

AD-A125 875

COMPUTATION OF PROPAGATION SPEED AND REFLECTION OF
AXIALLY SYMMETRIC WAVE. (U) NAVAL RESEARCH LAB
WASHINGTON DC P S DUBBELDAY ET AL. 25 AUG 82
NRL-MR-4885 F/G 11.

1/1

UNCLASSIFIED

F/G 11/4

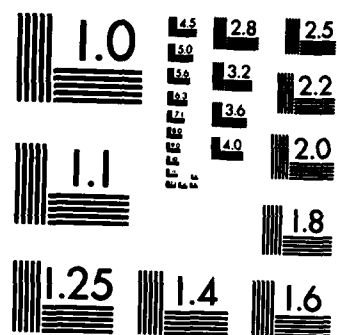
NL

END

FILMED

11

1998



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL MEMORANDUM REPORT 4885	2. GOVT ACCESSION NO. AD-A125 875	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (U) Computation of Propagation Speed and Reflection of Axially Symmetric Waves in Composite Cylinders, With Application to Impedance Tube and Calibrator		5. TYPE OF REPORT & PERIOD COVERED Interim report on a continuing problem
7. AUTHOR(s) Pieter S. Dubbelday and Clementina M. Ruggiero		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Underwater Sound Reference Detachment Naval Research Laboratory P.O. Box 8337, Orlando, FL 32856		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element: 61153N Task: RR011-08-42 NRL Work Unit: (59)0585
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE
		13. NUMBER OF PAGES 77
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Impedance tube Dispersion of Waves Hydroacoustic calibrator Propagation speed in composite cylinders Waveguides		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Waveguide theory is applied to the interpretation of data on reflection and propagation of sound in a water-filled impedance tube and in an acoustic calibrator. Computer programs are described and documented that: (1) relate propagation speed and attenuation for the sample in the impedance tube to the complex reflection coefficient; (2) relate propagation speed in the sample in the impedance tube to the elastic moduli of the sample material, both real and complex; and (3) compute the propagation speed in the acoustic calibrator. (continued on reverse)		

DD FORM 1473
1 JAN 73EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

UNCLASSIFIED

1 SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Item 20 continued.

A correction factor is developed to account for the lack of rigidity of the reflector in the impedance tube. The major conclusion with respect to the data reduction for the impedance tube is the observation that the relevant wave speed is not necessarily the dilatational wave speed, but is a function of the space between sample and tube wall. By modifying the experimental arrangement one may measure two separate elastic moduli.

UNCLASSIFIED

11 SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

CONTENTS

1.0.	INTRODUCTION	1
2.0.	REFLECTION COEFFICIENT IN IMPEDANCE TUBE	3
3.0.	THEORY OF WAVE PROPAGATION IN INFINITE CYLINDERS & CYLINDRICAL SHELLS	4
4.0.	PROPAGATION SPEED OF AXIALLY SYMMETRIC WAVES IN COMPOSITE CYLINDER	7
4.1.	Wave Speed in Sample in Impedance Tube	8
4.2.	Wave Speed in Calibrator	11
4.3.	Complex Wave Speed in Sample in Impedance Tube	17
5.0.	RESULTS AND DISCUSSION	17
5.1.	Impedance Tube	17
5.2.	Calibrator	24
6.0.	CONCLUSIONS.	27
	REFERENCES	28
	APPENDIX A - Computer Programs	31
	Program RTUBE	32
	Program WGUIDE	44
	Program IMPED	61
	APPENDIX B - Correction for Imperfect Reflector in Impedance Tube.	73

DTIC
ELECTE
S MAR 21 1983 **D**
B



Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

COMPUTATION OF PROPAGATION SPEED AND REFLECTION OF AXIALLY SYMMETRIC WAVES IN COMPOSITE CYLINDERS, WITH APPLICATION TO IMPEDANCE TUBE AND CALIBRATOR

1.0. INTRODUCTION

Various instruments used at the Underwater Sound Reference Detachment of the Naval Research Laboratory (NRL-USRD) are based on acoustic waveguide technology. Examples are the impedance tube [1], inertial calibrators [2,3], and the Low-Frequency Facility's systems [4,5]. A common feature of these instruments is the creation of an acoustic field with well-defined properties in a limited space, as contrasted with free-field measurements. The field description of this class of instruments is based on waveguide theory. In this report an analysis of the waveguide aspects of these instruments is presented. Discussion and documentation of the computer programs used in the data reduction are given. The correspondence between predictions from the analysis and the data will be a measure of the extent to which simple waveguide theory is adequate in describing the phenomena.

Waveguide theory is well established and described [6,7]. Some of its features are given in this report to serve as a basis for the understanding of the computer programs and quantitative conclusions. It is assumed that the wave propagation takes place in one dimension only. The field perpendicular to this direction is spatially limited by shape and properties of physical bodies. This does not exclude the possibility that particle velocity and pressure may be functions of more than one coordinate. It will be assumed, though, that at interfaces perpendicular to the direction of propagation, the boundary conditions are only imposed on averaged axial velocity and averaged pressure. Usually, one applies waveguide theory to cases where the cross-sectional dimension is small compared with the wavelength; a consequence is that the radial variation of the field variables is small. The analysis developed in this report does not account for the influence of finite cylinder length and its attending termination on the wave pattern. This influence will be less the larger the ratio of length to diameter of the various cylindrical elements of the system.

The hierarchy of computer programs documented here may be understood by reference to the sketches of impedance tube, Fig. 1, and free surface calibrator, Fig. 2. For the cylindrical sample in the impedance tube, Fig. 1, one wants to evaluate the propagation speed in the axial direction given the values of a pair of elastic moduli of the solid material. In general, propagation speed and elastic moduli are complex, accounting for the existence of dissipation of acoustic energy in the material. The reflection of a plane wave from the front face of the sample may then be computed. The propagation speed in the water will be influenced by the lack of rigidity of the cylinder wall. This problem is more serious in the case of the calibrator where the

wall thickness is considerably less than that of the impedance tube. This influence is numerically determined. The sequence of computations in data analysis of the impedance tube is just the opposite of the one sketched. One experimentally determines the complex reflection coefficient. From this a complex propagation speed is inferred and, in turn, this propagation speed is related to the elastic moduli of the solid material. This inverse procedure may be effected by a combination of root-search techniques and the programs presented here.

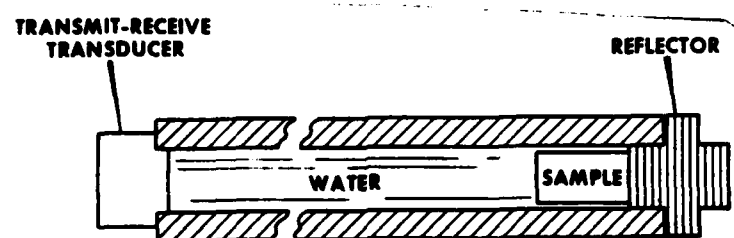


Fig. 1 - Section of acoustic impedance tube.

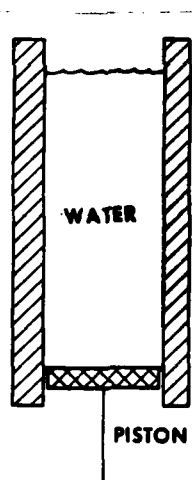


Fig. 2 - Principle of free surface acoustic calibration device.

In this report three computer programs are described and documented. Program RTUBE computes the complex propagation speed in the sample in the impedance tube from the measured complex reflection coefficient. Program WGUIDE, with various options, determines real propagation speed in the sample in the impedance tube and in the water in the calibrator from a pair of real elastic moduli characterizing the material of the sample and tube wall, respectively. Program IMPED computes the complex wave speed from two complex elastic moduli for the sample in the impedance tube.

In the next section the theory for reflection of the signal in the impedance tube is given. Then the theory of composite waveguides is developed and the various dispersion relations are derived applicable to the boundary conditions for impedance tube and calibrator. Examples are given for wave propagation in samples of steel and rubber in the impedance tube, and for the influence of the wall of the calibrator. The detailed discussion and documentation of the computer programs is given in Appendix A. Appendix B deals with the application of a correction factor to the measurement in the impedance tube that accounts for the lack of perfection of the reflector.

An important conclusion of this study is the observation that the sound speed in the sample in an impedance tube, ignoring end effects, is not necessarily the dilatational wave speed but a function of the size of the fluid-filled gap between sample and tube wall, and the elastic properties of the sample material. This effect may be utilized to obtain more information on the elastic moduli by varying the sample radius, and also by not admitting fluid to fill this gap.

2.0. REFLECTION COEFFICIENT IN IMPEDANCE TUBE

It is assumed that a wave propagates in the axial direction identified with the z coordinate and that the wave pattern is periodic in z . Thus the field variables will contain a factor $\exp i(\omega t - kz)$, where ω is the angular frequency and k the wave number; the phase speed c is related by $\omega = kc$. Because of the geometry, it is assumed that the field variables do not depend on the azimuth angle. The pressure and axial particle velocity are, in general, functions of the radial distance r . For the sake of the computation of the reflection coefficient, it is assumed that one may represent the wave as a plane wave with average values for pressure and velocity. The sample of length d is backed by a reflector with acoustic impedance Z_r . One can show [8] that the input impedance at the front end of the sample is

$$Z = (\rho_s c/S) \frac{Z_r + i(\rho_s c/S) \tan kd}{(\rho_s c/S) + i Z_r \tan kd}, \quad (1)$$

where ρ_s is the density of the sample
 c is the sound speed in the sample
 S is the cross-sectional area
 k is the wave number in the sample.

If one applies this same formula to the reflector behind the sample, where the impedance of the air backing the reflector may be ignored, one sees that the impedance of the reflector, Z_r , is given by $Z_r = i(\rho c)_r \tan k_r l$, where the subscript r refers to the reflector, and l is the length of the reflector. In practice, one chooses l such that $k l = \pi/2$ for some intermediate frequency and then the impedance Z_r is virtually infinite for a reasonably large bandwidth.

The reflection coefficient (complex) r of the sample is defined as the ratio of the amplitude of the reflected to that of the incoming wave. Equating impedances at the interface leads to the relation between r and the wave number k according to

$$\frac{Z}{(\rho_0 c_0 / S)} = \frac{1 + r}{1 - r} = -i(\rho_s / \rho_0) (k_0 d / kd) \cot(kd) , \quad (2)$$

where ρ_0 is the density of and k_0 the wave number in the fluid. The program RTUBE determines the complex root kd of this equation from the measured complex value of r .

The data reduction incorporated in program RTUBE is based on Eq. (2) which is derived under the assumption that the reflector is ideal; i.e., that the amplitude and phase of the reflected pressure wave are the same as those of the incident wave. To account for the lack of rigidity of the reflector, and to avoid measurement of absolute magnitude and phase, the wave reflected in the tube with the sample removed is measured. The reflection coefficient r is computed according to $r = (r_{sa} / r_{st}) \exp(-2ik d)$, where r_{sa} is the reflection with the sample in place, and r_{st} is the reflection without the sample. The exponential factor accounts for the fact that the sample is replaced by a column of water with a given phase shift. This r is only approximately correct; an additional correction factor is described in Appendix B.

3.0. THEORY OF WAVE PROPAGATION IN INFINITE CYLINDERS AND CYLINDRICAL SHELLS

In an elastic solid of infinite extent there are two different types of wave motion. One is an irrotational wave, usually denoted as a dilatational wave, and the other is an incompressible motion, or shear wave. Mathematically, they can be described by a scalar and vector potential ϕ and $\mathbf{A}$ respectively. In the axially symmetric situation of waveguides, only the azimuthal component of the vector potential is needed, henceforth indicated by the function $H(r, z)$.

The general relations used below can be found in Ref. 9. The particle displacement $\vec{u}$ is given in terms of the potentials by

$$\begin{aligned} u_r &= \frac{\partial \phi}{\partial r} - \frac{\partial H}{\partial z} \\ u_z &= \frac{\partial \phi}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (rH) . \end{aligned} \quad (3)$$

The stresses pertinent to the analysis for the case of axial symmetry are given in terms of the strains by

$$\begin{aligned} \tau_{rr} &= \lambda e + 2\mu e_{rr} , \\ \tau_{zz} &= \lambda e + 2\mu e_{zz} , \\ \tau_{\theta\theta} &= \lambda e + 2\mu e_{\theta\theta} , \end{aligned} \quad (4)$$

$$\tau_{rz} = 2\mu e_{rz} ,$$

where

$$e_{rr} = \frac{\partial u_r}{\partial r} ,$$

$$e_{\theta\theta} = \frac{u_r}{r} ,$$

$$e_{zz} = \frac{\partial u_z}{\partial z} ,$$

$$e_{rz} = \frac{1}{2} \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) ,$$

$$e = e_{rr} + e_{\theta\theta} + e_{zz} ,$$

and λ and μ are Lamé constants.

The equations of motion are

$$\begin{aligned} \frac{\partial \tau_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} &= \rho_s \frac{\partial^2 u_r}{\partial t^2} , \\ \frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \tau_{zz}}{\partial z} + \frac{1}{r} \tau_{rz} &= \rho_s \frac{\partial^2 u_z}{\partial t^2} . \end{aligned} \quad (5)$$

By combining Eqs. (3) through (5) one derives the following wave equations for the potentials ϕ and H

$$c_d^2 \Delta \phi = \frac{\partial^2 \phi}{\partial t^2} \quad (6)$$

and

$$c_s^2 (\Delta H - H/r^2) = \frac{\partial^2 H}{\partial t^2} , \quad (7)$$

where c_d is the wave speed for dilatational waves, $c_d^2 = (\lambda + 2\mu)/\rho_s$, c_s is the wave speed for shear waves, and $c_s^2 = \mu/\rho_s$; Δ is the Laplacean

operator for cylindrical coordinates $\Delta = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2}$.

If one assumes solutions for ϕ and H in the form of a wave traveling in the z direction, according to

$$\phi = f(r) e^{i(\omega t - kz)}$$

and (8)

$$H = g(r) e^{i(\omega t - kz)},$$

the functions $f(r)$ and $g(r)$ are solutions of

$$f'' + f'/r - q^2 f = 0 \quad (9)$$

and

$$g'' + g'/r - (s^2 + 1/r^2)g = 0, \quad (10)$$

where $q^2 = k^2 - k_d^2$ and $s^2 = k^2 - k_s^2$, k_d and k_s are the wave numbers for dilatational and shear waves. In terms of the variables q' and s' , defined by $(q')^2 = -q^2$ and $(s')^2 = -s^2$, the general solutions to Eqs. (9) and (10) are $f = A J_0(q'r) + B Y_0(q'r)$ and $g = C J_1(s'r) + D Y_1(s'r)$. J_n is the Bessel function of the first kind with order n , and Y_n is the Bessel function of the second kind with order n .

The compressional wave in the fluid is represented by a velocity potential ϕ_0 which is a solution of the wave equation

$$c_0^2 \Delta \phi_0 = \frac{\partial^2 \phi_0}{\partial t^2}, \quad (11)$$

where c_0 is the wave speed in the fluid.

The velocity components U_r and U_z of the particle velocity in the fluid follow from $U_r = \frac{\partial \phi_0}{\partial r}$ and $U_z = \frac{\partial \phi_0}{\partial z}$. The solution of Eq. (11) representing a traveling wave in the z direction is

$$\phi_0 = [E J_0(q'_0 r) + F Y_0(q'_0 r)] e^{i(\omega t - kz)},$$

where $(q'_0)^2 = k^2 - k_0^2$, k_0 is the wave number of the compressional wave in the fluid. Application of the boundary conditions leads to the dispersion relations for the phase speed c . For a wave in an infinite cylinder in

vacuum, the analysis has been performed by Pochhammer and Chree [10]. A comprehensive set of calculations of the phase speed based on the Pochhammer-Chree solution were performed by Bancroft [11].

4.0. PROPAGATION SPEED OF AXIALLY SYMMETRIC WAVES IN COMPOSITE CYLINDERS

The determination of phase speed in composite cylinders is accomplished by imposing the pertinent boundary conditions on the stresses and particle velocities derived in Section 3. An overview of the various options in the computer program is presented in Fig. 3.

The physical situation in the impedance tube is represented in Option 1. A solid cylinder is surrounded by a thin layer of fluid and the center wall of the tube is assumed to be rigid. For comparison, a free outer boundary is considered in Option 2, although this case can hardly be realized in practice. The calibrator situation is represented by Option 3, where the influence of a nonrigid wall on the fluid wave speed is evaluated. Options 4 and 5 reflect the situation where a soft coating is applied to the inside of the calibration tube in order to slow down the wave in the liquid; in Option 4 the coating is cemented to the wall, in Option 5 it is free to move tangentially at the wall. In Option 6 the wave speed in a fluid cylinder with rigid boundary is computed, in Option 7 the wave speed in a fluid cylinder with free boundary is computed.

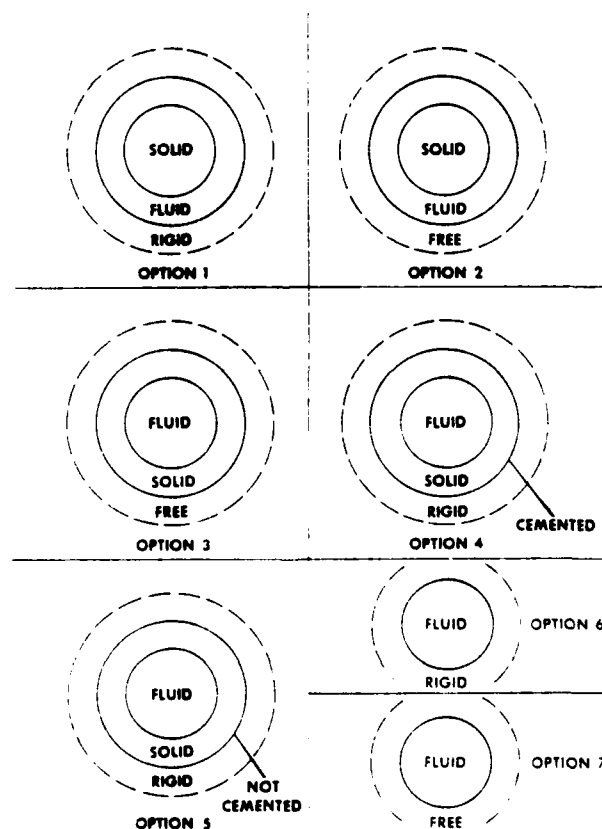


Fig. 3 - Cross section of waveguides in situations corresponding to options in program WGUIDE.

In the real variable program WGUIDE, the variables q' , s' , and q'_0 may be either real or imaginary. In order to avoid the necessity of complex calculations, the transformations $q' \rightarrow iq$; $s' \rightarrow is$, and $q'_0 \rightarrow iq_0$ are applied whenever imaginary values appear. The Bessel functions are transformed according to the identities $J_0(ix) = I_0(x)$, $J_1(ix) = iI_1(x)$, $Y_0(ix) = -K_0(x)$, and $Y_1(ix) = iK_1(x)$. $I_n(x)$ and $K_n(x)$ are modified Bessel functions [12].

4.1. Wave Speed in Sample in Impedance Tube

The following set of boundary conditions is applied to the field variables in the solid cylinder and surrounding fluid if the outer wall is rigid (Option 1).

- a. The normal stress τ_{rr} in the solid is equal to the negative of the pressure p in the fluid at the surface $r = a$, where a is the radius of the cylinder. The pressure follows from

$$\frac{\partial p}{\partial r} = -\rho_0 \frac{\partial u}{\partial t}. \quad \text{Thus } p = -\rho_0 \frac{\partial \phi_0}{\partial t} \text{ at } r = a.$$

- b. The tangential stress τ_{rz} is zero in the solid for $r = a$.
- c. The normal component of the particle velocity is continuous at the interface between solid and fluid at $r = a$.
- d. The normal component of the particle velocity in the fluid U_r is zero at $r = b$.

The coefficients of the amplitudes A , C , E , and F may be shown in a matrix, Table 1. Equating the determinant value of this matrix to zero yields the dispersion relation for the wave speed.

If the outer boundary is assumed free (Option 2), the fourth boundary condition is replaced by one where the pressure in the fluid is zero for $r = b$. The resulting matrix of the coefficients is shown in Table 2.

Table 1 - Matrix of coefficients of boundary conditions for impedance tube, Option 1.

Amplitudes +	A	-iC	E/(iω)	F/(iω)
Continuous normal stress at r = a	$(2k^2 - k_s^2)a^2J_0(q'a) + 2(q'a)J_1(q'a)$	$2(ka)\{J_1(s'a) - (s'a)J_0(s'a)\}$	$(\rho_0/\rho_s)(k_s a)^2 J_0(q'_0 a)$	$(k_s a)^2 (\rho_0/\rho_s) Y_0(q'_0 a)$
Zero tangential stress at r = a	$2(ka)(q'a)J_1(q'a)$	$(2k^2 - k_s^2)a^2 J_1(s'a)$	0	0
Continuous normal particle vel. at r = a	$(q'a)J_1(q'a)$	$(ka)J_1(s'a)$	$-(q'_0 a)J_1(q'_0 a)$	$-(q'_0 a)Y_1(q'_0 a)$
Zero normal particle vel. at r = b	0	0	$(q'_0 b)J_1(q'_0 b)$	$(q'_0 b)Y_1(q'_0 b)$

Table 2 - Matrix of coefficients of boundary conditions for impedance tube, Option 2.

Amplitudes →	A	-iC	E/(iω)	F/(iω)
Continuous normal stress at r = a	$(2k^2 - k_s^2)a^2J_0(q'a) + 2(q'a)J_1(q'a)$	$2(ka)J_1(s'a) - (s'a)J_0(s'a)$	$(k_s a)^2(\rho_0/\rho_s)J_0(q'_0 a)$	$(k_s a)^2(\rho_0/\rho_s)Y_0(q'_0 a)$
Zero tangential stress at r = a	$2(ka)(q'a)J_1(q'a)$	$(2k^2 - k_s^2)a^2J_1(s'a)$	0	0
Continuous normal particle vel. at r = a	$(q'a)J_1(q'a)$	$(ka)J_1(s'a)$	$-(q'_0 a)J_1(q'_0 a)$	$-(q'_0 a)Y_1(q'_0 a)$
Zero normal particle vel. at r = b	0	0	$J_0(q'_0 b)$	$Y_0(q'_0 b)$

4.2. Wave Speed in Calibrator

Option 3

For the case of the calibrator, one assumes that the fluid cylinder has a radius a and is bounded by a solid, but not rigid, wall with outer radius b . The following set of boundary conditions applies to the field variables in fluid and cylinder wall.

- a. The negative value of the pressure p is equal to the normal stress in the solid at $r = a$.
- b. The tangential stress in the solid is zero at $r = a$.
- c. The normal stress in the solid is zero at $r = b$.
- d. The tangential stress in the solid is zero at $r = b$.
- e. The normal component of the particle velocity in the fluid is equal to that in the solid at $r = a$.

The resulting coefficient matrix for amplitudes A, B, C, D, and F is shown in Table 3.

It was desirable to analyze the situation where a soft coating was applied to the inside of the calibrator with the purpose of reducing the effective wave speed in the liquid of the calibrator. If this coating is cemented to the wall, the following boundary conditions apply:

Table 3 - Matrix of coefficients of boundary conditions for calibrator, Option 3.

Amplitudes →	A	B	-iC	-iD	E/(im)
Continuous normal stress at $r = a$	$(2k^2 - k_0^2)a^2J_0(q'a) + 2(q'a)J_1(q'a)$	$(2k^2 - k_0^2)a^2Y_0(q'a) + 2(q'a)Y_1(q'a)$	$2(ka)[J_1(s'a) - (s'a)J_0(s'a)]$	$2(ka)[Y_1(s'a) - (s'a)Y_0(s'a)]$	$(\rho_0/\rho_a)(k_0a)^2J_0(q'a)$
Zero tangential stress at $r = a$	$2(ka)(q'a)J_1(q'a)$	$2(ka)(q'a)Y_1(q'a)$	$(2k^2 - k_0^2)a^2J_1(s'a)$	$(2k^2 - k_0^2)a^2Y_1(s'a)$	0
Zero normal stress at $r = b$	$(2k^2 - k_0^2)b^2J_0(q'b) + 2(q'b)J_1(q'b)$	$(2k^2 - k_0^2)b^2Y_0(q'b) + 2(q'b)Y_1(q'b)$	$2(kb)[J_1(s'b) - (s'b)J_0(s'b)]$	$2(kb)[Y_1(s'b) - (s'b)Y_0(s'b)]$	0
Zero tangential stress at $r = b$	$2(kb)(q'b)J_1(q'b)$	$2(kb)(q'b)Y_1(q'b)$	$(2k^2 - k_0^2)b^2J_1(s'b)$	$(2k^2 - k_0^2)b^2Y_1(s'b)$	0
Continuous normal particle vel. at $r = a$	$(q'a)J_1(q'a)$	$(q'a)Y_1(q'a)$	$(ka)J_1(s'a)$	$(ka)Y_1(s'a)$	$-(q'_0a)J_1(q'a)$

Option 4

- a. The negative value of the pressure p is equal to the normal stress in the solid at $r = a$.
- b. The tangential stress in the solid is zero at $r = a$.
- c. The normal component of the velocity is zero at $r = b$.
- d. The tangential component of the particle velocity is zero at $r = b$.
- e. The normal component of the particle velocity is continuous at $r = a$.

The resulting matrix of the coefficients of these boundary conditions is shown in Table 4.

Table 4 - Matrix of coefficients of boundary conditions for coating cemented to calibrator wall,
Option 4.

Amplitudes →	A	B	-iC	-iD	$V/(i\omega)$
Continuous normal stress at $r = a$	$(2k^2 - k_g^2)a^2J_0(q'a) + 2(q'a)J_1(q'a)$	$(2k^2 - k_g^2)a^2Y_0(q'a) + 2(q'a)Y_1(q'a)$	$2(ka)[J_1(s'a) - (s'a)J_0(s'a)]$	$2(ka)[Y_1(s'a) - (s'a)Y_0(s'a)]$	$(\rho_0/\rho_g)(k_g a)^2J_0(q'_0 a)$
Zero tangential stress at $r = a$	$2(ka)(q'a)J_1(q'a)$	$2(ka)(q'a)Y_1(q'a)$	$(2k^2 - k_g^2)a^2J_1(s'a)$	$(2k^2 - k_g^2)a^2Y_1(s'a)$	0
Zero normal particle vel. at $r = b$	$(q'b)J_1(q'b)$	$(q'b)Y_1(q'b)$	$(kb)J_1(s'b)$	$(kb)Y_1(s'b)$	0
Zero tangential particle vel. at $r = b$	$(kb)J_0(q'b)$	$(kb)Y_0(q'b)$	$-(s'b)J_0(s'b)$	$-(s'b)Y_0(s'b)$	0
Continuous normal particle vel. at $r = a$	$(q'a)J_1(q'a)$	$(q'a)Y_1(q'a)$	$(ka)J_1(s'a)$	$(ka)Y_1(s'a)$	$-(q'_0 a)J_1(q'_0 a)$

Option 5

If the coating is not cemented to the wall, but is free to move in a tangential direction, condition (d) is replaced by the requirement that the tangential stress in the solid is zero at $r = b$. The resulting matrix is given in Table 5.

Table 5 - Matrix of coefficients of boundary conditions for coating not cemented to calibrator wall, Option 4.

Amplitudes →	A	B	-iC	-iD	$\frac{E}{(1+\mu)}$
Continuous normal stress at $r = a$	$(2k^2 - k_0^2)a^2 J_0(q'a) + 2(q'a)J_1(q'a)$	$(2k^2 - k_0^2)a^2 Y_0(q'a) + 2(q'a)Y_1(q'a)$	$2(ka)[J_1(q'a) - (q'a)J_0(q'a)]$	$2(ka)[Y_1(q'a) - (q'a)Y_0(q'a)]$	$(\rho_0/\rho_s)(k_0 a)^2 J_0(q'_0 a)$
Zero tangential stress at $r = a$	$2(ka)(q'a)J_1(q'a)$	$2(ka)(q'a)Y_1(q'a)$	$(2k^2 - k_0^2)a^2 J_1(q'a)$	$(2k^2 - k_0^2)a^2 Y_1(q'a)$	0
Zero normal particle vel. at $r = b$	$(q'b)J_1(q'b)$	$(q'b)Y_1(q'b)$	$(kb)J_1(q'b)$	$(kb)Y_1(q'b)$	0
Zero tangential stress at $r = b$	$2(kb)(q'b)J_1(q'b)$	$2(kb)(q'b)Y_1(q'b)$	$(2k^2 - k_0^2)b^2 J_1(q'b)$	$(2k^2 - k_0^2)b^2 Y_1(q'b)$	0
Continuous normal particle vel. at $r = a$	$(q'a)J_1(q'a)$	$(q'a)Y_1(q'a)$	$(ka)J_1(q'a)$	$(ka)Y_1(q'a)$	$-(q'_0 a)J_1(q'_0 a)$

Empty cylinder

The phase speed in an empty solid cylinder may be readily derived from the matrix in Table 2 by omitting the fifth row and fifth column and setting the determinant value of the remaining 4x4 matrix equal to zero.

Other phase speeds of interest may be obtained. In a fluid column with free boundaries, the phase speed is given by

$$J_0(q'_0 a) = 0 \text{ or } (c/c_0)^2 = 1/[1 - j_{0,m}^2/(k_0 a)^2] \quad (12)$$

where $m = 1, 2, \dots$ and $j_{0,m}$ are the zeros of the J_0 Bessel function. Thus, there is a cutoff frequency given by

$$\omega_0 = j_{0,1} c_0 / a \quad (13)$$

below which no wave propagation is possible in a free fluid cylinder. Similarly, the phase speed of waves in a fluid cylinder bounded by rigid walls follows from

$$(q_0 a) J_1(q_0 a) = 0 \quad (14)$$

or

$$(c/c_0)^2 = 1/[1 - j_{1,m}^2/(k_0 a)^2] \quad (15)$$

where $m = 0, 1, 2, \dots$ and $j_{1,m}$ are the zeros of the J_1 Bessel function (including $j_{1,0} = 0$).

4.3. Complex Wave Speed in Sample in Impedance Tube

The measurements in the impedance tube consist of a complex reflection coefficient. Thus, it is important to determine the complex wave speed that is connected with a pair of complex elastic moduli. This is accomplished by the program IMPED. Its dispersion relation is the same as in WGUIDE option 1, but the roots are found by a complex root finder described in Ref. 13.

5.0. RESULTS AND DISCUSSION

5.1. Impedance Tube

The original incentive to develop the WGUIDE program was the need to determine exactly which propagation speed is measured in the impedance tube. It appears to be tacitly assumed that the dilatational speed is the quantity measured. For instance, in Ref. 14 the sound speed is graphed without further qualification; from the text one may infer that it is the dilatational wave

speed. The present analysis shows that it depends on the circumstances as to what speed applies for the sample in the impedance tube. This is shown in a sequence of figures. The first set shows the dispersion relation in a sample of steel with a density of 7700 kg/m^3 , Young's modulus $19.5 \times 10^{10} \text{ Pa}$, shear modulus $8.3 \times 10^{10} \text{ Pa}$, Poisson's ratio 0.37 (data from Ref. 15). The tube is filled with water with density 998 kg/m^3 and wave speed 1481 m/s . The abscissa $k_d a$ is a dimensionless measure of the frequency, with k_d the wave number for the dilatational wave in the solid and a the radius of the cylindrical sample; the ordinate is the ratio of the wave speed to the dilatational wave speed. For large values of $k_d a$ the wave speed approaches the Rayleigh wave speed. Figure 4 shows that for a sample in vacuum at low frequency the bar speed is approached, as expected. With water in the space between the tube wall and the sample, radii a and b , respectively, the wave speed in the sample at low frequency increases with decreasing value of b/a as seen in Figs. 5 and 6 and is indistinguishable from the dilatational wave speed ($c/c_d = 1$) for $b/a = 1.001$, Fig. 7. Notice that there is a "fluid mode" with low propagation speed in addition to various modes determined by the sample. The same behavior is shown by hard rubber as sample material, density 1100 kg/m^3 , shear modulus $0.1 \times 10^{10} \text{ Pa}$, bulk modulus $0.5 \times 10^{10} \text{ Pa}$. Here, however, a thicker layer of fluid is able to constrain the material to manifest dilatational wave speed than for steel (see Figs. 8 through 10). Even for $b/a = 1.2$, the speed is close to the dilatational wave speed (Fig. 9). The value for b/a is about 1.03 for the NRL-USRD impedance tube.

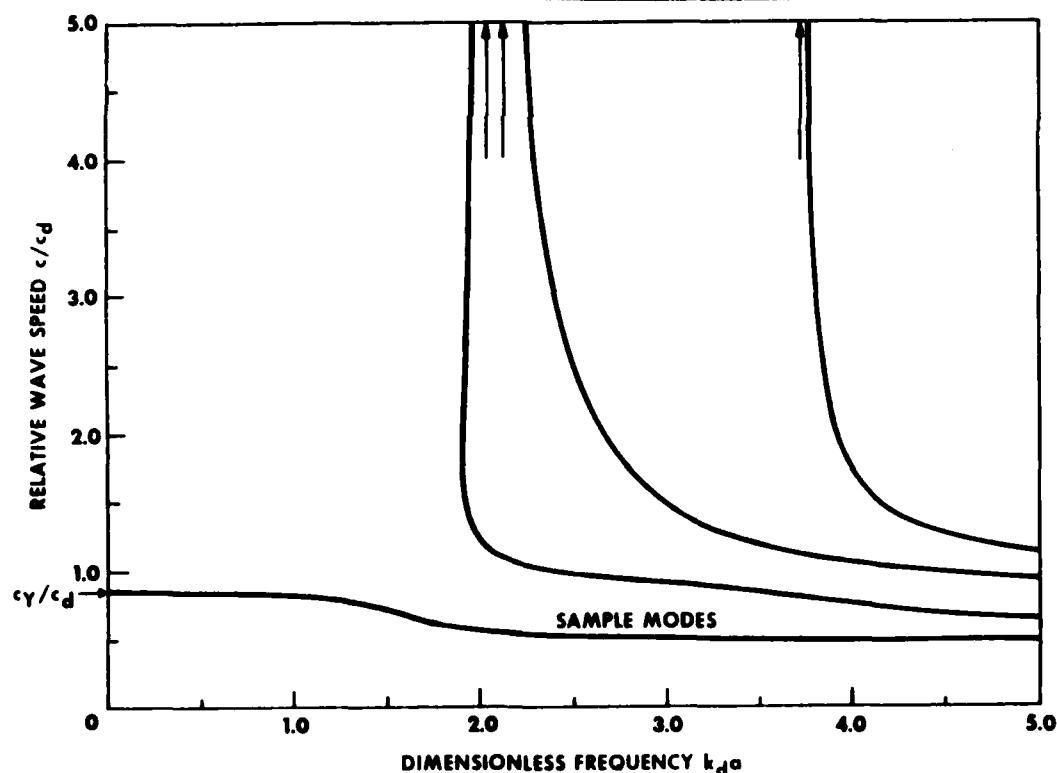


Fig. 4 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for steel cylinder in vacuum. Vertical arrows indicate asymptotes; c_d is the dilatational speed, c_y is bar speed.

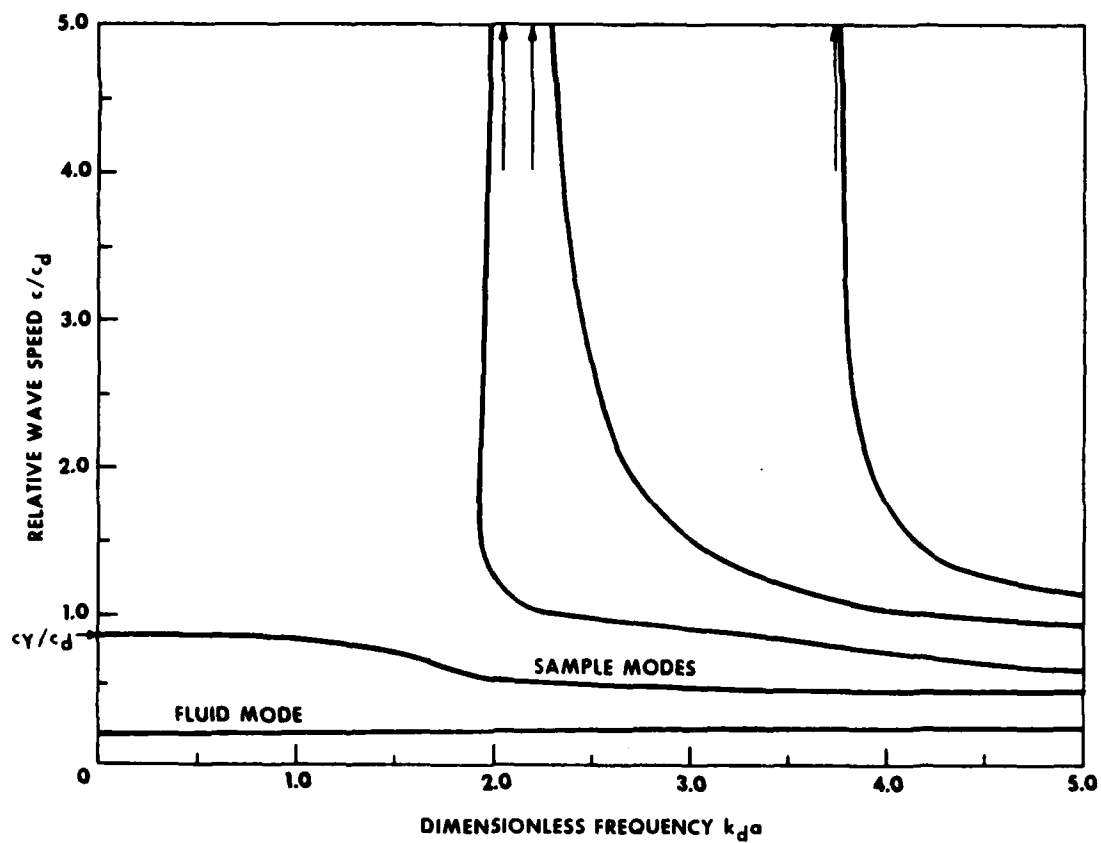


Fig. 5 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for steel cylinder in water, bounded by a concentric rigid wall. Ratio of the cylinder radii is 1.05. Vertical arrows indicate asymptotes; c_d is the dilatational speed, c_y is the bar speed.

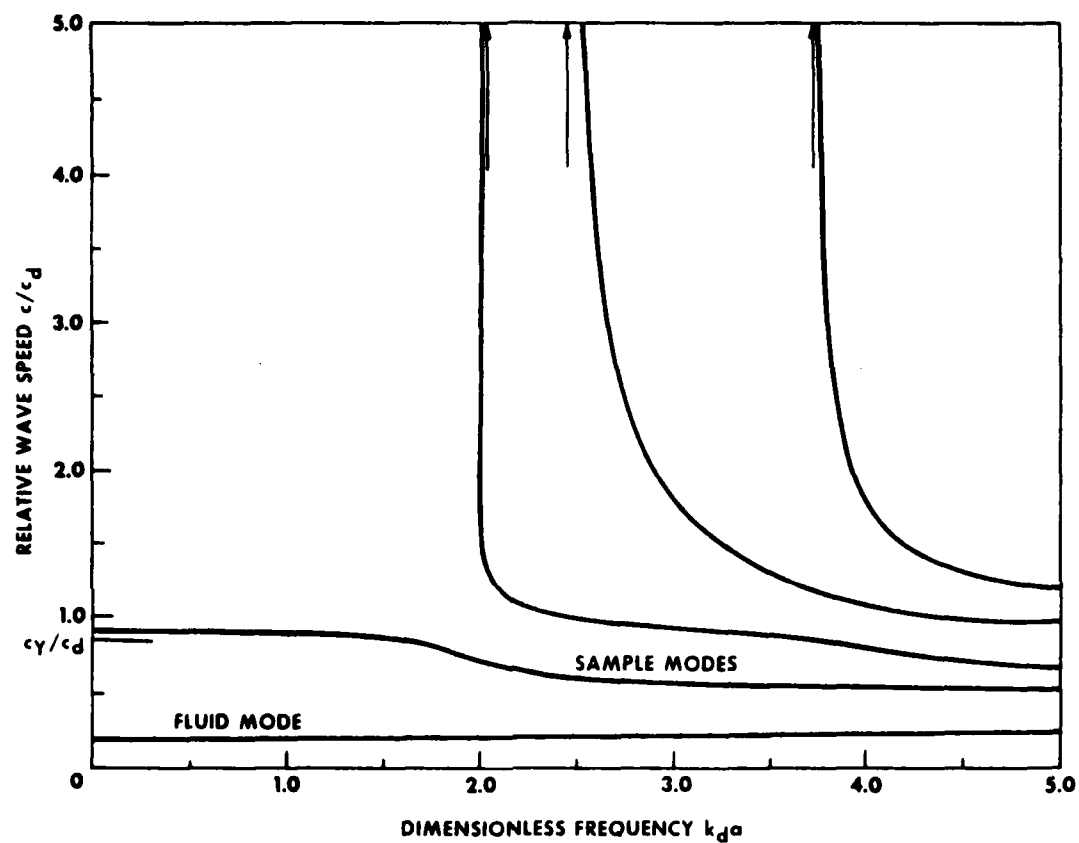


Fig. 6 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a steel cylinder in water, bounded by a concentric rigid wall. Ratio of the cylinder radii is 1.01. Vertical arrows indicate asymptotes; c_d is the dilatational speed, c_y is the bar speed.

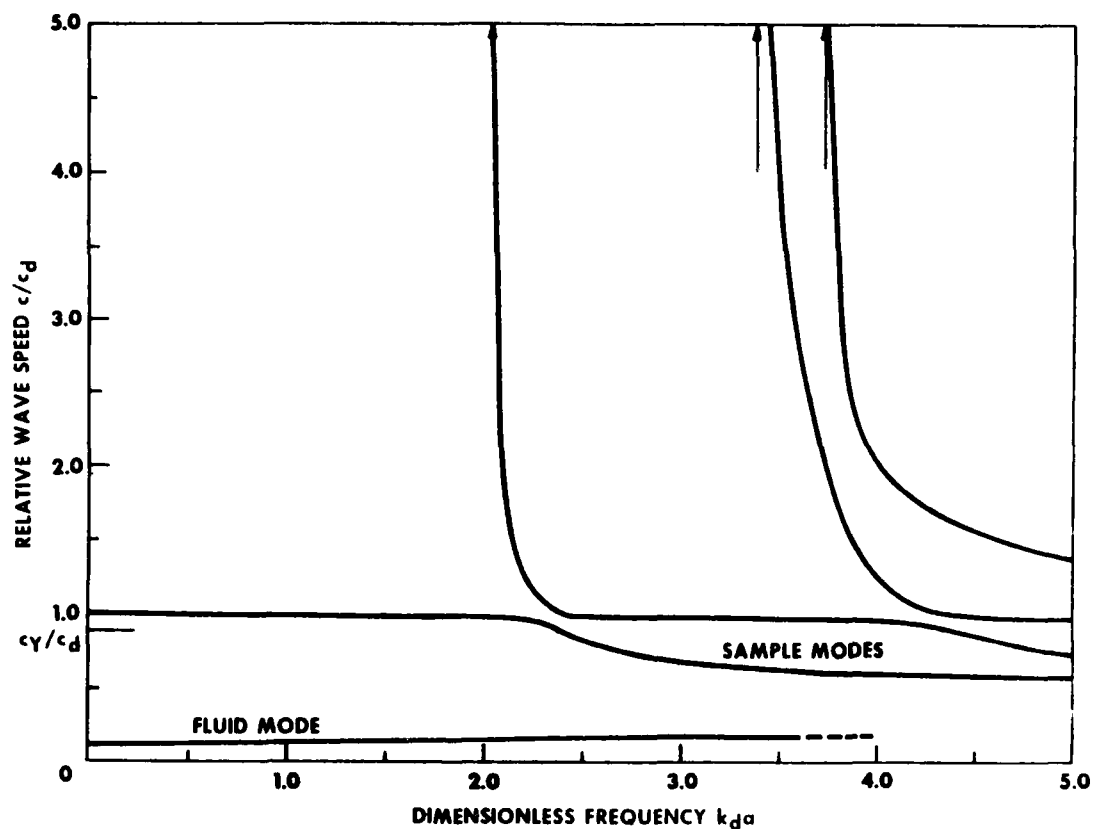


Fig. 7 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a steel cylinder in water bounded by a concentric rigid wall. Ratio of cylinder radii is 1.001. Vertical arrows indicate asymptotes; d_d is the dilatational wave speed, c_y is the bar speed.

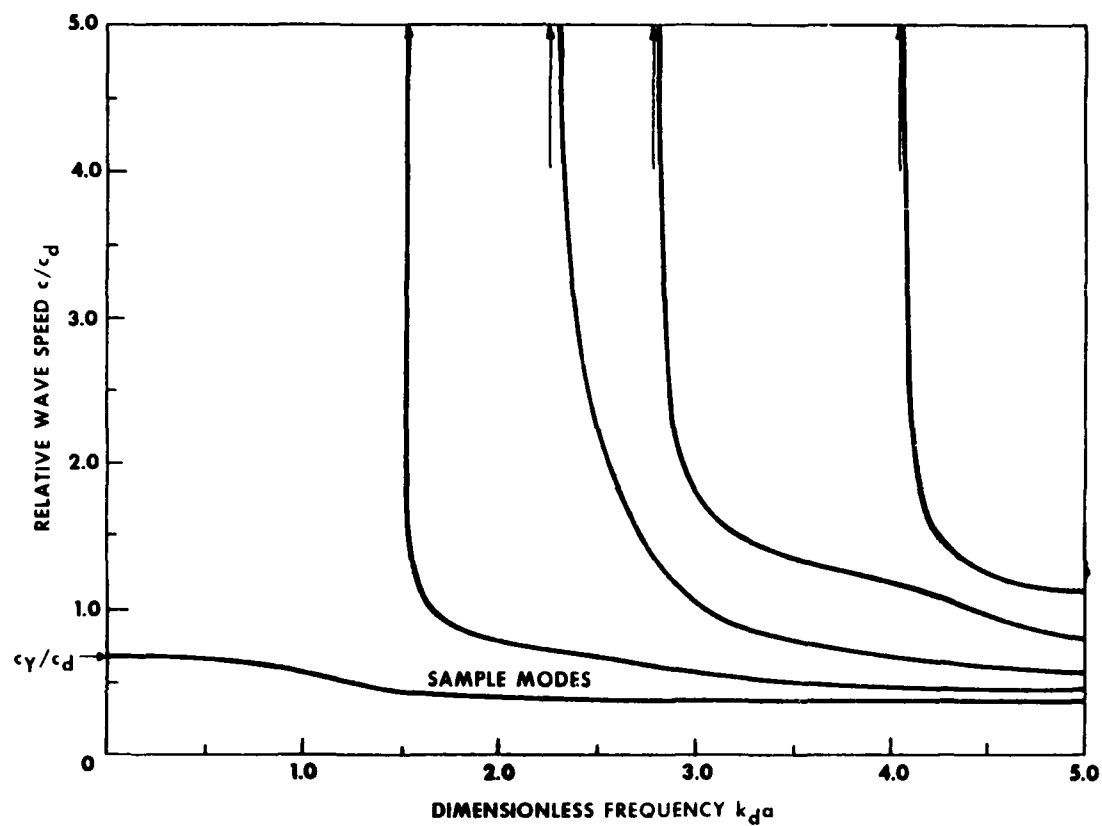


Fig. 8 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a rubber cylinder in vacuum. Vertical arrows indicate asymptotes; c_d is the dilatational wave speed, c_y is the bar speed.

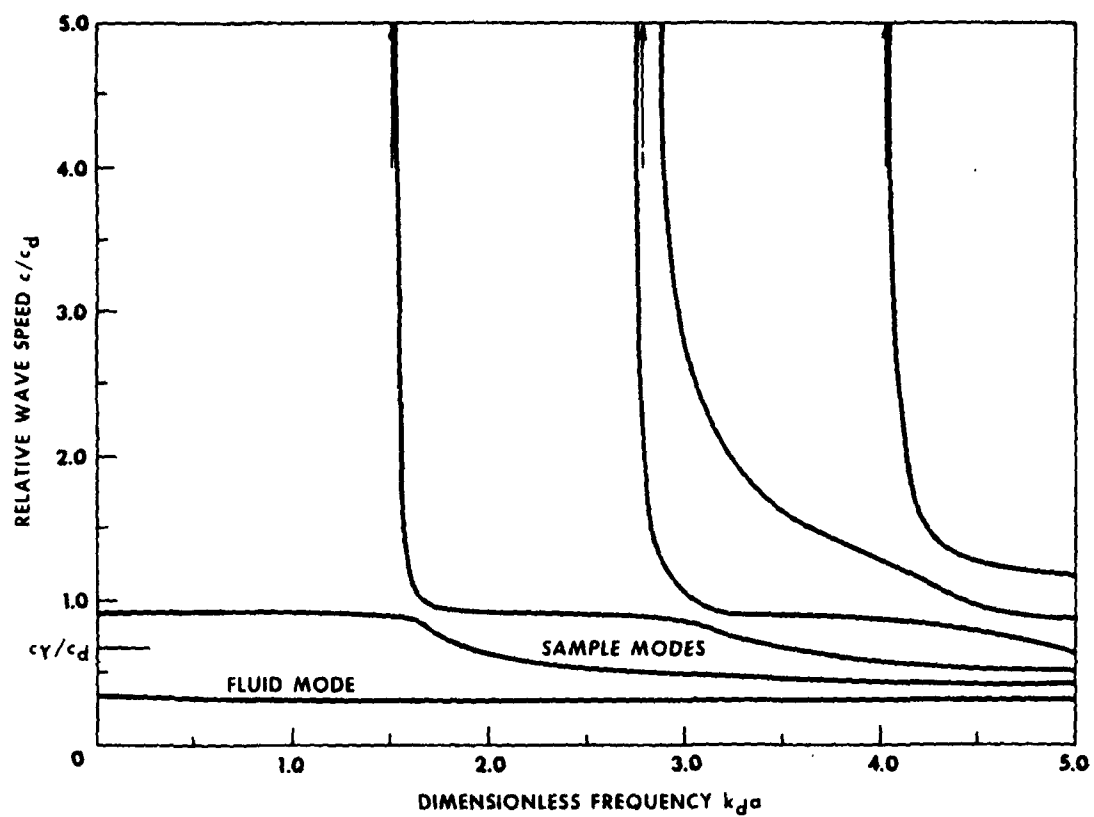


Fig. 9 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a rubber cylinder in water, bounded by a concentric rigid wall. Ratio of cylinder radii is 1.2. Vertical arrows indicate asymptotes; c_d is the dilatational wave speed, c_y is the bar speed.

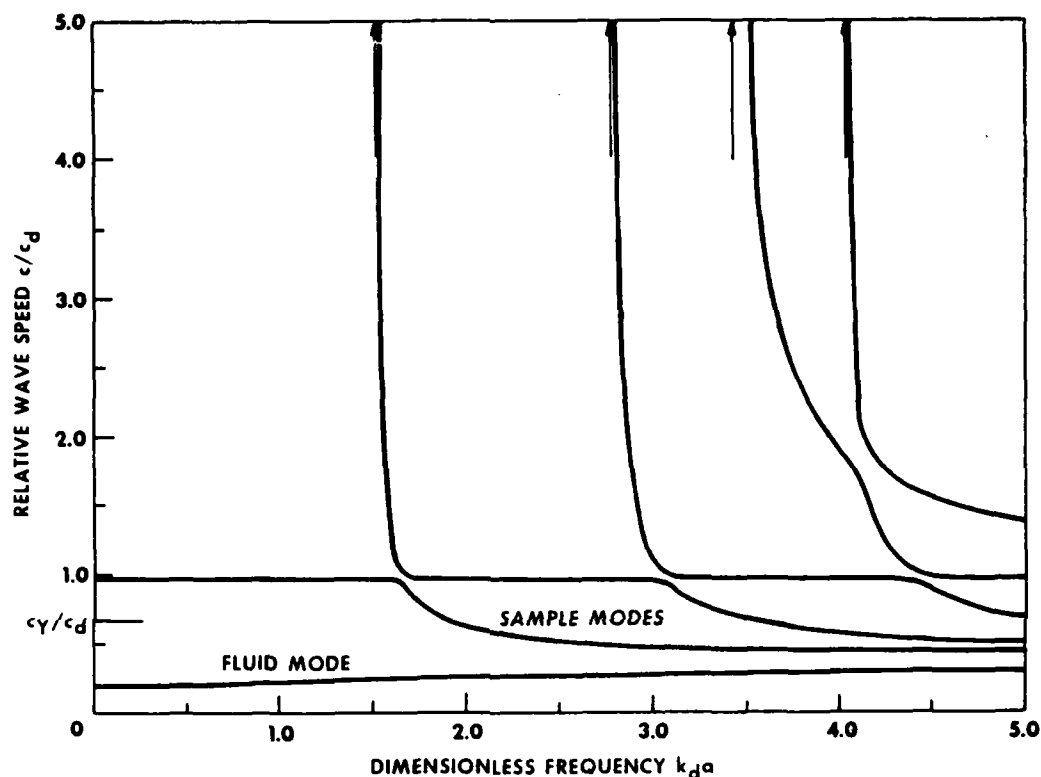


Fig. 10 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a rubber cylinder in water, bounded by a concentric rigid wall. Ratio of the cylinder radii is 1.05. Vertical arrows indicate asymptotes; c_d is the dilatational wave speed, c_y is the bar speed.

Generally, for elastomers, the measured wave speed is correctly identified with the dilatational wave speed since the compressibility and density of most rubbers are close to that of water, but this is not true for more rigid materials. A single measurement is not sufficient to determine the pair of elastic moduli needed to describe the material; by preventing the fill fluid to reach the gap between the sample and the tube or by varying the diameter of the sample, one may measure values for different combinations of the moduli from which the desired information may be extracted.

5.2. Calibrator

Two issues of interest were investigated with the WGUIDE program. In the first place, one may consider the influence of the walls on the sound speed in the water. One expects that the less rigid the wall the more the speed will

be reduced. As an example, the dispersion relation was computed for the case of the G40 calibrator developed at NRL-USRD. The following data apply: $a = 9.68$ cm, $b = 10.95$ cm, the material of the wall is aluminum with a density of 2700 kg/m^3 , shear modulus 2.4×10^{10} Pa, bulk modulus 7.5×10^{10} Pa and a dilatational wave speed of 6300 m/s (Ref. 15). The results are shown in Fig. 11. It is seen that at low frequency, ($k_d a = 0.1$, which corresponds to 1035 Hz), the ratio of the speed in the tube to that in water is 0.83 . Thus, there is a 17% reduction in speed due to the presence of the walls.

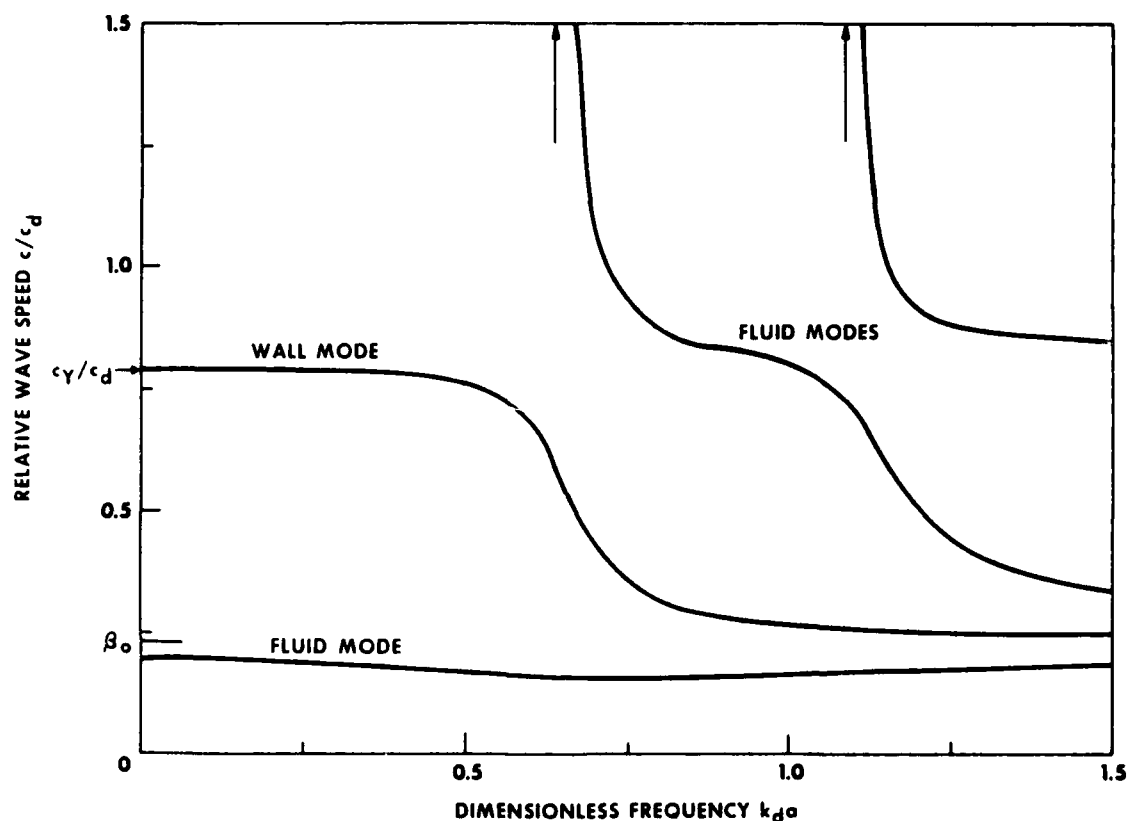


Fig. 11 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a water-filled tube of the same cross-sectional dimensions and material as the G40 calibrator. Ratio of the cylinder radii is 1.13. Vertical arrows indicate asymptotes; c_d is the dilatational wave speed, c_y is the bar speed, c_0 is the compressional wave speed in water.

The next question concerns the calibration of a hot-film particle motion hydrophone in the calibrator. In order to increase the particle velocity for a given pressure amplitude, one can reduce the propagation speed by choosing a less rigid wall material. Rubber might be too weak to support the water column, but a lucite wall might be realizable. Let the dimensions be the same as for the G40. The data for lucite are: density 1200 kg/m^3 , shear modulus 0.14×10^{10} Pa, bulk modulus 0.65×10^{10} , dilatational wave speed 2650 m/s . The speed in water for $k_d a = 0.1$ (1035 Hz) is reduced by a factor of 0.30 or a

speed of 440 m/s. Another possible way to reduce the wave speed in the tube is to insert a concentric layer of a suitable material inside the tube. The parameters influencing the reduction in wave speed are the elastic constants, density, and the ratio b/a . The results for a number of materials are shown in Table 6. Only polyethylene gives a sizable reduction in wave speed: a factor of 2. The data for glass, lead, rubber, and lucite were taken from Ref. 15, the data for polyethylene and polystyrene from Ref. 16.

Table 6 - Reduction in wave speed for various wall coatings in G40 calibrator (Frequency is 1035 Hz, $a = 7.0$ cm, $b = 9.8$ cm).

MATERIAL	G/K	c_o/c_d	ρ_o/ρ_s	c/c_o
Glass	0.641	0.265	0.434	0.99
Lead	0.131	0.722	0.088	0.86
Lucite	0.215	0.559	0.832	0.84
Polyethylene	0.085	0.760	0.890	0.54
Polystyrene	0.326	0.630	1.065	0.82
Rubber (hard)	0.200	0.617	0.907	0.79

Recent theoretical and experimental work in sound propagation in pipes with acoustic impedance comparable to that in water is found in Refs. 17 and 18.

For comparison with the literature, Fig. 12 shows the dispersion relation in a water-filled tube in addition to the curves for an empty tube, and a water column that has a rigid boundary and one that has a free boundary. The material of the wall is brass, as in Ref. 19, the results of which are reproduced in Ref. 20. The ratio of cylinder radii was chosen accordingly, $b/a = 1.13$. The material constants for brass are those given in Ref. 15. $\rho_s = 8500 \text{ kg/m}^3$, $G/K = 0.279$, and $c_d = 4700 \text{ m/s}$. The "longitudinal wave speed" is given in Ref. 19 as 3590 m/s which actually is closer to the bar speed, 3500 m/s , according to Ref. 16. The results in Fig. 12 show that the wave speed approaches the bar speed (subscript Y) for small $k_d a$, in disagreement with Ref. 19 but in accordance with Ref. 21.

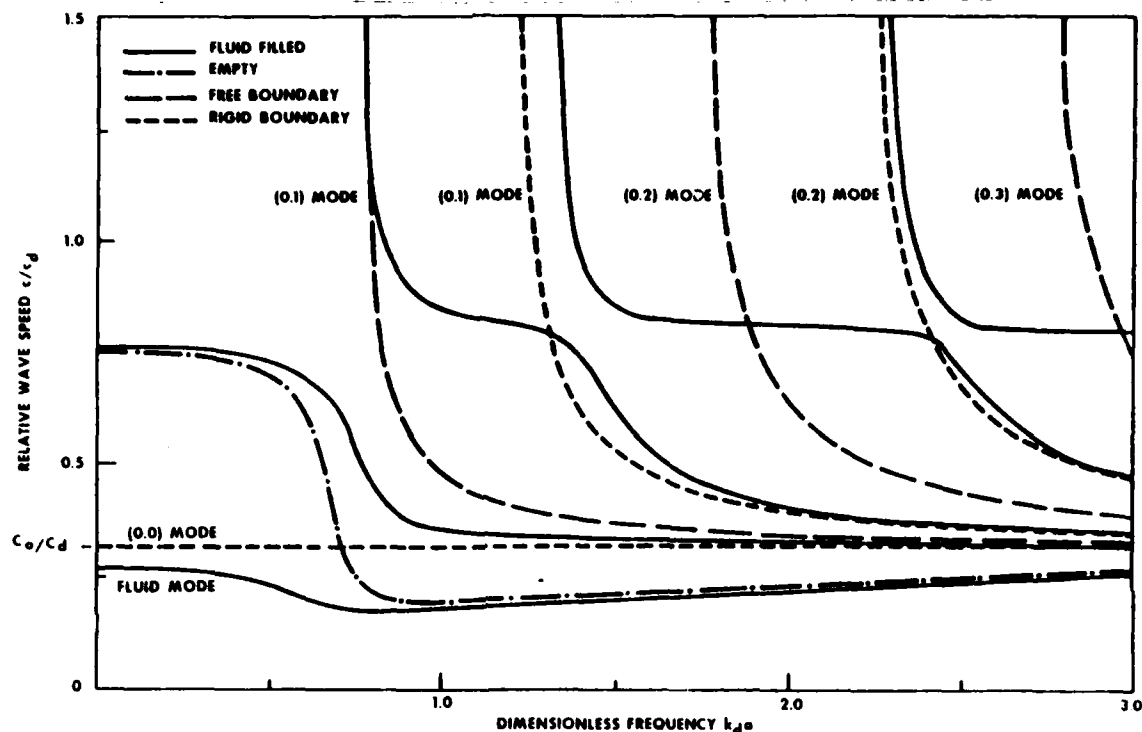


Fig. 12 - Relative wave speed c/c_d as a function of dimensionless frequency $k_d a$ for a water-filled brass tube, according to Ref. 21, with the wave speeds for the separate composing parts: tube and water column. The ratio of the cylinder radii is 1.13; c_d is the dilatational wave speed, c_y is the bar speed, and c_0 is the compressional speed in water. The mode designation for the fluid column follows the nomenclature of Ref. 6 by which the first entry gives the number of radial nodal lines, and the second the number of nodal circles.

6.0. CONCLUSIONS

The mathematical analysis and computer programs developed in this report may be used for determining spatial properties of the acoustic fields, including wave speed, in impedance tube and calibrator. Their application to the interpretation of data from these instruments is subject to the restrictive assumptions underlying the analysis. First, in relating the reflection coefficient to propagation speed in the impedance tube it is assumed that the radial variation of pressure and particle velocity is small or, in other words, that no other than the zero mode of the wave in the sample needs to be considered. In the second place, it is assumed throughout the

computation of wave speeds that the composite cylinders are infinite in the axial direction. This ignores the influence of the unavoidable termination in a finite cylinder on the wave pattern in the waveguide. The experimental efforts to terminate the waveguides by rigid reflectors or other idealized devices can ipso facto be only approximately successful. It is assumed that the idealization implied by the model of infinite cylinders will be more closely approached the larger the ratio of length to diameter of the sections of the waveguide.

Keeping this restriction in mind, one may conclude that it is not necessarily true that the wave speed measured by the impedance tube is the dilatational speed. It is rather a function of the thickness of the fluid-filled gap between sample and wall, in relation to the elastic properties of the sample material that determines the proper wave speed. As a consequence, one may measure different wave speeds by changing the radius of the sample, or by removing the fluid from the gap. Experimental study will have to decide in how far the finite length of the sample would interfere with the interpretation of the data.

REFERENCES

1. G. H. Sabin, "Acoustic Impedance Measurements at High Hydrostatic Pressures," J. Acoust. Soc. Am. 40, 1345-1353 (1966).
2. I. D. Groves Jr., "Twenty Years of Underwater Electroacoustic Standards," NRL Report 7735, 1974, pp. 61-65.
3. G. D. Hugus III and I. D. Groves, Jr., "Hydrophone Calibrator for Shipboard Use," J. Acoust. Soc. Am. 56, 70-74, (1974).
4. G. Beatty and J. F. Prandoni, "Underwater Sound Transducer Calibration Facility for the 10- to 4000-Hz Frequency Range at Hydrostatic Pressure to 10,000 psig," NRL Report 6965, August 1969.
5. H. J. Hebert and L. P. Browder, "Underwater Sound Transducer Calibration System for the 0.3- to 2000-Hz Frequency Range at Hydrostatic Pressure to 6.89 MPa.," NRL Report 7502, October 1972.
6. M. Redwood, *Mechanical Waveguides*, Pergamon Press, New York, 1960.
7. J. Miklowitz, *The Theory of Elastic Waves and Waveguides*, North Holland, Amsterdam, 1978.
8. L. E. Kinsler and A. R. Frey, *Fundamentals of Acoustics*, 2nd ed, Wiley, New York, 1962, p. 201.
9. I. S. Sokolnikoff, *Mathematical Theory of Elasticity*, McGraw Hill, 1956, pp. 183-184.
10. Ref. 6, Chapter 6.
11. D. Bancroft, "The Velocity of Longitudinal Waves in Cylindrical Bars," Phys Rev. 59, 588-593 (1941).

12. N. W. McLachlan, *Bessel Functions for Engineers*, Oxford University Press, 1955.
13. P. S. Dubbelday, "Complex Root-Finding Program with Application to the Dispersion Relation of Waves Propagating in a Fluid Plate," NRL Memorandum Report 4559, November 1981.
14. J. L. Lastinger and I. D. Groves, "Speed of Sound in Four Elastomers," NRL Memorandum Report 2363, December 1971.
15. Ref. 8, p. 502.
16. H. L. Anderson editor, *"Physics Vade Mecum,"* American Institute of Physics, NY (1981), p. 60.
17. R. A. Skop, "Sound Propagation Through Liquids in Viscoelastic Circular Cylinders", Shock-Vib. Bull. 51 Part 1, 217-223 (May 1981).
18. N. P. Horne and R. J. Hansen, "Sound propagation in a pipe containing a liquid of comparable acoustic impedance," J. Acoust. Soc. Am. 71, 1400 - 1405 (1982).
19. T. C. Lin and G. W. Morgan, "Wave propagation through fluid contained in a cylindrical elastic shell", J. Acoust. Soc. Am. 28, 1165-1176 (1956).
20. Ref. 6, p. 245.
21. I. Mirsky and G. Herrmann, "Axially Symmetric Motions of Thick Cylindrical Shells," J. Appl. Mech. 25, 97-102 (1958).

(BLANK PAGE)

APPENDIX A

Computer Programs

In this appendix the computer programs are described, and listings and examples are presented. An overview of the programs is given in Table A1.

Table A1 - Name and purpose of computer programs*

<u>NAME</u>	<u>TYPE</u>	<u>INPUT</u>	<u>OUTPUT</u>	<u>PURPOSE</u>
RTUBE	Complex	Reflection coefficient	Propagation speed	Impedance tube
WGUIDE Options 1 & 2	Real	G/K and c_o/c_d of sample	Propagation speed tube	Speed in sample in impedance
WGUIDE Options 3-7	Real	G/K and c_o/c_d of wall	Propagation speed	Speed in calibrator
IMPED	Complex	G/K and c_o/c_d of sample	Propagation speed	Speed in sample

* G, shear modulus; K, bulk modulus; c_o speed in water; c_d dilatational speed in solid.

DESCRIPTION OF PROGRAM RTUBE

The computer program RTUBE determines numerical values for the sound speed and attenuation of a sample in the acoustic impedance tube. The input to the program consists of the measured values of the amplitude and phase of the signal reflected by the sample and of the signal reflected without a sample in the tube, indicated by "standard". The program is written in FORTRAN IV and should run on any computer that accepts this language.

There are two major parts in RTUBE. First a real-valued seed or starting value for the sound speed is computed from the reactive part of the acoustic impedance.* This seed is entered into subroutine R0819, which contains the logic for the complex root search. The function, the roots of which are sought, is contained in subroutine DS.**

The real root finder determines the real part of (kd) corresponding to the reactive part X of the impedance Z in Eq. (2). according to the relation

$$\frac{X}{k_0 d} = -(\rho_s / \rho_0) \cot(kd) . \quad (A1)$$

Obviously, there is an infinite number of roots kd of this equation and one needs circumstantial evidence about the propagation speed in order to choose the correct one. In the iterative procedure used here, it is necessary to evaluate an inverse cotangent function which involves choosing the proper branch.

The propagation speed in the water is computed for a given temperature and pressure. The density of the fluid is 1.034 g/cm³, applicable to the glycol water mixture used in the experiment. For a different fill-fluid, this number should be changed accordingly.

Important Variables in RTUBE

Input Variables

AMAX	Maximum frequency (Hz)
AMIN	Minimum frequency (Hz)
DENS	ρ_s , density of material (g/cm ³)
PRES	p, pressure (MPa)

* C.M. Ruggiero, "Solution of Transcendental and Algebraic Equations with Application to Wave Propagation in Elastic Plates," NRL Memorandum Report 4449, November 1981.

** P.S. Dubbelday, "Complex Root-Finding Program with Application to the Dispersion Relation of Waves Propagating in a Fluid Plate," NRL Memorandum 4559, November 1981.

PSA	Phase of sample
PST	Phase of standard
STEP	Step size
TEMP	Temperature (°C)
THICK	Length of sample (cm)
VSA	Voltage of sample
VST	Voltage of standard

Computational Variables

A	$(\rho_o/\rho_s)(kd)$
ANS = ANS1	Counter, same as Z plus branch correction ($\pm n\pi$)
DIFF	Difference of last approximation and current approximation
E	Phase difference (in radians for signals with and without sample).
G	Phase difference corrected for absence of sample
I and X	Slopes computed at seed
J	Tolerance of relative error in root
J1	Number of frequencies
K	Counter
M	Reactance
N	Resistance
Q	$2 k_o d$, k_o is the wave number in water.
U	Number used in slope computation
V	Absolute value of reflection coefficient
W	Wave speed in water corrected for ambient temperature and pressure
Y	$\cot(kd)$

Output variables

AMGA Attenuation (Np/m)

FREQ Frequency (Hz)

SS Sound speed (m/s)

Major Computation Blocks in RTUBE, R0819

Descriptions of the major computation blocks in RTUBE are as follows:

<u>Computation Block</u>	<u>Line Number</u>	
	<u>From</u>	<u>To</u>
Input section	2	25
Slope computation	41	42
Iterative rootfinder	43	59
Output section (real roots)	64	66

Descriptions of the major computation blocks in R0819 are as follows:

<u>Computation Block</u>	<u>Line Number</u>	
	<u>From</u>	<u>To</u>
Set vertical pair	31	32
Test for initial location of vertical pair	39	40
Determination and movement of vertical test pairs	41 52	49 53
Check for iterations	62	63
Check for sign change in vertical pair	67	
Set horizontal pair	68	
Determination and movement of horizontal test pairs	74 86	75 88
Check for sign change in horizontal pair	102	
Check for desired accuracy	111	112
Reduce step size	114	

Horizontal vertical error (real & imaginary)	120	121
Advancing y value	124	
Output section	127	131


```

0044      IF (ABS(I) .LE. ABS(X))ASSIGN 100 TO JOB
0045      GO TO JOB
0046  95      Y=A* $\cos(ZNOT+PI*3.14159)$  / $\sin(ZNOT+PI*3.14159)$ 
0047      Z=Y/(-M)
0048      IF(K .GT. 1000)ASSIGN 100 TO JOB
0049      GO TO 110
0050  100     H=-M*(ZNOT+PI*3.14159)/A
0051      Z=ACOS(H/SQRT(1+H**2))
0052      IF(K .GT. 1000)ASSIGN 95 TO JOB
0053  110     DIFF=ZNOT-Z
0054      IF(ABS(DIFF) .LE. J) GO TO 200
0055      ZNOT=Z
0056      K=K+1
0057      IF(K .GT. 1001)ZNOT=ANS1
0058      IF (K .GT. 1001)K=1
0059      GOTO JOB
0060  200     ANS=Z+PI*3.14159
0061      IF(ANS1 .GT. (ANS))FI=1
0062      IF(ANS1 .GT. ANS)ZNOT=ANS1
0063      IF(ANS1 .GT. (ANS))GOTO JOB
0064      WRITE(5,210)ANS,M,N
0065  210     FORMAT(' REAL SEED, REACTANCE=, RESISTANCE= ',3F10.5)
0066      WRITE(1)FREQ,M,N,A,ANS
0067      ANS1=ANS
0068  500     CONTINUE
0069      CLOSE(UNIT=1)
0070      CALL R0819(J1,THICK)
0071      END

```

SUBROUTINE R0819(J1,THICK)

nnnnnnnnnnnnnn

```

0037      KUP=0
0038      KDOWN=0
0039      SIGN=AIMAG(F1)*REAL(F2)-REAL(F1)*AIMAG(F2)
0040      IF(SIGN)71,72,73
0041      71      FH1=-1.
0042              GO TO 74
0043      72      WRITE(5,302) SIGN
0044      302      FORMAT (' LEFT-RIGHT SIGN = ',E12.5)
0045              GO TO 92
0046      73      FH1=1.
0047      74      DELH=FH1*DEL
0048      75      Z1=Z1+DELH
0049              Z2=Z2+DELH
0050              F1=DS(Z1)
0051              F2=DS(Z2)
0052      SIGN=AIMAG(F1)*REAL(F2)-AIMAG(F2)*REAL(F1)
0053      IF (SIGN) 81,82,83
0054      81      FH2=-1.
0055              KL=KL+1
0056              IF (KAM-KL) 99,99,84
0057      99      WRITE(5,310)
0058      310      FORMAT (' EXIT LEFT')
0059              GO TO 100

0060      82      GO TO 92
0061      83      FH2 = 1.
0062              KR=KR+1
0063              IF(KAM-KR) 98,98,84
0064      98      WRITE (5,311)
0065      311      FORMAT(' EXIT RIGHT')
0066              GO TO 100
0067      84      IF (FH1*FH2) 92,75,75
0068      92      Z4=(Z1+Z2)/2.+DEL
0069      D      WRITE (5,886) KR,KL
0070      D886      FORMAT ('      KR=',I4,'      KL=',I4)
0071              KR=0
0072              KL=0
0073              Z3=(Z1+Z2)/2.-DEL
0074              F3=DS(Z3)
0075              F4=DS(Z4)
0076      SIGN=AIMAG(F3)*REAL(F4)-AIMAG(F4)*REAL(F3)
0077      IF (SIGN) 101,102,103
0078      101      FV1=1.
0079              GO TO 104
0080      102      WRITE (5,304) SIGN
0081              GO TO 122
0082      103      FV1=-1.
0083      104      DELV=FV1*DEL
0084      105      Z4=Z4+AI*DELV
0085              Z3=Z3+AI*DELV
0086              F3=DS(Z3)
0087              F4=DS(Z4)
0088      SIGN=AIMAG(F3)*REAL(F4)-AIMAG(F4)*REAL(F3)
0089      304      FORMAT(' UP-DOWN SIGN = ',E12.5)
0090      IF (SIGN) 111,112,113
0091      111      FV2 =1.

```

```

0090      KUP=KUP+1
0091      IF(KAM-KUP) 97,97,114
0092  97    WRITE(5,305)
0093  305   FORMAT (' EXIT UP')
0094      GO TO 100
0095  112   GO TO 122
0096  113   FV2=-1.
0097      KDOWN=KDOWN+1
0098      IF (KAM-KDOWN) 96,96,114
0099  96    WRITE(5,306)
0100  306   FORMAT(' EXIT DOWN')
0101      GO TO 100
0102  114   IF (FV1*FV2) 122,105,105
0103  122   REGA=REAL(Z1)
      D    WRITE(5,888) KUP,KDOWN
      D888  FORMAT (' KUP=',I4,' KDOWN=',I4)
0104      AMGA=AIMAG(Z3)
0105      KUP=0
0106      KDOWN=0
0107      IF (AMGA) 133,134,133
0108  134   WRITE (5,233)
0109  233   FORMAT(' AMGA IS ZERO')
0110      GO TO 131
0111  133   AMGA=ABS(AMGA)
0112      IF(DEL/AMGA-TOL)132,132,131
0113  131   CONTINUE
0114      DEL=DEL/REDF
0115      Z1=(Z3+Z4)/2.-AI*DEL
0116      Z2=(Z3+Z4)/2.+AI*DEL
0117      GO TO 65
0118  132   REGA=REAL (Z1) -DELH/2.
0119      AMGA=AIMAG(Z3)-DELU/2.
0120      DELH=ABS(DELH/2.)
0121      DELU=ABS(DELU/2.)
0122      IF(NR-NRV)145,145,146
0123  146   ZVAR=REGA+AMGA*AI
0124      RHO=RHO+RHM/ANR
0125      NRV=NRV+1
0126      GO TO 70
0127  145   WRITE (5,173)FREQ
0128  173   FORMAT(/,' SOUND SPEED AND ATTENUATION FOR FREQ =: ',F6.0)
0129      SS=(2*3.14159*FREQ)/REGA
0130      WRITE (5,174)SS,ABS(AMGA)
0131  174   FORMAT(2E15.5)
0132  10    CONTINUE
0133      CLOSE(UNIT=1)
0134  100   END

```

```

0001      FUNCTION DS(Z)
      C
      C      THIS IS A SPECIFIC FUNCTION TO BE CALCULATED WITH
      C      COMPLEX ROOTFINDER. THE COMPLEX ARGUMENT IS Z.
      C
      C      NOTE
      C      ALL DATA IS PASSED IN COMMON STATEMENT
0002      COMPLEX DS,Z,A11,A12,A22,ACOTZ,COTZ,SINZ,COSZ,CCOS,CSIN,S11,CMLX
0003      COMMON RHO,AKSD,THICK,A
0004      13      FORMAT(4F10.5,2E15.5)
0005      AI=CMPLX(0.0,1.0)
0006      A12= -THICK*Z
0007      A11=CMPLX(-AKSD,RHO)
0008      A22=A12*A11
0009      S11=THICK*Z
0010      COSZ=CCOS(S11)
0011      SINZ=CSIN(S11)
0012      COTZ=COSZ/SINZ
0013      ACOTZ=A*COTZ
0014      DS=ACOTZ+A22
0015      RETURN
0016      END

```

EXAMPLE:

```
RUN RTUBE <CR>
ENTER PRESSURE(MPA),LENGTH(CM),TEMP(C),DENSITY(G)
.61,12.09,23.9,1.13 <CR>

ENTER FREQUENCY (MIN,MAX,STEP)
4000,11000,1000 <CR>

FREQUENCY=      4000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.084,.077,269.91,287.41 <CR>
REAL SEED, REACTANCE=, RESISTANCE=      2.22014      0.77738      0.06983

FREQUENCY=      5000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.127,.112,216.6,237.94 <CR>
REAL SEED, REACTANCE=, RESISTANCE=      2.76703      2.61257      0.50517

FREQUENCY=      6000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.183,.148,176.49,192.52 <CR>
REAL SEED, REACTANCE=, RESISTANCE=      3.35705     -4.64150      4.21691

FREQUENCY=      7000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.231,.173,136.86,145.43 <CR>
REAL SEED, REACTANCE=, RESISTANCE=      3.77241     -1.44402      0.46431

FREQUENCY=      8000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.263,.177,101.86,111.03 <CR>
REAL SEED, REACTANCE=, RESISTANCE=      4.28325     -0.48575      0.24388

FREQUENCY=      9000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.308,.190,71.89,85.73 <CR>
REAL SEED, REACTANCE=, RESISTANCE=      4.79123      0.08435      0.23873

FREQUENCY=     10000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.338,.220,40.17,53.37 <CR>
REAL SEED, REACTANCE=, RESISTANCE=      5.24966      0.64459      0.30552

FREQUENCY=     11000.0

ENTER VOLT. STAND.,VOLT. SAMP, PHASE STAND.,PHASE SAMP
.339,.242,352.36,0.79 <CR>
REAL SEED, REACTANCE=, RESISTANCE=      5.72622      1.75329      0.75410
```

SOUND SPEED AND ATTENUATION FOR FREQ =:	4000.
0.13677E+04	0.45741E+00
SOUND SPEED AND ATTENUATION FOR FREQ =:	5000.
0.13658E+04	0.60050E+00
SOUND SPEED AND ATTENUATION FOR FREQ =:	6000.
0.13943E+04	0.88003E+00
SOUND SPEED AND ATTENUATION FOR FREQ =:	7000.
0.14196E+04	0.10834E+01
SOUND SPEED AND ATTENUATION FOR FREQ =:	8000.
0.14255E+04	0.14134E+01
SOUND SPEED AND ATTENUATION FOR FREQ =:	9000.
0.14290E+04	0.19029E+01
SOUND SPEED AND ATTENUATION FOR FREQ =:	10000.
0.14401E+04	0.19019E+01
SOUND SPEED AND ATTENUATION FOR FREQ =:	11000.
0.14429E+04	0.15798E+01

- 1) All underlined portions are user supplied.
- 2) <CR> indicates 'RETURN'

DESCRIPTION OF PROGRAM WGUIDE

The program WGUIDE computes the speed of axially symmetric waves propagating along the axial direction in composite infinite cylindrical waveguides, given the values of the ratio of shear modulus and bulk modulus of the solid and the ratio of the propagation speed in the fluid to the dilatational wave speed in the solid. To facilitate tracing the complete graph for the dispersion relation, including higher modes, two methods of scanning are possible. In the first one, the dimensionless frequency $k_d a$ (abscissa) is fixed and the program searches for all relative wave speeds (ordinate) between two limits. In the other method, the program steps through increasing or decreasing values for the dimensionless frequency and finds the smallest value for the relative wave speed that falls between limits, chosen such that the desired branch is bracketed. It is also possible to determine the vertical asymptotes. The application of the program is twofold: firstly, the program computes the wave speed in the sample inside an impedance tube which is assumed to be rigid and separated from the tube wall by a concentric layer of fluid; secondly, one may apply the program to assess the influence of nonrigidity of the wall for inertial calibrators.

The main program calls subroutine FUNC for Option numbers 1 through 5. FUNC, in turn, calls subroutine DI3 which contains the impedance tube dispersion relation, for Options 1 and 2. Number 1 assumes that the impedance tube wall is rigid, number 2 (rather unrealistically) assumes a free outer boundary. For Option numbers 3, 4, and 5, FUNC calls subroutine CALIB, which contains the dispersion relations for the inertial calibrator arrangement. Number 3 is for an elastic tube, fluid-filled or empty; number 4 is for a rigid tube with inner coating cemented to the wall; and number 5 is for a rigid tube with inner coating free to move tangentially with respect to the wall. If the Option number is 6 or 7, the main program calls subroutine FLUID; for number 6, it computes the wave speed for a fluid tube with rigid boundary. Note that the zero order is the compressional wave speed in the fluid. For the number 7, the wave speed in a fluid with free boundary (in this case there is a low-frequency cut-off) is calculated.

The program computes the location of vertical asymptotes by entering a negative value of AKDA. In Option 1, entering a negative value for RHM will prompt the computation of the wave speed in a solid cylinder in vacuum (indicated by "VACUUM TUBE" in the listing and printout). In Option 3, setting RHM equal to 0 will result in the computation of the wave speed in an empty cylindrical shell, indicated by "EMPTY TUBE" in print-out.

The Bessel function subroutines are documented in: System/360 Scientific subroutine Package Version II (c), International Business Machines Corporation 1966, 1967, 1968, White Plains, NY, Section - Special Operations and Functions, 363 - 367 (copies of these subroutines will not be found in this report; but the user may implement the needed Bessel function routines from the program library). Subroutine DET computes the determinant value of a matrix. The various results are stored by calling subroutine FILE, which is prompted by choosing Option number 9. The values of $\beta_Y = c_Y/c_d$, $\beta_s = c_s/c_d$, $\beta_R = c_R/c_d$, $\beta_o = c_o/c_d$, and the asymptotes are also stored in a

file separate from the dispersion results in order to discriminate between them. They can, however, appear on the same graph by running a plotting program named PLOTTER.*

FORTTRAN Variable Names for WGUIDE

Input Variables

KB: Option number determining mode of operation
 = 1 Sample in fluid tube with rigid boundary
 = 2 Sample in fluid tube with free boundary
 = 3 Fluid-filled elastic tube or empty tube
 = 4 Fluid-filled coated rigid tube (cemented)
 = 5 Fluid-filled coated rigid tube (not cemented)
 = 6 Fluid tube with rigid boundary
 = 7 Fluid tube with free boundary

GOK (G/K) ratio of shear modulus to bulk modulus

COCD (c_o/c_d) ratio of wave speed in fluid to dilatational wave speed in solid

RHM (ρ_o/ρ_s) ratio of fluid density to density of solid; negative value for solid cylinder in vacuum; zero for empty calibrator

AKDA ($k_d a$) dimensionless wave number of dilatational waves
 $k_d a = \frac{\omega a}{c_d}$

B1 or BAM Minimum ($k_d a$ if asymptote option, β otherwise)

B2 Maximum ($k_d a$ if asymptote option, β otherwise)

DB or DEL Step size of ($k_d a$ if asymptote option, β otherwise)

KO: Return option number
 = 1 program returns to entering input option number
 = 2 program returns to entering GOK
 = 3 program returns to entering COCD
 = 4 program returns to entering RHM
 = 5 program returns to entering MIN, MAX, STEP (Y value)
 = 6 program returns to entering BDA
 = 7 program returns to entering AKDA
 = 8 program returns to entering MIN, MAX, STEP (X value)
 = 9 stores data
 10 program exits

* C.M. Ruggiero and R.W. Anderson, "General Purpose High-Resolution Plotting Package for Tektronix 4662 Plotter and Compatible CRT Terminals," NRL Memorandum Report 4533, August 1981.

AKM Maximum $k_d a$
DKDA Step size of $k_d a$

Computational Variables

A Array that stores $k_d a$
B Array that stores β
D1 and D2, Values of the determinant
TOL Tolerance

Output Variables

AKF Asymptote at $k_d a$
ERR Error ($DEL/2$)
AKDA $k_d a$
BAF $\beta = c/c_d$, final value

Major Computation Blocks in WGUIDE

Descriptions of the major computation blocks in WGUIDE are as follows:

<u>Computation Block</u>	<u>Line Number</u>	
	<u>From</u>	<u>To</u>
Input Section	4	55
Computation of Asymptotes	61	78
Bisection Algorithm	89	111
Output Section	112	113

SOURCE LANGUAGE LISTING

```

C
C
C      WGUIDE
C      WRITTEN BY P. DUBBELDAY AND T. RUGGIERO
C
C      THE PURPOSE OF THIS PROGRAM IS TO CALCULATE THE SPEED
C      OF AXIAL WAVES IN CYLINDRICAL WAVE GUIDES.
C
C      C      SUBROUTINES: BESJ,BESY,BESK,DET,FILE,FUNC,INUE
C                  IO,DI3,CALIB,FLUID
C
0001      REAL A(500),B(500)
0002      COMMON AKA,AKDA,BAMS,ACC,COCD,BDA,RHM,KB,AA
0003      K9=0
0004      110      WRITE(5,120)
0005      120      FORMAT(' ENTER # CORRESPONDING TO OPTION DESIRED',
1/,'(1) SAMPLE IN FLUID TUBE WITH RIGID BOUNDARY',
2/,'(2) SAMPLE IN FLUID TUBE WITH FREE BOUNDARY',
3/,'(3) FLUID FILLED ELASTIC TUBE, OR EMPTY TUBE',
4/,'(4) FLUID FILLED COATED RIGID TUBE (CEMENTED)',
5/,'(5) FLUID FILLED COATED RIGID TUBE (NOT CEMENTED)',
6/,'(6) FLUID TUBE WITH RIGID BOUNDARY',
7/,'(7) FLUID TUBE WITH FREE BOUNDARY')
0006      READ (5,640) KB
0007      IF(KB .GE. 6)CALL FLUID(KB)
0008      TOL=.001
0009      ACC=.01
0010      130      WRITE (5,140)
0011      140      FORMAT(/,'$ENTER RATIO OF SHEAR TO BULK MODULUS (G/K) ')
0012      READ (5,170) GOK
0013      IF(K9 .EQ. 1)GO TO 300
0014      150      WRITE (5,160)
0015      160      FORMAT ('$ENTER RATIO OF FLUID TO SOLID SPEED (CO/CD) ')
0016      READ(5,170) COCD
0017      170      FORMAT (F15.0)
0018      IF(K9 .EQ. 1)GO TO 300
0019      190      WRITE(5,190)
0020      190      FORMAT(/,' ENTER RATIO OF FLUID TO SOLID DENSITY ',
1/,'$ (FOR EMPTY TUBE SET RHM=0,VACUUM TUBE SET RHM=NEG VALUE)
0021      READ(5,170)RHM
0022      IF(K9 .EQ. 1)GO TO 300
0023      BAMS=SQRT(GOK/(1.+4.*GOK/3.))
0024      KO=7
0025      BY=BAMS*3./SQRT(3.+GOK)
0026      POI=(3.-2.*GOK)/(6.+2.*GOK)
0027      BRE=(0.87+1.12*POI)*BAMS/(1.+POI)
0028      WRITE(5,200)BAMS,BY,BRE
0029      200      FORMAT(/,' SHEAR= ',E8.3,' YOUNG= ',E8.3,' RAYLEIGH= ',E8.3)
C
C      IN THE FIRST 3 STORAGE LOCATIONS ARE POINTS OF THE YOUNGS
C      MODULUS. IN THE 4TH LOCATION CO/CD IS STORED AT .1
0030      A(1)=0.5
0031      B(1)=COCD
0032      A(2)=1.
0033      B(2)=COCD
0034      A(3)=2.
0035      B(3)=COCD

```

```

0036      A(4)=.1
0037      B(4)=BY
      C
      C
      C      INPUT SECTION
0038      N=4
0039      210  WRITE(5,290)
0040      READ(5,260)AKDA
0041      IF(K9.EQ. 1)GOTO 300
0042      IF(AKDA .GE. 0)GO TO 230
0043      WRITE(5,220)
0044      220  FORMAT(/,'$ENTER MIN,MAX,STEP (HORIZONTAL SCAN) ')
0045      GO TO 250
0046      230  WRITE (5,240)
0047      240  FORMAT (/,'$ENTER MIN,MAX,STEP (VERTICAL SCAN) ')
0048      250  READ (5,260) B1,B2,DB
0049      260  FORMAT(3E15.0)
0050      IF(K9 .EQ. 1)GOTO 300
0051      270  WRITE (5,280)
0052      280  FORMAT (/,'$ENTER RATIO OF TUBE TO SAMPLE RADIUS (B/A) ')
0053      READ (5,170) BDA
0054      TYPE *, ' '
0055      290  FORMAT(/,'$ENTER X VALUE (SET KDA NEGATIVE FOR ASYMPTOTES) ')
0056      300  IF (AKDA) 310,440,440
0057      310  KO=8
0058      AKDA=B1
0059      320  DEL =DB
0060      N=N+1
      C
      C      FUNC IS A SUBROUTINE THAT CALLS THE CORRECT SUBROUTINE TO
      C      EVALUATE THE DETERMINANT. KB IS THE OPTION DESIRED.
      C
0061      CALL FUNC(-1,KB,D1)
0062      330  AKDA=AKDA +DEL
0063      CALL DECLAR
0064      340  CALL FUNC(-1,KB,D2)
0065      350  IF(D1*D2) 370,540,390
0066      360  GO TO 620
0067      370  IF (ABS(DEL/AKDA)-TOL) 420,380,380
0068      380  AKDA=AKDA-DEL
0069      DEL=DEL/10.
0070      GO TO 330
      C
      C      CHECKS IF X VALUE EXCEEDS MAXIMUM CHOSEN
      C
0071      390  IF (AKDA-B2) 330,400,400
0072      400  WRITE(5,410)
0073      410  FORMAT (NO MORE ASYMPTOTES ')
0074      GO TO 620
0075      420  ERR=DEL/2.
0076      ALF=AKDA-ERR
0077      WRITE (5,430) AKF,ERR
0078      430  FORMAT (ASYMPTOTE AT X = 'F10.5,' +/- 'E9.3)
      C
      C      THE ASYMPTOTE ARE FOUND AND POINTS STORED AT Y=1.25,1.5
      C

```

```

0079      A(N) = AKF
0080      B(N)=1.25
0081      N=N+1
0082      A(N)=AKF
0083      B(N)=1.5
0084      GO TO 320
0085      440      BAM=B1
0086      450      DEL=DB
0087      N=N+1
0088      CALL FUNC(BAM,KB,D1)
0089      460      BAM=BAM+DEL
0090      CALL DECLAR
0091      470      CALL FUNC(BAM,KB,D2)
0092      480      IF (D1*D2) 490,540,510
0093      490      IF (ABS(DEL/BAM)-TOL) 560,500,500
0094      500      BAM=BAM-DEL
0095      DEL=DEL/10.
0096      GO TO 460
0097      510      IF (BAM-B2) 460,520,520
0098      520      WRITE (5,530)
0099      530      FORMAT (' NO MORE ROOTS')
0100      GO TO 620

      C
      C      PRINTED WHEN EITHER DETERMINANT IS EQUAL TO ZERO
0101      540      WRITE (5,550)
0102      550      FORMAT (' D1*D2=ZERO')
0103      GO TO 620
0104      560      ERR=DEL/2.
0105      BAF=BAM-ERR
0106      570      IF(KO .NE. 7)GO TO 600
0107      580      WRITE (5,590) BAF,ERR
0108      590      FORMAT(' Y = ',F10.5,' ERROR = ',E9.2 )
0109      A(N)=AKDA
0110      B(N)=BAF
0111      GO TO 450
0112      600      WRITE (5,610) AKDA,BAF,ERR
0113      610      FORMAT(' X = ',F10.5,' Y = ',F10.5,' ERROR = ',E9.3)
0114      A(N)=AKDA
0115      B(N)=BAF
0116      AKDA=AKDA+DKDA
0117      IF ((AKM-AKDA)*DKDA) 620,440,440
0118      620      K9=1
      C      TO RECYCLE BACK OR EXIT CHOSE OPTION #
0119      WRITE(5,630)
0120      630      FORMAT(/,' ENTER # CORRESPONDING TO CHANGE '
1/, ' 1) OPTION'
2/, ' 2) SHEAR MODULUS/BULK MODULUS (G/K) '
3/, ' 3) SPEED IN FLUID/SPEED IN SOLID (CO/CD)'
4/, ' 4) FLUID DENSITY/SOLID DENSITY (RHM) '
5/, ' 5) MIN MAX, STEP (Y VALUES)'
6/, ' 6) TUBE RADIUS/SAMPLE RADIUS (B/A) '
7/, ' 7) X VALUE (KDA)'
8/, ' 8) MIN,MAX,STEP (X VALUES)'
9/, ' 9) STORE DATA'
9/, ' 10) EXIT FROM PROGRAM')
0121      N=N-1

0122      READ (5,640) KO
0123      640      FORMAT (I4)
0124      GO TO (110,130,150,180,230,270,210,660,650,690) KO
0125      650      CALL FILE(N,A,B)
0126      GO TO 620
0127      660      WRITE (5,670)
0128      670      FORMAT(/,'$ENTER X VALUES: START(KNOWN),END,STEP(+/-) ')
0129      READ (5,680) AKDA,AKM,DKDA
0130      680      FORMAT (3F15.0)
0131      GO TO 230
0132      690      CONTINUE
0133      END

```

SUBROUTINES

SUBROUTINE: DET

```

C
C      SUBROUTINE DET
C      DECEMBER, 1980
C      EDITED BY TINA RUGGIERO
C
0001      SUBROUTINE DET(N,A,D)
C
C      THIS SUBROUTINE USES THE METHOD OF GAUSSIAN ELIMINATION
C      TO CALCULATE THE DETERMINANT
C
0002      DOUBLE PRECISION A(5,5),B(5),X(5),D,SUM,C
0003      DIMENSION JPRM(5)
0004      D=1.0D 00
0005      DO 13 I=1,N
0006      X(I)=0.0D 00
0007      JPRM(I)=I
C
C      FIND THE ELEMENT OF MAXIMUM ABSOLUTE VALUE
C
0008      DO 1 K=1,N
0009      C=A(K,K)
0010      II=K
0011      JJ=K
0012      DO 2 J=K,N
0013      DO 2 I=K,N
0014      IF (DABS(C)-DABS(A(I,J)))3,2,2
0015      3 C=A(I,J)
0016      II=I
0017      JJ=J
0018      2 CONTINUE
0019      D=D*C
0020      IF(DABS(D))20,20,30
0021      20 WRITE(5,100)
0022      100 FORMAT(' MATRIX A(N,N) IS SINGULAR')
0023      D=0.0
0024      RETURN
0025      30 B(II)=B(II)/C
0026      KPD=K+1
C
C      DIVIDE EACH ELEMENT OF THE II TH ROW BY C
C
0027      DO 4 J=K,N
0028      4 A(II,J)=A(II,J)/C
0029      A(II,JJ)=1.0D 00
0030      IF (II.EQ.K)GO TO 60
C
C      SWITCH THE KTH ROW AND THE II TH ROW

```

```

C
0031      D=-D
0032      DO 5 J=K,N
0033      C=A(K,J)
0034      A(K,J)=A(II,J)
0035      5  A(II,J)=C
0036      C=B(K)
0037      B(K)=B(II)
0038      B(II)=C
0039      60 IF (JJ.EQ.K)GO TO 70

C
C      STORE THE LOCATION OF THE MAX PIVOT
C

0040      II=JPRM(JJ)
0041      JPRM(JJ)=JPRM(K)
0042      JPRM(K)=II

C
C      SWITCH THE KTH AND THE JJTH COLUMNS
C

0043      D=-D
0044      DO 6 I=1,N
0045      C=A(I,K)
0046      A(I,K)=A(I,JJ)
0047      6  A(I,JJ)=C
0048      70 IF (K.EQ.N)GO TO 1
0049      DO 7 I=KPO,N
0050      DO 8 J=KPO,N
0051      8  A(I,J)=A(I,J)-A(I,K)*A(K,J)
0052      B(I)=B(I)-A(I,K)*B(K)
0053      7  A(I,K)=0.0D 00
0054      1  CONTINUE
0055      END

```

SUBROUTINE: DI3

```

0001      SUBROUTINE DI3(BAM,ANS)
>
      C
      C      THIS SUBROUTINE COMPUTES 2 OPTIONS.
      C      OPTION 1 IS SOLID-FLUID-RIGID BOUNDARIES
      C      OPTION 2 IS SOLID-FLUID-FREE BOUNDARIES
      C
0002      COMMON AKA,AKDA,BAMS,ACC,COCD,BDA,RHM,KB
0003      IF (BAM) 31,32,32
0004      31      AKA=0.
0005              GO TO 33
0006      32      AK4=AKDA/BAM
0007      33      AKSA=AKDA/BAMS
0008              Q2A=AKA**2-AKDA**2
0009              S2A=AKA**2-AKSA**2
0010              IF (RHM) 65,65,40
0011      40      AKFA=AKDA/COCD
0012              QF2A=AKA**2-AKFA**2
0013              IF (QF2A) 61,60,60
0014      60      QFA=SQRT(QF2A)
0015              QFB=BDA*QFA

      C      COMPUTES BESSEL FUNCTIONS
0016      CALL BESK(QFA,0,YOQ,IER)
0017      CALL BESK(QFA,1,Y1Q,IER)
0018      CALL BESK(QFB,0,YOQB,IER)
0019      CALL BESK(QFB,1,Y1QB,IER)
0020      CALL IO(QFA,BFOQ)
0021      CALL IO(QFB,BFOQB)
0022      CALL INUE(QFA,1,BFOQ,BF1Q)
0023      CALL INUE(QFB,1,BFOQB,B1QB)
0024      QFA=-QFA
0025      QFB=-QFB
0026      YOQ=-YOQ
0027      YOQB=-YOQB
0028      GO TO 65
0029      61      QFA=SQRT(-QF2A)
0030              QFB=BDA*QFA
0031      CALL BESJ(QFA,0,BFOQ,ACC,IER)
0032      CALL BESJ(QFA,1,BF1Q,ACC,IER)
0033      CALL BESJ(QFB,0,BFOQB,ACC,IER)
0034      CALL BESJ(QFB,1,B1QB,ACC,IER)
0035      CALL BESY(QFA,0,YOQ,IER)
0036      CALL BESY(QFA,1,Y1Q,IER)
0037      CALL BESY(QFB,0,YOQB,IER)
0038      CALL BESY(QFB,1,Y1QB,IER)
0039      65      IF (Q2A) 81,80,80
0040      80      QA=SQRT(Q2A)
0041      CALL IO(QA,BOQ)
0042      CALL INUE(QA,1,BOQ,B1Q)
0043      QA=-QA
0044      GO TO 91
0045      81      QA=SQRT(-Q2A)
0046      CALL BESJ(QA,0,BOQ,ACC,IER)
      D      WRITE (5,85) IER
0047      85      FORMAT (' IER =: ',I2)
0048      CALL BESJ(QA,1,B1Q,ACC,IER)

```

```

0049      D      WRITE (5,85) IER
0050      91      IF (S2A) 71,70,70
0051      70      SA=SQRT(S2A)
0052      CALL IO(SA,BOS)
0053      CALL INUE(SA,1,BOS,B1S)
0054      GO TO 101
0055      71      SA=SQRT(-S2A)
0056      CALL BESJ(SA,0,BOS,ACC,IER)
0057      D      WRITE (5,85) IER
0058      101      CALL BESJ(SA,1,B1S,ACC,IER)
0059      D      WRITE (5,85) IER
0060      101      D11=B0Q*(2.*AKA**2-AKSA**2)+B1Q*2.*QA
0061      D12=2.*AKA*(B1S-SA*BOS)
0062      D21=2.*AKA*QA*B1Q
0063      D22=B1S*(2.*AKA**2-AKSA**2)
0064      IF (RHM) 21,21,22
0065      21      ANS=D11*D22-D12*D21
0066      GO TO 11
0067      22      D13=AKSA**2*RHM*BF0Q
0068      D14=AKSA**2*RHM*Y0Q
0069      D31=QA*B1Q
0070      D32=AKA*B1S
0071      D33=-QFA*BF1Q
0072      D34=-QFA*Y1Q
0073      IF (KB-1) 75,75,76
0074      76      D43=BF0QB
0075      D44=Y0QB
0076      GO TO 78
0077      75      D43=QFB*B1QB
0078      D44=QFB*Y1QB
0079      78      P12=D11*D22-D12*D21
0080      P34=D33*D44-D34*D43
0081      P23=D21*D32-D22*D31
0082      P14=D13*D44-D14*D43
0083      ANS=P12*P34+P23*P14
0084      RETURN
0085      END

```

SUBROUTINE: CALIB

```

0001      C      SUBROUTINE CALIB(BAM,ANS)
          C      THIS SUBROUTINE CORRESPONDS TO THE OPTIONS 3,4, AND 5.
          C      OPTION 3 IS WHERE FLUID SOLID AND FREE BOUNDARIES IS ANALYZED
          C      OPTION 4 IS WHERE FLUID-SOLID-RIGID BOUNDARIES(CEMENTED)
          C      OPTION 5 IS WHERE FLUID-SOLID-RIGID(NOT CEMENTED)
          C      IS ANALYZED.
0002      DOUBLE PRECISION AA(5,5)
0003      COMMON AKA,AKDA,BAMS,ACC,COCD,BDA,RHM,KB,AA
0004      IF(BAM)31,32,32
0005      31      AKA=0
0006      GO TO 33
0007      32      AKA=AKDA/BAM
0008      33      AKSA=AKDA/BAMS
0009      Q2A=AKA**2-AKDA**2
0010      S2A=AKA**2-AKSA**2
0011      AKFA=AKDA/COCD
0012      QF2A=AKA**2-AKFA**2
0013      AKB=AKA*BDA
0014      AKSB=AKSA*BDA
0015      Q2B=Q2A*BDA*BDA
0016      S2B=S2A*BDA*BDA
0017      IF(QF2A)61,60,60
0018      60      QFA=SQRT(QF2A)
          C      BESSEL FUNCTIONS ARE COMPUTED HERE
0019      CALL IO(QFA,BF0Q)
0020      CALL INUE(QFA,1,BF0Q,BF1Q)
0021      QFA=-QFA
0022      GO TO 65
0023      61      QFA=SQRT(-QF2A)
0024      CALL BESJ(QFA,0,BF0Q,ACC,IER)
0025      CALL BESJ(QFA,1,BF1Q,ACC,IER)
0026      65      IF(Q2A)81,80,80
0027      80      QA=SQRT(Q2A)
0028      QB=SQRT(Q2B)
0029      CALL IO(QA,B0Q)
0030      CALL IO(QB,BB0Q)
0031      CALL INUE(QA,1,B0Q,B1Q)
0032      CALL INUE(QB,1,BB0Q,BB1Q)
0033      CALL BESK(QA,0,B0A,IER)
0034      CALL BESK(QB,0,BB0A,IER)
0035      CALL BESK(QA,1,B1A,IER)
0036      CALL BESK(QB,1,BB1A,IER)
0037      QA=-QA
0038      QB=-QB
0039      B0A=-B0A
0040      BB0A=-BB0A
0041      GO TO 91
0042      81      QA=SQRT(-Q2A)
0043      QB=SQRT(-Q2B)
0044      CALL BESJ(QA,0,B0Q,ACC,IER)
0045      CALL BESJ(QB,0,BB0Q,ACC,IER)
0046      CALL BESJ(QA,1,B1Q,ACC,IER)
0047      CALL BESJ(QB,1,BB1Q,ACC,IER)
0048      CALL BESY(QA,0,B0A,IER)
0049      CALL BESY(QB,0,BB0A,IER)

```

```

0050      CALL BESY(QA,1,B1A,IER)
0051      CALL BESY(QB,1,B1B,IER)
0052      91      IF(S2A)71,70,70
0053      70      SA=SQRT(S2A)
0054      SB=SQRT(S2B)
0055      CALL IO(SA,BOS)
0056      CALL IO(SB,BBOS)
0057      CALL INUE(SA,1,BOS,B1S)
0058      CALL INUE(SB,1,BBOS,BB1S)
0059      CALL BESK(SA,0,BSA0,IER)
0060      CALL BESK(SB,0,BSB0,IER)
0061      CALL BESK(SA,1,BSA1,IER)
0062      75      CALL BESK(SB,1,BSB1,IER)
0063      77      BSA1=-BSA1
0064      BSB1=-BSB1
0065      GOTO 101
0066      71      SA=SQRT(-S2A)
0067      SB=SQRT(-S2B)
0068      CALL BESJ(SA,0,BOS,ACC,IER)
0069      CALL BESJ(SB,0,BBOS,ACC,IER)
0070      CALL BESJ(SA,1,B1S,ACC,IER)
0071      CALL BESJ(SB,1,BB1S,ACC,IER)
0072      CALL BESY(SA,0,BSA0,IER)
0073      CALL BESY(SB,0,BSB0,IER)
0074      CALL BESY(SA,1,BSA1,IER)
0075      85      CALL BESY(SB,1,BSB1,IER)
0076      87      CONTINUE
          C      THE DETERMINANT IS BEING FILLED
0077      101     AA(1,1)=B0Q*(2.*AKA**2-AKSA**2)+2.*(QA*B1Q)
0078      AA(1,2)=(2.*AKA**2-AKSA**2)*B0A+2.*QA*B1A
0079      AA(1,3)=2.*AKA*(B1S-SA*BOS)
0080      AA(1,4)=2.*AKA*(BSA1-SA*BSA0)*SA
0081      AA(1,5)=RHM*AKSA*AKSA*BF0Q
0082      AA(2,1)=2.*AKA*QA*B1Q
0083      AA(2,2)=2.*AKA*QA*B1A
0084      AA(2,3)=(2.*AKA*AKA-AKSA*AKSA)*B1S
0085      AA(2,4)=(2.*AKA*AKA-AKSA*AKSA)*BSA1*SA
0086      AA(2,5)=0.0
0087      IF(KB .EQ. 3)GO TO 120
0088      AA(3,1)=BB1Q*QB
0089      AA(3,2)=B1B*QB
0090      AA(3,3)=AKB*BB1S
0091      AA(3,4)=AKB*BSB1*SA
0092      GO TO 150
0093      120     AA(3,1)=BB0Q*(2.*AKB**2-AKSB**2)+2.*QB*BB1Q
0094      AA(3,2)=(2.*AKB*AKB-AKSB*AKSB)*B0B+2.*QB*B1B
0095      AA(3,3)=2.*AKB*(BB1S-SB*BBOS)
0096      AA(3,4)=2.*AKB*(BSB1-SB*BSB0)*SA
0097      150     AA(3,5)=0.0
0098      IF(KB .NE. 4)GO TO 160
0099      AA(4,1)=AKB*BB0Q
0100      AA(4,2)=AKB*B0B
0101      AA(4,3)=-SB*BBOS
0102      AA(4,4)=-SB*BSB0*SA
0103      GO TO 170
0104      160     AA(4,1)=2.*AKB*QB*BB1Q
0105      AA(4,2)=2.*AKB*QB*BB1B
0106      AA(4,3)=(2.*AKB*AKB-AKSB*AKSB)*BB1S
0107      AA(4,4)=(2.*AKB*AKB-AKSB*AKSB)*BSB1*SA
0108      170     AA(4,5)=0.0
0109      AA(5,1)=QA*B1Q
0110      AA(5,2)=QA*B1A
0111      AA(5,3)=AKA*B1S
0112      AA(5,4)=AKA*BSA1*SA
0113      AA(5,5)=-QFA*BF1Q
0114      IF (RHM .EQ. 0.0)AA(5,5)=1.0
0115      CALL DET(5,AA,ANS)
0116      RETURN
0117      END

```

SUBROUTINE: FILE

```

0001      SUBROUTINE FILE(N,FREQ,RATIO)
>
0002      BYTE FILEX(32)
0003      REAL FREQ(1000),RATIO(1000)
0004      WRITE(5,10)
0005      10  FORMAT(/,'FILE NAME FOR X,Y DATA:  ')
0006      READ(5,20)LEN,FILEX
0007      20  FORMAT(Q,32A1)
0008      FILEX(LEN + 1)=0
0009      OPEN(UNIT=2,NAME=FILEX,TYPE='NEW')
0010      DO 40 J=1,N
0011          WRITE(2,30)FREQ(J),RATIO(J)
0012      30  FORMAT(2E20.10)
0013      40  CONTINUE
0014      CLOSE(UNIT=2)
0015      CONTINUE
0016      WRITE(5,45)
0017      45  FORMAT(/,'FILE IS STORED IN 1 FILE WITH FORMAT (2E20.0) ')
0018      RETURN
0019      END

```

SUBROUTINE: FUNC

0001		SUBROUTINE FUNC(B,IOP,ANS)
0002		IF(IOP .GE. 3)GO TO 20
0003		CALL DI3(B,ANS)
0004		GO TO 50
0005	20	CALL CALIB(B,ANS)
0006	50	RETURN
0007		END

TEST METHODS AND RESULTS

```

RUN WGUIDE <CR>
ENTER # CORRESPONDING TO OPTION DESIRED
1) SAMPLE IN FLUID TUBE WITH RIGID BOUNDARY
2) SAMPLE IN FLUID TUBE WITH FREE BOUNDARY
3) FLUID FILLED ELASTIC TUBE, OR EMPTY TUBE
4) FLUID FILLED COATED RIGID TUBE (CEMENTED)
5) FLUID FILLED COATED RIGID TUBE (NOT CEMENTED)
6) FLUID TUBE WITH RIGID BOUNDARY
7) FLUID TUBE WITH FREE BOUNDARY
3 <CR>

ENTER RATIO OF SHEAR TO BULK MODULUS (G/K) .379 <CR>
ENTER RATIO OF FLUID TO SOLID SPEED (CO/CD) .315 <CR>

ENTER RATIO OF FLUID TO SOLID DENSITY
(FOR EMPTY TUBE SET RHM=0,VACUUM TUBE SET RHM=NEG VALUE) 0 <CR>

SHEAR= .451E+00 YOUNG= .747E+00 RAYLEIGH= .423E+00

ENTER X VALUE (SET KDA NEGATIVE FOR ASYMPTOTES) -1 <CR>

ENTER MIN,MAX,STEP (HORIZONTAL SCAN) .1,1.,.05 <CR>

ENTER RATIO OF TUBE TO SAMPLE RADIUS (B/A) 1.134 <CR>

ASYMPTOTE AT X = 0.75625 +/- 0.250E-03
NO MORE ASYMPTOTES

ENTER # CORRESPONDING TO CHANGE
1) OPTION
2) SHEAR MODULUS/BULK MODULUS (G/K)
3) SPEED IN FLUID/SPEED IN SOLID (CO/CD)
4) FLUID DENSITY/SOLID DENSITY (RHM)
5) MIN MAX, STEP (Y VALUES)
6) TUBE RADIUS/SAMPLE RADIUS (B/A)
7) X VALUE (KDA)
9) MIN,MAX,STEP (X VALUES)
9) STORE DATA
10) EXIT FROM PROGRAM
8 <CR>

ENTER X VALUES: START(KNOWN),END,STEP(+/-) .1,2.5,.5 <CR>

ENTER MIN,MAX,STEP (VERTICAL SCAN) .1,1.5,.1
X = 0.10000 Y = 0.74605 ERROR = 0.500E-04
X = 0.60000 Y = 0.63565 ERROR = 0.500E-04
X = 1.10000 Y = 0.19835 ERROR = 0.500E-04
X = 1.60000 Y = 0.21645 ERROR = 0.500E-04
X = 2.10000 Y = 0.23655 ERROR = 0.500E-04

ENTER # CORRESPONDING TO CHANGE
1) OPTION
2) SHEAR MODULUS/BULK MODULUS (G/K)
3) SPEED IN FLUID/SPEED IN SOLID (CO/CD)
4) FLUID DENSITY/SOLID DENSITY (RHM)
5) MIN MAX, STEP (Y VALUES)
6) TUBE RADIUS/SAMPLE RADIUS (B/A)
7) X VALUE (KDA)
9) MIN,MAX,STEP (X VALUES)
9) STORE DATA
10) EXIT FROM PROGRAM
9 <CR>

```

\$FILE NAME FOR X,Y DATA:
DATA <CR>

ENTER # CORRESPONDING TO CHANGE

- 1) OPTION
- 2) SHEAR MODULUS/BULK MODULUS (G/K)
- 3) SPEED IN FLUID/SPEED IN SOLID (CO/CD)
- 4) FLUID DENSITY/SOLID DENSITY (RHM)
- 5) MIN MAX, STEP (Y VALUES)
- 6) TUBE RADIUS/SAMPLE RADIUS (B/A)
- 7) X VALUE (KDA)
- 8) MIN,MAX,STEP (X VALUES)
- 9) STORE DATA
- 10) EXIT FROM PROGRAM
- 10 <CR>

- 1) All underlined portions are user supplied.
- 2) <CR> indicates 'RETURN'

DESCRIPTION OF PROGRAM IMPED

The program IMPED computes values for the complex dimensionless propagation speed in the solid sample (assumed infinite in length) inside the impedance tube. Input to the program are the complex values for the ratio of shear modulus to bulk modulus G/K , and the speed in the fluid divided by the dilatational wave speed in the solid, c_o/c_d . Thus, it may be considered as the complex counterpart to program WGUIDE, Option 1. The root finding technique is identical to that used in program RTUBE.*

Subroutines for Bessel functions were written by direct series expansion since the real part of the argument is small enough that a small number of terms suffices. The names of the subroutines are CBJO, CBJ1, CBYO, and CBY1, where the last two letters of each name indicate kind and order of the Bessel function.

The output produced by the program is in the form of real and imaginary parts of the relative wave speed $\beta = c/c_d$, each with its own error limit, corresponding to the input error limits set separately for the two terms. The attenuation constant may be readily calculated by $\alpha = -k \beta_2 / (\beta_1^2 + \beta_2^2)$ where α is the attenuation per unit of length and β_1 and β_2 are the real and imaginary parts of β which is equal to c/c_d . If the propagation of the wave in water is assumed to be without loss (c_o real), the input value of AKDA should have the same ratio of imaginary to real part as COCD.

Important Variables in IMPED

Input Variables

AKDA	$(k_d a)$, Dimensionless wave number of dilatational waves, = $(\omega a)/c_d$ (real & complex)
AMI	# of iterations
ANR	# of divisions of imaginary part
BAM	(c/c_d) relative wave speed; enter real part of c/c_d (ZNOT)
BDA	(b/a) b = radius of tube a = radius of cylindrical sample
COCD	(c_o/c_d) - wave speed in fluid divided by dilatational wave speed in solid (real and complex)
DELZ	(used in computation of stepsize)
GOK	(G/K) (real and complex) G = shear modulus, K = bulk modulus

* P.S. Dubbelday, "Complex Root-Finding Program with Application to the Dispersion Relation of Waves Propagating in a Fluid Plate," NRL Memorandum Report 4559, November 1981.

RHM	(ρ_o/ρ_s) ratio of density of fluid to density of solid; if RHM < 0 solid is in vacuum
TOL1	Tolerance of real part
TOL2	Tolerance of imaginary part

Computational Variables

AI	i, imaginary unit
AKA	(ka) , dimensionless wave number, $= \frac{\omega a}{c}$ a = radius of cylinder ω = angular frequency c = propagation speed
AKDA	$(k_d a)$ Dimensionless wave number of dilatational waves, $= \omega a/c_d$, c_d = dilatational wave speed
AKFA	$k_o a$ = Dimensionless wave number in fluid
AKSA	$(k_s a)$, Dimensionless wave number of shear waves $k_s a = \omega a/c_s$
ANR	# of divisions of imaginary part
AMGA	Imaginary part of wave speed β
BAMS	(c_s/c_d) , relative wave speed of shear waves
DEL	Real stepsize
DELH, DELV	Step sizes for movement of test pairs, horizontally and vertically.
DELP	Intermediate symbol of stepsize DEL.
DELZ	Δz , used in computation of approximation to $\partial f/\partial z$
DFDZ	Approximation for $\partial f/\partial z$.
DS(Z):	Function subprogram
DZ	Complex stepsize
F1	Function evaluated at Z1: $f(z_1)$
F2	Function evaluated at Z2: $f(z_2)$
F3	Function evaluated at Z3: $f(z_3)$
F4	Function evaluated at Z4: $f(z_4)$

FV1, FV2 FH1, FH2	Flags to indicate direction of motion of test pairs
FSTEP	Factor to adjust step size DEL
IOP	Return option number = 1 Program returns to entering COCD, GOK, AKDA = 2 Program returns to entering Real Seed, # of iterations, DELZ, BDA = 3 Program returns to entering AWR, TOL (Real), TOL (Imag.), RHM = 4 Program exits
KAM	= AMI
KR, KL KUP, KDOWN	Counters in loops to check number of iterations
NR	= ANR
NRV	Counter in advancing the parameter RHO
Q2A	$(q_a)^2 = (k_a)^2 - (k_d a)^2$
QF2A	$(q_o a)^2 = (k_a)^2 - (k_o a)^2$
QF2B	$(q_o b)^2 = (k_b)^2 - (k_o a)^2$
REGA	Real part of relative wave speed β
RHM	Nominal density of fluid loading the plate, divided by density of plate material.
RHO	Stepwise varied value of relative density, varying from zero to RHM
SIGN	Expression, the algebraic sign of which determines location of root relative to test pair.
TOL	Limit for relative error in the root.
Z	Variable for root used in calling subroutines
Z1, Z2	Vertical test pair
Z3, Z4	Horizontal test pair
ZNOT	Value for the real root of the dispersion relation from a real root finder, serving as the seed for starting the program.
ZVAR	Intermediate value of root, serving as seed for the next step, in parameter RHO

ZVAR Intermediate value of root, serving as seed for the next
step, in parameter RHO

Output Variables

AMGA Imaginary part of root
ZEGA Real part of root

Major Computation Block in IMPED

Refer to major computation blocks in R0819

```

C
C
C      COMPLEX ROOTFINDING ROUTINE
C      WRITTEN BY TINA RUGGIERO
C      DECEMBER 17,1981
C
C      THIS COMPLEX ROUTINE INTRODUCED BY DUBBELDAY
C      ALLOWS THE USER TO SOLVE FOR BOTH THE REAL AND IMAGINARY
C      PARTS OF A COMPLEX ROOT, GIVEN A REAL ROOT IN THE
C      VICINITY AS A SEED.
C
C      SUBROUTINES USED:
C      CDI3 (CONTAIN FUNCTION DESCRIBED BY DETERMINANT OF A SET OF
C            BOUNDARY CONDITIONS)
C      CBJ0 (BESSEL FUNCTION (J0))
C      CBJ1 (BESSEL FUNCTION (J1))
C      CBY0 (BESSEL FUNCTION (Y0))
C      CBY1 (BESSEL FUNCTION (Y1))
C
0001      COMPLEX Z,DSP,AI,DFDZ,DZ,ZVAR,Z1,Z2,Z3,Z4,F1,F2,F3,F4,
0002      1 GOK,DSP2,CDI3,CMFX,AKDA,AKA,BAMS,COCD,HGOK,HAKDA,HCOCD
COMMON AKA,AKDA,COCD,BDA,RHM,GOK
C
C      INPUT SECTION
0003      5      WRITE(5,10)
0004      10     FORMAT(/'%ENTER COCD(R,I),GOK(R,I),AKDA(R,I)  ')
0005      READ(5,20)HCOCD,HGOK,HAKDA
0006      20     FORMAT(6F10.0)
0007      25     WRITE(5,30)
0008      30     FORMAT(/'%ENTER REAL SEED,% OF ITER.,DELZ,BDA  ')
0009      READ(5,40)ZNOT,AMI,DELZ,BDA
0010      40     FORMAT(4F10.0)
0011      KAM=AMI
0012      BAMS=CSQRT(GOK/(1.+4.*GOK/3.))
0013      45     WRITE(5,50)
0014      50     FORMAT(/'%ENTER ANR,TOL (REAL),TOL (IMAG),RHM  ')
0015      READ(5,60)ANR,TOL2,TOL1,RHM
0016      60     FORMAT(4F10.0)
0017      AI=CMPLX(0.,1.0)
0018      GOK=HGOK
0019      COCD=HCOCD
0020      AKDA=HCOCD
0021      RGOK=REAL(GOK)
0022      RAKDA=REAL(AKDA)
0023      YGOK=AIMAG(GOK)
0024      YAKDA=AIMAG(AKDA)
0025      RCOCD=REAL(COCD)
0026      YCOCD=AIMAG(COCD)
0027      NR=INT(ANR)
0028      NRV=0
0029      ZVAR=ZNOT
0030      GOK=CMPLX(RGOK,0.0)
0031      AKDA=CMPLX(RAKDA,0.0)
0032      COCD=CMPLX(RCOCD,0.0)
0033      Z=ZVAR+DELZ
0034      DFDZ=CDI3(Z)/DELZ

```

```

0035      GOK=GOK+AI*YGOK/ANR
0036      AKDA=AKDA+AI*YAKDA/ANR
0037      COCD=COCD+AI*YCOCD/ANR
0038      NRV=NRV+1
0039      DZ=-CDI3(ZVAR)/DFDZ
0040      X=ABS(REAL(DZ))
0041      Y=ABS(AIMAG(DZ))

      C
      C      DEL1 AND DEL2 ARE HORIZONTAL AND VERTICAL STEP SIZES
0042      DEL1=Y
0043      DEL2=X

0044      WRITE(5,16)DEL1,DEL2
0045      16      FORMAT(/,' DEL1= ',E8.2,'      DEL2= ',E8.2)
0046      WRITE(5,57)
0047      TYPE *,'-----'
0048      DELP=DEL1
0049      DELQ=DEL2
      C      FIND DIRECTION OF INITIAL VERTICAL PAIR
0050      70      DEL2=DELP
0051      DEL1=DELQ
0052      64      Z1=ZVAR-AI*DEL1
0053      Z2=ZVAR+AI*DEL1
0054      65      F1=CDI3(Z1)
0055      F2=CDI3(Z2)
0056      KR=0
0057      KL=0
0058      KUP=0
0059      KDOWN=0
      C      THE RESULT OF SIGN IS TO DETERMINE DIRECTION OF PAIR
0060      SIGN=AIMAG(F1)*REAL(F2)-REAL(F1)*AIMAG(F2)
0061      IF (SIGN) 71,72,73

0062      71      FH1=-1
0063      GO TO 74
0064      72      TYPE *,'LEFT RIGHT SIGN'
0065      GO TO 100
0066      73      FH1=1
0067      74      DELH=FH1*DEL1
      C      LOOP TO FIND VERTICAL LINES BRACKETING ROOT

0068      75      Z1=Z1+DELH
0069      Z2=Z2+DELH
0070      F1=CDI3(Z1)
0071      F2=CDI3(Z2)
      C
      C      SIGN IS USED TO DETERMINE DIRECTION PAIR IS MOVED
0072      SIGN=AIMAG(F1)*REAL(F2)-AIMAG(F2)*REAL(F1)
0073      IF (SIGN) 81,72,83
0074      81      FH2=-1
0075      KL=KL+1
0076      IF(KAM-KL)99,99,84
0077      99      TYPE *,'EXIT LEFT'
0078      GO TO 100
0079      83      FH2=1
0080      KR=KR+1

```

```

0081      IF(KAM-KR)98,98,84
0082      98      TYPE *, 'EXIT RIGHT'
0083      GO TO 100
0084      84      IF(FH1*FH2)92,75,75
C      FIND HORIZONTAL TEST PAIR DIRECTION
0085      92      Z4=(Z1+Z2)/2.+DEL2
0086      KR=0
0087      KL=0
0088      Z3=(Z1+Z2)/2.-DEL2
0089      F3=CDI3(Z3)
0090      F4=CDI3(Z4)
0091      SIGN=AIMAG(F3)*REAL(F4)-AIMAG(F4)*REAL(F3)
0092      IF(SIGN)101,102,103
0093      101      FV1=1.
0094      GOTO 104
0095      102      TYPE *, 'UP-DOWN SIGN= ZERO'
0096      GOTO 100
0097      103      FV1=-1
0098      104      DELV=FV1*DEL2
C      LOOP TO FIND HORIZONTAL LINES BRACKETING ROOT
0099      105      Z4=Z4+AI*DELV
0100      Z3=Z3+AI*DELV
0101      F3=CDI3(Z3)
0102      F4=CDI3(Z4)
0103      SIGN=AIMAG(F3)*REAL(F4)-AIMAG(F4)*REAL(F3)
0104      IF(SIGN)111,102,113
0105      111      FV2=1.
0106      KUP=KUP+1
0107      IF(KAM-KUP)97,97,114
C
0108      97      TYPE *, 'EXIT UP'
0109      GOTO 100
0110      113      FV2=-1.
0111      KDOWN=KDOWN+1
0112      IF(KAM-KDOWN)96,96,114
0113      96      TYPE *, 'EXIT DOWN'
0114      GO TO 100
0115      114      IF(FV1*FV2)122,105,105
0116      122      REGA=REAL(Z1)
0117      AMGA=AIMAG(Z3)
0118      KUP=0
0119      KDOWN=0
0120      133      AMGA=ABS(AMGA)
0121      DREG=ABS(REGA-ZNOT)
C
C      CHECK IF REQUIRED PRECISION REACHED
0122      IF(DEL1/AMGA-TOL1 .LE. 0)GOTO 132
0123      IF(DEL2/REGA-TOL2 .LE. 0)GO TO 135
0124      DEL2=DEL2/10.
0125      GO TO 92
0126      135      DEL1=DEL1/10.
0127      DEL2=DEL2/10.
0128      Z1=(Z3+Z4)/2.-AI*DEL1
0129      Z2=(Z3+Z4)/2.+AI*DEL1
0130      GOTO 65
0131      132      REGA=REAL(Z1)-DELH/2.

```

```

0132      AMGA=AIMAG(Z3)-DELV/2.
0133      DELH=ABS(DELH/2.)
0134      C
0134      C      DELV=ABS(DELV/2.)
0134      C
0134      C      OUTPUT SECTION
0135      57      FORMAT(/,'      REAL',7X,'IMAGINARY',5X,'REAL ERROR',5X,'IMAG ERROR',
0136      WRITE(5,58)REGA,AMGA,DELH,DELV
0137      58      FORMAT(E10.4,4X,E10.4,4X,E10.4,6X,E10.4)
0137      C
0137      C      CHECK IF FINAL GOK AND AKDA IS REACHED
0138      IF(NR-NRV)145,145,146
0139      146      ZVAR=REGA+AMGA*AI
0140      GOK=GOK+AI*YGOK/ANR
0141      AKDA=AKDA+AI*YAKDA/ANR
0142      COCD=COCD+AI*YCOCD/ANR
0143      NRV=NRV+1
0144      GOTO 70
0145      145      CONTINUE
0146      TYPE *, ' '
0147      100      TYPE *, ' ENTER OPTION # (1,2,3,4) '
0148      READ(5,160)IOP
0149      160      FORMAT(I2)
0150      GO TO (5,25,45,550),IOP
0151      550      END

```

```

0001      FUNCTION CDI3(BAM)
0002      COMMON AKA,AKDA,COCD,BDA,RHM,GOK
0003      COMPLEX AKA,AKDA,BAMS,COCD,AKSA,Q2A,S2A,AKFA,QF2A,QFA,QFB,BF0Q,
1      BF1Q,B1QB,Y0Q,Y1Q,Y1QB,QA,B0Q,B1Q,SA,B0S,B1S,D11,D12,D21,D22,
1      D13,D14,D31,D32,D33,D34,D43,D44,P12,P34,P23,P14,CDI3,QP2A,AI,
1      QPF2A,SP2A,QPFA,BAM,D13,CSQRT,GOK
0004      BAMS=CSQRT(GOK/(1.+4.*GOK/3.))
0005      AI=CHPLX(0.0,1.0)
0006      AKA=AKDA/BAM
0007      AKSA=AKDA/BAMS
0008      QP2A=AKDA**2-AKA**2
0009      SP2A=AKSA**2-AKA**2
0010      IF(RHM)65,65,40
0011      40      AKFA=AKDA/COCD
0012      QPF2A=AKFA**2-AKA**2
0013      IF(REAL(QPF2A))45,46,46
0014      46      QPFA=CSQRT(QPF2A)
0015      GO TO 50
0016      45      QPFA=-AI*CSQRT(-QPF2A)
0017      50      QFB=BDA*QPFA
0018      IF(ABS(QFB).GT. 10.0)GOTO 1000
0019      CALL CBJ0(QPFA,BF0Q)
0020      CALL CBJ1(QPFA,BF1Q)
0021      CALL CBJ1(QFB,B1QB)
0022      CALL CBY0(QPFA,Y0Q)
0023      CALL CBY1(QPFA,Y1Q)
0024      CALL CBY1(QFB,Y1QB)
0025      65      IF(REAL(QP2A))81,80,80
0026      80      QA=CSQRT(QP2A)
0027      GO TO 85
0028      81      QA=-AI*CSQRT(-QP2A)
0029      85      CALL CBJ0(QA,B0Q)
0030      CALL CBJ1(QA,B1Q)
0031      IF(REAL(SP2A))71,70,70
0032      70      SA=CSQRT(SP2A)
0033      GO TO 90
0034      71      SA=-AI*CSQRT(-SP2A)
0035      90      CALL CBJ0(SA,B0S)
0036      CALL CBJ1(SA,B1S)
0037      D11=B0Q*(2.*AKA**2-AKSA**2)+B1Q*2.*QA
0038      D12=2.*AKA*(B1S-SA*B0S)
0039      D21=2.*AKA*QA*B1Q
0040      D22=B1S*(2.*AKA**2-AKSA**2)
0041      IF (RHM)21,21,22
0042      21      CDI3=D11*D22-D12*D21
0043      GO TO 11
0044      22      D13=AKSA**2*RHM*BF0Q
0045      D14=AKSA**2*RHM*Y0Q
0046      D31=QA*B1Q
0047      D32=AKA*B1S
0048      D33=-QPFA*BF1Q
0049      D34=-QPFA*Y1Q
0050      D43=QFB*B1QB
0051      D44=QFB*Y1QB
0052      P12=D11*D22-D12*D21
0053      P34=D33*D44-D34*D43
0054      P23=D21*D32-D22*D31
0055      P14=D13*D44-D14*D43
0056      CDI3=P12*P34+P23*P14
0057      11      RETURN
0058      1000     TYPE *, 'ARGUMENT FOR BESSEL FUNCTIONS, TOO LARGE'
0059      2000     END

```

```

0001      SUBROUTINE CBJ1(Z,ANS)
>
      C
      C      BESSEL FUNCTION J1
      C      (COMPUTED BY SERIES)
      C
0002      COMPLEX Z,FUNC,ANS,CHPLX,FIRST
0003      CALL ERRSET(73,,.FALSE.,,.FALSE.,)
0004      TOL=.000001
0005      8      ERROR=1.0
0006      FUNC=Z/2
0007      ANS=Z/2
0008      RHO=CABS(Z)
0009      DO 30 R=2,1000
0010      FIRST=-1.0*FUNC*(Z/2.0)**2/(R*R)
0011      FUNC=FIRST*2.0*R/(2*R-2.0)
0012      ANS=ANS+FUNC
0013      ERROR=ERROR*(RHO/2.0)**2/(R*R)
0014      IF(ERROR/CABS(ANS)-TOL)50,50,30
0015      30      CONTINUE
0016      50      RETURN
0017      END

```

```

0001      SUBROUTINE CBJO(Z,ANS)
>
      C
      C      BESSEL FUNCTION JO
      C      (COMPUTED BY SERIES)
      C
0002      COMPLEX Z,FUNC,ANS,CHPLX
0003      8      ANS=CHPLX(1.0,0.0)
0004      TOL=.000001
0005      ERROR=1.0
0006      FUNC=CHPLX(1.0,0.0)
0007      RHO=CABS(Z)
0008      DO 30 R=1,1000
0009      FUNC=(-1.0)*FUNC*(Z/2.0)**2/(R*R)
0010      ANS=ANS+FUNC
0011      IF (CABS(ANS) .EQ. 0.0)GO TO 30
0012      ERROR=ERROR*(RHO/2.0)**2/(R*R)
0013      IF(ERROR/SQRT(REAL(ANS)**2+AIMAG(ANS)**2)-TOL)50,50,30
0014      30      CONTINUE
0015      50      RETURN
0016      END

```

RUN IMPED

ENTER COCD(R,I),GOK(R,I),AKDA(R,I) .2358,.002358,.4541,.004541,.2,.002 <CR>

ENTER REAL SEED,# OF ITER.,DELZ,BDA .97875,200,.01,1.001 <CR>

ENTER ANR,TOL (REAL),TOL(IMAG),RHM 3,.01,.001,.12805 <CR>

DEL1= 0.24E-03 DEL2= 0.16E-05

REAL	IMAGINARY	REAL ERROR	IMAG ERROR
------	-----------	------------	------------

.9788E+00	0.2313E-03	0.8249E-07	0.1217E-04
-----------	------------	------------	------------

.9788E+00	0.4870E-03	0.8249E-07	0.1217E-04
-----------	------------	------------	------------

.9788E+00	0.7183E-03	0.8249E-07	0.1217E-04
-----------	------------	------------	------------

ENTER OPTION # (1,2,3,4)

4 <CR>

- 1) All underlined portions are user supplied.
- 2) <CR> indicates 'RETURN'

APPENDIX B

Correction for Imperfect Reflector in Impedance Tube

The analysis of the role of the reflector in the impedance tube starts from the observation that there are traveling waves in both directions in the water of the impedance tube, in the sample, and in the reflector, each with its own amplitude and phase. By equating stress and particle velocity at each of the interfaces one may infer the reflection coefficient for the following cases: r_{sa} is the reflection coefficient without the sample in place, r_{st} is the reflection coefficient without the sample. One would like to relate these measured quantities to the reflection coefficient r used in Eq. (2), which implies a perfect reflector. The length l of the reflector is chosen such that $k_r l$ is close to $\pi/2$. The subscript r refers to the reflector. Define δ by $k_r l = \pi/2 + \delta$, δ is a small complex number. One finds that

$$r_{sa} = \frac{1 - i(\tan k_s d)(\rho c)_o/(\rho c)_s - i\epsilon[1 - i(\tan k_s d)(\rho c)_s/(\rho c)_o]}{1 + i(\tan k_s d)(\rho c)_o/(\rho c)_s + i\epsilon[1 + i(\tan k_s d)(\rho c)_s/(\rho c)_o]} \quad (B1)$$

$$r_{st} = e^{-2ik_o d} \frac{1 - i\epsilon}{1 + i\epsilon} \quad (B2)$$

$$r = \frac{1 - i(\tan k_s d)(\rho c)_o/(\rho c)_s}{1 + i(\tan k_s d)(\rho c)_o/(\rho c)_s} \quad (B3)$$

where the parameter ϵ is defined by $\epsilon = (\tan \delta)(\rho c)_o/(\rho c)_r$.

In the present data analysis, the reflection coefficient r in Eq. (2) is replaced by r_c , defined by

$$r_c = \exp(-2ik_o d) r_{sa} / r_{st} \quad (B4)$$

The correction factor applied to r_{sa} according to this equation may be expressed in terms of the small parameter ϵ by

$$\exp(-2ik_o d) / r_{st} \approx 1 + 2i\epsilon \quad (B5)$$

The proper correction factor is r/r_{sa} , which may be expressed to the same order by

$$r/r_{sa} \approx 1 + 2iF\epsilon, \quad (B6)$$

where the factor F is given by

$$F = 1/\left\{\cos^2 kd + \sin^2 kd[(\rho c)_o/(\rho c)_s]^2\right\}. \quad (B7)$$

This factor F may be noticeably different from one, if the complex specific acoustic impedance of the sample material is not very close to that of the water in the tube. Even though densities and propagation speeds of sample and water might be close, as is the case for elastomers, a difference in attenuation still would make this factor unequal to one.

This proposed correction does not consider the effect of the termination of the tube by the reflector and the consequent deviation from the idealized situation of an infinite waveguide.

4-8
DT