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<p>This report describes a civilian manpower planning model for scientific and engineering personnel in Navy R&D Centers that is capable of providing alternative projections by age and grade based on various policy scenarios. The projections include the total number of personnel, the number of personnel to be hired, and expected levels of promotion and attrition. The model has been used for the offices of the Assistant Secretary of the Navy and the Chief of Naval Material to estimate the effect of high-grade limitations on the Navy R&D Centers.</p>		

FOREWORD

During fiscal years 1974 through 1981, Navy research and development (R&D) centers were limited as to the number of high-grade positions they could have. The Assistant Secretary of the Navy (Research, Engineering and Systems) expressed concern that these limitations would have a long-term impact on the R&D centers and could contribute to the decline in productivity and motivation of Navy professional engineers and scientists for years to come. Accordingly, the Assistant Secretary of the Navy and the Chief of Naval Material requested the Navy Personnel Research and Development Center to assess the impact of these limitations on personnel attrition and, consequently, on the ability of the Navy laboratories to perform their missions.

Results, which were published in the first report in this series (NPRDC SR 82-36), showed that the high-grade limitation was closely related to increases in professional attrition. This report describes a model that was developed to forecast the change in the professional grade structure in the Navy R&D Centers resulting from policy changes with respect to promotion. A forecast of high-grade vacancies is a part of the output from this model.

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FEBRUARY 1983

**CIVILIAN MANPOWER PLANNING MODEL
FOR SCIENTIFIC AND ENGINEERING
PERSONNEL IN THE NAVY R&D CENTERS**

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**NAVY PERSONNEL RESEARCH
AND
DEVELOPMENT CENTER
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**CIVILIAN MANPOWER PLANNING MODEL FOR SCIENTIFIC
AND ENGINEERING PERSONNEL IN THE NAVY R&D CENTERS**

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SUMMARY

Problem

Since the Congress and the Department of Defense set limits on the number of high-grade positions in the Navy's research and development (R&D) centers, the promotion rate of the professionals from GS-12 to GS-13 has declined. Consequently, the attrition rate at both the GS-12 level and the lower levels has increased substantially.

To assess the effect of high-grade limitations on the attrition rate, it is necessary to measure the quantitative response of attrition to promotion opportunity. Also, a civilian manpower planning model is needed to assess the impacts of alternative manpower policies on the scientific and engineering (S&E) personnel structure and flows in Navy R&D centers.

Objective

The purpose of this effort was to develop a manpower planning model to project the effect of high-grade limitations on Navy R&D centers.

Approach

The number of S&E vacancies in the Navy R&D centers is affected by a variety of personnel flows, including the number of accessions, voluntary quits, promotions, and retirements. Changes in Navy personnel policies, among other factors, affect personnel flows that, in turn, affect the age and grade structures of S&E professionals. Therefore, policy variables were included in developing the manpower planning model. The techniques used in this effort include probability and ratio estimation and regression analysis.

Results

A civilian manpower planning model was developed to forecast professional personnel flows and levels in the Navy R&D centers. The model provides alternative projections by age and grade based on various policy scenarios. The projections include the total number of personnel, the number of personnel to be hired, and expected levels of promotion and attrition.

Conclusions

The civilian manpower planning model developed can be used to assist the offices of the Assistant Secretary of the Navy and the Chief of Naval Material in evaluating the effect of high-grade ceilings on the capability of Navy R&D centers. In addition, the model can be used by Navy manpower managers in evaluating alternative manpower strategies.

Recommendations

1. For more general use, the model should be expanded to cover job classifications other than S&E professionals and Navy activities other than Navy R&D centers.
2. An enhanced version of the model should incorporate cohort-based personnel flows and demand-driven hiring parameters.

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INTRODUCTION

Problem

Since the mid-1970s, Navy R&D centers have operated with restrictions on the total number of permanent employees at the GS-13, 14, and 15 levels. Specifically, they were required to reduce the number of high grades by roughly 2 percent a year for the 3 years from 1978 through 1980 (Public Law 95-79). This requirement produced a steady decline in the number of high-grades in the Navy R&D centers.

The former Assistant Secretary of the Navy (Research, Engineering, and Systems) expressed concern that "the long-term impacts of these constraints will have a debilitating effect on the Navy research and development establishment and, in the long run, could contribute to the professional demise of these organizations." Consequently, the Assistant Secretary of the Navy and the Chief of Naval Material requested the Navy Personnel Research and Development Center to evaluate the impact of these limitations on personnel attrition and on the ability of the R&D centers to perform their missions.

Since the establishment of an annual limit on the number of high grades, there has been a reduction in the number of professional high-grade positions in the Navy R&D Centers and, therefore, promotion opportunity. To estimate how changes in promotion to high grade affects other personnel flows (e.g., attrition), as well as the grade and age structures in the Navy R&D centers, it is necessary to develop appropriate analytic techniques.

Background

Much of the previous work in modeling civilian personnel flows emphasized attrition. Clark (1977) derived survival curves to estimate the attrition for federal civil service employees, based on length of service (LOS). Chipman (1981) applied a similar technique to a Navy research laboratory by using longitudinal data. Grinold and Marshall (1977) described the use of transition probabilities to estimate manpower flows. Bartholomew (1973) and Bartholomew and Forbes (1979) described a technique for modifying the Markov processes to include the promotion rate into the transition probabilities. In terms of Navy civilian personnel, Charnes, Cooper, Lewis, and Niehaus (1979) developed a variety of civilian personnel models emphasizing equal employment opportunity issues.

Objective

The objective of this effort was to develop a civilian manpower planning model for scientific and engineering (S&E) personnel in the Navy R&D centers. Such a model would be needed to project high-grade vacancies, high-grade promotion opportunity, and the relationship of high-grade promotion opportunity to attrition of GS-5 through GS-12 S&Es.

APPROACH

To estimate the future high-grade vacancies in the Navy R&D centers, an analysis and projection of personnel flows (personnel gains and losses) are first required. A personnel gain or loss in a given year affects the total number of personnel in that year as well as the personnel gains or losses for later years. Furthermore, forecasting high-grade vacancies requires analysis of high-grade flows as well as flows for other grades. A personnel gain to a high grade may well be a personnel loss to the next lower grade. Any personnel gain or loss to one grade may affect the number of personnel in the grades immediately above and below for the given year as well as for future years.

Figure 1, which provides a partial network of personnel flows, shows personnel flows from one grade to another and from one year to another as a set of interconnected personnel stocks and flows. The network shows that the promotion of an additional person from GS-12 to GS-13 in 1978 would affect the estimates for the total number of GS-12s and GS-13s in 1979. Since the total number of personnel is one factor to be used to estimate personnel flows in this model, a promotion from GS-12 to GS-13 in 1978 would indirectly affect the estimates of personnel levels and flows in the following years.

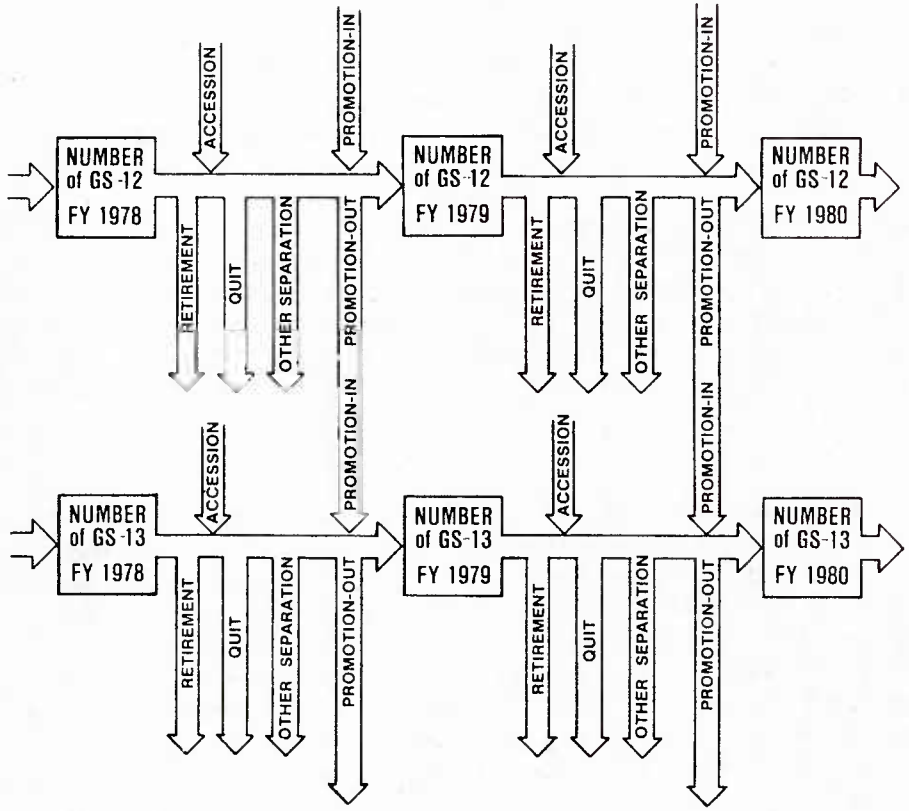


Figure 1. System relationships of growth and losses in the number of personnel.

Retirement is a major factor influencing high-grade vacancies. Since the number of retirements is generally affected by the age structure of the personnel, the approach used projects personnel flows by age.

Policies limiting the number of high-grade positions or changing the minimum age for retirement may affect the age and grade distribution of the work force. Thus, the approach also considers the impact of policy changes on personnel flows by age and grade.

In the model, personnel flows are divided into six categories: accessions, promotions-in, retirements, voluntary quits, promotions-out, and other separations. The mathematical formulation of the model is provided in the appendix.

RESULTS

The first step in developing the model was to select a base year for estimating personnel flows. Fiscal years 1979 and 1980 were selected as base years and used to determine the values for end strength, personnel gains, and personnel losses. The next step was to determine policy variables. Two examples of forecasts based on different assumptions are discussed below.

Impact of a Fixed Ceiling for High Grades

A ceiling for the number of high grades would affect not only high grades but also other grades. Other variables in the model are also affected. By assuming that the total number of high grades in future years will equal the actual number of high grades at the end of FY 1980, a set of projections can be derived. The projections are for 6 years, including estimates for the number of personnel to be accessed, promoted, attrited, and retired by grade and by age, as well as the total number of personnel by the same categories.

The empirical results show that the change from a policy of annual reductions in the number of high grades (FY 1974 to FY 1980) to the policy of a constant ceiling (current policy) increases the amount of projected high-grade vacancies and, consequently, promotion opportunity for GS-12 professionals.

Figure 2, which illustrates the change in age distribution for high-grade professionals over time, shows that the segment of the curve representing the age below 50 moved up and to the right from 1974 to 1980. It is projected that this segment will continue to shift in a similar direction from 1980 to 1986. The aging of the high-grade work force may be due to the restrictions on promotions of GS-12 professionals in the past and the low attrition rate of high-grade professionals.

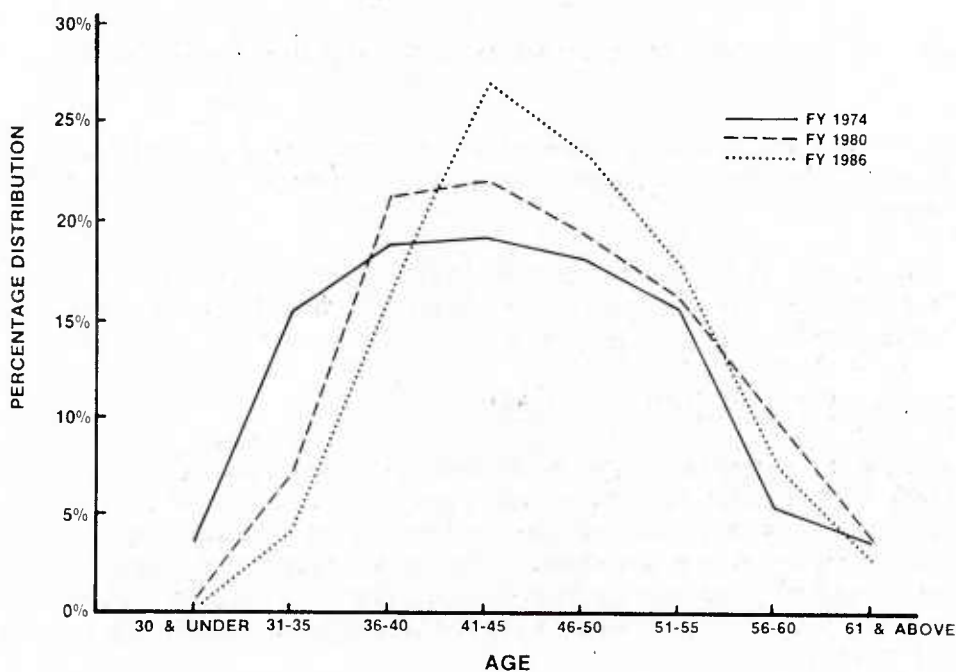


Figure 2. Age distribution of high-grade professionals.

Figure 3 shows how the retirements of high-grade professionals have fluctuated over time. High-grade retirements are projected to decrease from the FY 1980 peak level. Retirement is the major factor creating vacancies for high-grade promotion. Thus, when there are no further reductions in the number of high grades, additional high-grade vacancies are created.

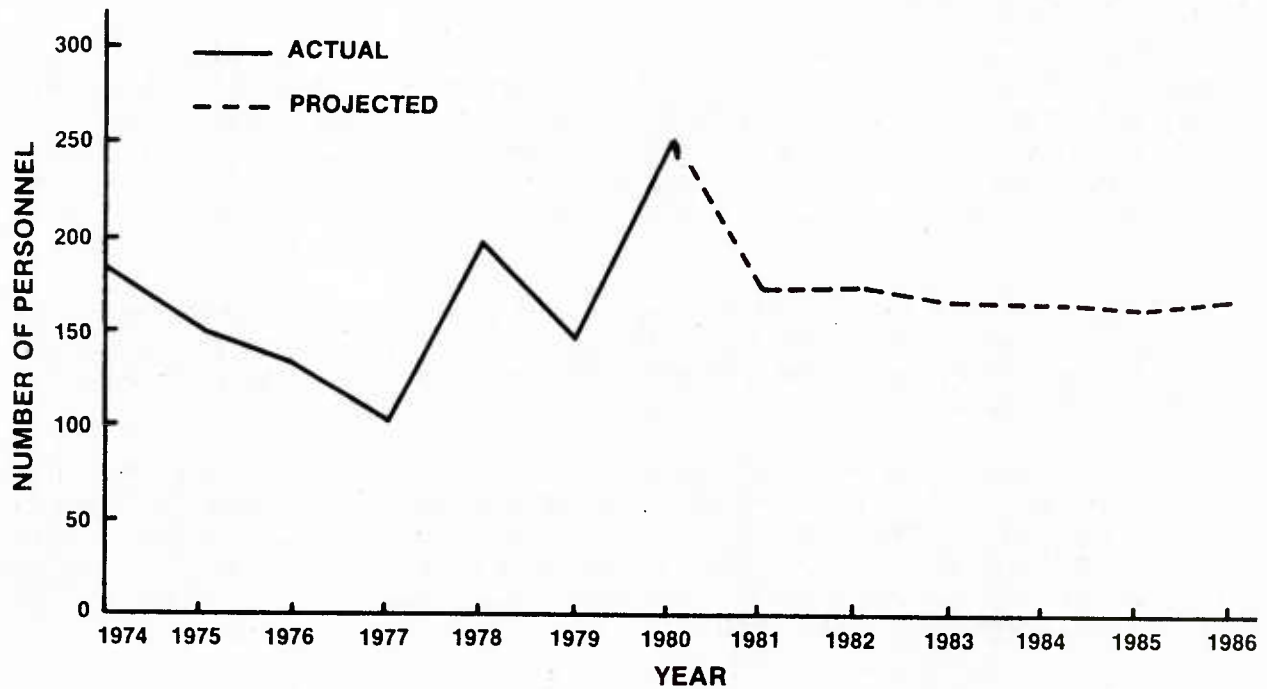


Figure 3. Projections of retirements for high-grade professionals.

Figure 4 shows that the projected promotion rate for GS-12 professionals in 1981 is somewhat higher than that for 1980, even though the number of high-grade professionals projected to retire is lower.

The grade distribution in Figure 5 indicates that a policy of fixing the number of high grades at the FY 1980 level will stabilize the escalating share of GS-12s and will stop the declining share of high grades in the workforce.

Impact of Raising the Minimum Retirement Age

Another example of using the model to analyze personnel policy is to estimate the impact of a change in the minimum retirement age. If it is assumed that the minimum retirement age is raised from 55 to 60, with a fixed ceiling for the number of high grades, the results shown in Figure 6 are obtained. This shows that the number of high-grade retirements is projected to decrease by two-thirds. The decrease in high-grade retirements would make the projected promotion rate for GS-12 professionals decrease by one-half, as shown in Figure 7.

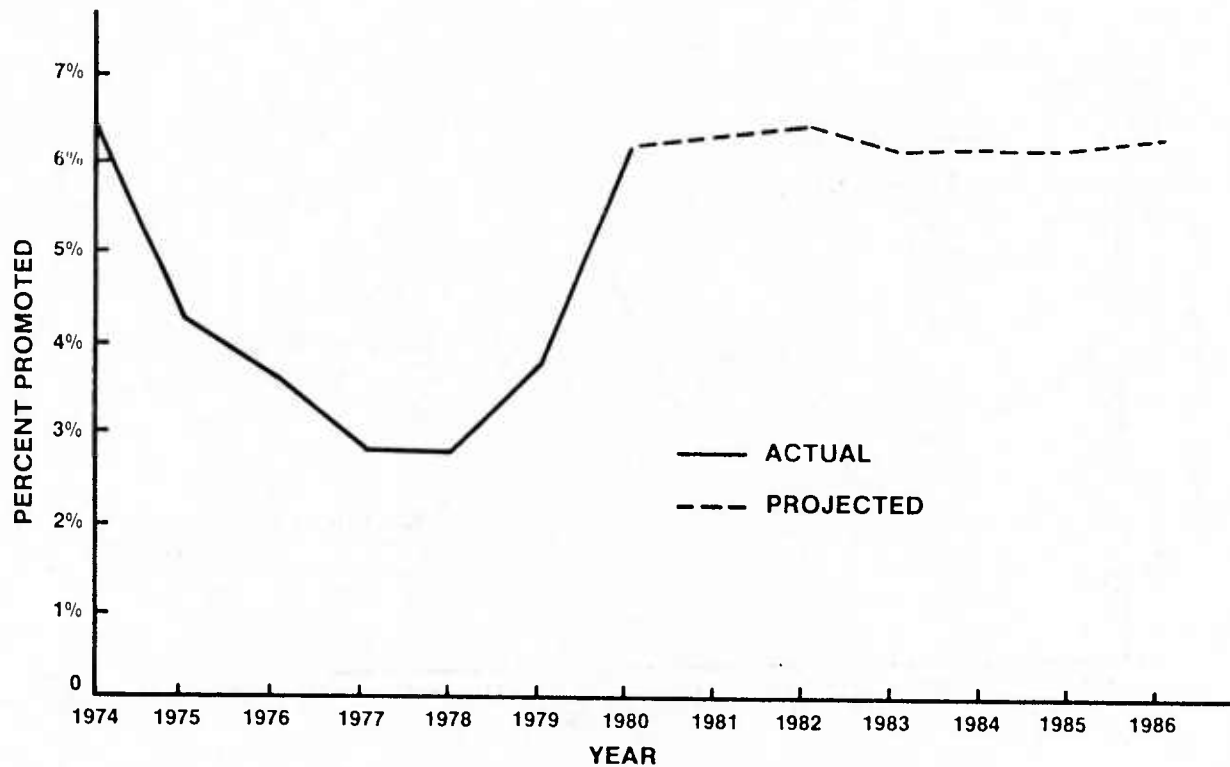


Figure 4. Projections of the promotion rate for GS-12 professionals.

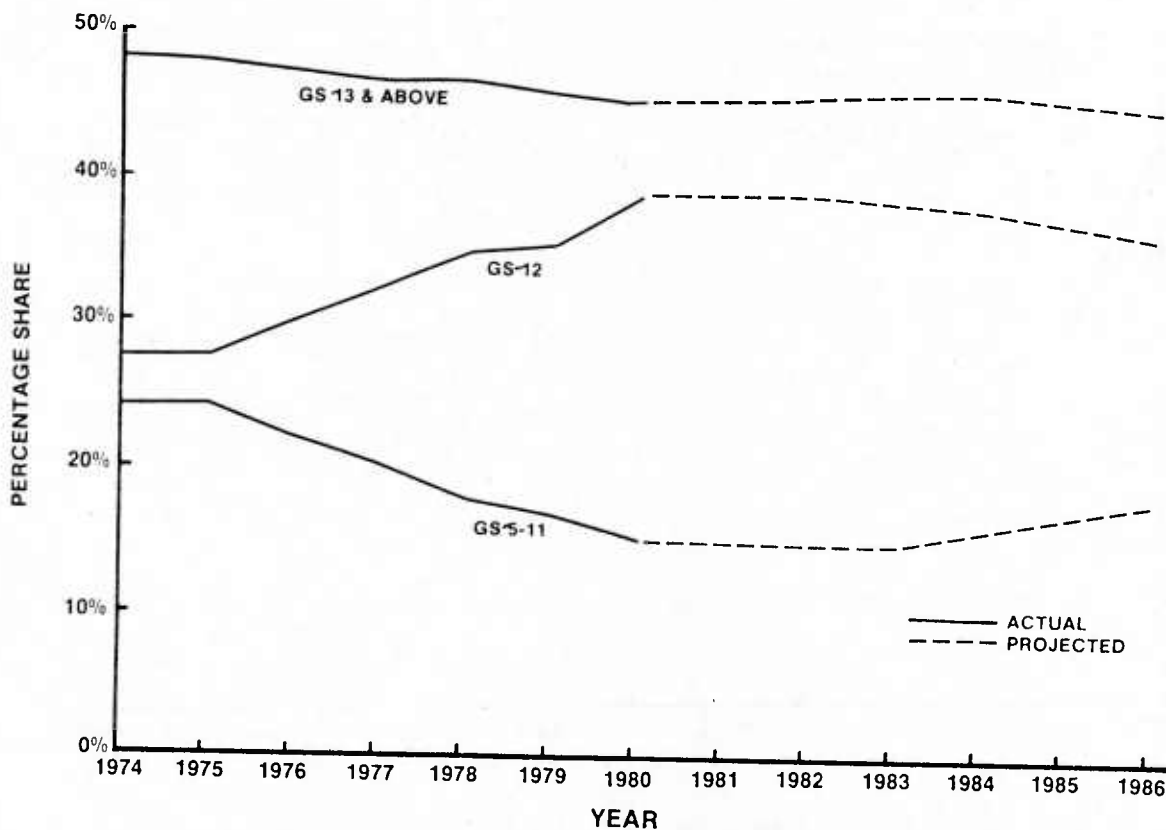


Figure 5. Projections of grade distribution for professionals.

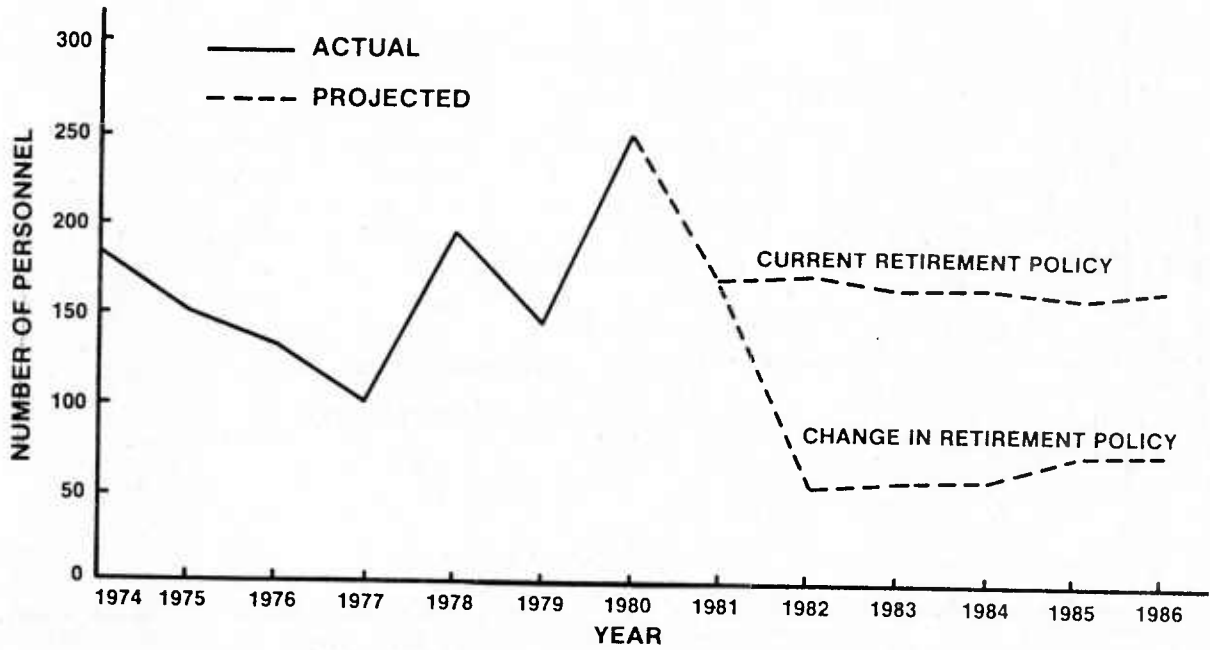


Figure 6. Effects of raising minimum retirement age on high-grade retirements

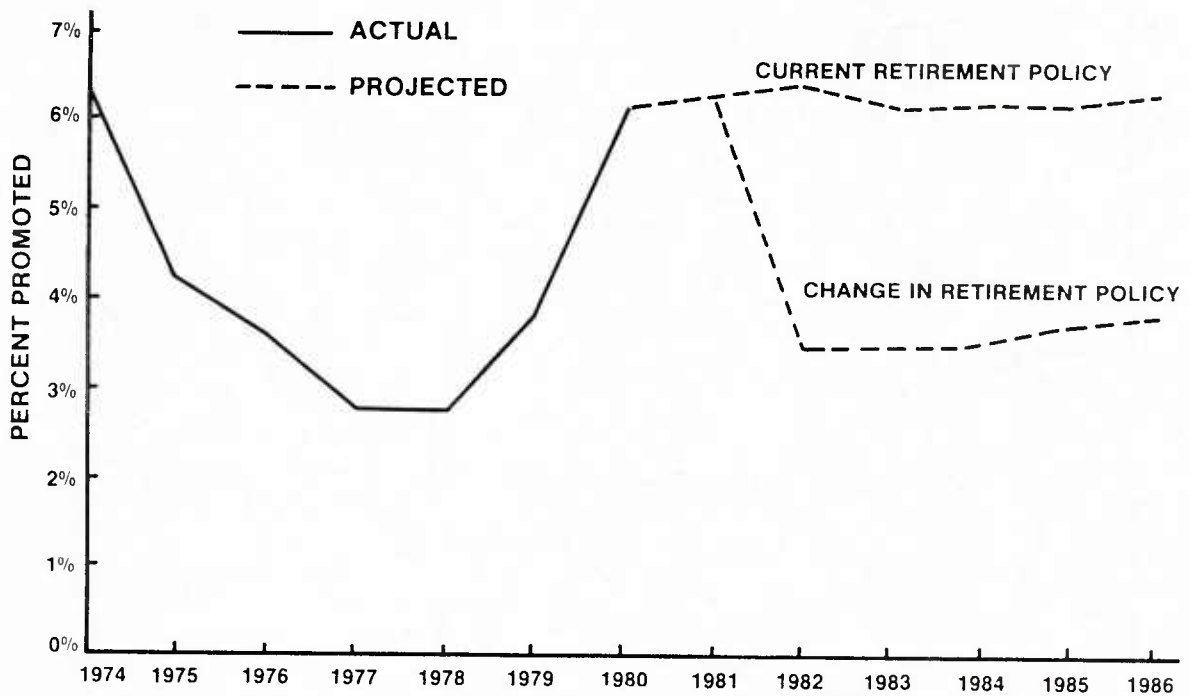


Figure 7. Effects of raising minimum retirement age on GS-12 promotion rates.

CONCLUSIONS AND DISCUSSION

A civilian manpower planning model was developed for use in evaluating policies affecting S&E personnel in the Navy R&D centers. The model is capable of providing alternative projections by age and grade based on various policy scenarios. The projections include the total number of personnel, the number of personnel to be hired, and expected levels of promotion and attrition. The model can be used by Navy manpower managers in evaluating alternative manpower strategies. It has been used to assist the offices of the Assistant Secretary of the Navy and the Chief of Naval Material in measuring the impact of high-grade ceilings on the Navy R&D centers.

While the initial purpose of this effort was limited to forecasting high-grade vacancies, a more general civilian manpower model was needed to estimate the effects of high-grade policies on other grades. Although the model was developed for a restricted class of personnel (S&E personnel in the Navy R&D centers), the concept of the model can be applied to other job classifications and other kinds of activities.

RECOMMENDATIONS

1. For more general use, the model should be expanded to cover job classifications other than S&E professionals and Navy activities other than Navy R&D centers.
2. An enhanced version of the model should incorporate cohort-based personnel flows and demand-driven hiring parameters.

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APPENDIX
MATHEMATICAL FORMULATION

MATHEMATICAL FORMULATION

Definition of Variables

Let Y represent personnel stock or inventory
X represent personnel gains
Z represent personnel losses

In detail,

$Y_{ij}(T)$ = The number of employees with age i and grade j at the end of year T,

$X_{ijk}(t)$ = The number of employee gains with age i and grade j through hiring during year t, or promotions from grade j-1 during year t,

$Z_{ijm}(t)$ = The number of employee losses with age i and grade j through retirements, voluntary quits, other separations during year t, or promotions to grade j+1 during year t,

where, $t \in (T-1, T)$,

i represents age (for example, i=30 represents Age 30),

j represents grade,
in detail,

j=1 for GS-5/6
j=2 for GS-7/8
j=3 for GS-9/10
j=4 for GS-11
j=5 for GS-12
j=6 for GS-13 and above,

k represents types of gains,
in detail,

k=1 for accessions
k=2 for promotion-in, and

m represents type of losses,
in detail,

m=1 for retirements
m=2 for voluntary quits
m=3 for other separations
m=4 for promotion-out.

Assumptions

In order to make projections, it is important to observe what has happened in the past. For certain types of personnel flows, continuous historical data were used as assumptions for projections. For other types of personnel flows, data for a given year or an average over a few years were used as a forecasting base. The base year information is essential to derive sets of probabilities and ratios that are used as assumptions for projections.

Let * in the parentheses represent average of data over selected years (base year). It is assumed that:

1. The probability of personnel gain of type k for employees with age i and grade j can be represented by:

$$a_{ijk} = \frac{X_{ijk}^{(*)}}{Y_{ij}^{(*)}}$$

It is assumed that personnel gains are estimated as a fraction of beginning inventory. It implies that personnel demand and policy during the base year will remain the same in future years.

2. The probability of personnel losses of type m for employees with age i and grade j can be calculated by using:

$$b_{ijm} = \frac{Z_{ijm}^{(*)}}{Y_{ij}^{(*)}}$$

It is assumed that personnel losses (except voluntary quits) are estimated as a fraction of beginning inventory. They reflect the loss rates during the base year.

3. The ratio used in determining the age distribution of voluntary quits for GS-12 in the base year was computed by using:

$$c_{ijm} = \frac{Z_{ijm}^{(*)}}{Z_{jm}^{(*)}} \quad \text{for } m=2.$$

In an earlier effort, a relationship was developed between the rate of voluntary quits and promotions (Liang, 1982). The ratio here is used to compute the age breakdown for voluntary quits after the total number of voluntary quits for GS-12 is derived by using the previously developed relationship.

4. The ratio used in determining the age distribution of voluntary quits for each grade below GS-12 level can be computed by:

$$d_{ijm} = \frac{Z_{ijm}^{(*)}}{Z_{..m}^{(*)}} \quad \text{for } j \leq 4.$$

This ratio is used to estimate the age and grade breakdown after the total number of voluntary quits for GS-5/11 is derived.

Equations for Estimating Personnel Losses

Retirement

The number of employees retired in year t with age i and grade j can be estimated by using the base-year probability for retirements as follows:

$$Z_{ijm}(t) = b_{ijm} Y_{ij}(T-1) \tag{1}$$

where $m=1.$

This equation indicates that the age distribution of beginning inventory affects retirement.

Voluntary Quits

In an earlier effort, an equation was derived to describe the quantitative relationship between the average promotion rate over the past 2 years, P, and the current attrition rate, S (Liang, 1982). The attrition rate for GS-12 employees was estimated by using the average promotion rate over the past 2 years, as follows:

$$S = 2.38 P^{-0.86}.$$

By converting S and P into X, Y, and Z, the above equation can be rewritten as:

$$\frac{Z_{\cdot j2}(t)}{Y_{\cdot j}(T-1)} = 2.38 P^{-0.86}$$

where,

$$P = \frac{Z_{\cdot j4}(t-1) + Z_{\cdot j4}(t-2)}{Y_{\cdot j}(T-2) + Y_{\cdot j}(T-3)}$$

and j = 5 (for GS-12).

Therefore, the total number of GS-12 employees attrited can be obtained using the following equation:

$$Z_{\cdot j2}(t) = 2.38 P^{-0.86} Y_{\cdot j}(T-1) \quad \text{for } j = 5.$$

The number of GS-12 attrited by age can be distributed by using the base year ratio:

$$\begin{aligned} Z_{ij2}(t) &= c_{ij2} Z_{\cdot j2}(t) \\ &= 2.38 c_{ij2} P^{-0.86} Y_{\cdot j}(T-1) \end{aligned} \quad (2)$$

for j = 5.

The procedure of estimating attrition for GS-5/11 differs from that for GS-12. The attrition rate for GS-5/11 can be estimated by using the attrition rates for GS-5/12 and GS-12. An equation was developed to show how the attrition rate for GS-5/12, S, was affected by the promotion rate for GS-12, P, (Liang, 1982).

With $S = 2.54 P^{-0.66}$ and,

using the notation in this report, the equation for GS-5/12 attrition rate can be expressed by:

$$\frac{\sum_{j=1}^5 Z_{\cdot j2}(t)}{\sum_{j=1}^5 Y_{\cdot j}(T-1)} = 2.54 P^{-0.66}.$$

Thus, the number of GS-5/12 employees attrited is

$$\sum_{j=1}^5 Z_{\cdot j2}(t) = 2.54 P^{-0.66} \sum_{j=1}^5 Y_{\cdot j}(T-1).$$

The number of GS-5/11 attrited is computed as the difference between the number of GS-5/12 attrited and the number of GS-12 attrited below:

$$\begin{aligned} \sum_{j=1}^4 Z_{\cdot j2}(t) &= \sum_{j=1}^5 Z_{\cdot j2}(t) - Z_{\cdot 52}(t) \\ &= 2.54 P^{0.66} \sum_{j=1}^5 Y_{\cdot j}(T-1) \\ &\quad - 2.38 P^{-0.86} Y_{\cdot 5}(T-1). \end{aligned}$$

Then, the age and grade breakdown for the employees attrited below the GS-12 level can be estimated by using the ratio of attrition for each grade level by age to the total attrition for all grade levels below GS-12 in the base year:

$$\begin{aligned} Z_{ij2}(t) &= d_{ij2} \sum_{j=1}^4 Z_{\cdot j2}(t) \\ &= d_{ij2} (2.45 P^{-0.66}) \sum_{j=1}^5 Y_{\cdot j}(T-1) \\ &\quad - 2.38 P^{0.86} Y_{\cdot 5}(T-1) \end{aligned} \tag{3}$$

for $j \leq 4$.

The number of high-grade voluntary quits was estimated by using the base year probability for high grades (i.e., the ratio of the total number of voluntary quits to the total number of employees in the base year) below:

$$\begin{aligned} Z_{ij2} &= b_{ij2} Y_{ij}(T-1) \end{aligned} \tag{4}$$

for $j = 6$.

Other Separations

The category of other separations includes all separations besides retirement and voluntary quits. It can be estimated by:

$$\begin{aligned} Z_{ijm}(t) &= b_{ijm} Y_{ij}(T-1) \end{aligned} \tag{5}$$

for $m = 3$.

Promotion-out

The promotion to grade $j+1$ from grade j can be estimated by:

$$\begin{aligned} Z_{ijm}(t) &= b_{ijm} Y_{ij}(T-1) \end{aligned} \tag{6}$$

for $m = 4$.

Equations for Estimating Personnel Gains

Accessions

The estimate for accession can be shown by using the probability of accession for the base year as follows:

$$X_{ijk}(t) = a_{ijk} Y_{ij}(T-1) \quad (7)$$

for $k = 1$.

Promotion-in

The promotion-in grade $j-1$ to grade j equals to promotion-out from grade $j-1$ to grade j . That is,

$$X_{ijk}(t) = Z_{i(j-1)m}(t) \quad (8)$$

for $k = 2$ and $m = 4$.

Equation for Total End Strength

Based on equations (1) to (8), the total number of employees at the end of year is,

$$Y_{(i+1)j}(T) = Y_{ij}(T-1) + X_{ij}(t) - Z_{ij}(t). \quad (9)$$

Policy Variables

Equations (1) through (9) can be used to estimate various types of personnel gains and losses and the total number of employees by age, grade, and year. However, to measure the impact of policy changes, policy variables had to be included in the model. The policy variables used in this model include a ceiling on the number of high grades and a possible increase in minimum retirement age. The model was modified to adapt to changes in these policies.

Ceiling on the Number of High Grades

Let

$C(T)$ = The ceiling on the number of high grades in year T ; and $Y_{\cdot 6}(T) \leq C(T)$.

When

$$Y_{\cdot 6}(T) < C(T),$$

the above ceiling would not affect equations (1) through (9).

But, if

$$Y_{\cdot 6}(T) \geq C(T),$$

then, let

$$Y_{\cdot 6}(T) = C(T).$$

Equation (9) would have to be modified as follows:

$$C(T) = Y_{\cdot 6}(T-1) + X_{\cdot 6}(t) - Z_{\cdot 6}(t). \quad (10)$$

By using a prime to represent adjusted estimates, the modified equation for computing the level of promotion for GS-12 professionals can be obtained by rearranging equation (10).

$$\begin{aligned} X'_{\cdot 62}(t) &= C(T) - Y_{\cdot 6}(T-1) - X_{\cdot 61}(t) \\ &+ Z_{\cdot 6}(t). \end{aligned} \quad (11)$$

Then we distribute $X'_{\cdot 62}(t)$ into individual age groups by using ratios for age distribution and equation (11), and we obtain

$$\begin{aligned} X'_{i62}(t) &= \frac{X_{i62}(t)}{X_{\cdot 62}(t)} (X'_{\cdot 62}(t)) \\ &= \frac{X_{i62}(t)}{X_{\cdot 62}(t)} (C(T) - Y_{\cdot 6}(T-1) + Z_{\cdot 6}(t) \\ &- X_{\cdot 61}(t)). \end{aligned} \quad (12)$$

Based on equation (9), the modified estimate for the total number of high-grade personnel by age can be derived as follows:

$$Y'_{(i+1)6}(T) = Y_{i6}(T-1) + X_{i61}(t) - Z_{i6}(t) + X'_{i62}(t). \quad (13)$$

By inserting equation (12) into equation (13), we obtain a modified estimate for the total number of high grades by age at the end of the year as follows:

$$\begin{aligned} Y'_{(i+1)6}(T) &= Y_{i6}(T-1) + X_{i61}(t) - Z_{i6}(t) \\ &+ \frac{X_{i62}(t)}{X_{\cdot 62}(t)} (C(T) - Y_{\cdot 6}(T-1) + Z_{\cdot 6}(t) - X_{\cdot 61}(t)). \end{aligned} \quad (14)$$

It should be noted that a ceiling affects estimates of inventory. Consequently, the effect of the ceiling will pass on to every estimate from equation (1) through equation (9).

Change in Minimum Retirement Age

The issue of increasing minimum retirement age of federal employees has been discussed in public forums. The increase in the minimum retirement age could reduce the number of retirements, at least in the short run. Assuming that the minimum retirement age will be increased by n years, equation (1) can be modified as follows:

$$Z_{ij1}(t) = b_{(i-n)j1} Y_{ij}(T-1). \quad (15)$$

The change in the minimum retirement age will affect the number of employees retired, $Z_{ij1}(t)$. The number of employees retired will affect the estimate for the total number of personnel through equation (9). Consequently, it affects every variable from equation (1) through equation (15).

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Director of Manpower Analysis (ODASN(M))
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Chief of Naval Operations (OP-01), (OP-11), (OP-12) (2), (OP-13), (OP-14), (OP-15), (OP-115) (2), (OP-140), (OP-140F2), (OP-141), (OP-142), (OP-964D), (OP-987H)
Chief of Naval Material (NMAT 00), (NMAT 01M1), (NMAT 05)
Chief of Naval Research (Code 200), (Code 440) (3), (Code 442), (Code 442PT)
Chief of Information (OI-213)
Chief of Naval Education and Training (02), (N-5), (022)
Chief of Naval Technical Training (016)
Commandant of the Marine Corps (MPI-20)
Commander Naval Air Development Center
Commander Naval Weapons Center
Commander David W. Taylor Naval Ship Research and Development Center
Commander Naval Surface Weapons Center
Commander Naval Ocean Systems Center
Commanding Officer, Naval Coastal Systems Center
Director, Naval Civilian Personnel Command
Superintendent, Naval Postgraduate School
Commander, Army Research Institute for the Behavioral and Social Sciences, Alexandria (PERI-ASL)
Chief, Army Research Institute Field Unit, Fort Harrison
Commander, Air Force Human Resources Laboratory, Brooks Air Force Base (Manpower and Personnel Division)
Commander, Air Force Human Resources Laboratory, Brooks Air Force Base (Scientific and Technical Information Office)
President, National Defense University (3)
Director, Office of Personnel Management (Planning and Evaluation)
Director, Science and Technology, Library of Congress
Defense Technical Information Center (DDA) (12)