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SACLANT ASW RESEARCH CENTRE MEMORANDUM

A COMPARISON OF SOME SIGNAL-PROCESSING ALGORITHMS TO SUPPRESS TOW-VESSEL NOISE IN A TOWED ARRAY, WITH RESULTS FROM A SHALLOW-WATER TRIAL

by

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--
--       ARE PRESENTED ON THE OUTPUTS FROM CONVENTIONAL BEAMFORMING AND
--       MAXIMUM-LIKELIHOOD BEAMFORMING AND ON SOME BASIC TECHNIQUES FOR
--       SUPPRESSING TOWSHIP NOISE. THE SIGNAL PROCESSING PACKAGE IS
--       IMPLEMENTED IN SOFTWARE AND PROVIDES THE VARIOUS FORMS OF
--       PROCESSING BY USING THE ARRAY COVARIANCE MATRIX AND EIGENVALUE/
--       VECTOR ANALYSIS. POSSIBLE FUTURE WORK ON THESE TOPICS IS PROPOSED.
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A COMPARISON OF SOME SIGNAL-PROCESSING ALGORITHMS TO SUPPRESS TOW-VESSEL NOISE IN A TOWED ARRAY, WITH RESULTS FROM A SHALLOW-WATER TRIAL

by

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1 October 1982

This memorandum has been prepared within the SACLANTCEN Systems Research Division as part of Project 02.

L.F. Whicker
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A COMPARISON OF SOME SIGNAL-PROCESSING ALGORITHMS TO SUPPRESS TOW-VESSEL NOISE IN A TOWED ARRAY, WITH RESULTS FROM A SHALLOW-WATER SEA TRIAL

by

Ulrich E. Rupe and Even B. Lunde

ABSTRACT

A short trial has been conducted in shallow water using a towed array and a number of signal-processing techniques. Results are presented on the outputs from conventional beamforming and maximum-likelihood beamforming and on some basic techniques for suppressing towship noise. The signal-processing package is implemented in software and provides the various forms of processing by using the array covariance matrix and eigenvalue/vector analysis. Possible future work on these topics is proposed.

INTRODUCTION

The self-noise of the platform can enter a hull-mounted sonar array from different directions and in various frequency bands, thereby presenting a number of problems in the reception of signals during the initial processes of target detection, classification, and tracking when signal levels are low. The advent of the towed array has minimized these problems considerably and, in essence, reduced them to simply one of a noise source ahead of the array.

However, depending on the characteristics of the towed-array system (the scope of the tow cable, the beamwidth of the array, etc.) and the environmental conditions in the area of operation (depth of water, type of sea bottom, etc.), the towship noise can appear in a number of beams in the ahead ±90° sector and cause serious operational limitations.

This memorandum compares a number of signal-processing techniques by describing results obtained from a sea-trial in shallow water using a towed-array system.

The general headings are:

a. The presence of a number of signal paths from the towship to the array.

b. The effect of the basic techniques for suppressing towship noise.

c. Target indication and bearing estimation.

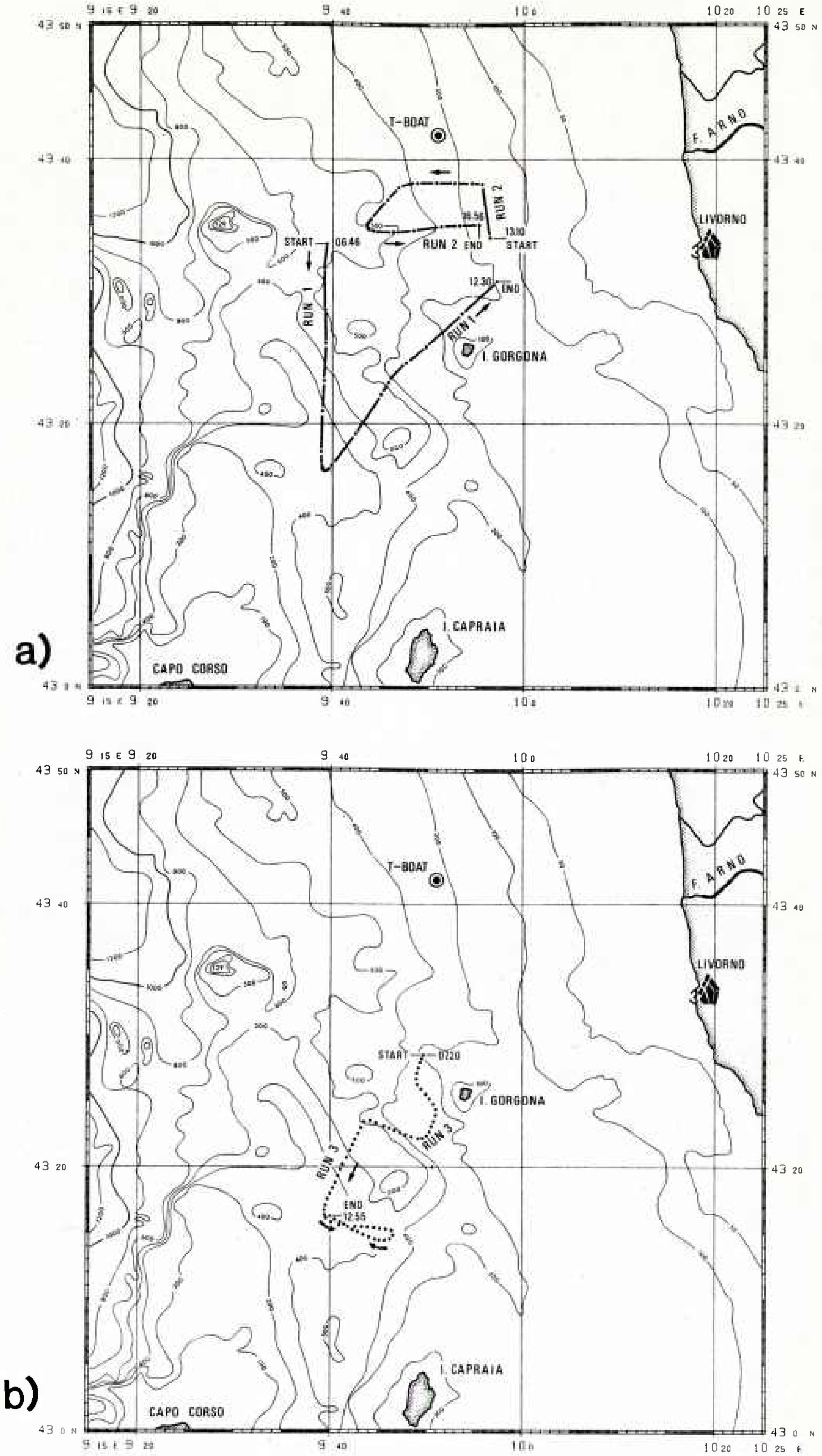


FIG. 1 SEA TRIALS
a) Runs 1 and 2
b) Run 3

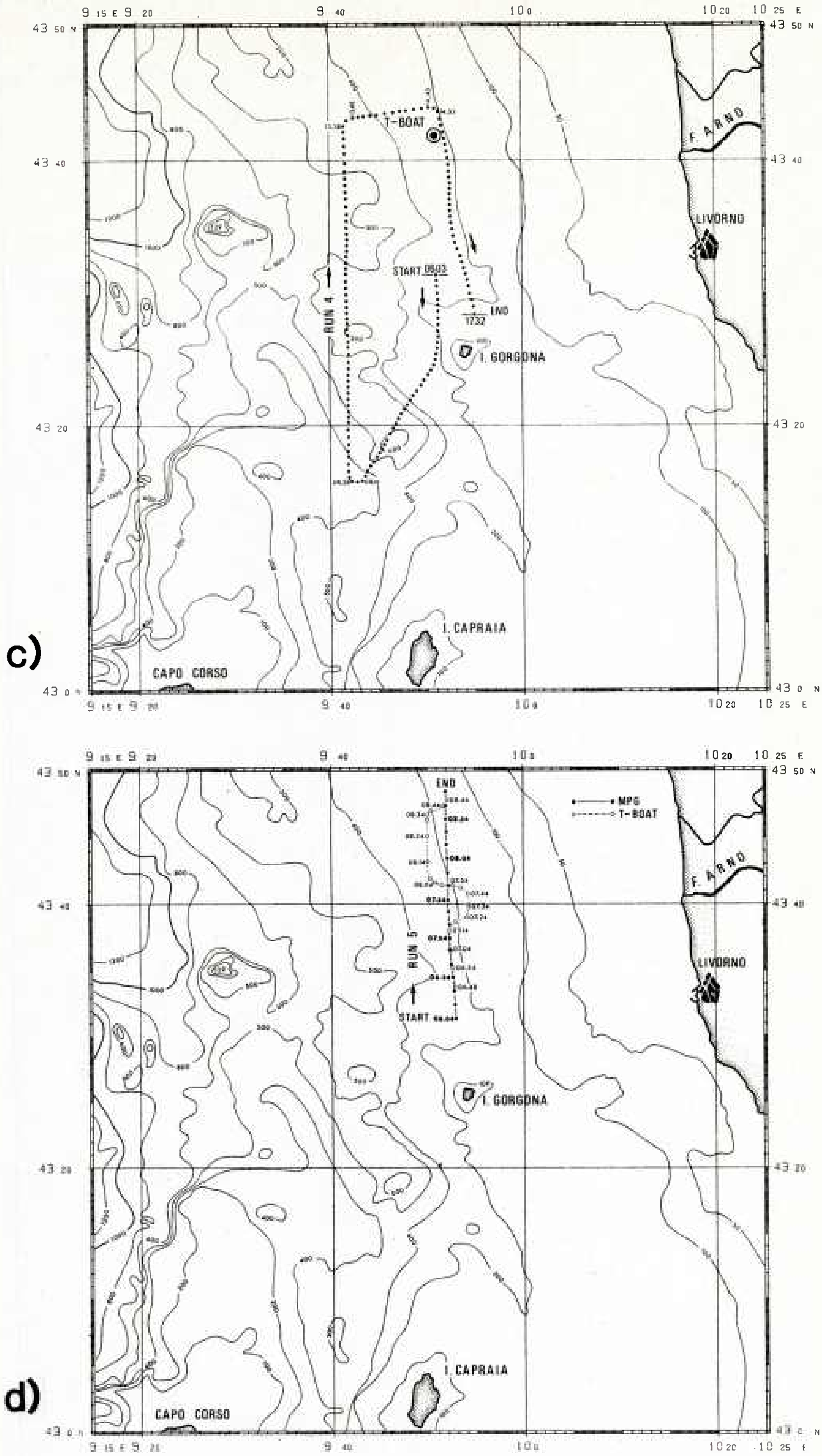


FIG. 1 SEA TRIALS
c) Run 4
d) Run 5

The conclusions and recommendations outline the proposals and arrangements for further studies and sea-trials with the aim of reducing the operational limitations caused by towship noise and improving the estimation of target bearing.

1 METHOD

1.1 Sea Trials

Sea-trials were conducted in the area off the northwestern Italian coast indicated on Fig. 1. Some parts of the area have a constant water depth of 150 m; in other parts the depth increases from 150 m to as much as 500 m. Bottom data are assumed to be the same as those measured by SACLANTCEN in an area a few kilometres further south (Area D1 of <1>). A 3½ m core (Core 127) in that area was mainly of silt with some shells, overlain in the upper 50 cm by sand and then by clay. The relative sound speed was 1.025 and the relative density was about 1.8. No data on bottom-reflection losses are available. For the theoretical predictions of transmission loss in the experimental area, sound-speed data have been taken from <2>.

During the trials the sea-state was SS-0 most of the time and never higher than SS-1. The towed array was deployed from SACLANTCEN's research vessel MARIA PAOLINA G. (MPG) as shown in Fig. 2 and most of the sea-trials were conducted at a speed of 5 kn. SACLANTCEN's workboat MANNING transmitted 334 Hz signals from a source suspended at depths of either 50 or 100 m. These signals were used primarily as a CW source with different source levels.

Figure 1 shows the relative positions of the MARIA PAOLINA G. and MANNING for the five runs constituting the trials. In Runs 1 to 4 (Figs. 1a, b, c) the MANNING was at anchor whilst transmitting; during Run 5 (Fig. 1d) it was underway. The various runs placed the MANNING on all possible bearings and at ranges from 1 km to 30 km. Figure 3 gives the sound-speed profiles recorded during the sea-trial. They show that a shallow sound channel usually existed at about 100 m.

1.2 Towed-Array System

SACLANTCEN's Prakla-Seismic towed array was used, with a maximum tow-cable scope of 700 m. The hydrophone spacing employed was 1.96 m, corresponding to $\lambda/2$ at approximately 385 Hz.

An additional hydrophone, referred to as the reference hydrophone, was used to investigate one of the basic methods of suppressing towship noise. This could be clamped onto the tow-cable at distances of up to 200 m behind the towship (Fig. 2). No streamlined body was available for this additional hydrophone during this sea trial and heavy flow noise was expected to degrade the correlation with the array signals. The signal from the reference hydrophone was brought in-board for use in the signal-processing techniques (see Ch. 3).

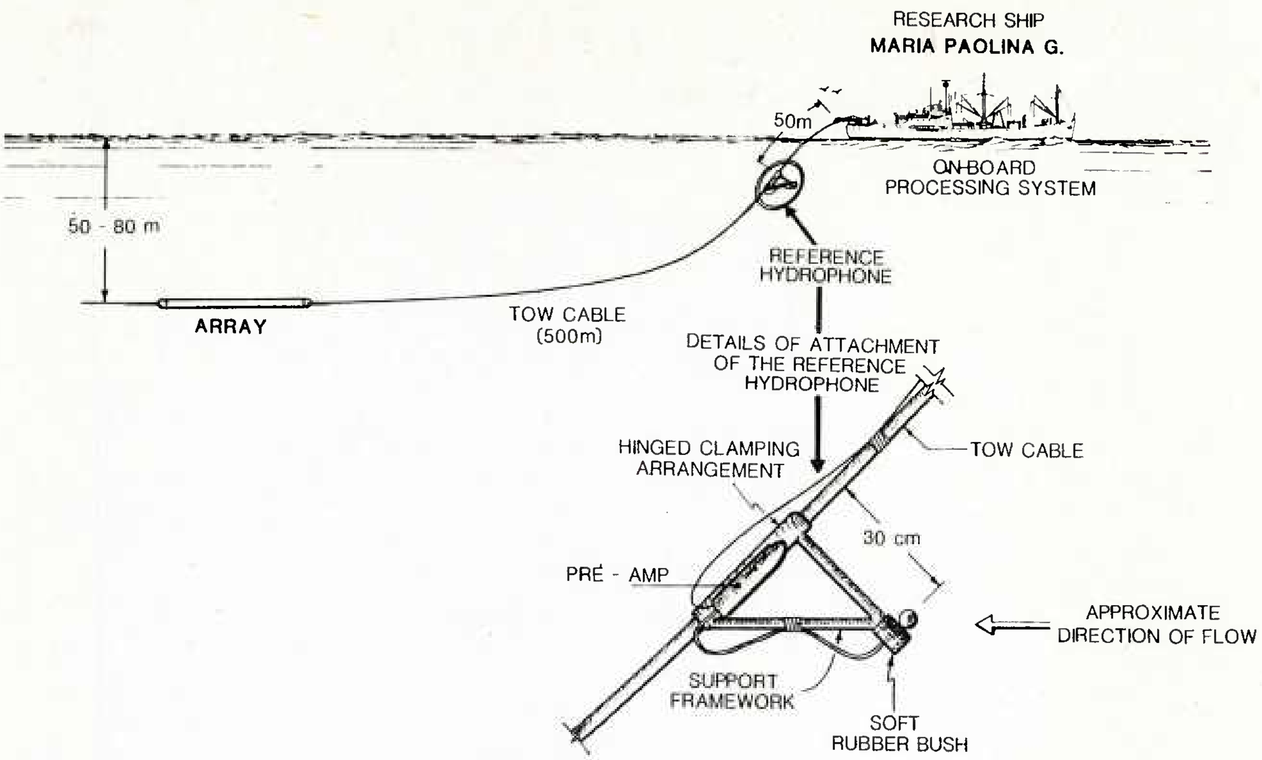


FIG. 2 DEPLOYMENT OF TOWED ARRAY DURING TRIALS

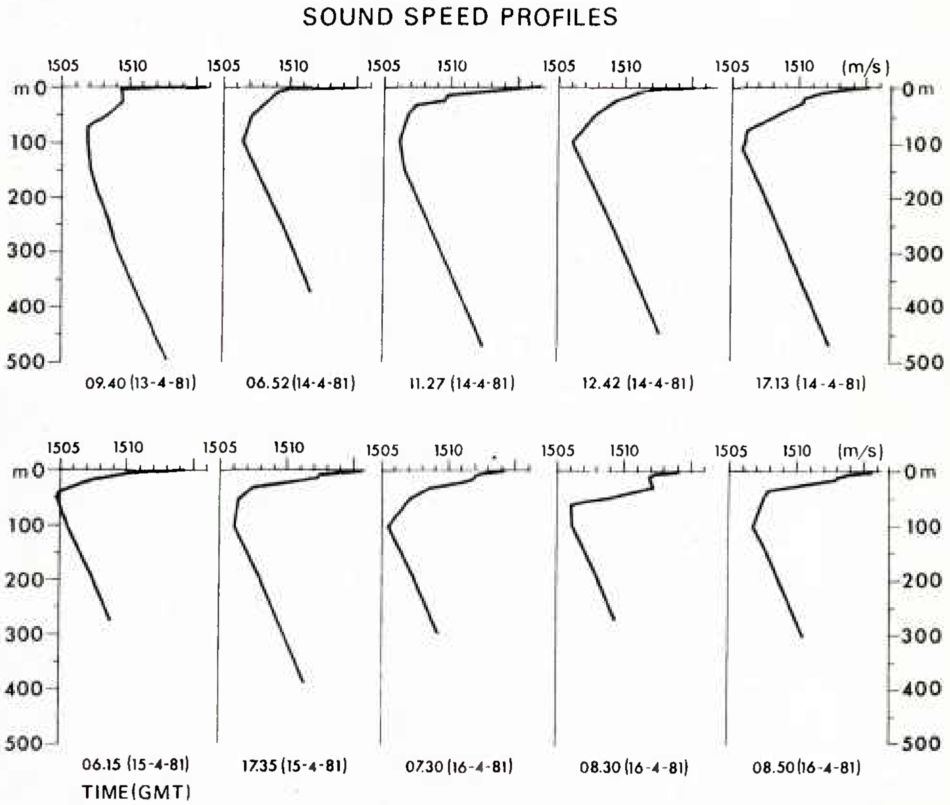


FIG. 3 SOUND-SPEED PROFILES DURING TRIALS

1.3 The On-Board Processing and Analysis System

A block diagram of the on-board processing and analysis system used in the sea-trials is shown in Fig. 4. It consisted of two main parts: the on-line monitoring system and the off-line processing system, as described in more detail in the following chapters.

2 ON-LINE DATA MONITORING

The on-line data monitoring system on board the MPG provided a display of either the output of a conventional beamforming system (CBF) or of the maximum-likelihood method (MLM) of beamforming for a single selected frequency. The display, which used the standard computer line-printer in a novel manner, gave an immediate visual indication of the power in the CBF or MLM beams. The component parts of the system are described below.

2.1 Signal Conditioning Unit

The lowest cutoff frequency of the available anti-aliasing lowpass filter bank was 1 kHz. As the frequency range of interest was below 350 Hz, a sampling frequency of ≧ 2.9 kHz was needed to have an aliasing level of ≦ -40 dB. For reasons given below, the chosen sampling frequency of the A/D converter was 12.288 kHz.

2.2 Digital Lowpass Filter Bank

The Plessey beamformer was used as a lowpass filter bank in order to reduce the sampling frequency by implementing 48-point linear-phase finite-impulse-response (FIR) filters with 3.072 kHz (= 12.288/4 kHz) input sampling frequency, and 819.2 Hz (= 12.288/15 kHz) output sampling frequency. The beamformer was also used to delay the data from the reference hydrophone.

2.3 On-Line Processing and Monitoring

A MAP array processor and an HP minicomputer were used for on-line processing and monitoring. The first operation was a 1024-point discrete fourier transform (DFT), implemented by a fast fourier transform (FFT), which gave a resolution about 0.8 Hz (= 819.2/1024 Hz), and a data window of 1.25 s length. The data collection and FFT processing had to be done in sequence, thereby giving an "acquisition" time of approximately 2.4 s. In other words, a time window of 1.25 s was repeated each 2.4 s, yielding a time gap of about 1.15 s. The DFT outputs for ten selected frequencies were written on magnetic tape for use by the off-line signal-processing system.

Flexibility in the choice of the parameters for the calculation of the covariance matrix was obtained by the scheme shown in Table 1. The covariance matrices are calculated for a selected number of frequencies

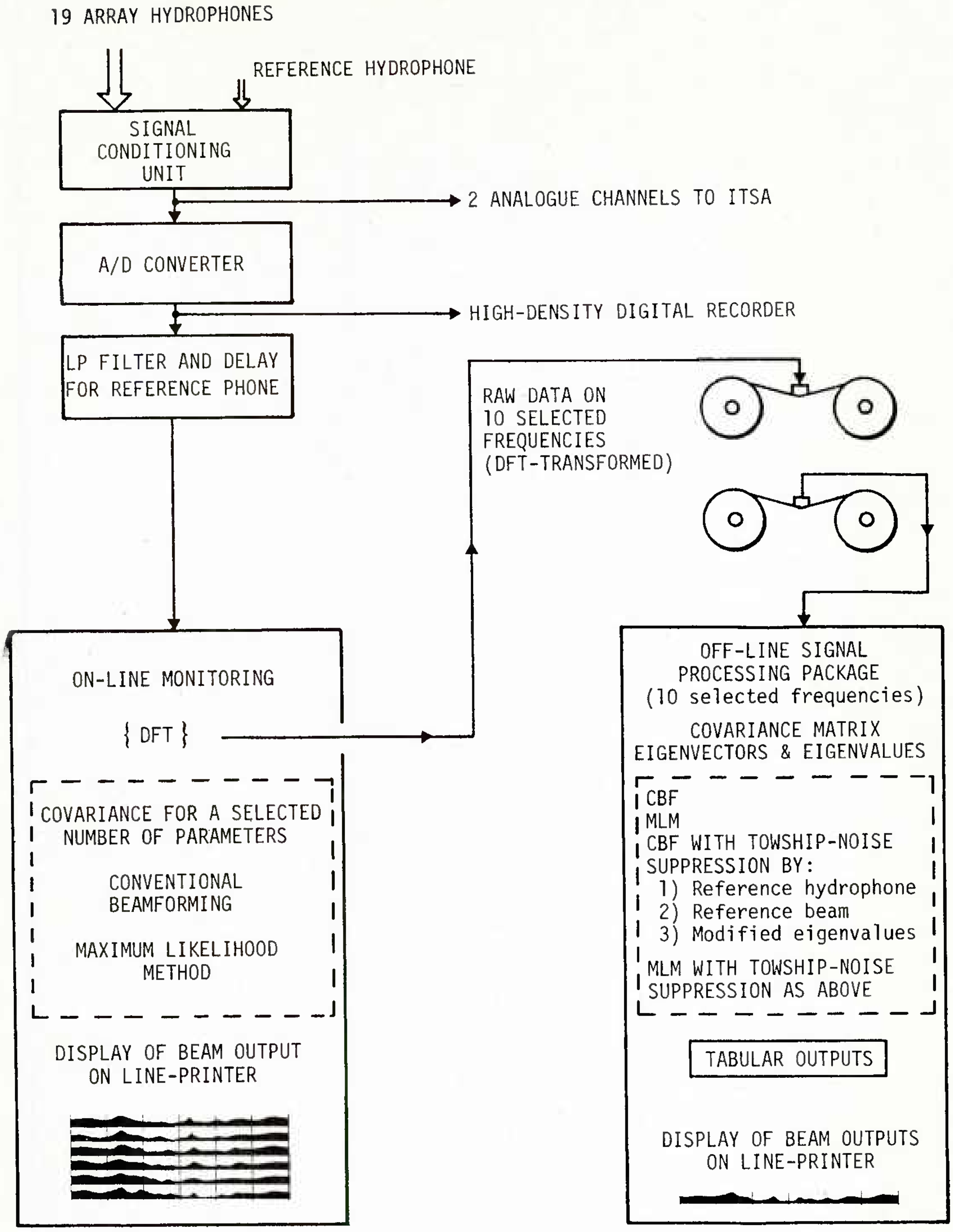


FIG. 4 ON-BOARD PROCESSING AND ANALYSIS SYSTEM

TABLE 1

CALCULATION OF THE NARROW-BAND SPATIAL COVARIANCE MATRIX

```
DO    50    NS                                    No. of averages
  DO    20    NE                                  No. of hydrophones

          CALL        X(T)                        Time-domain data for the
                                                  phone   (NE)th

          CALL        FFT                         Frequency-domain data in terms
                                                  of real and imaginary parts

    DO    10    NF                                No. of selected frequencies

          STORE REAL PART AND
          IMAGINARY PART FOR
          EACH FREQUENCY NF
          AND HYDROPHONE NE

    10    CONTINUE
  20    CONTINUE

  DO    40    NF                                  No. of selected frequencies
  LP = 1
    DO    30          K1 = 1,NE
    DO    30          K2 = 1,NE
    JF(K2. LE. K1) GOTO 30
    AR                 =                          Real part of        signal A
    AI                 =                          Imaginary part of   signal A
    BR                 =                          Real part of        signal B
    BI                 =                          Imaginary part of   signal B
    RC                 =     AR*BR+AI*BI
    AC                 =     AR*BI-AI*BR
    RCS(NF, LP)        =     RCS(NF, LP) + RC
    ACS(NF, LP)        =     ACS(NF, LP) + AC
    LP                 =     LP + 1
    30    CONTINUE
  40    CONTINUE

50    CONTINUE

          NC = NE *(NE-1)/2                       No. of combinations

DO    60    K          =     1, NF
DO    60    L          =     1, NC
RCS(K, L)              =     RCS(K, L)/NS
ACS(K, L)              =     ACS(K, L)/NS
60 CONTINUE
```

from filtered and fourier-transformed time-domain data of each hydrophone channel. The required average time is obtained by repeating this operation.

Thus for one selected frequency out of the ten, the output vector:

$$X_S = \left[X_{1,S}\ X_{2,S}\ \ldots\ldots\ldots,\ X_{N,S} \right]^T$$

was used to build up an estimate of the covariance matrix:

$$R_j = \frac{1}{M} \sum_{i=1}^{M} X(Mj+i) \cdot X^*(Mj+i) \quad , \qquad \text{(Eq. 1)}$$

where N is the number of hydrophones used to build up the output vector X_S for the time index S and j is the number of matrices calculated.

The matrix R_j was used to produce the mean power outputs for 181 beams, one for every degree, from one endfire to the other. These outputs were converted to decibel levels and then displayed on a line printer. An example for conventional beamforming is shown in Fig. 5. The bearing is marked above the top-line for a narrow-band processing of 334 Hz. All other outputs were produced with the off-line processing system. Dependent on the number of acquired time-series, such an output was produced for monitoring purpose at intervals of 1 to 2 minutes. The beamforming was either conventional:

$$P_{k,j} = B_k^* R_J B_k \qquad k = 1, 2, \ldots 181 \qquad \text{(Eq. 2)}$$

or "maximum-likelihood":

$$P_{k,j} = \left(B_k^* R_j^{-1} B_k \right)^{-1} \qquad k = 1, 2, \ldots 181 \quad , \qquad \text{(Eq. 3)}$$

where B_k is the steering vector determining the beam direction:

$$B_k = \left[\ldots\ldots,\ \exp\left(-j2\pi \frac{x_n}{\lambda} \cos \beta_k \right),\ \ldots\ldots \right]^T \quad , \qquad \text{(Eq. 4)}$$

in which x_n is the coordinate of hydrophone n and β_k is beam direction.

Every time the covariance matrix were calculated, the "acquisition" time was increased by the order of 2 to 3 seconds. Depending on the number of acquisitions, the beam outputs were calculated and printed from the averaged matrix. The minimum number of averages had to equal the number of hydrophones.

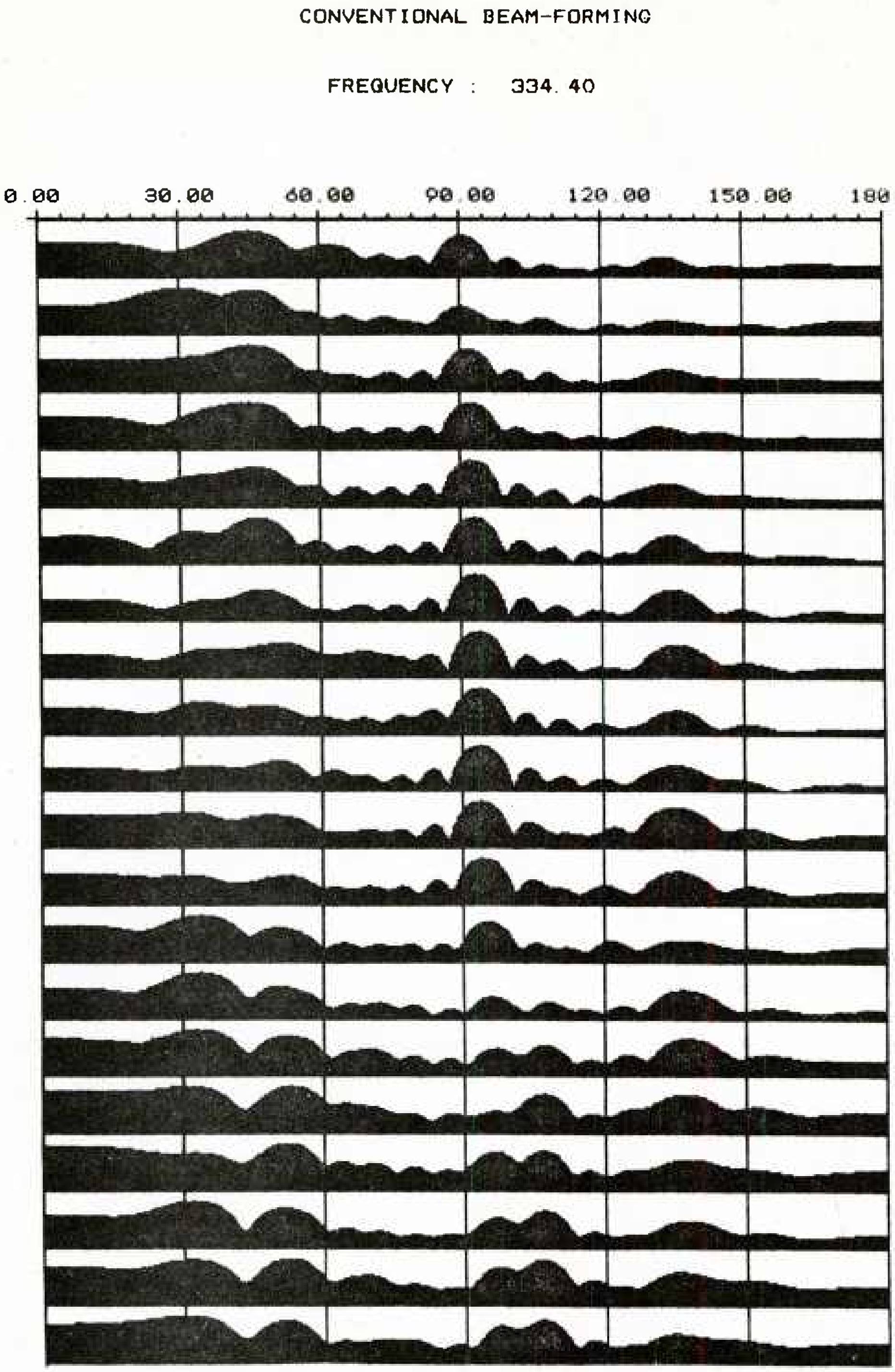


FIG. 5 *EXAMPLE OF LINE-PRINTER OUTPUT FOR CONVENTIONAL BEAMFORMING*

3 OFF-LINE PROCESSING AND ANALYSIS SYSTEM

The off-line processing and analysis system used tapes of discrete fourier transform (DFT) data produced by the on-line system for ten preselected frequencies and gave, as outputs, the line-printer display mentioned above for the on-line monitoring system and tabulated data relating to the various parameters of interest. The off-line analysis provided the above outputs for the following signal-processing techniques:

a. Conventional beamforming with no shading (CBF).

b. Maximum-likelihood method of beamforming (MLB).

c. CBF and MLB with basic techniques for suppressing towship noise by:

    1) Use of a reference hydrophone.
    2) Use of a reference beam.
    3) Use of modified eigenvalues.

3.1 General

The off-line processing system consisted of an HP minicomputer with vector instruction set (VIS) option <3>, a magnetic tape station, a line printer, and a terminal. The input data from magnetic tape were from 19 array hydrophones and one reference hydrophone, quadrature bandpass-filtered around ten selected frequencies and with a bandwidth of 0.8 Hz. These data were then processed by a signal-processing program, and the output written on disc. A program produced plots of these data on the line printer in the same format as the on-line monitoring program (Fig. 5).

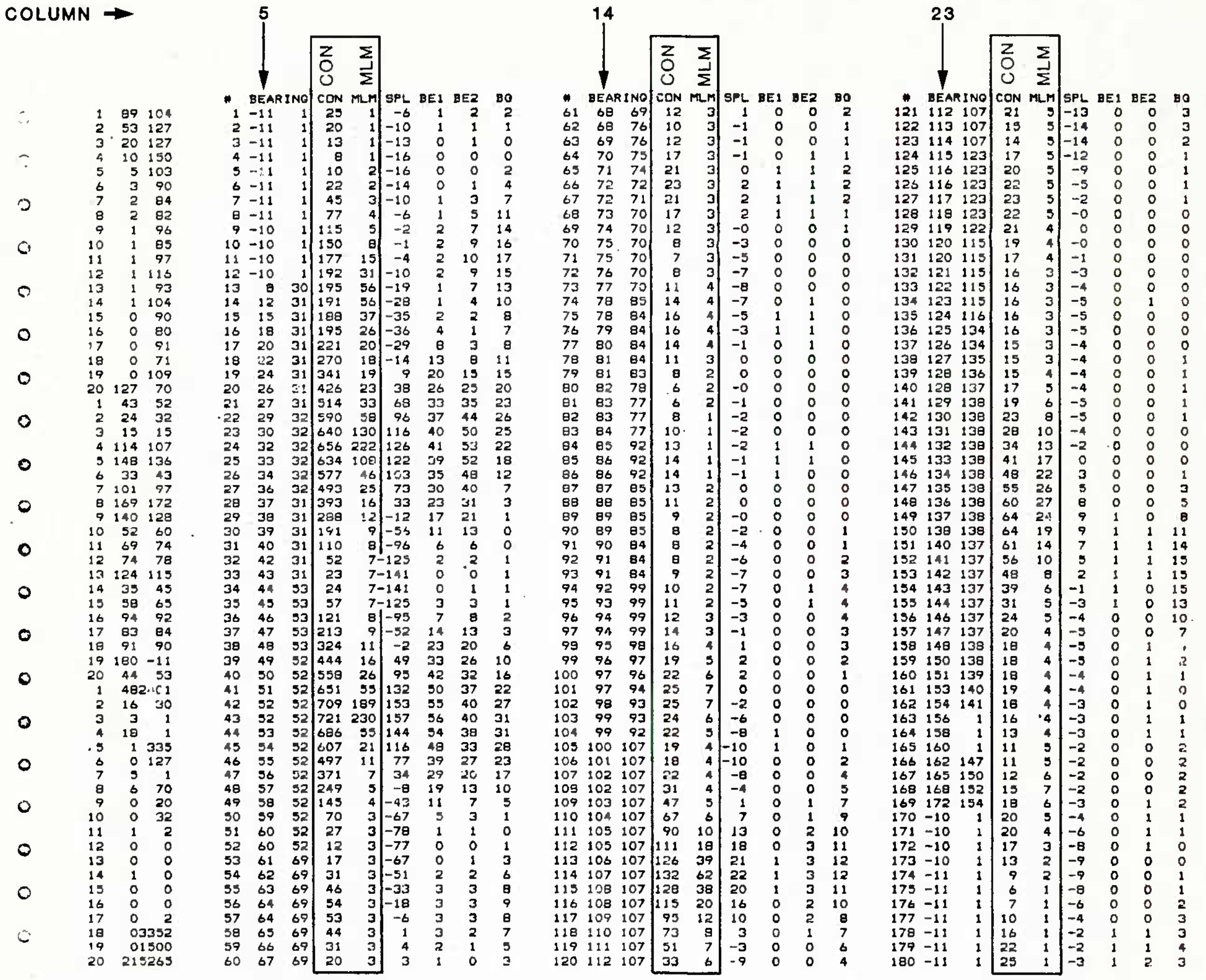
In addition a tabular output was produced, as shown in Fig. 6, for each line-printer output. This tabular output shows a whole range of data; for example, columns 4, 13 and 22 give the beam numbers from 1 to 180, columns 5, 14 and 23 give the beam angles for the frequency in use, and the columns under CON and MLM give the values of $P_{k,j}$ (Eqs. 2 and 3) for the conventional and "maximum-likelihood" beam processing.

The first step in the processing program is to create the following quantities:

$$R_{j,q} = \frac{1}{M} \sum_{i=1}^{M} X\left(Mj+i,q\right)\cdot X^{*}\left(Mj+i,q\right) \qquad \text{(Eq. 5)}$$

$$Q_{j,q} = \frac{1}{M} \sum_{i=1}^{M} X\left(Mj+i,q\right)\cdot y^{*}\left(Mj+i,q\right) \qquad \text{(Eq. 6)}$$

$$r_{j,q} = \frac{1}{M} \sum_{i=1}^{M} y\left(Mj+i,q\right)\cdot y^{*}\left(Mj+i,q\right) \qquad \text{(Eq. 7)}$$



```
COLUMN →                      5                                         14                                        23
                                        CON  MLM                                  CON  MLM                                  CON  MLM
                     #  BEARING         CON  MLM  SPL  BE1  BE2   BO      #  BEARING  CON  MLM  SPL  BE1  BE2   BO      #  BEARING  CON  MLM  SPL  BE1  BE2   BO
  1   89  104        1  -11    1   25    1   -6    1    2    2     61   68   69   12    3    1    0    0    2    121  112  107   21    5  -13    0    0    3
  2   53  127        2  -11    1   20    1  -10    1    1    1     62   68   76   10    3   -1    0    0    1    122  113  107   15    5  -14    0    0    3
  3   20  127        3  -11    1   13    1  -13    0    1    0     63   69   76   12    3   -1    0    0    1    123  114  107   14    5  -14    0    0    2
  4   10  150        4  -11    1    8    1  -16    0    0    0     64   70   75   17    3   -1    0    1    1    124  115  123   17    5  -12    0    0    1
  5    5  103        5  -11    1   10    2  -16    0    0    2     65   71   74   21    3    0    1    1    2    125  116  123   20    5   -9    0    0    1
  6    3   90        6  -11    1   22    2  -14    0    1    4     66   72   72   23    3    2    1    1    2    126  116  123   22    5   -5    0    0    1
  7    2   84        7  -11    1   45    3  -10    1    3    7     67   72   71   21    3    2    1    1    2    127  117  123   23    5   -2    0    0    1
  8    2   82        8  -11    1   77    4   -6    1    5   11     68   73   70   17    3    2    1    1    1    128  118  123   22    5   -0    0    0    0
  9    1   96        9  -10    1  115    5   -2    2    7   14     69   74   70   12    3   -0    0    0    1    129  119  122   21    4    0    0    0    0
 10    1   85       10  -10    1  150    8   -1    2    9   16     70   75   70    8    3   -3    0    0    0    130  120  115   19    4   -0    0    0    0
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 13    1   93       13    8   30  195   56  -19    1    7   13     73   77   70   11    4   -8    0    0    0    133  122  115   16    3   -4    0    0    0
 14    1  104       14   12   31  191   56  -28    1    4   10     74   78   85   14    4   -7    0    1    0    134  123  115   16    3   -5    0    1    0
 15    0   90       15   15   31  188   37  -35    2    2    8     75   78   84   16    4   -5    1    1    0    135  124  116   16    3   -5    0    0    0
 16    0   80       16   18   31  195   26  -36    4    1    7     76   79   84   16    4   -3    1    1    0    136  125  134   16    3   -5    0    0    0
 17    0   91       17   20   31  221   20  -29    8    3    8     77   80   84   14    4   -1    0    1    0    137  126  134   15    3   -4    0    0    0
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  1   43   52       21   27   31  514   33   68   33   35   23     81   83   77    6    2   -1    0    0    0    141  129  138   19    6   -5    0    0    1
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  1   48201         41   51   52  651   55  132   50   37   22    101   97   94   25    7    0    0    0    0    161  153  140   19    4   -4    0    1    0
  2   16   30       42   52   52  709  189  153   55   40   27    102   98   93   25    7   -2    0    0    0    162  154  141   18    4   -3    0    1    0
  3    3    1       43   52   52  721  230  157   56   40   31    103   99   93   24    6   -6    0    0    0    163  156    1   16    4   -3    0    1    1
  4   18    1       44   53   52  686   55  144   54   38   31    104   99   92   22    5   -8    1    0    0    164  158    1   13    4   -3    0    1    1
  5    1  335       45   54   52  607   21  116   48   33   28    105  100  107   19    4  -10    1    0    1    165  160    1   11    5   -2    0    0    2
  6    0  127       46   55   52  497   11   77   39   27   23    106  101  107   18    4  -10    0    0    2    166  162  147   11    5   -2    0    0    2
  7    5    1       47   56   52  371    7   34   29   20   17    107  102  107   22    4   -8    0    0    4    167  165  150   12    6   -2    0    0    2
  8    6   70       48   57   52  249    5   -8   19   13   10    108  102  107   31    4   -4    0    0    5    168  168  152   15    7   -2    0    0    2
  9    0   20       49   58   52  145    4  -43   11    7    5    109  103  107   47    5    1    0    1    7    169  172  154   18    6   -3    0    1    2
 10    0   32       50   59   52   70    3  -67    5    3    1    110  104  107   67    6    7    0    1    9    170  -10    1   20    5   -4    0    1    1
 11    1    2       51   60   52   27    3  -78    1    1    0    111  105  107   90   10   13    0    2   10    171  -10    1   20    4   -6    0    1    1
 12    0    0       52   60   52   12    3  -77    0    0    1    112  105  107  111   18   18    0    3   11    172  -10    1   17    3   -8    0    1    0
 13    0    0       53   61   69   17    3  -67    0    1    3    113  106  107  126   39   21    1    3   12    173  -10    1   13    2   -9    0    0    0
 14    1    0       54   62   69   31    3  -51    2    2    6    114  107  107  132   62   22    1    3   12    174  -11    1    9    2   -9    0    0    1
 15    0    0       55   63   69   46    3  -33    3    3    8    115  108  107  128   38   20    1    3   11    175  -11    1    6    1   -8    0    0    1
 16    0    0       56   64   69   54    3  -18    3    3    9    116  108  107  115   20   16    0    2   10    176  -11    1    7    1   -6    0    0    2
 17    0    2       57   64   69   53    3   -6    3    3    8    117  109  107   95   12   10    0    2    8    177  -11    1   10    1   -4    0    0    3
 18     03352       58   65   69   44    3    1    3    2    7    118  110  107   73    8    3    0    1    7    178  -11    1   16    1   -2    1    1    3
 19     01500       59   66   69   31    3    4    2    1    5    119  111  107   51    7   -3    0    0    6    179  -11    1   22    1   -2    1    1    4
 20    215265       60   67   69   20    3    3    1    0    3    120  112  107   33    6   -9    0    0    4    180  -11    1   25    1   -3    1    2    3
```

FIG. 6 TABULAR OUTPUT CORRESPONDING TO LINE-PRINTER OUTPUT where

X : Data vector from array hydrophones
y : Data scalar from reference hydrophone
R : Covariance matrix
Q : "Reference vector"
r : "Reference scalar"
q : Frequency index
i : Acquisition time index
j . Time index after averaging
M : Number of acquisitions

The indices are omitted hereafter.

The next step was to obtain the eigenvalues and eigenvectors of R. As shown in App. A, R can be written

$$R = \sum_{n=1}^{N} \lambda_n E_n E_n^* \quad , \qquad \text{(Eq. 8)}$$

where λ_n are eigenvalues

E_n are eigenvectors

$n = 1, 2 \ldots N$

and N is the number of the array hydrophones

There were three reasons for introducing the "eigen-quantities". Firstly, these quantities might be of interest on their own. Several signal-processing methods are based on explicit knowledge of these quantities. Only one was implemented in this version of the program, the so-called modified eigenvalue method.

Secondly, by calculating these eigen-quantities, one might gain some feeling for their behaviour under more or less complex sound-field conditions.

Thirdly, as shown in App. A and B, the ten signal-processing modes implemented for this sea trial could all be formulated and calculated in terms of the eigen-quantities. The signal processing in the off-line program was therefore based on the use of the eigen-quantities, whereas the on-line program used a matrix inversion for the maximum-likelihood technique. A check on the two programs could thus be made by comparing the MLM columns of the outputs illustrated in Fig. 6. The following sections describe these methods both with and without the use of eigenvalues and eigenvectors.

3.2 Conventional Beamforming (CB)

The mean power output from an unweighted (unshaded) conventional beam can be written as (Sect. B.1 of App. B):

$$P = B^{*} R B \quad .$$

Introducing eigenvalues and eigenvectors:

$$P = \sum_{n=1}^{N} \lambda_n \left| B^{*} E_n \right|^2 \quad . \qquad \text{(Eq. 9)}$$

3.3 "Maximum-Likelihood" Beamforming (MLB)

The mean power output can in this case be written as (Sect. B.2 of App. B):

$$P = \left(B^{*} R^{-1} B \right)^{-1} \quad , \qquad \text{(Eq. 10)}$$

and with eigenvalues and eigenvectors:

$$P = \left(\sum_{n=1}^{N} \lambda_n^{-1} \left| B^{*} E_n \right|^2 \right)^{-1} \quad . \qquad \text{(Eq. 11)}$$

3.4 Conventional beamforming plus modified eigenvalue noise suppression (CB + ME)

The eigenvalues of R were modified according to some rules. One such rule was to make eigenvalue λ_i equal to the smallest eigenvalue if the corresponding eigenvalue E_i correlated well with the steering vector representing the direction of the towship. Then:

$$P = \sum_{n=1}^{N} \lambda_n' \left| B^{*} E_n \right|^2 \quad . \qquad \text{(Eq. 12)}$$

3.5 Maximum likelihood plus modified eigenvalue (ML + ME)

This is a combination of ML beamforming with modified eigenvalues

$$P = \left(\sum_{n=1}^{N} \lambda'^{-1} \left| B^{*} E_n \right|^2 \right)^{-1} \quad . \qquad \text{(Eq. 13)}$$

3.6 Conventional beamforming plus reference beam noise suppression (CB+RB)

A reference beam direction with steering vector B_r is chosen according to some rule and this is used to calculate (Sect. B.3 of App. B):

$$P = \left(B^*RB\right) \left[1 - \left|B^*RB_r\right|^2 \left(B^*RB \cdot B_r^* R\ B_r\right)^{-1}\right]. \qquad \text{(Eq. 14)}$$

And with eigen-quantities:

$$B^*\ R\ B = \sum_{n=1}^{N} \lambda_n \left|B^*E_n\right|^2 \quad , \qquad \text{(Eq. 15)}$$

$$B_r^*\ R\ B_r = \sum_{n=1}^{N} \lambda_n \left|B_r^*E_n\right|^2 \quad , \qquad \text{(Eq. 16)}$$

$$B^*\ R\ B_r = \sum_{n=1}^{N} \lambda_n \left(B^*E\right) \left(B_r^*\ E_n\right)^* \quad . \qquad \text{(Eq. 17)}$$

3.7 "Maximum likelihood" plus reference beam noise suppression (ML + RB)

Also, from Sect. B.3 of App. B, we have:

$$P = \left(B^*R^{-1}B\right)^{-1} \left[1 - \left|B^*R^{-1}B_r\right|^2 \left(B^*R^{-1}B \cdot B_r^*R^{-1}B_r\right)^{-1}\right] \quad . \qquad \text{(Eq. 18)}$$

And with eigen-quantities:

$$B^*R^{-1}B = \sum_{n=1}^{N} \lambda_n^{-1} \left|B^*E_n\right|^2 \quad , \qquad \text{(Eq. 19)}$$

$$B_r^*R^{-1}B_r = \sum_{n=1}^{N} \lambda_n^{-1} \left|B_r^*E_n\right|^2 \quad , \qquad \text{(Eq. 20)}$$

$$B^*R\ B_r = \sum_{n=1}^{N} \lambda_n^{-1} \left(B^*E_n\right) \left(B_r^*E_n\right)^* \quad . \qquad \text{(Eq. 21)}$$

3.8 Conventional beamforming plus reference hydrophone noise suppression (CB + RH)

The mean output power in this mode is (Sect. B.4 of App. B):

$$P = B^*R\ B \cdot \left[1 - \left|B^*Q\right|^2 \left(B^*R\ B \cdot r\right)^{-1}\right] \qquad \text{(Eq. 22)}$$

With eigenvector and eigenvalues, this was implemented using:

$$B^*R\,B = \sum_{n=1}^{N} \lambda_n \left|B^*E_n\right|^2 \quad , \qquad \text{(Eq. 23)}$$

$$B^*\,Q = \sum_{n=1}^{N} (B^*E_n)\,(Q^*E_n)^* \quad . \qquad \text{(Eq. 24)}$$

3.9 "Maximum Likelihood" plus reference-hydrophone noise suppression (ML + RH)

With reference to Sect. B.4 of App. B:

$$P = (B^*R^{-1}B)^{-1}\left[1+\left|B^*R^{-1}Q\right|^2 \{B^*R^{-1}B(r - Q^*R^{-1}Q)\}^{-1}\right] \quad . \qquad \text{(Eq. 25)}$$

With eigen-quantities

$$B^*R^{-1}B = \sum_{n=1}^{N} \lambda_n^{-1} \left|B^*E_n\right|^2 \quad , \qquad \text{(Eq. 26)}$$

$$B^*R^{-1}Q = \sum_{n=1}^{N} \lambda_n^{-1} (B^*E_n)\,(Q^*E_n)^* \quad , \qquad \text{(Eq. 27)}$$

$$Q^*R^{-1}Q = \sum_{n=1}^{N} \lambda_n^{-1} \left|Q^*E_n\right|^2 \quad . \qquad \text{(Eq. 28)}$$

4 RESULTS

This chapter presents and discusses sample data from all runs except Run 2, which are similar to those from Runs 1, 3 and 4. Some data are presented from a previous sea-trial to show the interference effects in the signal paths from a remotely deployed transducer to an array.

4.1 Signal path from towship to array in shallow-water

The geometry of the trials is shown on the left of Table 2. Typical geometry with an operational towed array is, however, more likely to be as shown on the right of the table.

TABLE 2

GEOMETRY OF TRIALS AND TYPICAL GEOMETRY

	Trials	Typical
Source depth	3 m	2 to 5 m
Tow-cable scope	200 to 700 m	500 to 2000 m
Array length	40 m	50 to 200 m
Array depth	50 to 100 m	-
Water depth	100 to 500 m	-

Together with the type of sea-bottom, the sea-state, and the sound-speed profile, the geometry defines the towship's noise field at an array towed in shallow water. It will thus be appreciated that the noise field at the towed array can be very complex and difficult to predict. Thus care must be taken in using any of the various available techniques.

Ray-tracing and normal-mode propagation are derived theoretically in App. D, indicating that ray-tracing cannot describe accurate low-frequency propagation in shallow water. For ray-tracing there is a general rule of thumb that the water depth should be greater than 10λ. Table 3 indicates some safe limits for the use of ray tracing.

TABLE 3

MINIMUM WATER DEPTH (10λ) FOR SAFE USE OF RAY TRACING

Frequency (Hz)	Minimum depth (m)
50	300
100	150
250	60
500	30

Thus, for example, ray tracing for frequencies of 100 Hz should not be used in water shallower than 150 m; equally, in a water depth of 150 m ray tracing should not be used for frequencies below 100 Hz.

In the case of the normal-mode solution there is a difficulty at short range because of the incomplete description of the acoustical field in terms of the normal-mode solution, which is valid for farfield only. It is usual to discard the short-range continuous field. Thus there is a minimum range below which the discrete modes alone do not sufficiently describe the field. This minimum range is usually taken as 3 to 5 water depths. Thus in 150 m water depth, ray tracing would be satisfactory at frequencies above 100 Hz out to ranges of between 450 to 750 m. Beyond this range the normal-mode solution should be used, particularly for shallow water. As, in fact, different modes travel with different velocities and arrive at different angles, a mismatch of the array to the acoustical field might become a serious problem for signal processing at low frequencies. This is evidently true for the endfire direction of a horizontal array <5>. A short explanation of the need to distinguish between rays and modes for array processing at low frequencies in shallow water can be given readily by use of Eq. 4, this being the steering vector determining the beam direction. The factor $2\pi x_n/\lambda$ in that equation, which is essentially the array aperture size in wave length, can be rewritten as $\omega x_n/c$, where c is the phase velocity at the receiving hydrophone and is a constant value at one receiver for all the rays impinging on this receiver. However for all modes exhibiting their individual phase velocities, this conventional array processing will be matched to one velocity only, hence with increasing dispersion the array will become increasingly mismatched.

It is not the intention of this present memorandum to make an extensive analysis of the towship's noise field at the array (such an analysis should probably use the fast field program or the range-dependent parabolic equation techniques <4>), but merely to show that the noise field is complex, with a covariance matrix of rank >1. This is the same as saying that the noise from one source arrives from different directions at the same receiver when added coherently. The resolution of various paths from the towship with the array was shown in <5>, using different signal-processing techniques. With those methods and a fairly good signal-to-noise ratio, only four phones of a towed array were used to resolve the towship's noise directions.

An additional indication of the complex nature of acoustic signals in shallow water was obtained as a side result of a previous sea trial. Figure 7 gives an experimental example of mode interference over a signal path of 3 to 4 km. The range from the transmitting site to the receiving

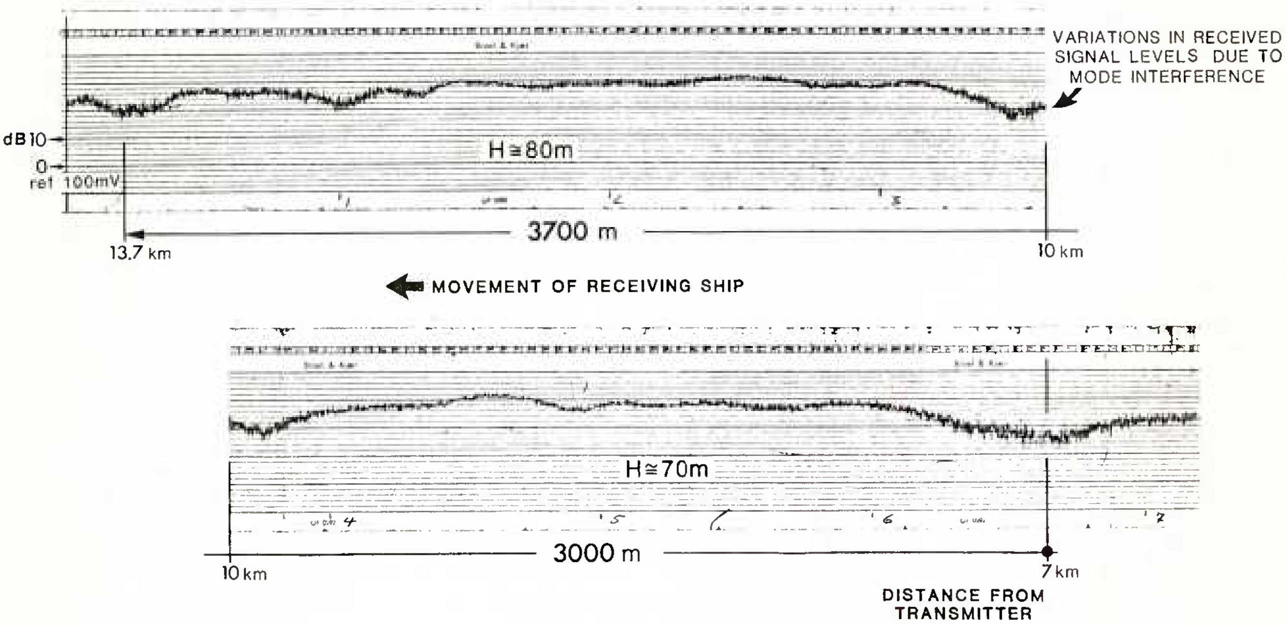

FIG. 7 EXAMPLE OF MODE INTERFERENCE AT 334 Hz RECEIVED WITH A TOWED ARRAY (taken from earlier experiments)

ship varied from 7 to about 14 km during the measurements. For this distance the modes can be assumed to be well separated. A strong, periodically repeated change (~10 dB) of the received signal level was observed while the ship was approaching the MANNING's position, where a transducer submerged at 30 m water depth transmitted a 334 Hz CW signal. The array was towed at 45 m depth in a shallow-water area of about 80 m water depth with a slightly upward-sloping bottom. The interference wavelength is calculated using the approximation for a 'soft' bottom of $\lambda_{n,m} \approx 8\, H^2 f/c_1(n^2-m^2)$. From a comparison of the experimental and theoretical results for the first three modes, it is believed that the first and second modes have the strongest influence on this change in level. Table 4 compares the calculated and measured interference wavelength.

TABLE 4

INTERFERENCE WAVELENGTH $\lambda_{n,m}$ FOR TWO DIFFERENT WATERDEPTHS

THEORETICALLY

Mode	H = 70 m	H = 80 m
1+2	2909 m	3800 m
2+3	1745 m	2280 m
1+3	1091 m	1425 m

OBSERVED

H = 70 m	H = 80 m
~ 3000 m	~ 3700 m

No further investigation was made during the sea trial. The observed values shown in Fig. 7 indicate the range from own ship to the transmitting site.

In the present sea-trial the water depth was generally greater than 150 m, the frequency over 100 Hz, and the cable scope of the towed array less than 750 m. Therefore straight-line ray tracing rather than normal-mode arrivals was used to estimate the angle of arrival at the array of the most likely paths <7>:

Direct (D)
Bottom-bounce (B)
Bottom/surface (BS)
Bottom/surface/bottom (BSB)
Bottom/surface/bottom/surface (BSBS)

Figure 8 shows the Maximum Likelihood Beamformer (MLB) output and the corresponding water depths and arrival angles for (a) 160 Hz in Run 1, (b) 335.2 Hz in Run 3, and (c) 280 Hz in Run 4. The angles given on the sea-depth profile correspond to the paths quoted above.

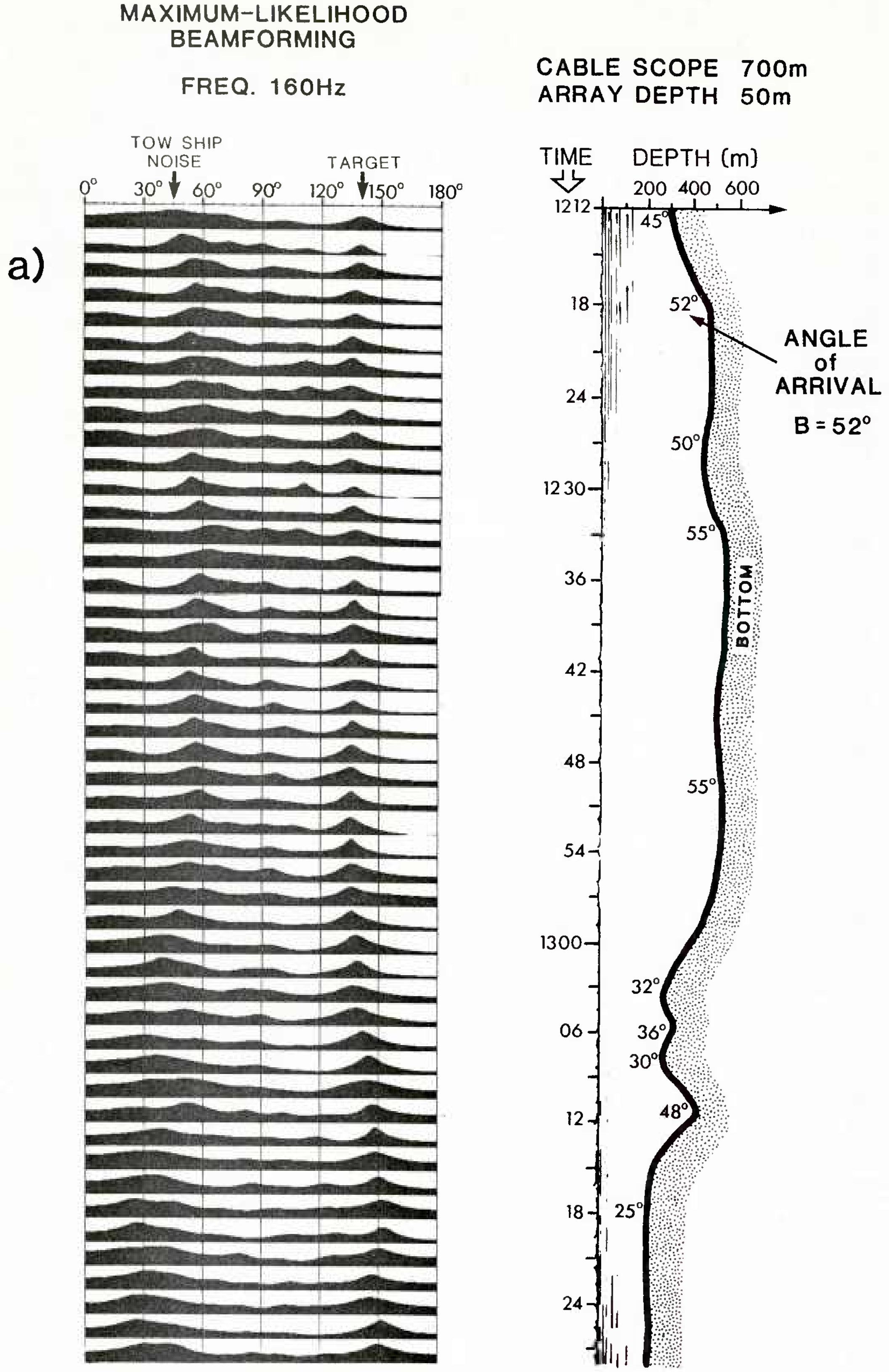

FIG. 8a MAXIMUM LIKELIHOOD BEAMFORMING OUTPUT SHOWING CORRESPONDING WATER DEPTHS AND ARRIVAL ANGLES AT THE BOTTOM
160 Hz (Run 1)

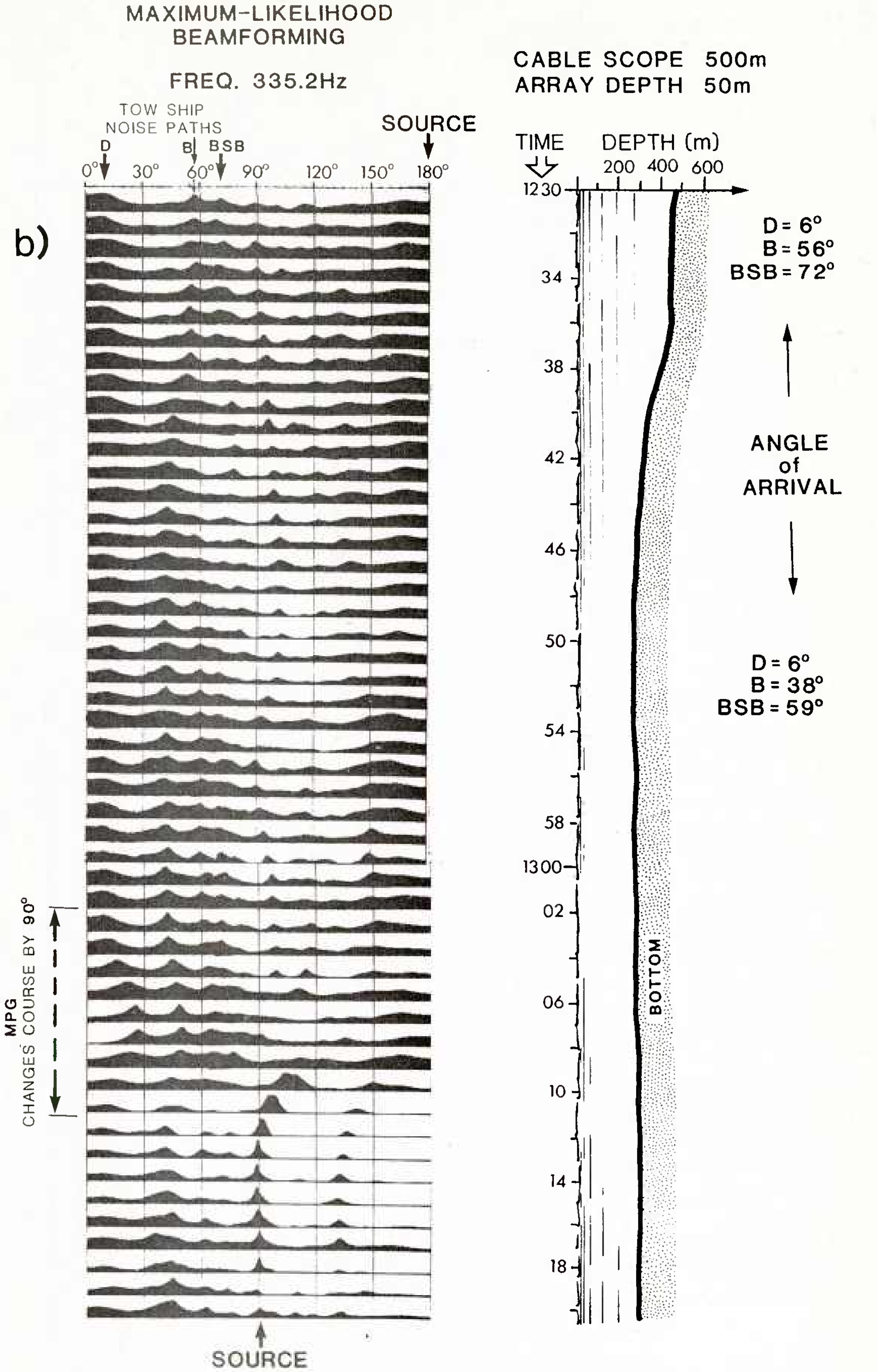

FIG. 8b MAXIMUM LIKELIHOOD BEAMFORMING OUTPUT SHOWING CORRESPONDING WATER DEPTHS AND ARRIVAL ANGLES AT THE BOTTOM
335.2 Hz (Run 3)

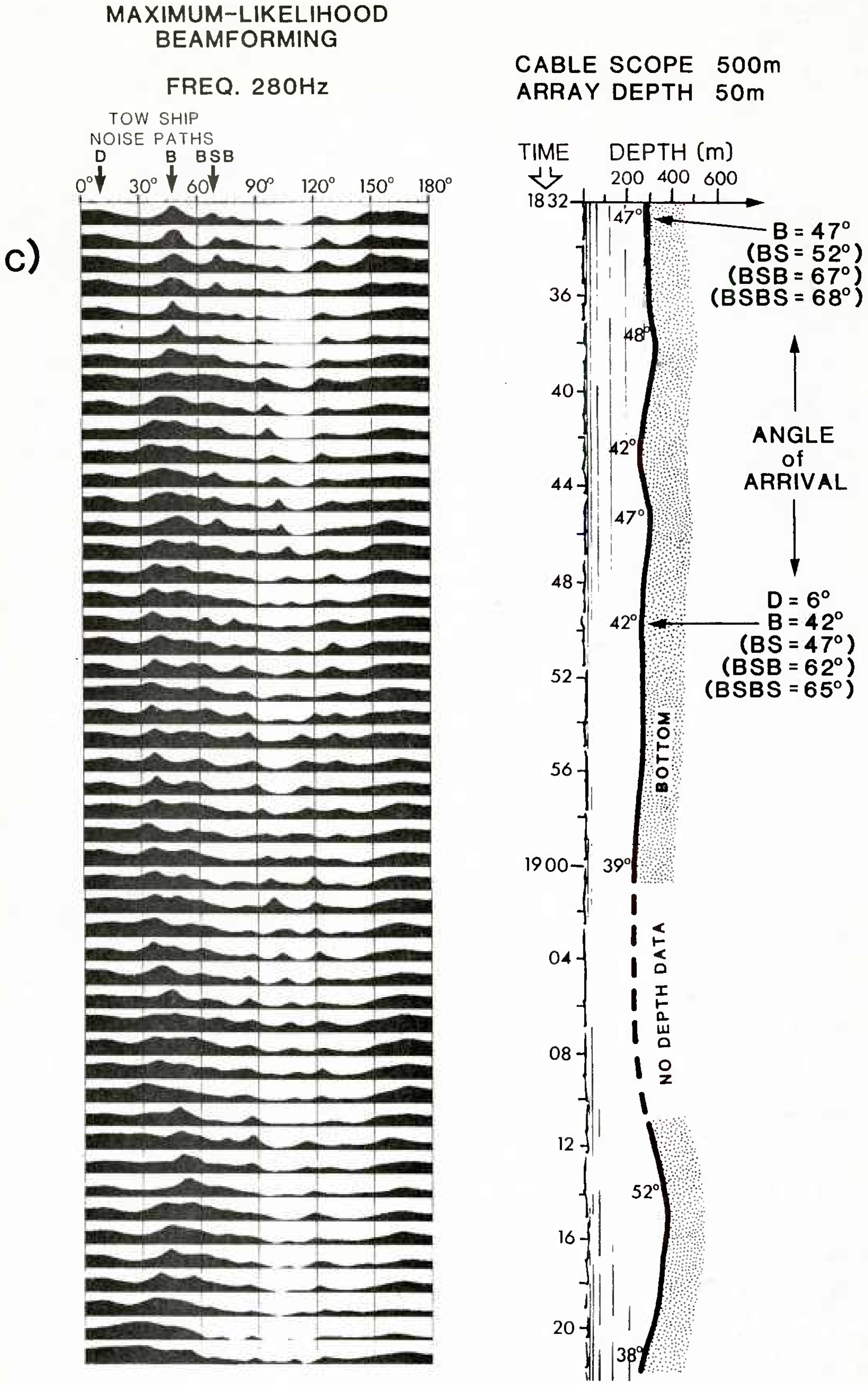

FIG. 8c MAXIMUM LIKELIHOOD BEAMFORMING OUTPUT SHOWING CORRESPONDING WATER DEPTHS AND ARRIVAL ANGLES AT THE BOTTOM 280 Hz (Run 4)

It is seen from these figures that the bottom-bounce path (B) is usually the dominant one, although the direct path (D) and the bottom/surface/bottom path (BSB) are also observed, particularly at the higher frequency when the angular resolution is better.

Figure 8b includes data obtained whilst the towship was making a 90° turn and it is seen how the array at 500 m scope lagged behind the MPG during the turn so that the direct path made a bigger angle with the array. These direct-path data indicate how, at this frequency, when the array is looking end-fire it can indicate the vertical angle of arrival of a signal. For example, the direct path under the steady-state conditions of Run 3 arrives at about 5° (arcsin 40/450), as shown in Fig. 8b.

4.2 Towship-noise suppression

This discussion starts with some mathematical observations about the three methods implemented:

a. Modified eigenvalues
b. Reference beam
c. Reference hydrophone

The question is to determine how the methods modify the covariance matrix R given by:

$$R = \sum_{n=1}^{N} \lambda_n E_n E_n^* \quad , \qquad \text{(Eq. 29)}$$

The method of modified eigenvalues, as described in Eq. 12, results in a modified matrix R' given by

$$R' = \sum_{n=1}^{N} \lambda'_n E_n E_n^* \quad . \qquad \text{(Eq. 30)}$$

which can be written as:

$$\begin{aligned} R' &= R - \sum_{n=1}^{N} \left(\lambda_n - \lambda'_n\right) E_n E_n^* \\ &= R - R_{T,1} \end{aligned} \quad . \qquad \text{(Eq. 31)}$$

The method with reference beam

$$R' = R - \frac{(RW_r)(RW_r)^*}{W_r^* \, R \, W_r} = R - \mu_2 V_2 V_2^* = R - R_{T,2} \qquad \text{(Eq. 32)}$$

The method with reference hydrophone

$$R' = R - \frac{Q\,Q^*}{r_y} = R - \mu_3 V_3 V_3{}^* = R - R_{T,3} \quad . \qquad \text{(Eq. 33)}$$

$R_{T,1}$, $R_{T,2}$ and $R_{T,3}$ respectively, are what the three methods estimate as the contribution to the covariance matrix R from the towship. According to definition (App. D), $R_{T,1}$ will have rank M if M eigevalues are modified, while $R_{T,2}$ and $R_{T,3}$ are seen always to have rank 1 (only one non-zero eigenvalue). But it is shown in App. C that the rank is 1 only for a completely coherent reception. Hence the two last methods may work only if the real contribution $R_{T,0}$ has rank 1. The experimental results, however, indicate strongly that this is not so.

The three methods are discussed individually below. Figure 9 gives the line-printer outputs supporting the discussion, although no data are shown for the Reference Hydrophone Technique (Sect. 4.2.3).

4.2.1 The method with modified eigenvalues

If M eigenvalues are modified, then:

$$R_{T,1} = \sum_{n=1}^{N} (\lambda_n - \lambda'_n)\, E_n E_n^* \qquad n \in \{n_1, n_2, \ldots, n_m\} \quad . \qquad \text{(Eq. 34)}$$

Hence this method can easily create a modification of any rank. But that is all; almost as expected, the method is more or less useless. It was implemented simply because it was so simple to implement. The obvious reason that it does not work well is that there is not a one-to-one correspondence between eigenvalues vectors and sources. The contribution from one source is usually distributed on many eigenvalues, and one eigenvalue is a combination of contributions from several sources. The only simple case is with orthogonal sources.

Still, the method gave insight. It was quite interesting to see the "correlation" between the beam-steering vectors and the eigenvectors:

$$C_{ij} = \left| B_i^* \, E_j \right|^2 \qquad 0 \leqq C_{ij} \leqq 1 \qquad \text{(Eq. 35)}$$

when both B and E are normalized. During the sea-trial, C_{ij} was printed out for the two eigenvectors belonging to the two largest eigenvalues (part of the tabulated data given to create Fig. 6). When signals were transmitted from the source below the workboat it was often possible to

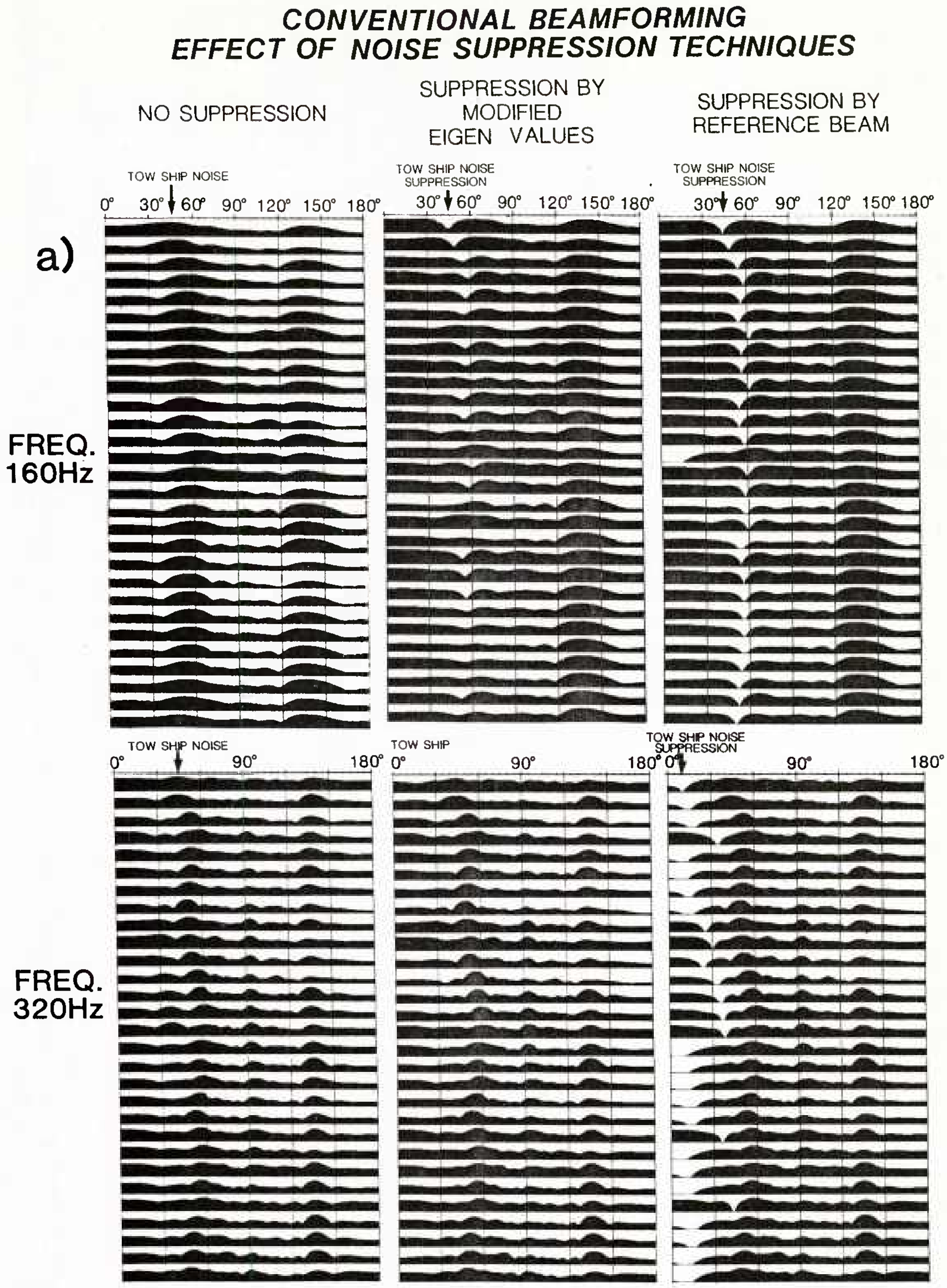

FIG. 9a *COMPARISON OF NOISE SUPPRESSION TECHNIQUES*
Run 1 (160 and 320 Hz) processed with conventional beamforming

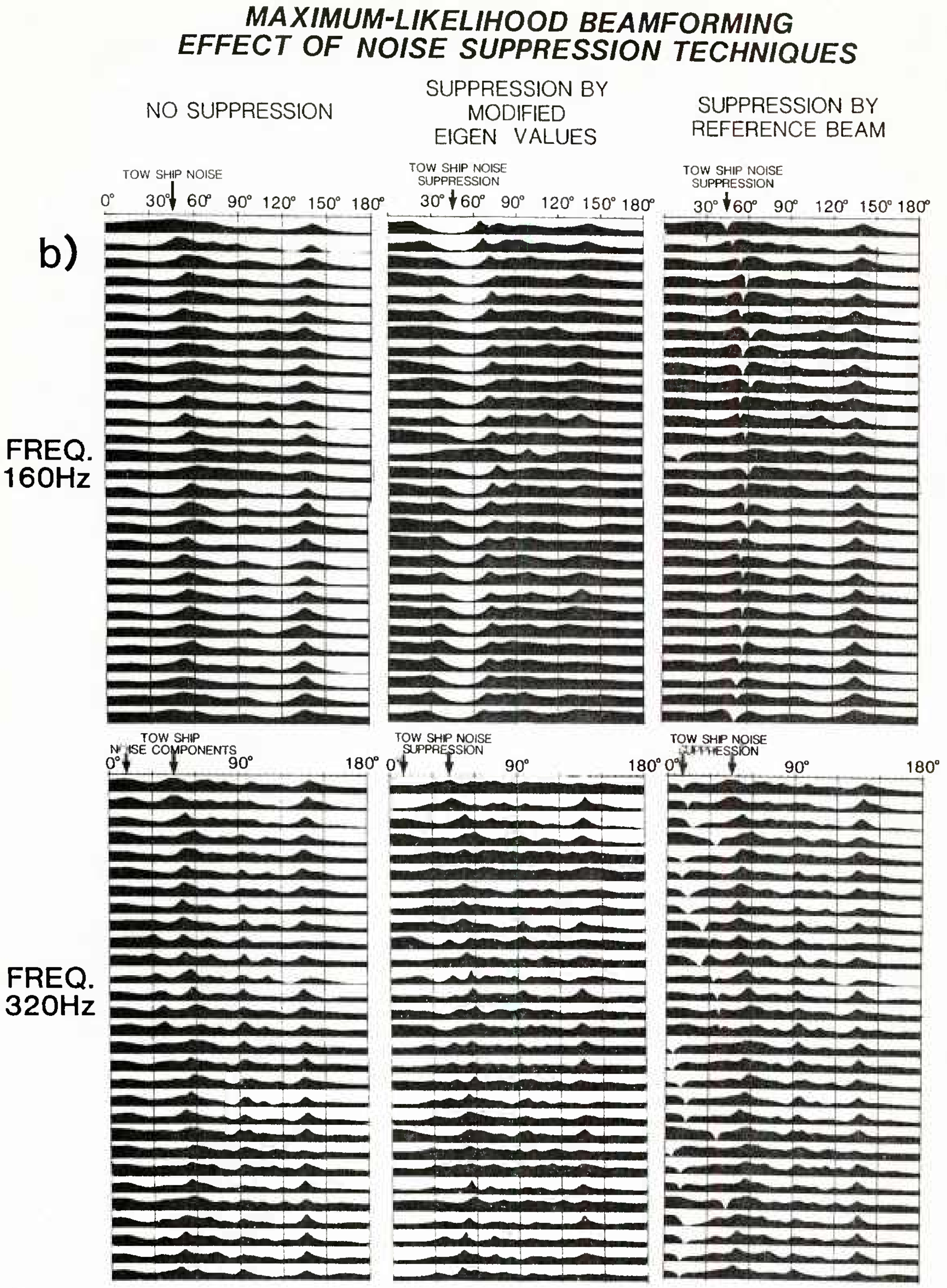

FIG. 9b COMPARISON OF NOISE SUPPRESSION TECHNIQUES
Run 1 (160 and 320 Hz) processed with maximum likelihood beamforming

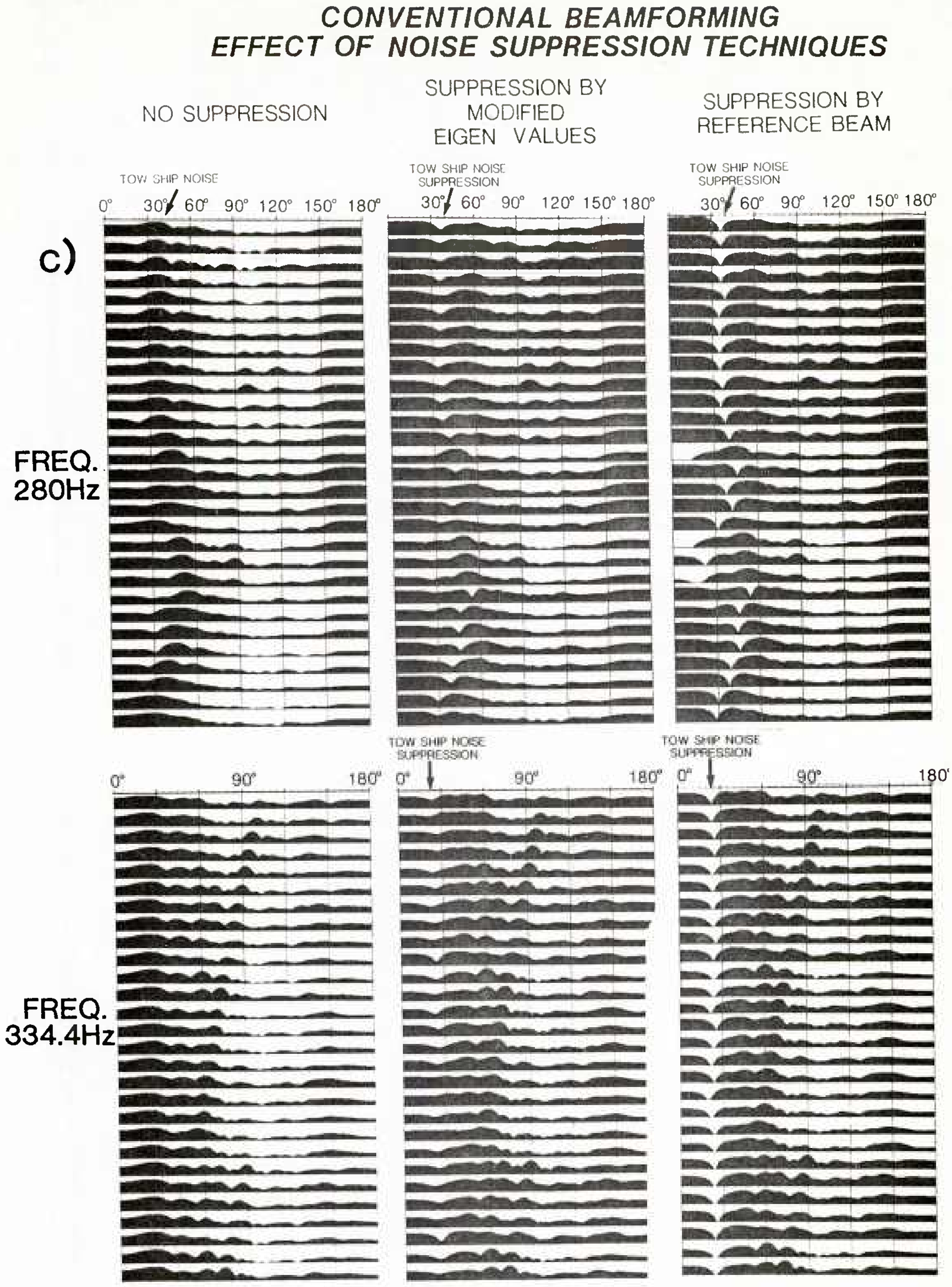

FIG. 9c *COMPARISON OF NOISE SUPPRESSION TECHNIQUES*
Run 2 (280 and 334.4 Hz) processed with conventional beamforming

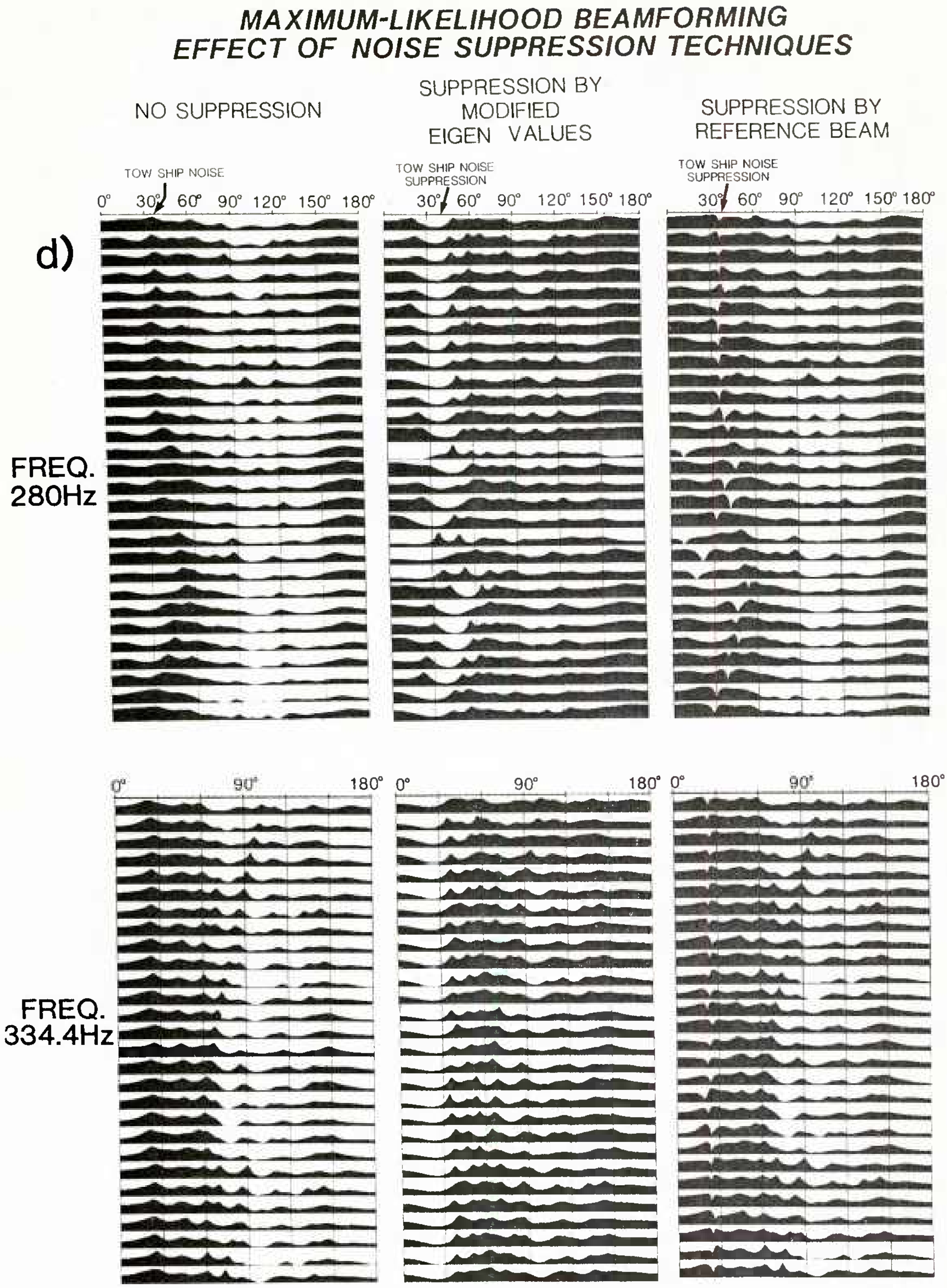

FIG. 9d COMPARISON OF NOISE SUPPRESSION TECHNIQUES
Run 2 (280 and 334.4 Hz) processed with maximum likelihood beamforming

observe how the two eigenvalues E_1 and E_2 were correlated with beam directions corresponding both to the towship noise and to the workboat source. This means that we could easily suppress the towship noise by modifying the first three to four eigenvalues, but at the same time the other signals would also be suppressed in an uncontrolled, random, and untolerable way.

At this point it may be appropriate to speculate why the towship noise seen by the array towed 500 m behind the ship has a covariance matrix of rank 3 to 4. There are three possibilities:

a) That the geometry of the array relative to the ship is changing during the averaging time of the covariance matrix. This time was 40 to 60 s in the sea-trials, which is obviously long compared with the possible snaking of the array. The solution would be to use a much shorter averaging time and, if possible, to overlap the FFT-windows in the filtering process before the averaging, so as to use the data efficiently.

b) That the different paths of the multipath propagation from the towship to the array have different travel times. As the continuous background spectrum of the towship has a short correlation time, the contribution at a given time from the different paths may have a reduced cross-correlation, which would mean a non-coherent signal from the towship and thus a covariance matrix of rank higher than one.

c) That the influence of the medium and its boundaries is reducing the coherence, even for an originally stable line (CW).

4.2.2 The method with reference beam

This method estimates the covariance matrix of towship noise to be

$$R_{T,2} = \frac{(RW_r)(RW_r)^*}{W_r^* RW_r} \cdot \mu_2 V_2 V_2^* \quad , \qquad \text{(Eq. 36)}$$

where

$$W_r = \begin{cases} B_r & \text{for CBF} \\ (B_r^* R^{-1} B_r)^{-1} R^{-1} B_r & \text{for MLBF} \end{cases} .$$

In both cases the model assumes that the towship noise arrives as a coherent plane wave at the array. This might work well in deep water; in shallow water it definitely did not. The towship noise was certainly suppressed in a bearing interval around the reference-beam direction (given by B_r), but the rest of the towship noise was more or less unchanged.

Secondly, if there was any signal (target) in the reference direction, it would also be cancelled. The plots indicate multipath arrival at different vertical arrival angles. Hence, even if the towship noise is coherent, it would not fit a plane-wave model. Making it even more difficult, there are strong indications (from study of eigenvalues and vectors) that the towship-noise signal is not coherent over the array.

It might be possible to work out an ad hoc algorithm by choosing more than one reference beam and cross correlating between the reference beams, but it is not obvious that the outcome would be worth the effort.

4.2.3 The method with reference hydrophone

This method estimates the covariance matrix of the towship noise to be

$$R_{T,3} = \frac{Q\,Q^*}{r_y} = \mu_3 V_3 V_3^* \quad , \qquad \text{(Eq. 37)}$$

where

$$Q = \frac{1}{M} \sum_{i=1}^{M} X_i y_i^*$$

and

$$r_y = \frac{1}{M} \sum_{i=1}^{M} |y_i|^2 \quad . \qquad \text{(Eq. 38)}$$

As seen, the method assumes the covariance to be of rank 1, which means a coherent signal. But it is an improvement (theoretically, at least) from the reference-beam method in that it does not assume anything about the shape of the wavefront. Furthermore, even a signal with the same directivity pattern as the towship noise should be only slightly reduced if:

$$\left(\sigma^2_{\text{ref}} \big/ \sigma^2_{\text{array}}\right)_{\text{Towship}} \gg \left(\sigma^2_{\text{ref}} \big/ \sigma^2_{\text{array}}\right)_{\text{Target}} \quad ,$$

where

σ^2_{ref} = Towship or Target power on the reference hydrophone

σ^2_{array} = Towship or Target power on the array hydrophone.

This is exactly the reason why the reference hydrophone has to be closer to the towship than to the array. From a theoretical point of view this should be the best of the three implemented methods. Although this sounds promising, the results so far show that the method in its present shape does not work at all. There are two possible reasons for this:

a) The first is that the reference hydrophone was rather simply designed and thus created flow noise. This would especially influence its power estimate r_y. This estimate was therefore modified by a factor smaller than 1, to see what would happen:

$$r_y{}' = q_n r_y, \quad \text{where} \quad 0 \leqq q \leqq 1.$$

This was done at different frequencies, including the frequency of 334 Hz transmitted by the workboat.

The result was disastrous. The workboat signal disappeared almost completely, while the towship noise was still there, reduced only for some bearings. Looking a little closer at the mathematics, let B_T be a direction where we can identify towship noise, and B_S be the direction of the workboat source. Then, in obvious notation:

$$P_T = B_T^* \, R \, B_T \quad , \tag{Eq. 39}$$

$$P_T' = B_T^* \left(R - \frac{Q \, Q^*}{r_y} \right) B_T = B_T^* \, R \, B - \frac{\left| B_T^* \, Q \right|^2}{r_y} \quad , \tag{Eq. 40}$$

$$\frac{P_T'}{P_T} = 1 - \frac{1}{r_y} \, \frac{\left| B_T^* \, Q \right|^2}{B_T^* \, R \, B_T} \quad . \tag{Eq. 41}$$

Similarly:

$$\frac{P_S'}{P_S} = 1 - \frac{1}{r_y} \, \frac{\left| B_S^* \, Q \right|^2}{B_S^* \, R \, B_S} \quad . \tag{Eq. 42}$$

To obtain a good suppression of the towship noise relative to the suppression of the workboat signal, we must have:

$$\frac{P_T'}{P_T} \ll \frac{P_S'}{P_S} \quad , \tag{Eq. 43}$$

which is the same as:

$$\frac{\left| B_T^* \, Q \right|^2}{B_T^* \, R \, B_T} \gg \frac{\left| B_S^* \, Q \right|^2}{B_S^* \, R \, B_S} \quad . \tag{Eq. 44}$$

This is in many (maybe most) situations, not the case.

The second possibility is that the change of the geometry relevant to towship noise (towship source, reference hydrophone, array) during the averaging time may have been much more damaging than the changes in geometry relevant to the workboat signal. There would always have been some heaving movement of the towship that could have changed the position of the noise source and the nature of the multipath pattern of its noise propagation. The implemented averaging time was certainly too long to allow an adjustment to the dynamics.

Even if the geometry was completely frozen, the present implementation might still not work well. The reason is, as mentioned earlier, that the rank of the towship noise at the array was higher than 1, at least of the order 3 to 4 at 334 Hz. Hence it cannot be suppressed by a rank-1 matrix.

There are two ways to increase the rank of the reference system. One is to increase the number of reference hydrophones. This might help to some extent, but it will not help if the reason for reduced coherence is the different travel times along the different paths. Another possibility is to work with several delayed versions of the reference hydrophone output, to create a vector:

$$y(t) = \left\{ \begin{array}{l} y(t-\tau_1) \\ y(t-\tau_2) \\ y(t-\tau_L) \end{array} \right. . \qquad \text{(Eq. 45)}$$

By going through the mathematics, this will result in:

$$R' = R_{xx} - R_{xy} R_{yy}^{-1} R_{yx} = R-R_{T,y} , \qquad \text{(Eq. 46)}$$

where

$$R_{xy} = \frac{1}{M} \sum_{i=1}^{M} X_i Y_i^* , \quad \text{etc.} \qquad \text{(Eq. 47)}$$

The dimension L of Y should be such that R_{YY} has full rank. Otherwise it cannot be inverted, and in fact has more elements than can be used. Ideally the rank of R_{YY} should be the same as the rank of the towship-noise contribution to R.

The problems of flow noise could be overcome by creating a new reference-sensor system with reduced flow noise and low, or almost no, sensitivity to acceleration (vibration).

4.3 Target indication and bearing estimation

As with the techniques for the suppression of towship noise, only qualitative comments can be given on the performance (Fig. 10) of the two forms of processing (CBF, MLB) for providing an indication of the presence of a target of opportunity and an estimate of the target bearing for frequencies of 112 Hz and 334.4 Hz. Figure 10b also includes the presence of a 334.4 CW source. Points to note from Figs. 10a and b are:

a. The CBF processing does not include any shading, therefore the sidelobes are high. However, CBF is not a serious contender for providing target data.

b. The MLB beamforming gives a sharp and precise display, although at times low-level 'targets' are suppressed, due to the maximum level used on the normalizing factor in the display.

CONCLUSIONS

a. A detailed analysis should be made of the signal paths between the towship and the array. An eigenvalue analysis indicated that the multipath propagation is reducing the coherence (Sect. 4.2). This may require the application of the FFP technique and normal-mode propagation (Sect. 4.1).

b. Because of the lack of a one-to-one correspondence between eigenvalue/vector and source, except when the sources are orthogonal, the modified eigenvalue technique for suppressing towship noise does not appear to be feasible (Ch. 3). However, the eigenvalues/vectors give useful information on certain characteristics of signals and will therefore be used in the future signal-processing software.

c. The use of a single reference beam for suppressing towship noise is not satisfactory; it assumes that the towship noise arrives at the array as a coherent plane wave and that there is no target signal in that direction (Ch. 3). It would be possible to select more than one beam and produce an ad hoc algorithm but its usefulness under the full range of operating conditions would be questionable. No further work is proposed at this stage.

d. The reference-hydrophone technique did not work because of the oversimplified nature of the present technique and the high level of flow-noise in the reference hydrophone (Ch. 3). A technique to provide a more accurate representation of the towship noise is being developed for future sea trials, and the hydrophone design is being improved to keep the flow-noise level to a minimum using a separate streamer for the reference hydrophone.

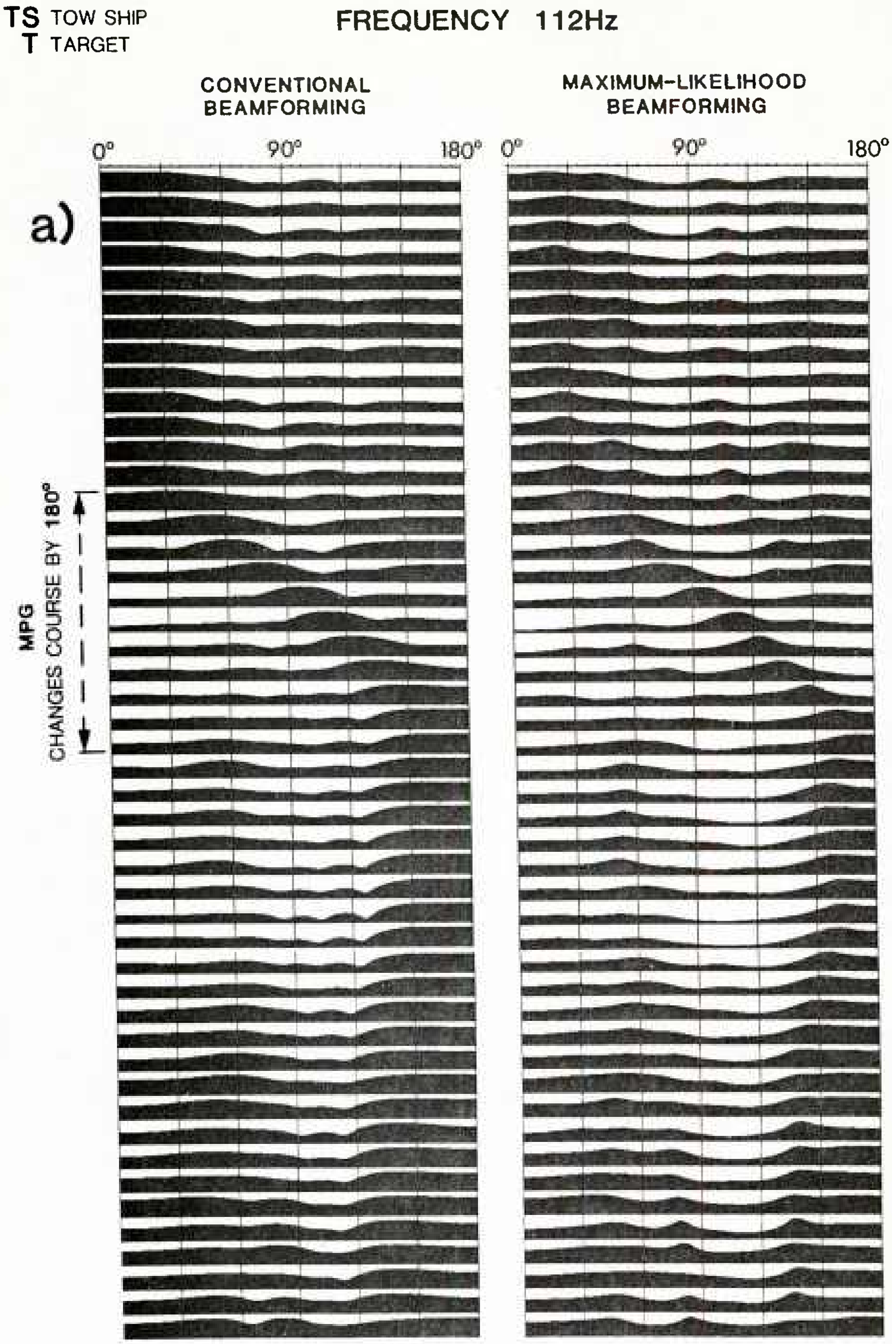

FIG. 10a *COMPARISON OF TARGET-BEARING INDICATION BY CONVENTIONAL BEAMFORMING AND MAXIMUM LIKELIHOOD BEAMFORMING*
Low frequency, 112 Hz, Target only

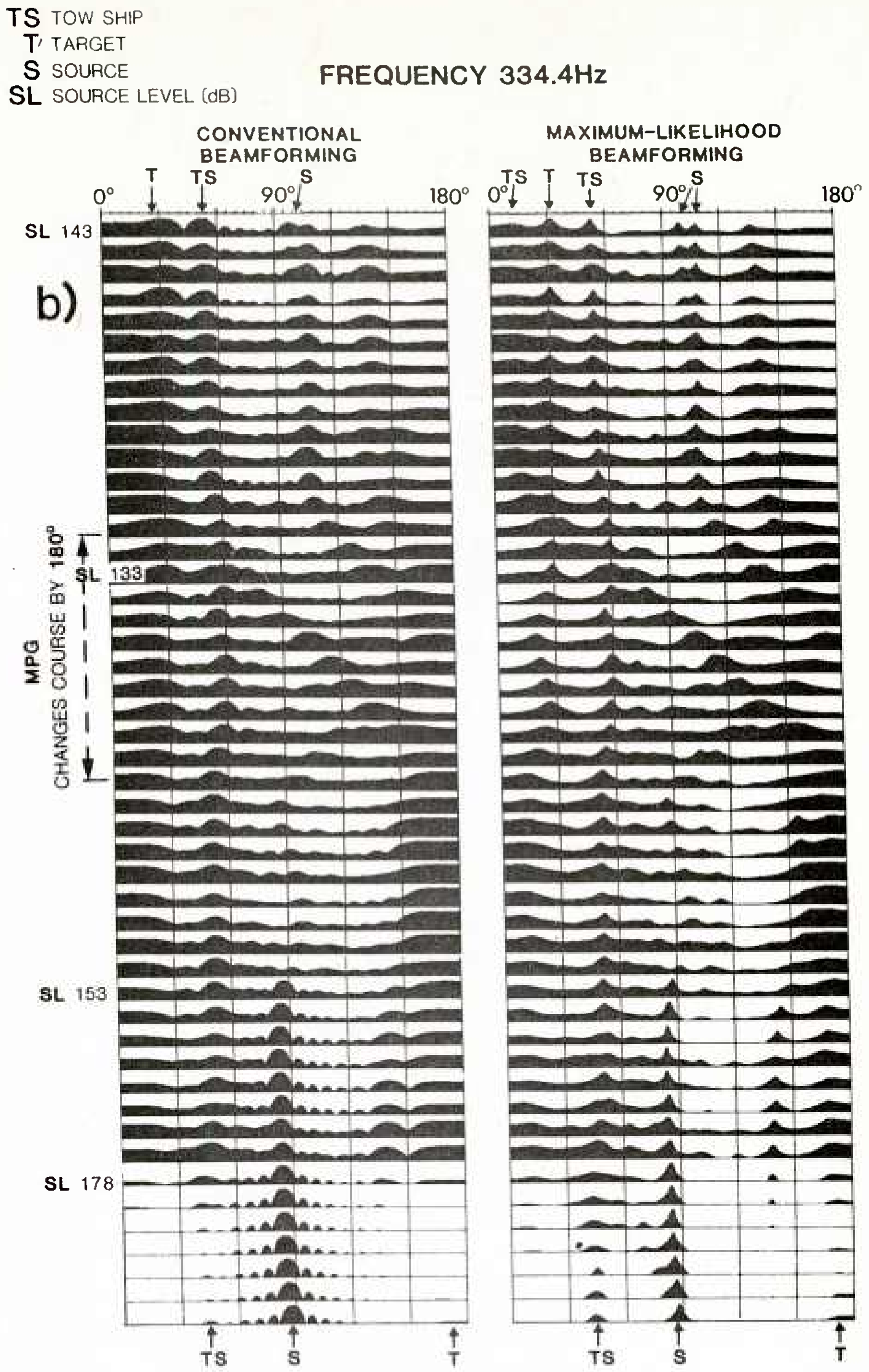

FIG. 10b *COMPARISON OF TARGET-BEARING INDICATION BY CONVENTIONAL BEAMFORMING AND MAXIMUM LIKELIHOOD BEAMFORMING*
High frequency, 334.4 Hz, Target plus CW source

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Electronic/Acoustic	E. Muzi	Assistance with sea trials.
Engineering Department	P. Saia	Assistance with sea trials.

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APPENDICES

APPENDIX A

COVARIANCE MATRIX ESTIMATE

This appendix gives seven definitions with their associated properties and "proofs" relevant to our use of the covariance matrix estimate, R. The main emphasis is on eigenvalues and eigenvectors.

The square matrix R of dimension N is defined by:

$$R = \frac{1}{M} \sum_{i=1}^{M} X_i \, X_i^* \qquad , \qquad \text{(Eq. A.1)}$$

where X will be called a data vector, and is of dimension N, and <u>the star indicates a complex conjugate transpose.</u>

A.1 DEFINITION

A square matrix P is called hermitian if it is equal to its own complex conjugate transpose, P*.

Property 1: R is a hermitian matrix. (P1)

Proof:

$$R^* = \frac{1}{M}\left(\sum_{i=1}^{M} X_i \, X_i^*\right)^* = \frac{1}{M}\sum_{i=1}^{M}\left(X_i^*\right)^*\left(X_i\right)^* = \frac{1}{M}\sum_{i=1}^{M} X_i \, X_i^* = R \qquad \text{(Eq. A.2)}$$

Property 2 $Z = Y^*RY$ is a real scalar for all vectors Y: (P2)

Proof:

$$Z^* = (Y^*RY)^* = Y^*R^*\,Y$$

By use of (P1):

$$Z^* = Y^*R^*Y = Y^*RY = Z \quad \text{q.e.d.}$$

A.2 DEFINITION

A square hermitian matrix P is positive definite if, for all vectors Y except the null vector, $Y^*PY > 0$, and non-negative definite if $Y^*PY \geq 0$.

Property 3: R is a non-negative definite matrix. (P3)

Proof:

$$Z = Y^*RY = \frac{1}{M} \sum_{i=1}^{M} Y^*X_iX_i^*Y = \frac{1}{M} \sum_{i=1}^{M} (Y^*X_i)(Y^*X_i)^* \qquad \text{(Eq. A.3)}$$

$$= \frac{1}{M} \sum_{i=1}^{M} \left| Y^*X_i \right|^2 \geqq 0 \qquad \text{q.e.d.}$$

A.3 DEFINITION

If E is a vector, but not a null vector, such that $PE = \lambda E$, then E is called an eigenvector, and λ the corresponding eigenvalue. The eigenvectors are always assumed to be normalized: $E^*E = 1$.

Property 4: the eigenvalues of R are real and non-negative. (P4)

Proof:

$$\lambda E = RE$$

Premultiplying with E^*:

$$\lambda = \lambda E^* E = E^* R E$$

According to (P2), λ is real, and according to (P3), λ is non-negative.

A.4 DEFINITION

Two vectors X and Y, not being null-vectors, are orthogonal if $X^*Y = 0$. They are called orthonormal if both X and Y are normalized by having unit length.

Property 5: Eigenvectors of R with different eigenvalues are orthogonal.

Proof:

$$\lambda_m E_n^* E_m = E_n^* \lambda_m E_m = E_n^* R E_m$$

By exchanging indices:

$$\lambda_n E_m^* E_n = E_m^* R E_n$$

Then take the complex conjugate transpose of both sides of the last equation and use (P1) and (P4):

$$\lambda_n E_n^* E_m = E_n^* R E_m$$

Combining the first and last equations of the proof:

$$(\lambda_m - \lambda_n) E_n^* E_m = 0$$

As the eigenvalues are assumed different $\lambda_n \neq \lambda_m$:

$$E_n^* E_m = 0 \qquad \text{q.e.d.}$$

Property 6: There are N eigenvalues of R, which may not be all distinct (different from each other), and they are a solution to

$$\det(R-\lambda I) = 0$$

Proof:

$$RE = \lambda E$$
$$(R-\lambda I) E = 0$$

This can be true for E not being a null vector if, and only if, the determinant of $(R-\lambda I)$ is zero, $\det(R-\lambda I) = 0$.

This is a polynome in λ of degree N, and has N solutions, which may not be distinct. These N solutions are hence eigenvalues of R.

We know already that if eigenvalues are different, their corresponding eigenvectors are orthonormal. If there are k eigenvalues with the same value, we will state without proof that it is always possible to find exactly k orthonormal vectors that are eigenvectors of R with this same value on their eigenvalues. Hence we state without proof the following property:

Property 7: R has always N orthonormal eigenvectors. (P7)

Property 8: The eigenvectors with positive eigenvalues are linear combinations of the data vectors.

Proof:

$$\lambda E = RE = \frac{1}{M} \sum_{i=1}^{M} X_i X_i^* E = \frac{1}{M} \sum_{i=1}^{M} (X_i^* E) X_i \qquad \text{q.e.d.}$$

Property 9: Eigenvectors, which are linear combinations of the data vectors, have positive eigenvalues. (P9)

Proof:

$$\lambda = \lambda E^*E = E^*RE = \frac{1}{M}\sum_{i=1}^{M} E^*X_iX_i^*E = \frac{1}{M}\sum_{i=1}^{M} \left|X_i^*E\right|^2 > 0 \qquad \text{q.e.d.}$$

A.5 DEFINITION

The k vectors Y are said to be linearly independent if

$$\sum_{i=1}^{K} c_iY_i = 0 \qquad \text{only if} \quad c_i = 0 \quad \text{for } i = 1, \ldots, k$$

Property 10: If the number of independent data vectors is M, and M < N, N - M eigenvectors are orthogonal to the data vectors, and their corresponding eigenvalues are zero. (P10)

"Proof":

From M independent data vector, we state without proof that M eigenvectors be constructed:

$$E_m = \sum_{i=1}^{M} c_iX_i \qquad m = 1, 2, \ldots, M$$

or, in a more compact form:

$$\left[E_1, E_2, \ldots, E_M\right] = \left[X_1, X_2, \ldots, X_M\right] C$$

As the M eigenvectors are also independent, C must be non-singular, and C^{-1} exists:

$$\left[X_1, X_2, \ldots, X_M\right] = \left[E_1, E_2, \ldots, E_n\right] C^{-1}$$

Premultiply with an eigenvector E_n $n = M+1, \ldots, N$

$$\left[E_n^*X_1, E_n^*X_2, \ldots, E_n^*X_n\right] = \left[E_n^*E_1, E_n^*E_2, \ldots, E_n^*E_m\right] C^{-1}$$

$$= \left[0, 0, \ldots, 0\right] C^{-1} = 0$$

by using (P7). Hence

$$E_n^*X_i = 0 \quad \text{for} \quad i = 1, 2, \ldots, M \qquad n = M+1, M+2, \ldots, N \qquad \text{q.e.d.}$$

The corresponding eigenvalues are:

$$\lambda_n = \lambda_n E_n^* E_n = E_n^* R\ E_n = \frac{1}{M} \sum_{i=1}^{M} \left| E_n^* X_i \right|^2 = 0 \qquad \text{q.e.d.}$$

A.6 DEFINITION

A square matrix P is singular if its determinant is zero, and its inverse P^{-1} does not exist.

Property 11: If the number of independent data vectors M is less than N, R is singular. (P11)

Proof: From (P10) it follows that if $M < N$, $N - M$ eigenvalues are zero. But the eigenvalues are solutions to $\det(R-\lambda I) = 0$.

Hence for $\lambda = 0$, $\det(R) = 0$ and R is singular.

Property 12: If R has N independent data vectors, it is non-singular and its inverse R^{-1} does exist. (P12)

Proof: The proof follows directly from (P8) and (P9) for $M = N$ and these data vectors are independent.

A.7 DEFINITION

A square non-singular matrix P is unitarian if its inverse P^{-1} is equal to its complex conjugate transpose P^*.

Property 13: The square matrix $S = [E_1, E_2, \ldots, E_n]$ constructed from the N eigenvectors of R, are unitarian. (P13)

Proof: Because the eigenvectors are independent, S is non-singular, and its inverse S^{-1} exists. Let us introduce:

$$\begin{aligned} T &= (S^{-1})^* \\ &= \left[T_1, T_2, \ldots, T_N\right] \end{aligned}$$

where T_i is a vector.

These vectors can be expressed as linear functions of the eigenvectors:

$$T_i = \sum_{j=1}^{N} \alpha_{ij} E_j$$

From $T^*S = S^{-1}S = I$ it follows that:

$$T_i{}^*E_n = \sum_{j=1}^{N} \alpha^*_{ij}\, E^*_j\, E_n = \alpha^*_{in} \quad = 1 \text{ if } i = n$$

$$= 0 \text{ if } i \neq n$$

From this follows:

$$T_i = E_i \quad \text{and} \quad T = S = (S^{-1})^* \qquad \text{q.e.d.}$$

Property 14: R can be written as $R = SD\,S^*$, where $D = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$ is a diagonal matrix. (P14)

Proof:

$RE_i = \lambda_i\, E_i$ $\quad i = 1, 2, \ldots, N$ can be combined to:

$$RS = SD$$

As S is non-singular:

$$R = S\, D\, S^{-1}$$

and from (P13):

$$R = S\, D\, S^* \qquad \text{q.e.d.}$$

Property 15: If R is non-singular, its inverse can be written:

$$R^{-1} = S\, D^{-1}\, S^* \quad . \qquad \text{(P15)}$$

Proof:

$$R^{-1} = (S\, D\, S^*)^{-1} = (S^*)^{-1}\, D^{-1}\, S^{-1}$$

Using (P13), this is

$$R^{-1} = S\, D^{-1}\, S^* \qquad \text{q.e.d.}$$

Property 16: Let n be an integer, then

$$R^n = S\,D^n\,S^*$$

Proof:

For $n \geqq 1$

$$R^n = (SDS^*)\,(SDS^*)\,\ldots\,(SDS^*) = SDS^{-1}\;SDS^{-1}\;\ldots\;SDS^* = SD^nS^* \qquad \text{q.e.d.}$$

For $n \leqq -1$

$$R^{-n} = (SD^{-1}S^*)(SD^{-1}S^*)\ldots(SD^{-1}S^*) = SD^{-1}S^{-1}S\,D^{-1}S^{-1}\ldots SD^{-1} = SD^{-n}S^* \qquad \text{q.e.d.}$$

For $n = 0$

$$R^o = I = SS^{-1} = SS^* = SD^oS^* \qquad \text{q.e.d.}$$

Property 17: Let n be an integer, then

$$R^n = \sum_{i=1}^{N} \lambda_i^n\,E_i\,E_i^*$$

Proof:

$$D^n = [\mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)]^n = \mathrm{diag}(\lambda_1^n, \lambda_2^n, \ldots, \lambda_N^n)$$

by direct use of matrix multiplication. Hence:

$$R^n = [E_1, E_2, \ldots, E_N] \begin{bmatrix} \lambda_1^n & & & & \\ & \lambda_2^n & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot\;\lambda_N^n \end{bmatrix} \begin{bmatrix} E_1^* \\ E_2^* \\ \cdot \\ \cdot \\ \cdot \\ E_N^* \end{bmatrix}$$

$$= [E_1, E_2, \ldots, E_N] \begin{bmatrix} \lambda_1^n E_1^* \\ \lambda_2^n E_2^* \\ \cdot \\ \cdot \\ \lambda_N^n E_N^* \end{bmatrix} = \sum_{i=1}^{N} \lambda_i^n\,E_i\,E_i^* \qquad \text{q.e.d.}$$

Property 18: For all X and Y:

$$X^* R^n Y = \sum_{i=1}^{N} \lambda_i^n \left(X^* E_i\right)\left(E_i^* Y\right)$$

Proof: This follows directly from (P17).

APPENDIX B

BEAMFORMING AND NOISE SUPPRESSION

B.1 CONVENTIONAL BEAMFORMING (CB)

Let X_i be a narrowband complex data vector of dimension N (N array hydrophones) from a frequency band around f, where i is a time index. Then the covariance matrix estimate R is defined by:

$$R = \frac{1}{M} \sum_{i=1}^{M} X_i X_i^* \qquad M \geqq N \quad , \qquad \text{(Eq. B.1)}$$

where 1/M is added for convenience.

A steering vector B is defined by:

$$B^T = \left[1, \ldots, e^{-j2\pi \frac{x_n}{\lambda} \cos \beta} \right] \frac{1}{\sqrt{N}} \quad ,$$

where x_n is the position of the n-th hydrophone, β is the bearing angle measured from ahead endfire, and $\sqrt{N}$ is introduced to normalize B: $B^*B = 1$.

A "conventional" beam is then:

$Z_{k,i} = B_k^* X_i$, where i and k are time and bearing indices.

The power output is:

$$P_{k,i} = \left| Z_{k,i} \right|^2 = Z_{k,i} Z_{k,i}^* = B_k^* X_i X_i^* B_k \quad .$$

The mean (in time) conventional beam power output is:

$$P_k = \frac{1}{M} \sum_{i=1}^{M} P_{k,i} = \frac{1}{M} B_k^* \sum_{i=1}^{M} X_i X_i^* B_k = B_k^* R B_k \quad .$$

Using eigenvalues and eigenvectors, this can be written:

$$P_k = \sum_{n=1}^{N} \lambda_n (B_k^* E_n)(E_n^* B_k) = \sum_{n=1}^{N} \lambda_n \left| B_k^* E_n \right|^2$$

B.2 MAXIMUM LIKELIHOOD BEAMFORMING (MLB)

Instead of the steering vector B , use a weight vector W to form a beam:

$$Z_{k,i} = W_k^* X_i \ .$$

The so-called maximum-likelihood beamforming is obtained by choosing a weight vector W that is minimizing

$$W_k^* R W_k \quad \text{constrained by} \quad W_k^* B_k = B_k^* W_k = 1$$

$$Y_k = W_k^* R W_k = (W_k^* - W_o^*) R (W_k - W_o) + W_k^* R W_o + W_o^* R W_k - W_o^* R W_o$$

The constraint can now be introduced by choosing

$$Y_k = (W_k - c R^{-1} B_k)^* R(W_k - c R^{-1} B_k) + c + c^* - cc^* B_k^* R^{-1} B_k$$

Assuming R to be positive definite, this has a minimum for:

$$W_k = c R^{-1} B_k$$

The constant c is determined from the constraint

$$B_k^* W_k = c B_k^* R^{-1} B_k = 1$$

$$W_k = (B_k^* R^{-1} B_k)^{-1} R^{-1} B_k$$

The ML beamformer output will be:

$$Z_{k,i} = (B_k^* R^{-1} B_k)^{-1} (B_k^* R^{-1} X_i)$$

and the mean power output will be:

$$P_k = W_k^* R W_k = (B_k^* R^{-1} B_k)^{-1} (B_k^* R^{-1} R R^{-1} B_k) (B_k^* R^{-1} B_k)$$

$$= (B_k^* R^{-1} B_k)^{-1}$$

Using eigenvectors and eigenvalues of R:

$$P_k = \left[\sum_{n=1}^{N} \lambda_n^{-1} (B_k^* E_n) (E_n^* B_k) \right]^{-1} = \left(\sum_{n=1}^{N} \lambda_n^{-1} \left| B_k^* E_n \right|^2 \right)^{-1}$$

B.3 NOISE SUPPRESSION WITH REFERENCE BEAM

Let there be a reference beam:

$$Y_i = W_r^* X_i \quad \text{where} \quad W_r = B_r \qquad \text{for CBF}$$

$$= (B_r^* R^{-1} B_r)^{-1} R^{-1} B_r \qquad \text{for MLBF}$$

This reference beam is used to correct the beams:

$$Z'_{k,i} = Z_{k,i} - V_k Y_i$$

V_k is now chosen to minimize:

$$P'_k = \frac{1}{M} \sum_{i=1}^{M} \left| Z'_{k,i} \right|^2 = \frac{1}{M} \sum_{i=1}^{M} (Z_{k,i} - V_k Y_i)(Z_{k,i} - V_k Y_i)^*$$

$$= \frac{1}{M} \sum_{i=1}^{M} (Z_{k,i} Z^*_{k,i} - Z_{k,i} Y_i^* V_k^* - V_k Y_i Z^*_{k,i} + V_k V_k^* Y_i Y_i^*)$$

$$= r_z - r_{zy} V_k^* - V_k r^*_{zy} + V_k V_k^* r_y$$

with obvious notation. This can be written:

$$P'_k = \left(V_k - \frac{r_{zy}}{r_y}\right)\left(V_k - \frac{r_{zy}}{r_y}\right)^* r_y + r_z - \frac{r_{zy}\, r_{zy}^*}{r_y}$$

This has a minimum for:

$$V_k = \frac{r_{zy}}{r_y} = \frac{W_k^* R W_r}{W_r^* R W_r}$$

where W_k is defined similar to W_r for CBF and MLBF.

The corrected linear and mean power outputs are:

$$Z'_{k,i} = W_k^* X_i - \frac{W_k^* R W_r}{W_r^* R W_r} W_r^* X_i = \left(W_k^* - \frac{W_k^* R W_r}{W_r^* R W_r} W_r^* \right) X_i$$

$$P'_k = W_k^* R W_k - \left(W_r^* R W_r \right)^{-1} \left| W_k^* R W_r \right|^2$$

for CBF:

$$P_k' = B_k^* R B_k - (B_r^* R B_r)^{-1} \left| B_k^* R B_r \right|^2$$

Using eigenvalues and eigenvectors:

$$B_k^* R B_k = \sum_{n=1}^{N} \lambda_n \left| B_k^* E_n \right|^2$$

$$B_r^* R B_r = \sum_{n=1}^{N} \lambda_n \left| B_r^* E_n \right|^2$$

$$\left| B_k^* R B_r \right| = \sum_{n=1}^{N} \lambda_n (B_k^* E_n)(B_r^* E_n)^*$$

For MLBF:

$$P_k' = (B_k^* R^{-1} B_k)^{-1} - (B_r R^{-1} B_r)^{-1} (B_k R^{-1} B_k)^{-2} \left| B_k^* R^{-1} B_r \right|^2$$

And with eigenvalues and eigenvectors:

$$B_k^* R^{-1} B_k = \sum_{n=1}^{N} \lambda_n^{-1} \left| B_k^* E_n \right|^2$$

$$B_r^* R^{-1} B_r = \sum_{n=1}^{N} \lambda_n^{-1} \left| B_r^* E_n \right|^2$$

$$B_k^* R^{-1} B_r = \left| \sum_{n=1}^{N} \lambda_n^{-1} \left(B_k^* E_n\right)\left(B_r^* E_n\right)^* \right|$$

B.4 NOISE SUPPRESSION WITH REFERENCE HYDROPHONE

Let there be a reference hydrophone with output y. This reference hydrophone output is used to correct the array hydrophone outputs:

$$X_i' = X_i - Wy_i$$

W is now chosen to minimize:

$$p' = \frac{1}{M} \sum_{i=1}^{M} \left| X_i - Wy_i \right|^2 = \frac{1}{M} \sum_{i=1}^{M} (X_i - Wy_i)^* (X_i - Wy_i)$$

$$= \frac{1}{M}\,\mathrm{tr}\left[\sum_{i=1}^{M} (X_i - Wy_i)(X_i - Wy_i)^*\right]$$

$$= \frac{1}{M}\,\mathrm{tr}\left[\sum_{i=1}^{M} (X_i\, X_i^* - X_i\, y_i^*\, W^* - W\, y_i\, X_i^* + W\, W^*\, y_i\, y_i^*)\right]$$

$$= \mathrm{tr}\;(R - QW^* - WQ^* + WW^*\, r_y)$$

with obvious notation. This can be written:

$$p' = \mathrm{tr}\left[\left(W - \frac{Q}{r_y}\right)\left(W - \frac{Q}{r_y}\right)^* r_y + R - \frac{QQ^*}{r_y}\right]$$

This has a minimum for:

$$W = \frac{Q}{r_y}$$

The corrected data vector and covariance matrix will be:

$$X_i' = X_i - \frac{Q}{r_y}\; y_i$$

$$R' = \frac{1}{M}\sum_{i=1}^{M} X_i'\, X_i'^{\,*} = R - \frac{QQ^*}{r_y}$$

The new inverse matrix will be using the matrix inversion lemma:

$$R'^{-1} = \left(R - \frac{QQ^*}{r_y}\right)^{-1} = R^{-1} + (r_y - Q^*R^{-1}Q)^{-1}(R^{-1}\; QQ^*\; R^{-1})$$

The mean output power using CBF:

$$p_k' = B_k^* R'\, B_k = B_k^* R B_k - r_y^{-1} \left| B_k^* Q \right|^2$$

Using eigenvalues and eigenvectors:

$$B_k^* R B_k = \sum_{n=1}^{N} \lambda_n \left| B_k^* E_n \right|^2$$

$$\left| B_k^* Q \right| = \sum_{n=1}^{N} \lambda_n^{\circ} (B_k^* E_n) Q^* E_n)$$

This way of writing B^*Q, as $B^*IQ = B^*R^{\circ}Q$ may seem artificial, but may be useful from a software point of view.

The mean output power using MLBF:

$$p_k' = (B_k^* R'^{-1} B_k)^{-1} = \left[B_k^* R^{-1} B_k + (r_y - Q^*R^{-1}Q)^{-1} \left| B_k^* R^{-1} Q \right|^2 \right]^{-1}$$

Using eigenvectors and eigenvalues:

$$B_k^* R^{-1} B_k = \sum_{n=1}^{N} \lambda_n^{-1} \left| B_k^* E_n \right|^2$$

$$Q^* R^{-1} Q = \sum_{n=1}^{N} \lambda_n^{-1} \left| Q^* E_n \right|^2$$

$$B_k^* R^{-1} Q = \left| \sum_{n=1}^{N} \lambda_n^{-1} (B_k^* E_n) (Q^* E_n)^* \right|$$

APPENDIX C

THE RANK OF A SOURCE'S CONTRIBUTION TO THE COVARIANCE MATRIX

From Appendix A, we know that the covariance matrix R can be written:

$$R = S\Lambda S^* = \sum_{i=1}^{N} \lambda_i E_i E_i^*$$

If only the first M eigenvalues are non-zero, R is singular, and is said to have rank M:

$$R = \sum_{i=1}^{M} \lambda_i E_i E_i^*$$

Let us look at the contribution from one source. If its contribution is completely coherent over the total array, then:

$$E\{x_i x_j^*\} = (E\{|x_i|^2\} \cdot E\{|x_j|^2\})^{\frac{1}{2}} e^{i(\psi_i - \psi_j)} = V_i V_j^*$$

leading to:

$$R_S = E\{XX^*\} = \sigma_s^2 VV^*$$

where V is a normalized vector (V*V = 1). Now:

$$R_s E = \sigma_s^2 V(V^*E) \begin{cases} = \sigma_s^2 V & \text{if } E^*V = 1 \quad (E = V) \\ = 0 & \text{if } E^*V = 0 \quad (E \perp V) \end{cases}$$

This means that there is only one non-zero eigenvalue $\lambda_i = \sigma_s^2$ with corresponding eigenvector $E_1 = V$ and the rank of R_s is 1.

If the contribution from the source is completely incoherent, then:

$$E\{x_i x_j^*\} \begin{cases} = \sigma_i^2 & \text{if } i = j \\ = 0 & \text{if } i \neq j \end{cases}$$

Then:

$$R_S = \text{diag}\,\{\sigma_1^2 \ldots \sigma_i^2 \ldots \sigma_N^2\}$$

As $\det R_S = \prod_{i=1}^{N} \sigma_i^2 > 0$, this matrix has always full rank, or rank equal to N.

Hence a real source will give a contribution to the matrix with a rank from 1 in the case of complete coherence, and increasing rank with decreasing coherence. We will define the field from the source to have rank M if:

$$
\begin{aligned}
10 \lg\left(\frac{\lambda_i}{\lambda_1}\right) &\geqq a \qquad \text{for} \quad i \leqq M \\
&< a \qquad \text{for} \quad i > M
\end{aligned}
$$

where a is a chosen threshold (as an example, -10 dB) and the eigenvalues are arranged in decreasing order.

APPENDIX D

RAY THEORY AND NORMAL-MODE THEORY

The acoustic propagation is usually expressed in terms of normal modes for the low-frequency propagation or in terms of rays for the high-frequency range. Ray theory and normal-mode theory is briefly reviewed here. A numerical comparison and overview is in reference <6>.

D.1 RAY THEORY

The limiting form of the exact wave theory for $\lambda \to 0$, i.e. $f \to \infty$ is called ray theory and the propagation is found to be frequency-independent for this limit. To achieve the limit the wave equation for the sound pressure

$$\nabla^2 p - c(z)^{-2} \frac{\partial p}{\partial t} = 0 \qquad \text{(Eq. D.1)}$$

yields for $p \sim \exp(-i\omega t)$

$$\nabla^2 p + k(z)^2 p = 0 \qquad \text{(Eq. D.2)}$$

where $k(z)$ is the wavenumber.

The "eikonal-solution"

$$p = A(r,z) \exp[i\, S(r,z)] \qquad \text{(Eq. D.3)}$$

permits for slowly varying $A(r,z)$ the "eikonal equation"

$$(\vec{\nabla} S)^2 = k(z)^2 \qquad \text{(Eq. D.4)}$$

For the derived solution for p the phase function $S(r,z)$ permits the surfaces of constant phase, and

$$S(r,z) = \text{const.} \qquad \text{(Eq. D.5)}$$

are the wavefronts of the sound field. The vector $\vec{\nabla} S$ are normal to the wavefront and chosen to form tangents to space curves, the so-called "rays".

The path element along a ray in range and depth coordinates (r,z) is given by:

$$d\ell = \sqrt{1 + (dr/dz)^2}\ dz \qquad \text{(Eq. D.6)}$$

and the angle θ which the ray makes with the horizontal can be found as

$$\frac{dz}{d\ell} = \sin\theta\ ;\quad \frac{dr}{d\ell} = \cos\theta\ ;\quad \frac{dr}{dz} = \cot\theta \qquad \text{(Eq. D.7)}$$

The unit vector pointing normal to the surface $S(r,z) = \text{const.}$ and being parallel to the vector $\vec{\nabla}S$ is defined

$$\vec{n} = (dr/d\ell,\ dz/d\ell) \qquad \text{(Eq. D.8)}$$

Together with Eq. D.4 follows

$$k\,\frac{dr}{d\ell} = \frac{\partial S}{\partial r}\ ;\quad k\,\frac{dz}{d\ell} = \frac{\partial S}{\partial z} \qquad \text{(Eq. D.9)}$$

with the operator $(d/d\ell)$

$$\frac{d}{d\ell}\left(k\,\frac{dr}{d\ell}\right) = \frac{\partial}{\partial r}\,\frac{dS}{d\ell} \qquad \text{(Eq. D.10)}$$

and with

$$\frac{dS}{d\ell} = \frac{\partial S}{\partial r}\,\frac{dr}{d\ell} + \frac{\partial S}{\partial z}\,\frac{dz}{d\ell}$$

one has by using Eq. D.9

$$\frac{dS}{d\ell} = \left(k\,\frac{dr}{d\ell}\right)^2 + k\left(\frac{dz}{d\ell}\right)^2 = k\,n^2 \qquad \text{(Eq. D.11)}$$

with Eq. D.10 again

$$\frac{d}{d\ell}\left(k\,\frac{dr}{d\ell}\right) = \frac{d}{dr}\left(k\cos\theta\right) = 0 \qquad \text{(Eq. D.12)}$$

Going along $d\ell$, i.e. a ray, one has

$$k(z)\cos\theta = \text{const. or the Snell's law} \qquad \text{(Eq. D.13)}$$

$$\cos\theta/c(z) = \cos\theta_0/c_0 = \text{const.} \qquad \text{(Eq. D.14)}$$

This law determines the variation of grazing angle θ (the angle relative to the horizontal) along the ray corresponding to the variation of sound speed with depth z, and relating it to the corresponding quantities θ_o, C_o at the source. From this and Eqs. D.6, D.7 the ray equation is derived

$$r(z) = \int_{z_o}^{z} \frac{dz}{\sqrt{(c_o/c(z) \cdot \cos\theta)^2 - 1}} \qquad \text{(Eq. D.15)}$$

for a ray coming from the source at a grazing angle θ_o. Solving this equation for a constant-gradient profile, yields:

$$r(z) = \int_{z_o}^{z} \frac{z\,dz}{\sqrt{\alpha^2 - z^2}} = -\sqrt{\alpha^2 - z^2} \qquad \text{(Eq. D.16)}$$

with

$$c(z) = bz$$

and

$$\alpha = \frac{\cos\theta_o}{b\,c_o}$$

This permits

$$\alpha^2 = r^2 + z^2 \qquad \text{(Eq. D.17)}$$

with proper choice of integration constants

Hence for a small gradient, the "rays" are circles with large radius, for a large gradient they are circles with small radius and for isovelocity profile the rays become straight lines. This concept can explain the sound channeling around the SOFAR channel axis.

Effects of boundary occur, when upward curving rays are reflected by the sea surface and downward rays are reflected by the sea bottom. Bottom reflections are treated by this concept assuming equal grazing angles of incidence and reflections, but with reduced amplitudes after reflection. The sea surface is almost a perfect reflector, for a low frequency (< 100 Hz), but the phase is changed by π after the reflection. If a ray has touched a caustic, which is a curve forming a geometrical envelope of rays, the phase is changed by $\pi/2$. In fact modified ray theory does consider the caustic correction.

In calculating the phase explicitly between source and receiver along the rays, Eq. D.4 yields by integrating over a ray-path.

$$S = \int k\,d\ell = \omega \int \frac{d\ell}{c} = \omega \int dt = \omega T \qquad \text{(Eq. D.18)}$$

where T is the travel time along the ray. Each ray amplitude has to have a phase factor exp(iωT) for coherent summation. In writing Eq. D.18 explicitly

$$\omega T = \omega \int \frac{dz}{c \sin \theta} = \omega \int \frac{dz}{c \sin \theta}(\sin \theta^2 + \cos^2 \theta)$$

$$= \omega \int \frac{\sin \theta}{c} dz + \omega \int \frac{\cos^2 \theta}{c \sin \theta} dz$$

$$= \quad I_1 \quad + \quad I_2 \qquad \text{(Eq. D.19)}$$

$$I_1 = \omega \int \frac{dz}{c} \sqrt{1 - \cos^2 \theta} \qquad \text{(Eq. D.19a)}$$

and with

$\cos \theta_o = n \cos \theta$

$n = c_o/c(z)$

$k_o = \omega/c_o$

we have

$$I_1 = k_o \int \sqrt{n^2 - \cos^2 \theta_o}\ dz \qquad \text{(Eq. D.19b)}$$

For the second integral

$$I_2 = \omega \int \frac{dz}{c \sin \theta} \cos^2 \theta\ ' \qquad \text{(Eq. D.20a)}$$

follows with, Eq. D.7, $dz/d\ell = \sin \theta$

$$I_2 = \omega \int \frac{\cos^2 \theta}{c} d\ell \qquad \text{(Eq. D.20b)}$$

With the horizontal component $k_r = k \cos \theta$ of the wavenumber $k = \omega/c$ one finds from Snell's law the horizontal component

$$k \cos \theta = k_o \cos \theta_o = \text{const.}$$

being range-independent, i.e. a constant along the ray. This yields for Eq. D.20b)

$$I_3 = \int k_r dr = (k_o \cos \theta_o) r \qquad \text{(Eq. D.20c)}$$

The total phase along the ray can be expressed

$$\omega T = (k_o \cos \theta_o)\ r + k_o \int_{z_o}^{z} \sqrt{n^2 - \cos^2 \theta}\ dz$$

$$= \int k_r dr + \int k_z dz \qquad \text{(Eq. D.21)}$$

where the vertical wavenumber

$$k_z = k \sin\theta = \frac{\omega}{c_o} \sqrt{n^2 - \cos^2 \theta}$$

is depth dependent through $n = c_o/c(z)$

From this the well known relation

$$k^2 = k_r^2 + k_z \qquad \text{(Eq. D.22)}$$

is found.

D.2 NORMAL MODE

With a monopole source at $r = 0$ and $z = z_o$ the normal mode solution for the wave equation is

$$\nabla^2 p + kp = - \frac{\delta(r)\ \delta(z-z_o)}{2\pi r} \qquad \text{(Eq. D.23)}$$

with separation of variables, the solution is of the form

$$p(r,z) = \sum_n R_n(r)\ D_n(z) \qquad \text{(Eq. D.24)}$$

with some constants a_n and k_n we can take the range function

$$R_n(r) = a_n \frac{i}{4} H_o^{(1)} (k_n r) \qquad \text{(Eq. D.25)}$$

which is the Green's function of the wave equation, satisfying

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dR_n}{dr} \right) + k_n^{-2} R_n = - \frac{\delta(r)}{2\pi r} a_n \qquad \text{(Eq. D.26)}$$

and for cylindrical symmetry one has

$$\nabla^2 p = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{r \partial} \right) + \frac{\partial^2 p}{\partial z^2} \qquad \text{(Eq. D.27)}$$

Inserting Eqs. D.25 and D.26 in Eqs. D.23 and D.24 yields

$$\sum_n \left[-k_n^2 R_n D_n + R_n \frac{d^2 D_n}{dz^2} + \frac{\omega^2}{c^2} R_n D_n \right]$$

$$= \frac{\delta(r)}{2\pi r} \sum_n a_n D_n(z) - \frac{\delta(r)\,\delta(z-z_o)}{2\pi r} \qquad \text{(Eq. D.28)}$$

The right hand side vanishes for

$$a_n = D_n(z_o) \qquad \text{(Eq. D.29)}$$

and using for depth-function $D_n(z)$

$$\sum_n D_n(z)\, D_n(z_o) = \delta(z-z_o) \qquad \text{(Eq. D.30)}$$

This function satisfies the depth equation.

$$\frac{d^2 D_n}{dz^2} + \left(\frac{\omega^2}{c(z)} - k_n^2 \right) D_n = 0 \qquad \text{(Eq. D.31)}$$

This "eigenvalue equation" permits solutions for its characteristic "eigenvalues" k_n under the constraint that

$$D_n(z_s) = D_n(z_\infty) = 0 \qquad \text{(Eq. D. 32)}$$

The eigenvalue k_n forms a "discrete spectrum" or a continuum.
The solution for an outgoing wave is now obtained from Eq.D.24.

$$p(r,z) = e^{-i\omega t} \frac{i}{4} \sum_{n=1}^{\infty} H_o^{(1)}(k_n r)\, D_n(z_o)\, D_n(z), \qquad \text{(Eq. D.33)}$$

known as normal mode series. At large ranges $r \to \infty$

$$H_o^{(1)}(k_n r) \to \sqrt{\frac{2}{\pi k_n r}}\; e^{i(k_n r - \frac{\pi}{4})} \qquad \text{(Eq. D. 34)}$$

corresponds to cylindrical waves,
showing that the horizontal wave-numbers are constant, as they are in ray theory, too. In other words, they are range-independent. The ray-mode equivalence in terms of an "equivalent ray angle" may help interpretations. The horizontal wave number k_n for the mode solution is assumed to be equal to the horizontal wave number $k_r = \omega \cos\theta/c$ in ray theory.

Hence the n^{th} normal mode might be associated with an "equivalent ray angle θ" of that mode by

$$\theta_n(z) = \cos^{-1} \frac{k_n \, c(z)}{\omega} \, , \qquad \text{(Eq. D.35)}$$

showing the difference that at given depth z , arrivals in terms of rays are continuous and in terms of modes are a discrete set of arrivals. Introducing a vertical wavenumber one obtains a depth equation,

$$\frac{d^2}{dz^2} D_n(z) + k_v^2 \; D_n(z) = 0 \qquad \text{(Eq. D.36)}$$

which can be solved analytically for certain profiles.

The Eq. D.31 might be rewritten by introducing the "potential function"

$$V(z) = k_o^2 - k^2(z) \qquad \text{(Eq. D.37)}$$

with $k_o = \omega/c_o$ at channel axis and the new eigenvalue

$$E_n = k_o^2 - k_n^2 \qquad \text{(Eq. D.38)}$$

the form of the depth equation is found

$$\frac{d^2 \, D_n(z)}{dz^2} + \left[E_n - V(z) \right] \; D_n(z) = 0 \qquad \text{(Eq. D.39)}$$

Hence for a given sound speed profile a related potential function exists. The infinity value of this potential determines the boundary between continuous and discrete mode propagation and is given by

$$V(\infty) = \omega^2 \left(\frac{1}{{c_o}^2} - \frac{1}{c_\infty^{o}} \right) \qquad \text{(Eq. D.40)}$$

The turning point for mode functions occurs if $E_n = V(z)$, whereas beyond this depth z the vertical wavenumber is negative and the depth function becomes exponentially decaying. For $E_n > V_\infty$ the waves are always travelling towards $z \rightarrow \infty$, hence leaking into the bottom. This is known as "leaky modes".

The mode cut-off occurs when $E_n > \omega^2/c_o^2$, which is higher than the margin of the continuum since the latter is given by $E_n > V_\infty$.

From this short description it is generally concluded that in shallow water the boundary conditions at the bottom might cause an early cutoff, and only few modes propagate. In deep water at very low frequencies only few modes are trapped in the "potential pot". In both cases ray theory cannot describe accurately the acoustic propagation.

APPENDIX E

LISTINGS OF THE COMPUTER PROGRAMS USED FOR THE SIGNAL PROCESSING

E.1 MAIN PROGRAM FOR OFF-LINE PROCESSING

The purpose of the off-line processing is described in Chapter 3. The following is a listing of its main program.

```
FTN4,L
$EMA(RBE,5)
      PROGRAM LUNDE(3,99),**** SPG BEAMFORMING 3/3/81   ******
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C     PROGRAM FOR CONVENTIONAL AND MAXIMUM LIKELIHOOD BEAMFORMING    C
C     COMBINED WITH DIFFERENT METHODS FOR OWN VESSEL NOISE REDUCTION C
C                                                                    C
C     DVELOPED BY E.B. LUNDE (SPG)                                   C
C             MARCH 1981                                             C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     ARRAYS ARE DEFINED SO THAT THERE CAN BE MAXIMUM
C     19 ARRAY HYDROPHONES + 1 REFERENCE HYDROPHONE,
C     181 BEAMS, 10 FREQUENCIES, AND 10 PROCESSING MODES
C
C        PARAMETER VALUES :
C
C     NH - NUMBER OF HYDROPHONES
C     NB - NUMBER OF BEAMS
C     NF - NUMBER OF FREQUENCY CELLS
C     F  - ARRAY WITH FREQUENCY VALUES
C     C  - SOUND SPEED (m/sec)
C     A  - HYDROPHONE SEPARATION (m)
C     NA - NUMBER OF AVERAGE IN THE RAW MATRIX
C     NT - NUMBER OF TOTAL AVERAGES PER TAPE
C
C  NH,NB,NF,F,A,C  PARAMETERS WILL BE SET BY THE
C     ACQUSITION PROGRAM AND WILL BE IN THE MAGNETIC
C     TAPE HEADER BLOCK.
C
C     PARAMETERS NA , NT WILL BE TYPED BY THE OPERATOR .
C
C                V. DUARTE (COM) 05/03/81/
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DIMENSION R(400,2),E(400,2),EV(20,4)
      DIMENSION BGR(182),PR(20,6)
      DIMENSION B(20,2),BE(20,2),BSQ(20)
      DIMENSION NN(5),IDCB(144)
      DIMENSION TAB(181,2)
      DIMENSION P(182,10),PSP(6)
      DIMENSION F(10),KMAX(20),H(20,2,2)
      DIMENSION IB(815),FIB(12),RB(183)
      DIMENSION RSP(36,2),BSP(11,11)
      EQUIVALENCE (P(1,1),E(1,1),IDCB(1)),(P(1,6),R(1,1))
      EQUIVALENCE (IB(1),RB(1),H(1,1,1)),(IB(19),FIB(1))
      EQUIVALENCE (IB(161),B(1,1)),(IB(241),BE(1,1))
      EQUIVALENCE (IB(321),BSQ(1)),(IB(361),EV(1,1))
      EQUIVALENCE (IB(521),BGR(1)),(BSP(1,1),PR(1,1))
C
      COMMON/RBE/LREC(15),TREC(400),RF(2,4000),
     $BEW(4000,2),BSQW(4000)
```

```
C
      DATA NP/10/
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
   10 CONTINUE
      CALL EXEC(3,407B)
      CALL EXEC(1,7,IB,527)
      WRITE(1,1000) (IB(I),I=10,13)
 1000 FORMAT("  TAPE HEADER : ",4A2,/,"  OK ? (YE/NO) _")
      READ(1,1001) IP
 1001 FORMAT(2A2)
      IF(IP.EQ.2HYE) GO TO 30
      WRITE(1,1002)
 1002 FORMAT(/," INSERT NEW TAPE AND TYPE "GO" TO PROCEED _")
   20 CONTINUE
      READ(1,1001) IP
      IF(IP.EQ.2HGO) GO TO 10
      GO TO 20
   30 CONTINUE
      WRITE(1,1035)
 1035 FORMAT(/," MAG TAPE NUMBER ? _")
      READ(1,*) MGTP
      NH=IB(16)-1
      NB=IB(17)
      NF=IB(18)
      A=FIB(NF+1)
      C=FIB(NF+2)
      WRITE(1,1003) NH,NB,NF,A,C
 1003 FORMAT(/,"  NUMBER OF HYDROPHONES = ",I2,/,
     $"  NUMBER OF BEAMS = ",I3,/,
     $"  NUMBER OF FREQUENCIES = ",I2,/,
     $"  HYDROPHONE SEPARATION = ",F6.4,/,
     $"  SPEED OF SOUND (m/s) = ",F12.4,/)
      DO 40 I=1,NF
      F(I)=FIB(I)
      WRITE(1,1004) I,F(I)
 1004 FORMAT(2X,I2,4X,F8.4," Hz ")
   40 CONTINUE
      WRITE(1,1005)
 1005 FORMAT(/,"  OK ? (YE/NO) _")
      READ(1,1001) IP
      IF(IP.NE.2HYE) STOP 001
 4141 CONTINUE
      WRITE(1,1006)
 1006 FORMAT(/,"  ENTER STARTING BLOCK NUMBER ON M.T.  _")
      READ(1,*) IC
      IF(IC.GT.11400) GO TO 4141
 1106 CONTINUE
      WRITE(1,1007)
 1007 FORMAT(/,"  NUMBER OF AVERAGES ? _")
      READ(1,*) NA
      IF(NA.GE.20) GO TO 50
      WRITE(1,1107)
 1107 FORMAT(/,"  INPUT ERROR ",/," 20<= NA <=600 ")
      GO TO 1106
   50 CONTINUE
      WRITE(1,1008)
 1008 FORMAT(/,"  NUMBER OF AVERAGES PER TAPE ? _")
      READ(1,*) NT
      IF(NT.LE.30.AND.NT.GT.0) GO TO 1108
      WRITE(1,1118)
 1118 FORMAT(/,"  INPUT ERROR ",/,"  0< NA*NT <= 600 ")
      GO TO 1106
 1108 CONTINUE
```

```
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     WRITE ON FILE " >LUNDE " THE FOLLOWING INFO. :
C
C           NT - NUMBER OF AVARAGES PER TAPE
C           NF -   "    OF FREQUENCIES
C           NP -   "    OF PROCESSINGS
C           F1 - FREQUENCY # 1
C           :       :      : :
C           Fn -    "      " n   FOR n=NF
C
C    THIS FILE WILL BE READ BY THE PLOTTING PROGRAM
C
C
      CALL OPEN(IDCB,IERR,6H>LUNDE)
      IF(IERR.LT.0) STOP 100
      CALL CODE
      WRITE(IB,1109) NT,NF,NP
 1109 FORMAT(2X,3I4)
      CALL WRITF(IDCB,IERR,IB,8)
      IF(IERR.LT.0) STOP 101
      CALL CODE
      WRITE(IB,1119) (F(I),I=1,10)
 1119 FORMAT(10(2X,F6.2))
      CALL WRITF(IDCB,IERR,IB,NF*4)
      IF(IERR.LT.0) STOP 102
      CALL CLOSE(IDCB)
C
C
C     END OF INPUT PARAMETER PHASE [VD].
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SOME USEFUL CONSTANTS
C
      NFE=NF+1
      NBE=NB+1
      NBR=NB-1
      NBH=NB/2+1
      NHE=NH+1
      NHD=NH*2
      NHED=NHE*2
      NHSQ=NH*NH
      NHESQ=NHE*NHE
      NSP=NH/2
      ALN10=ALOG(10.)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATE TABLE USED FOR STEERING VECTOR GENERATION
C
      PI=4.*ATAN(1.)
      ARG=2.*PI/(1.*NBR)
      SNH=1./SQRT(1.*NH)
      DO 81 JB=1,NB
      JBM=JB-1
      TAB(JB,1)=COS(ARG*JBM)*SNH
      TAB(JB,2)=SIN(ARG*JBM)*SNH
   81 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     FIND THE MAG TAPE START POINT
C
C
      IF(IC.LE.1) GO TO 83
   82 CONTINUE
      CALL EXEC(1,7,IB,815)
      IF(IB(1).EQ.IC) GO TO 83
      GO TO 82
   83 CONTINUE
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     START OF MAIN LOOP
C
      DO 191 JT=1,NT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CLEAR THE CORRELATION MATRICES IN EMA
C
      CALL WSMY(0.,RF(1,1),1,RF(1,1),1,8000)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCC
CCC   READ RECORDS FROM TAPE
CCC
      DO 101 JA=1,NA
      CALL LEMA1(7,IERR)
      IF(IERR.LT.0) STOP 555
CCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SUBTRACT 1/48 OF X(NH) FROM X(NHE), THIS BECAUSE OF
C     THE METHOD USED IN THE BEAMFORMER TO DELAY X(NHE)
C
      G=-1./48.
      DO 22 KA=1,2
      LM=NHD+KA
      LH=LM-2
      CALL WPIV(G,TREC(LH),NHED,TREC(LM),NHED,TREC(LM),NHED,NF)
   22 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATE THE OUTER PRODUCT OF THE VECTOR X
C     AND ADD THE RESULTING MATRIX TO MATRIX RF
C
      DO 23 JF=1,NF
      LF=(JF-1)*NHED+1
      MF=(JF-1)*NHESQ+1
      CALL WVMOV(TREC(LF),1,R(1,1),1,NHED)
      DO 24 JH=1,NHE
      LH=(JH-1)*2
      LFH=LF+LH
      MFH=MF+(JH-1)*NHE
      YR=R(LH+1,1)
      YI=R(LH+2,1)
      CALL WPIV(YR,TREC(LF),1,RF(1,MFH),1,RF(1,MFH),1,NHED)
      CALL WPIV(YI,TREC(LF+1),2,RF(1,MFH),2,RF(1,MFH),2,NHE)
      CALL WPIV(-YI,TREC(LF),2,RF(2,MFH),2,RF(2,MFH),2,NHE)
   24 CONTINUE
   23 CONTINUE
  101 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     START OF FREQUENCY LOOP
C
      DO 21 JF=1,NF
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     READ THE CORR MATRIX AT THE ACTUAL FREQUENCY FROM EMA
C     TO THE MATRIX R, NORMALIZE AND MOVE IT TO MATRIX E
C
      LF=(JF-1)*NHESQ+1
      DO 115 KA=1,2
      CALL WVMOV(RF(KA,LF),2,R(1,KA),1,NHESQ)
  115 CONTINUE
      CALL VSUM(TR,R(1,1),NHE+1,NH)
```

```
      POW=TR/(1.*NH)
      POWI=1./POW
      DO 111 JH=1,NHE
      LH=(JH-1)*NHE+1
      MH=(JH-1)*NH+1
      CALL VSMY(POWI,R(LH,1),1,E(MH,1),1,NH)
      CALL VSMY(-POWI,R(LH,2),1,E(MH,2),1,NH)
  113 CONTINUE
  111 CONTINUE
      POW=POW/(1.*NA)
      QS=R(NHESQ,1)*POWI
      CALL VSMY(100.,E(1,1),NHE,PR(1,2),1,NH)
      PR(NHE,2)=QS*100.
      PR(NHE,1)=POW
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     PRELIMINARY SPLIT BEAM PROCESSING
C
      NN(1)=-NH+1
      NN(2)=-NH+NSP*NH+NSP+1
      NN(3)=-NH+NSP*NH-NSP-1
      NN(4)=-NH+NSP*NH+1
      DO 701 JSP=1,4
      LINC=NHE
      IF (JSP.EQ.3) LINC=-NHE
      LSP=(JSP-1)*NSP
      DO 702 JH=1,NSP
      NN(JSP)=NN(JSP)+NH
      NEL=NSP+1-JH
      IF (JSP.EQ.3) NEL=JH-1
      DO 703 KA=1,2
      CALL VSUM(RSP(JH+LSP,KA),E(NN(JSP),KA),LINC,NEL)
  703 CONTINUE
  702 CONTINUE
  701 CONTINUE
C     WRITE(6,822) ((RSP(JSP,KA),KA=1,2),JSP=1,36)
  822 FORMAT(2F10.3)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     USE A SUBROUTINE TO FIND EIGENVALUES AND EIGENVECTORS OF R
C     THE EIGENVALUES ARE STORED IN DECREASING ORDER IN VECTOR EV
C     AND THE RESPECTIVEIGENVECTORS IN MATRIX E
C
      NH1=NH
      CALL EIGEN(E(1,1),E(1,2),R(1,1),R(1,2),NHSQ,EV(1,1),NH1)
      CALL VSMY(10.,EV(1,1),1,PR(1,1),1,NH)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     INVERT THE EIGENVALUES
C
      CALL VSDV(1.,EV(1,1),1,EV(1,2),1,NH)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     START OF PRELIMINARY BEAM LOOP
C
      DO 41 JB=1,NBE
      IF (JB.EQ.NBE) GO TO 223
C
C
C     CREATE STEERING VECTOR FROM TABLE
C
      NBH=NB/2+1
      K=NBH-JB
      IF (K.LT.0) K=K+NBR
      L=0
      DO 33 JH=1,NH
      DO 32 KA=1,2
      B(JH,KA)=TAB(L+1,KA)
```

```
   32 CONTINUE
      L=L+K
      IF (L. GE. NBR) L=L-NBR
   33 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATE BEAM DIRECTION OR RELATIVE WAVENUMBER
C
      AL=A*F(JF)/C
      JJ=1
      ARG=(NBH-JB)/(AL*NBR)
  225 CONTINUE
      AARG=ABS(ARG)
      IF (AARG. LE. 1. ) GO TO 222
      BGR(JB)=-10. *AARG
      GO TO 223
  222 CONTINUE
      ARH=SQRT(1. -ARG*ARG)
      BGR(JB)=ATAN2(ARH, ARG)*180. /PI
  223 CONTINUE
C     IF (JJ. EQ. 2) GO TO 224
C     JJ=2
C     IF (JB. LE. NBH) ARG=ARG-1. /AL
C     IF (JB. GT. NBH) ARG=ARG+1. /AL
C     GO TO 225
C 224 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATE THE VECTORS B'E, B'Q, Q'E, AND Q'Q, AND STORE IN EMA
C
      IF (JB. NE. NBE) GO TO 47
      DO 48 KA=1,2
      CALL VMOV(E(NHSQ+1,KA),1,B(1,KA),1,NH)
C     NSB(NBE,KA)=0
   48 CONTINUE
   47 CONTINUE
      LB=(JB-1)*NHE+1
      DO 42 JH=1,NHE
      LH=(JH-1)*NH+1
      DO 43 KA=1,2
      DO 44 KB=1,2
      CALL VDOT(H(JH,KA,KB),B(1,KA),1,E(LH,KB),1,NH)
   44 CONTINUE
   43 CONTINUE
   42 CONTINUE
      CALL VADD(H(1,1,1),1,H(1,2,2),1,BE(1,1),1,NHE)
      CALL VSUB(H(1,2,1),1,H(1,1,2),1,BE(1,2),1,NHE)
      DO 45 KA=1,2
      CALL VWMOV(BE(1,KA),1,BEW(LB,KA),1,NHE)
   45 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATE THE SQUARE AMPLITUDE OF EACH ELEMENT OF MATRIX BE
C     AND STORE IN EMA
C
      DO 46 KA=1,2
      CALL VMPY(BE(1,KA),1,BE(1,KA),1,H(1,KA,1),1,NHE)
   46 CONTINUE
      CALL VADD(H(1,1,1),1,H(1,2,1),1,BSQ(1),1,NHE)
      CALL VWMOV(BSQ(1),1,BSQW(LB),1,NHE)
      P(JB,7)=100. *BSQ(1)
      P(JB,8)=100. *BSQ(2)
      P(JB,9)=100. *BSQ(NHE)
      IF (JB. LT. NBE) GO TO 715
      QNRM=SQRT(BSQ(NHE))
      QNRMI=1. /QNRM
      CALL VSMY(QNRMI,P(1,9),1,P(1,9),1,NB)
      CALL VSMY(QNRMI*100.,BSQ(1),1,PR(1,5),1,NHE)
  715 CONTINUE
C     WRITE(6,819) JB,WA,WB,(BSQ(JH),JH=1,NHE),JB
```

```
C 819 FORMAT(X,I3,2F8.1,2X,20F4.3,2X,I6)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CONTINUE SPLIT BEAM PROCESSING
C
      DO 711  JSP=1,3
      NEL=NSP-1
      IF (JSP.EQ.3) NEL=NSP*2-1
      LSP=(JSP-1)*NSP+1
      DO 712 KA=1,2
      DO 713 KB=1,2
      CALL VDOT(H(JSP,KA,KB),B(2,KA),1,RSP(LSP+1,KB),1,NEL)
  713 CONTINUE
  712 CONTINUE
      PSP(JSP)=RSP(LSP,1)/(1.*NH)+2.*(H(JSP,1,1)-H(JSP,2,2))*SNH
  711 CONTINUE
      PSP(3)=PSP(3)/2.
      PSP(4)=(H(3,2,1)+H(3,1,2))*SNH
      PSP(5)=PSP(3)/SQRT(PSP(1)*PSP(2))
      PSP(6)=PSP(1)+PSP(2)+2.*PSP(3)
      P(JB,6)=PSP(3)*100.
      P(JB,10)=1.
      IF (BGR(JB).LT.0.) GO TO 718
      DEL=ATAN2(PSP(4),PSP(3))/(2.*PI*AL*NSP)
      DCOS=COS(BGR(JB)*PI/180.)
      RCOS=DCOS-DEL
      IF (ABS(RCOS).GT.1.) GO TO 718
      RSIN=SQRT(1.-RCOS*RCOS)
      P(JB,10)=ATAN2(RSIN,RCOS)*180./PI
  718 CONTINUE
      IF (ISSW(3).GE.0) GO TO 719
      WRITE(6,817) JB,BGR(JB),(PSP(JM),JM=1,6),JB
  817 FORMAT(X,I3,4X,F10.2,4X,6F10.4,4X,I6)
  719 CONTINUE
C
  41  CONTINUE
C
C     END OF PRELIMINARY BEAM LOOP
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     WRITE(6,818) JA,JF,POW,QNRM,QS
C 818 FORMAT(2I10,3F10.3)
      IF (ISSW(3).LT.0) WRITE(6,535)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     FIND THE BEAM-INDEX ASSOCIATED WITH EACH EIGENVALUE
C
      DO 51 JH=1,NHE
      CALL WMAX(KMAX(JH),BSQW(JH),NHE,NB)
      PR(JH,3)=KMAX(JH)
      PR(JH,4)=BGR(KMAX(JH))
   51 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     PRINT THE EIGENVALUES AND THE ASSOCIATED BEAM-INDECES
C
C     WRITE(6,810) (JH,JH=1,NH)
      WRITE(1,810) (JH,JH=1,NH)
C     WRITE(6,810) (KMAX(JH),JH=1,NH)
      WRITE(1,810) (KMAX(JH),JH=1,NH)
C     WRITE(6,810) (PR(JH,1),JH=1,NH)
      WRITE(1,810) (PR(JH,1),JH=1,NH)
  810 FORMAT(X,19I4)
  811 FORMAT(X,F5.3,18F4.3)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     FIND REF BEAM DIRECTION
C
```

```
      BDEL=-1.
      DO 56 JB=1,NB
      IF (BGR(JB).LE.0.) GO TO 56
      ADEL=P(JB,10)-BGR(JB)
      IF (ADEL.LE.0..AND.BDEL.GT.0.) GO TO 57
      BDEL=ADEL
   56 CONTINUE
   57 CONTINUE
      KREF=JB
      IF (BDEL.LT.-ADEL) KREF=JB-1
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     FIND THE EIGENVALUE(S) TO BE MODIFIED
C     AND MODIFY THEM, THEN INVERT THEM
C
      CALL VMOV(EV(1,1),1,EV(1,3),1,NH)
      JJ=11
      DO 444 JH=1,NH
      IF (IABS(KMAX(JH)-KREF).GT.3) GO TO 444
      EV(JH,3)=EV(NH,3)
      PR(JJ,6)=JH
      JJ=JJ+1
  444 CONTINUE
      CALL VSDV(1.,EV(1,3),1,EV(1,4),1,NH)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CALCULATE CONVENTIONAL AND MAXIMUM LIKELIHOOD BEAMFORMRING
C
      DO 61 JB=1,NBE
      LB=(JB-1)*NHE+1
      CALL WVMOV(BSQW(LB),1,BSQ(1),1,NHE)
      DO 62 JM=1,4
      CALL VDOT(P(JB,JM),EV(1,JM),1,BSQ(1),1,NH)
   62 CONTINUE
      DO 66 JH=1,NH
      KMAX(JH)=100.*BSQ(JH)
      IF (JB.EQ.NBE) KMAX(JH)=100.*BSQ(JH)/QNRM
   66 CONTINUE
      KMAX(NHE)=100.*BSQ(NHE)/QNRM
      IF (JB.EQ.NBE) KMAX(NHE)=100.*BSQ(NHE)/(QNRM*QNRM)
      IF (ISSW(0).GE.0) GO TO 69
      WRITE(6,815) JB,BGR(JB),(KMAX(JH),JH=1,NHE),JB
  815 FORMAT(X,I3,4X,F10.2,4X,20I4,4X,I6)
   69 CONTINUE
   61 CONTINUE
      CALL VMIN(MIN,P(1,1),1,NB)
      CALL VSDV(1.,P(1,2),1,P(1,2),1,NB)
      CALL VSDV(1.,P(1,4),1,P(1,4),1,NB)
      QR=QS-P(NBE,2)
C     WRITE(6,845) QS,P(NBE,2),QR
C 845 FORMAT(3F10.3)
      IF (ISSW(0).LT.0) WRITE(6,535)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     MAIN PRINT OUTPUT
C
      DO 536 KA=1,2
      CALL VSMY(100.,P(1,KA),1,P(1,KA),1,NB)
  536 CONTINUE
      PR(1,6)=IC
      PR(2,6)=NA
      PR(3,6)=JT
      PR(4,6)=JF
      PR(5,6)=F(JF)
      PR(6,6)=POW
      PR(7,6)=QNRM*100./(1.*NH)
      PR(8,6)=QS*100.
      PR(9,6)=QR*100.
      PR(10,6)=BGR(KREF)
      PR(20,6)=MGTP
```

```
      PR(19,6)=C
      PR(18,6)=F(JF)*10.
      PR(17,6)=A
      DO 876 JH=JJ,16
      PR(JH,6)=0.
  876 CONTINUE
      WRITE(6,538)
  538 FORMAT(12X,3(4X,"  #  BEARING CON MLM SPL BE1 BE2  BQ"))
      DO 531 I=1,3
      M=(I-1)*2+1
      DO 533 J=1,20
      IJ=(I-1)*20+J
      DO 534 K=1,3
      NN(K)=(K-1)*60+IJ
  534 CONTINUE

      WRITE(6,9) J,PR(J,M),PR(J,M+1),(NN(K),BGR(NN(K)),P(NN(K),10),
      (P(NN(K),L),L=1,2),(P(NN(K),L),L=6,9),K=1,3)
    9 FORMAT(3I4,3(4X,9I4))
  533 CONTINUE
  531 CONTINUE
      DO 537 KA=1,2
      CALL VSMY(.01,P(1,KA),1,P(1,KA),1,NB)
  537 CONTINUE
      DO 676 JB=1,NB
      PH=2.5*ALN10/(1.+.5*ABS(P(JB,10)-BGR(JB)))
      P(JB,9)=EXP(PH*P(JB,6)/250.)
      P(JB,10)=EXP(PH)
  676 CONTINUE
      WRITE(6,535)
  535 FORMAT(1H1)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C    SELECT REFERENCE BEAM DIRECTION AND DO REFERENCE
C    BEAM NOISE SUPPRESSION .
C
C    NREF - REFERENCE BEAM NUMBER
C
      NREF=KREF
      LREF=(NREF-1)*NHE+1
      DO 96 KA=1,2
      CALL WVMOV(BEW(LREF,KA),1,BE(1,KA),1,NH)
   96 CONTINUE
      DO 97 JB=1,NB
      LB=(JB-1)*NHE+1
      DO 98 KA=1,2
      CALL WVMOV(BEW(LB,KA),1,B(1,KA),1,NH)
      DO 99 KB=1,2
      CALL VMPY(B(1,KA),1,BE(1,KB),1,H(1,KA,KB),1,NH)
   99 CONTINUE
   98 CONTINUE
      CALL VADD(H(1,1,1),1,H(1,2,2),1,H(1,1,1),1,NH)
      CALL VSUB(H(1,1,2),1,H(1,2,1),1,H(1,2,1),1,NH)
      DO 294 KM=1,2
      JM=KM+4
      DO 293 KA=1,2
      CALL VDOT(H(KA,1,2),EV(1,KM),1,H(1,KA,1),1,NH)
  293 CONTINUE
      CALL VDOT(P(JB,JM),H(1,1,2),1,H(1,1,2),1,2)
  294 CONTINUE
   97 CONTINUE
      SC=1./P(NREF,1)
      CALL VSMY(SC,P(1,5),1,P(1,5),1,NB)
      CALL VSMY(P(NREF,2),P(1,6),1,P(1,6),1,NB)
      CALL VMPY(P(1,6),1,P(1,2),1,P(1,6),1,NB)
      CALL VMPY(P(1,6),1,P(1,2),1,P(1,6),1,NB)
      DO 292 KM=1,2
      JM=KM+4
      CALL VSUB(P(1,KM),1,P(1,JM),1,P(1,JM),1,NB)
  292 CONTINUE
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     DO REFERENCE HYDROPHONE NOISE SUPPRESSION
C
      CALL VMOV(1.,0,EV(1,1),1,NH)
      LREF=NB*NHE+1
      DO 601 KA=1,2
      CALL WVMOV(BEW(LREF,KA),1,BE(1,KA),1,NH)
  601 CONTINUE
      DO 602 JB=1,NB
      LB=(JB-1)*NHE+1
      DO 603 KA=1,2
      CALL WVMOV(BEW(LB,KA),1,B(1,KA),1,NH)
      DO 604 KB=1,2
      CALL VMPY(B(1,KA),1,BE(1,KB),1,H(1,KA,KB),1,NH)
  604 CONTINUE
  603 CONTINUE
      CALL VADD(H(1,1,1),1,H(1,2,2),1,H(1,1,1),1,NH)
      CALL VSUB(H(1,1,2),1,H(1,2,1),1,H(1,2,1),1,NH)
      DO 605 KM=1,2
      JM=KM+6
      DO 606 KA=1,2
      CALL VDOT(H(KA,1,2),EV(1,KM),1,H(1,KA,1),1,NH)
  606 CONTINUE
      CALL VDOT(P(JB,JM),H(1,1,2),1,H(1,1,2),1,2)
  605 CONTINUE
  602 CONTINUE
      SC=1./QS
      CALL VSMY(SC,P(1,7),1,P(1,7),1,NB)
      SC=-1./QR
      CALL VSMY(SC,P(1,8),1,P(1,8),1,NB)
      CALL VSDV(1.,P(1,2),1,P(1,2),1,NB)
      DO 607 KM=1,2
      JM=KM+6
      CALL VSUB(P(1,KM),1,P(1,JM),1,P(1,JM),1,NB)
  607 CONTINUE
      CALL VSDV(1.,P(1,2),1,P(1,2),1,NB)
      CALL VSDV(1.,P(1,8),1,P(1,8),1,NB)
      IF (ISSW(1).GE.0) GO TO 609
      DO 281 JB=1,NB
      WRITE(6,816) JB,BGR(JB),(P(JB,JM),JM=1,10),JB
  816 FORMAT(X,I3,4X,F10.2,4X,10F10.4,4X,I6)
  281 CONTINUE
      WRITE(6,535)
  609 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     MOVE P(JB,JM),JB=1,NB AND JM=1,2 TO DISC
C
      JT1=JT
      IF(JT.GT.16) JT1=JT-16
      LD=(JF-1)*2
      ISEC=(JT1-1)*6
      ITRK1=LD*NP
      DO 400 JD=1,NP
      ITRK=ITRK1+(JD-1)*2
      IF(JT.GT.16) ITRK=ITRK+1
C     DO 500 ID=1,NB
C     RB(ID)=P(ID,JD)
C 500 CONTINUE
C     WRITE(6,599) (P(JB,JD),JB=1,NBE)
      CALL INTER(P(1,JD),RB(1),F(JF),NB)
      CALL VMAX(NMAX,RB(1),1,NB)
      RB(182)=RB(NMAX)
      RB(183)=POW
C     NBC=NB+2
C     WRITE(6,599) (RB(JB),JB=1,NBC)
C 599 FORMAT(10F8.3)
      CALL EXEC(2,14,IB,384,ITRK,ISEC)
  400 CONTINUE
```

```
C
  21  CONTINUE
C
C     END OF FREQUENCY LOOP
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C
C     END OF MAIN LOOP
C
  191 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     ALL DATA ARE PROCESSED AND THE RESULTS ARE ON DISC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     END OF MAIN PROGRAM
C
      IMERD=LREC(1)
      WRITE(1,9191) IMERD
 9191 FORMAT(/," LAST BLOCK : ",I6)
C
C
      END
FTN4 COMPILER: HP92060-16092 REV.  2001 (791101)
**  NO WARNINGS **  NO ERRORS **   PROGRAM = 10953      COMMON = 00000
```

E.2 SUBROUTINE EIGEN

The EIGEN subroutine calculates the eigenvalues and eigenvectors of a Hermitian matrix. A listing of the subroutine follows.

```
FTN4,L
      SUBROUTINE EIGEN(ARE,AIM,ERE,EIM,ISIZE,EIGV,N)
      DIMENSION ARE(ISIZE),AIM(ISIZE),ERE(ISIZE),EIM(ISIZE)
      DIMENSION EIGV(20),H(20,4)
      INTEGER P,Q,NN(20)
C
C     DATA FOR BREAKING OFF
C
      NNN=N
C
      IOW=N
C
      EPS=.1E-9
C
C     D  UNITY MATRIX ERE,EIM,FOR EIGENVECTORS
C
      DO 1 I=1,N
      DO 1 J=1,N
      IJ=(I-1)*N+J
      JI=(J-1)*N+I
      ERE(IJ)=0.
      EIM(IJ)=0.
      IF (I.GT.J) ARE(IJ)=ARE(JI)*ARE(JI)+AIM(JI)*AIM(JI)
1     CONTINUE
      DO 2 I=1,N
      K9=(I-1)*N+I
      ERE(K9)=1.
2     CONTINUE
C
C     BEGIN OF ITERATION
C
      DO 100 IT=1,1000
C
C     COMPUTATION OF SQUARE SUM OF NONDIAGONAL ELEMENTS
C     AND DETERMINATION OF MAX NONDIAGONAL ELEMENT
C
C     SW=0.
      H(1,1)=0.
      DO 101 I=2,N
      NEL=I-1
      IJ=(I-1)*N+1
      CALL VMAX(NN(I),ARE(IJ),1,NEL)
      H(I,1)=ARE(IJ-1+NN(I))
  101 CONTINUE
      CALL VMAX(Q,H(1,1),1,N)
      P=NN(Q)
      SW=SQRT(H(Q,1))
      NNN=IT
      IF(SW.LT.EPS) GOTO 899
C
C     PIVOT-ELEMENT FOUND COMPUTATION ROTATION PARAMETERS
C
      NP=N*P
      NQ=N*Q
      NPP=NP-N+P
      NPQ=NP-N+Q
      NQP=NQ-N+P
      NQQ=NQ-N+Q
      APP=ARE(NPP)
      AQQ=ARE(NQQ)
      APQ=ARE(NPQ)
      BPQ=AIM(NPQ)
      RO=SQRT(APQ*APQ+BPQ*BPQ)
      ROD=2.*RO
      C2X=(AQQ-APP)/ROD
      VZ=-1.
```

```
      IF(C2X.GE.0) VZ=1.
      CX=C2X+VZ*SQRT(C2X*C2X+1.)
      C2X=CX*CX
      CC=C2X/(1.+C2X)
      C=SQRT(CC)
      S=C/CX
      SS=S*S
      SC=S*C
      SR=APQ*S/RO
      SI=-BPQ*S/RO
C
C     ROTATION
C
      NEL=P-1
      IP=P
      IQ=Q
      CALL VSMY(SI,AIM(IQ),N,H(1,1),1,NEL)
      CALL VSMY(-SR,AIM(IQ),N,H(1,2),1,NEL)
      CALL VSMY(SI,AIM(IP),N,H(1,3),1,NEL)
      CALL VSMY(SR,AIM(IP),N,H(1,4),1,NEL)
      CALL VPIV(-SR,ARE(IQ),N,H(1,1),1,H(1,1),1,NEL)
      CALL VPIV(-SI,ARE(IQ),N,H(1,2),1,H(1,2),1,NEL)
      CALL VPIV(SR,ARE(IP),N,H(1,3),1,H(1,3),1,NEL)
      CALL VPIV(-SI,ARE(IP),N,H(1,4),1,H(1,4),1,NEL)
      CALL VPIV(C,ARE(IP),N,H(1,1),1,ARE(IP),N,NEL)
      CALL VPIV(C,AIM(IP),N,H(1,2),1,AIM(IP),N,NEL)
      CALL VPIV(C,ARE(IQ),N,H(1,3),1,ARE(IQ),N,NEL)
      CALL VPIV(C,AIM(IQ),N,H(1,4),1,AIM(IQ),N,NEL)
      JP=NP-N+1
      JQ=NQ-N+1
      CALL VMPY(ARE(IP),N,ARE(IP),N,ARE(JP),1,NEL)
      CALL VMPY(AIM(IP),N,AIM(IP),N,AIM(JP),1,NEL)
      CALL VMPY(ARE(IQ),N,ARE(IQ),N,ARE(JQ),1,NEL)
      CALL VMPY(AIM(IQ),N,AIM(IQ),N,AIM(JQ),1,NEL)
      CALL VADD(ARE(JP),1,AIM(JP),1,ARE(JP),1,NEL)
      CALL VADD(ARE(JQ),1,AIM(JQ),1,ARE(JQ),1,NEL)
      NEL=Q-P-1
      IP=NPP+1
      IQ=NP+Q
      CALL VSMY(SI,AIM(IQ),N,H(1,1),1,NEL)
      CALL VSMY(SR,AIM(IQ),N,H(1,2),1,NEL)
      CALL VSMY(-SI,AIM(IP),1,H(1,3),1,NEL)
      CALL VSMY(-SR,AIM(IP),1,H(1,4),1,NEL)
      CALL VPIV(-SR,ARE(IQ),N,H(1,1),1,H(1,1),1,NEL)
      CALL VPIV(SI,ARE(IQ),N,H(1,2),1,H(1,2),1,NEL)
      CALL VPIV(SR,ARE(IP),1,H(1,3),1,H(1,3),1,NEL)
      CALL VPIV(-SI,ARE(IP),1,H(1,4),1,H(1,4),1,NEL)
      CALL VPIV(C,ARE(IP),1,H(1,1),1,ARE(IP),1,NEL)
      CALL VPIV(C,AIM(IP),1,H(1,2),1,AIM(IP),1,NEL)
      CALL VPIV(C,ARE(IQ),N,H(1,3),1,ARE(IQ),N,NEL)
      CALL VPIV(C,AIM(IQ),N,H(1,4),1,AIM(IQ),N,NEL)
      JP=NP+P
      JQ=NQP+1
      CALL VMPY(ARE(IP),1,ARE(IP),1,ARE(JP),N,NEL)
      CALL VMPY(AIM(IP),1,AIM(IP),1,AIM(JP),N,NEL)
      CALL VMPY(ARE(IQ),N,ARE(IQ),N,ARE(JQ),1,NEL)
      CALL VMPY(AIM(IQ),N,AIM(IQ),N,AIM(JQ),1,NEL)
      CALL VADD(ARE(JP),N,AIM(JP),N,ARE(JP),N,NEL)
      CALL VADD(ARE(JQ),1,AIM(JQ),1,ARE(JQ),1,NEL)
      NEL=N-Q
      IP=NPQ+1
      IQ=NQQ+1
      CALL VSMY(-SI,AIM(IQ),1,H(1,1),1,NEL)
      CALL VSMY(-SR,AIM(IQ),1,H(1,2),1,NEL)
      CALL VSMY(-SI,AIM(IP),1,H(1,3),1,NEL)
      CALL VSMY(SR,AIM(IP),1,H(1,4),1,NEL)
      CALL VPIV(-SR,ARE(IQ),1,H(1,1),1,H(1,1),1,NEL)
      CALL VPIV(SI,ARE(IQ),1,H(1,2),1,H(1,2),1,NEL)
      CALL VPIV(SR,ARE(IP),1,H(1,3),1,H(1,3),1,NEL)
      CALL VPIV(SI,ARE(IP),1,H(1,4),1,H(1,4),1,NEL)
      CALL VPIV(C,ARE(IP),1,H(1,1),1,ARE(IP),1,NEL)
      CALL VPIV(C,AIM(IP),1,H(1,2),1,AIM(IP),1,NEL)
```

```
      CALL VPIV(C,ARE(IQ),1,H(1,3),1,ARE(IQ),1,NEL)
      CALL VPIV(C,AIM(IQ),1,H(1,4),1,AIM(IQ),1,NEL)
      JP=NQ+P
      JQ=NQ+Q
      CALL VMPY(ARE(IP),1,ARE(IP),1,ARE(JP),N,NEL)
      CALL VMPY(AIM(IP),1,AIM(IP),1,AIM(JP),N,NEL)
      CALL VMPY(ARE(IQ),1,ARE(IQ),1,ARE(JQ),N,NEL)
      CALL VMPY(AIM(IQ),1,AIM(IQ),1,AIM(JQ),N,NEL)
      CALL VADD(ARE(JP),N,AIM(JP),N,ARE(JP),N,NEL)
      CALL VADD(ARE(JQ),N,AIM(JQ),N,ARE(JQ),N,NEL)
C
C  E.B. L. 18/02/82
C
      ARE(NPP)=APP*CC+AQQ*SS-ROD*SC
      ARE(NQQ)=APP*SS+AQQ*CC+ROD*SC
      ARE(NPQ)=0.
      AIM(NPQ)=0.
      ARE(NQP)=0.
C
C     COMPUTATION OF EIGENVECTORS
C
      IP=NP-N+1
      IQ=NQ-N+1
      CALL VSMY(SI,EIM(IQ),1,H(1,1),1,N)
      CALL VSMY(-SR,EIM(IQ),1,H(1,2),1,N)
      CALL VSMY(SI,EIM(IP),1,H(1,3),1,N)
      CALL VSMY(SR,EIM(IP),1,H(1,4),1,N)
      CALL VPIV(-SR,ERE(IQ),1,H(1,1),1,H(1,1),1,N)
      CALL VPIV(-SI,ERE(IQ),1,H(1,2),1,H(1,2),1,N)
      CALL VPIV(SR,ERE(IP),1,H(1,3),1,H(1,3),1,N)
      CALL VPIV(-SI,ERE(IP),1,H(1,4),1,H(1,4),1,N)
      CALL VPIV(C,ERE(IP),1,H(1,1),1,ERE(IP),1,N)
      CALL VPIV(C,EIM(IP),1,H(1,2),1,EIM(IP),1,N)
      CALL VPIV(C,ERE(IQ),1,H(1,3),1,ERE(IQ),1,N)
      CALL VPIV(C,EIM(IQ),1,H(1,4),1,EIM(IQ),1,N)
 1006 FORMAT(2X,'ARE=',4(F10.3,3X),5X,'AIM=',4(F10.3,3X))
 1007 FORMAT(2X,'ERE=',4(F10.3,3X),5X,'EIM=',4(F10.3,3X))
C
C     ROTATION END
100   CONTINUE
C
C     200 ITERATIONS PERFORMED SET N=-N
C
      NNN=-N
C     WRITE(1,1002) NNN
C1002 FORMAT(10X,'NNN=-N=',I3)
C
C     SORT OF EIGENVALUES WITH INCREASING ABS VALUES
C
899   LL=1
900   AK=-1.E04
      DO 901 I=1,N
      K2=(I-1)*N+I
901   AK=AMAX1(AK,ARE(K2))
      DO 902 I=1,N
      K1=(I-1)*N+I
      IF(AK.EQ.ARE(K1)) GOTO 903
902   CONTINUE
C
C     FOUND STORE IN EIGV MARC POSITION
C
903   NN(LL)=I
      EIGV(LL)=AK
      LL=LL+1
      K11=(I-1)*N+I
      ARE(K11)=-1.E05
      IF(LL-N) 900,900,904
C
C     EIGENVALUES ARE SORTED IN EIGV. SORT OF COLUMNS
C     OF ERE, EIM IN THE SAME ORDER AND STORE IN ARE,AIM.
C
```

```
 904    DO 905 I=1,N
        KO=(I-1)*N+1
        K1O=(NN(I)-1)*N+1
        CALL VMOV(ERE(K1O),1,ARE(KO),1,N)
        CALL VSMY(-1.,EIM(K1O),1,AIM(KO),1,N)
 905    CONTINUE
        N=NNN
        ICL=15414B
        IBL=3407B
        CALL EXEC(2,1001B,ICL,1)
        CALL EXEC(2,1001B,IBL,1)
        WRITE(1,1003) N
  1003 FORMAT(10X,'OUT OF EIGEN   N=NNN=',I3)
        WRITE(1,2004) (EIGV(I),I=1,IOW)
  2004 FORMAT(5X,'EIGV=   ',F15.5)
 C
 C      RETURN BREAKING OFF
 C
        RETURN
        END
   FTN4 COMPILER: HP92060-16092 REV. 2001 (791101)
   **  NO WARNINGS **  NO ERRORS **   PROGRAM = 02935       COMMON = 00000
```

E.3 ASSEMBLER PROGRAM FOR DATA TRANSFER

The following is a listing of a program for transferring data from magnetic tape into a part of the off-line HP computer memory called "EMA".

```
0001                  ASMB,B,L,T
0002   00000                 NAM LEMA1,7     **** COM,V.DUARTE REV.00   05/03/81   ****
0003                         ENT LEMA1
0004                         EXT .EMAP,.ENTR,EXEC
0005                         EXT RBE
0006*
0007**********************************************************************
0008*
0009*         C O M
0010*
0011*                  V.  DUARTE    MARCH 81     REV 00
0012*
0013*        CALLING SEQUENCE :
0014*
0015*                CALL LEMA1(LU,IERR)
0016*
0017*                   LU - MAGNETIC TAPE LOGIC UNIT
0018*                   IERR - RETURN IF ERROR
0019*
0020*       PURPOSE :
0021*
0022*           ROUTINE TO TRANSFER DATA FROM MAGNETIC TAPE INTO
0023*      AN "EMA" ARRAY. "EMA" ARRAY SHOULD BE DECLARED  AS THE
0024*      FIRST OF THE "EMA" AREA.
0025*
0026**********************************************************************
0027*
0028   00000 000000   LU     NOP               MT LOGIC UNIT
0029   00001 000000   IERR   NOP               ERROR RETURN
0030   00002 000000   LEMA1  NOP               ROUTINE ENTRY POINT
0031   00003 016002X         JSB .ENTR         GET INPUT PARAMETER
0032   00004 000000R         DEF LU
0033   00005 016001X         JSB .EMAP         GET USER MAP ADDRESS
0034   00006 000012R         DEF *+4
0035   00007 000004X         DEF RBE           OF CORRESPONDENT EMA
0036   00010 000032R         DEF TABLE         AREA ARRAY
0037   00011 000037R         DEF INDEX
0038   00012 026030R  RTN    JMP ERROR         SEND ERROR MESSAGE TO MAIN
0039   00013 076041R         STB BUF           SAVE ADDRESS
0040   00014 162000R         LDA LU,I          PREPARE CONTROL WORD
0041   00015 042043R         ADA =B100         FOR EXEC CALL AS BINARY READ
0042   00016 072000R         STA LU
0043   00017 016003X         JSB EXEC          READ DATA FROM MT
0044   00020 000025R         DEF *+5
0045   00021 000040R         DEF CODE
0046   00022 000000R         DEF LU
0047   00023 100041R         DEF BUF,I
0048   00024 000042R         DEF NWORD
0049   00025 002400          CLA
0050   00026 172001R  ENDI   STA IERR,I
0051   00027 126002R         JMP LEMA1,I
0052   00030 003400   ERROR  CCA
0053   00031 026026R         JMP ENDI
0054   00032 000001   TABLE  DEC 1
0055   00033 177777          DEC -1
0056   00034 000001          DEC 1
0057   00035 000000          DEC 0
0058   00036 000000          DEC 0
0059   00037 000001   INDEX  DEC 1
0060   00040 000001   CODE   DEC 1
0061   00041 000000   BUF    NOP
0062   00042 001457   NWORD  DEC 815
       00043 000100
0063                         END
**   NO ERRORS *TOTAL **RTE ASMB 92067-16011**
```

E.4 PROGRAM FOR ON-LINE PROCESSING

The purpose of the on-line processing is described in Chapter 2. The following is a listing of the program.

```
&SPMBM T=00004 IS ON CR00013 USING 00056 BLKS R=0000

FTN4,L
$EMA(IBEMA,5)
      PROGRAM SUPVR
      DIMENSION SF(20),NDXSF(20)
      DIMENSION IHEAD(15),IB1(4),RFQ(22)
C     DIMENSION ITIME(10)
      DIMENSION POWR(184),NAM1(3),IFQ(2)
      DIMENSION IDCB(10)
      COMMON /IBEMA/ IBB(20480)
      COMPLEX CX
      COMMON LUMT,IHEAD,CX(400)
      EQUIVALENCE (IB1(1),CX(1)),(RFQ(1),IB1(4))
      EQUIVALENCE (IFQ(1),SF(1))
      EQUIVALENCE (AFRST,IDCB(3)),(ALAST,IDCB(5)),(DBMIN,IDCB(7)),
     *            (DBMAX,IDCB(9)),(NPTS,POWR(1)),(PMAX,POWR(2))
      DATA IDCB(1),DBMIN,DBMAX/0,-30.0,0.0/
      DATA NAM1/2HBF,2HPL,2HT /
      DATA IBUS2,IBUS3/128,192/
      DATA IEX,IST/2HEX,2HST/
      DO 15 K=1,15
15    IHEAD(K)=0
      LUMT=10
10    WRITE(1,100)
100   FORMAT('NUMBER OF SAMPLES(POWER OF 2)?')
      READ(1,*) NS
      I=1
20    IF(I-NS) 25,40,30
25    I=I+I
      GOTO 20
30    WRITE(1,200)
200   FORMAT('NUMBER OF SAMPLES MUST BE A POWER OF 2'/
     *         'PLEASE ENTER A NEW VALUE')
      GOTO 10
40    RNS=NS
      WRITE(1,300)
300   FORMAT('NUMBER OF HYDROPHONES,SPACING(METRES)?')
      READ(1,*) NH,D
      IF(NH .LE. 0) GOTO 10
      RNH=NH
      BS=RNS*RNH
      IF(BS .LE. 40960.0) GOTO 45
      WRITE(1,400)
400   FORMAT('CAPACITY OF MAP MEMORY BUS 2 EXCEEDED'/
     * 'REDUCE NUMBER OF SAMPLES AND/OR HYDROPHONES')
      GOTO 40
45    WRITE(1,450)
450   FORMAT('NUMBER OF BEAMS,INITIAL ANGLE & FINAL ANGLE(DEGREES)?')
      READ(1,*) NBMS,AFRST,ALAST
C
C     COMPUTE SPACE REQUIRED FOR CO-VARIANCE MATRICES
C     AND STEERING VECTORS FOR A SINGLE FREQUENCY
C
      SP=2.0*RNH*(NH+NBMS)-2*NBMS
      IF(SP+BS .LE. 40960.0) GOTO 46
      WRITE(1,470)
470   FORMAT('NO SPACE FOR CO-VARIANCE MATRICES & STEERING VECTORS')
      CALL EXEC(6)
46    WRITE(1,460)
460   FORMAT('SAMPLING FREQUENCY?')
      READ(1,*) SFQ
      DF=SFQ/NS
      FQMAX=DF*(NS/2-1)
50    M=40960.0-BS
```

```
      NN=2*NH*NH
C     NN=SIZE OF EACH CO-VARIANCE MATRIX
      L=20
60    WRITE(1,600)
600   FORMAT('HOW MANY FREQUENCIES DO YOU WISH TO SELECT?')
      READ(1,*) NF
      IF(NF .LE. 0) GOTO 40
      IF(NF .LE. L) GOTO 70
      WRITE(1,700)
700   FORMAT('MAXIMUM NUMBER OF FRQUENCIES EXCEEDED'/
     *  'PLEASE ENTER A NEW VALUE')
      GOTO 60
70    RNF=NF
      WRITE(1,800) DF,FQMAX
800   FORMAT('FREQUENCY RESOLUTION =',F8.2,' HZ'/
     *  'MAXIMUM FREQUENCY IS ',F6.1,' HZ'/
     *  'INPUT SELECTED FREQUENCIES USING FREE FIELD FORMAT'/
     *  'USE SPACE OR COMMA TO SEPARATE DATA ITEMS'/
     *  'TERMINATE EACH LINE(EXCEPT THE LAST ONE) BY /'/
     *  'FIRST FREQUENCY SPECIFIED SHOULD BE THE ONE'/
     *  'FOR IMMEDIATE PROCESSING AND DISPLAY')
      READ(1,*) (SF(I),I=1,NF)
      DO 72 I=1,NF
      IF(SF(I) .GT. FQMAX) GOTO 73
      K=SF(I)/DF+0.5
      SF(I)=K*DF
      NDXSF(I)=K+1
72    CONTINUE
      GOTO 74
73    WRITE(1,480)
480   FORMAT('MAXIMUM FREQUENCY EXCEEDED'/
     * 'PLEASE SELECT A NEW SET')
      GOTO 70
74    WRITE(1,500)
500   FORMAT('SPECIFY PROCESSING TYPE'/'1=CBF'/'2=CBS'/'3=MLM'/'4=MLS')
      READ(1,*) ITYPE
      IF(ITYPE .LT. 1 .OR. ITYPE .GT. 4) GOTO 74
      WRITE(1,1000)
1000  FORMAT('NUMBER OF DATA ACQUISITIONS?')
      READ(1,*) NA
C
C     PASS FIRST BUFFER TO PLOT PROGRAM
C     THEN SCHEDULE PLOT PROGRAM
C
      NPTS=NBMS
      ILEN=2*(NBMS+3)
      ICLS=0
      CALL EXEC(20,0,IDCB,10,0,0,ICLS)
      CALL EXEC(10,NAM1,ICLS,ITYPE,IFQ(1),IFQ(2),ILEN)
C
      WRITE(1,75)
75    FORMAT('RUN-ID(1-8 CHARS)?')
      READ(1,76) (IHEAD(K),K=10,13)
76    FORMAT(4A2)
D     WRITE(6,950)
D950  FORMAT(1H ,'FREQUENCY    INDEX')
D     DO 75 I=1,NF
D     WRITE(6,980) SF(I),NDXSF(I)
D980  FORMAT(1H ,F9.1,I9)
D75   CONTINUE
      WRITE(1,1100)
1100  FORMAT('TO TERMINATE THE PROGRAM HIT ANY KEY'/
     *         'THEN GIVE THE COMMAND BR,SUPVR')
C
C     THE DATA ACQUISITION PROGRAM IS EXPECTED TO PLACE INFORMATION
C     INTO THE FIRST NS*NH WORDS(32 BITS EACH) OF MEMORY BUS 2
C     SPACE FOR COVARIANCE MATRIX IS ALLOCATED AT HIGH END OF BUS 2
C     BUT LAST WORD IS LEFT FREE TO AVOID BUFFER ADDRESS OUTSIDE LIMITS
C
      BSCV=NN
      BACV=81918.0-2.0*BSCV
      CALL MPOPN(6)
```

```
C
C     SET UP THE FOLLOWING CONSTANTS IN THE SCALAR TABLE
C     50 = RNH*RNH,USED IN MLM & MLS CALCULATIONS
C     51 = 1.0/(RNH-1.0),USED IN CALCULATING AVERAGE POWER
C     52-53 = CMPLX(1.0,0.0),USED BY CSDOT
C     54 = 1.0/(NA*NS*NS),USED TO NORMALISE CO-VARIANCE MATRIX
C     55 = 100.0,USED TO SCALE RESULTS BEFORE PRINTING
C
      CX(1)=CMPLX(RNH*RNH,1.0/(RNH-1.0))
      CX(2)=1.0
      CX(3)=CMPLX(1.0/(NA*RNS*RNS),100.0)
      CALL MPWST(50,CX,6,1)
C
C     CALCULATION OF STEERING VECTORS
C
      PI=3.14159265
      AINC=(ALAST-AFRST)/(NBMS-1)
      A=AFRST
C
      BSSV=2.0*(RNH-1.0)*NBMS
      BASV=BACV-2.0*BSSV
D     WRITE(6,86) BASV
D86   FORMAT(' STEERING VECTORS START AT BUS2 ADDRESS:  ',F6.1)
      LROW=NBMS
      ROWL=LROW
C
C     CONFIGURE FIRST ROW AS A COMPLEX VECTOR
C     AND FILL WITH THE CONSTANT (1.0,0.0)
C
      CALL MPCLB(21,BASV,ROWL,1,NH-1,0)
      DO 210 I=1,LROW
      CX(I)=1.0
210   CONTINUE
      CALL MPWDB(21,CX,1,1)
C
C     CONFIGURE SECOND ROW AND INITIALIZE TO
C     EXP(J*2*PI*F*D/C*COS(ANGLE))
C
      BA=BASV+4.0
      CALL MPCLB(21,BA,ROWL,1,NH-1,0)
      K=1
      CONST=2.0*PI*D*SF(1)/1500.0
      DO 220 J=1,NBMS
      F=CONST*COS(PI*A/180.0)
      CX(K)=CMPLX(COS(F),SIN(F))
      K=K+1
      A=A+AINC
220   CONTINUE
      CALL MPWDB(21,CX,1,1)
C
C     SET UP REMAINING ROWS OF MATRIX
C     ROW(K)=ROW(2)*ROW(K-1)
C
      CALL MPEQB(22,21)
      LST=NH-1
      DO 240 I=3,LST
      BA=BA+4.0
      CALL MPCLB(23,BA,ROWL,1,NH-1,0)
      CALL CVMUL(23,0,21,0,22)
      CALL MPEQB(22,23)
240   CONTINUE
C
C     SET UP COSINE TABLE ON BUS 3 FOR USE BY FFT FUNCTION
C
      CTS=NS/2
      CALL MPCLB(30,0.0,CTS,0,1,0)
      CALL VCOST(30,CTS)
C
C     SET LST TO NO. OF HYDROPHONES-1
C     SET RNK=NO. OF 16-BIT HALFWORDS PER COMPLETE COVARIANCE MATRIX
C     SET SNK=NO. OF 16-BIT HALFWORDS PER EXTRACTED COVARIANCE MATRIX
C     INITIALIZE BAC TO START ADDRESS OF FIRST COVARIANCE MATRIX
C     INITIALIZE DAC TO START ADDRESS OF FIRST EXTRACTED MATRIX
C     INITIALIZE BAQ TO START ADDRESS OF FIRST Q-VECTOR(COLUMN)
```

```
C
      LST=NH-1
      RNK=4. 0*RNH*RNH
      SNK=4. 0*(LST*LST)
      BAC=BACV
      DAC=RNS
      BAQ=BACV+4. 0*RNH*(RNH-1. 0)
C
C     COMPUTE ADDRESSES OF FOLLOWING VECTORS
C     AR=ADDRESS OF FIRST COLUMN OF COVARIANCE MATRIX
C     AQ=ADDRESS OF  Q    COLUMN OF COVARIANCE MATRIX
C     AC=ADDRESS OF FIRST COLUMN OF EXTRACTED MATRIX
C
      AR=BAC
      AQ=BAQ
      AC=DAC
D     WRITE(6, 330) BAC, BAQ, DAC
D330  FORMAT(1H ,3F10. 1)
C
      INIT=1
      KK=1
      K=1
C
C     CONFIGURE STEERING VECTORS AS A SINGLE COMPLEX BUFFER
C
      CALL MPCLB(IBUS2+7, BASV, 0. 5*BSSV, 1, 1, 0)
C
C     CONFIGURE EACH ROW OF COVARIANCE MATRIX ON BUS 3( R OR, INV(R))
C     AS A COMPLEX VECTOR USING BUFFER NUMBERS FROM 32 ONWARDS
C
      DO 426 J=1, LST
      CALL MPCLB(IBUS3+31+J, AC+4*(J-1), RNH-1. 0, 1, LST, 0)
426   CONTINUE
      CALL BFPRM(LOBF, NRCH, XF, IER)
      IF(IER .NE. 0) STOP 1
C
C     PREPARE TO WRITE FIRST BLOCK ON MAG. TAPE
C
777   IHEAD(1)=1
      IB1(1)=NH
      IB1(2)=NBMS
      IB1(3)=NF
      DO 78 J=1, NF
      RFQ(J)=SF(J)
78    CONTINUE
      RFQ(NF+1)=D
      RFQ(NF+2)=1500. 0
C
C     SWITCH M. T. LU# FROM 7 TO 10 OR VICE VERSA
C
      LUMT=17-LUMT
      CALL EXEC(3, LUMT+400B)
      CALL EXEC(2, LUMT, IHEAD, 527)
      IHEAD(1)=2
C
C
C     CLEAR THE COVARIANCE MATRIX BEFORE THE
C     NEXT CYCLE OF ACCUMULATION AND PROCESSING
C
77    CALL MPCLB(20, BACV, BSCV, 0, 1, 0)
      CALL VCLR(20)
C
C     TT=0. 0
      DO 80 I=1, NA
C     CALL EXEC(11, ITIME)
C     CALL MBCOV(7, INIT, XF, IEND)
C     IF(IEND .LT. 0) GOTO 99
      CALL MBC2(20, LOBF, NRCH)
C     CALL EXEC(11, ITIME(6))
C     IDIFF=ITIME(8)-ITIME(3)
C     IF(IDIFF .LT. 0) IDIFF=IDIFF+60
C     IDIFF=IDIFF*60+ITIME(7)-ITIME(2)
C     RDIFF=IDIFF+(ITIME(6)-ITIME(1))*0. 01
C     TT=TT+RDIFF
```

```
      CALL COVAR(NS,NH,K,NDXSF,NF,XF)
      K=0
      INIT=0
80    CONTINUE
C     WRITE(6,81) TT
C81   FORMAT(' M.T. READ TIME = ',F8.2)
C
C     CALL EXEC(11,ITIME)
C
C     CONFIGURE COVARIANCE MATRIX AS A REAL VECTOR THEN
C     NORMALIZE & CONJUGATE BY NEGATING THE ODD PART
C
      CALL MPCLB(20,BACV,BSCV,0,1,0)
      CALL VSMA1(20,54,20,0)
      CALL MPODD(8,20)
      CALL VNEG(8)
C
C     CONFIGURE FIRST COLUMN OF COVARIANCE MATRIX AS A REAL VECTOR
C     SO COLUMNS CAN BE MOVED ONE BY ONE TO BUS 3 USING "VMOV"
C
      CALL MPCLB(IBUS2+8,BACV,2.0*RNH,0,1,0)
      CALL MPCLB(IBUS3+9,RNS,2.0*LST,0,1,0)
C
C     CREATE FUNCTION LIST TO EXTRACT A COLUMN OF A COVARIANCE MATRIX
C
      IF(KK .EQ. 0) GOTO 85
      CALL MPBFL(1)
      CALL VMOV(9,8)
      CALL MPSBA(8,2*NH)
      CALL MPSBA(9,2*LST)
      CALL MPEFL(1)
C
85    CALL MPFOR(1,1,LST,1)
C
C     PRINT THE COVARIANCE MATRIX JUST TRANSFERRED
C
      IF(ISSW(0) .LT. 0) CALL CVMPR(31,LST,AC,SF(1))
C
C     CONFIGURE THE FOLLOWING BUFFERS
C     21 = RESULT OF INV(R).Q(COMPLEX)
C     22 = RESULT OF R.B(COMPLEX) OR INV(R).B(COMPLEX)
C     23 = RESULTS MATRIX(REAL)
C
      BA=10.0*NBMS
      STEP=4.0*(RNH-1.0)
      CALL MPCLB(21,BA,RNH-1.0,1,1,0)
      CALL MPCSO(22,58,NH-1,1)
      CALL MPCLM(23,2.0*NBMS,4,NBMS)
C
C     CONFIGURE LOGICAL BUFFER CONSISTING OF REAL PARTS
C     OF THE DIAGONAL ELEMENTS(EXCEPT LAST ONE)
C     OF THE COVARIANCE MATRIX
C     THEN COMPUTE AVERAGE SUM IN SCALAR 132
C     AND TAKE RECIPROCAL OF THIS VALUE
C
      CALL MPCLB(20,BACV,RNH-1.0,0,2*(NH+1),0)
      CALL SSUM(132,20,51)
      CALL SDIV(129,1,132)
C
C     CONFIGURE FIRST STEERING VECTOR
C
      CALL MPCLB(25,BASV,RNH-1.0,1,1,0)
C
D     WRITE(6,415)
D415  FORMAT(1H1)
      CALL MPCOL(26,1,23)
C
C     CONFIGURE Q-VECTOR(COLUMN)
C
      CALL MPCLB(24,AQ,RNH-1.0,1,1,0)
C
      IF(ITYPE .LE. 2) GOTO 425
C
```

```
      MAP=MPCLA(IBUS3+31,AC,LST,LST,1,0)
      CALL CMINV(31,124,2)
C
C     CMINV GIVES -INV(R) SO CONFIGURE THE
C     AREA AS A REAL BUFFER & MULTIPLY BY -1
C
      CALL MPCLB(31,AC,2.0*(LST*LST),0,1,0)
      CALL VSMA1(31,3,31,0)
C
C     COMPUTE INV(R).Q IN SCALAR TABLE
C     AND SAVE RESULT IN BUFFER NO. 21
C
      DO 420 J=1,LST
      CALL CSDOT(54+2*J,J+31,52,24)
420   CONTINUE
      CALL MPEQB(27,21)
      CALL MPTSB(27,56,LST)
C
C     CONFIGURE ROW VECTOR Q* AND
C     COMPUTE Q*.INV(R).Q IN SCALARS 56-57
C
      BA=AR+STEP
      CALL MPCLB(27,BA,RNH-1.0,1,NH,0)
      CALL CSDOT(56,27,52,21)
C
C     EXTRACT POWER OF REFERENCE PHONE TO SCALAR 126
C
425   CONTINUE
      CALL MPEQB(27,24)
      CALL MPSBA(27,LST)
      CALL MPTBS(27,126,1)
C
C     COMPUTE Q-Q*.INV(R).Q IN SCALAR 127
C
      CALL SSUB(127,126,56)
C
      IF(KK .EQ. 0) GOTO 448
      KK=0
C
C     CREATE FUNCTION LIST TO PERFORM PROCESSING FOR ONE BEAM
C
      CALL MPBFL(2)
C
      IF(ITYPE .GE. 3) GOTO 435
C
C     CONFIGURE FIRST ROW OF COVARIANCE MATRIX
C     COMPUTE R.B IN SCALAR TABLE
C
      DO 430 J=1,LST
      CALL CSDOT(56+2*J,J+31,52,25)
430   CONTINUE
C
C     TAKE CONJUGATE OF STEERING VECTOR
C
      CALL CVCNJ(25)
C
C     COMPUTE B*.R.B IN SCALARS 120-121
C     COMPUTE B*.Q IN SCALARS 56-57
C     THEN (SCLR(56)**2+SCLR(57)**2)/SCLR(126)
C
      CALL CSDOT(120,25,52,22)
      CALL CSDOT(56,25,52,24)
      CALL SMUL(56,56,56)
      CALL SMUL(57,57,57)
      CALL SADD(56,56,57)
      CALL SDIV(56,56,126)
      CALL SSUB(121,120,56)
      GOTO 445
C
435   CONTINUE
C
C     COMPUTE INV(R).B IN SCALAR TABLE
```

```
C
      DO 440 J=1,LST
      CALL CSDOT(56+2*J,J+31,52,25)
440   CONTINUE
C
C     TAKE CONJUGATE OF STEERING VECTOR
C
      CALL CVCNJ(25)
C
C     COMPUTE B*.INV(R).B IN SCALARS 122-123
C     COMPUT B*.INV(R).Q IN SCALARS 56-57
C     THEN (SCLR(56)**2+SCLR(57)**2)/SCLR(127)
C
      CALL CSDOT(122,25,52,22)
      CALL CSDOT(56,25,52,21)
      CALL SMUL(56,56,56)
      CALL SMUL(57,57,57)
      CALL SADD(56,56,57)
      CALL SDIV(56,56,127)
      CALL SADD(123,122,56)
      CALL SDIV(122,50,122)
      CALL SDIV(123,50,123)
C
445   CONTINUE
C
C     OUR 4 RESULTS ARE NOW IN SCALARS 120-123
C     SO TRANSFER THEM TO NEXT COLUMN OF MATRIX
C
      CALL MPTSB(26,120,4)
C
      CALL MPSBA(25,LST)
      CALL MPEFL(2)
C
448   CALL MPIBC(0)
      CALL MPFOR(1,1,NBMS,2)
      CALL MPRBC(0)
C
C     AFTER EXECUTION OF FUNCTION LIST 2 FOR EACH BEAM
C     ALL THE STEERING VECTORS HAVE BEEN CONJUGATED
C     SO RESTORE THEM READY FOR THE NEXT CYCLE
C
      CALL CVCNJ(7)
C
C     PROCESSING FOR SELECTED FREQUENCY COMPLETE
C     FOR SELECTED COLUMN OF RESULTS MATRIX
C     MULTIPLY RESULTS BY 1.0/(AVERAGE POWER)
C     FIND MAXIMUM VALUE AND COMPUTE 1.0/MAX
C     THEN MULTIPLY EACH ELEMENT BY THIS VALUE
C     CONVERT THESE VALUES TO DECIBELS.
C     TRANSFER TO HOST & PLOT
C
      CALL MPROW(26,ITYPE,23)
      CALL VSMA1(26,129,26,0)
      CALL SMAX(131,26,127)
      CALL MPRST(131,PMAX,2,1)
      CALL SDIV(127,1,131)
      CALL VSMA1(26,127,26,0)
      CALL VLOGH(26,31,26)
      CALL MPRDB(26,POWR(4),1,1)
      J=3+NBMS
      DO 449 I=4,J
      POWR(I)=RNORM(POWR(I))-5.0
449   CONTINUE
      CALL EXEC(20,0,POWR,ILEN,0,0,ICLS)
C     CALL EXEC(11,ITIME(6))
C     WRITE(6,446) (ITIME(J),J=10,1,-1)
C446  FORMAT(' TIME: ',10I6)
C
      IF(IFBRK(0)) 90,77
90    WRITE(1,452)
452   FORMAT(' EX = EXIT PROGRAM'/' ST = SWITCH TAPES')
      READ(1,453) IANS
```

```
453   FORMAT(A2)
      IF(IANS .EQ. IEX) GOTO 99
      IF(IANS .NE. IST) GOTO 90
C
C     WRITE EOF ON M.T. & REWIND
C
      CALL EXEC(3,LUMT+100B)
      CALL EXEC(3,LUMT+400B)
      GOTO 777
99    CALL MPCLS(0)
      CALL EXEC(3,LUMT+100B)
      NPTS=-1
      CALL EXEC(20,0,POWR,2,0,0,ICLS)
      CALL EXEC(6)
      END
      END$
```

```
&CVM8I T=00004 IS ON CR00013 USING 00017 BLKS R=0000
```

```
FTN4,L
      SUBROUTINE COVAR(NS,NH,K,ISF,NSF,XF)
      DIMENSION ISF(50)
      COMMON LUMT,IHEAD(15),HRB(800)
      DATA IBUS2/128/,IBUS3/192/
      IF(K .EQ. 0) GOTO 100
      RNS=NS
      RNH=NH
      BS=RNS*RNH
      CVSIZ=2.0*RNH*RNH
      ISIZ=2*NH*NSF
      BACV=81918.0-2.0*CVSIZ
      ACM=RNS+4.0*RNH*NSF
C
C     SET SCALE FACTOR(XF) IN SCALAR 128
C     RAW DATA IS MULTIPLIED BY XF BEFORE FFT
C
      CALL MPWST(128,XF,1,1)
C
C     CONFIGURE THE FOLLOWING PERMANENT BUFFERS
C      1 = CONSTANT COMPLEX VECTOR(CURRENT VALUE OF SCALARS 56-57)
C      2 = DATA AREA AS A SINGLE REAL BUFFER
C     29 = DATA AREA AS A REAL MATRIX
C     10 = SOURCE BUFFER(REAL) FOR FFT
C     11 = COSINE TABLE FOR FFT
C     12 = COMPLEX BUFFER FOR RESULT OF FFT
C     30 = MATRIX OF SELECTED FREQUENCIES
C     28 = COVARIANCE MATRIX FOR SELECTED FREQUENCY
C     16 = FIRST COLUMN OF COVARIANCE INCREMENT MATRIX
C     17 = COVARIANCE INCREMENT(CONFIGURED AS REAL VECTOR)
C
      CALL MPDCV(1,56,NH,1)
      CALL MPCLB(IBUS2+2,0.0,BS,0,1,0)
      CALL MPCLM(29,0.0,NH,NS)
      CALL MPCLB(IBUS3+10,RNS,RNS,0,1,0)
      CALL MPCLB(IBUS3+11,0.0,0.5*RNS,0,1,0)
      CALL MPCLB(IBUS3+12,RNS,0.5*RNS,1,1,0)
      CALL MPCLM(30,RNS,2*NH,NSF)
      CALL MPCLB(28,BACV,CVSIZ,0,1,0)
      CALL MPCLB(IBUS3+16,ACM,RNH,1,1,0)
      CALL MPCLB(IBUS3+17,ACM,CVSIZ,0,1,0)
C
C     CREATE FUNCTION LIST 14 TO MOVE ONE ROW TO BUS 3,
C     PERFORM FFT AND MOVE RESULT BACK TO ORIGINAL ROW
C
      CALL MPBFL(14)
      CALL VMOV(10,3)
      CALL FFTR3(12,10,11)
      CALL VMOV(3,10)
      CALL MPNXR(3,29)
      CALL MPEFL(14)
```

```
C
C     CREATE FUNCTION LIST 15 TO MOVE
C     2 COLUMNS CORRESPONDING TO ONE FREQUENCY
C
      CALL MPBFL(15)
      CALL MPEVN(15,13)
      CALL MPODD(14,13)
      CALL VMOV(15,4)
      CALL VMOV(14,5)
      CALL MPNXC(13,30)
      CALL MPEFL(15)
C
C     CREATE FUNCTION LIST 13 TO SELECT & MOVE
C     TWO COLUMNS OF FREQUENCY DATA
C
      CALL MPBFL(13)
      DO 25 I=1,NSF
      J=2*ISF(I)-1
      CALL MPCOL(4,J,29)
      CALL MPCOL(5,J+1,29)
      CALL MPXFL(15)
25    CONTINUE
      CALL MPEFL(13)
C
C     CREATE FUNCTION LIST 12 TO COMPUTE INCREMENT FOR
C     ONE COLUMN OF A COVARIANCE MATRIX
C
      CALL MPBFL(12)
      CALL MPTBS(19,56,1)
      CALL CCVML(15,0,1,0,18)
      CALL MPSBA(15,NH)
      CALL MPEFL(12)
C
C     CREATE FUNCTION LIST 11 TO CALCULATE THE
C     INCREMENTS & UPDATE A COVARIANCE MATRIX
C
      CALL MPBFL(11)
C
C     MULTIPLY SELECTED COLUMN BY COMPLEX CONJUGATE OF EACH ELEMENT
C     AND SAVE THE VECTOR SO FORMED IN BUFFER 15
C
      CALL MPEQB(15,16)
      CALL MPFOR(1,1,NH,12)
      CALL VAD(28,17)
C
      CALL MPEFL(11)
C
C     CREATE FUNCTION LIST 9 TO PERFORM COMPLETE PROCESSING
C
      CALL MPBFL(9)
C
C     RE-DEFINE THE FOLLOWING BUFFER
C     18 = FIRST COLUMN OF FREQUENCY MATRIX
C
      CALL MPCLB(IBUS3+18,RNS,RNH,1,1,0)
C
C     CONVERT DATA TO REAL AND SCALE
C
      CALL VFLT(2,39,20,0)
      CALL VSMA1(2,128,2,0)
C
C     DEFINE FIRST ROW OF DATA
C     & PERFORM FFT ON EACH ROW
C
      CALL MPROW(3,1,29)
      CALL MPFOR(1,1,NH,14)
C
C     DEFINE FIRST COLUMN OF SELECTED FREQUENCIES
C     AND EXTRACT THE SELECTED FREQUENCY DATA
C
```

```
         CALL MPCOL(13,1,30)
         CALL MPXFL(13)
   C
   C     SELECT FIRST COVARIANCE MATRIX
   C     AND UPDATE EACH COVARIANCE MATRIX
   C
         CALL MPEQB(19,18)
         CALL MPXFL(11)
         CALL MPEFL(9)
   C
   C     REDEFINE DATA AREA AS AN INTEGER BUFFER
   C
   100   CALL MPCLB(IBUS2+20,0.0,BS,2,2,0)
         CALL MPXFL(9)
         CALL MPCLB(31,RNS,1.0*ISIZ,0,1,0)
         CALL MPRDB(31,HRB,1,1)
         DO 150 J=1,ISIZ
         HRB(J)=RNORM(HRB(J))
   150   CONTINUE
         CALL EXEC(2,LUMT,IHEAD,2*ISIZ+15)
         IHEAD(1)=IHEAD(1)+1
   C
         RETURN
         END
         END$
```

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