} + \frac{S_u}{\sqrt{n}} t_{\alpha/2}(n-1) \equiv \bar{q}_{JK} \qquad (1.31)$$

where $t_{\alpha/2}(n-1)$ is the $(1-\alpha/2)\cdot 100$ percent point of Student's t (the standard, central, distribution). Also we can use $z_{\alpha/2}$ as before as an option.

(6) This means that, with approximate $(1-\alpha)\cdot 100\%$ confidence, the probability of survival is between the two confidence limits that follow:

$$\int_{\bar{q}_{JK}}^{\infty} e^{-\frac{1}{2}z^2} \frac{dz}{\sqrt{2\pi}} \le P(x>h) \le \int_{\underline{q}_{JK}}^{\infty} e^{-\frac{1}{2}z^2} \frac{dz}{\sqrt{2\pi}} \qquad (1.32)$$

This procedure, based on the m.l.e., has been theoretically validated for large n. It competes with the delta method, but is somewhat more difficult to carry out.

5. <u>Bootstrap Method (BT)</u>

The bootstrap method (see Efron (1979) [Ref. 4]) is similar to the jackknife method, but differs in being a resampling procedure. The procedure is given as below for our case:

(1) Calculate

$$\hat{q} = q_n(y_1, y_2, \ldots, y_n) = \frac{\ln h - \bar{y}}{S_y} .$$

This is the m.l.e. using original data, same as before.

(2) Draw a "Bootstrap sample", using $y_1, y_2, \ldots, y_n$ as basic distribution, value each having probability 1/n

$$y_1^*, y_2^*, y_3^*, \ldots, y_n^*$$

and calculate $u = q_n(y_1^*, y_2^*, \ldots, y_n^*) = \dfrac{\ln h - \bar{y}^*}{S_y{}^*}$

(3) Independently repeat step (2) a large number of times, B, obtaining "bootstrap replications" u_i, $i = 1,2,\ldots,B$, and calculate

$$\bar{u} = \frac{1}{B} \sum_{i=1}^{B} u_i \tag{1.35}$$

$$S_U{}^2 = \frac{1}{B-1} \sum_{i=1}^{B} (u_i - \bar{u})^2 \tag{1.36}$$

(4) Approximate $(1-\alpha)\cdot 100\%$ confidence limits for q. Here are four different approaches:

(a) Non-Parametric Approach (BT1): Take the order statistics of bootstrap sample

$$u_{(1)} < u_{(2)} < \ldots < u_{(B)}$$

then let $j = \left[\frac{\alpha}{2}\cdot B\right]$, and take as confidence limits for q:

$$\underline{q}_{BT} \equiv u_{(J)} \leq q \leq u_{(B-J+1)} \equiv \bar{q}_{BT} \tag{1.37}$$

(b) Normal Approximation Approach (BT2): If we assume the bootstrap sample is approximately normally distributed then approximate confidence limits for q can be set down:

$$\underline{q}_{BT} \equiv \bar{u} - z_{1-\alpha/2}\cdot S_u \leq q \leq \bar{u} + z_{1-\alpha/2}\cdot S_u \equiv \bar{q}_{BT} \tag{1.38}$$

(c) Bias-Adjusted Non-Parametric Approach (BT3): The bootstrap estimate of bias is

$$\widehat{BIAS} = \bar{u} - \hat{q}$$

where $\hat{q}$ is the estimate of q from the original data. Confidence limits for this case:

$$\underline{q}_{BT} \equiv u_{(J)} - \widehat{BIAS} \leq q \leq u_{(B-J+1)} - \widehat{BIAS} \equiv \bar{q}_{BT} \qquad (1.39)$$

(d) Bias-Adjusted Normal Approximation Approach (BT4): In this approach the confidence limits are:

$$\underline{q}_{BT} \equiv (\hat{q}-\widehat{BIAS}) - z_{1-\alpha/2} \cdot S_u \leq q \leq (\hat{q}-\widehat{BIAS}) + z_{1-\alpha/2} \cdot S_u \equiv \bar{q}_{BT} \qquad (1.40)$$

Some simulation results for these four approaches will be presented in the analysis section. Identification for these cases are BT1, BT2, BT3, BT4.

II. SIMULATION PROCEDURE

A simulation procedure has been used to compare the three methods for obtaining confidence intervals for the probability a repair time exceeds h = 4 hours in the case in which the log-normal assumption is met and other cases in which it isn't (the exponential and long-tailed exponential). Specifically, simulation has been used to compute

(a) The actual coverage of the true survival probability by the confidence intervals given by the procedures under study, when the nominal coverage is $(1-\alpha)\cdot 100\%$.

(b) Measure of confidence interval size: the expected width and standard deviation of width.

The simulation programs were written in FORTRAN IV, and the simulations have been carried out on the IBM 3033 at the Naval Postgraduate School. The Naval Postgraduate School LLRANDOM package was used, along with the International Mathematical and Statistical Library (IMSL) random generator; 1000 replications were used to evaluate each procedure in each distributional situation. Also B = 200 bootstrap replications were taken for each trial, four sample sizes, n = 10, 20, 30, 40, and h = 4 hours and three distributions: Log-normal, Exponential, and a Long-tailed Exponential were investigated.

An outline of the simulation procedure now follows:

(A) Log-normal: In this case the basic variables, the down or repair times, are i.i.d. log-normal (see Appendix B).

This case was simulated for the following "population" parameter values:

(1) $\mu = 1.$ $\sigma^2 = 1$

(2) $\mu = 1.$ $\sigma^2 = \ln 2$

(3) $\mu = 1.$ $\sigma^2 = 3\ln 2$

(4) $\mu = 1.$ $\sigma^2 = \ln 2/3.$

The procedures of the previous section were used to obtain confidence intervals for the probability a repair time exceeds $h = 4$ hours.

(B) Stretched long-tailed exponential: Down or repair times come independently from a stretched long-tailed exponential; see Appendix B. The simulated data was treated as if it was a sample from a log-normal distribution and procedures of the previous section were carried out. In this simulation we used the log transformation to tend to convert the long-tailed exponential observations towards normality (symmetrize them). The stretched long-tailed exponential model is:

$$X = AZ(1 + CZ) \tag{2.1}$$

X is stretched long-tailed exponential where Z has the Expon(1) distribution. Simulation was carried out for $A = 3.225$, $C = 0.1948$. These values were taken in order to compare the results with Exponential ($\lambda = 0.22$) case.

(C) Exponential down or repair time: In this situation simulations were carried out for two cases. First taking the log of exponential down or repair times, and treating these as having the normal distribution; that is treating the data as log-normally distributed. Second, taking the p power of data values, and then treating the transformed values as normally distributed. Also Appendix C gives an algorithm for estimating the p value from the data. Simulations are also carried out for $p = 0.33$ (the classical Wilson-Hilferty value), and for three λ values.

III. ANALYSIS

The methods for obtaining 95% confidence intervals for the probability that a repair time exceeds h = 4 hours described in chapter 1 were performed on simulated data having various distributions; these distributions were described in chapter 2. Simulation results for each method are shown in Tables 1 to 11.

If we examine these tables case by case, we can find these results:

(A) Log-normal data: All three methods work very well for this case except BT1 and BT3; they seem to consistently have less than nominal coverage. The simple delta method exhibits good coverage, and always has relatively small average width, and also low standard deviation. It seems to work as well as JK and BT4 for small sample sizes (n = 10, 20). In large sample sizes all methods agree in their coverage except for BT1. The method BT4 always appears to exhibit over-coverage.

(B) Stretched long-tailed exponential data: Table 5 shows that JK and BT4 exhibit over-coverage when the sample size n = 10. JK, DL, BT4 appear to exhibit correct coverage for n = 20, 30, 40, the others don't; especially at sample size n = 40 they are very poor. Also there is decreased coverage for all methods when sample size increases; the

JK and BT4 methods have lower average width than does DL, when sample size increases. This is results of the bias.

(C) Exponential case: Tables 6, 7, 8 show that the log-transformation may give very poor results for the exponential case, especially at Table 7. If we examine Table 6 and Table 8, these tables show DL, JK, BT4 work well for small sample sizes. Tables 9, 10 and 11 indicate that the power transformation works better than the log transformation. The JK, BT2, and BT4 methods always have better coverage than the DL method. Also all methods agree in their coverage when the sample size increases, as was true for the log-normal case. Generally JK and BT4 exhibit acceptable coverage.

Table 1: Simulation results for log-Normal(μ=1.,σ=1.) case.

h =4.0

Sample Size	Method	Coverage	Average Width	Std.Dev Width
10	DL	0.9420	0.4353	0.0409
	JK	0.9520	0.4985	0.1202
	BT1	0.8940	0.4688	0.0979
	BT2	0.9480	0.5105	0.1131
	BT3	0.9150	0.4857	0.0940
	BT4	0.9720	0.5380	0.1205
20	DL	0.9380	0.3221	0.0184
	JK	0.9560	0.3455	0.0510
	BT1	0.9060	0.3339	0.0461
	BT2	0.9400	0.3389	0.0452
	BT3	0.9290	0.3406	0.0469
	BT4	0.9620	0.3495	0.0491
30	DL	0.9450	0.2670	0.0114
	JK	0.9520	0.2795	0.0302
	BT1	0.9240	0.2748	0.0306
	BT2	0.9430	0.2737	0.0296
	BT3	0.9380	0.2772	0.0308
	BT4	0.9550	0.2804	0.0298
40	DL	0.9500	0.2329	0.0084
	JK	0.9560	0.2408	0.0215
	BT1	0.9280	0.2375	0.0246
	BT2	0.9360	0.2369	0.0227
	BT3	0.9460	0.2385	0.0234
	BT4	0.9610	0.2405	0.0226

Table 2: Simulation results for log-Normal(μ=1.,σ=.83) case.

h=4.0

Sample Size	Method	Coverage	Average Width	Std.Dev Width
10	DL	0.9420	0.4278	0.0468
	JK	0.9550	0.4931	0.1284
	BT1	0.8750	0.4464	0.1069
	BT2	0.9390	0.4952	0.1151
	BT3	0.9190	0.4717	0.1002
	BT4	0.9750	0.5352	0.1281
20	DL	0.9370	0.3165	0.0218
	JK	0.9520	0.3407	0.0567
	BT1	0.9130	0.3259	0.0506
	BT2	0.9380	0.3330	0.0499
	BT3	0.9280	0.3325	0.0506
	BT4	0.9600	0.3452	0.0538
30	DL	0.9450	0.2624	0.0137
	JK	0.9520	0.2755	0.0342
	BT1	0.9300	0.2681	0.0332
	BT2	0.9430	0.2704	0.0316
	BT3	0.9390	0.2713	0.0334
	BT4	0.9540	0.2764	0.0330
40	DL	0.9520	0.2288	0.0101
	JK	0.9580	0.2369	0.0246
	BT1	0.9310	0.2316	0.0250
	BT2	0.9440	0.2332	0.0240
	BT3	0.9470	0.2335	0.0253
	BT4	0.9560	0.2363	0.0249

Table 3: Simulation results for log-Normal(μ=1.,σ=.48) case.

h=4.0

Sample Size	Method	Coverage	Average Width	Std.Dev Width
10	DL	0.9430	0.3829	0.0700
	JK	0.9580	0.4621	0.1640
	BT1	0.8530	0.3526	0.1263
	BT2	0.9490	0.4404	0.1440
	BT3	0.9190	0.3944	0.1218
	BT4	0.9810	0.5180	0.1615
20	DL	0.9460	0.2810	0.0366
	JK	0.9520	0.3104	0.0771
	BT1	0.9020	0.2710	0.0630
	BT2	0.9380	0.2896	0.0646
	BT3	0.9290	0.2845	0.0633
	B[illegible]	0.9580	0.3167	0.0714
30	DL	0.9460	0.2325	0.0241
	JK	0.9480	0.2483	0.0480
	BT1	0.9220	0.2275	0.0417
	BT2	0.9420	0.2360	0.0421
	BT3	0.9350	0.2346	0.0421
	BT4	0.9510	0.2500	0.0446
40	DL	0.9580	0.2022	0.0181
	JK	0.9560	0.2118	0.0349
	BT1	0.9260	0.1979	0.0317
	BT2	0.9480	0.2033	0.0315
	BT3	0.9450	0.2024	0.0322
	BT4	0.9550	0.2121	0.0333

Table 4: Simulation results for log-Normal (μ=1., σ=1.44) case.

h=4.0

Sample Size	Method	Coverage	Average Width	Std.Dev Width
10	DL	0.9420	0.4440	0.0324
	JK	0.9520	0.5046	0.1093
	BT1	0.8880	0.4851	0.0886
	BT2	0.9430	0.5168	0.0988
	BT3	0.9210	0.5027	0.0849
	BT4	0.9730	0.5417	0.1101
20	DL	0.9410	0.3285	0.0135
	JK	0.9550	0.3508	0.0423
	BT1	0.9180	0.3464	0.0410
	BT2	0.9400	0.3483	0.0394
	BT3	0.9290	0.3499	0.0413
	BT4	0.9620	0.3544	0.0421
30	DL	0.9440	0.2724	0.0081
	JK	0.9520	0.2862	0.0241
	BT1	0.9280	0.2825	0.0271
	BT2	0.9390	0.2822	0.0245
	BT3	0.9360	0.2841	0.0273
	BT4	0.9560	0.2849	0.0254
40	DL	0.9520	0.2375	0.0059
	JK	0.9600	0.2451	0.0165
	BT1	0.9390	0.2435	0.0209
	BT2	0.9480	0.2434	0.0186
	BT3	0.9490	0.2444	0.0210
	BT4	0.9570	0.2449	0.0191

Table 5: Simulation results for Stretched Long-tailed Exponential.

$h=4.0$ $\lambda=3.225$ $C=0.1948$

Sample Size	Method	Coverage	Average Width	Std.Dev Width
10	DL	0.9530	0.4341	0.0350
	JK	0.9720	0.4973	0.1094
	BT1	0.8550	0.4404	0.1052
	BT2	0.9280	0.4632	0.1055
	BT3	0.9090	0.4563	0.1001
	BT4	0.9850	0.4890	0.1058
20	DL	0.9650	0.3195	0.0170
	JK	0.9590	0.3170	0.0499
	BT1	0.8910	0.3010	0.0485
	BT2	0.9170	0.3015	0.0460
	BT3	0.9200	0.3044	0.0476
	BT4	0.9590	0.3079	0.0453
30	DL	0.9570	0.2642	0.0110
	JK	0.9460	0.2490	0.0321
	BT1	0.9020	0.2424	0.0333
	BT2	0.9150	0.2415	0.0313
	BT3	0.9170	0.2440	0.0329
	BT4	0.9420	0.2445	0.0309
40	DL	0.9430	0.2301	0.0087
	JK	0.9160	0.2115	0.0245
	BT1	0.8750	0.2076	0.0260
	BT2	0.8940	0.2063	0.0244
	BT3	0.8950	0.2085	0.0258
	BT4	0.9240	0.2087	0.0241

Table 6: Simulation results for EXP(λ) case using Log transformation.

h=4.0 λ=0.22 P(X>4.0)=0.4148

Sample Size	Method	Coverage	Average Width	Std.Dev Width	Point Estimate
10	DL	0.9480	0.4432	0.0265	0.3731
	JK	0.9290	0.4476	0.1000	0.3689
	BT1	0.8450	0.4650	0.0989	0.3707
	BT2	0.9090	0.4808	0.1043	0.3707
	BT3	0.9010	0.4772	0.0974	0.3707
	BT4	0.9740	0.4975	0.1036	0.3763
20	DL	0.9520	0.3261	0.0127	0.3685
	JK	0.9060	0.3025	0.0473	0.3665
	BT1	0.8680	0.3133	0.0489	0.3670
	BT2	0.8840	0.3105	0.0457	0.3670
	BT3	0.8930	0.3151	0.0483	0.3670
	BT4	0.9310	0.3136	0.0445	0.3702
30	DL	0.9380	0.2697	0.0081	0.3661
	JK	0.8790	0.2416	0.0310	0.3651
	BT1	0.8530	0.2497	0.0338	0.3649
	BT2	0.8680	0.2467	0.0311	0.3649
	BT3	0.8810	0.2504	0.0335	0.3649
	BT4	0.8990	0.2481	0.0306	0.3674
40	DL	0.9010	0.2349	0.0065	0.3650
	JK	0.8280	0.2068	0.0237	0.3644
	BT1	0.8110	0.2124	0.0265	0.3641
	BT2	0.8160	0.2105	0.0245	0.3641
	BT3	0.8250	0.2128	0.0263	0.3641
	BT4	0.8440	0.2112	0.0241	0.3660

Table 7: Simulation results for EXP(λ) case using Log transformation.

h=4.0 λ=0.13 P(X>4.0)=0.5945

Sample Size	Method	Coverage	Average Width	Std.Dev Width	Point Estimate
10	DL	0.9100	0.4475	0.0298	0.5492
	JK	0.9010	0.5282	0.1199	0.5218
	BT1	0.8690	0.5064	0.0922	0.5718
	BT2	0.9110	0.5490	0.1102	0.5718
	BT3	0.9020	0.5280	0.0932	0.5718
	BT4	0.9410	0.5728	0.1213	0.5245
20	DL	0.8810	0.3329	0.0103	0.5385
	JK	0.8660	0.3774	0.0665	0.5250
	BT1	0.8970	0.3705	0.0462	0.5500
	BT2	0.8820	0.3740	0.0521	0.5500
	BT3	0.9010	0.3750	0.0502	0.5500
	BT4	0.8840	0.3796	0.0585	0.5267
30	DL	0.8470	0.2763	0.0057	0.5343
	JK	0.8390	0.3053	0.0409	0.5256
	BT1	0.8840	0.3031	0.0319	0.5421
	BT2	0.8600	0.3018	0.0322	0.5421
	BT3	0.8890	0.3048	0.0336	0.5421
	BT4	0.8460	0.3043	0.0354	0.5264
40	DL	0.7900	0.2411	0.0035	0.5315
	JK	0.7810	0.2615	0.0293	0.5251
	BT1	0.8330	0.2594	0.0266	0.5375
	BT2	0.8120	0.2594	0.0256	0.5375
	BT3	0.8370	0.2604	0.0276	0.5375
	BT4	0.7860	0.2607	0.0273	0.5255

Table 8: Simulation results for EXP(λ) case using Log transformation.

$h=4.0 \quad \lambda=0.26 \quad P(X>4.0)=0.3535$

Sample Size	Method	Coverage	Average Width	Std.Dev Width	Point Estimate
10	DL	0.9630	0.4322	0.0343	0.3197
	JK	0.9530	0.4188	0.0994	0.3225
	BT1	0.8460	0.4276	0.1072	0.3097
	BT2	0.9260	0.4478	0.1062	0.3097
	BT3	0.9060	0.4427	0.1021	0.3097
	BT4	0.9880	0.4726	0.1052	0.3311
20	DL	0.9660	0.3173	0.0171	0.3178
	JK	0.9380	0.2789	0.0458	0.3192
	BT1	0.8790	0.2859	0.0489	0.3124
	BT2	0.8980	0.2857	0.0459	0.3124
	BT3	0.9070	0.2841	0.0478	0.3124
	BT4	0.9500	0.2917	0.0447	0.3232
30	DL	0.9550	0.2621	0.0111	0.3161
	JK	0.9160	0.2228	0.0298	0.3174
	BT1	0.8700	0.2282	0.0332	0.3122
	BT2	0.8890	0.2271	0.0309	0.3122
	BT3	0.8960	0.2298	0.0327	0.3122
	BT4	0.9350	0.2301	0.0302	0.3199
40	DL	0.9370	0.2282	0.0088	0.3155
	JK	0.8840	0.1910	0.0229	0.3167
	BT1	0.8390	0.1947	0.0258	0.3127
	BT2	0.8510	0.1938	0.0241	0.3127
	BT3	0.8650	0.1956	0.0255	0.3127
	BT4	0.8940	0.1956	0.0236	0.3185

Table 9: Simulation results for EXP(λ) case using X^P transformation.

$h=4.0 \quad \lambda=0.22 \quad p=0.33$

Sample Size	Method	Coverage	Average Width	Std.Dev Width
10	DL	0.9300	0.4460	0.0302
	JK	0.9410	0.5038	0.0944
	BT1	0.8690	0.4941	0.0857
	BT2	0.9330	0.5234	0.0929
	BT3	0.9050	0.5101	0.0829
	BT4	0.9760	0.5459	0.0990
20	DL	0.9390	0.3307	0.0118
	JK	0.9610	0.3556	0.0363
	BT1	0.9150	0.3515	0.0356
	BT2	0.9450	0.3536	0.0335
	BT3	0.9350	0.3545	0.0359
	BT4	0.9630	0.3588	0.0360
30	DL	0.9410	0.2742	0.0069
	JK	0.9570	0.2888	0.0200
	BT1	0.9210	0.2875	0.0250
	BT2	0.9420	0.2874	0.0220
	BT3	0.9310	0.2887	0.0254
	BT4	0.9570	0.2897	0.0232
40	DL	0.9270	0.2390	0.0055
	JK	0.9440	0.2491	0.0133
	BT1	0.9200	0.2480	0.0193
	BT2	0.9370	0.2479	0.0163
	BT3	0.9270	0.2487	0.0194
	BT4	0.9440	0.2492	0.0167

Table 10: Simulation results for EXP(λ) case using X^P transformation.

$h=4.0 \quad \lambda=0.13 \quad p=0.33$

Sample Size	Method	Coverage	Average Width	Std.Dev Width
10	DL	0.9350	0.4440	0.0291
	JK	0.9530	0.4911	0.0912
	BT1	0.8670	0.4804	0.0896
	BT2	0.9490	0.5133	0.0933
	BT3	0.9070	0.4968	0.0843
	BT4	0.9760	0.5377	0.1005
20	DL	0.9480	0.3282	0.0127
	JK	0.9670	0.3411	0.0351
	BT1	0.9150	0.3409	0.0403
	BT2	0.9480	0.3427	0.0372
	BT3	0.9350	0.3439	0.0403
	BT4	0.9700	0.3479	0.0385
30	DL	0.9510	0.2720	0.0084
	JK	0.9610	0.2762	0.0195
	BT1	0.9180	0.2775	0.0274
	BT2	0.9450	0.2770	0.0235
	BT3	0.9360	0.2788	0.0273
	BT4	0.9710	0.2792	0.0237
40	DL	0.9420	0.2372	0.0060
	JK	0.9520	0.2380	0.0140
	BT1	0.9130	0.2385	0.0213
	BT2	0.9370	0.2386	0.0178
	BT3	0.9220	0.2392	0.0212
	BT4	0.9530	0.2399	0.0179

Table 11: Simulation results for EXP(λ) case using X^p transformation.

$h=4.0 \quad \lambda=0.26 \quad p=0.33$

Sample Size	Method	Coverage	Average Width	Std.Dev Width
10	DL	0.9270	0.4346	0.0420
	JK	0.9390	0.4970	0.1106
	BT1	0.8660	0.4677	0.1027
	BT2	0.9310	0.5082	0.1072
	BT3	0.9100	0.4892	0.0974
	BT4	0.9760	0.5415	0.1127
20	DL	0.9380	0.3224	0.0186
	JK	0.9570	0.3505	0.0482
	BT1	0.9060	0.3381	0.0436
	BT2	0.9420	0.3439	0.0423
	BT3	0.9290	0.3434	0.0435
	BT4	0.9610	0.3534	0.0453
30	DL	0.9380	0.2672	0.0117
	JK	0.9500	0.2845	0.0285
	BT1	0.9220	0.2781	0.0294
	BT2	0.9430	0.2805	0.0277
	BT3	0.9290	0.2806	0.0297
	BT4	0.9530	0.2851	0.0291
40	DL	0.9240	0.2329	0.0094
	JK	0.9390	0.2454	0.0198
	BT1	0.9150	0.2412	0.0225
	BT2	0.9290	0.2422	0.0207
	BT3	0.9210	0.2427	0.0226
	BT4	0.9410	0.2451	0.0212

IV. EXAMPLE: APPLICATION TO OPERATIONAL DATA

In this chapter four methods were applied to a real data set. The methods are Binomial (BN), Delta (DL), Jackknife (JK), Bootstrap (see section I-b); specifically, the data refer to recovery times from loss of offsite power at nuclear plants. The problem was to estimate survival probabilities for h = 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 (hours). Data points (n = 42) are shown in appendix D. We initially applied several statistical goodness-of-fit tests to inquire into the evidence for the adequacy of the Log-Normal distribution as a model for these data. The results of these goodness of fit tests are as follows:

(1) Chi-square test: See Arnold, D. [Ref. 5]; this accepts the log-normal model at the significance level $\alpha = 0.05$.

(2) Kolmogorov-Smirnov test: See Arnold, D. [Ref. 5]; this test rejects the log-normal model with the tabulated value C = 0.1367 and test statistic D = 0.21 for $\alpha = 0.05$.

(3) Wilk-Shapiro test: See Hahn, G. J. [Ref. 6]; this test accepts log-normal model for $\alpha = 0.05$.

We applied four estimation methods to these data, utilizing the log-normal assumption. The results are shown in Table 12.

Table 12: Recovery Time Example Results (h in hours)

H	Method	Upper Conf. Limits	Point Estimation	Lower Conf. Limits	Width
1.5	BN	0.476	0.333	0.191	0.285
	DL	0.445	0.324	0.219	0.226
	JK	0.432	0.324	0.230	0.202
	BT1	0.435	0.318	0.215	0.220
	BT2	0.438	0.318	0.214	0.224
	BT3	0.441	0.318	0.215	0.226
	BT4	0.451	0.330	0.224	0.227
2.0	BN	0.422	0.286	0.149	0.273
	DL	0.400	0.280	0.182	0.218
	JK	0.384	0.282	0.195	0.189
	BT1	0.379	0.278	0.198	0.180
	BT2	0.374	0.278	0.197	0.177
	BT3	0.381	0.278	0.198	0.182
	BT4	0.378	0.282	0.200	0.178
2.5	BN	0.422	0.286	0.149	0.273
	DL	0.366	0.249	0.155	0.211
	JK	0.348	0.250	0.169	0.179
	BT1	0.330	0.250	0.175	0.155
	BT2	0.345	0.250	0.171	0.174
	BT3	0.329	0.250	0.175	0.153
	BT4	0.342	0.247	0.169	0.173
3.0	BN	0.395	0.262	0.129	0.266
	DL	0.339	0.224	0.135	0.204
	JK	0.320	0.226	0.150	0.170
	BT1	0.309	0.223	0.135	0.174
	BT2	0.324	0.223	0.143	0.182
	BT3	0.311	0.223	0.135	0.176
	BT4	0.327	0.225	0.144	0.183

3.5	BN	0.395	0.262	0.129	0.266
	DL	0.318	0.204	0.120	0.198
	JK	0.298	0.207	0.135	0.163
	BT1	0.287	0.204	0.140	0.147
	BT2	0.289	0.204	0.136	0.152
	BT3	0.288	0.204	0.140	0.147
	BT4	0.290	0.205	0.137	0.153
4.0	BN	0.367	0.238	0.109	0.258
	DL	0.299	0.188	0.107	0.192
	JK	0.279	0.191	0.122	0.157
	BT1	0.263	0.185	0.113	0.150
	BT2	0.273	0.185	0.117	0.156
	BT3	0.267	0.185	0.113	0.154
	BT4	0.282	0.192	0.122	0.160

V. CONCLUSIONS

The Delta, Jackknife and Bootstrap methods applied to the log-normal model work well when down or repair times are truly log-normal. Especially notice that DL, JK, and BT4 seem to work much better than BT1, BT2, BT3. Recall that these procedures do not appear sensitive to the population variance; see section III. It is comparatively easy to use JK and BT4 when sample size is small (n = 10, 20). The delta method is always convenient, but especially when the sample is large (n = 40 or more) because it is a very simple procedure to apply, requiring much less computation than the others. As Table 12 shows, the Binomial method gives some idea of the survival probability for practical purposes. Note that Binomial confidence limits are much wider than those that assume the log-normal model.

Use of the log transform on exponential data produces biased estimates of survival. Use of the power transformation with (p = 1/3) always gives a better coverage of the survival probability when data are exponential. One procedure was described in Appendix C for estimating the p value from data. Table 13 gives simulation results for the exponential case. As our results show, this procedure is not estimating p value correctly. If this procedure were to work correctly (if it could be calibrated) then we could use the

' transformation (for converting data towards the normal ›rm) without making any assumption (e.g. this data coming :om exponential or gamma or log-normal, etc.). Then after ıis is done, methods DL, JK, BT4 might produce considerably ›tter confidence limits for the actual survival probability.

APPENDIX A

COMPUTER PROGRAMS

Simulation programs consist of two main programs for three methods (DL, JK, BT). These main programs compute survival probability confidence limits, and scores the coverage for each replication. Then, after 1000 replications the program computes the statistics of these parameters and prints out the results.

There is another program, called SURVP. This program computes point estimates, confidence limits, and widths of confidence limits on survival probability, using the BN, DL, JK, BT procedures on a given data set, under the log-normal model assumption.

Variables List:

R = Down or repair times.

R1 = Log of down or repair times.

RBAR = Mean of down or repair times.

RSD = Standard deviation of down or repair times.

GHAT = Point estimation of q parameter (see 1.17) for delta method

GJK = Point estimation of q parameter for jackknife method.

GBOOT = Point estimation of q parameter for bootstrap method.

VARG = Variance of point estimation for delta method.

SE = Standard error of point estimation for jackknife method.

PHAT = Point estimation of survival probability.

BUP = Upper confidence limit estimation of q parameter.

BLOW = Lower confidence limit estimation of q parameter.

CUP = Upper confidence limit estimation of survival probability.

CLOW = Lower confidence limit estimation of survival probability.

AINT = Width of estimated confidence limits of survival probability.

G = Pseudovalues.

F = Bootstrap replications.

N = Number of data points.

N1 = Number of replications.

N2 = Number of bootstrap replications.

```
C
C     THIS PROGRAM COMPUTES POINT ESTIMATES AND CONFIDENCE
C     LIMITS OF SURVAVIL PROBABILITY USING BN,DL,JK,BT
C     PROCEDURES ON A GIVEN DATA SET UNDER THE LOG-NORMAL
C     MODEL ASSUMPTION.
C
      DIMENSION R(100),R1(100),IOPT(5),STAT(5),A(100),B(100),G(100),F(10
     *0),IR(100),GH(400)
      DOUBLE PRECISION DSEED
      N=42
      N2=200
      IOPT(1)=1
      IOPT(5)=1
      IOPT(4)=1
      NA=N-1
      DSEED=5927.0D0
C
C     READ DATA
C
      READ(5,1000) (R(I),I=1,N)
 1000 FORMAT(7F10.5)
      WRITE(6,901)
  901 FORMAT('1',5X,'H',3X,'METHOD',3X,'CUP',6X,'PHAT',4X,'CLOW',4X,'CUP
     *-CLOW',3X,/,5X,'-----------------------------------------------------')
      DO 900 IE=1,6
      H=1.+(FLOAT(IE)/2.)
C
C     BINOMIAL METHOD
C
      ICON=0
      DO 800 I=1,N
      IF(R(I).GE.H) ICON=ICON+1
  800 CONTINUE
      PHAT=FLOAT(ICON)/N
C
C     COMPUTE CONFIDENCE LIMITS
C
      CUP=PHAT+(1.96*((PHAT*(1.-PHAT)/N)**.5))
      CLOW=PHAT-(1.96*((PHAT*(1.-PHAT)/N)**.5))
      AINT=CUP-CLOW
C
C     OUTPUT FOR BINOMIAL METHOD
C
      WRITE(6,200) H,CUP,PHAT,CLOW,AINT
  200 FORMAT('0',5X,F3.1,3X,'BN',3X,F6.3,3X,F6.3,3X,F6.3,3X,F6.3)
C
C     DELTA METHOD
C
```

```
      DO 700 I=1,N
      R1(I)=ALOG(R(I))
  700 CONTINUE
      CALL BETUGR(R1,N,IOPT,STAT,IER)
      RBAR=STAT(1)
      RSD=STAT(5)**.5
      GHAT=(ALOG(H)-RBAR)/RSD
      VARG=(GHAT**2./((N-1)*2.))+(1./FLOAT(N))
C
C     COMPUTE CONFIDENCE LIMITS
C
      BUP=GHAT+(1.96*(VARG**.5))
      BLOW=GHAT-(1.96*(VARG**.5))
      CALL MDNOR(GHAT,E1)
      PHAT=1.-E1
      CALL MDNOR(BUP,E2)
      CLOW=1.-E2
      CALL MDNOR(BLOW,E3)
      CUP=1.-E3
      AINT=CUP-CLOW
C
C     OUTPUT FOR DELTA METHOD
C
      WRITE(6,201) CUP,PHAT,CLOW,AINT
  201 FORMAT('0',11X,'DL',3X,F6.3,3X,F6.3,3X,F6.3,3X,F6.3)
C
C     JACK-KNIFE METHOD
C
      DO 11 J=1,N
      DO 10 I=1,N
      A(I)=R1(I)
   10 CONTINUE
      A(J)=9999.
      I=0
      K=0
  100 I=I+1
      IF(K.GT.0) GO TO 102
      IF(A(I).EQ.9999.) GO TO 101
      B(I)=A(I)
      GO TO 100
  101 K=I
      IF(K.GT.NA) GO TO 103
      I=I+1
      B(K)=A(I)
      GO TO 100
  102 K=K+1
      IF(K.GT.NA) GO TO 103
      B(K)=A(I)
```

```
      GO TO 100
  103 CALL BEIUGR(B,NA,IOPT,STAT,IER)
C
C     CALCULATE PSEDOVALUES
C
      BAR=STAT(1)
      SD1=STAT(5)**.5
      G(J)=(N*GHAT)-(NA*(ALOG(H)-BAR)/SD1)
   11 CONTINUE
      CALL BEIUGR(G,N,IOPT,STAT,IER)
      GJK=STAT(1)
      SE=(STAT(5)/N)**.5
      BUP=GJK+(1.96*SE)
      BLOW=GJK-(1.96*SE)
      CALL MDNOR(GJK,Z1)
      PHAT=1.-Z1
      CALL MCNOR(BUP,Z2)
      CLOW=1.-Z2
      CALL MDNOR(BLOW,Z3)
      CUP=1.-Z3
      AINT=CUP-CLOW
C
C     OUTPUT FOR JACKKNIFE METHOD
C
      WRITE(6,202) CUP,PHAT,CLOW,AINT
  202 FORMAT('0',11X,'JK',3X,F6.3,3X,F6.3,3X,F6.3,3X,F6.3)
C
C     BOOTSTRAP METHOD
C
      CALL VSRTA(R1,N)
C
C     RESAMPLING
C
      DO 32 JJ=1,N2
      CALL GGUD(DSEED,N,N,IR)
      DO 33 J1=1,N
C
C     BOOTSTRAP SAMPLE
C
      F(J1)=R1(IR(J1))
   33 CONTINUE
      CALL BEIUGR(F,N,IOPT,STAT,IER)
C
C     BOOTSTRAP REPLICATIONS
C
      GH(JJ)=(ALOG(H)-STAT(1))/(STAT(5)**.5)
   32 CONTINUE
      CALL BEIUGR(GH,N2,IOPT,STAT,IER)
```

```
      BIAS=STAT(1)-GHAT
      GBOOT=(2.*GHAT)-STAT(1)
      CALL VSRTA(GH,N2)
      DO 154 ICA=1,4
      CALL CONF(BUP,BLOW,GHAT,PHAT,GBOOT,GH,BIAS,STAT,ICA)
      CALL MDNOR(BLOW,S2)
      CUP=1.-S2
      CALL MDNOR(BUP,S3)
      CLOW=1.-S3
      AINT=CUP-CLOW
C
C     OUTPUT FOR BOOTSTRAP METHOD
C
      WRITE(6,203) ICA,CUP,PHAT,CLOW,AINT
  203 FORMAT('0',11X,'BT',I1,2X,F6.3,3X,F6.3,3X,F6.3,3X,F6.3)
  154 CONTINUE
  900 CONTINUE
      STOP
      END
C
C     THIS SUBROUTINE GIVES CONF. LIMITS FOR BT1,BT2,BT3,BT4
C
      SUBROUTINE CONF(BUP,BLOW,GHAT,PHAT,GBOOT,GH,BIAS,STAT,ICA)
      REAL GH(200),STAT(5)
      IF(ICA.GT.1) GO TO 150
      BLOW=GH(5)
      BUP=GH(195)
      CALL MDNOR(STAT(1),S1)
      PHAT=1.-S1
      GO TO 153
  150 IF(ICA.GT.2) GO TO 151
      BLOW=STAT(1)-(1.96*(STAT(5)**.5))
      BUP=STAT(1)+(1.96*(STAT(5)**.5))
      GO TO 153
  151 IF(ICA.GT.3) GO TO 152
      BLOW=GH(5)-BIAS
      BUP=GH(195)-BIAS
      GO TO 153
  152 BLOW=GBOOT-(1.96*(STAT(5)**.5))
      BUP=GBOOT+(1.96*(STAT(5)**.5))
      CALL MDNOR(GBOOT,S1)
      PHAT=1.-S1
  153 RETURN
      END
```

```
C
C      THIS PROGRAM SIMULATES LOG-NORMAL CASE USING
C      DL AND JK PROCEDURES.
C
C
C       MAIN
C
       DIMENSION R(50),IOPT(5),STAT(5),BUP(1000),BLOW(1000),PUP(1000),PLO
      *W(1000),AIN(1000),A(100),B(100),G(100)
       EX=1.
       SD=0.48
       N1=1000
       DO 800 LL=1,4
       WRITE(6,100) EX,SD
   100 FORMAT('1',25X,'NORMAL(',F3.1,',',F4.2,')')
       WRITE(6,101)
   101 FORMAT('0',///,15X,'SAMPLE',25X,'AVERAGE',2X,'VARIANCE')
       WRITE(6,102)
   102 FORMAT(5X,'METHOD',5X,'SIZE',5X,' H ',5X,'COVERAGE',5X,'WIDTH',5X,
      1'WIDTH')
       N=LL*10
       DO 801 KA=6,6
       H=1.+(FLOAT(KA)/2.)
       GREL=(ALOG(H)-EX)/SD
       CALL DELT(N,N1,EX,SD,H,GREL)
       CALL JAC(N,N1,EX,SD,H,GREL)
   801 CONTINUE
   800 CONTINUE
       STOP
       END
C
C      DELTA METHOD
C
       SUBROUTINE DELT(N,N1,EX,SD,H,GREL)
       REAL R(50),BUP(1000),BLOW(1000),PUP(1000),PLOW(1000),AIN(1000),STA
      2T(5)
       INTEGER IOPT(5),N,N1
       IOPT(1)=1
       IOPT(5)=1
       IX=56662
       J=0
       DO 11 I=1,N1
       CALL LNORM(IX,R,N,16807,0)
       DO 12 II=1,N
       R(II)=(SD*R(II))+EX
    12 CONTINUE
       CALL BEIUGR(R,N,IOPT,STAT,IER)
       RBAR=STAT(1)
```

```
      RSD=STAT(5)**.5
      GHAT=(ALOG(H)-RBAR)/RSD
      VARG=(GHAT**2/((N-1.)*2.))+(1./FLOAT(N))
      BUP(I)=GHAT+(1.96*(VARG**.5))
      BLOW(I)=GHAT-(1.96*(VARG**.5))
      CL=BLOW(I)
      CALL MDNOR(CL,E1)
      PLOW(I)=1.-E1
      CU=BUP(I)
      CALL MDNOR(CU,E2)
      PUP(I)=1.-E2
      AIN(I)=E2-E1
      IF(GREL.GE.BLOW(I).AND.GREL.LE.BUP(I)) J=J+1
   11 CONTINUE
      PROB=FLOAT(J)/N1
      CALL BEIUGR(PLOW,N1,IOPT,STAT,IER)
      TPLOW=STAT(1)
      CALL BEIUGR(PUP,N1,IOPT,STAT,IER)
      TPUP=STAT(1)
      CALL BEIUGR(AIN,N1,IOPT,STAT,IER)
      TINT=STAT(1)
      SINT=STAT(5)**.5
      CALL BEIUGR(BUP,N1,IOPT,STAT,IER)
      CU1=STAT(1)
      CALL BEIUGR(BLOW,N1,IOPT,STAT,IER)
      CL1=STAT(1)
      WRITE(6,103) N,H,PROB,TINT,SINT
  103 FORMAT('0',7X,'DL',7X,I2,6X,F3.1,5X,F6.4,6X,F6.4,5X,F6.4)
      RETURN
      END
C
C     JACKKNIFE METHOD
C
      SUBROUTINE JAC(N,N1,EX,SD,H,GREL)
      REAL R(50),BUP(1000),BLOW(1000),PUP(1000),PLOW(1000),AIN(1000),A(1
     *00),B(100),G(100),PROB,STAT(5)
      INTEGER IOPT(5),N,N1,NA
      IOPT(1)=1
      IOPT(5)=1
      IX=56662
      NA=N-1
      JJ=0
      DO 21 KB=1,N1
      CALL LNORM(IX,R,N,16807,0)
      DO 22 KC=1,N
      R(KC)=(SD*R(KC))+EX
   22 CONTINUE
      CALL BEIUGR(R,N,IOPT,STAT,IER)
```

```
      RBAR=STAT(1)
      RSD=STAT(5)**.5
      GHAT=(ALOG(H)-RBAR)/RSD
      DO 11 J=1,N
      DO 10 I=1,N
      A(I)=R(I)
   10 CONTINUE
      A(J)=9999.
      I=0
      K=0
  100 I=I+1
      IF(K.GT.0) GO TO 102
      IF(A(I).EQ.9999.) GO TO 101
      B(I)=A(I)
      GO TO 100
  101 K=I
      IF(K.GT.NA) GO TO 103
      I=I+1
      B(K)=A(I)
      GO TO 100
  102 K=K+1
      IF(K.GT.NA) GO TO 103
      B(K)=A(I)
      GO TO 100
  103 CALL BEIUGR(B,NA,IOPT,STAT,IER)
C
C     CALCULATE PSEDOVALUES.
C
      BAR=STAT(1)
      SD1=STAT(5)**.5
      G(J)=(N*GHAT)-(NA*(ALOG(H)-BAR)/SD1)
   11 CONTINUE
      CALL BEIUGR(G,N,IOPT,STAT,IER)
      GJK=STAT(1)
      SE=(STAT(5)/N)**.5
C     F=FLOAT(NA)
C     CALL MDSTI(.05,F,X,IER)
      X=1.96
      BUP(KB)=GJK+(X*SE)
      BLOW(KB)=GJK-(X*SE)
      CL=BLOW(KB)
      CALL MDNOR(CL,E1)
      PLOW(KB)=1.-E1
      CU=BUP(KB)
      CALL MDNOR(CU,E2)
      PUP(KB)=1.-E2
      AIN(KB)=E2-E1
      IF(GREL.GE.BLOW(KB).AND.GREL.LE.BUP(KB)) JJ=JJ+1
```

```
 21 CONTINUE
    PROB=FLOAT(JJ)/N1
    CALL BEIUGR(PLOW,N1,IOPT,STAT,IER)
    TPLOW=STAT(1)
    CALL BEIUGR(PUP,N1,IOPT,STAT,IER)
    TPUP=STAT(1)
    CALL BEIUGR(AIN,N1,IOPT,STAT,IER)
    TINT=STAT(1)
    SINT=STAT(5)**.5
    CALL BEIUGR(BUP,N1,IOPT,STAT,IER)
    CU1=STAT(1)
    CALL BEIUGR(BLOW,N1,IOPT,STAT,IER)
    CL1=STAT(1)
    WRITE(6,503) N,H,PROB,TINT,SINT
503 FORMAT('0',7X,'JK',7X,I2,6X,F3.1,5X,F6.4,6X,F6.4,5X,F6.4)
    RETURN
    END
```

```
C
C     THIS PROGRAM SIMULATES LOG-NORMAL CASE
C     USING BT4 PROCEDURES.
C
      DIMENSION R(90),IOPT(5),STAT(5),BUP(1000),BLOW(1000),PUP(1000),PLO
     *W(1000),AIN(1000),F(100),GH(400),IR(100)
      DOUBLE PRECISION DSEED
      DO 800 LL=1,4
      N=LL*10
      N1=1000
      N2=200
      K=N
      NR=N
      EX=1.
      SD=0.48
      IOPT(1)=1
      IOPT(5)=1
      WRITE(6,100)
  100 FORMAT('0',10X,'BOOTSTRAP  METHOD',/,11X,'-------------------')
      WRITE(6,101) N1,N,EX,SD
  101 FORMAT('0',5X,'REPETITION=',I4,//,5X,'SAMPLE SIZE=',I2,//,5X,'EXPE
     1CTATION=',F3.1,//,7X,'STAND DEV=',F4.2,//)
      WRITE(6,102)
  102 FORMAT('0',6X,'H',4X,'LOWER G',1X,'REAL G',2X,'UPPER G',2X,'PROB',
     25X,'PUP',5X,'PLOW',5X,'PUP-PLOW',5X,'SD(PUP-PLOW)',6X,/,'---------
     3-------------------------------------------------------------------
     4---------------')
      DO 10 JA=6,6
      DSEED=6759.0D0
      H=1.+(FLOAT(JA)/2.)
      GREL=(ALOG(H)-EX)/SD
      IX=56662
      J=0
C
C     GENERATE NORMAL(EX,SD)
C
      DO 30 I=1,N1
      CALL LNORM(IX,R,N,1,1)
      DO 31 II=1,N
      R(II)=(SD*R(II))+EX
   31 CONTINUE
      CALL BEIUGR(R,N,IOPT,STAT,IER)
      GHAT=(ALOG(H)-STAT(1))/(STAT(5)**.5)
      CALL VSRTA(R,N)
C
C     RESAMPLING
C
      DO 32 JJ=1,N2
```

```
      CALL GGUD(DSEED,K,NR,IR)
      DO 33 J1=1,N
C
C     BOOTSTRAP SAMPLE
C
      F(J1)=R(IR(J1))
   33 CONTINUE
      CALL BEIUGR(F,N,IOPT,STAT,IER)
C
C     BOOTSTRAP REPLICATIONS
C
      GH(JJ)=(ALOG(H)-STAT(1))/(STAT(5)**.5)
   32 CONTINUE
      CALL BEIUGR(GH,N2,IOPT,STAT,IER)
      BIAS=STAT(1)-GHAT
      GBOOT=(2.*GHAT)-STAT(1)
      CALL VSRTA(GH,N2)
C
C     COMPUTE CONFIDENCE LIMITS
C
      BLOW(I)=GBOOT-(1.96*(STAT(5)**.5))
      BUP(I)=GBOOT+(1.96*(STAT(5)**.5))
      CL=BLOW(I)
      CALL MDNOR(CL,E1)
      PLOW(I)=1.-E1
      CU=BUP(I)
      CALL MDNOR(CU,E2)
      PUP(I)=1.-E2
      AIN(I)=E2-E1
      IF(GREL.GE.BLOW(I).AND.GREL.LE.BUP(I)) J=J+1
   30 CONTINUE
      PROB=FLOAT(J)/N1
      CALL BEIUGR(PLOW,N1,IOPT,STAT,IER)
      TPLOW=STAT(1)
      CALL BEIUGR(PUP,N1,IOPT,STAT,IER)
      TPUP=STAT(1)
      CALL BEIUGR(AIN,N1,IOPT,STAT,IER)
      TINT=STAT(1)
      SINT=STAT(5)**.5
      CALL BEIUGR(BUP,N1,IOPT,STAT,IER)
      CU1=STAT(1)
      CALL BEIUGR(BLOW,N1,IOPT,STAT,IER)
      CL1=STAT(1)
C
C     OUTPUT
C
      WRITE(6,103) H,CL1,GREL,CU1,PROB,TPLOW,TPUP,TINT,SINT
  103 FORMAT('0',5X,F3.1,3X,F5.2,3X,F5.2,3X,F5.2,3X,F6.4,3X,F6.4,3X,F6.4
```

```
     *,3X,F6.4,9X,F6.4)
  10 CONTINUE
 800 CONTINUE
     STOP
     END
```

APPENDIX B

LOG-NORMAL AND STRETCHED LONG-TAILED EXPONENTIAL DISTRIBUTION

(1) Let x be log-normal random variable which ln(x) ι,σ^2) k moment of x as follows:

$$E[x^k] = \exp(k\mu + 1/2 \cdot k^2\sigma^2)$$

ː [Ref. 7] so

$$E[x] = \exp(\mu + 1/2 \cdot \sigma^2)$$

$$E[x^2] = \exp(2\mu + 2 \cdot \sigma^2)$$

$$Var[x] = (E[x])^2 \ (e^{\sigma^2} - 1)$$

"coefficient of variation" as follows:

$$\frac{Var[x]}{(E[x])^2} = e^{\sigma^2} - 1$$

(2) Let W be stretched long-tailed exponential variable ιch W = A·z(1 + C·z) where z is unit exponential and A and ιre constant. If we write CDF for this distribution as:

$$P(W \le w) = P[A \cdot z(1 + C \cdot z) \le w] = P[Z \le z(w)]$$

= A·z(1 + C·z). If we solve this equation for z we can get ɾ) as

$$z(w) = \frac{-A \pm \sqrt{A^2 + 4ACw}}{2AC}$$

$$E[w] = A(1 + 2C)$$

$$E[w^2] = A^2[2 + 12C + 24C]$$

$$Var[w] = A^2[1 + 8C + 20C]$$

$$(cv)^2 = \frac{Var[w]}{(E[w])^2} = \frac{1 + 8C + 20C^2}{1 + 4C + 4C^2}$$

If we look at w as a log-normal (μ,σ^2) variable then

$$\frac{1 + 8C + 20C^2}{1 + 4C + 4C^2} = e^{\sigma^2} - 1$$

We can get C value from this equation, then

$$E[w] = A(1 + 2C) = e^{\mu + \frac{1}{2}\sigma^2}$$

gives A value.

APPENDIX C

POWER TRANSFORMATION

The problem is in the x^p transformation (toward the normal form) finding the p value for given data. One method for finding the p value is as follows [Ref. 8]:

$x_1, x_2, \ldots, x_n$ data points and M = Median of this data.

(1) Take order statistic of given data

$$x_{(1)} < x_{(2)} < \cdots < x_{(n)}$$

(2) Then compute q_j values as

$$q_j = 1 - p_j = \left[\frac{\left(x_{(n-j+1)} - M\right) - \left(M - x_{(j)}\right)}{\left(x_{(n-j+1)} - M\right)^2 + \left(M - x_{(j)}\right)^2}\right] 2 \cdot M \quad j = 1,2,\ldots,n/2$$

where M is the median of the data.

(3) Take the median or mean of q_j:

$$\tilde{q} = \text{median}(q_j) \text{ or } \tilde{q} = \frac{1}{[n/2]} \sum_{j=1}^{n/2} q_j$$

(4) Then we can get p value as $\tilde{p} = 1 - \tilde{q}$.

Table 13 gives simulation results for this algorithm for the exponential case. For the exponential case the best p value is p = 1/3. Simulation results do not give this value so this algorithm is not working correctly.

Table 13: Simulation results for P algorithm using Exponential data.

λ=0.22

Sample Size	Average of P	Variance of P	Skewness of P
10	0.7434	0.9174	0.7535
20	0.6039	0.3507	1.3270
30	0.5336	0.2455	1.3619
40	0.5121	0.1872	1.8234
50	0.4903	0.1397	1.4981
60	0.4859	0.1122	1.4225
70	0.4658	0.0912	1.5475
80	0.4571	0.0814	1.6633
90	0.4498	0.0710	1.5987
100	0.4470	0.0669	1.8234
110	0.4403	0.0580	1.9510
120	0.4323	0.0515	1.4446
130	0.4298	0.0434	1.3154
140	0.4263	0.0452	1.4466
150	0.4196	0.0394	1.5704

APPENDIX D

MEAN SQUARE ERROR OF SURVIVAL PROBABILITY

Mean-square errors were calculated for exponential case using the log transformation for three values and h = 4.0. The procedure is as follows:

(1) Generate exponential sample $x_1, x_2, \ldots, x_n$, $n = 10, 20, \ldots, 250$.

(2) Find actual survival probability as:

$$P(x>h) = e^{-\lambda h}$$

(3) Estimate survival probability (incorrectly) as:

$$\hat{P}(x>h) = \int_{\frac{\ln h - \hat{\mu}}{\hat{\sigma}}}^{\infty} e^{-\frac{1}{2}z^2} \frac{dz}{\sqrt{2\pi}}$$

(4) Calculate $(P(h) - \hat{P}(h))^2$

(5) Repeat this procedure 1000 times.

(6) Calculate $\widehat{MSE}$ as:

$$\widehat{MSE} = \frac{1}{1000} \sum_{i=1}^{1000} [P(h) - \hat{P}_i(h)]^2$$

Simulation results for mean square error of survival probability were shown in Tables 14, 15 and 16.

Table 14: Mean Square Error of Survival probability for EXP(λ) case using Log transformation.

$h=4.0 \quad \lambda=0.22$

Sample Size	Mean Square Error(MSE)	Square Root of MSE
10	0.0166	0.1288
20	0.0085	0.0920
30	0.0063	0.0795
40	0.0056	0.0746
50	0.0049	0.0699
60	0.0045	0.0673
70	0.0041	0.0642
80	0.0040	0.0635
90	0.0038	0.0614
100	0.0038	0.0612
110	0.0037	0.0604
120	0.0035	0.0594
130	0.0034	0.0585
140	0.0033	0.0578
150	0.0033	0.0576
160	0.0033	0.0572
170	0.0032	0.0564
180	0.0031	0.0560
190	0.0032	0.0562
200	0.0032	0.0563
210	0.0031	0.0561
220	0.0031	0.0557
230	0.0031	0.0558
240	0.0031	0.0557
250	0.0031	0.0553

Table 15: Mean Square Error of Survival probability for EXP(λ) case using Log transformation.

$h=4.0 \quad \lambda=0.13$

Sample Size	Mean Square Error (MSE)	Square Root of MSE
10	0.0234	0.1531
20	0.0128	0.1132
30	0.0098	0.0991
40	0.0089	0.0942
50	0.0081	0.0897
60	0.0076	0.0869
70	0.0070	0.0834
80	0.0068	0.0824
90	0.0065	0.0803
100	0.0064	0.0802
110	0.0063	0.0792
120	0.0061	0.0781
130	0.0059	0.0770
140	0.0059	0.0766
150	0.0058	0.0762
160	0.0058	0.0758
170	0.0057	0.0753
180	0.0055	0.0745
190	0.0056	0.0746
200	0.0056	0.0747
210	0.0056	0.0747
220	0.0055	0.0744
230	0.0056	0.0745
240	0.0055	0.0743
250	0.0054	0.0737

Table 16: Mean Square Error of Survival probability for EXP(λ) case using Log transformation.

h=4.0 λ=0.25

Sample Size	Mean Square Error (MSE)	Square Root of MSE
10	0.0135	0.1160
20	0.0066	0.0812
30	0.0047	0.0688
40	0.0041	0.0637
50	0.0034	0.0585
60	0.0031	0.0559
70	0.0028	0.0525
80	0.0027	0.0519
90	0.0025	0.0495
100	0.0024	0.0492
110	0.0023	0.0483
120	0.0022	0.0472
130	0.0022	0.0465
140	0.0021	0.0456
150	0.0021	0.0453
160	0.0020	0.0448
170	0.0019	0.0439
180	0.0019	0.0435
190	0.0019	0.0437
200	0.0019	0.0438
210	0.0019	0.0435
220	0.0019	0.0431
230	0.0019	0.0431
240	0.0018	0.0429
250	0.0018	0.0425

APPENDIX E

DATA POINTS FOR EXAMPLE

These are recovery times (hours) from LOSP at nuclear plants:

24.6160	25.6660	11.0830	0.0038	0.3333
0.6166	1.5000	1.1833	0.0333	0.0500
0.2666	5.8000	4.9830	1.8330	0.5000
6.4660	0.2833	1.0000	0.9000	0.1666
0.6666	0.4333	0.1333	0.0166	5.5833
0.4833	0.0028	0.9333	0.2333	4.9833
0.1500	2.6660	4.7500	0.1333	1.0166
0.0666	0.0166	8.9000	3.5000	0.9166
0.3333	0.0041			

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