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**STRESS INTENSITY FACTORS FOR RADIAL
CRACKS IN A PRE-STRESSED, THICK-WALLED
CYLINDER OF STRAIN-HARDENING MATERIALS**

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A simple method which combines the weight function technique and finite element results is used to obtain mode I stress intensity factor solutions for radially cracked cylinders subjected to a high internal pressure. The method is especially effective for cylinders having residual stresses due to a manufacturing pre-stress process to increase the maximum pressure the cylinder can contain, and to improve the cylinder's useful life against fatigue and (CONT'D ON REVERSE)		

20. ABSTRACT (CONT'D)

fracture. The method is quite general for various assumptions involving the plastic stress-strain relations, the yield condition, the strain-hardening, and the compressibility of the cylinder material.

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INTRODUCTION

A cylinder is probably the most common configuration of all pressure vessels. The large tensile stress in the circumferential direction at the bore limits the maximum pressure a cylinder can contain and causes radial cracks to initiate. A cross section of a radially cracked cylinder subjected to an internal pressure p_i is shown in Figure 1 with four uniformly distributed cracks of equal depth emanating from the bore and from the outer surface. Due to modern design requirement of lighter weight with improved performance the useful life of a high pressure container is often determined by its fatigue life and the resistance to fracture. This has prompted the recent interest in the prediction of fatigue life^{1,2} and the estimation of stress intensity factors²⁻⁸ of a radially cracked cylinder. To increase the maximum pressure and to retard the initiation and propagation of radial cracks effectively, a high compressive residual stress is introduced near the bore through various pre-stress techniques such as autofrettage and shrink fit. The residual stress in the cylinder has increased difficulties in the fatigue life prediction and the stress intensity factor estimation.

One of the difficulties is the disagreement among results obtained by different investigators for the residual stress distribution in the cylinder after the autofrettage process.⁹⁻¹¹ This discrepancy in residual stress is a result of different assumptions for the material property (compressibility), the material's response to yield (the Mises' or Tresca's yield criterion), the end conditions (plane-strain, open-end or closed-end) and the theory of

References are listed at the end of this report.

plasticity (flow theory or deformation theory). For an incompressible, elastic ideally-plastic material obeying the yield criterion of Tresca or von Mises, the plane-strain residual stresses in an autofrettaged cylinder are given in closed form expressions.⁹ Based on these residual stresses, methods of computing stress intensity factors for multiple radial cracks in a cylinder are outlined in recent papers.^{12,13} The residual stress distribution with less restrictive assumptions is usually given in a numerical form, e.g. in Reference 11, instead of closed form. A method is needed to calculate stress intensity factors for radially cracked cylinders having residual stress distribution existing in a more realistic material with less restrictive assumptions. In this report a simple approximation is devised so that the method which combines the finite element and weight function methods given in Reference 12 can be applied successfully.

FINITE ELEMENTS FOR RADIALY CRACKED CYLINDERS

Due to the advancement of high speed computers, the finite element technique has become increasingly important and popular as a numerical method in structural analysis. In the earlier stage of direct application of finite elements to fracture mechanics, highly refined element meshes were used in the vicinity of a crack tip, e.g. in Reference 14. It required a large computer core and excessive manpower to generate and verify input data. The accuracy of results was not certain. To obviate these drawbacks, higher order finite elements are often used and special crack-tip elements are developed and used to replace a large number of refined elements in the region surrounding a crack tip. For a uniform array of radial cracks emanating from the bore of a

hollow cylinder, quite accurate stress intensity factors have been obtained⁷ using a maximum of seventeen 12-node quadrilateral isoparametric element. In addition to the use of higher order elements, the use of special crack-tip elements developed in Reference 15 aided in the solution. These elements are obtained by collapsing the regular quadrilateral elements around a crack tip into triangular elements with the crack tip as the common vertex and by shifting the side nodes of all sides of triangles radiating from the crack tip from the usual 1/3 and 2/3 locations to the new 1/9 and 4/9 locations measured from the common vertex. It has been shown in Reference 15 that the strain field in the special crack-tip elements near the vertex is dominantly a function of the inverse square root of distance from the vertex, providing all nodes coinciding with the crack tip are tied together (having the same displacement during deformation). Various techniques have been studied¹⁶ to estimate the stress intensity factors from the finite element results of nodal displacements of crack-tip elements. A simple and effective way for mode I crack is to use the normal component of nodal displacement v at the node on the crack face and nearest to the crack tip by the formula

$$K_I = (2\pi/r)^{1/2} [2G/(1+\kappa)]v \quad (1)$$

where r is the distance between the node and the crack tip, $\kappa = 3 - 4\nu$, for the case of plane strain the $\kappa = (3-\nu)/(1-\nu)$, for the case of plane stress, where G and ν are shear modulus and Poisson's ratio of the material respectively.

This special element and a similarly developed collapsed quarter-point special element for 8-node quadratic elements have been both implemented in the general purpose program NASTRAN. Both 12-node and 8-node quadrilateral

isoparametric elements with their special crack-tip elements have been used in the study of radially cracked cylinders with the results being quite good.^{7,17}

APES is a computer program which uses specifically 12-node quadrilateral isoparametric elements.¹⁸ If a 12-node element is chosen, the APES is more convenient than NASTRAN because of its time saving features in the preparation of input data. The foregoing special collapsed elements have been implemented in APES as an alternate method in elastic fracture analysis. The original APES program has two special crack-tip elements; the present authors prefer the enriched elements in which the displacement at an arbitrary point in local element coordinates (s,t) is given in matrix form by

$$u(s,t) = [P(s,t)]\{\alpha\} + K_I f(s,t) + K_{II} g(s,t) \quad (2)$$

where the first term at the right hand side is the usual displacement function, f and g are well known classical displacements near the crack tip for mode I and mode II cracks.¹⁹ Stress intensity factors K_I and K_{II} are additional unknowns in this finite element formulation. This technique of direct computation of K_I and K_{II} using the finite element method was due to Benzley.²⁰ Very good results have been obtained using special enriched elements in the APES program for radially cracked cylinders.¹²

Using high order elements with either collapsed or enriched crack-tip elements, rather complex fracture problems can be modelled by only a few elements^{12,18} and the accuracy is acceptable for practical engineering applications. A large class of engineering mechanics problems can be solved with NASTRAN or APES with reasonable cost and time. However, both computer programs lack the feature of handling initial stress. In order to use APES and NASTRAN for autofrettaged cylinders, a thermal simulation technique²⁶

is used to replace residual stresses in a cylinder by simulated thermal loads.¹²

WEIGHT FUNCTION METHOD

Many approximate methods²⁻⁸ have been introduced for stress intensity factor solutions for radially cracked rings. A brief review of the weight function method is given in the following paragraphs. This method can be used along with the finite element method to obtain new stress intensity factors for a crack under different loads by a simple algebraic equation.

According to Bueckner²¹ and Rice,²² a weight function is a universal function which depends only on geometry. If the mode I stress intensity factor $K^{(1)}$ and displacement field $u^{(1)}$ associated with the symmetric load system 1 are known, the weight function for the cracked geometry is

$$h = \frac{H}{2K^{(1)}(c)} \frac{\partial u^{(1)}(c)}{\partial c} \quad (3)$$

where $H = E$ for plane stress and $H = E/(1-\nu^2)$ for plane strain, c is the crack depth. Once h is determined, the mode I stress intensity factor induced by any other symmetric load system $\underline{t}$ and $\underline{f}$ is given by

$$K = \int_{\Gamma} (\underline{t} \cdot \underline{h}) d\Gamma + \int_A (\underline{f} \cdot \underline{h}) dA$$

where $\underline{t}$ is the stress vector acting on boundary Γ around the crack tip and $\underline{f}$ is the body force in region A defined by Γ . This equation can be reduced to

$$K = \frac{H}{K^{(1)}} \int_0^c p_c(x) \frac{\partial v^{(1)}}{\partial c} dx \quad (4)$$

for radially cracked rings with x being a distance measured along the crack from the base toward the tip. The crack face pressure $p_c(x)$ can be found from the hoop stress (at the site of radial cracks) in an uncracked ring subjected to the loading of interest. Even though the numerical values of $K(1)$ and $v(1)$, the normal component of displacement, are known, the partial derivative $\partial v(1)/\partial c$ is usually unknown. A technique of computing $\partial v/\partial c$ was devised in Reference 5 by assuming the crack face displacement v to be a conic section given by Orange.²³ Another method developed in Reference 12 made no assumptions on v or $\partial v/\partial c$ but utilized the finite element method to compute several stress intensity factors each associated with a simple loading system. For a new load, the new K is expressed in terms of known values of K . For instance, the hoop stress in an uncracked cylinder subjected to an internal pressure p_1 is

$$\sigma_\theta(r) = \frac{p_1}{b^2-1} \left(1 + \frac{b^2}{r^2}\right) \quad (5)$$

where b is the outside radius, r is a general radius, both normalized with respect to the inner radius a which is taken as unit length. Substituting Eq. (5) as p_c into Eq. (4) with $r(x) = 1 + x$ for inner cracks and $r(x) = b - x$ for outer cracks, we have

$$\frac{K(p_1)}{p_1} = \frac{1}{b^2-1} K_c(1) + \frac{b^2}{b^2-1} K_c(r^{-2}) \quad (6)$$

where

$$K_c(1) = \frac{H}{K(1)} \int_0^c \frac{\partial v(1)}{\partial c} dx \quad (7)$$

$$K_c(r^{-2}) = \frac{H}{K(1)} \int_0^c [r(x)]^{-2} \frac{\partial v(1)}{\partial c} dx \quad (8)$$

are called functional stress intensity factors.

Similarly we get

$$\frac{K(p_0)}{p_0} = \frac{b^2}{b^2-1} K_c(1) + \frac{b^2}{b^2-1} K_c(r^{-2}) \quad (9)$$

for the same cylinder subjected to uniform tension p_0 on outer cylindrical surface. The finite element results of $K(p_1)/p_1$ and $K(p_0)/p_0$ enable us to compute $K_c(1)$ and $K_c(r^{-2})$. For any new loading, if the hoop stress can be expressed in the form of

$$p_c(r) = c_0 + c_1 r^{-2}$$

then the stress intensity factor for the new loading is given by

$$K = c_0 K_c(1) + c_1 K_c(r^{-2})$$

Extending the idea to other loadings we can obtain many other functional stress intensity factors such as $K_c(\log r)$ from thermal loads and $K_c(r^2)$ from centrifugal forces.

RESIDUAL STRESSES IN AN AUTOFRETTAGED CYLINDER

The residual stress distribution in an autofrettaged thick-walled cylinder has been studied by a large number of investigators. There is considerable disagreement in their results due to different assumptions which must be made in order to make the problem mathematically tractable. Detailed discussions of the results and the associated assumptions are given in References 9 and 24. Under the combination of assumptions that the material is incompressible, elastic-perfectly plastic and obeys the Mises' yield criterion and that the cylinder is under the condition of plane strain, the following closed form solution for residual stresses is obtained for an elastically unloaded cylinder after partial autofrettage:

$$\sigma_r(r) = \frac{\sigma_0}{\sqrt{3}} \left\{ 2 \log \frac{r}{\rho} - 1 + \frac{\rho^2}{b^2} - P_1 \left(\frac{1}{b^2} - \frac{1}{r^2} \right) \right\} \quad 1 < r < \rho \quad (10)$$

$$\frac{\sigma_0}{\sqrt{3}} (\rho^2 - P_1) \left(\frac{1}{b^2} - \frac{1}{r^2} \right) \quad \rho < r < b \quad (11)$$

$$\sigma_\theta(r) = \frac{\sigma_0}{\sqrt{3}} \left\{ 2 \log \frac{r}{\rho} + 1 + \frac{\rho^2}{b^2} - P_1 \left(\frac{1}{b^2} + \frac{1}{r^2} \right) \right\} \quad 1 < r < \rho \quad (12)$$

$$\frac{\sigma_0}{\sqrt{3}} (\rho^2 - P_1) \left(\frac{1}{b^2} + \frac{1}{r^2} \right) \quad \rho < r < b \quad (13)$$

where ρ is the radius of the elastic-plastic interface during pressurization, σ_0 is the uniaxial yield stress in tension and compression, and

$$P_1 = P_1(\rho) = \frac{b^2}{b^2 - 1} \left(1 - \frac{\rho^2}{b^2} + 2 \log \rho \right) \quad (14)$$

This stress distribution will be used as a basis to develop a method to compute stress intensity factors for cracks in such a stress field. The presence of cracks in an autofrettaged cylinder may cause redistribution of residual stresses²⁵ and the stress intensity factors associated with cracks in a residual stress field can be calculated by the same finite element code APES with the residual stress replaced by an equivalent thermal load applied to an unstressed cylinder. The thermal simulation technique is discussed in Reference 26. It has been shown that the thermal stresses in the cylinder subjected to a thermal load

$$T(r) = \begin{cases} T_0 - \frac{(T_0 - T_\rho)}{\log \rho} \log r & 1 \leq r \leq \rho \\ T_\rho & \rho \leq r \leq b \end{cases} \quad (15)$$

are equivalent to the residual stresses (10) - (13) if the temperature gradient and the yield stress satisfy

$$\frac{E\alpha(T_0 - T_\rho)}{2(1-\nu)\log \rho} = \frac{2\sigma_0}{\sqrt{3}} \quad (16)$$

where T_0 and T_ρ are the temperatures at the bore, and $r = \rho$ respectively, E is Young's modulus, and α is the coefficient of linear thermal expansion. When $\rho = b$ the cylinder is fully autofrettaged, and stress intensity factors can be obtained from finite element computations for various crack configurations. Substituting σ_θ from Eq. (12) as $p_c(x)$ into Eq. (4) we have

$$\frac{K(\rho=b)}{\sigma_0} = \frac{1}{\sqrt{3}} \{ [2 - P_1(b)] K_c(1) - P_1(b) K_c(r^{-2}) + 2K_c(\log r) \} \quad (17)$$

where

$$K_c(\log r) = \frac{H}{K(1)} \int_0^c \log(r(x)) \frac{\partial v(1)}{\partial c} dx \quad (18)$$

Equation (17) can be used to find the functional stress intensity factor $K_c(\log r)$. For an inner radial crack with the crack tip r_c in the range $1 \leq r_c \leq \rho$ in a partially autofrettaged cylinder, the stress intensity factor due to residual stresses can be directly computed from

$$\frac{K(\rho)}{\sigma_0} = \frac{1}{\sqrt{3}} \{ [2 - P_1(\rho)] K_c(1) - P_1(\rho) K_c(r^{-2}) + 2K_c(\log r) \} \quad (19)$$

With the crack emanating from the outside surface and in the range $\rho < r \leq b$, Eqs. (4) and (13) give us the following equation

$$\frac{K(\rho)}{\sigma_0} = \frac{1}{\sqrt{3}} [\rho^2 - P_1(\rho)][b^{-2}K_C(1) + K_C(r^{-2})] \quad (20)$$

Equations (19) and (20) are not valid when a crack crosses the elastic-plastic interface. Modifications are given in References 12 and 17 for a crack slightly crossing the interface.

As mentioned previously, the residual stresses given by Eqs. (10) through (13) are obtained based on very idealized assumptions. In practice, a high strength steel is often used for high pressure vessels. This exhibits considerable volume change under elastic strain and the end condition in the process of autofrettage is either open or closed instead of plane strain. The more reliable and complicated von Mises' criterion must be used. The more rigorous incremental plastic stress-strain relation of Prandtl-Reuss should be used in the plastic region. The ideally plastic assumption should be modified to include the strain-hardening effect. Due to the complexity of the problem, no closed form solution can be expected and certain numerical methods must be used. Both finite element method²⁸ and finite difference method^{10,29} have been successfully developed for the problem. One of the authors has made modifications in both the finite element method³⁰ and the finite-difference method¹¹ and obtained good numerical results for a partially autofrettaged cylinder of $b = 2$, $\nu = 0.3$ for different values of E_t/E where E_t is the tangent-modulus. The following tables show a numerical comparison of residual stresses σ_θ/σ_0 and σ_r/σ_0 obtained from the finite-difference method with $E_t/E = 0.1$ ¹¹ and the corresponding values computed from Eqs. (10) through (13) under

idealized assumptions. Table I is for the 100 percent autofrettaged case and Table II for 60 percent. It can be seen that both the compressive and tensile residual stresses are smaller in a strain-hardening material than in a more restricted ideal plastic material. In other words, the effect of autofrettage is somewhat exaggerated with the use of Eqs. (12) and (13). It would be desirable to calculate stress intensity factors for cracks in a residual stress field which is given numerically at discrete points.

TABLE I. RESIDUAL STRESSES σ_θ AND σ_r FOR 100% AUTOFRETTAGED CYLINDERS, $b = 2$
FINITE DIFFERENCE RESULTS ARE BASED ON $\nu = 0.3$, $E_t/E = 0.1$

r	$\sigma_\theta(r)/\sigma_0$		$\sigma_r(r)/\sigma_0$	
	Finite-Difference	Eq. (12)	Finite-Difference	Eq. (10)
1.0	-0.85591	-0.97964	0.	0.
1.1	-0.59733	-0.68437	-0.06543	-0.07516
1.2	-0.38574	-0.44303	-0.10050	-0.11555
1.3	-0.20843	-0.24089	-0.11530	-0.13267
1.4	-0.05708	-0.06842	-0.11631	-0.13417
1.5	0.07388	0.08142	-0.10781	-0.12468
1.6	0.18825	0.21338	-0.09272	-0.10759
1.7	0.28866	0.33098	-0.07312	-0.08519
1.8	0.37700	0.43687	-0.05045	-0.05908
1.9	0.45473	0.53306	-0.02582	-0.03041
2.0	0.52301	0.62111	0.	0.

TABLE II. RESIDUAL STRESSES σ_θ AND σ_r FOR 60% AUTOFRETTAGED CYLINDERS
FINITE DIFFERENCE RESULTS ARE BASED ON $\nu = 0.3$, $E_t/E = 0.1$

r	$\sigma_\theta(r)/\sigma_0$		$\sigma_r(r)/\sigma_0$	
	Finite-Difference	Eq. (12)	Finite-Difference	Eq. (10)
1.0	-0.73976	-0.84679	0.	0.
1.1	-0.49080	-0.56305	-0.05545	-0.06363
1.2	-0.28651	-0.33048	-0.08288	-0.09526
1.3	-0.11534	-0.13525	-0.09171	-0.10564
1.4	-0.03011	-0.03190	-0.08801	-0.10164
1.5	0.15479	0.17737	-0.07582	-0.08778
1.6	0.26220	0.30575	-0.05791	-0.06712
1.7	0.24381	0.28446	-0.03952	-0.04583
1.8	0.22841	0.26662	-0.02412	-0.02799
1.9	0.21539	0.25153	-0.01110	-0.01289
2.0	0.20429	0.23864	0.	0.

AN APPROXIMATE METHOD FOR STRESS INTENSITY FACTORS

The approximate method is based on an assumption that the numerical values of residual stresses can be closely approximated by expressions similar to Eqs. (10) through (13) with different values of coefficients. We assume

$$\sigma_r(r)/\sigma_0 = A_r + B_r/r^2 + C_r \log r \quad 1 \leq r \leq \rho \quad (21)$$

$$D_r + E_r/r^2 \quad \rho \leq r \leq b \quad (22)$$

$$\sigma_\theta(r)/\sigma_0 = A_\theta + B_\theta/r^2 + C_\theta \log r \quad 1 \leq r \leq \rho \quad (23)$$

$$D_\theta + E_\theta/r^2 \quad \rho \leq r \leq b \quad (24)$$

The coefficients $A_r, \dots, E_r$, and $A_\theta, \dots, E_\theta$ are to be found from the given values $\sigma_r^*(r)/\sigma_0$ and $\sigma_\theta^*(r)/\sigma_0$ of residual stresses such that the total square-error for the stresses

$$\epsilon_j = \sum_{i=1}^N [\sigma_j^*(r_i) - \sigma_j(r_i)]^2 \quad j = r \text{ or } \theta \quad (25)$$

is a minimum.

Once the expressions (23) and (24) are obtained then stress intensity factors can be obtained from either

$$\frac{K(\rho)}{\sigma_0 \sqrt{\pi c}} = \frac{1}{\sqrt{3\pi c}} [A_\theta K_c(1) + B_\theta K_c(r^{-2}) + C_\theta K_c(\log r)] \quad (26)$$

or

$$\frac{K(\rho)}{\sigma_0 \sqrt{\pi c}} = \frac{1}{\sqrt{3\pi c}} [D_\theta K_c(1) + E_\theta K_c(r^{-2})] \quad (27)$$

depending on whether the crack starts from inner or outer surface. As in Eqs. (19) and (20), these expressions are not valid when cracks cross the elastic-plastic interface.

The accuracy of Eqs. (26) and (27) is dependent on how well the least square approximation of σ_θ represents the given distribution of σ_θ^* . Table III shows the given residual stresses σ_θ^*/σ_0 and σ_r^*/σ_0 and the corresponding approximations computed from Eqs. (23) and (21) respectively for a 100 percent autofrettaged cylinder with an outside diameter twice that of the inside diameter. The coefficients used in Eqs. (23) and (21), obtained from the least-square approximation at fifty one equal spaced points, are $A_\theta = 0.170$, $B_\theta = 1.030$, $C_\theta = 0.890$, $A_r = -0.926$, $B_r = 0.925$, and $C_r = 1.003$. It can be seen from Table III that the approximations are close to the given values for all r with relative errors in general less than one percent. For 60 percent

TABLE III. COMPARISON OF GIVEN VALUES OF $\sigma_{\theta}(r)/\sigma_0$ AND $\sigma_r(r)/\sigma_0$ AT VARIOUS VALUES OF r WITH THEIR CORRESPONDING LEAST SQUARE APPROXIMATIONS FOR A 100% AUTOFRETTAGED CYLINDER OF $b = 2$.

r	$\sigma_{\theta}(r)/\sigma_0$		$\sigma_r(r)/\sigma_0$	
	Given ¹¹	Approx. by (23)	Given	Approx. by (21)
1.0	-0.85591	-0.86039	0.	-0.00037
1.1	-0.59733	-0.59673	-0.06543	-0.06540
1.2	-0.38574	-0.38328	-0.10050	-0.10029
1.3	-0.20843	-0.20620	-0.11530	-0.11509
1.4	-0.05708	-0.05626	-0.11631	-0.11620
1.5	0.07388	0.07290	-0.10781	-0.10786
1.6	0.18825	0.18579	-0.09272	-0.09294
1.7	0.28866	0.28571	-0.07312	-0.07342
1.8	0.37700	0.37509	-0.05045	-0.05069
1.9	0.45473	0.45580	-0.02582	-0.02574
2.0	0.52301	0.52928	-0.	-0.00071

autofrettage, the coefficients are: $A_{\theta} = 0.205$, $B_{\theta} = -0.946$, $C_{\theta} = 0.912$, $D_{\theta} = 0.101$, $E_{\theta} = 0.412$, $A_r = -0.873$, $B_r = 0.873$, $C_r = 1.009$, $D_r = 0.103$, $E_r = -0.412$. The given values of $\sigma_{\theta}^*(r)/\sigma_0$ and $\sigma_r^*(r)/\sigma_0$ and their approximate values are shown in Table IV. The agreement is excellent. At the elastic-plastic interface small discontinuities exist due to two approximate expressions in the elastic and plastic regions. Similar agreement has been obtained for other degrees of autofrettage. The numerical agreement indicates that the assumption made in the expressions of Eqs. (21) through (24) is valid.

TABLE IV. COMPARISON OF GIVEN VALUES OF $\sigma_{\theta}(r)/\sigma_0$ AND $\sigma_r(r)/\sigma_0$ AT VARIOUS VALUES OF r WITH THEIR CORRESPONDING LEAST SQUARE APPROXIMATIONS FOR A 60% AUTOFRETTAGED CYLINDER OF $b = 2$.

r	$\sigma_{\theta}(r)/\sigma_0$		$\sigma_r(r)/\sigma_0$	
	Given ¹¹	Approx. by (23) or (24)	Given	Approx. by (21) or (22)
1.0	-0.73976	-0.74126	0.	-0.00010
1.1	-0.49080	-0.49009	-0.05545	-0.05540
1.2	-0.28651	-0.28581	-0.08288	-0.08282
1.3	-0.11534	-0.11558	-0.09171	-0.09172
1.4	-0.03011	-0.02915	-0.08801	-0.08809
1.5	0.15479	0.15431	-0.07582	-0.07587
1.6	0.26220	0.26411	-0.05791	-0.05773
1.6	0.26220	0.26219	-0.05791	-0.05790
1.7	0.24381	0.24382	-0.03952	-0.03953
1.8	0.22841	0.22842	-0.02412	-0.02413
1.9	0.21539	0.21540	-0.01110	-0.01111
2.0	0.20429	0.20427	-0.	-0.00002

The use of Eq. (26) or (27) for the computations of stress intensity factors depends on the functional stress intensity factors $K_c(1)$, $K_c(r^{-2})$, and $K_c(\log r)$. They are functions of the wall ratio of the cylinder b , the number of cracks N , the depth of cracks c , and the geometrical location of radial cracks, either emanating from the inner or outer surface. For $b = 2$, they are given in Reference 12 for ID cracks with $N = 1$ to 40 and $c = 0.05$ to 0.3. Results for OD cracks are given in Reference 27 for $N = 1$ to 20 with $c = 0.1$ to 0.3. With the coefficients obtained from the least square approximation for any given residual stress distribution, stress intensity factors can be calculated from Eq. (26) for ID cracks and Eq. (27) for OD cracks. A typical dimensionless plot of $K/\sigma_0\sqrt{\pi c}$ versus c/t , where $t = b-1$ is the wall thickness, is shown in Figure 2 for ID radial cracks in an unpressurized tube due to residual stresses resulting from 100 percent autofrettage. The solid lines are the results for a strain-hardening material while the dotted lines are results previously obtained in an idealized elastic-perfectly plastic material. A similar plot for a 60 percent autofrettaged cylinder is shown in Figure 3. In Figures 2 and 3, the negative values are not actual stress intensity factors at the tips of inner radial cracks. It is not physically possible to have a negative stress intensity factor since a negative value would require a crack face displacement in a direction which would have one face penetrating the opposite crack face. The stress intensity factor is actually zero when crack faces are closed due to compressive residual stress normal to the crack face. If a pressure is applied at the bore and its magnitude is gradually increased, the stress intensity factor remains zero as long as the crack remains closed. It becomes positive when the tensile stress

normal to the crack face, due to the internal pressure, is larger than the compressive residual stress. For computational purposes, it is convenient to compute stress intensity factors due to internal pressure and to residual stress separately, and to yield the resultant stress intensity factor by combining these two results. The negative values in Figures 2 and 3 are only intermediate results. The larger magnitude of negative values will result in smaller values in the final stress intensity factors. Hence, the resultant stress intensity factors in a pressurized autofrettaged cylinder predicted from the assumption of ideal material behavior, are lower than those in the strain-hardening material used in our numerical examples. As a consequence the fatigue life will be less in the strain-hardening material than that predicted using the ideal material.

For radial cracks starting from the outer surface the results are shown in Figures 4 and 5 for 100 percent and 60 percent autofrettage respectively. The stress intensity factors due to residual stress are about 12-15 percent less in strain-hardening material when $E_t/E = 0.1$ than in ideal material. The final stress intensity factors at outer cracks in a pressurized autofrettaged cylinder is less than those predicted using the ideal material.

CONCLUSION

A closed form solution of residual stress in a cylinder undergoing autofrettage is valid only for idealized material. For more realistic materials which obey the Mises' yield criterion, the Prandtl-Reuss incremental stress-strain relation and have strain-hardening, residual stresses can be calculated numerically. It has been shown in this report that the numerical

results for residual stress can be approximated in an expression similar to the closed form solution for the idealized material model with constant coefficients determined by the least square technique. This enables the method of functional stress intensity factors developed for residual stress in idealized materials to be applicable to residual stresses given in numerical form.

In the numerical example when $E_t/E = 0.1$ is used, the autofrettage effect on ID cracks is 12-15 percent less effective for strain-hardening material with $E_t/E = 0.1$ than in the idealized material. Similarly the autofrettage produces about 15 percent less adverse effect on OD cracks in strain-hardening materials. For high strength gun steel, the ratio E_t/E is less than 0.1, the strain-hardening effect on stress intensity factors of radial cracks would be expected to be less.

The numerical results given in Reference 11 considered E_t as a constant and there was no Bauschinger effect. Since there is a noticeable Bauschinger effect in a high strength gun steel according to Milligan's study,³¹ further work is planned to include Bauschinger effect and to consider E_t not a constant. In such cases it remains to be seen whether the numerical results of residual stresses can still be approximated closely by Eqs. (21) to (24). If errors in such approximations are acceptable, the method for the computation of stress intensity factors used in this report can still be applicable.

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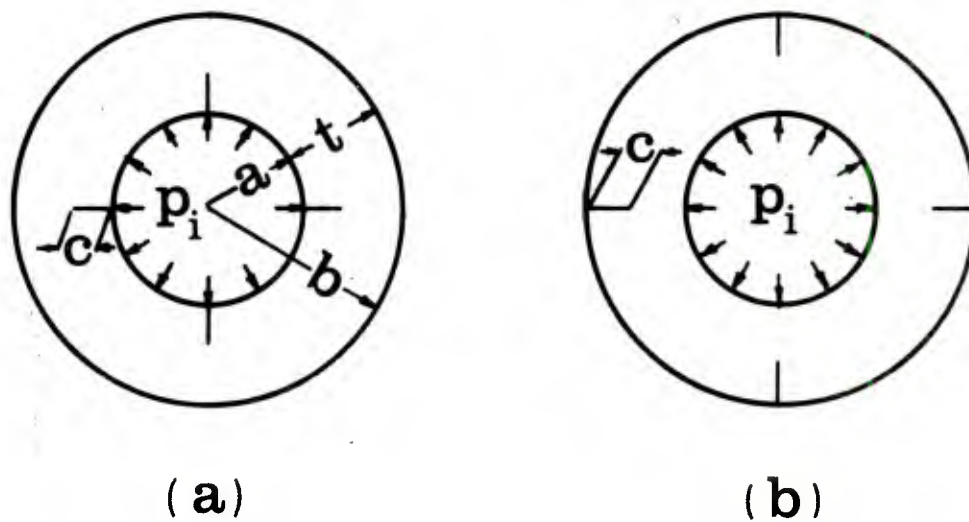


FIGURE 1. Cross-section of a hollow cylinder under internal pressure P_i with N uniformly distributed radial cracks of equal depth emanating from (a) the bore and (b) the outer surface.

Dotted lines-- idealized materials

Solid lines---strain hardening materials with $E_t/E = 0.1$, $\nu = 0.3$

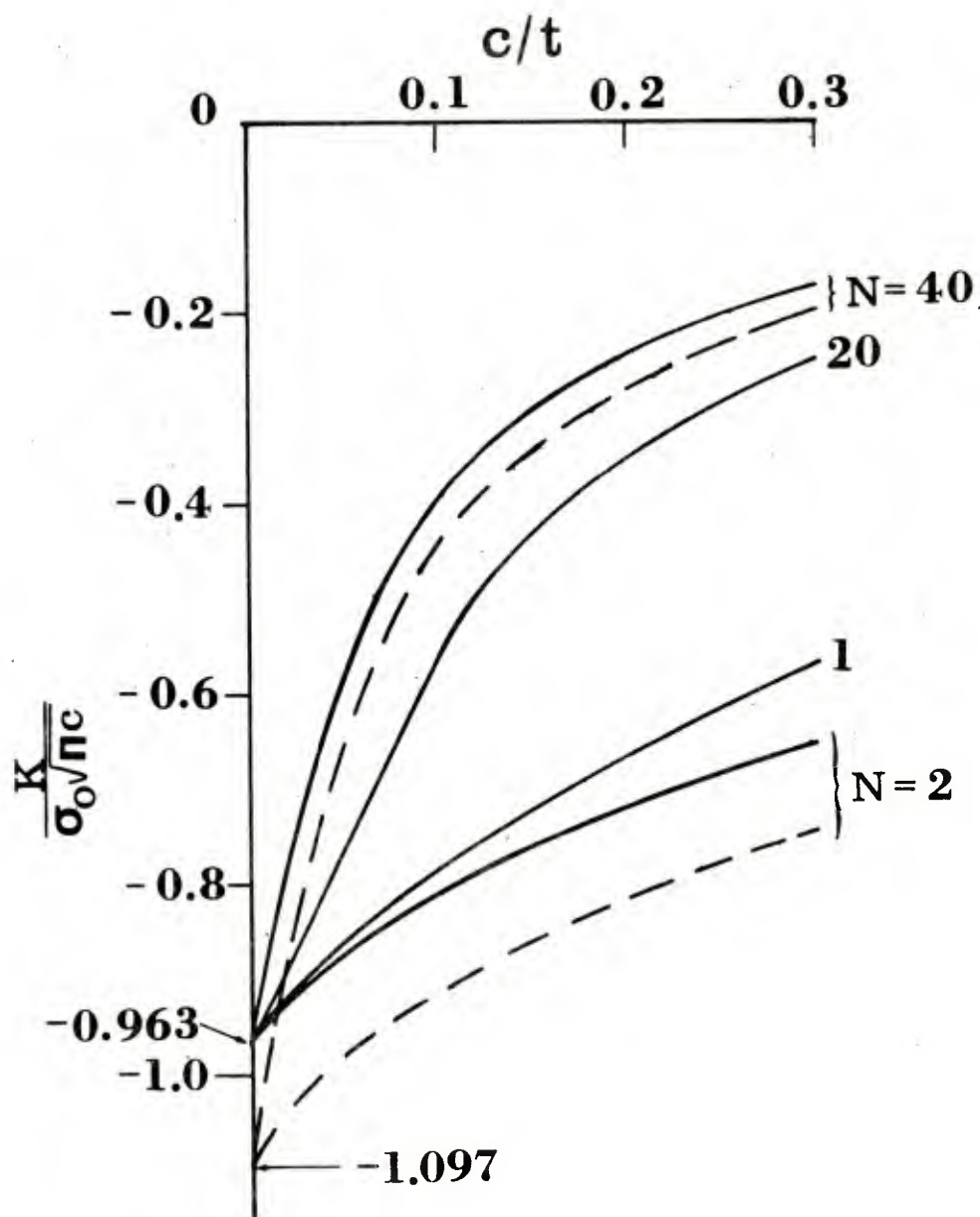


FIGURE 2. Stress intensity factors as a function of c/t for N radial cracks starting at the inner surface of a 100 percent autofrettaged, thick-walled cylinder.

Dotted materials-- idealized materials
Solid lines---strain-hardening materials with $E_t/E = 0.1, \nu = 0.3$

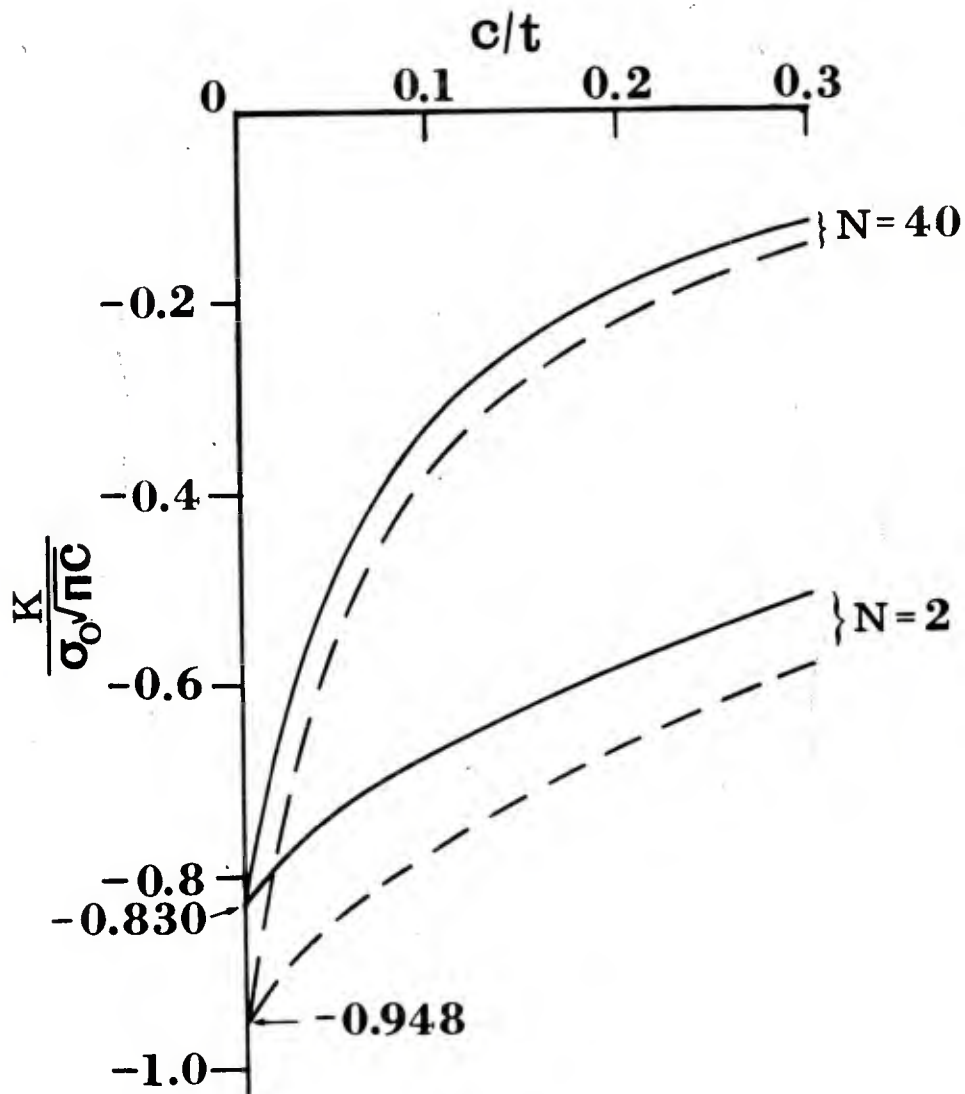


FIGURE 3. Stress intensity factors as a function of c/t for N radial cracks starting at the inner surface of a 60 percent autofrettaged, thick-walled cylinder.

Dotted lines-- idealized materials
Solid lines--- strain-hardening materials with $E_t/E = 0.1, \nu = 0.3$

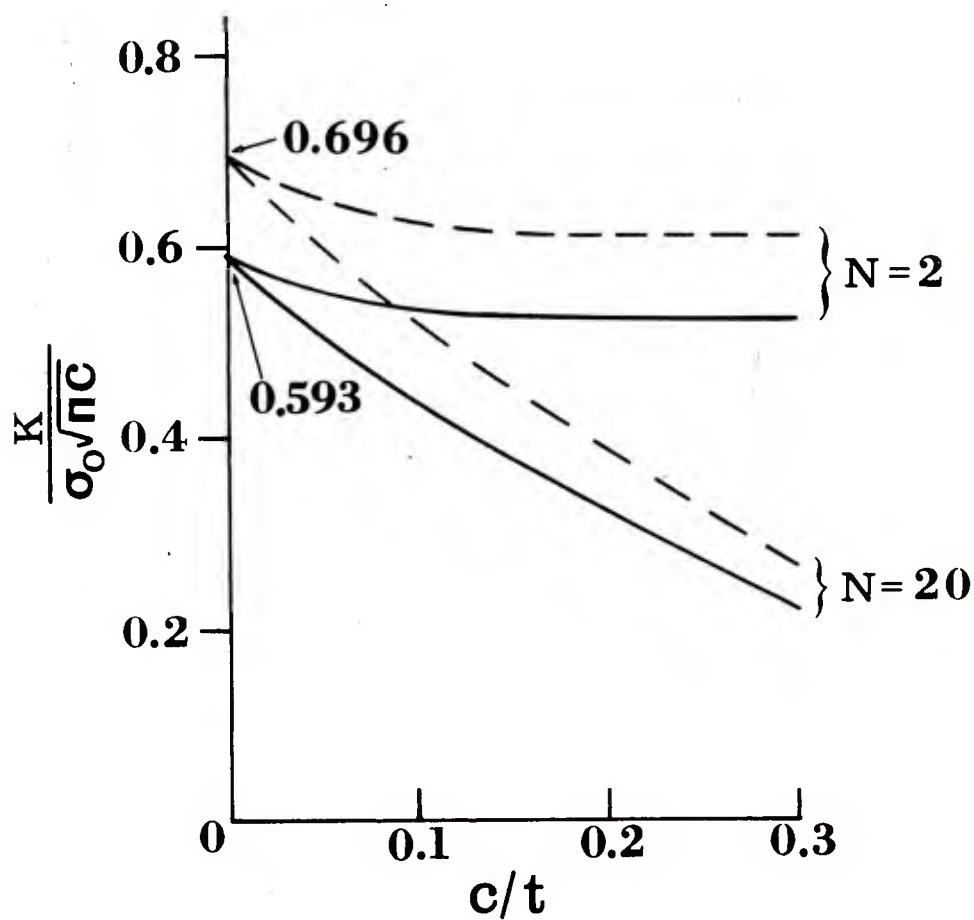


FIGURE 4. Stress intensity factors as a function of c/t for N radial cracks starting at the outer surface of a 100 percent auto-frettaged, thick-walled cylinder.

Dotted lines-- idealized materials
Solid lines-- strain-hardening materials with $E_t/E = 0.1, \nu = 0.3$

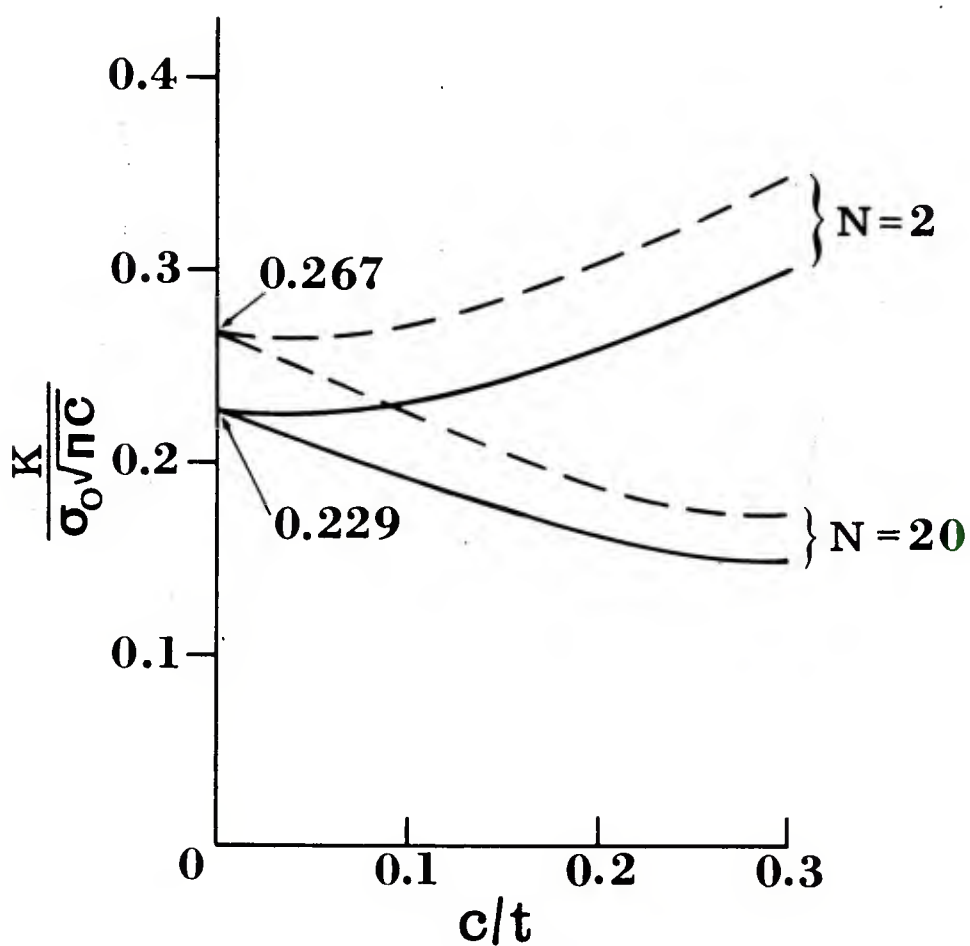


FIGURE 5. Stress intensity factors as a function of c/t for N radial cracks starting at the outer surface of a 60 percent autofrettaged, thick-walled cylinder.

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