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A MODEL OF THE TRAILING EDGE SEPARATION ON AN AIRFOIL

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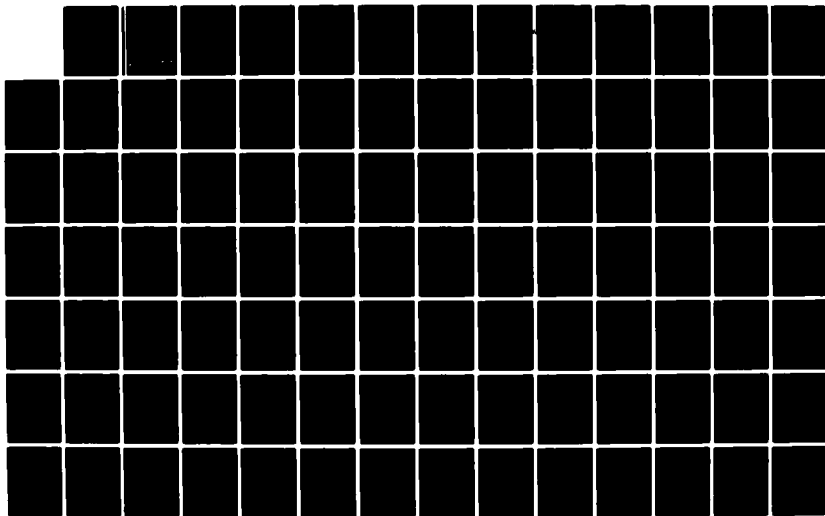
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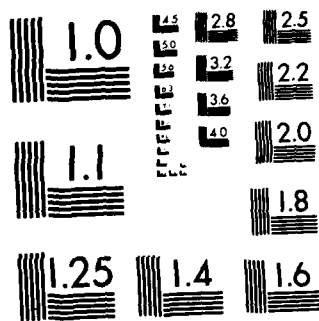
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A MODEL OF THE TRAILING EDGE SEPARATION ON AN AIRFOIL

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FEBRUARY 1983

FINAL SCIENTIFIC REPORT, 15 MAY 1982 - 14 FEBRUARY 1983

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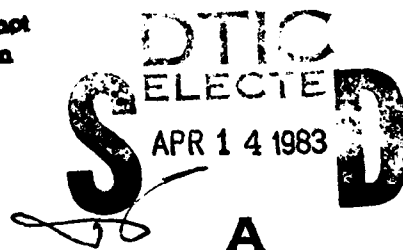
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| 1. REPORT NUMBER<br><b>EOARD-TR-83E71 -</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER                                                                              |
| 4. TITLE (and Subtitle)<br><b>A MODEL OF THE TRAILING EDGE SEPARATION<br/>ON AN AIRFOIL</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                       | 5. TYPE OF REPORT & PERIOD COVERED<br><b>Final Scient. Rep.<br/>15 May 82 - 14 Feb 83</b>                  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                       | 6. PERFORMING ORG. REPORT NUMBER                                                                           |
| 7. AUTHOR(s)<br><b>Greg ZILLIAC</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                       | 8. CONTRACT OR GRANT NUMBER(s)<br><b>AFOSR 82-0204</b>                                                     |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br><b>von Karman Institute for Fluid Dynamics,<br/>Chaussée de Waterloo, 72,<br/>B-1640 Rhode-Saint-Genèse, Belgium</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                       | 10. PROGRAM ELEMENT, PROJECT, TASK<br>AREA & WORK UNIT NUMBERS<br><b>P.E. 61102F<br/>Proj/Task 2301/D1</b> |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br><b>European Office of Aerospace R&amp;D/CA<br/>Box 14<br/>FPO New York 09510</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                       | 12. REPORT DATE<br><b>February 1983</b>                                                                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                       | 13. NUMBER OF PAGES<br><b>91</b>                                                                           |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)<br><b>European Office of Aerospace Research and<br/>Development/CA<br/>Box 14<br/>FPO New York 09510</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                       | 15. SECURITY CLASS. (of this report)<br><b>UNCLASSIFIED</b>                                                |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                       | 15a. DECLASSIFICATION/DOWNGRADING<br>SCHEDULE                                                              |
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| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)<br><br><b>Approved for public release; distribution unlimited.</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                       |                                                                                                            |
| 18. SUPPLEMENTARY NOTES                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                       |                                                                                                            |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)<br><br><b>BOUNDARY LAYER SEPARATION; LIFT PREDICTION; TURBULENT BOUNDARY LAYER;<br/>TWO DIMENSIONAL FLOW; SEPARATED FLOW</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                       |                                                                                                            |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br><br><b>A model of the turbulent boundary layer separation of a two<br/>dimensional airfoil has been investigated. The model consists of<br/>distributing sources on the contour in the separated region of<br/>sufficient strength to maintain the constant pressure characteris-<br/>tic of trailing edge boundary layer separation. A model for the<br/>attached boundary layer has also been included. An accurate esti-<br/>mation of maximum lift, pitching moment and pressure distribution<br/>have been obtained. A correct drag prediction has not been<br/>accomplished.</b> |                       |                                                                                                            |

EOARD-TR-83E7

This report has been reviewed by the EOARD Information Office and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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ACKNOWLEDGEMENTS

There are many people who deserve recognition for their contributions to this work. The basic framework of the computer programs existed when I started to work on this project. These programs were written or modified mainly by S. Huo, and R.A. Van den Braembussche among others.

I would also like to give additional thanks to R.A. Van den Braembussche, who spend a lot of time helping me become familiar with the methods used.

Thanks also to Sharon Stanaway for her moral support and assistance in turning the lights off after long nights at the video terminal.

ABSTRACT

A model of the turbulent boundary layer separation of a two dimensional airfoil has been investigated. The model consists of distributing sources on the contour in the separated region of sufficient strength to maintain the constant pressure characteristic of trailing edge boundary layer separation. A model for the attached boundary layer has also been included. An accurate estimation of maximum lift, pitching moment and pressure distribution have been obtained. A correct drag prediction has not been accomplished.

LIST OF SYMBOLS

|             |                                                     |
|-------------|-----------------------------------------------------|
| $c_D$       | energy dissipation coefficient                      |
| $c_f$       | skin friction coefficient                           |
| $c$         | airfoil chord                                       |
| $f_x, f_y$  | singularity influence functions                     |
| $m$         | exponent in Faulkner-Skan velocity profile          |
| $r$         | transformed boundary layer variable                 |
| $S$         | distance along contour non dimensionalized by chord |
| $S^*$       | distance along contour in separated region          |
| $u_e$       | local external velocity                             |
| $u_\tau$    | skin friction velocity                              |
| $v$         | local velocity on airfoil in the separated region   |
| $w_L$       | local velocity on airfoil                           |
| $z$         | point on airfoil where velocity is calculated       |
| $B( , )$    | boundary layer function                             |
| $C'( , )$   | boundary layer function                             |
| $C_d$       | two dimensional drag coefficient                    |
| $C_l$       | two dimensional lift coefficient                    |
| $C_m$       | two dimensional pitching moment coefficient         |
| $C_p$       | pressure coefficient                                |
| $F( )$      | Faulkner-Skan velocity profile function             |
| $G$         | function used in boundary layer analyses            |
| $H_{12}$    | momentum thickness form factor = $\delta^*/\theta$  |
| $H_{32}$    | energy thickness form factor = $\theta_H/\theta$    |
| $K$         | singularity influence matrix                        |
| $L$         | Le Foll's form factor                               |
| $p$         | pressure                                            |
| $Re_\theta$ | Reynolds number based on the momentum thickness     |

|               |                                                                      |
|---------------|----------------------------------------------------------------------|
| TEPS,TESS     | points just before airfoil trailing edge                             |
| U             | component of the complex velocity in x-direction                     |
| V             | component of the complex velocity in y-direction                     |
| W             | complex velocity                                                     |
| X             | Le Foll's Reynolds number                                            |
| $\alpha$      | airfoil angle of attack or uniform stream basic flow vortex strength |
| $\beta$       | uniform stream basic flow vortex strength                            |
| $\gamma$      | total vortex strength                                                |
| $\delta$      | outflow basic flow vortex strength                                   |
| $\delta^*$    | boundary layer displacement thickness                                |
| $\zeta$       | point where singularity is located                                   |
| $\eta$        | Blasius variable                                                     |
| $\theta$      | boundary layer momentum thickness                                    |
| $\theta_H$    | boundary layer energy thickness                                      |
| $\lambda$     | Le Foll's parameter for the velocity profile                         |
| $\mu$         | circulatory basic flow vortex strength                               |
| $\nu$         | kinematic viscosity                                                  |
| $\xi, \eta$   | coordinate of the singularity point location                         |
| $\rho_\infty$ | freestream density                                                   |
| $\tau$        | shearing stress                                                      |
| $\phi$        | airfoil transformed coordinates                                      |
| $\Gamma$      | total circulation                                                    |
| $\pi$         | pressure gradient                                                    |

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- 2 The airfoil coordinate system
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## 1. INTRODUCTION

Analytical solutions to the problem of boundary layer separation have long been sought after but few have been successful. The Navier-Stokes equations which govern flow separation have not been solved for high Reynolds number flows, hence existing analytical solutions must involve assumptions concerning the nature of separation. Empiricism and/or experimental results are often used. In these cases, the question arises as to whether we are obtaining solutions which accurately reflect the physics of the problem, or are we modeling a phenomenon. This paper deals with the latter of these possibilities.

One of the first and arguably the best analytical model of turbulent boundary layer separation was developed Jacob (Ref. 1). This model consists of using inviscid singularity methods to model a viscous phenomenon. Sources are distributed in the dead air region to maintain constant pressure (in this region). A constant pressure, dead air region is a characteristic of turbulent boundary layer separation on an airfoil. The boundary layer and outer inviscid region must simultaneously be accounted for to complete the picture. Jacob has used this model to achieve very accurate prediction of maximum lift.

A second model which has been used with similar success is the model proposed by Dvorak and Maskew (Ref. 2). Their model consists of placing vortices on the free shear layer. The strength of the vortices is such as to obtain constant pressure in the separated region.

There are many other methods which have a stronger physical basis, but surprisingly, do not give results which can compare with the quality of results of Jacob and Dvorak's techniques.

In this investigation, Jacob's model has been chosen, because it has been in existence longer and has proven its validity as a model.

It is the object of this investigation to develop the method to the point where an accurate estimation of the lift, drag, pitching moment and pressure distribution can be obtained for the turbulent boundary layer separation of a two dimensional airfoil.

The importance of this investigation is that a correct prediction of the maximum lift coefficient is required. An inaccurate estimation of  $C_{l_{max}}$  can have drastic effects on the payload capacity of an aircraft (under estimation of  $C_{l_{max}}$  by 0.1 can mean up to a 20% reduction in payload).

Two points must be emphasized before continuing. First, this method is only applicable as a design tool and will not give any insight into the mechanisms of separation. Secondly, separation is a three dimensional, and quite often unsteady phenomenon. This has been demonstrated by wind tunnel tests which have shown that under optimum conditions, it is difficult to obtain a truly two dimensional and steady measurement. Hence, the results of this research cannot directly be extended to a three dimensional situation.

## 2. THEORETICAL FOUNDATION OF THE MODEL

### 2.1 The model

The model is shown in figure 1. It consists of breaking down the separated flow over an airfoil into three regions :

Region 1 : the outer inviscid region which is modelled by line vortices placed on the contour of the airfoil;

Region 2 : the boundary layer characteristics which are determined by an integral boundary method (similar to Le Foll's method). The boundary layer effect is included by distributing sources to compensate for the displacement thickness.

Region 3 : the separated region is modelled by sources distributed on the contour of sufficient strength to maintain constant pressure on the airfoil in this region.

### 2.2 The inviscid and separated regimes

#### 2.2.1 Governing equations

The governing equation for the model can be derived from the boundary contours. Three boundary conditions are required for the analysis. Application of these conditions gives a Glauert type integral expression for the velocity on the contour :

- (1) Uniform stream far upstream.
- (2) Normal velocity on the airfoil surface is zero (kinematic condition).
- (3) An equivalent Kutta condition.

In addition to the boundary conditions, an additional criterium that the pressure is constant in the separated region must be enforced. Conditions 1 & 2 give rise to the integral expression for the vortex strength in function of blade geometry, uniform velocity, and the source distribution.

$$u_{\infty} \frac{dy}{dS} - v_{\infty} \frac{dx}{dS} = \frac{1}{2\pi} \oint \gamma(\zeta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) + \frac{1}{2\pi} \oint Q(\zeta) \left[ -f_x(z, \zeta) \frac{dx}{dS} + f_y(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) \quad (2.1)$$

where:

$$f_y(z, \zeta) = \frac{(x-\xi)}{(x-\xi)^2 + (y-\eta)^2}$$

$$f_x(z, \zeta) = \frac{-(y-\eta)}{(x-\xi)^2 + (y-\eta)^2}$$

Martensen (Ref. 3) has shown that the flow over an airfoil may be considered as the superposition of four basic flows, a uniform stream in the x-direction, a uniform stream in the y-direction, a circulatory flow, and an outflow. Expression (2.1) becomes four integral expressions for the vortex strength of the basic flows.

Uniform stream in x-direction :

$$\frac{1}{2\pi} \oint \alpha(\zeta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) = \frac{dy}{dS} \quad (2.2)$$

Uniform stream in y-direction :

$$\frac{1}{2\pi} \oint \beta(\zeta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dx}{dS} \right] dS(\zeta) = \frac{dx}{dS} \quad (2.3)$$

Circulatory flow :

$$\frac{1}{2} \oint \mu(\zeta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) = 0 \quad (2.4)$$

Outflow :

$$\begin{aligned} \frac{1}{2\pi} \oint \delta(\theta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) \\ = - \frac{1}{2\pi} \oint Q(\zeta) \left[ f_x(z, \zeta) \frac{dx}{dS} - f_y(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) \end{aligned} \quad (2.5)$$

Equation (2.1) has been split into one homogeneous and three non-homogeneous equations. The total solution is given by :

$$\gamma(\zeta) = U_\infty \gamma(\zeta) - V_\infty \beta(\zeta) + \Gamma \mu(\zeta) + \delta(\zeta) \quad (2.6)$$

The following supplementary equations are required because the circulatory basic flow is the only basic flow which contributes to the circulation (note  $\Gamma$  in equation 2.6) :

$$\oint \gamma(\zeta) dS(\zeta) = 0 \quad (2.7)$$

$$\oint \beta(\zeta) dS(\zeta) = 0 \quad (2.8)$$

$$\oint \mu(\zeta) dS(\zeta) = 1 \quad (2.9)$$

$$\oint \delta(\zeta) dS(\zeta) = 0 \quad (2.10)$$

Applying the equivalent Kutta condition determines the magnitude of the total circulation :

$$\gamma(\text{TEPS}) = - \gamma(\text{TESS}) \quad (2.11)$$

Solving for the circulation :

$$\Gamma = - \frac{U_\infty [\alpha(\text{TEPS}) + \alpha(\text{TESS})] + V_\infty [\beta(\text{TEPS}) + \beta(\text{TESS})] + [\delta(\text{TEPS}) + \delta(\text{TESS})]}{[\mu(\text{TEPS}) + \mu(\text{TESS})]}$$

The local velocity on the contour is given by :

$$W_L = \sqrt{\gamma(\zeta)^2 + Q(\zeta)^2} \quad (2.13)$$

This expression contains a contribution due to the vortices and due to the sources (sources are distributed along the contour for the boundary layer model as well as the separation model).

It appears that the complexity of the problem has been increased by separating the flow into four basic flows and that is true. The major advantage of this approach is that the velocity on the contour for any angle of attack can be determined by substitution of the corresponding freestream velocity components in equation (2.6). One limitation is that if the source distributions is changed, the whole problem must be resolved.

The solution of these integral equations is not straightforward. The equations become singular when calculating the influence of a particular vortex (or source) at its origin. This difficulty can be avoided as shown in appendix A.

### 2.2.2 Numerical considerations

Attention must now be turned to the numerical side of the problem. The profile is placed in the x-y plane as shown in figure 2. A coordinate transformation is used :

$$x = \frac{c}{2} (1 - \cos \phi) \quad (2.14)$$

This transformation allows a greater number of singularities to be placed near the leading and trailing edges where the slope of the contour changes rapidly. The contour is divided into N segments of constant included angle  $\Delta\phi$ . On each segment, a line vortex of constant strength is placed. Line vortices have an advantage over point vortices in that a line vortex distribution more closely approximates the continuous vortex

distribution of the integral equations. The integral expressions are replaced by the following summations :

Uniform stream in x-direction :

$$\frac{1}{2\pi} \sum_{i=1}^N \alpha(\zeta_i) K(z_j, \zeta_i, \Delta\zeta_i) = \frac{dy_j}{dS} \quad (2.15)$$

Uniform stream in y-direction :

$$\frac{1}{2\pi} \sum_{i=1}^N \beta(\zeta_i) K(z_j, \zeta_i, \Delta\zeta_i) = \frac{dx_j}{dS} \quad (2.16)$$

Circulatory flow :

$$\frac{1}{2\pi} \sum_{i=1}^N \mu(\zeta_i) K(z_j, \zeta_i, \Delta\zeta_i) = 0 \quad (2.17)$$

Outflow :

$$\frac{1}{2\pi} \sum_{i=1}^N \delta(\zeta_i) K(z_j, \zeta_i, \Delta\zeta_i) = \frac{1}{2\pi} \sum_a^b Q(\zeta_i) K(z_j, \zeta_i, \Delta\zeta_i) \quad (2.18)$$

where the influence function K is given in appendix A.

The supplementary conditions become :

$$\sum_{i=1}^N \alpha(\zeta_i) \Delta S(\zeta_i) = 0 \quad (2.20)$$

$$\sum_{i=1}^N \beta(\zeta_i) \Delta S(\zeta_i) = 0 \quad (2.21)$$

$$\sum_{i=1}^N \mu(\zeta_i) \Delta S(\zeta_i) = 1 \quad (2.22)$$

$$\sum_{i=1}^N \delta(\zeta_i) \Delta S(\zeta_i) = 0 \quad (2.23)$$

Supposing that equations (2.15) through (2.19) must be satisfied in the points  $j=1, N$  where the vortices are located and together with equations (2.20) through (2.23) one obtains a set of  $4(N+1)$  equations with  $4N$  unknowns. The unknowns are the vortex strength of the basic flows. However, a system of equations derived from an integral equation of the first kind is often ill-conditioned. This can be avoided if the kinetic condition is also applied at the intersection points of the line vortex. This leads to a set of  $4(2N+1)$  equations with  $4N$  unknowns, which can be treated by a least square technique. See Appendix A for further details.

### 2.3 The boundary layer analysis

#### 2.3.1 The integral boundary layer approach

The method used for the boundary layer region is by S. Huo (Ref. 4) and is similar to Le Foll's method. Huo's method is based on two assumptions :

- (1) Local boundary layer velocity profile similarity.
- (2) The velocity profile is assumed to be a one parameter profile.

The integral boundary layer equations are :

Momentum :

$$d(\theta u_e^2) + \delta^* u_e du_e = c_f \frac{u_e^2}{2} dS \quad (2.24)$$

Energy :

$$d\left(\theta_H \frac{u_e^3}{2}\right) = c_D u_e^3 dS \quad (2.25)$$

where the skin friction coefficient is :

$$c_f = \frac{2\tau_w}{\rho u_e^2} \quad (2.26)$$

and the energy dissipation coefficient is :

$$c_D = \frac{2}{\rho u_\infty^3} \int_0^\infty \tau(y) \frac{\partial u}{\partial y} dy \quad (2.27)$$

The unknowns in equations (2.24) and (2.25) are  $\delta^*, \theta, \theta_H, c_f$  and  $c_D$ . There are two equations at hand with five unknowns. In order to solve these equations, the following information is necessary :

- (1) A velocity profile law in the boundary layer.
- (2) A skin friction law.
- (3) A relation for the dissipation coefficient.

The velocity profile law will enable the determination of the relation between the shape factors  $H_{12}$  and  $H_{32}$ . Therefore, knowing two of the characteristic thicknesses of the boundary layer from equations (2.24) and (2.25) will allow the calculation of the other thicknesses. The skin friction and dissipation coefficients will be determined from the skin friction law and dissipation relation respectively. Hence, the complete boundary layer characteristics can be computed.

Le Foll bases his method on the idea that the boundary layer can be described in terms of a form factor and a characteristic length instead of the velocity and length. The main advantage of this approach is that the inverse procedure of specifying boundary layer characteristics and their solving for the corresponding velocity distribution is easily achieved. The inverse problem is not of interest in this investigation, but the method is rapidly applicable to the direct problem.

The parameters  $L$  (Truckenbrodt's parameter) and  $X$  are used by Le Foll to specify the boundary layer characteristics:

$$dL = \frac{1}{(H_{12}-1)} \frac{dH_{32}}{H_{32}} \quad X = \ln Re_{\theta_H} + 2L \quad (2.28)$$

As they are directly related to each other,  $L$  and  $X$  can, therefore, replace  $H_{12}$  and  $Re_{\theta}$  to characterize a boundary layer.

### 2.3.2 The laminar boundary layer

The assumed velocity profile is the Faulkner-Skan profile :

$$\frac{u}{u_e} = F(n,m) \quad n = y \sqrt{\frac{u_e}{S}} \quad u_e = S^m \quad (2.29)$$

Stewartson's transformation has been used with the Faulkner-Skan profile to extend the boundary layer calculation into the compressible regime.

The compressibility aspect of boundary layer separation is not included in the separation model, but it was felt that since a compressible boundary layer method was available and programmed), compressibility should be (and was) included in the boundary layer model.

The skin friction and dissipation relation are functions of the assumed velocity profile. For the laminar boundary layer :

$$c_f = - \frac{4a_2(m)}{\sqrt{Re_x}} \quad c_D = \frac{F_1(m)}{\sqrt{Re_x}} \quad (2.30)$$

To simplify the calculation, another change of variable is introduced :

$$r = \frac{Re_0 H e^{2L}}{Re_{ref}} \quad (2.31)$$

The momentum and energy integral equations become :

$$dr = u_e C(L,r) dS = \frac{u_e C'(L)}{r Re_{ref}} dS \quad (2.32)$$

$$\frac{dL}{dS} = \frac{1}{u_e} \frac{du_e}{dS} - B(L,r) \frac{1}{r} \frac{dr}{dS} \quad (2.33)$$

The function  $C'(L,r)$  and  $B(L,r)$  are given in reference 5.

Equations (2.32) and (2.33) are two first order differential equations which are solved by a fourth order Runge-Kutta procedure.

### 2.3.3 Transition and separation

Transition from laminar to turbulent flow can occur if the boundary layer becomes unstable (dictated by Schlichting's stability curve). The unstable region is :

$$L + 0.011 - 0.005 X < 0 \quad (2.34)$$

Whether transition will actually occur depends on the turbulence level, pressure gradient, and  $Re_0$ . A transition criterium has been developed by B. Roberts (Ref. 6) based on the experimental results of P.S. Granville. This criteria predicts transition if  $\Delta Re_0$  (the difference in Reynolds number between point in instability and point of transition) is larger than a specified amount (see Ref. 6).

The boundary layer may separate (laminar separation) before transition. Zero wall shear stress is used as the separation criteria. In the calculation of L, the integration constant is chosen so that L equals zero at separation. This corresponds to :

$$H_{12} = 4.03$$

$$H_{32} = 1.515$$

Since the inviscid model makes no allowances for laminar separation regions, some approximations must be introduced. In all cases laminar separation bubbles followed by turbulent reattachment is suspected, the length of the bubble is neglected and a turbulent boundary layer is assumed from the point of laminar separation.

#### 2.3.4 The turbulent boundary layer

The assumed velocity profile is a modified Falkner-Skan profile :

$$\frac{u}{u_e} = 1 - \frac{u_\tau}{u_e} \frac{g(\eta, 2)}{K} \quad (2.35)$$

The function  $g(\eta, Z)$  in equation (2.35) is determined from a combination of experimental results and theory (see Ref. 5 for further details). The turbulent boundary layer is also applicable to compressible flow by the inclusion of Van Driest's generalized velocity.

The skin friction coefficient is determined from :

$$\frac{u_\tau}{u_e} = \frac{K}{\lambda - .5 - \ln \left( \frac{2/3 + \lambda/2}{K} \right) + KC + \ln Re_\delta^*} \quad (2.36)$$

The momentum and energy integral equations may be combined to obtain :

$$c_D = f(\delta, \theta, \theta_H) c_f \quad (2.37)$$

If for a given Mach number, similar solutions exist, a relation between the pressure gradient and velocity profile parameter  $G$  must also exist. Assume that :

$$\pi = \left( \frac{G+1.635}{6.1} \right)^2 - 1.8 \quad (2.38)$$

where :

$$\pi = \frac{\delta_i^*}{\tau_w} \frac{dp}{dS}$$

and :

$$G = \frac{u_e}{u_\tau} \left( 1 - \frac{1}{H_{12f}} \right)$$

with these assumptions, sufficient information is available to solve the turbulent boundary layer characteristics. See reference 5 for further details.

Turbulent boundary layer separation is also indicated when  $L$  equals 0. At this point, the boundary layer calculation is stopped.

#### 2.4 The source distribution

A source distribution is required for the separation region model. This distribution must be chosen in such a way as to maintain constant pressure (constant resultant velocity) in the separation region. Equation (2.1) which is in effect an expression for the resultant velocity on the airfoil indicates

that the source distribution should be a function of :

- (1) profile geometry;
- (2) upstream condition;
- (3) angle of attack;
- (4) separation point location;
- (5) velocity distribution.

This is a paradoxical situation. The source distribution depends on the velocity on the contour which in turn depends on the source distribution. Also, for a given airfoil, the fundamental relationship for the source distribution must be general enough to allow for changes in angle of attack. To change the source function for different angles of attack would defeat the purpose of using a functional relationship.

Figure 3 shows a typical source distribution used in this study. The functional relationship is :

$$Q = 2.5S^*V_{sep.}\sin\alpha \quad 0 \leq S^* \leq 0.4$$

$$Q = V_{sep.}\sin\alpha$$

$$Q = V_{sep.}\sin\alpha - (V_{sep.}\sin\alpha - 0.2)(5S^* - 3)^2 / 4$$

where :

$S^*$  is the distance along the contour in the separated region

$V_{sep.}$  is the velocity in the separated region

This relationship is dependent on angle of attack, velocity in the separated point and location of separation point. It does not include influence of the velocity distribution (over the remainder of the airfoil) on the separated region or influence of the profile geometry. A function which includes all of these aspects was too difficult to determine.

A completely different approach to the problem, which avoids functional source distributions altogether was attempted. This involves an iterative procedure to find a source distribution maintained constant velocity in the separated region, but this procedure failed to converge.

Another possible approach would be to a priori specify the velocity in the separated region, and then analytically solve the inverse problem to find the source strength. There have been recent research efforts at VKI to model wing tunnel wall interference effects using source distributions determined by an inverse procedure. This research has shown that a solution of a nearly singular system of equations is necessary to find the appropriate source distribution. This sheds a little light on the question why the iterative processes failed.

In the unseparated region a source distribution for the boundary layer model is determined from :

$$Q = \frac{d}{dS} (u_e \delta^*) \quad (2.40)$$

This relationship determines the source strength required to compensate for the displacement thickness. This approach (the blowing approach) is considered easier to use than the alternative approach of using a displacement surface for a couple of reasons : the profile geometry does not have to be recalculated. The blowing approach is also more compatible with the separated flow model.

## 2.5 Forces and moment on the airfoil

### 2.5.1 The pressure coefficient

The pressure coefficient is computed from the classical relationship :

$$c_p = 1 - \frac{w_L^2}{u_\infty^2} \quad (2.41)$$

where :

$$c_p = \frac{p - p_\infty}{q_\infty} \quad q_\infty = \frac{1}{2} \rho u_\infty^2 \quad (2.42)$$

These expressions are a result of Bernoulli's equation and are valid because this is an inviscid model. It is assumed that the pressure on the airfoil is the same as the pressure along the free streamline. Also, no wake effects have been included.

### 2.5.2 Lift coefficient

The lift coefficient can be computed directly from the Kutta Joukowski law :

$$\Gamma = \oint \gamma ds$$

$$L = \Gamma \rho u_\infty \quad (2.43)$$

$$c_l = 2\Gamma$$

### 2.5.3 Drag coefficient

An accurate determination of the drag for an airfoil with turbulent boundary layer separation is very difficult due to the following :

(1) Integral boundary layer methods often do not predict the separation point exactly because these methods are not valid close to the separation points.

(2) Singularity methods are not accurate near the airfoil trailing edge.

(3) Surface roughness and freestream turbulence have a large influence.

(4) Wake effects have an effect especially when there is separation.

(5) The equivalent Kutta condition does not model the physical situation exactly.

(6) Deviations from constant pressure in the separated region can have a significant effect.

With these considerations in mind, the total drag is composed of a pressure contribution due to incomplete pressure recovery, and a viscous contribution due to the viscous shearing stresses. The viscous drag is :

$$C_{d_v} = \frac{D_v}{\frac{1}{2} \rho u_\infty^2 c} = \cos \alpha \int_0^{2\pi} c_f(\phi) \frac{dx}{d\phi} \left( \frac{w_L}{u_\infty} \right)^2 d\phi + \sin \alpha \int_0^{2\pi} c_f(\phi) \frac{dy}{d\phi} \left( \frac{w_L}{u_\infty} \right)^2 d\phi \quad (2.44)$$

The pressure drag is :

$$C_{d_p} = \frac{D_p}{\frac{1}{2} \rho u_\infty^2 c} = - \cos \alpha \int_0^{2\pi} C_p(\phi) \frac{dy}{d\phi} d\phi + \sin \alpha \int_0^{2\pi} C_p(\phi) \frac{dx}{d\phi} d\phi \quad (2.45)$$

The contribution of the separated region to the viscous drag is zero ( $c_f$  is zero in this region).

#### 2.5.4 Pitching moment coefficient

The pitching moment is the integral of the pressure distribution over the contour and is given by :

$$C_{m_{1/4c}} = \frac{M}{\frac{1}{2} \rho u_{\infty}^2 c^2} = - \int_0^{2\pi} C_p(0.25-x) \frac{dx}{d\phi} d\phi + \int_0^{2\pi} C_p(0.25-x) \frac{dx}{d\phi} d\phi \quad (2.46)$$

Expression (2.46) is the pitching moment coefficient about the quarter chord.

## 2.6 The computer program

Four computer programs have been written which perform the calculations described in the preceeding sections. The basic computational procedure is outlined in figure 4 and listings of the program along with examples of the input and output are in Appendix B.

The calculations have been split into four programs to facilitate ease of use and also to allow modification of the results of each program if required. Duplication of computation is also avoided, for example, if for a given airfoil, the angle of attack is changed, the calculation may start with the second program (instead of the first program).

### 3. RESULTS

The separated turbulent boundary layer for the GA(W-1) and the NACA 2412 airfoil have been investigated. The results of these investigations are presented in figures 5 through 18.

For the GA(W-1) airfoil, figures 5 and 6 show the inviscid and viscous pressure coefficient for low angles of attack. Notice that the viscous calculated pressure coefficient agrees very well with the experimental results (Ref. 7) except near the leading edge where there is a short laminar separation bubble.

At angles of attack greater than four degrees, computational difficulties arose. The boundary layer program was originally written and calibrated for turbomachinery applications and in cases where the pressure peaks near the leading edge become too great, the boundary layer calculation failed. A second problem was a short laminar separation bubble which existed throughout the angle of attack range.

Figures 7-14 show the results of the calculation where the experimental separation point has been used. The boundary layer model has not been included. The source distribution used for the separation region has been optimized for an angle of attack of about 18 degrees, hence for other angles of attack, the pressure distribution is not as constant (in the separation region) as it is for 18 degrees.

Figures 15 and 16 show the lift and pitching moment coefficient versus angle of attack respectively. Agreement with the experimental results is quite good. There is a deviation of the computed pitching moment coefficient at the high angles of attack. This is due to the calculated pressure distribution which differs slightly from the experimental pressure distribution for the high angle of attack cases. Since the boundary layer program failed, a calculation of the drag was not achieved.

Figure 17 shows the results of a computed pressure distribution for NACA 2414 airfoil. A very flat pressure distribution was obtained for this angle of attack. The lift curve for this airfoil is presented in figure 18. To obtain results for the NACA 2412 which included a prediction of the separation point the computational procedure was again modified.

(1) The extremely high pressure peak near the leading edge was lowered (first five percent of airfoil).

(2) An estimated separation point was used.

(3) The boundary layer program was then used to validate the estimated separation point location.

It was felt that lowering the pressure peak was justifiable because the boundary layer program predicted laminar separation in the first five per cent. A short laminar separation bubble would lower the pressure peak and since there is no laminar separation model, the peak was lowered by using engineering judgement. Figure 17 shows that a good prediction of  $C_{l_{max}}$  was obtained.

#### 4. CONCLUSIONS AND RECOMMENDATIONS

The following can be concluded about the investigation :

- (1) The model appears to be a valid model for turbulent boundary layer separation.
- (2) Could improve the method of determining the source distribution for the separated region. The inverse problem is a possibility. It will be very difficult to define an iterative procedure to find the source strength because of convergence problems.
- (3) The boundary layer program should be extended so that it can handle airfoils with high pressure peaks and at high Reynolds number.
- (4) A laminar separation model should be included.
- (5) The lift, pitching moment, and pressure distribution have been accurately predicted for two airfoils.
- (6) The drag has not been predicted.
- (7) A wake model should be incorporated.
- (8) Extension of the computer model to compressible flow is worth considering.

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APPENDIX A - DERIVATION AND SOLUTION  
OF THE INTEGRAL EQUATIONS

Derivation of the integral equations

This appendix contains a detailed derivation of integral equations. Attention is also given to the solution of these equations.

The complex velocity at point  $x, y$  induced by a vortex at  $\xi, \eta$  is :

$$\bar{W}(z) = u - iv = - \frac{i}{2\pi} \frac{\gamma(\rho)}{X + iY} \quad (A.1)$$

where :

$$X = x - \xi \quad \text{and} \quad Y = y - \eta$$

The complex velocity due to a distribution of vortices on a closed contour is :

$$\bar{W}(z) = - \frac{i}{2\pi} \oint \frac{\gamma(\zeta)}{X + iY} dS(\zeta) \quad (A.2)$$

separating the complex velocity into its components :

$$U(z) = - \frac{1}{2\pi} \oint \frac{Y\gamma(\delta)}{X^2 + Y^2} dS(\zeta) \quad (A.3)$$

$$V(z) = \frac{1}{2\pi} \oint \frac{X\gamma(\rho)}{X^2 + Y^2} \quad (A.4)$$

A uniform stream must be superimposed :

$$\bar{W}_\infty = U_\infty + iV_\infty \quad (A.4)$$

$$U(z) = U_{\infty} - \frac{1}{2\pi} \oint \frac{Y\gamma(\zeta)}{X^2+Y^2} dS(\zeta) \quad (A.6)$$

$$V(z) = V_{\infty} + \frac{1}{2\pi} \oint \frac{X\gamma(\zeta)}{X^2+Y^2} dS(\zeta) \quad (A.7)$$

Sources are placed on the contour for the boundary layer model.

$$W(z) = \frac{1}{2\pi} \frac{Q(\zeta)}{X+iY} \quad (A.8)$$

For sources distributed on a contour, the complex velocity is :

$$W(z) = \frac{1}{2\pi} \oint \frac{Q(\zeta)}{X+iY} dS(\zeta) \quad (A.9)$$

Superimposing the source distribution :

$$U(z) = U_{\infty} - \frac{1}{2\pi} \oint \frac{Y\gamma'(\zeta)}{X^2+Y^2} dS(\zeta) + \frac{1}{2\pi} \oint \frac{XQ(\zeta)}{X^2+Y^2} dS(\zeta) \quad (A.10)$$

$$V(z) = V_{\infty} + \frac{1}{2\pi} \oint \frac{Y\gamma(\zeta)}{X^2+Y^2} dS(\zeta) - \frac{1}{2\pi} \oint \frac{YQ(\zeta)}{X^2+Y^2} dS(\zeta) \quad (A.11)$$

The kinetic boundary condition that the velocity normal to the airfoil surface is zero is enforced. This is equivalent to stating that the surface is a streamline.

$$\frac{\partial \psi}{\partial S} = 0$$

$$\frac{\partial \psi}{\partial S} = \frac{\partial \psi}{\partial x} \frac{dx}{dS} + \frac{\partial \psi}{\partial y} \frac{dy}{dS}$$

$$0 = U(z) \frac{dy}{dS} - V(z) \frac{dx}{dS} \quad (A.12)$$

Introducing (A.10) and (A.11) one obtains :

$$\begin{aligned} 0 = U_{\infty} \frac{dy}{dS} - \left[ \frac{1}{2\pi} \oint \frac{Y\gamma(\zeta)}{X^2+Y^2} dS(\zeta) \right] \frac{dy}{dS} \\ + \left[ \frac{1}{2\pi} \oint \frac{XQ(\zeta)}{X^2+Y^2} dS(\zeta) \right] \frac{dy}{dS} \\ - V_{\infty} \frac{dx}{dS} - \left[ \frac{1}{2\pi} \oint \frac{X\gamma(\zeta)}{X^2+Y^2} dS(\zeta) \right] \frac{dx}{dS} \\ + \left[ \frac{1}{2\pi} \oint \frac{YQ(\zeta)}{X^2+Y^2} dS(\zeta) \right] \frac{dx}{dS} \end{aligned} \quad (A.13)$$

Since  $dS(\zeta)$  is not a function of  $x, y$  the derivatives  $\frac{dy}{dS}$  and  $\frac{dx}{dS}$  can be taken inside the integral.

$$\begin{aligned} U_{\infty} \frac{dy}{dS} - V_{\infty} \frac{dx}{dS} = \frac{1}{2\pi} \oint \gamma(\zeta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) \\ + \frac{1}{2\pi} \oint Q(\zeta) \left[ f_x(z, \zeta) \frac{dx}{dS} - f_y(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) \end{aligned} \quad (A.14)$$

where :

$$f_y(z, \zeta) = \frac{X}{X^2+Y^2} \quad f_x(z, \zeta) = - \frac{Y}{X^2+Y^2}$$

Integral expression (A.14) can be split into one homogeneous and three non-homogeneous equations corresponding to the four basic flows described in chapter 2.

$$\frac{1}{2\pi} \oint \alpha(\zeta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) = \frac{dy}{dS} \quad (A.15)$$

$$\frac{1}{2\pi} \oint \beta(\zeta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) = \frac{dx}{dS} \quad (A.16)$$

$$\frac{1}{2\pi} \oint \mu(\zeta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dy}{dS} \right] dS = 0 \quad (A.17)$$

$$\frac{1}{2\pi} \oint \delta(\zeta) \left[ f_y(z, \zeta) \frac{dx}{dS} - f_x(z, \zeta) \frac{dy}{dS} \right] dS(\zeta) \quad (A.18)$$

$$= - \frac{1}{2\pi} \oint Q(\zeta) \left[ f_x(z, \zeta) \frac{dx}{dS} - f_y(z, \zeta) \frac{dy}{dS} \right] dS(\zeta)$$

The total vortex strength is the superposition of the four basic vortex strengths.

$$\gamma(\zeta) = U_\infty \alpha(\zeta) - V_\infty \beta(\zeta) + \Gamma \mu(\zeta) + \delta(\zeta) \quad (A.19)$$

Equations (A.15) through (A.18) are obtained by substituting equation (A.19) into (A.14) and collecting like terms.

The far field boundary condition (uniform stream far upstream) has been implicitly satisfied. This can be demonstrated by taking the limit of (A.10) and (A.11). As  $x$  approaches infinity, the velocity components become equivalent to the imposed uniform stream. This result shows that the singularities do not have an influence far upstream.

The supplementary equations (as described in chapter 2) are :

$$\oint \alpha(\zeta) dS(\zeta) = 0 \quad (A.20)$$

$$\oint \beta(\zeta) dS(\zeta) = 0 \quad (A.21)$$

$$\oint \mu(\zeta) dS(\zeta) = 1 \quad (A.22)$$

$$\oint \delta(\zeta) dS(\zeta) = 0 \quad (A.23)$$

The equivalent Kutta condition fixes the circulation :

$$\gamma(\text{TEPS}) = - \gamma(\text{TESS}) \quad (A.24)$$

Using equations (A.19 and A.24) gives :

$$\Gamma = \frac{-U_{\infty} [\alpha(\text{TEPS}) + \alpha(\text{TESS})] + V_{\infty} [\beta(\text{TEPS}) + \beta(\text{TESS})] + [\delta(\text{TEPS}) + \delta(\text{TESS})]}{[\mu(\text{TEPS}) + \mu(\text{TESS})]} \quad (A.25)$$

### Solution of the integral equations

The solution of the integral equations is complicated by the fact that expressions (A.15) through (A.18) are singular integrals. Before the discussion of the singular integrals, a coordinate transformation which allows greater numerical accuracy will be performed :

$$x = \frac{c}{2} (1 - \cos \phi) \quad (A.26)$$

Hence :

$$\frac{dy}{dS} = \frac{dy}{d\phi} \frac{d\phi}{dS} \quad \frac{dx}{dS} = \frac{dx}{d\phi} \frac{d\phi}{dS}$$

$$dS(z) = \sqrt{\left(\frac{\partial x}{\partial \phi}\right)^2 + \left(\frac{\partial y}{\partial \phi}\right)^2} d\phi(z) \quad (A.27)$$

Similarity :

$$dS(\zeta) = \sqrt{\left(\frac{\partial \xi}{\partial \phi}\right)^2 + \left(\frac{\partial \eta}{\partial \phi}\right)^2} d\phi(\zeta) \quad (A.28)$$

A new variable will be introduced :

$$\gamma'(\zeta) = \gamma(\zeta) \sqrt{\left(\frac{\partial \xi}{\partial \phi}\right)^2 + \left(\frac{\partial \eta}{\partial \phi}\right)^2} \quad (A.29)$$

This definition of the vortex strength has no physical significance, it is just used to simplify the derivation. This definition also applies to the basic flow vortex strengths.

#### The singular integral

Expressions (A.15) through (A.18) are singular integrals. This computational difficulty can be avoided by finding a function whose derivative is equal to the integral. Thus the integrals will become equivalent to the function evaluated between the limits of integration. The function  $I(z, \zeta)$  must be determined such that :

$$\frac{dI(z, \zeta)}{d\phi(\zeta)} = \left[ f_y(z, \zeta) \frac{dx}{d\phi} - f_x(z, \zeta) \frac{dy}{d\phi} \right] \quad (A.30)$$

Make the following assumptions :

- (1)  $\gamma'(\zeta)$  is constant over the distance  $\phi - \frac{\Delta\phi}{2}$  to  $\phi + \frac{\Delta\phi}{2}$  so that the vortex strength in (A.15) through (A.18) can be taken out from under the integral sign.
- (2) Also assume that  $\frac{\partial x}{\partial \phi(\gamma)}$  and  $\frac{\partial y}{\partial \phi(\zeta)}$  are constant in that region.

Define the following two functions :

$$F(z, \zeta) = \operatorname{atan} \frac{X}{Y}$$

$$G(z, \zeta) = \frac{1}{2} \ln (X^2 + Y^2)$$

Hence :

$$\frac{dG(z, \zeta)}{d\phi(\zeta)} = \frac{-X \frac{d\xi}{d\phi(\zeta)} - Y \frac{d\eta}{d\phi(\zeta)}}{X^2 + Y^2}$$

$$\frac{dF(z, \zeta)}{d\phi(\zeta)} = \frac{X \frac{d\eta}{d\phi(\zeta)} - Y \frac{d\xi}{d\phi(\zeta)}}{X^2 + Y^2}$$

Now multiply  $\frac{dF}{d\phi(\zeta)}$  by  $C_1$  and  $\frac{dG}{d\phi(\zeta)}$  by  $C_2$  where :

$$C_1 = \frac{\frac{d\xi}{d\phi(\zeta)} \frac{dx}{d\phi(z)} - \frac{dx}{d\phi(z)} \frac{d\eta}{d\phi(\zeta)}}{\left(\frac{d\xi}{d\phi(\zeta)}\right)^2 + \left(\frac{d\eta}{d\phi(\zeta)}\right)^2}$$

$$C_2 = \frac{\frac{d\xi}{d\phi(\zeta)} \frac{dx}{d\phi(z)} + \frac{dy}{d\phi(z)} \frac{d\eta}{d\phi(\zeta)}}{\left(\frac{d\xi}{d\phi(\zeta)}\right)^2 + \left(\frac{d\eta}{d\phi(\zeta)}\right)^2}$$

After simplification, it can be shown that the function  $I(z, \zeta)$  in (A.30) has been obtained.

$$\frac{dI(z, \zeta)}{d\phi(\zeta)} = C_1 \frac{dF}{d\phi(\zeta)} + C_2 \frac{dG}{d\phi(\zeta)} \quad (A.31)$$

$$I(z, \zeta) = C_1 F(z, \zeta) + C_2 G(z, \zeta) \quad (A.32)$$

Hence the integral equations (A.15) through (A.18) reduce to the following summations :

$$\frac{1}{2\pi} \sum_{i=1}^N \alpha'(\zeta_i) K(z, \zeta_i, \Delta\zeta_i) = \frac{dy_j}{d\phi} \quad (A.33)$$

$$\frac{1}{2\pi} \sum_{i=1}^N \beta'(\zeta_i) K(z, \zeta_i, \Delta\zeta_i) = \frac{dx_j}{d\phi} \quad (A.34)$$

$$\frac{1}{2\pi} \sum_{i=1}^N \mu'(\zeta_i) K(z, \zeta_i, \Delta\zeta_i) = 0 \quad (A.35)$$

$$\frac{1}{2\pi} \sum_{i=1}^N \delta'(\zeta_i) K(z, \zeta_i, \Delta\zeta_i) \quad (A.36)$$

$$= - \frac{1}{2\pi} \oint Q'(\zeta) \left[ f_x(z, \zeta) \frac{dx}{d\phi} - f_y \frac{dy}{d\phi} \right] dS(\zeta)$$

where :

$$K(z, \zeta_i, \Delta\zeta_i) = I \left[ z, \zeta_i + \frac{\Delta\zeta_i}{2} \right] - I \left[ z, \zeta_i - \frac{\Delta\zeta_i}{2} \right] \quad (A.37)$$

The supplementary conditions are :

$$\sum_{i=1}^N \alpha(\zeta_i) \Delta S(\zeta_i) = 0 \quad (A.38)$$

$$\sum_{i=1}^N \beta(\zeta_i) \Delta S(\zeta_i) = 0 \quad (A.39)$$

$$\sum_{i=1}^N \mu(\zeta_i) \Delta S(\zeta_i) = 1 \quad (A.40)$$

$$\sum_{i=1}^N \delta(\zeta_i) \Delta S(\zeta_i) = 0 \quad (A.41)$$

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*****
CURROUTINE SFC07
*****
      THIS SUBROUTINE SETS UP THE DATA FOR THE B. L. CALCULATION AND
      CALCULATES THE STRENGTHS OF THE SOURCES OF THE MODELLING OF THE
      HOUDARY LAYER AND ALSO THE SEPARATION REGION. THERE IS SOME TRICKY
      MANIPULATION OF ARRAYS IN ORDER TO GET THE DATA IN THE CORRECT
      FORM FOR THE B. L. SUBPROGRAM. THE B. L. PROGRAM REQUIRES THE
      VELOCITY, PRESSURE, ARC LENGTH, AND RADII OF CURVATURE. THESE MUST
      BE READ IN STARTING FROM THE STAGNATION POINT. THE PROGRAM SOLVES
      FOR THE HOUDARY LAYER ON THE SUCTION SIDE AND THEN IN A SECOND
      CALL FOR THE HOUDARY LAYER ON THE PRESSURE SIDE. THE B. L.
      PROGRAM ALSO RETURNS THE LOCATION OF THE SEPARATION POINT. IF THE
      IS NO SEPARATION POINT, THE PROGRAM RETURNS SEP=0.
      THERE IS NO LIMITATION ON WHERE SEPARATION OCCURS PROVIDING A PROPER
      MODEL (SOURCE DISTRIBUTION) IS GIVEN. HOWEVER THERE IS A LIMITATION
      ON THE STAGNATION POINT LOCATION. THE STAGNATION POINT MUST BE ON
      THE PRESSURE GIBE OF AT THE L.E. BUT NOT ON THE SUCTION SIDE.
      *****
      COMBDA/ACCF/MIN2,TFL,KPCF(10),NLP,P1
      COMBDA/CCCF/X(40),Y(40),XPC(10),YP(40),RC(480)
      COMBDA/ERFC/SSEP,SI,SSEP,JJ,AM,INEP,ITERBL,ITMAX,IERRNT,
      ILOVE,FULLC,SSEPS,SSIFC,CUY(480)
      COMBDA/GRFC/GIX,GIV,GZA,CZY
      COMBDA/PRFC/XSAVE(240),CISAVE(240)
      COMBDA/GPFC/DELTS(480),SDELTS(480),CODEL(480),DCUDS(480),IDS,
      TIME(1490)
      COMBDA/IRFC/CDFR(150)
      COMBDA/AS(32),CUMD(32),DK(32),REFEF,PAREF,TRCF
      COMBDA/CCF,RCC(32),RCF,CLD(32),KEI,MDEF,IDCM
      COMBDA/SWMC,NR
      DIMENSION CUXDEL(480),SF(480),SQ(480)
      NITEM=1
      NITEM=1
      NITEM=1

```

[illegible][illegible]



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C*****
SUBROUTINE SFC05
COMMON /CFC2/ I02(240,240)
COMMON/ACFC/ N1,I1,I2,EFCF,U(10),NIR,FI
COMMON/CCFC/X(480),I(480),AP(480),YP(480),NC(480)
COMMON/ESC/ C1X,C1Y,C2X,C2Y
COMMON/ESFC/ SC(N0),H1,SSSEP,JI,AM,INEP,ITERBL,ITMAX,IPRNT,
1 TURP,TDLL,SSPPSS,SSPPSS,CDY(480)
COMMON/FSFC/ ASAVE(240),CUSAVE(240)
COMMON/ESFC/ G
COMMON/MSFC/ SQ(480),CU(240),SP(480),CO(4,240)
DIMENSION CS(480),SS(480),CUX(480),CUY(480),S(240)
N1=N1
N2=N1+1
N3=N1/2+1
N4=N1/2
N5=N1
N6=N1+1
N7=N1+1
N8=N1+1
N9=N1+1
N10=N1+1
N11=N1+1
N12=N1+1
N13=N1+1
N14=N1+1
N15=N1+1
N16=N1+1
N17=N1+1
N18=N1+1
N19=N1+1
N20=N1+1
N21=N1+1
N22=N1+1
N23=N1+1
N24=N1+1
N25=N1+1
N26=N1+1
N27=N1+1
N28=N1+1
N29=N1+1
N30=N1+1
N31=N1+1
N32=N1+1
N33=N1+1
N34=N1+1
N35=N1+1
N36=N1+1
N37=N1+1
N38=N1+1
N39=N1+1
N40=N1+1
N41=N1+1
N42=N1+1
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N44=N1+1
N45=N1+1
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N98=N1+1
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N100=N1+1
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N330=N1+1
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N332=N1+1
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N341=N1+1
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N345=N1+1
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N348=N1+1
N349=N1+1
N350=N1+1
N351=N1+1
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N373=N1+1
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N422=N1+1
N423=N1+1
N424=N1+1
N425=N1+1
N426=N1+1
N427=N1+1
N428=N1+1
N429=N1+1
N430=N1+1
N431=N1+1
N432=N1+1
N433=N1+1
N434=N1+1
N435=N1+1
N436=N1+1
N437=N1+1
N438=N1+1
N439=N
```

```

153 CONTINUE
   IF(J=1,EO,2) GO TO 157
   CU(1)=CU(1,1)+C1X-CU(2,1)*C1Y+G*CU(3,1)+CU(4,1)
157 CONTINUE
184 CONTINUE
   IF(J=1,EO,1) GO TO 98
   GO TO 97
98 WRITE(2,110) B1K
   GO TO 99
97 CONTINUE
   DO 70 K=1,N
   IF(CF=1) 72,70,71
72 JK=K
   GO TO 73
71 JK=K-1
73 JK=2,PI*(C-4-X(K))/T
   YK=-PI*Y(A)/T
   C1Y=C1X-CU(JK)*PI*(1+0)*S1H(YK)/(CUSH(AK))-COS(YK)
   C1Y=C1Y+CU(JK)*PI*(1+0)*S1H(AK)/(CUSH(AK))-COS(YK)
   AK=2,PI*(1/5-X(N))/T
   C2X=C1X-CU(JK)*PI*(1+0)*S1H(YK)/(CUSH(AK))-COS(YK)
   C2X=C2X+CU(JK)*PI*(1+0)*S1H(AK)/(CUSH(AK))-COS(YK)
70 CONTINUE
   B1R=A1X*(C1Y/C1X)
   B1R=B1R+180./PI
   C1X=SGN(C1X)*C1Y+C1Y
   B2P=A1X*(C2Y/C2X)
   B2P=B2P+180./PI
   C2X=SGN(C2X)*C2Y+C2Y
   PRINT(2,74)PIR,C1X,B2P,C2X
99 CONTINUE
20 DO 20 I=1,N2
   SO(1)=SGH(XP(I)*XF(I)+YF(I)*YF(I))
   DF=3.14159/FLOAT(N2)
   SP(1)=0.
   DO 11 I=2,NT
   J=I-1
   JL=J+1
   SP(1)=SP(J)+SO(J)*DF+SO(JL)*2*DF+SO(L)*LF
   L=L+2-I
   JL=L+1
   J=L+1
   IF(J=N-1)10,12,12
12 J=1
10 SP(1)=SP(J)+SO(J)*DF+SO(JL)*2*DF+SO(L)*LF
11 CONTINUE
   DO 63 I=1,NH
   L=L+2-I
   JL=L+1
   CU(L)=CU(L-1)
   DO 1 I=1,N
   CU(1)=CU(I)/SO(I)
   DO 30 ID=1,N
   K=1L
   AF(K)/=((XF(K)+XP(K))*YF(K))*0.5)
   AG(K)=1P(K)/((XP(K)+XP(K))*YF(K))*0.5)
   CH(K)=CU(K)*SS(K)+G(CD)*SS(K)
   CU(K)=CU(K)*SS(K)+G(CD)*SS(K)
   CO(K)=CU(K)*CUX(K)+CUY(K)*CUY(N)*0.5*CH(K)/AFSCU(N)
30 CO(K)=CU(K)*CUX(K)+CUY(K)*CUY(N)*0.5*CH(K)/AFSCU(N)
   CHECK FOR CONVERGENCE OF THE HOMOLOGY LAYER CALCULATION.
   IRE=2
   DO 31 I=1,N

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[illegible]





[illegible]

```

13 V(J)=R(11)
   CALL LARAH(2,V,N,SA,DRA)
   DO 14 J=1,NP
     IT=I-4*J
14 V(J)=R(11)
   CALL LARAH(2,V,N,SA,RCA)
   C MODIF 3 2 74
     IF=2
     NEND
   C
   DO 11 J=1,NP
     IT=I-4*J
   C MODIF 3 2 74
     Z(J)=S(11)
   C
11 V(J)=R(11)
   CALL LARAH(2,V,N,SA,DOA)
12 C 7.11.10E
   END
C** TIME RING
SUBROUTINE MAGH(PRT1,Y,DERY,NDIM,INLF,AUX,CUL)
  DIMENSION Y(2),DERY(2),AUX(8,2),A(4),B(4),C(4),PRMT(5),CUL(2)
  DO 1 I=1,NP
    1 A(I)=9.12E-06000667*DERY(1)
    X=PRMT(1)
    XEND=PRMT(2)
    N=PRMT(3)
    PRMT(5)=2
    CALL PSY(X,Y,DERY)
  C FROM TEST
  C
  IF(0*(XEND-X))3N,37,2
C PREPARATIONS FOR RUNGE-KUTTA METHOD
2 A(1)=2
  A(2)=2248932
  A(3)=1.79107
  A(4)=1.060057
  C(1)=2
  C(2)=1
  C(3)=1
  C(4)=2
C PREPARATIONS FOR FIRST RUNGE-KUTTA STEP
DO 3 I=1,NP
  AUX(1,I)=Y(1)
  AUX(2,I)=DERY(1)
  AUX(3,I)=0
  AUX(4,I)=0
  3 INTCU
  NEND
  INLF=1
  ISTER=0
  ISTER=0
C START OF A RUNGE-KUTTA STEP
4 IF((X+H-XEND)*H)7,6,5
5 H=H*10-X
6 IER=1
C RECORDING OF INITIAL VALUES OF THIS STEP
7 CALL DMUC(X,Y,DERY,IREC,ADIM,PRMT,CUL)
8 ISTER=0
9 ISTER=1STEP+1
C START OF INNERMOST RUNGE-KUTTA LOOP
DO 10 J=1
  AJEA(J)
  HJER(J)
  CJEC(J)
  DO 11 I=1,NDIM
    FIER=DERY(I)
    K2ER2=HJ*AJEA(6,I)
    Y(I)=Y(I)+H2
    K2ER2=H2+H2
11 AUX(6,I)=AUX(6,I)+H2-CJ*H1
12 IF(J-4)12,15,15
12 JEJ=1
12 IF(J-3)13,19,13
13 X=X+5*H
14 CALL PSY(X,Y,DERY)
14 GO TO 10
  C END OF INNERMOST RUNGE-KUTTA LOOP
  C TEST OF ACCURACY
15 IF(ITER)16,16,20
  INCASE ITER=9, THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY
16 DO 17 I=1,NDIM
17 AUX(4,I)=Y(I)
  ITER=1
  ISTER=1STEP+1STEP-2
18 INLF=INLF+1
  X=X+H
  H=5*H
  DO 19 I=1,NDIM
    Y(I)=AUX(1,I)
    DERY(I)=AUX(2,I)
19 AUX(6,I)=AUX(3,I)
  GO TO 9
  C INCASE ITER=1, TESTING OF ACCURACY IS POSSIBLE
  C
20 INCM=1STEP/2
  IF(1STEP-1MOD-IP00)21,23,21
21 CALL PSY(X,Y,DERY)
  DO 22 I=1,NDIM
    AUX(5,I)=Y(I)
    AUX(7,I)=DERY(I)
  GO TO 9
  C COMPUTATION OF TEST VALUE DELI

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[illegible]





```

181 IF (AL) 15, 15, 13
15 IF (OU-0.2) 16, 182, 671
13 DALS=DALS
DALS=0.005*AA-0.011-AL
TPM=3.1E-7/(M*RE)
ITEST=HLCAL
IF (DALS) 151, 151, 14
151 ALKA=0.
ISCHI=1
GO TO 91
14 ITEST=ISTAR
GO TO (141, 142), ISCHI
141 ISCHI=2
IUIP=2
RT=RT
AK=RE*THC*THC*DU*SGRT(1.-GU*AMC12)*(1.-GU*AMC12*U*U)**((2.*G-1.)/(
1G-1.))
X11=X-1.E-5
X12=1.E-5
X1=X
142 AK=AF
AK=RE*THC*THC*DU*SGRT(1.-GU*AMC12)*(1.-GU*AMC12*U*U)**((2.*G-1.)/(
1G-1.))
ATRA=AF*0.5*(AK+AK1)*(X-X1)
X12=X1L*(X-X1)*(1.-GU*AMC12*U*U)**((3.*G-1.)*.5/(G-1.))
X1=X
AK=ATRA/X1L
IF (AK) 1421, 1422, 1422
1421 RT=RT+GS*400.*(EXP(80.*AKK)+1.)
GO TO 1423
1422 RT=RT+SS*32./(0.04-AKK)
1423 IF (RT-RT1) 91, 103, 103
103 IUIP=3
GO TO 10
182 AG=0
WRITE(2, 103)
C*****
WRITE(6, 104)
C*****
H32=1.5195
WRITE(2, 104)
C MODIF TO 8.278
DSC=DSC+MOD*GUP
IF (IPRT, ED, 1) WRITE(2, 102) X, H32, DSC, U
C*****
WRITE(2, 102) X, H32, DSC, U
IUIP=2
GO TO 1000
071 IUIP=3
WRITE(2, 109)
C*****
WRITE(6, 109)
C*****
GO TO 1000
999 IUIP=1
IUIP=PT.EQ.1 WRITE(2, 106)
1000 IUIP=1
106 FORMAT(//, 'LAMINAR BOUNDARY LAYER//', LFOU, ' + ST*AMTSCN//',
' RT= ', RT, ' TUN= ', TUN, ' VISC= ', VISC, ' LAMINAR MACH NUMBER= ', F7.3/
3X, ' H32= ', H32, ' IUIP= ', IUIP, ' DSC= ', DSC, ' H32= ', H32, ' IUIP= ', IUIP, '
101 FORMAT(2, 3(F9.4, 3X), 2E12.3, F9.4, 3X, F12.3, F9.4)
102 FORMAT(//, ' THE PROGRAM DIVIDES ADD IS STOPPED')
103 FORMAT(//, ' CONDITIONS AT TRANSITION POINT//', F10.5, 2E12.3, F10.5)
104 FORMAT(//, ' LAMINAR SEPARATION//')
105 FORMAT(//, ' TRANSITION//')
106 FORMAT(//, ' FULL LAMINAR E.D.//')

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[illegible]

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[illegible]

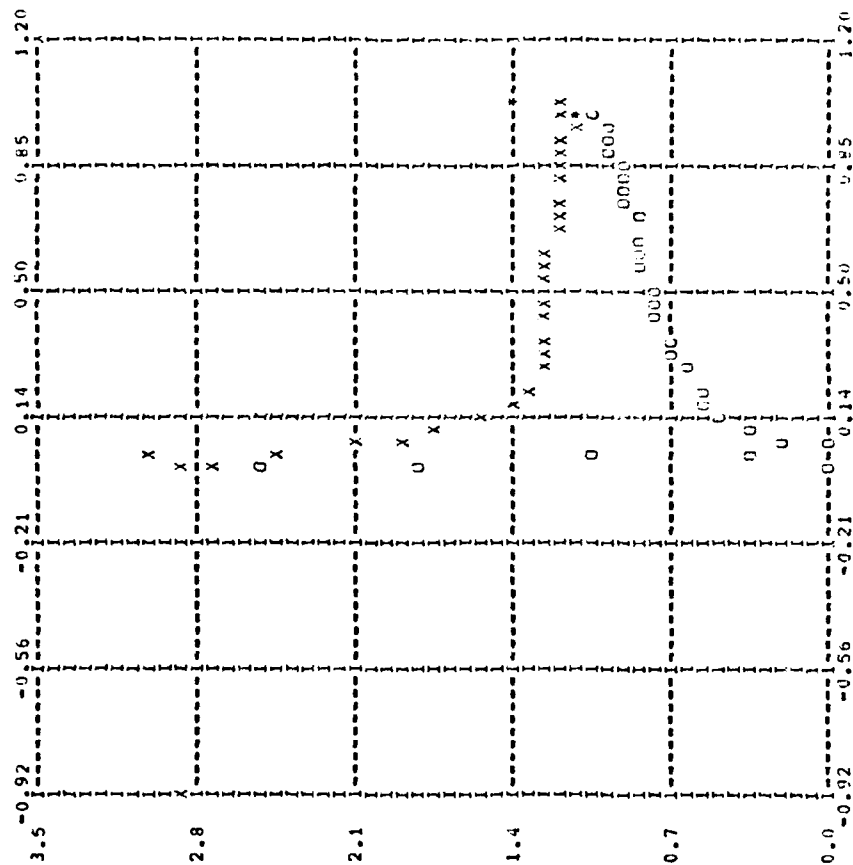




|                     |        |        |       |       |        |
|---------------------|--------|--------|-------|-------|--------|
| 90                  | 1.017  | -1.221 | 0.990 | 1.003 | 1.144  |
| 98                  | 1.024  | -1.319 | 0.998 | 1.010 | 1.366  |
| SLY/Q1 DISTRIBUTION |        |        |       |       |        |
|                     | UCTION | S/C    | LOC   | S     | LOC    |
| 00                  | 0.000  | 51.353 | 0.000 | 0.000 | 51.353 |
| 01                  | 0.003  | 9.359  | 0.001 | 0.003 | 9.359  |
| 02                  | 0.010  | 7.315  | 0.005 | 0.010 | 7.315  |
| 03                  | 0.023  | 6.445  | 0.013 | 0.023 | 6.445  |
| 04                  | 0.039  | 5.613  | 0.025 | 0.039 | 5.613  |
| 05                  | 0.061  | 4.834  | 0.043 | 0.061 | 4.834  |
| 06                  | 0.087  | 4.108  | 0.065 | 0.087 | 4.108  |
| 07                  | 0.115  | 3.434  | 0.091 | 0.115 | 3.434  |
| 08                  | 0.145  | 2.812  | 0.121 | 0.145 | 2.812  |
| 09                  | 0.177  | 2.242  | 0.153 | 0.177 | 2.242  |
| 10                  | 0.212  | 1.724  | 0.187 | 0.212 | 1.724  |
| 11                  | 0.249  | 1.256  | 0.229 | 0.249 | 1.256  |
| 12                  | 0.289  | 0.838  | 0.269 | 0.289 | 0.838  |
| 13                  | 0.331  | 0.470  | 0.314 | 0.331 | 0.470  |
| 14                  | 0.375  | 0.253  | 0.360 | 0.375 | 0.253  |
| 15                  | 0.421  | 0.135  | 0.407 | 0.421 | 0.135  |
| 16                  | 0.469  | 0.055  | 0.457 | 0.469 | 0.055  |
| 17                  | 0.517  | 0.032  | 0.505 | 0.517 | 0.032  |
| 18                  | 0.564  | 0.011  | 0.553 | 0.564 | 0.011  |
| 19                  | 0.610  | 0.003  | 0.600 | 0.610 | 0.003  |
| 20                  | 0.656  | 0.000  | 0.644 | 0.656 | 0.000  |
| 21                  | 0.700  | 0.000  | 0.686 | 0.700 | 0.000  |
| 22                  | 0.743  | 0.000  | 0.726 | 0.743 | 0.000  |
| 23                  | 0.785  | 0.000  | 0.764 | 0.785 | 0.000  |
| 24                  | 0.826  | 0.000  | 0.799 | 0.826 | 0.000  |
| 25                  | 0.866  | 0.000  | 0.832 | 0.866 | 0.000  |
| 26                  | 0.905  | 0.000  | 0.862 | 0.905 | 0.000  |
| 27                  | 0.943  | 0.000  | 0.889 | 0.943 | 0.000  |
| 28                  | 0.979  | 0.000  | 0.913 | 0.979 | 0.000  |
| 29                  | 1.014  | 0.000  | 0.934 | 1.014 | 0.000  |
| 30                  | 1.047  | 0.000  | 0.951 | 1.047 | 0.000  |
| 31                  | 1.079  | 0.000  | 0.964 | 1.079 | 0.000  |
| 32                  | 1.109  | 0.000  | 0.974 | 1.109 | 0.000  |
| 33                  | 1.137  | 0.000  | 0.981 | 1.137 | 0.000  |
| 34                  | 1.164  | 0.000  | 0.985 | 1.164 | 0.000  |
| 35                  | 1.189  | 0.000  | 0.987 | 1.189 | 0.000  |
| 36                  | 1.212  | 0.000  | 0.988 | 1.212 | 0.000  |
| 37                  | 1.233  | 0.000  | 0.988 | 1.233 | 0.000  |
| 38                  | 1.252  | 0.000  | 0.987 | 1.252 | 0.000  |
| 39                  | 1.269  | 0.000  | 0.985 | 1.269 | 0.000  |
| 40                  | 1.284  | 0.000  | 0.981 | 1.284 | 0.000  |
| 41                  | 1.297  | 0.000  | 0.976 | 1.297 | 0.000  |
| 42                  | 1.309  | 0.000  | 0.969 | 1.309 | 0.000  |
| 43                  | 1.319  | 0.000  | 0.960 | 1.319 | 0.000  |
| 44                  | 1.327  | 0.000  | 0.949 | 1.327 | 0.000  |
| 45                  | 1.334  | 0.000  | 0.936 | 1.334 | 0.000  |
| 46                  | 1.339  | 0.000  | 0.921 | 1.339 | 0.000  |
| 47                  | 1.342  | 0.000  | 0.894 | 1.342 | 0.000  |
| 48                  | 1.343  | 0.000  | 0.855 | 1.343 | 0.000  |
| 49                  | 1.342  | 0.000  | 0.803 | 1.342 | 0.000  |
| 50                  | 1.339  | 0.000  | 0.739 | 1.339 | 0.000  |
| 51                  | 1.333  | 0.000  | 0.664 | 1.333 | 0.000  |
| 52                  | 1.324  | 0.000  | 0.579 | 1.324 | 0.000  |
| 53                  | 1.312  | 0.000  | 0.486 | 1.312 | 0.000  |
| 54                  | 1.297  | 0.000  | 0.387 | 1.297 | 0.000  |
| 55                  | 1.279  | 0.000  | 0.284 | 1.279 | 0.000  |
| 56                  | 1.258  | 0.000  | 0.178 | 1.258 | 0.000  |
| 57                  | 1.234  | 0.000  | 0.070 | 1.234 | 0.000  |
| 58                  | 1.207  | 0.000  | 0.000 | 1.207 | 0.000  |
| 59                  | 1.177  | 0.000  | 0.000 | 1.177 | 0.000  |
| 60                  | 1.144  | 0.000  | 0.000 | 1.144 | 0.000  |

2412 TEST CASE  
.28319 FL= 0.07448 N= 64

NON DIMENSIONAL VELOCITY DISTRIBUTION



# THE MODEL

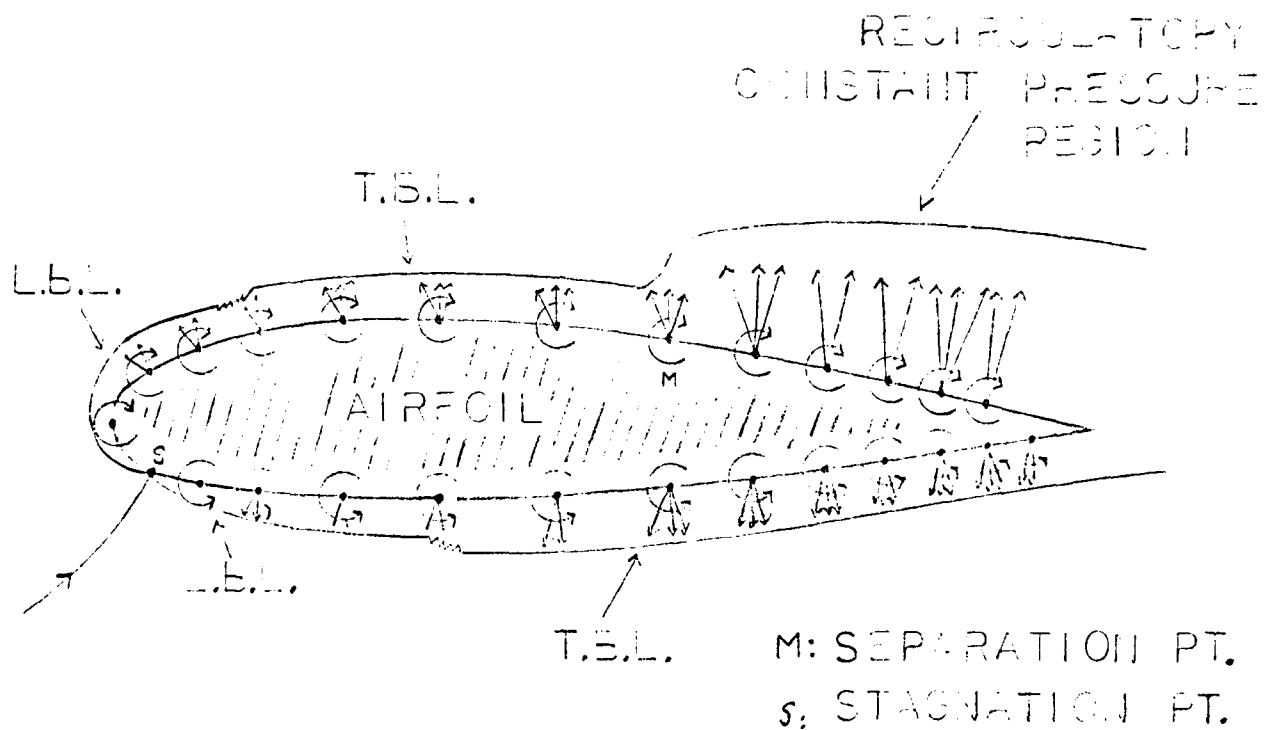


Fig. 1. The Model

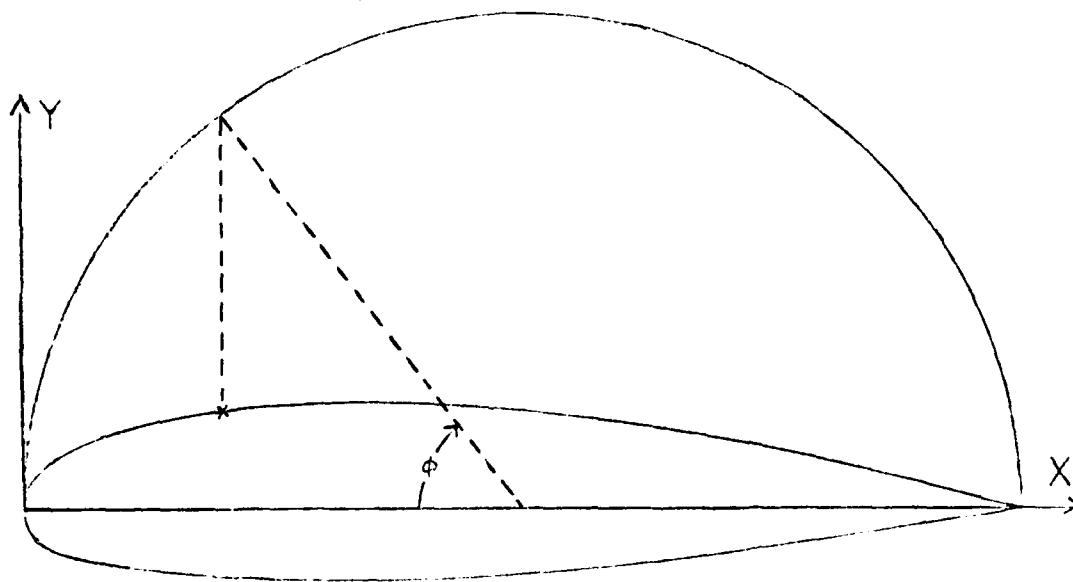


Fig. 2. The Airfoil Coordinate System

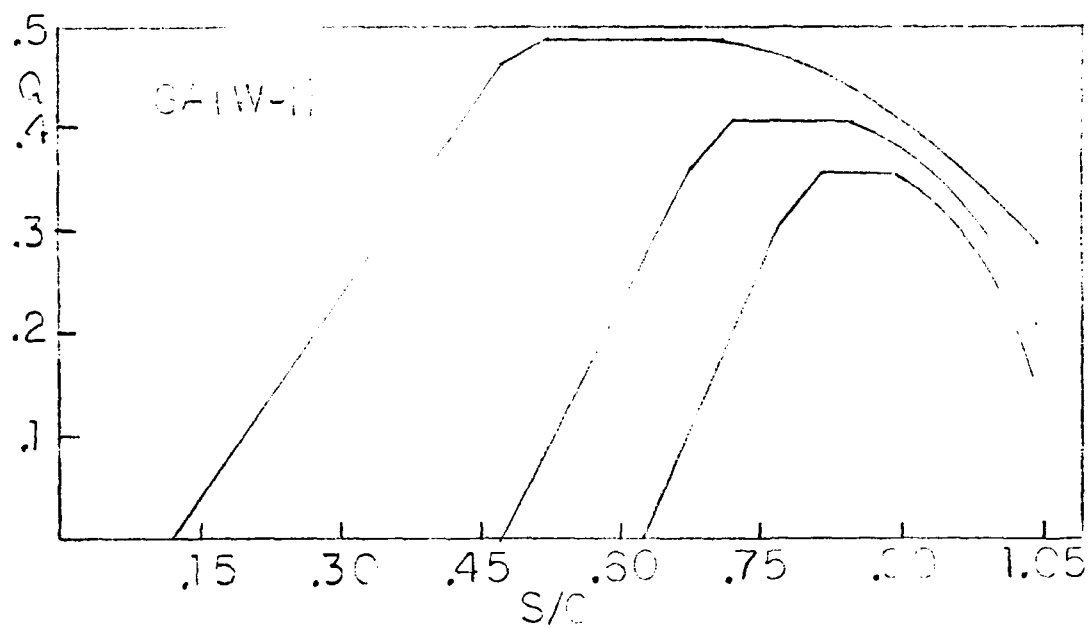
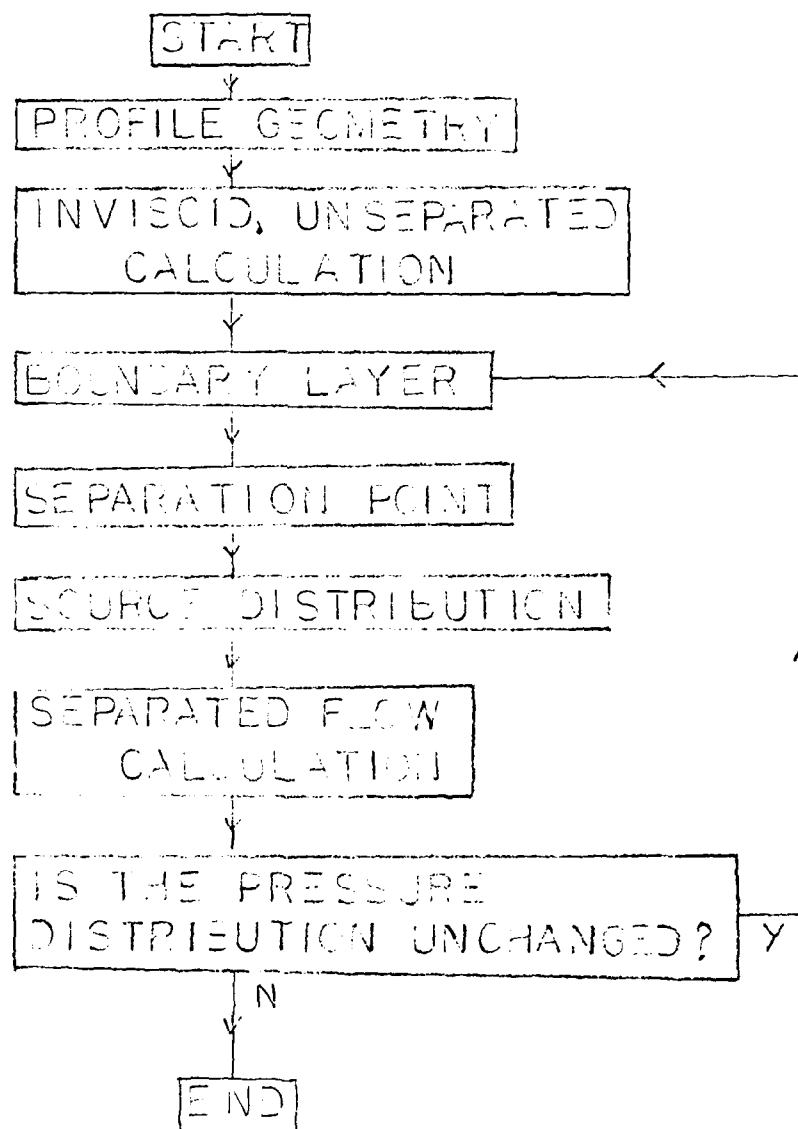


Fig. 3. Source Distributions for the GA(W-1)

## THE COMPUTER PROGRAMS

THE FOLLOWING FLOW CHART ILLUSTRATES THE COMPUTATIONAL PROCEDURE



Fig, 4. The Basic Computational Procedure

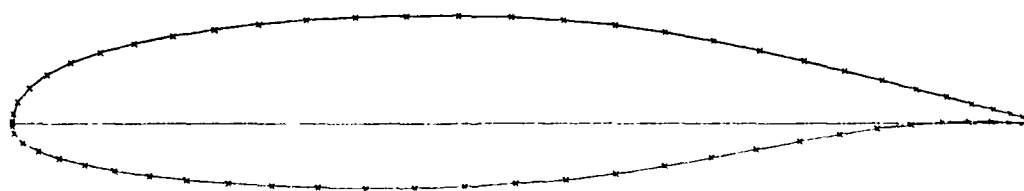
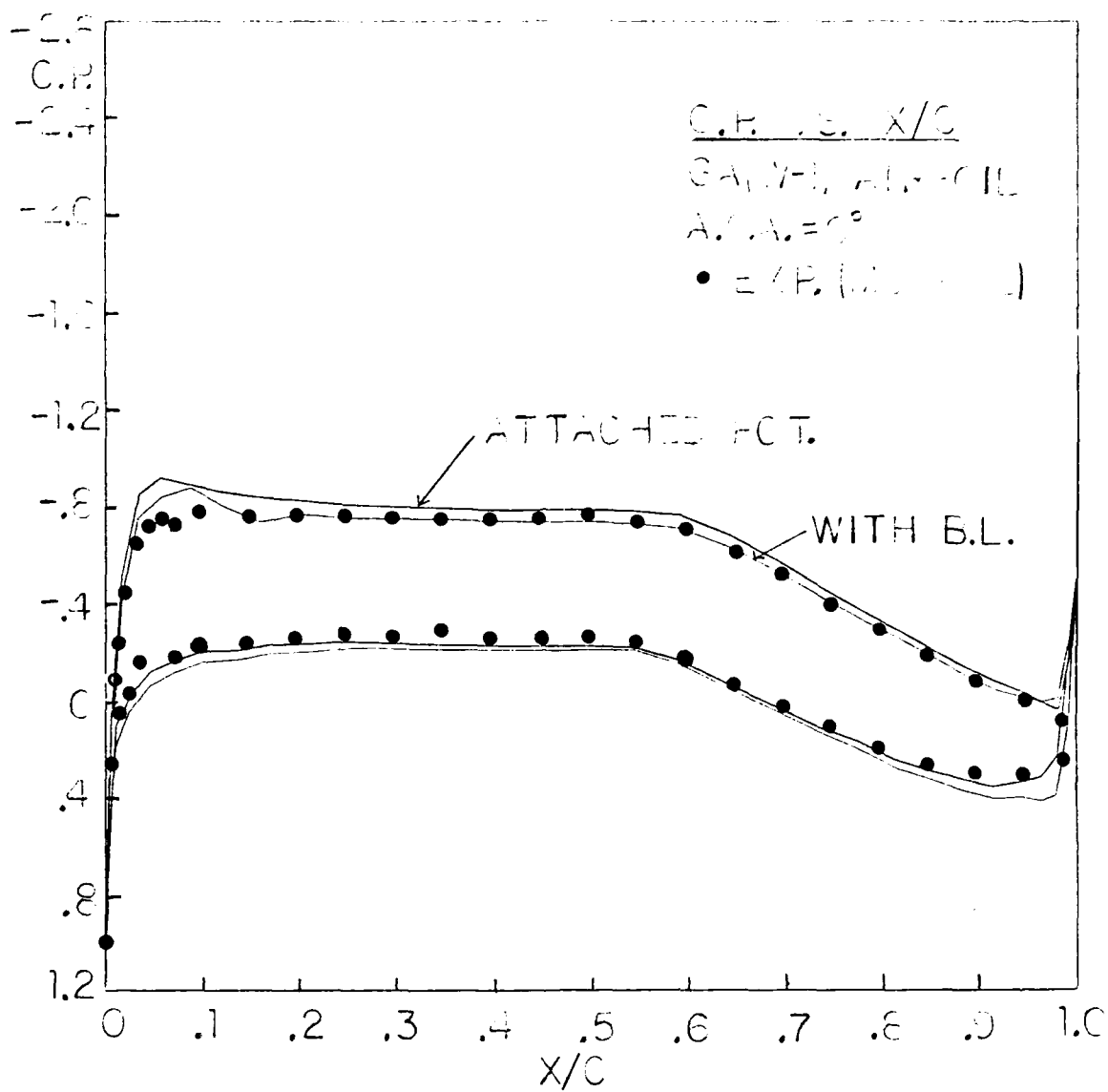


Fig. 5, Pressure Coefficient for GA(W-1)

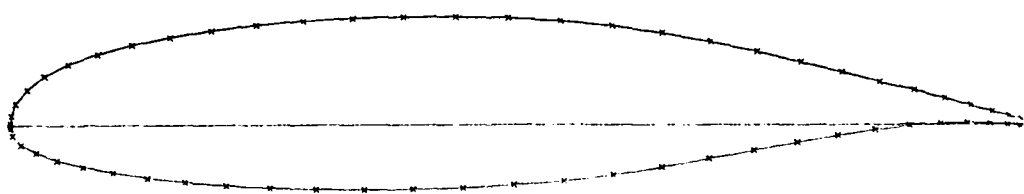
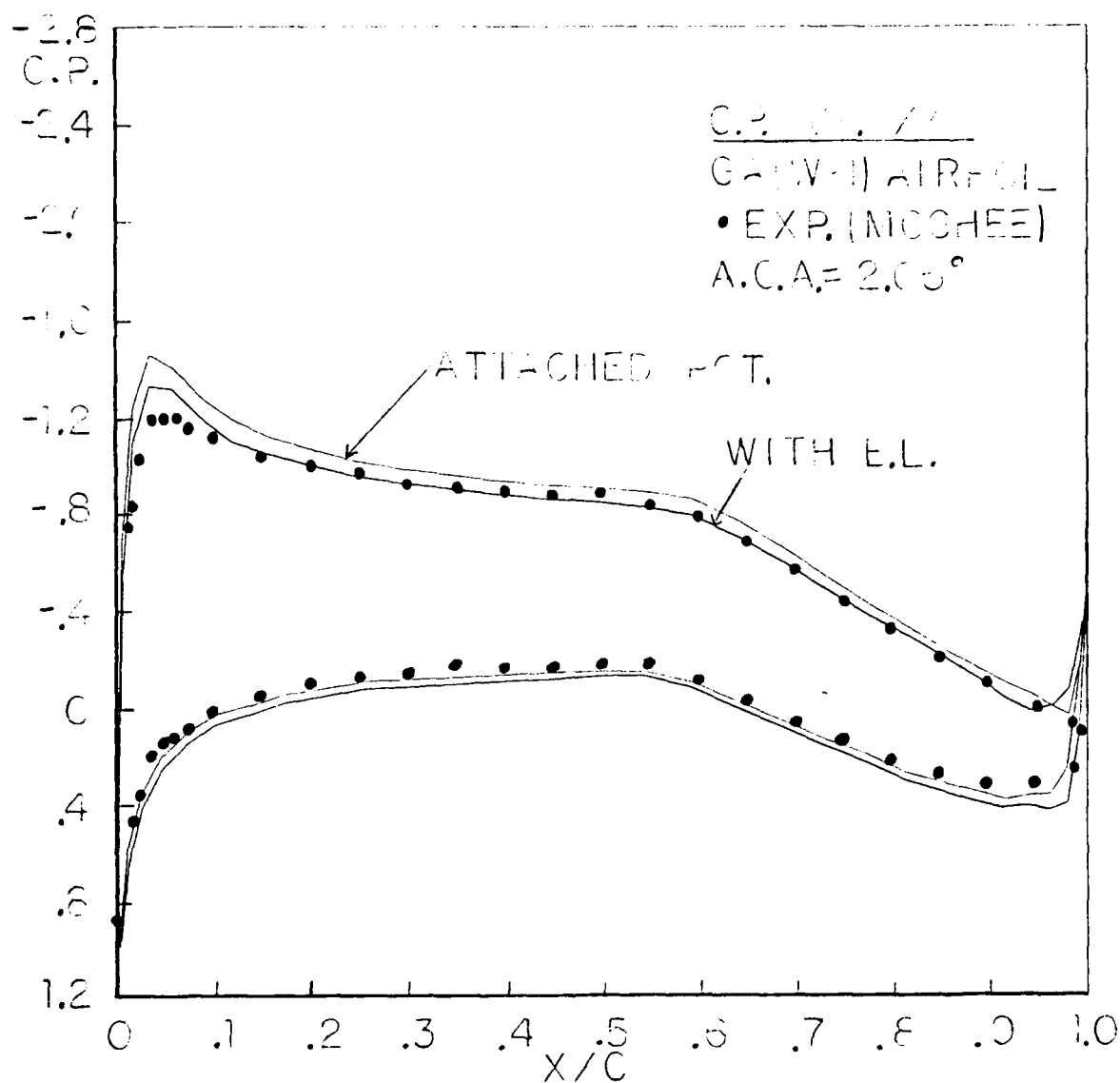


Fig. 6, Pressure Coefficient for GA(W-1)

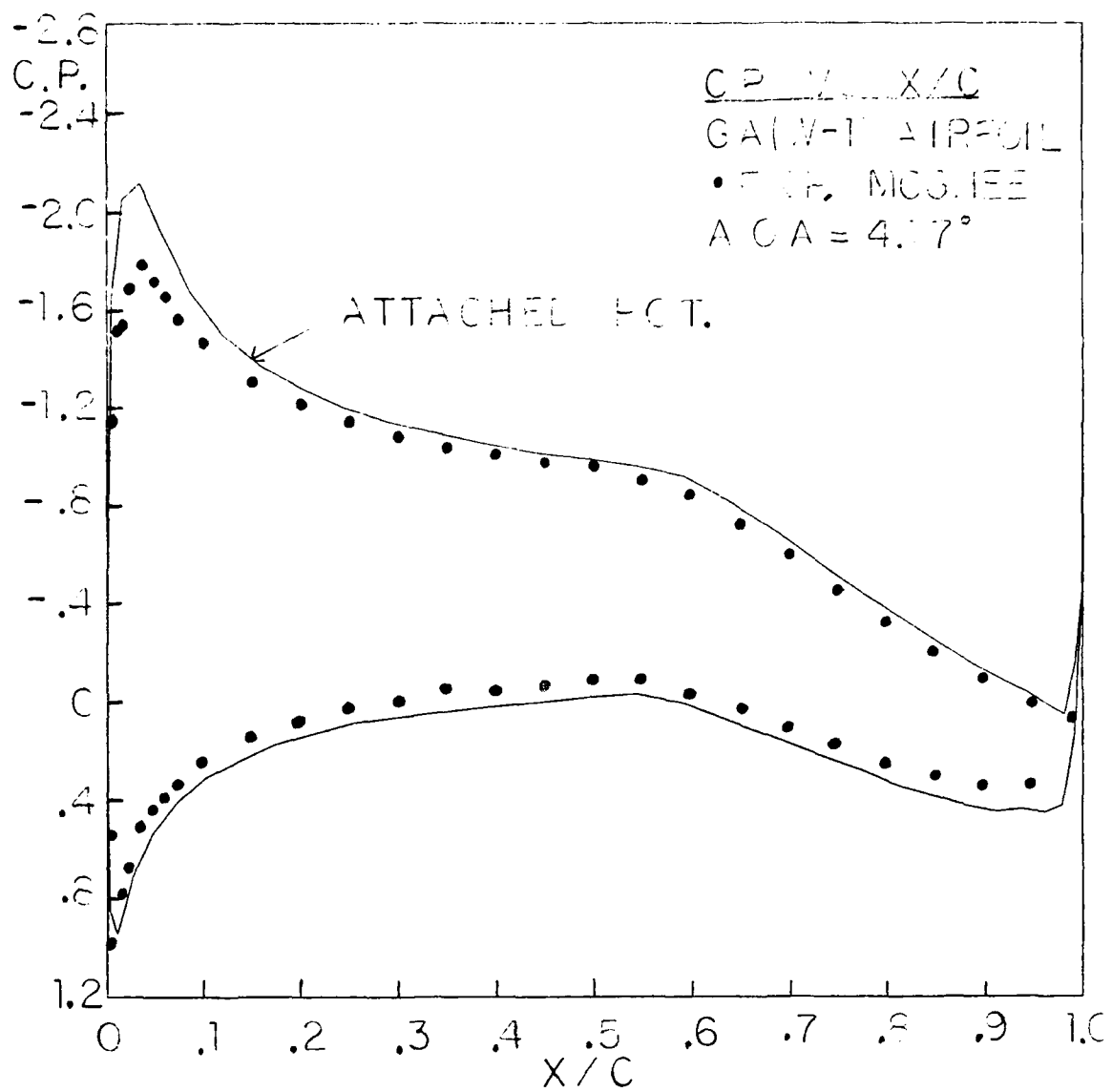


Fig. 7, Pressure Coefficient for GA(W-1)

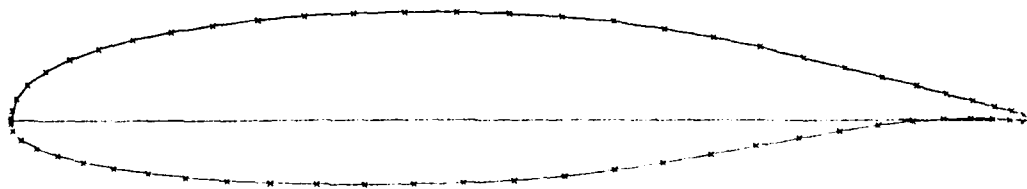
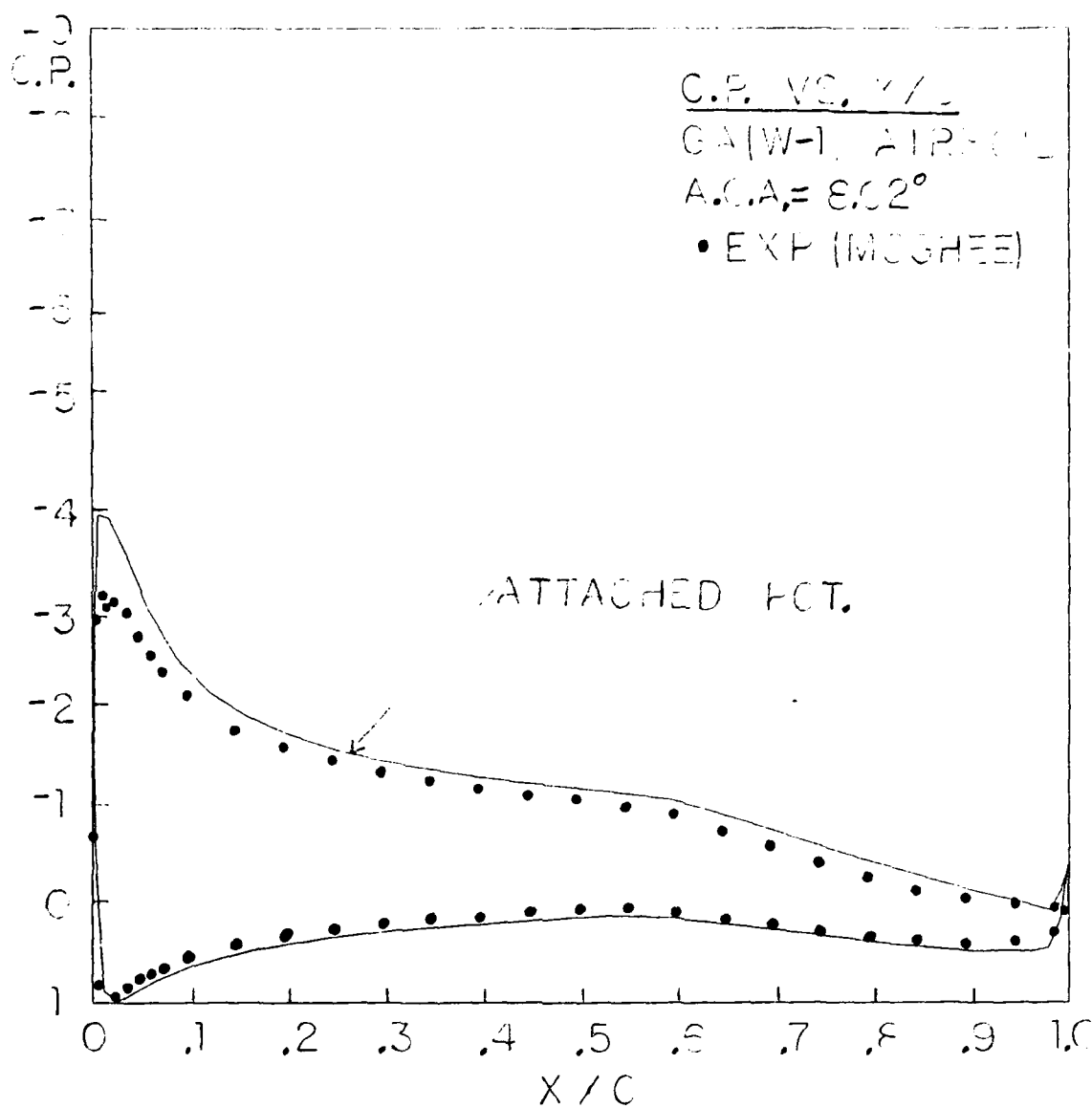


Fig. 8. Pressure Coefficient for GA(W-1)

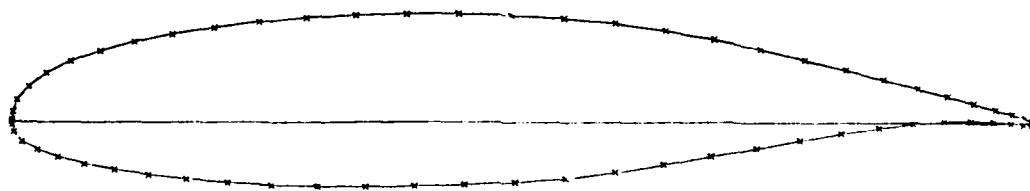
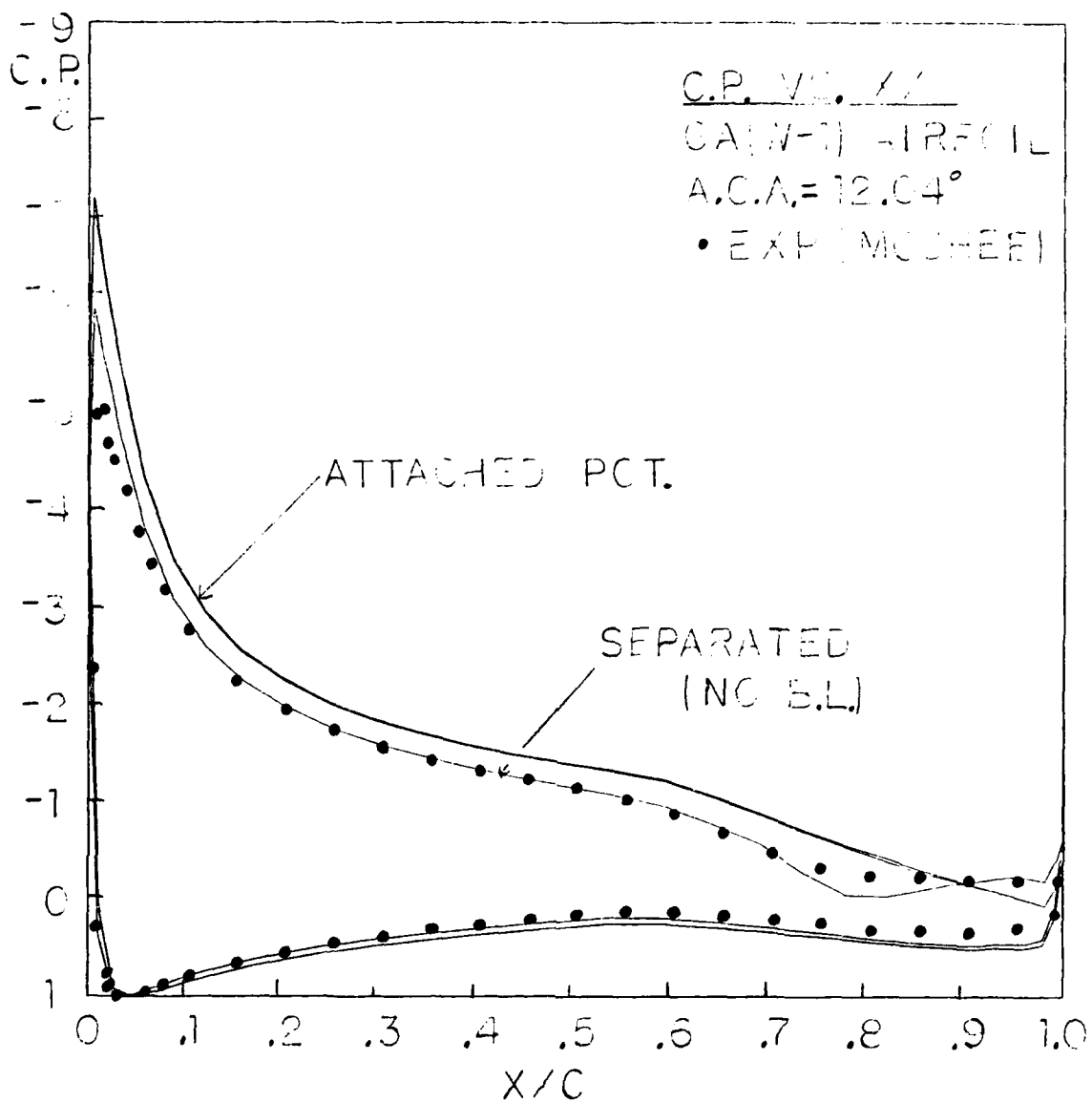


Fig. 9. Pressure Coefficient for GA(W-1)

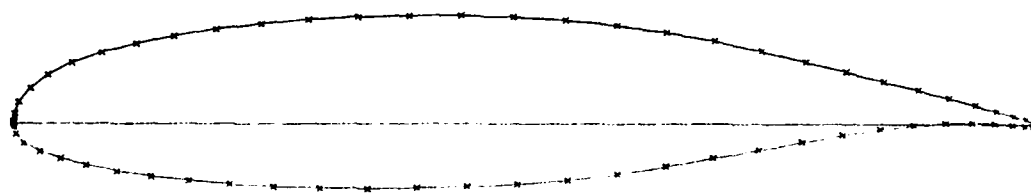
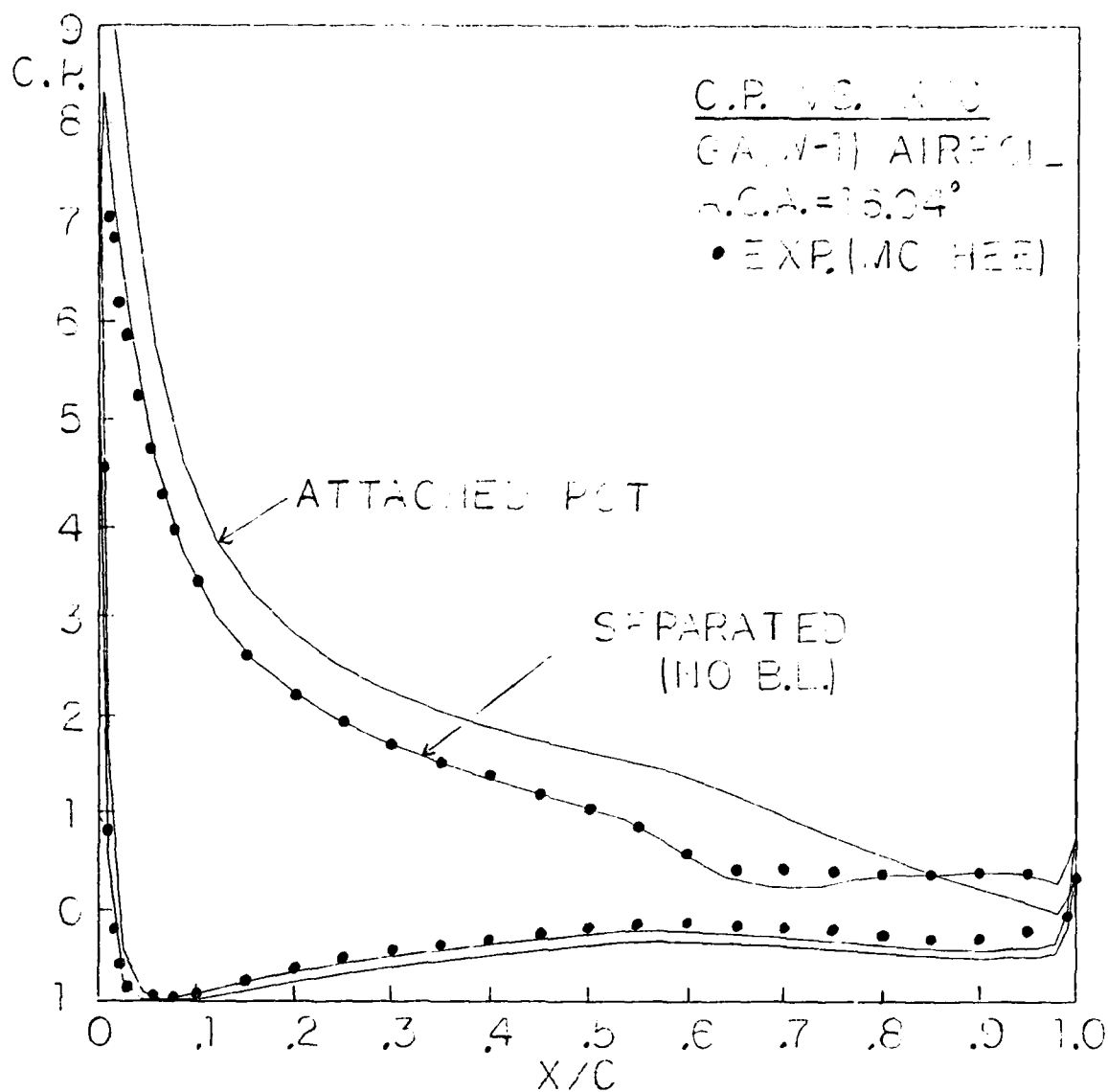


Fig. 10. Pressure coefficient for GA(W-1)

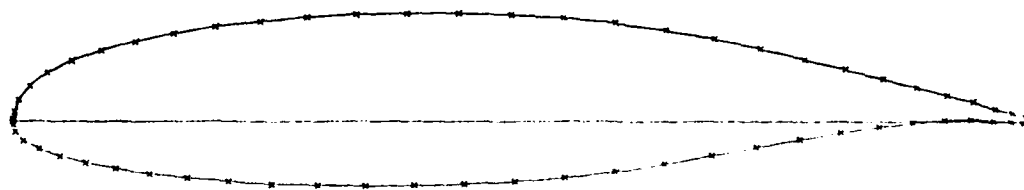
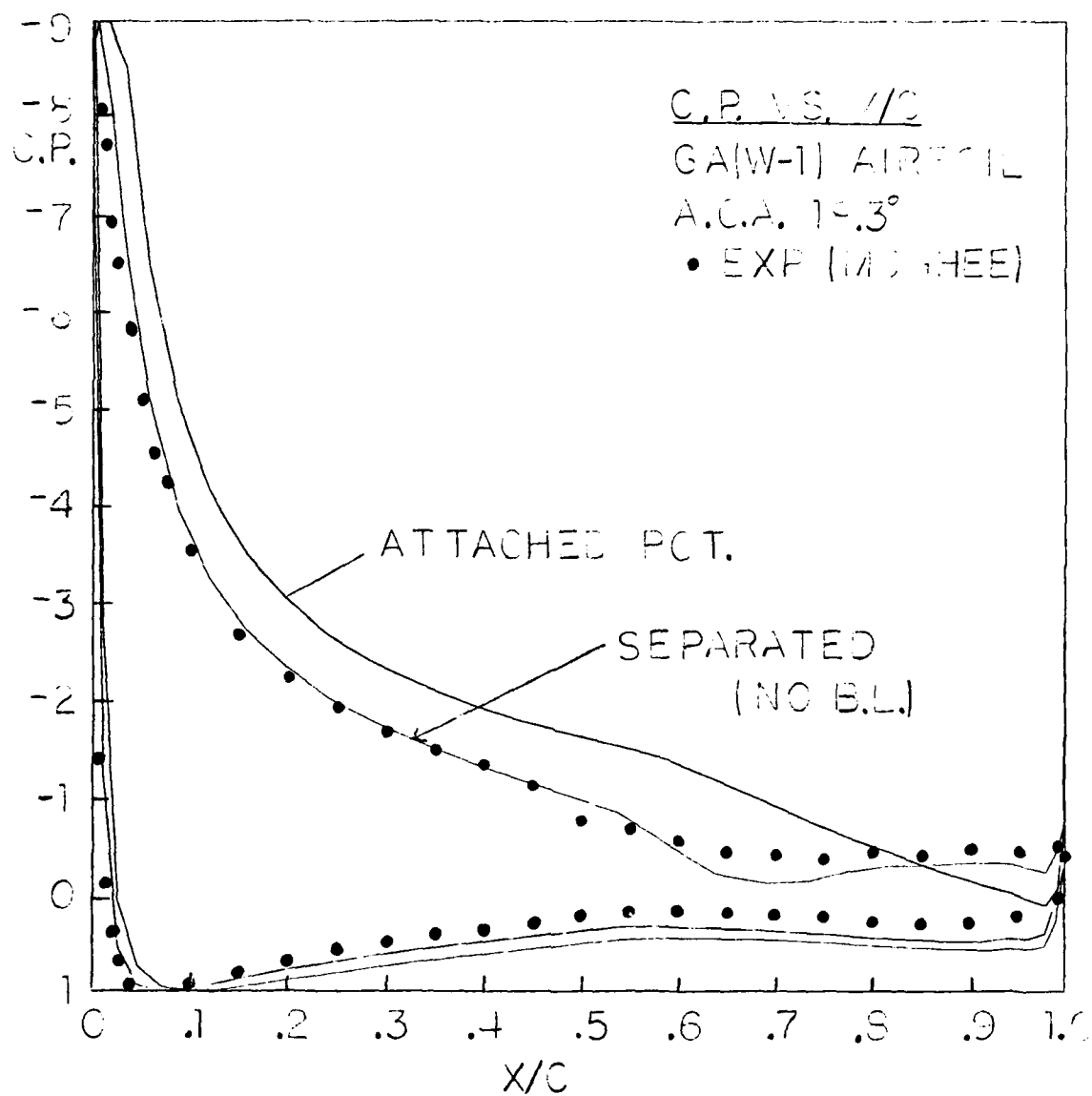


Fig. 11. Pressure Coefficient for GA(W-1)

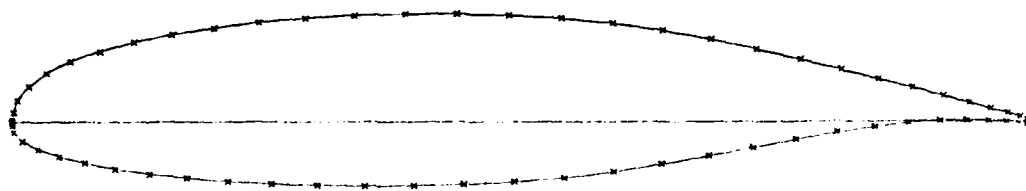
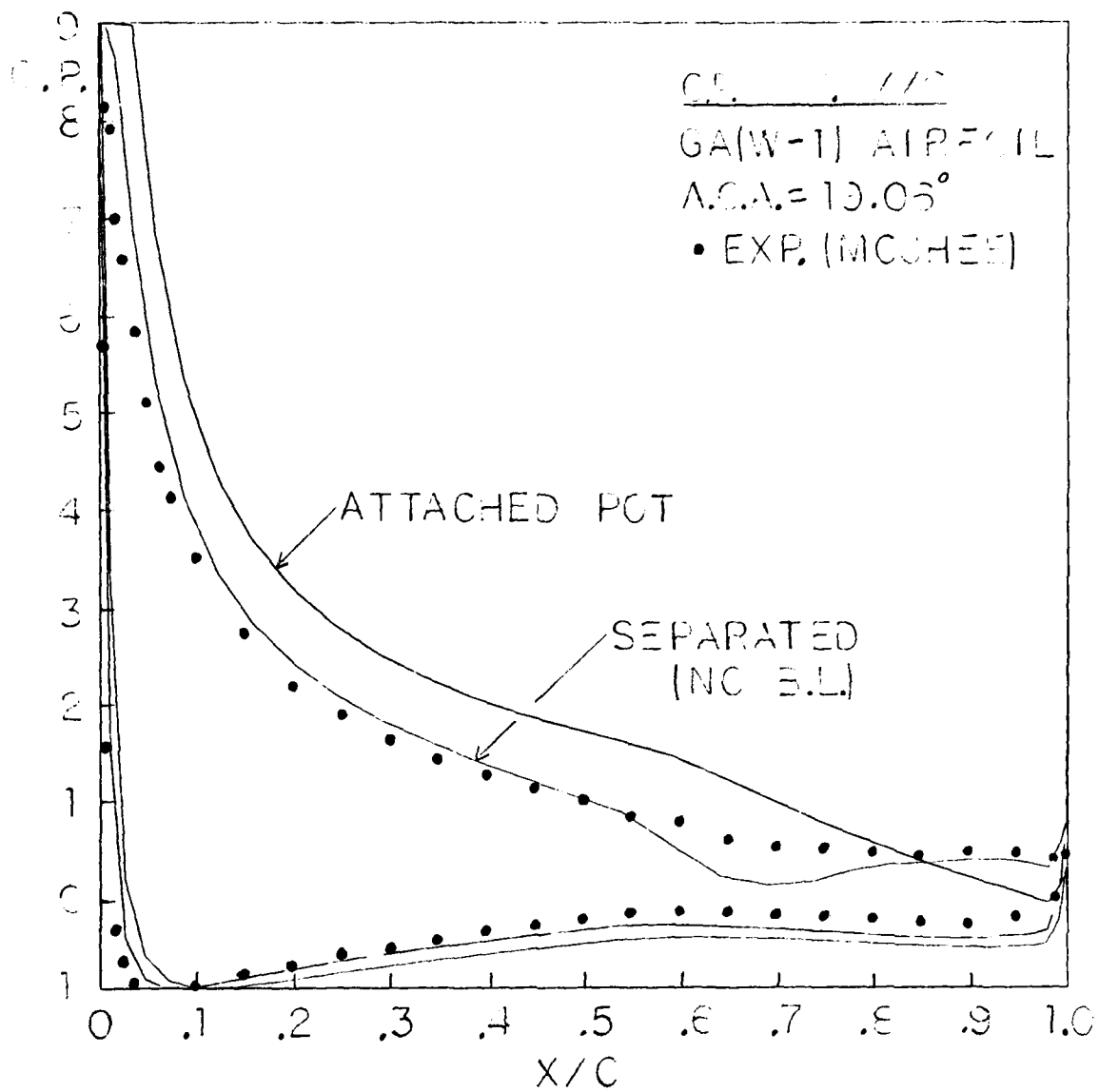


Fig. 12, Pressure Coefficient for GA(W-1)

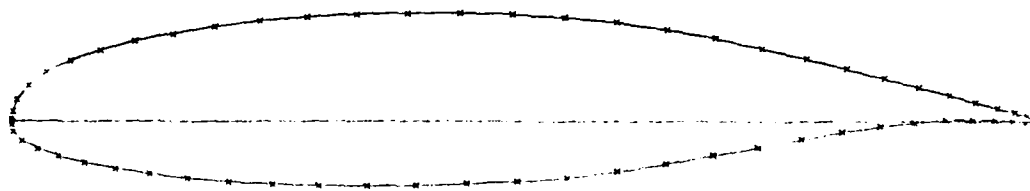
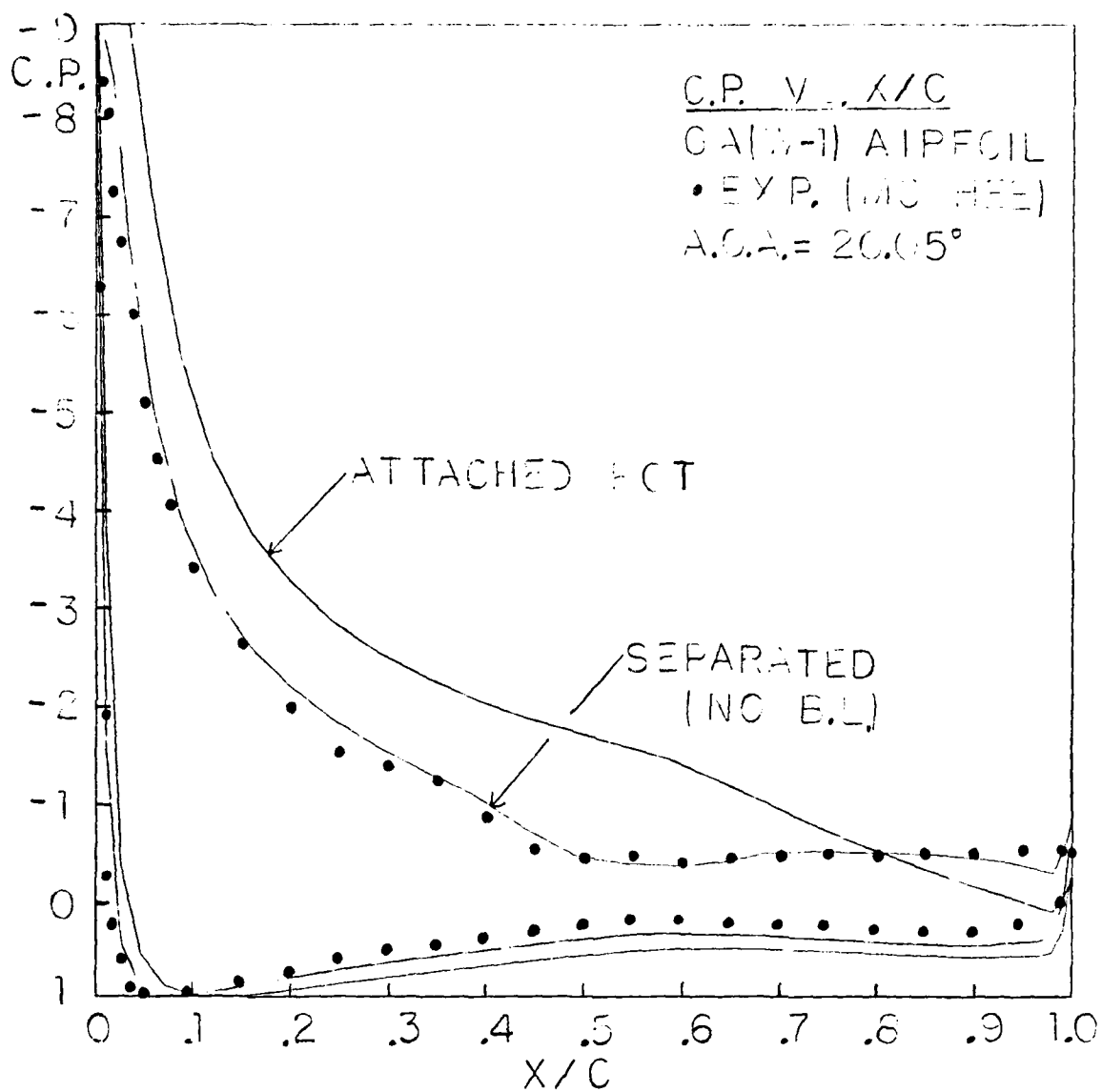


Fig. 13, Pressure Coefficient for GA(W-1)

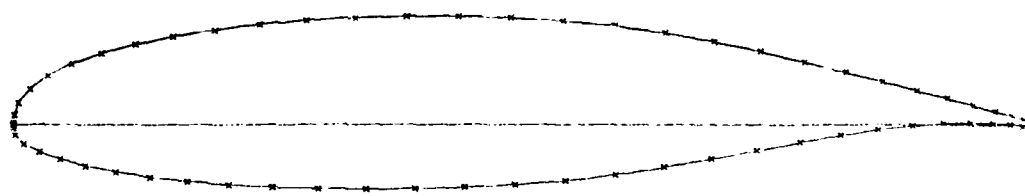
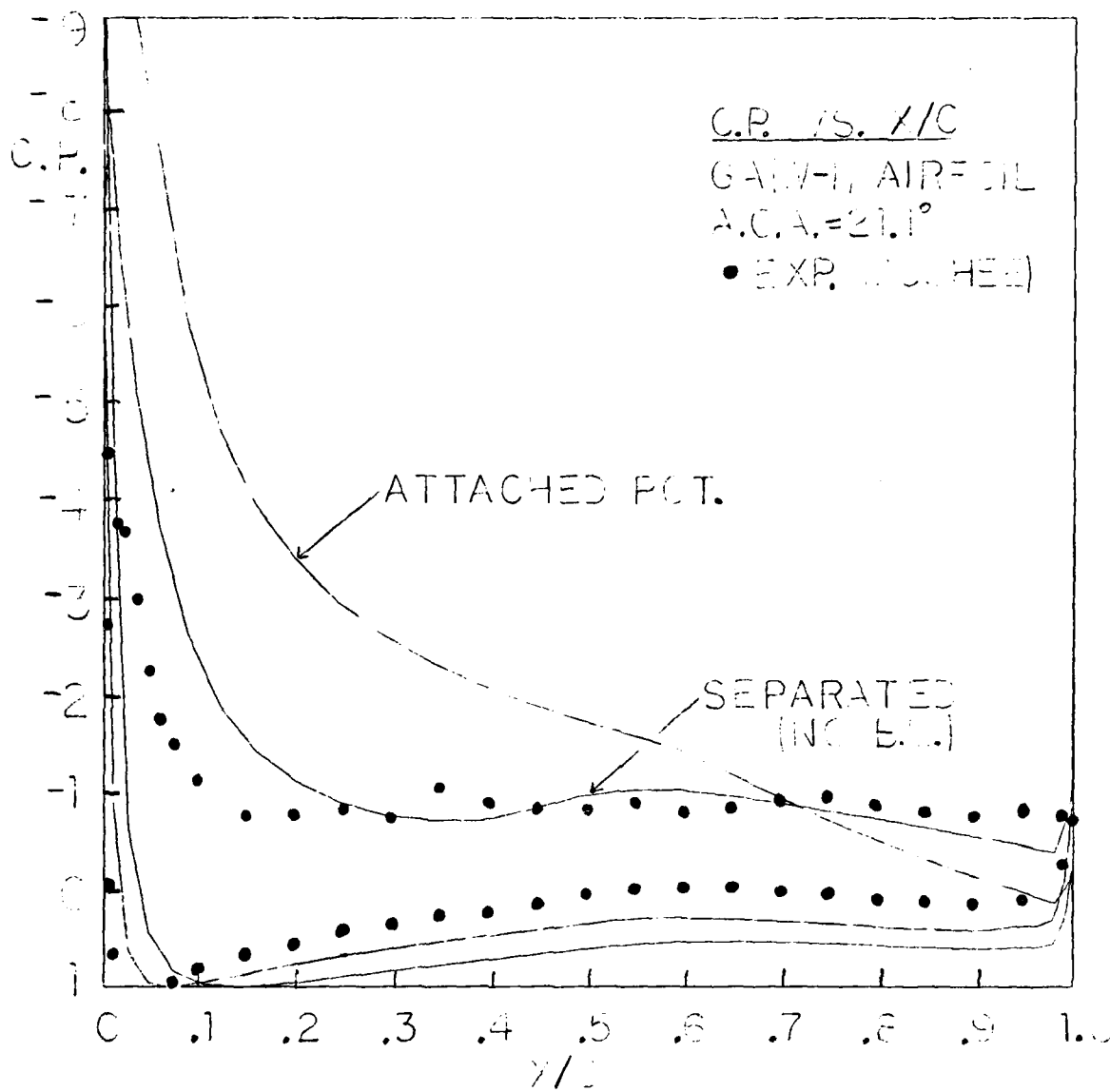


Fig. 14. Pressure Coefficient for GA(W-1)

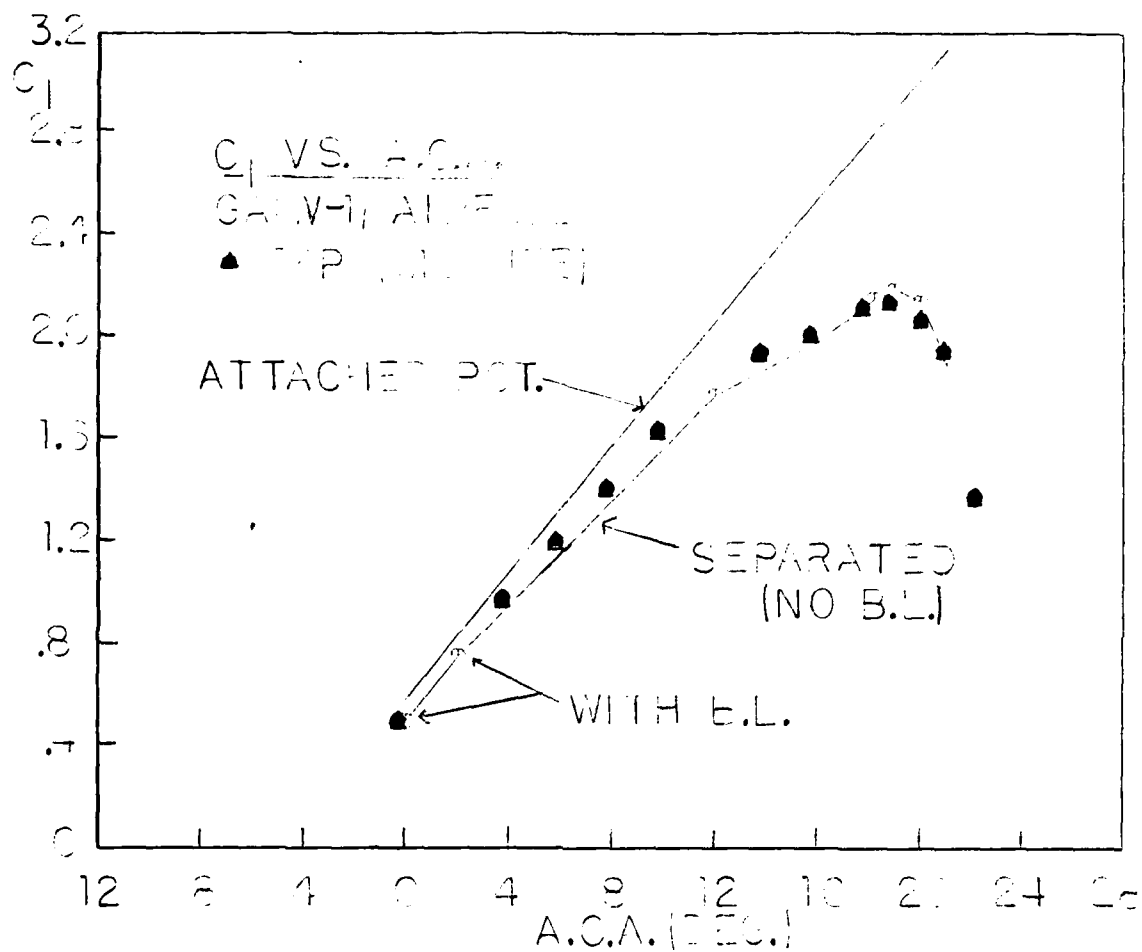


Fig. 15, Lift Coefficient for GA(W-1)

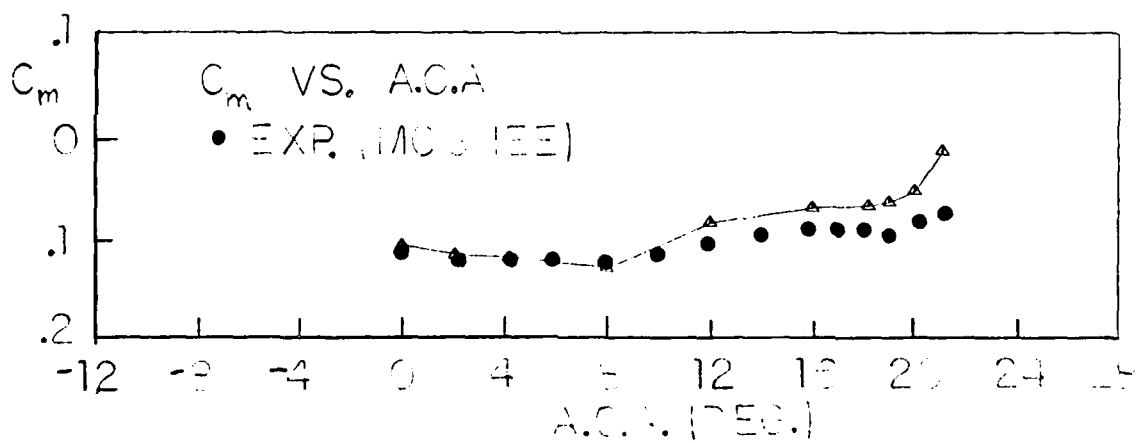


Fig. 16, Pitching Moment Coefficient for GA(W-1)

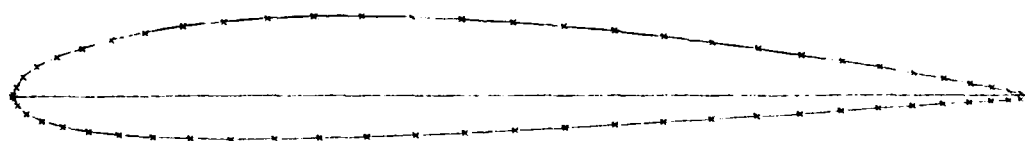
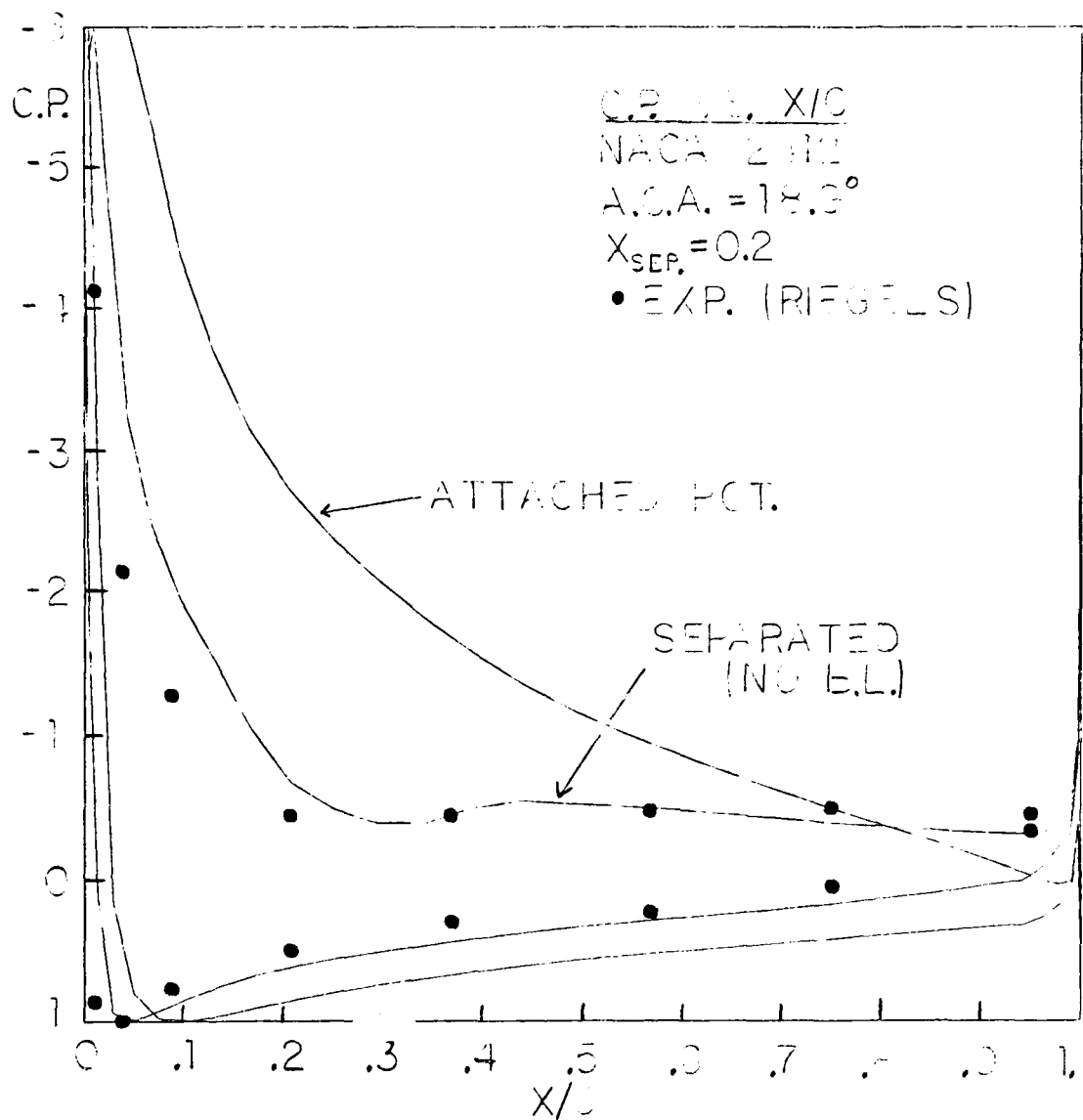


Fig. 17. Pressure Coefficient for NACA 2412

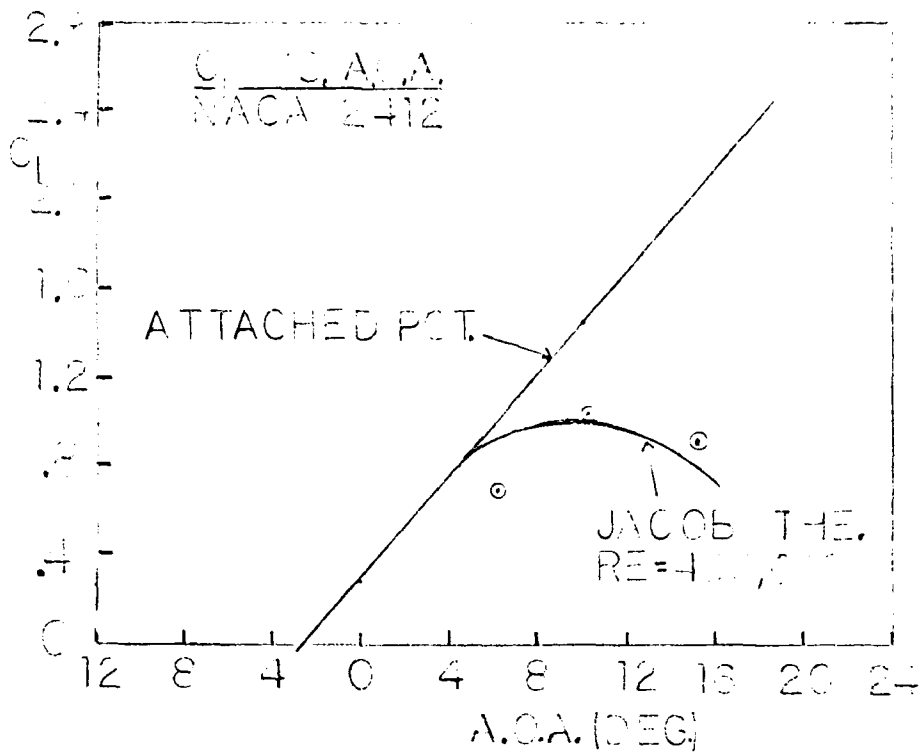


Fig. 18. Lift Coefficient for NACA 2412

END 5-83