AEDC-TR-83-3

Rotational and Irrotational Freestream Disturbances Interacting Inviscidly with a Semi-Infinite Plate

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April 1983

Final Report for Period March 1979 – August 1982

Approved for public release, distribution unlimited

ARNOLD ENGINEERING DEVELOPMENT CENTER
ARNOLD AIR FORCE STATION, TENNESSEE
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

This final report was submitted by United Research Corporation, 428 Hill Street, Suite 21, Santa Monica, California 90405, under contract F40600-79-C-0002, with the Arnold Engineering Development Center, Arnold Air Force Station, Tennessee 37389. Dr. Keith Kushman, AEDC/DOTA, was the AEDC Project Manager.

APPROVAL STATEMENT

This report has been reviewed and approved.

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Deputy for Operations

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1 REPORT NUMBER AEDC-TR-83-3	2 GOVT ACCESSION NO.	3 RECIPIENT'S CATALOG NUMBER
4 TITLE (and Subtitle) ROTATIONAL AND IRROTATIONAL FREESTREAM DISTURBANCES INTERACTING INVISCIDLY WITH A SEMI-INFINITE PLATE		5 TYPE OF REPORT & PERIOD COVERED Final Report, March 1979 - August 1982
		6 PERFORMING ORG REPORT NUMBER URC-TR-82-05
7 AUTHOR(s) Harold L. Rogler, United Research Corporation, and Eli Reshotko, Consultant		8. CONTRACT OR GRANT NUMBER(s) F40600-79-C-0002
9 PERFORMING ORGANIZATION NAME AND ADDRESS United Research Corporation 428 Hill Street, Suite 21 Santa Monica, California 90405		10 PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element 65807F
11 CONTROLLING OFFICE NAME AND ADDRESS Arnold Engineering Development Center/DOS Air Force Systems Command Arnold Air Force Station, Tennessee 37389		12 REPORT DATE April 1983
		13 NUMBER OF PAGES 42
14 MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15 SECURITY CLASS (of this report) UNCLASSIFIED
		15a DECLASSIFICATION/DOWNGRADING SCHEDULE N/A

16 DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17 DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18 SUPPLEMENTARY NOTES

Available in Defense Technical Information Center (DTIC).

19 KEY WORDS (Continue on reverse side if necessary and identify by block number)

freestream disturbances
unsteady fluctuations
vortical disturbances
irrotational disturbances
potential disturbances
leading edge effects
standing waves
traveling waves
pressure fluctuations
unsteady aerodynamics
wave excitation

20 ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report is part of a search for all possible classes of linear disturbances that can arise in a flowfield and thus serve as descriptive elements for a general disturbance flowfield. This report examines the velocity and pressure fields of disturbances that exist in a full two-dimensional unsteady incompressible inviscid flowfield about a semi-infinite flat plate. The permissible classes of disturbances include rotational as well as

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20. ABSTRACT, Concluded.

irrotational disturbances. Because of the linearity of the treatment, superpositions of these are also included. In general, the disturbances do not vanish far from the plate and in some instances are unbounded at infinite distances from the plate. These unbounded disturbances are retained, since they are relevant to applications involving finite domains, for example, in wind tunnels.

In this elliptic problem, the semi-infinite plate exerts an irrotational upstream influence which is composed of a pattern of standing waves. Downstream of the leading edge, the flowfield is also altered by an irrotational flow, but this irrotational flow is composed of both standing and traveling waves. The details of the flow along the plate are influenced by both the normal and longitudinal velocity fluctuations of the freestream disturbance. The pressure fluctuation is also influenced by the phase speed of the disturbance.

PREFACE

This research was supported by the Arnold Engineering Development Center of the Air Force Systems Command under Contract F40600-79-C-0002 with United Research Corporation and by the Air Force Office of Scientific Research under Grants AFOSR-74-2577A, B with Case Western Reserve University in Cleveland, Ohio. Dr. Keith Kushman was the Technical Monitor of the contract with AEDC. Mrs. Ernestine Badman was the Contracting Officer.

The authors thank Dr. Keith Kushman for his review of this report.

This is one in a series of AEDC Technical Reports prepared by United Research Corporation, Santa Monica, California, which document the solutions of the Orr-Sommerfeld equation and how these oscillations are initiated in a boundary layer. The reports in this series are as follows:

AEDC-TR-83-4
Exponentially Varying, Standing Waves
in Parallel-Flow Boundary Layers

AEDC-TR-83-7
Waves Which Travel Upstream in Boundary Layers

AEDC-TR-83-8
Spatially-Decaying Arrays of Rectangular Vortices
Interacting with Falkner-Skan Boundary Layers

AEDC-TR-83-9
The Boundary-Value Problem
for Two-Dimensional Fluctuations in Boundary Layers

AEDC-TR-83-3
Rotational and Irrotational Freestream Disturbances
Interacting Inviscidly with a Semi-Infinite Plate (This report)

AEDC-TR-83-10
Nonperiodic Fluctuations Induced by Surface Waviness
Near the Leading Edge of a Model

The last report presents the spectrum of standing waves excited by surface waviness. This spectrum, with appropriate nondimensionalizations, also applies to the case of freestream disturbances encountering a semi-infinite plate.

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1. INTRODUCTION

This report is part of a search for all possible classes of linear disturbances that can arise in a flowfield and thus serve as descriptive elements for a general disturbance flowfield. In the past, considerable attention has been given to limited classes of disturbances, for example the Tollmien-Schlichting waves which are traveling wave eigenmodes having vorticity transverse to the flow direction. But these waves do not represent all the possibilities.

In flows over immersed bodies or in channels, the oncoming flow usually contains small amplitude fluctuations of velocity and pressure that we identify as freestream turbulence. These can have significant influence on the boundary layer development particularly with respect to the transition from laminar to turbulent flow. It is therefore important to determine the structural possibilities for such flows so that the significant features among them can be identified and included in more complete analytical investigations of factors relating to transition.

Many flow disturbances are most easily identified in the uniform inviscid flow external to the boundary layers developing on the solid boundaries. If the boundary layers are thin enough, then they are benign with respect to the external flow and the impermeability of the solid boundary is the appropriate boundary condition for the external flow. These are the premises of the present paper and they will be pursued with the realization that there can be alterations to these disturbances in a viscous flow. There may of course be additional disturbances in a viscous flow that do not have inviscid counterparts and would therefore not be found by the present procedure.

This report in particular will examine the velocity and pressure fields of disturbances that exist in a full two-dimensional incompressible flowfield about a semi-infinite flat plate. All of these disturbances have their three-dimensional counterparts but the two dimensional aspects have yet to be elaborated.

It will be shown that the permissible classes include rotational as well as irrotational disturbances. Because of the linearity of the treatment, superpositions of these are also included. An example of a rotational disturbance field as treated herein has been developed in Refs. 1 and 2. In general, the disturbances presently considered do not vanish far from the plate and in several instances are unbounded at infinite distances from the plate. These are retained since they are relevant to applications involving finite domains, for example in wind tunnels.

This report therefore will (1) demonstrate how the solutions of Refs. 1 and 2 for <u>square vortices</u> convecting downstream and beside a semi-infinite plate can be adapted to other forms of freestream disturbances which also might propagate at speeds different from the freestream speed, (2) present results for several rotational and irrotational, two-dimensional, incompressible disturbances, and (3) demonstrate similarities and differences in the pressure fluctuations for the different disturbance fields.

2. GENERATION OF SOLUTIONS

2.1 SOLUTION FOR AN ARRAY OF SQUARE VORTICES INTERACTING WITH A SEMI-INFINITE PLATE

By conformal mapping, the streamfunction was derived for a low-intensity, high-Reynolds number array of two-dimensional square vortices convecting downstream in a uniform mean flow and along a semi-infinite flat plate (Refs. 1 and 2). For regions neither "very near" the leading edge and plate, nor "very-far" downstream where nonlinear and viscous effects become significant, as discussed in Ref. 2, the disturbance streamfunction is

$$\psi(x,y,t;y_1) = \psi^{(a)}(x-t,\pi y+y_1) - \psi^{(i)}(z,t,y_1) \qquad (2.1a)$$

where

$$\psi^{(a)} = \frac{1}{\pi}\sin(\pi y+y_1)\sin\pi(x-t) \qquad (2.1b)$$

is the streamfunction for an array of square vortices which convects with the mean flow. The superscript (a) emphasizes that this is the disturbance when the plate is <u>absent</u>, i.e., it is the "freestream" disturbance. These disturbances are a non-decaying form of a spatially-decaying array of vortices analyzed in Ref. 3.

The second streamfunction in Eqn.(2.1a) is

$$\begin{aligned}\psi^{(i)} = \frac{\sin y_1}{\pi}\Big\{&\cos\pi t\,\mathcal{R}\big[-\cos\pi z\,S_2(-\pi z) - \sin\pi z\,C_2(-\pi z)\\ &+\cos\pi z\,C_2(-\pi z) - \sin\pi z\,S_2(-\pi z) + \sin\pi z\big]\\ &-\sin\pi t\,\mathcal{R}\big[-\cos\pi z\,S_2(-\pi z) - \sin\pi z\,C_2(-\pi z)\\ &-\cos\pi z\,C_2(-\pi z) + \sin\pi z\,S_2(-\pi z) + \cos\pi z\big]\Big\}\end{aligned} \qquad (2.1c)$$

where

$$\begin{matrix}S_2\\C_2\end{matrix}(z) = \frac{1}{(2\pi)^{1/2}}\int_0^z t^{-1/2}\begin{matrix}\sin\\\cos\end{matrix}(t)\,dt \qquad (2.1d)$$

are the sine and cosine Fresnel integrals, and z=x+iy.

$\psi^{(i)}$ is the "impermeability" streamfunction which, when subtracted from $\psi^{(a)}$, satisfies the impermeability condition along the plate. $\psi^{(i)}$ is irrotational with streamlines as plotted in Fig. 1. $\mathcal{R}$[ ] denotes the real part of [ ]. y_1 is the phase angle which controls the y-orientation of the array relative to the x-axis or plate. x and y have been nondimensionalized against the half-wavelength or "diameter", Λ, of a vortex. The time has been nondimensionalized against Λ/U_∞ which represents the time for a fluid particle to convect a distance of

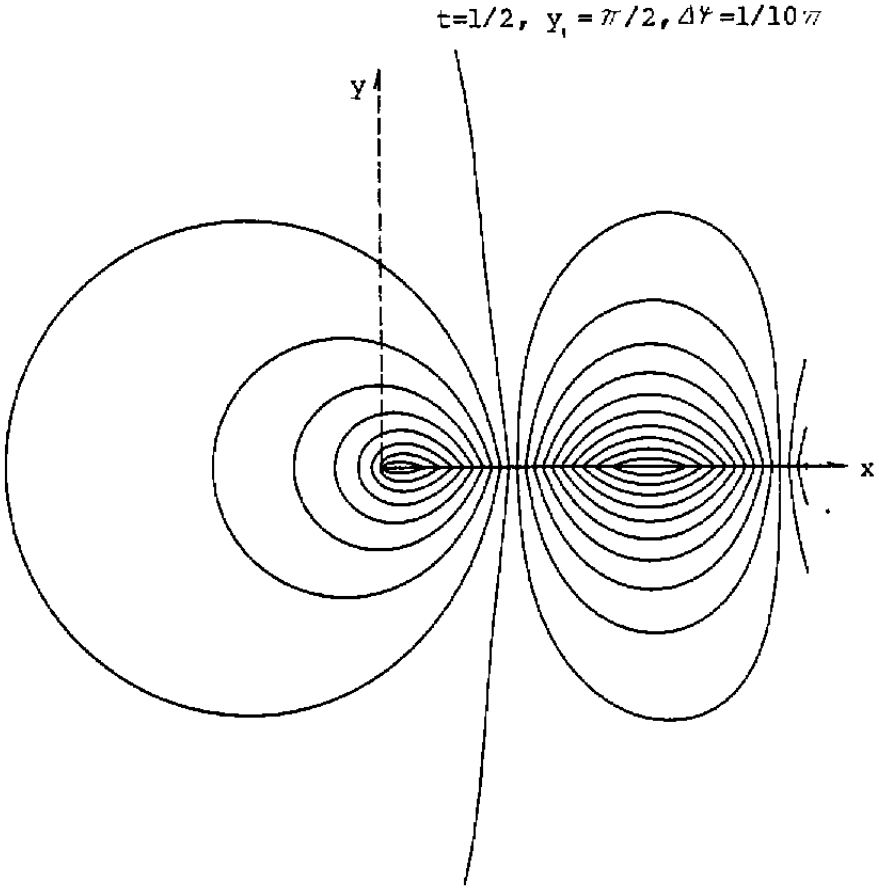

Figure 1. Streamlines for the impermeability flow $\psi^{(1)}$. This irrotational flow represents the adjustment to the freestream disturbance caused by the semi-infinite plate.

Λ at the freestream speed, U_∞, and the streamfunctions have been nondimensionalized against $q\Lambda$ where q is the maximum normal speed of the freestream disturbance.

While $\psi^{(i)}$ is a traveling, oscillating function along the plate for x>0 (see Fig. 1), it is not generally periodic in x or y. Upstream of the plate, this flow does not oscillate. Hence, it cannot be thought of as a function with a single wavelength and speed. The abrupt change in boundary conditions at x=0+ has introduced a leading edge effect that can be viewed as a standing wave pattern that decays with distance from the leading edge superposed on the traveling wave for x>0.

For later use, the disturbance velocities associated with this impermeability flowfield are

$$u^{(i)} = -\psi_y^{(i)} = -i \sin y_1 F(z,t) \quad \text{(longitudinal velocity)} \quad (2.2)$$

$$v^{(i)} = \psi_x^{(i)} = \sin y_1 F(z,t) \quad \text{(normal velocity)} \quad (2.3)$$

where
$$F = \sin\pi(z-t)\left[S_2(-\pi z) - C_2(-\pi z)\right] - \cos\pi(z-t)\left[S_2(-\pi z) + C_2(-\pi z) - 1\right] - \frac{\sin\pi t + \cos\pi t}{\pi(-2z)^{1/2}} \quad (2.4a)$$

The quantity $\sin y_1$ is the ratio of the normal velocity along the x-axis to the maximum normal velocity in the freestream vortex.

Computational experience has shown that the direct calculation of (2.4a) for $|z|$ greater than about 1.8 is not satisfactory when using 32 bit arithmetic. When using 64 bit arithmetic, $|z|$ was limited to about 5.2. However, solution (2.4a) can be rearranged as

$$F(z,t) = \sin\pi t\left[f - g - \frac{1}{\pi(-2z)^{1/2}}\right] + \cos\pi t\left[f + g - \frac{1}{\pi(-2z)^{1/2}}\right] \quad (2.4b)$$

where asymptotic series for large $|z|$ are presented in Ref. 6 for the functions, f and g, which are related to the Fresnel integrals as defined in the Nomenclature. Using this relation, the normal velocity has been calculated for $|z|=400$, or 200 x-wavelengths from the leading edge. The normal velocity and its sine transform are plotted in Ref. 7. There are differences in the characteristic velocity in the nondimensionalizations of the present work and Ref. 7.

The expressions for $\psi^{(a)}$ and $\psi^{(i)}$ are applicable for arrays of square vortices and considerable information has been assembled concerning the interaction of such arrays with plates and boundary layers. However, other forms of freestream disturbances exist, such as arrays of rectangular vortices, oblique plane waves of vorticity, irrotational disturbances, and many others. Extensions to solution (2.1a,b,c) are now considered which greatly broaden the class of permissible disturbances.

2.2 SOLUTION FOR OTHER ROTATIONAL AND IRROTATIONAL DISTURBANCES INTERACTING WITH A SEMI-INFINITE PLATE

In Refs. 1 and 2, the impermeability condition is satisfied by superimposing a second flowfield with normal velocity along the plate ($y = 0$, $x > 0$ but not upstream of the leading edge) which is equal to the normal velocity of the freestream disturbance evaluated along the x-axis. The difference between these two velocities then vanishes along the plate, $v^{(a)} - v^{(i)} = 0$, for $y = 0$ and $x \geq 0$ and thus impermeability is satisfied downstream of the leading edge.

This procedure can also be applied to freestream disturbances other than arrays of square vortices, including irrotational disturbances. In fact, the impermeability solution (2.1c) can be applied either directly or with minor modification to all freestream disturbances with normal component of velocity varying sinusoidally along the x-axis as

$$[v^{(a)}]_{y=0} = [\partial\psi^{(a)}/\partial x]_{y=0} = \sin y_1 \cos\pi(x-t) \qquad (2.5)$$

For some disturbances, y_1 is a dummy parameter which will be set equal to $\pi/2$ and the normal velocity fluctuation thus has magnitude 1 along the x-axis far-upstream of the plate. Condition (2.5) and the streamfunction (2.1c) are sufficient to consider how other freestream disturbances interact with a semi-infinite plate. Examples of rotational disturbances are presented in Section 3, while examples of irrotational disturbances are given in Section 4. The case of a row of potential vortices is presented in Section 5.

3. ROTATIONAL DISTURBANCES

3.1 ARRAYS OF COUNTER-ROTATING RECTANGULAR VORTICES

The streamfunction for an array of rectangular vortices convecting with the mean flow is

$$\psi^{(a)}(x-t, \beta y+y_1) = \frac{1}{\pi} \sin(\beta y+y_1) \sin\pi(x-t) \qquad (3.1)$$

where β is the y-wavenumber. If $\beta=\pi$, then the vortices are square. All lengths have been nondimensionalized with respect to the x-length of the vortex. The characteristic disturbance speed, q, is the maximum y-component. The derivative, $\partial\psi^{(a)}/\partial x = v^{(a)}$, satisfies Eqn.(2.5). Hence the streamfunction for an array of rectangular vortices convecting past a semi-infinite plate is

$$\psi(x, y, t, y_1, \beta) = \psi^{(a)}(x-t, \beta y+y_1) - \psi^{(i)}(z, t, y_1) \qquad (3.2)$$

where $\psi^{(i)}$ is given by Eqn.(2.1c). The streamlines for $\beta=3\pi/2$ and $\beta=2\pi/3$ are shown in Figs. 2 and 3 respectively. The solution far-downstream of the leading edge is presented in Section 6. Although $y_1=\pi/2$ in these figures (the plate bisects one row of vortices), y_1 can be assigned any value but can be restricted to the range $0\leq y_1<2\pi$ since its influence is periodic. As well as shifting the vortices in the y-direction, the parameter y_1 also scales the amplitude of $\psi^{(i)}$ by the magnitude $\sin y_1$.

Note in Figs. 2 and 3 the large disturbance velocities near the leading edge as indicated by the closeness of the streamlines. Also note from the streamline distortion that the plate exerts an upstream influence in this elliptic problem. The major downstream influence of the leading edge is limited to approximately one vortex length. Thereafter, the flowfield adjusts slowly to a new pattern which convects downstream without further change. For the case shown in Fig. 2, note that some of the fluid in Row 2 of the vortices is absorbed by Rows 1 and 3 as the fluid convects past the leading edge. The result is that slender, slowly-rotating vortices form near the wall and convect downstream. A similar behavior is seen in Fig. 3.

The array of rectangular vortices (3.1) can be decomposed into two oblique, plane-waves of vorticity

$$\psi^{(a)} = -\frac{1}{2\pi} \cos[\pi(x-t)+\beta y+y_1] + \frac{1}{2\pi} \cos[\pi(x-t)-(\beta y+y_1)] \qquad (3.3)$$

with the argument of each shifted by the phase angle y_1.

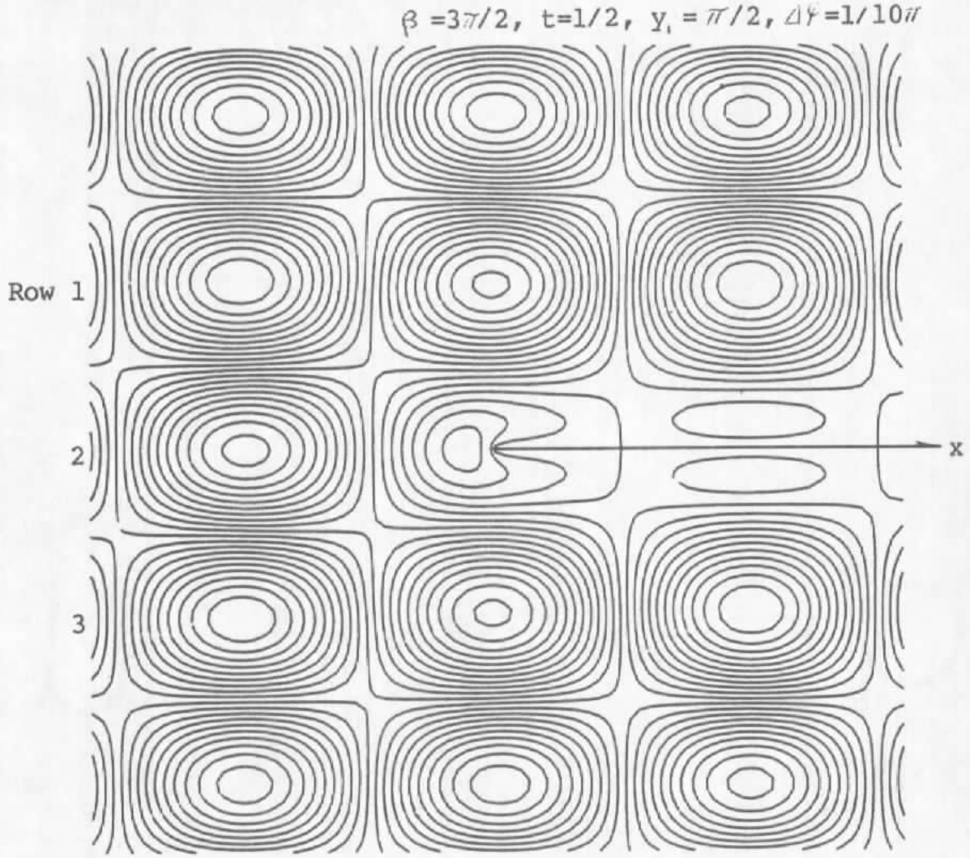

Figure 2. Streamlines for an array of rectangular vortices propagating downstream past the leading edge and along a semi-infinite plate.

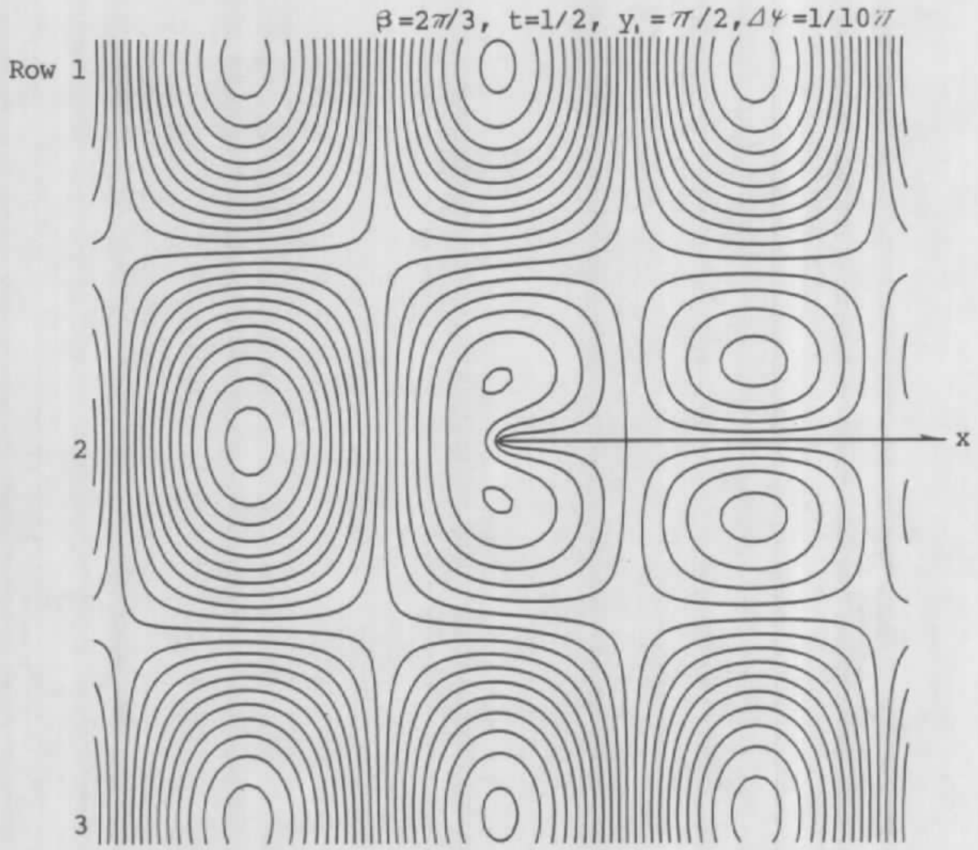

Figure 3. Streamlines for an array of rectangular vortices propagating downstream past the leading edge and along a semi-intinite plate. As in Fig. 2, the plate bisects one row of vortices, although the y-wavenumber is different.

3.2 PLANE-WAVES OF VORTICITY

For this case, the phase angle y_1 is eliminated in Eqn.(2.5) by setting $y_1 = \pi/2$. Then we seek a plane-wave streamfunction which satisfies the requirement

$$[\partial\psi^{(a)}/\partial x]_{y=0} = \cos\pi(x-t) \tag{3.4}$$

where x has been nondimensionalized against the half-wavelength in the x-direction. An oblique plane-wave of vorticity which satisfies the above requirement is

$$\psi^{(a)}(x-t, \beta y) = \frac{1}{\pi}\sin[\pi(x-t)+\beta y] \tag{3.5}$$

where the y-wavenumber is arbitrary. Thus the streamfunction for an oblique plane-wave of vorticity convecting downstream past the leading edge and along a semi-infinite plate is

$$\psi(x,y,t,\beta) = \psi^{(a)}(x-t,\beta y) - \psi^{(i)}(x,y,t;y_1=\pi/2) \tag{3.6}$$

The streamlines for the cases of a longitudinal plane wave (β=0) and an oblique plane wave inclined at 45° ($\beta=\pi$) are shown respectively in Figs. 4 and 5.

Again note the high velocities near the leading edge and note that a slender vortex forms about a half-wavelength upstream of the leading edge. Downstream there evolves a pattern of U-shaped streamlines. For the oblique-plane wave, the "stagnation-points" at the wall where the <u>disturbances vanish</u> are not exactly opposite each other.

Just as it was shown in Eqn.(3.3) that an array of rectangular vortices can be decomposed into two oblique plane-waves of vorticity, it can be shown that an oblique-plane wave can be decomposed into two arrays of rectangular vortices. Eqn.(3.5) can be expanded, yielding

$$\psi^{(a)} = \frac{1}{\pi}\sin\pi(x-t)\cos\beta y + \frac{1}{\pi}\cos\pi(x-t)\sin\beta y \tag{3.7}$$

where each term represents an array of rectangular vortices. The two arrays are shifted by one-fourth of an x-wavelength and one-fourth of a y-wavelength relative to one another.

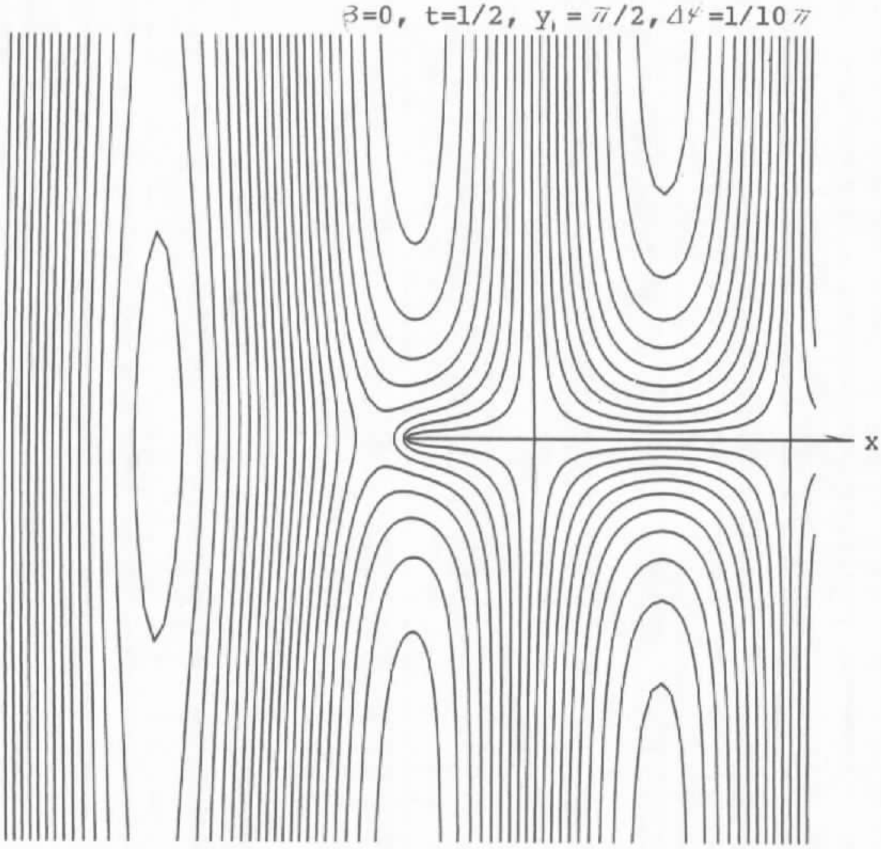

Figure 4. Streamlines for a longitudinal plane-wave of vorticity propagating downstream past the leading edge and along a semi-infinite plate.

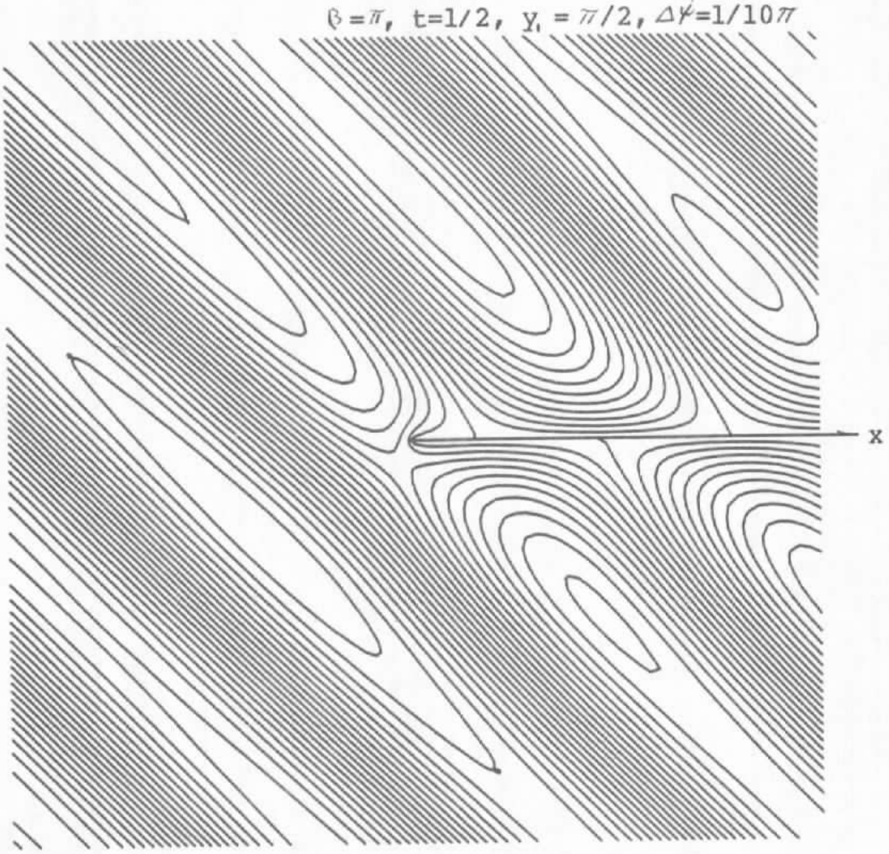

Figure 5. Streamlines for an oblique plane-wave of vorticity propagating downstream past the leading edge and along a semi-infinite plate.

4. IRROTATIONAL, INCOMPRESSIBLE DISTURBANCES

4.1 INTRODUCTION

Previously, rotational freestream disturbances were considered in the forms of arrays of vortices and plane-waves of vorticity. Now we consider a class of irrotational fluctuations and their interactions with semi-infinite plates. Attention is restricted to fluctuations which satisfy Eqn.(2.5) with $y_1 = \pi/2$, but which also satisfy Laplace's equation

$$\nabla^2 \psi^{(a)} = 0 \tag{4.1}$$

Temporarily these disturbances will be assumed to propagate at the freestream speed, but this restriction will be eliminated at the close of the following section. Irrotational disturbances which propagate in wind tunnels with wavy or porous walls have been considered in Ref. 4.

4.2 "ONE-SIDED" IRROTATIONAL DISTURBANCE

By inspection or via a sine transform of Laplace's equation subject to condition (2.5), an irrotational streamfunction which oscillates neutrally in the x-direction is

$$\psi^{(a)} = \frac{1}{\pi} e^{\pi y} \sin \pi(x-t) \tag{4.2}$$

with velocities

$$u^{(a)} = -\psi_y^{(a)} = -e^{\pi y} \sin \pi(x-t) \tag{4.3}$$

and

$$v^{(a)} = \psi_x^{(a)} = e^{\pi y} \cos \pi(x-t) \tag{4.4}$$

Either a negative or positive exponent is acceptable, denoting decay or growth of the amplitude in the y-direction. A positive sign is specified here. In an engineering device, the amplitude would be finite if the y-dimension were limited by channel walls or by the vorticity source of the fluctuations. Rather than studying such bounded flows with their additional geometrical parameters, the simpler unbounded case is considered here.

Analogous to the case of plane-waves of vorticity, we set $y_1 = \pi/2$ in Eqn.(2.1c) and thus obtain the streamfunction for an irrotational disturbance propagating along a semi-infinite plate

$$\psi(x,y,t) = \psi^{(a)}(x-t,\, y) - \psi^{(i)}(x,y,t;\, y_1 = \pi/2) \tag{4.5}$$

The streamlines for this flowfield are plotted in Fig. 6. Note the apparent undisturbed region beneath the plate and far-downstream of the leading edge. In the quadrant x>0,y<0, only a pattern of standing waves exists; there are no traveling waves. In section 6, the

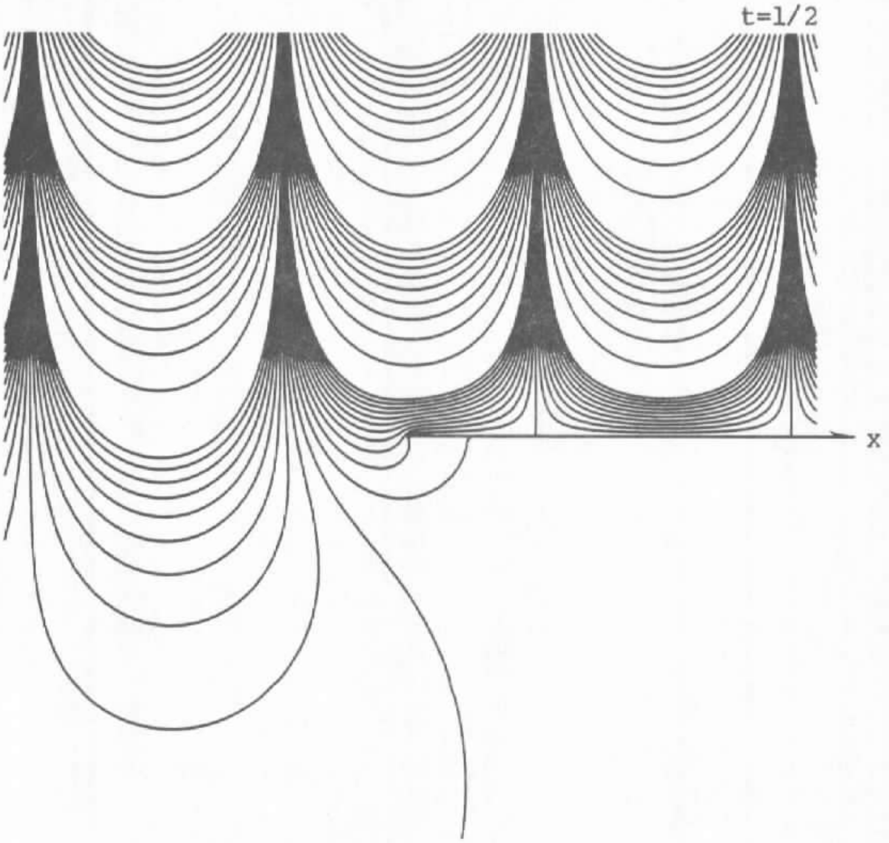

Figure 6. Streamlines for a "one-sided" irrotational disturbance propagating past the leading edge and along a semi-infinite plate. Because of the large variation of the velocity in this figure, the interval between streamlines has been assigned the three values $\Delta\psi=1/10\pi$, $1/\pi$ and $10/\pi$.

fluctuations are shown analytically to vanish far-downstream of the leading edge and below the plate.

The time t has been nondimensionalized with respect to the characteristic time Λ/U_∞ and the disturbances propagate at the freestream speed. If the disturbances propagate at a speed different from U_∞, then the above solutions apply if the time is nondimensionalized against Λ/c, except for the stationary case of c=0. The instantaneous streamline pattern would be identical, but the freestream disturbance would propagate at dimensional speed c rather than U_∞. As shown later, the phase speed affects the fluctuating pressure.

This irrotational solution is valid for <u>all amplitudes and all distances downstream</u>, except that the amplitude cannot be so large that compressibilty effects are significant. In contrast, for rotational disturbances, the solution is valid only for amplitudes and distances downstream sufficiently small such that the cumulative effects of the nonlinear rearrangement of vorticity are negligible. Secondly, except for the unsteady viscous layer associated with the no-slip condition at the plate surface, the viscous terms for the irrotational disturbances are identically zero outside of those layers. This characteristic differs from the case of rotational disturbances which decay via viscous dissipation in the streamwise direction.

The case of the plate perpendicular to the "wave front" of the freestream disturbance has been considered here. A skewing or oblique penetration is also possible. Also possible is the limiting case of irrotational disturbances which oscillate neutrally in the y-direction but grow or decay exponentially in the x-direction. These interesting and practical cases will require further study.

4.3 "TWO-SIDED" IRROTATIONAL DISTURBANCES YIELDING ONLY A NORMAL VELOCITY ALONG THE X-AXIS

With careful tailoring of the amplitudes and phases, we shall now superimpose two of the basic disturbances of the "one-sided" form to yield irrotational disturbances which are unbounded both above and below the plate, and which have special properties along the x-axis.

The superposition of two streamfunctions with one decaying in the positive y-direction and the other decaying in the negative y-direction yields

$$\psi^{(a)} = \frac{1}{\pi} \sin \pi(x-t) \left[\frac{e^{-\pi y} + e^{\pi y}}{2}\right] = \frac{1}{\pi} \sin \pi(x-t) \cosh(\pi y) \quad (4.6)$$

The velocities are $u^{(a)} = - \sin \pi(x-t) \sinh(\pi y)$ (4.7)

$$v^{(a)} = \cos\pi(x-t)\cosh(\pi y) \tag{4.8}$$

Along the x-axis, $u^{(a)} = 0$ and $v^{(a)} = \cos\pi(x-t)$. Hence, an irrotational disturbance has been specified with purely normal or "upwash" velocities along the x-axis and no longitudinal fluctuations. In general at other y-values, both velocities are nonzero.

The streamlines for this freestream disturbance interacting with a semi-infinite plate are shown in Fig. 7.

4.4 "TWO-SIDED" IRROTATIONAL DISTURBANCES YIELDING ONLY AN OBLIQUE VELOCITY ALONG THE X-AXIS

For an oblique disturbance, $u^{(a)}$ is proportional to $v^{(a)}$. An oblique plane-wave of vorticity was considered in Section 3.2 where the same constant of proportionality $A=u^{(a)}/v^{(a)}$ applies over the entire flowfield of the the freestream disturbance, $\psi^{(a)}$. In a more restricted sense, oblique disturbances arise in irrotational flows as well, except that the two velocities are proportional only along some line, say the x-axis:

$$u^{(a)}(x,0,t) = Av^{(a)}(x,0,t) \tag{4.9}$$

Such an irrotational disturbance can be created by superimposing two irrotational waves, shifted relative to one another, and which decay in the opposite directions. The streamfunction and velocities are

$$\psi^{(a)} = \left\{ e^{-\pi y}\sin[\pi(x-t)-\theta] + e^{\pi y}\sin[\pi(x-t)+\theta]\right\}/2\pi\cos\theta \tag{4.10}$$

$$u^{(a)} = \left\{ e^{-\pi y}\sin[\pi(x-t)-\theta] - e^{\pi y}\sin[\pi(x-t)+\theta]\right\}/2\cos\theta \tag{4.11}$$

$$v^{(a)} = \left\{ e^{-\pi y}\cos[\pi(x-t)-\theta] + e^{\pi y}\cos[\pi(x-t)+\theta]\right\}/2\cos\theta \tag{4.12}$$

The coefficient $(2\pi\cos\theta)^{-1}$ has been adjusted so that condition (2.5) is satisfied. When $\cos\theta=\pi/2$, Eqn.(2.5) is not required, since for that case, purely longitudinal disturbances arise along the x-axis. When the two disturbances are in phase, then $\theta=0$ and $u^{(a)}=0$, which is the case of a normal velocity fluctuation considered in Section 4.3.

For a general angle θ, the proportionality constant is

$$A = -\tan\theta \tag{4.13}$$

The disturbance streamfunction which satisfies impermeability at the plate is

$$\psi = \psi^{(a)} - \psi^{(i)}(y_1 = \pi/2) \tag{4.14}$$

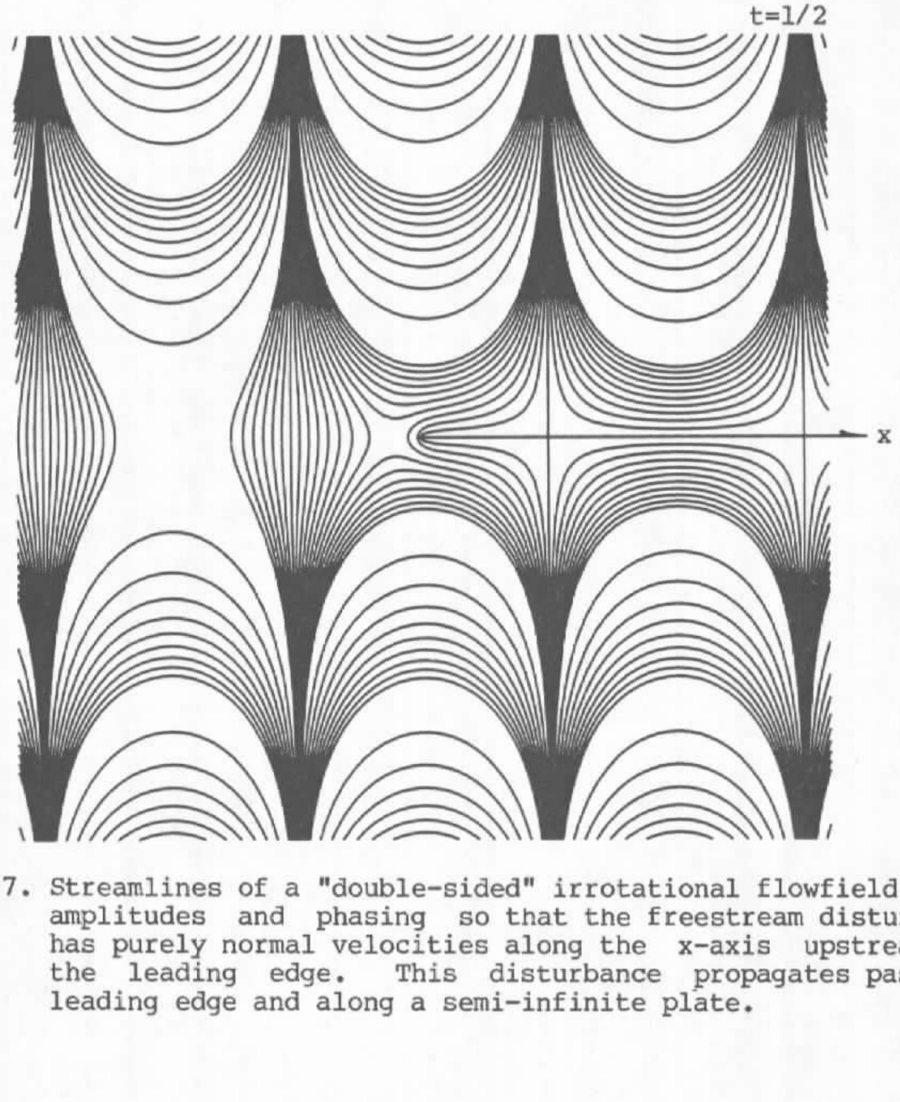

Figure 7. Streamlines of a "double-sided" irrotational flowfield with amplitudes and phasing so that the freestream disturbance has purely normal velocities along the x-axis upstream of the leading edge. This disturbance propagates past the leading edge and along a semi-infinite plate.

The streamlines for the case of $\theta=\pi/4$ are plotted in Figure 8. Upstream of the plate, a region of oblique streamlines exists. Far-downstream of the leading edge, the patterns on opposite sides of the plate are offset by the phase angle $2\theta=\pi/2$, and adjust to two independent patterns which subsequently propagate without further change in structure.

4.5 "TWO-SIDED" IRROTATIONAL DISTURBANCE YIELDING ONLY A LONGITUDINAL VELOCITY ALONG THE X-AXIS

The examples of irrotational disturbances considered here will conclude with another limiting case: a disturbance field with purely longitudinal disturbances along the x-axis

$$\psi^{(a)} = \frac{1}{\pi} \sin\pi(x-t) \sinh(\pi y) \tag{4.15}$$

with velocities

$$u^{(a)} = -\sin\pi(x-t) \cosh(\pi y) \tag{4.16}$$

and

$$v^{(a)} = \cos\pi(x-t) \sinh(\pi y) \tag{4.17}$$

where $u^{(a)} = \sin\pi(x-t)$ and $v^{(a)} = 0$ along the x-axis, but $v^{(a)}$ is generally nonzero at other y-values. Since impermeability is naturally satisfied by this freestream disturbance, then Eqn.(2.5) can neither be satisfied nor is it required. The interaction between the plate and freestream disturbance is trivial, $\psi^{(i)} = 0$, and the streamfunction is $\psi = \psi^{(a)}$ with streamlines as plotted in Fig. 9. The streamlines associated with the freestream disturbance merely propagate along the plate without any distortion.

While these examples of superposition yield special, limiting flows along the x-axis, the superposition of various irrotational disturbances with different amplitudes, phases and x-wavenumbers can yield an unlimited variety of fluctuations. Fourier analysis can be used in more general cases.

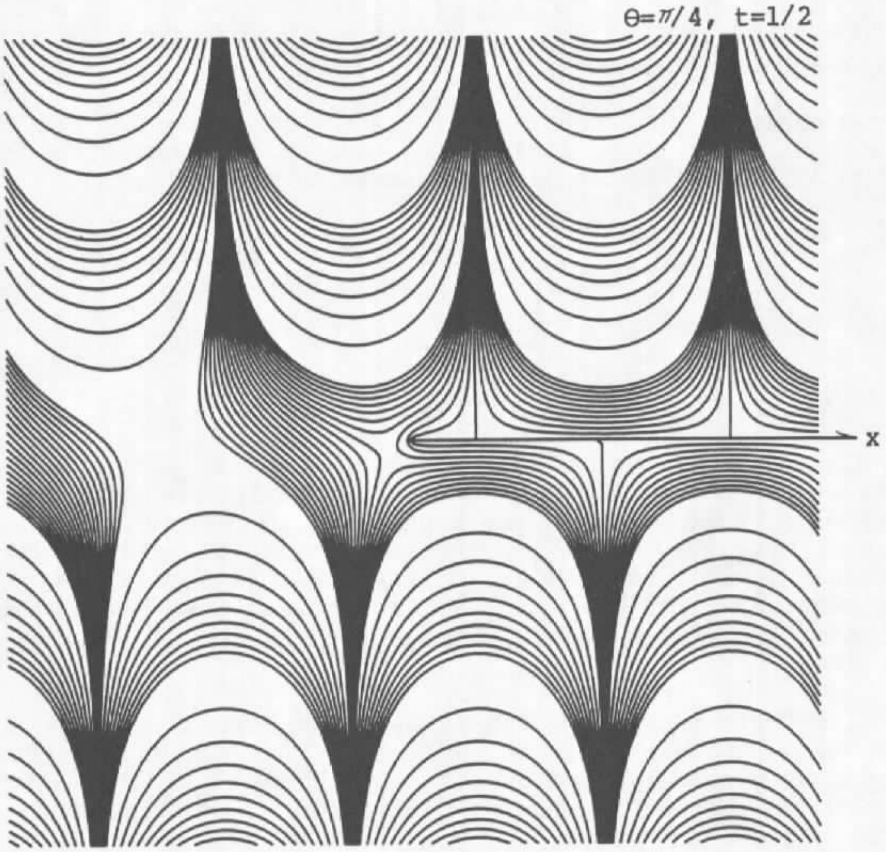

Figure 8. Streamlines for a "double-sided" irrotational flowfield with oblique velocities along the x-axis far upstream of the leading edge. This disturbance propagates past the leading edge and along a semi-infinite plate.

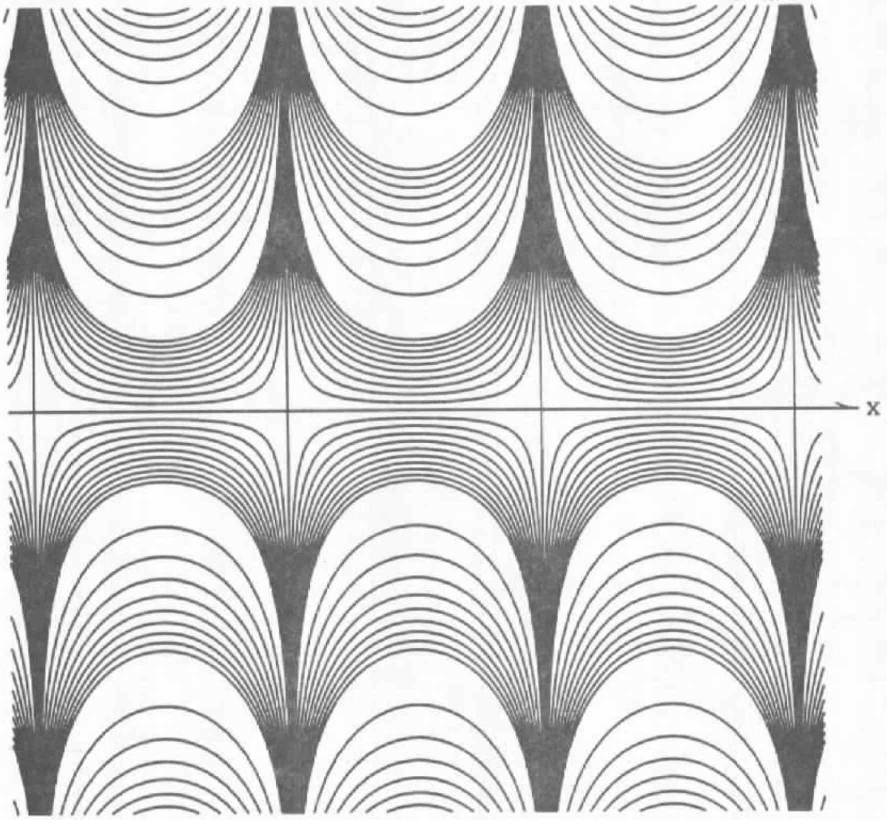

Figure 9. Streamlines for a "double-sided" irrotational disturbance field with a purely longitudinal velocity along the x-axis. A plate aligned with the x-axis does not influence tne flowfield.

5. ROW OF POTENTIAL VORTICES

The previous examples of freestream disturbances have had only one x-wavenumber. As an example of a periodic disturbance with a discrete series of x-wavenumbers and with singularities in the flow, we will analyze the case of a row of potential vortices propagating downstream. If these vortices model little patches of "weak" vorticity convecting along in the flow, then they convect with the mean flow at speed $c=U_\infty$. However, if they model a set of bound circulations such as airfoils translating along in the x-direction, then the phase speed c can be different from the freestream speed. Even negative (upstream) speeds are possible. As long as the proper speed is specified or incorporated into the nondimensionalization of time, then the mechanism of propagation is not of central importance here.

Beginning with the complex potential $W = \phi + i\psi$ for an infinite row of equi-distant vortices, each with strength κ, and positioned at the coordinates (0,0), ($\pm$a,0), ($\pm$2a,0), ...

$$W = \frac{i\kappa}{2a} \ln \sin \frac{\pi}{a}(x+iy) \tag{5.1}$$

as given by Lamb (Ref. 5, page 224), then the streamfunction is

$$\psi = \frac{\kappa}{2\pi} \ln \left\{ \sin^2 \frac{\pi x}{a} \cosh^2 \frac{\pi y}{a} + \cos^2 \frac{\pi x}{a} \sinh^2 \frac{\pi y}{a} \right\} \tag{5.2}$$

This streamfunction will be modified to describe a row of traveling, alternating vortices along the line $y = y_2$. Eqn.(5.2) can be modified by (a) accounting for the motion of the vortices by replacing x by x-t, (b) accounting for the alternating circulations by setting up two streamfunctions where the streamfunction with positive circulation is written down by replacing x-t by x-t+a/4 and the streamfunction for the vortices with negative circulation is written down by replacing x-t by x-t-a/4 and replacing κ by $-\kappa$, and (c) shifting the row to lie along $y=y_2$ by replacing y by $y-y_2$. After some manipulation, the result is

$$\psi^{(a)} = \frac{\kappa}{4\pi} \ln \left\{ \frac{\cosh[2\pi(y-y_2)/a] + \sin[2\pi(x-t)/a]}{\cosh[2\pi(y-y_2)/a] - \sin[2\pi(x-t)/a]} \right\} \tag{5.3}$$

These vortices translate to the right at dimensional speed c; this speed does not directly appear because time has been nondimensionalized against a/2c. This streamfunction is periodic with period a. Its average vanishes.

Nondimensionalizing y and y_2 against a/2, letting $A = \cosh[\pi(y-y_2)]$ and defining $X=2(x-t)/a$, then for y fixed, the streamfunction behaves as

$$\psi^{(a)} = 4\pi \Psi^{(a)}/\kappa = \ln\left[(A + \sin\pi X)/(A - \sin\pi X)\right] \tag{5.4}$$

This function was fast Fourier transformed along the x-axis to yield the series representation

$$\psi^{(a)} = 0.17286\sin\pi X - 0.10760(10^{-3})\sin 3\pi X + 0.11920(10^{-6})\sin 5\pi X + O(10^{-8})$$

for $y_z = 1$. For each term in this series, the streamfunction was scaled and an impermeability streamfunction included so that each Fourier mode satisfied impermeability. A streamline pattern associated with this freestream disturbance interacting with a semi-infinite plate is shown in Figure 10.

In comparing Figs.6 and 10, note the similarity of the pattern for $y<1/2$. Beneath the singularities, there is a superposition of waves, and the largest x-wavelength is the period a of the row of vortices. The other wavelengths are smaller and decay exponentially away from the vortices more rapidly. The mode with the largest x-wavelength survives the furthest from the row of vortices.

This case of a row of vortices is an example of a flowfield which is unbounded in extent $(-\infty<y<\infty)$, but where a region exists $(y<y_z)$ where it is appropriate to introduce an exponentially growing freestream disturbance, $\exp(+\pi y)$, which is irrotational.

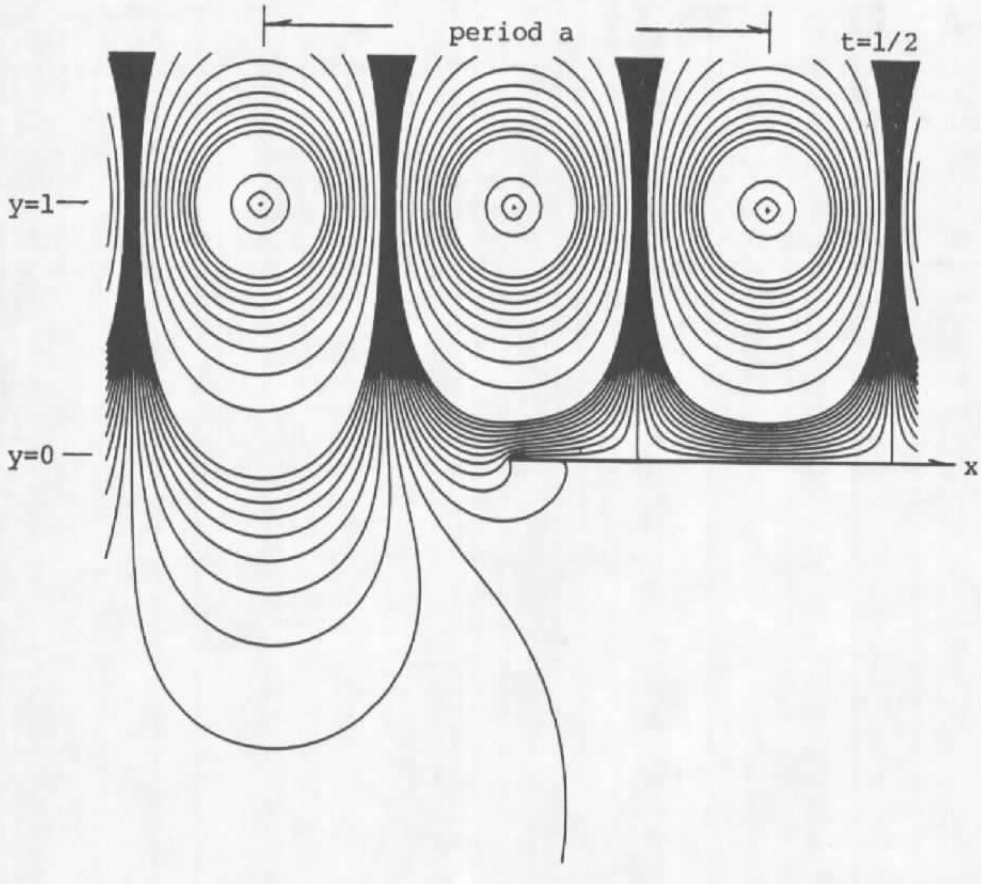

Figure 10. Streamlines for a row of potential vortices convecting downstream past the leading edge and along a semi-infinite plate.

6. THE FAR-DOWNSTREAM SOLUTIONS

As can be observed in Figures 1-10, at a distance of about one-half x-wavelength downstream of the leading edge, a (nearly) periodic flowfield has formed which propagates along at (almost) the same speed as the freestream disturbances without any further significant change in pattern. By letting $x \to \infty$ in Eqn.(2.1c) or by direct solution of the mathematical system which neglects the leading edge

$$
\begin{aligned}
&\nabla^2 \psi^{(i)} = 0 \\
&\psi^{(i)} = \psi^{(a)} \quad \text{at } y=0 \text{ for } -\infty<x<+\infty \\
&\psi^{(i)} \text{ is bounded as } y \to \infty
\end{aligned}
\qquad (6.1)
$$

as presented in Ref. 1, the solution for $\psi^{(i)}$ far downstream of the leading edge is

$$\psi^{(i)}(x-t, y, y_1) = \frac{1}{\pi} \sin \pi y_1 \, e^{-\pi|y|} \sin \pi(x-t) \text{ for } -\infty<y<+\infty \qquad (6.2)$$

where $|y|$ denotes the absolute value of y. The streamlines for this irrotational flow are plotted in Fig.11a. Note that this lateral influence is dictated by the x-wavenumber (with dimensionless value π). Solution (6.2) is a traveling-wave version of the classical Kelvin-Helmholtz solution (Ref.5) for flow past a wavy wall.

Hence, if solution (6.2) is used for $\psi^{(i)}$ rather than (2.1c), then the "far-downstream" solution is again of form

$$\psi \to \psi^{(a)} - \psi^{(i)}(\text{eqn. } 6.2) \quad \text{as } x \to \infty \qquad (6.3)$$

where the $\psi^{(a)}$ for the various freestream disturbances are the same as given in earlier sections. The streamlines for these far-downstream solutions, as shown earlier in Figures 2-10 with a leading edge, are plotted in Fig. 11b-f and Fig. 12. By comparing the figures with the same freestream disturbance, note how rapidly the streamlines adjust downstream of the leading edge to qualitatively resemble the "far-downstream" patterns. Fig.11g is the pressure associated with the flow in Fig. 11f; this pressure will be discussed in Section 7.3.

The blockage of the flow by the semi-infinite plate, as can be clearly seen in Figure 6, can be easily demonstrated by the far-downstream solution for the "one-sided" irrotational disturbance considered in Section 4.2. The far-downstream solution is

$$\psi \to \frac{1}{\pi} \sin \pi(x-t) \left[e^{\pi y} - e^{-\pi|y|}\right] \quad \text{as } x \to \infty \text{ and for } -\infty<y<\infty \qquad (6.4)$$

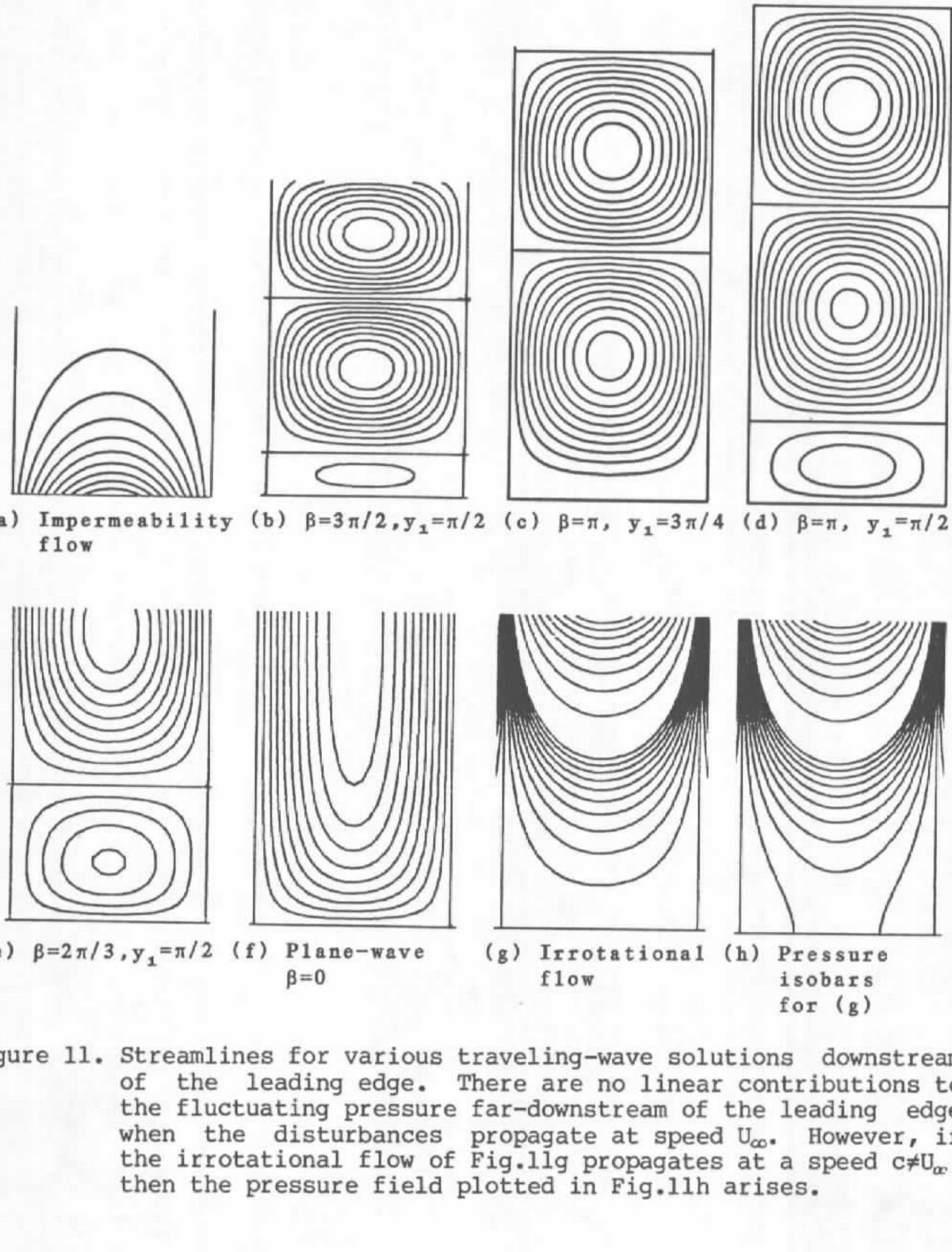

Figure 11. Streamlines for various traveling-wave solutions downstream of the leading edge. There are no linear contributions to the fluctuating pressure far-downstream of the leading edge when the disturbances propagate at speed U_∞. However, if the irrotational flow of Fig.11g propagates at a speed $c \neq U_\infty$, then the pressure field plotted in Fig.11h arises.

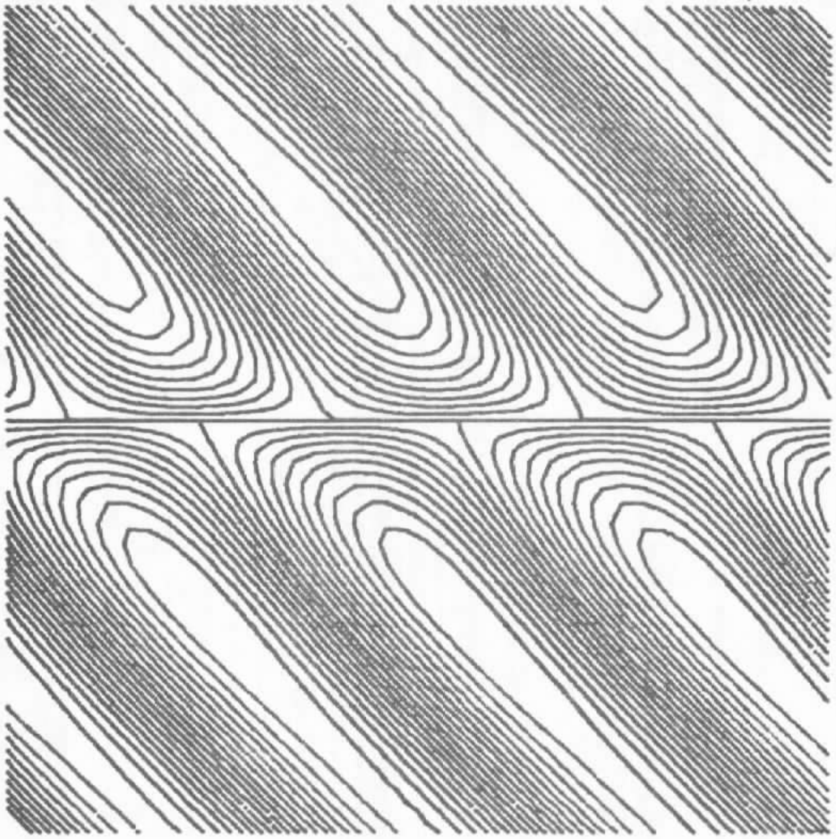

Figure 12. Streamlines for an oblique plane-wave of vorticity far-downstream of the leading edge of a semi-infinite plate.

Consequently for $y \leq 0$ below the plate, the two exponentials cancel and $\psi = 0$, i.e., the flow is blocked by the plate. Far above the plate, the second exponential vanishes and we are left with our original exponentially-growing freestream disturbance (4.2).

These far-downstream solutions are the only ones which survive for large x. However, recent evidence has shown that <u>these solutions are the only traveling wave solutions in the half-plane downstream of the leading edge</u> for the case of an inviscid uniform mean flow. The effects of the leading edge can be described in terms of a superposition of standing waves. Hence for all positive x, the disturbances consist of this far-downstream traveling wave solution and standing waves. Far-downstream of the leading edge, the standing waves vanish, and only the traveling wave survives. Study into this description of the disturbance field is continuing.

7. THE FLUCTUATING PRESSURE FIELD

7.1 PRESSURE FIELD FOR ROTATIONAL DISTURBANCES

The fluctuating pressure in a linear analysis of arrays of vortices and plane-waves of vorticity interacting with a semi-infinite plate is the same as for square vortices

$$p = p^{*}/\rho q U_{\infty} = -\phi_t^{(i)} + u^{(i)} = \mathcal{R}\left\{ \frac{i \sin\gamma_1 (\cos\pi t + \sin\pi t)}{\pi(-2z)^{1/2}} \right\} \quad (7.1)$$

The pressure has been nondimensionalized against the product $\rho q U_{\infty}$. Dimensionally, this expression indicates that the pressure fluctuation is proportional to the rate at which the vortices are convected downstream and "collide" with the plate, U_{∞}, and the y-momentum (per unit volume) as the fluid decelerates and satisfies impermeability, $\rho q \sin\gamma_1$. Along the plate, this pressure decays as $x^{-1/2}$. The nonlinear terms have been neglected in the Bernoulli equation, consistent with the linear analysis.

The contours of constant pressure are plotted in Fig. 13 as also given in Ref. 2. For reasons clarified in the next section, the pressure should be nondimensionalized against $\rho q c$, where c is the propagation speed of the disturbances. For weak rotational disturbances, $c=U_{\infty}$.

7.2 PRESSURE FIELD FOR IRROTATIONAL DISTURBANCES

If the flow is irrotational, then the x and y momentum equations for the disturbances can be combined to yield the unsteady Bernoulli equation

$$-\phi_t + \frac{U_{\infty}}{c} u + O(q/c) + p = 0 \quad (7.2)$$

in dimensionless form where the characteristic time is Λ/c and the characteristic pressure is $\rho q c$. Of course, if $c=0$, this characteristic pressure is not appropriate. ϕ is the velocity potential related to the longitudinal velocity by $u=-\partial\phi/\partial x$. The quadratic terms will be eliminated so that a meaningful comparison is possible with solution (7.1) for rotational fluctuations.

Note that the order of the quadratic terms is $q/c=(q/U_{\infty})(U_{\infty}/c)$ which may be significant for low phase speeds, $c \ll U_{\infty}$, even though q/U_{∞} may be quite small.

To better identify the various contributions to the pressure, we now represent the velocity as $u=u^{(a)}-u^{(i)}$ and the potential as $\phi=\phi^{(a)}-\phi^{(i)}$. The pressure is therefore

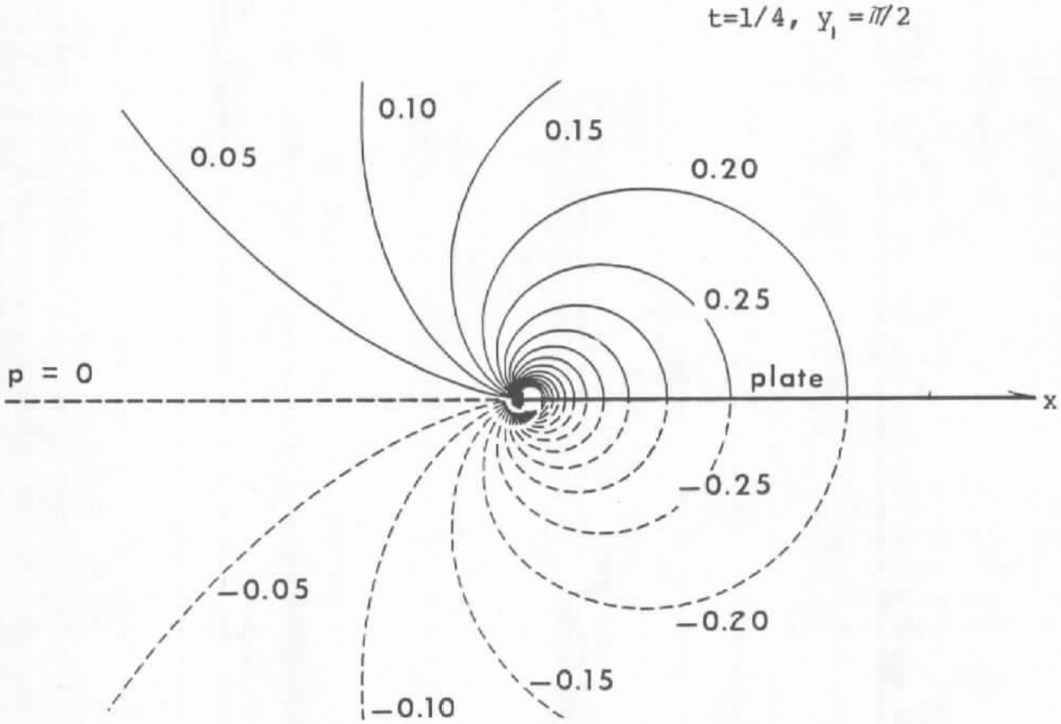

Figure 13. Isopressure contours for the pressure p_1 arising when rotational or irrotational freestream disturbances propagate downstream past the leading edge of a semi-infinite plate.

$$p = \varphi_t^{(a)} - \frac{U_\infty}{c} u^{(a)} - \varphi_t^{(i)} + \frac{U_\infty}{c} u^{(i)} \tag{7.3}$$

which can be rewritten as

$$p = \underbrace{\varphi_t^{(a)} + \varphi_x^{(a)}}_{p_0} \underbrace{- \varphi_t^{(i)} + u^{(i)}}_{p_1} + \underbrace{\left(1 - \frac{U_\infty}{c}\right)\left(u^{(a)} - u^{(i)}\right)}_{p_2} \tag{7.4}$$

Since the potential $\varphi^{(a)}$ of the "freestream" disturbances propagates at speed unity (corresponding to dimensional speed c), then $\varphi^{(a)}$=fct(x-t). Consequently, terms (p_0) vanish.

Terms (p_1) are identical with the pressure field associated with rotational disturbances, except for the nondimensionalization against ρqc rather than ρqU_∞. The pressure isobars are those plotted in Fig. 13.

Terms (p_2) represent a contribution to the pressure when $c \neq U_\infty$. If $c=U_\infty$, then that contribution vanishes, and the pressure is the same for rotational and irrotational disturbances if nonlinear and viscous effects can be neglected. This term depends on both the details of the freestream disturbance through the term $u^{(a)}$, as well as the impermeability flow $u^{(i)}$.

Both contributions in p_2 can be expressed analytically. The freestream disturbance $u^{(a)}$ has been given in Eqn.(4.3) for the one-sided disturbance, in Eqn.(4.7) for the disturbance with only an upwash velocity along the x-axis, in Eqn.(4.11) for the disturbance with an oblique velocity along the x-axis, and in Eqn.(4.16) for the disturbance with only a longitudinal velocity along the x-axis.

The expression for $u^{(i)}$ is given by Eqn.(2.2) with $y_1=\pi/2$, i.e.,

$$u^{(i)} = -i\Big\{\sin\pi(z-t)\left[S_2(-\pi z) - C_2(-\pi z)\right] - \cos\pi(z-t)\left[S_2(-\pi z) + C_2(-\pi z) - 1\right] - \frac{\sin\pi t + \cos\pi t}{\pi(-2z)^{1/2}}\Big\} \tag{7.5}$$

This expression is of universal form.

Hence, the pressure is given in terms of two universal functions, p_1 and $u^{(i)}$, which depend on the freestream disturbance only through the magnitude and phase of the <u>normal velocity along the x-axis</u>, and one simple analytical expression $u^{(a)}(x,y,t)$ that is intimately linked with the structure and variation of the longitudinal freestream disturbance.

The second universal function due to the longitudinal velocity $u^{(i)}$ is plotted in Figure 14.

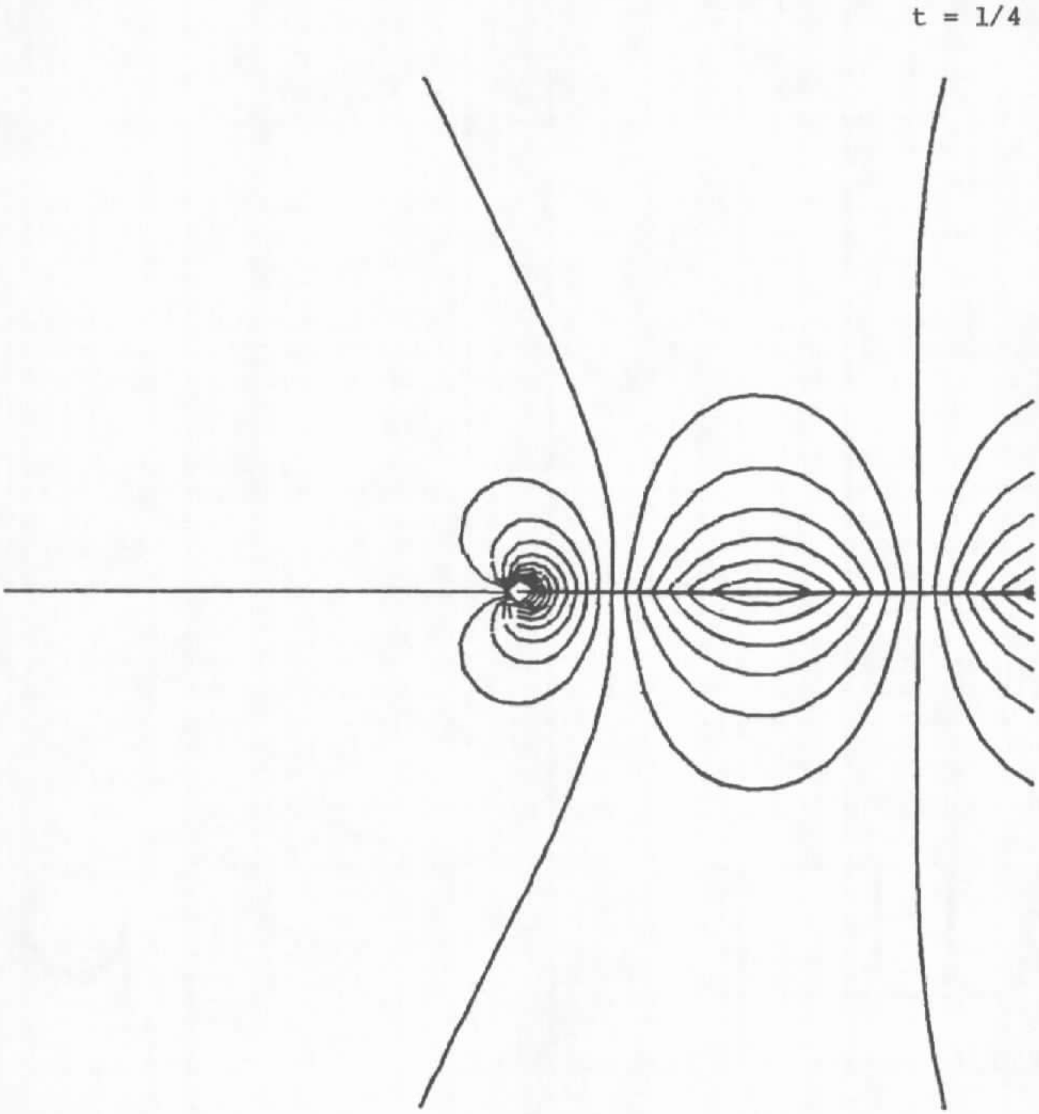

Figure 14. Pressure isobars for the contribution, $u^{(i)}$, to the pressure p_2. This contribution is of universal form, and is significant when the disturbance does not propagate at the freestream speed, $c \neq U_\infty$.

7.3 PRESSURE FAR-DOWNSTREAM OF THE LEADING EDGE

Far downstream of the leading edge, p_1 vanishes. The only linear contribution which remains is

$$p_2 = \left(1 - \frac{U_\infty}{c}\right)\left(u^{(a)} - u^{(i)}\right) \tag{7.6}$$

where the analytical forms for $u^{(a)}$ have been cited previously, and $\psi^{(i)}$ is given by Eqn.(6.2). To illustrate the behavior of this pressure, consider the one-sided irrotational disturbance

$$\psi^{(a)} = \frac{1}{\pi} e^{\pm \pi y} \sin \pi(x-t) \tag{7.7}$$

which decays in the negative or positive y-direction depending on our choice of sign. Then

$$\psi = \psi^{(a)} - \psi^{(i)} = \frac{1}{\pi} \sin \pi(x-t)\left[e^{\pm \pi y} - e^{-\pi|y|}\right] \tag{7.8}$$

and the longitudinal velocity is

$$u = -\psi_y = -\sin \pi(x-t)\left[\pm e^{\pm \pi y} + \delta e^{-\pi|y|}\right] \tag{7.9}$$

where $\delta = +1$ for $y \geq 0+$ and $\delta = -1$ for $y \leq 0-$. Hence the dimensional pressure far-downstream of the leading edge is

$$p_2^* = \rho q (c - U_\infty) \sin \pi(x-t)\left[\pm e^{\pm \pi y} + \delta e^{-\pi|y|}\right] \tag{7.10}$$

No matter which sign is selected on the exponent, no pressure fluctuation appears on one side of the plate <u>far-downstream of the leading edge</u>. This can be expected since Fig. 6 suggests that there is no flow on one side of the plate far-downstream of the leading edge.

On the other side of the plate, the pressure increases away from the plate in an unbounded manner. The pressure isobars are plotted in Fig. 11h.

In summary of this chapter, there are three linear contributions to the pressure which can be expressed dimensionally as

$$p^*(x,y,t) = \underbrace{\rho q c p_1(x,y,t)}_{\substack{\text{Eqn.(7.1)} \\ \text{Figure 13}}} + \rho q(c-U)\,[\underbrace{u^{(a)}(x,y,t)}_{(4.3,4.7,4.11,4.14)} - \underbrace{u^{(i)}(x,y,t)}_{\substack{(7.5) \\ \text{Figure 14}}}] \tag{7.11}$$

The first and third terms are universal functions which depend only on the amplitude and phase of the normal velocity of the freestream disturbance along the x-axis, $v^{(a)}(x,0,t)$. The second term is not universal in nature, and is proportional to the longitudinal velocity of the freestream disturbance. If the disturbances convect with the freestream speed, $c=U_\infty$, then only the normal velocity along the flight path influences the pressure everywhere. If $c \neq U_\infty$, much more information is required: the longitudinal velocity of the freestream disturbance $u^{(a)}(x,y,t)$ must be known everywhere that the pressure is needed, $p(x,y,t)$.

8. SUMMARY, DISCUSSION AND CONCLUSIONS

The solution for an array of square vortices convecting downstream with the freestream speed and interacting with a semi-infinite plate (Ref. 1) has been modified for more general rotational disturbances (including rectangular vortices and plane waves of vorticity) and irrotational disturbances (which can propagate at speeds different from the freestream). The disturbance streamfunction which satisfies impermeability downstream of the leading edge is of the form

$$\psi = \psi^{(a)} - \psi^{(i)}$$

where $\psi^{(a)}$ is the streamfunction of the freestream disturbance and $\psi^{(i)}$ is an irrotational distortion to that flow caused by the impermeability of the plate. The normal velocity, $v^{(a)} = \psi_x^{(a)}$, of the freestream disturbance varies sinusoidally along the x-axis. If $v^{(a)} = 0$ along the x-axis, then the plate does not affect the freestream disturbance, and $\psi^{(i)} = 0$.

For rotational disturbances, the solution is limited to low intensities and short distances downstream because of the linearization introduced. For irrotational, inviscid and unseparated disturbances, the solution is exact for all intensities and all distances downstream since superposition of irrotational flows is permissible for all amplitudes.

Far downstream of the leading edge, the influence of the plate in the y direction consists of the irrotational flow which decays away from either side of the plate

$$\psi^{(i)} = \psi^{(a)}(x,0,t)\, e^{-\pi|y|} \qquad \text{for } x>>1$$

Since it is possible for irrotational freestream disturbances to also decay in the y-direction at this rate, then the plate can act to effectively block freestream disturbances on one side of the plate, except for the standing waves near the leading edge. Hence, what survives far-downstream of the leading edge depends on some additional information such as the nature of the freestream disturbance and where the vorticity lies which is inducing the flowfield, as well as the velocities along the flight path.

For rotational disturbances, the fluctuating pressure is of dimensional form

$$p^* = \rho q U_\infty p_1(x,y,t) + O(\rho q^2)$$

where q is the characteristic normal velocity fluctuation along the x-axis of the freestream disturbance. p_1 is a dimensionless fluctuating pressure which is unbounded at the leading edge and vanishes far-away. This pressure is "universal" function which depends only on the x-wavelength and phasing of the normal velocity fluctuation along the x-axis, $v(x,0,t)$. The other details of the freestream disturbance elsewhere in the flow do not influence this pressure.

For irrotational disturbances propagating at speed c, the pressure is of dimensional form

$$p^*(x,y,t) = \rho q c p_1(x,y,t) + \rho q (c - U_\infty)\,[u^{(a)}(x,y,t) - u^{(i)}(x,y,t)]$$

The function p_1 is the same universal function applicable for rotational or irrotational disturbances. The longitudinal velocity $u^{(i)}$ is also a universal function independent of the form of the freestream disturbance. Unfortunately, $u^{(a)}(x,y,t)$ is the longitudinal velocity fluctuation in the absence of a plate, which must be known everywhere and at all time x,y,t that the pressure is needed. If $c=U_\infty$, then this function does not influence the pressure, which reduces to the same as for rotational disturbances above.

The freestream disturbances assumed here have been described as a single Fourier component, except for the case of the potential vortices where the velocity was represented as a series of waves to illustrate the use of superposition. General disturbances would require decomposition into the basic constituents, including the case of z-wavenumber zero as assumed here. For turbulence or other 3-D disturbances, the 3-D irrotational alteration to the freestream disturbance would be required to account for the 3-D flow about the leading edge.

Numerical solutions of the unsteady, incompressible, viscous, linearized, 2-D equations with a developing boundary layer were obtained by Kachanov, et al. (Ref. 8). The disturbance was a vortex sheet convecting downstream at speed U_∞ on one side of the plate. This disturbance is related to the "one-sided" irrotational disturbance of Section 4.2. The present authors have analyzed the cases of a vortex sheet, "half-arrays" of rectangular vortices, and Kármán vortex streets interacting with semi-infinite plates. For brevity, however, these cases are not included in this report.

Rockwell (Ref. 9) recently surveyed the literature related to vortical fluctuations impinging on leading edges and corners. For an oscillating shear layer impinging on a 90 degree corner, experimental data for the pressure fluctuation shows a rise in pressure along the plate as the corner is approached. The present authors believe that the pressure fluctuation associated with standing waves contributes to that pressure fluctuation, which also has the characteristics of a traveling wave.

This elliptic analysis is an example where only a single Fourier component is specified far-upstream of the leading edge. However, because of a change in the boundary condition at the leading edge, a pattern of growing standing waves appears upstream of the leading edge which reflects the upstream influence of the plate. For $x \geq 0$, downstream of the leading edge, a decaying standing wave pattern appears, as well as the freestream disturbance modified by an irrotational traveling wave. Through superposition, all these solutions in their appropriate domains join smoothly along the y-axis. Hence, this analysis is an example of the "excitation" of various waves with amplitudes and phases which can be linked to the properties of the freestream disturbance.

Elliptic problems of this nature are central to the problem of linking freestream disturbances with the various waves possible inside and above boundary layers, including the Tollmien-Schlichting wave and other instabilities, the continuous spectra of freestream vorticity fluctuations, and the standing waves. Accounting for the elliptic upstream influence for freestream disturbances as they interact with a plate is necessary for the later description of what happens in the boundary layer. The procedures, results, and interpretations from the present analyses add to the description of the processes leading to boundary layer transition. The similarities and differences between the various freestream disturbances in their Fourier descriptions helps identify basic classes of problems and eliminate redundancy in future studies. The pressure gradient and longitudinal velocity fluctuation from this analysis appear in analyses of unsteady boundary layers as the forcing function in the boundary layer equation and the outer boundary condition on the velocity.

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NOMENCLATURE

English

$a/2$	separation distance between between neighboring potential vortices with opposite senses of rotation
A	constant of proportionality in Eqn.(4.9); the quantity $\cosh[\pi(y-y_2)]$ for y fixed in Eqn.(5.4)
c	phase speed
cosh, sinh	hyperbolic cosine, hyperbolic sine
$C(z)$	Cosine Fresnel integral, $C(z)=\int_0^z \cos(\pi t^2/2)\,dt = C_2(\pi z^2/2)$
$C_2(z)$	Cosine Fresnel integral(2), $C_2(z)=\frac{1}{(2\pi)^{1/2}}\int_0^z t^{-1/2}\cos(t)\,dt$
f	function related to the Fresnel integrals by $f(z) = [\frac{1}{2} - S(z)]\cos(\pi z^2/2)-[\frac{1}{2} - C(z)]\sin(\pi z^2/2)$
$F(z,t)$	function defined in Eqn.(2.4a)
g	function related to the Fresnel integrals by $g(z) = [\frac{1}{2} - C(z)]\cos(\pi z^2/2)+[\frac{1}{2} - S(z)]\sin(\pi z^2/2)$
i	$(-1)^{1/2}$
ln	logarithm to base e
p	disturbance pressure
p_0, p_1, p_2	contributions to the pressure fluctuation as defined by Eqn.(7.4)
q	characteristic disturbance velocity
$S(z)$	Sine Fresnel integral, $S(z)=\int_0^z \sin(\pi t^2/2)\,dt$
$S_2(z)$	Sine Fresnel integral(2), $S_2(z)=\frac{1}{(2\pi)^{1/2}}\int_0^z t^{-1/2}\sin(t)\,dt$
t	time
u, v	disturbance velocities in the x and y directions
U_∞	mean x-velocity in the freestream
W	$\varphi + i\psi$, complex potential
x	coordinate parallel to plate and in streamwise direction
X	the quantity $2(x-t)/a$ in Eqn.(5.4)

y	coordinate normal to the plate
y_1	phase angle controlling the shifting of vortex arrays in the y-direction
y_2	the row of potential vortices lie along the line $y=y_2$
z	$x+iy$

Greek and Script

β	y-wavenumber
δ	the constant with value +1 for $y \geq 0+$ and -1 for $y \leq 0-$
∇^2	Laplacian operator
2θ	relative phase shift between the two irrotational freestream disturbances
κ	circulation of a potential vortex
Λ	half-wavelength; width of a rectangular vortex
ρ	mass density of the fluid
Φ	velocity potential related to velocities by $u_j = -\partial\Phi/\partial x_j$
ψ	disturbance streamfunction $= \psi^{(a)} - \psi^{(i)}$ which satisfies impermeability along the semi-infinite plate and reduces to the freestream disturbance far-upstream from the plate.
$\psi^{(a)}$	disturbance streamfunction in the <u>absence of the plate</u>, i.e. the "freestream disturbance"
$\psi^{(i)}$	the streamfunction for an irrotational flow which, when subtracted from the "freestream disturbance", $\psi^{(a)}$, causes the impermeability condition to be satisfied for $x \geq 0$, $y=0$

Superscripts, Subscripts, and Miscellaneous Notation

a	in the <u>a</u>bsence of the plate
$\|y\|$	absolute value of y
*	dimensional
i	associated with the irrotational flow, $\psi^{(i)}$
$\overline{(\ )}$	time average over one time period
$\mathcal{R}[\ ]$	real part of [ ]