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MECHANICS OF CRACKS SCREENED BY DISLOCATIONS†

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Abstract—We develop a general theorem for the force on singularities in an elastic two-dimensional medium in terms of the residues of the stress at the singularities. This theorem is then applied to a sharp crack screened by a cloud of dislocations. A total fracture criterion can then be derived in principle by specifying the local cleavage condition at the crack and the lattice resistance of the dislocations. The COD of the crack is shown to be given by the total screening Burgers vector of the dislocation cloud and the wake of a moving crack is discussed in terms of the resistance to moving the screening cloud. Finally, limitations of the model are discussed in terms of the geometrical effects on the cleavage plane caused by blunting the crack.

Résumé—Nous développons un théorème général concernant la force sur des singularités dans un milieu élastique bidimensionnel à partir des résidus de la contrainte sur ces singularités. On applique ce théorème à une fissure étroite écrantée par un nuage de dislocations. En principe, on peut obtenir un critère de rupture totale en précisant la condition locale de clivage sur la fissure et la résistance du réseau sur les dislocations. On montre que le nuage de la fissure est défini par le vecteur de Burgers d'écrantage total de ce nuage de dislocations et l'on discute la mise en mouvement d'une fissure en fonction de la résistance que l'on doit surmonter pour déplacer cette atmosphère. Enfin, on discute les limitations du modèle en termes des effets géométriques sur le plan de clivage, provoqués par une fissure qui s'émousse.

Zusammenfassung—Wir formulieren ein allgemeines Theorem für die Kraft, die auf Singularitäten in einem elastischen zweidimensionalen Medium wirkt, auf der Basis der Residuen der Spannung an den Singularitäten. Dieses Theorem wird auf einen scharfen Riß, der von einer Versetzungswolke abgeschirmt ist, angewendet. Ein Bruchkriterium kann im Prinzip abgeleitet werden, wenn die lokalen Spaltungsbedingungen am Riß und der Gleitwiderstand für die Versetzungen angegeben wird. Die kritische Rißöffnungs-verschiebung its bestimmt durch den Gesamtburgersvektor der Versetzungswolke. Der Verlauf eines sich bewegenden Risses wird anhand des Widerstandes diskutiert, den die Abschirmwolke bei ihrer Bewegung überwinden muß. Zuletzt werden die Grenzen des Modelles an den geometrischen Effekten auf der Spaltungsebene, die durch Abstumpfung des Risses entstehen, aufgezeigt.

1. INTRODUCTION

In this paper we shall develop a general theorem for the force on singularities in an elastic two-dimensional medium in terms of the residues of the stress at the singularity (in the sense of complex function theory), and apply this theorem to a sharp crack shielded by a distribution of dislocations in anti-plane strain (mode III fracture). From this application, a dislocation basis for fracture mechanics can be developed which permits more satisfactory approaches to fracture criteria than is possible through continuum fracture mechanics alone.

The theory is based on the properties of cracks and dislocations described as singularities in an elastic field, and thus addresses only the behavior of sharp cracks with their associated shielding clouds of dislocations. In the paper we shall nevertheless discuss how a significant crack opening displacement (COD) can be inferred for such a crack and how sharp cracks governed by cleavage processes at their tips are related to the fully ductile variety.

Our motivation for developing such a theory based on discrete elastic singularities is that in continuum theory there is no satisfactory way to develop a fracture criterion based on the microstructure of the material. Further, in the J-integral theory, the medium is required to possess an energy density function which is single valued at all points. This requirement makes it impossible to consider a moving crack rigorously, because as the crack moves, those portions of the medium sufficiently close to the crack are first loaded up as the crack approaches, undergo plastic deformation and finally unload as the crack passes. Since by definition, plastic deformation is highly hysteretic, such a loading cycle will violate the assumption of single valuedness, and make the J-integral in-

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applicable. This limitation is well known in the fracture mechanics community [1] and its implications have been intensively studied, because the concept of specifying the critical onset of fracture in terms of J_{IC} requires taking account of the effects of unloading [2].

In a second example, if the maximum yield strength at the tip of the crack is finite, as it must be in any real material, Rice [1] has shown that the force on the bare crack is effectively zero, because all the work done during crack growth is entirely absorbed by the plastic deformation, and that a fracture criterion cannot be constructed by analogy to the Griffith energy balance concepts of linear elastic brittle fracture. This is an intuitively unfortunate result, because then the crack opening processes cannot be separated physically or mathematically from the plastic work processes in the 'process zone'.

On the other hand, if one could build a theory which described the deformation aspects of fracture in terms of the behavior of the dislocations which in fact constitute the plastic zone, one might hope to circumvent these difficulties which are inherent in the continuum approximation. Further, because a fracture mechanics based on dislocation concepts would be more fundamentally grounded, one might hope to build a framework within which to address more satisfactorily such questions as the materials parameters controlling the fracture criterion, and the time and temperature properties of fracture, at least qualitatively. We are guided by the hope that by focussing on the dislocation aspects of fracture, one may be able to develop an alternative picture which can usefully complement the usual continuum approximation.

The beginnings of a fracture mechanics based on dislocation theory have already been made. Rice [3] has derived an expression for the force on a dislocation near a crack, and for the change of the stress intensity factor of the crack induced by the dislocation. Thomson [4], Weertman [5] and Thomson and Fuller [6] have introduced the idea of a dislocation shielded crack, with an elastic enclave near the tip and Hart [7] has developed a continuum dislocation screening theory. Majumdar and Burns [8] have derived stress distributions and discussed interaction forces for a variety of crack-dislocation configurations. Finally, Chang and Ohr [9] have discussed the dislocation free zone surrounding a crack tip.

2. THE FORCE ON AN ELASTIC SINGULARITY

In this section we show how the expression for the force on an elastic singularity in terms of a contour integral of Eshelby's energy-momentum tensor assumes a particularly simple form in the complex-variable notation possible for two-dimensional linear-elastic fields. This result is given here for the case of general anisotropy, and can be used to develop simple expressions for energy release rates associated with motion of defects of all kinds. Budiansky and Rice [10] have derived corresponding expressions for the forces on simple cracks for the isotropic plane and anti-plane strain cases, but they have not put their results into the form of the residues at the singularities as we shall do, nor have they included dislocations in their treatment. In subsequent sections of this paper we shall use the extremely simple form for the anti-plane strain case.

For a field of displacements u_i with associated stress σ_{ij}, dependent only on x_1 and x_2, Eshelby [11] has derived the expression

$$g_i = \oint_S P_{ij} n_j \, ds \tag{1}$$

for the force on a singularity or singularities contained within the contour S in the x_1x_2 plane, where n_j is the outward unit normal to S, and the energy-momentum tensor P_{ij} is given, for linear elasticity, by

$$P_{ij} = \tfrac{1}{2}\sigma_{kl}u_{k,l}\delta_{ij} - \sigma_{kj}u_{k,i}. \tag{2}$$

The usual suffix notations for summations and derivatives are assumed. These forces may be caused by stresses exerted on the external surfaces of the medium, or alternatively by other singularities such as dislocations, cracks, etc., which happen to lie outside S. When applied to a crack, g is the same as Irwin's crack extension force. δ_{kj} is the stress, and u_k the displacement. In the usual way, this theorem is applied to elastic cracks by noting that $P_{ij}(n_j^I + n_j^{II}) = 0$, where n_j^I and n_j^{II} refer to normal vectors on opposite sides of the mathematical slit representing the cleavage surface. Thus $\int P_{ij}n_j \, ds = 0$, when integrated over the two cleavage surfaces, provided the fracture surfaces are free of applied forces. Thus, a contour for a slit crack which ends at opposite points of the cleavage surface is equivalent to one which follows the cleavage surface to the crack tip and back (see Fig. 1).

Eshelby *et al.* [12] first gave general complex-variable expressions for displacement and stress fields in equilibrium under anisotropic elasticity.

In the variant notation of Stroh [13] these expressions take the form

$$u_i = 2\,\mathrm{Re}\left\{\sum_{\alpha=1}^{3} A_{i\alpha} f_\alpha(z_\alpha)\right\} \tag{3a}$$

$$\sigma_{i1} = 2\,\mathrm{Re}\left\{-\sum_{\alpha=1}^{3} P_\alpha L_{i\alpha} f_\alpha'(z_\alpha)\right\} \tag{3b}$$

$$\sigma_{i2} = 2\,\mathrm{Re}\left\{\sum_{\alpha=1}^{3} L_{i\alpha} f_\alpha'(z_\alpha)\right\} \tag{3c}$$

where $f_\alpha(z_\alpha)$ are analytic functions of $z_\alpha = x_1 + P_\alpha x_2$, $f_\alpha'(z_\alpha) \equiv \mathrm{d}f_\alpha(z_\alpha)/\mathrm{d}z_\alpha$ and the complex constants P_α, $A_{i\alpha}$ and $L_{i\alpha}$ are functions of the elastic constants. After manipulations involving relations between $A_{i\alpha}$ and $L_{i\alpha}$ not given here, the required components of

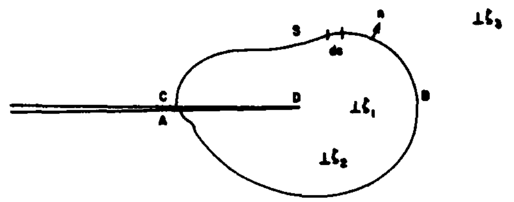

Fig. 1. A contour around a slit crack. Since the integral for $\bar{g}$ over the cleavage surface (ADC) contained within S is zero when no forces are exerted on that portion of the cleavage surface, S may be considered to consist only of the portion, ABC. S may enclose other singularities beside the crack, and still others may lie external to S.

done during crack growth is entirely absorbed by the P_{ij} are found to be

$$P_{11} = -P_{22} = 2\,\mathrm{Re}\left\{\sum_{\alpha=1}^{3} P_\alpha L_{k\alpha} A_{k\alpha} [f'_\alpha(z_\alpha)]^2\right\} \quad (4a)$$

$$P_{12} = 2\,\mathrm{Re}\left\{-\sum_{\alpha=1}^{3} L_{k\alpha} A_{k\alpha} [f'_\alpha(z_\alpha)]^2\right\} \quad (4b)$$

$$P_{21} = 2\,\mathrm{Re}\left\{\sum_{\alpha=1}^{3} P_\alpha^2 L_{k\alpha} A_{k\alpha} [f'_\alpha(z_\alpha)]^2\right\}. \quad (4c)$$

Substitution into (1) yields

$$g_i = 2\,\mathrm{Re}\left\{\sum_{\alpha=1}^{3} P_{i\alpha} L_{k\alpha} A_{k\alpha} \times \left(\pm \oint_{S_\alpha} [f'_\alpha(z_\alpha)]^2\,\mathrm{d}z_\alpha\right)\right\} \quad (5)$$

where

$$P_{1\alpha} \equiv 1 \qquad P_{2\alpha} \equiv P_\alpha$$

the $\pm$ sign is the sign of $\mathrm{Im}(P_\alpha)$, and S_α are the contours in the Z_α planes corresponding to S. Provided S lies entirely in elastic material in equilibrium the $(f'_\alpha)^2$ are analytic along S_α and (5) can be expressed

$$g_i = 4\pi\,\mathrm{Im}\left\{\sum_{\alpha=1}^{3} \pm P_{i\alpha} L_{k\alpha} A_{k\alpha} \times \sum \mathrm{Res}\,[f'_\alpha(z_\alpha)]^2\right\} \quad (6)$$

where ΣRes denotes the sum of the residues of the squared stress function at all its poles inside S_α.

In the limit of isotropy, the plane and anti-plane strain problems decouple; anti-plane solutions are given in terms of just one stress function, f_3. We have

$$P_3 \to i;\ L_{k3} \to \tfrac{1}{2}\delta_{k3};\ A_{k3} \to -\delta_{k3}/2\mu$$

where μ is the shear modulus, equations (3b) and (3c) become

$$\sigma_{32} + i\sigma_{31} = f'_3(z) \equiv \sigma(z) \quad (7)$$

where $z = x + iy$, and equations (5) and (6) become

$$\bar{g} \equiv g_1 - ig_2 = \frac{1}{2\mu i}\oint_S \sigma^2\,\mathrm{d}z \quad (8)$$

$$= \frac{\pi}{\mu}\sum \mathrm{Res}\,(\sigma^2). \quad (9)$$

As an elementary example, for a semi-infinite slit crack on $x_2 = 0$, $x_1 < 0$, loaded in mode III,

$$\sigma(z) = K/\sqrt{2\pi z} \quad (10)$$

where K is the stress intensity factor. In this case, σ^2 is analytic even on the negative real axis where σ is discontinuous, and (9) gives for the crack extension force

$$g_1 = K^2/2\mu. \quad (11)$$

Similarly, for an isolated screw dislocation under a remote uniform stress,

$$\sigma = \sigma_1 + \mu\mathbf{b}/2\pi z \quad (12)$$

and the force is given by

$$\bar{g} = \sigma_1\mathbf{b}. \quad (13)$$

Although these simple results are familiar, the real advantage of the present formulation becomes more apparent when collections of defects, such as cracks with accompanying dislocations, are considered. The additional terms in the complex stress functions necessary to satisfy the combined boundary conditions can be readily identified with interaction and image forces. Strain energy release rates for cooperative motion of defects can be readily evaluated by summing residues at the associated singularities. The celebrated path independence of the J-integral ($\equiv g_1$) is clearly identified with the path independence of contour integrals of meromorphic functions provided the poles enclosed remain constant (Cauchy residue theorem).

3. STRESS AND FORCE CALCULATIONS FOR SCREENED CRACKS

We now wish to apply the above theorem to the case of mode III fracture with contained yield, treating the screening dislocations as discrete entities.

We postulate a crack which in the complex plane $x + iy$ occupies the negative real axis, and at first consider just one dislocation, at the point ζ, Fig. 2. With full anisotropy, Sinclair and Hirth [14] have

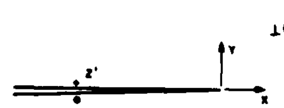

Fig. 2. Schematic of coordinate system in $z = x + iy$ plane. A dislocation at ζ exerts stresses on the cleavage surface which must be cancelled by a distribution of sources at z' on the cleavage surface.

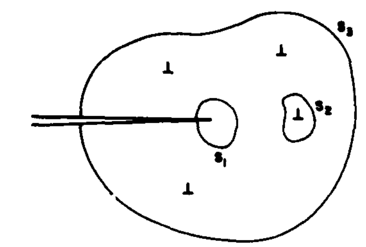

Fig. 3. The crack plus dislocation screen. S_1 encloses only the crack tip, and leads to $\bar{g}_c$; S_2 surrounds a single dislocation in the screen, and leads to $\bar{g}_d$; and S_3 surrounds the entire configuration, and leads to $\bar{g}_{total}$.

solved the corresponding problem for a finite length crack, and this can be taken to the limit of infinite crack length after a suitable coordinate shift to center one tip at the origin. Alternatively, Majumdar and Burns [8] have solved the present case directly in anti-plane strain. By either route, we obtain for the complex stress function.

$$\sigma = \frac{\mu \mathbf{b}}{4\pi\sqrt{z}}\left\{\frac{1}{\sqrt{z}-\sqrt{\zeta}} - \frac{1}{\sqrt{z}+\sqrt{\bar{\zeta}}}\right\}$$
$$= \frac{\mu \mathbf{b}}{4\pi}\left\{\frac{1}{z-\zeta} - \frac{1}{z-\bar{\zeta}} + \sqrt{\frac{\zeta}{z}}\frac{1}{z-\zeta} + \sqrt{\frac{\bar{\zeta}}{z}}\frac{1}{z-\bar{\zeta}}\right\}. \tag{14}$$

In the expanded second form, the first term in the braces is that for a dislocation without crack; the second is an image term, the third represents the fact that any external stress applied to the crack generates a crack-like singularity at the tip and the fourth is the image term for the third.

We can now obtain the stress at any point z due to the combination of (1) a slit core crack at the origin, and (2) screening dislocations distributed at discrete points, ζ_j, by adding the stresses of the individual contributions

$$\sigma = \frac{K}{\sqrt{2\pi z}} + \sum_j \frac{\mu \mathbf{b}_j}{4\pi}\left\{\frac{1}{z-\zeta_j} - \frac{1}{z-\bar{\zeta}_j} + \sqrt{\frac{\zeta_j}{z}}\frac{1}{z-\zeta_j} + \sqrt{\frac{\bar{\zeta}_j}{z}}\frac{1}{z-\bar{\zeta}_j}\right\}. \tag{15}$$

Equation (15) reduces to a simpler form if the distribution is symmetric about the x-axis, i.e. if for every dislocation at ζ_j there is a matching one at $\bar{\zeta}_j$ with the same $\mathbf{b}$.

$$\sigma = \frac{K}{\sqrt{2\pi z}} + \sum_j \frac{\mu \mathbf{b}_j}{2\pi}\sqrt{\frac{\zeta_j}{z}}\frac{1}{z-\zeta_j} \tag{16}$$

This is the result given in integral form by Hart [7].

Having a rigorous expression for the stress at any point, we can now apply (15) to obtain the force on any element or collection of elements of the core crack and its screen by considering contours, S, which enclose these and only these singularities. First, however, we write the familiar expression for a simple bare crack without any dislocation screen present. That is, the total stress is given by equation (10). Then for any contour surrounding the origin

$$\bar{g}_{\text{bare crack}} = \frac{K^2}{2\mu}. \tag{17}$$

With the screen present, Fig. 3, and evaluating equation (9) with (15) for the residue at $z = 0$ corresponding to a contour surrounding the origin, but enclosing no dislocations, we obtain the force on the core crack by itself in the presence of the screen

$$\bar{g}_c = \frac{1}{2\mu}\left\{K - \sum_j \frac{\mu \mathbf{b}_j}{2\sqrt{2\pi}}\left(\frac{1}{\sqrt{\zeta_j}} + \frac{1}{\sqrt{\bar{\zeta}_j}}\right)\right\}^2. \tag{18}$$

This result states that the crack extension force is the same as the Irwin–Orowan result, except that the core crack is shielded from the external stress field by its dislocation screen. It is suggestive to write this as

$$\bar{g}_c = \frac{1}{2\mu}\mathscr{K}^2;\quad \mathscr{K} = K - K_d$$
$$K_d = \sum_j \frac{\mu \mathbf{b}_j}{2\sqrt{2\pi}}\left(\frac{1}{\sqrt{\zeta_j}} + \frac{1}{\sqrt{\bar{\zeta}_j}}\right). \tag{19}$$

We emphasize that $\bar{g}_c$ is the local force to extend the core crack, independent of the motion of the dislocation screen. The shielding effect predicted by equation (19) has been discussed in various forms by all the previous authors listed in Refs [3–8].

The force on one of the dislocations of the screen is obtained from equations (15) and (9) by identifying the residue at the position of one of the dislocations, say that at ζ. Then for a contour surrounding ζ and no other singular points, we have

$$\bar{g}_d(\zeta) = \frac{K\mathbf{b}}{\sqrt{2\pi\zeta}} - \frac{\mu \mathbf{b}^2}{4\pi}\left\{\frac{1}{2\zeta} + \frac{1}{\zeta-\bar{\zeta}} - \sqrt{\frac{\bar{\zeta}}{\zeta}}\frac{1}{\zeta-\bar{\zeta}}\right\}$$
$$+ \sum_j{}'\left(\frac{1}{\zeta-\zeta_j} - \frac{1}{\zeta-\bar{\zeta}_j} + \sqrt{\frac{\zeta_j}{\zeta}}\frac{1}{\zeta-\zeta_j} + \sqrt{\frac{\bar{\zeta}_j}{\zeta}}\frac{1}{\zeta-\bar{\zeta}_j}\right)\frac{\mathbf{b}\mathbf{b}_j\mu}{4\pi}. \tag{20}$$

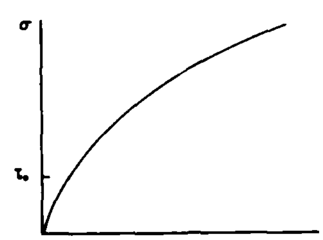

Fig. 4. Schematic stress-strain law, σ vs ϵ, with τ_0 the critical yield stress.

The summation sign is primed to remind us that the sum is not taken over the dislocation at ζ. This expression can also be written in terms of $\mathscr{K}$

$$\bar{g}_d(\zeta) = \frac{\mathscr{K}\mathbf{b}}{\sqrt{2\pi\zeta}} + \frac{\mu \mathbf{b}^2}{4\pi}\left\{\frac{1}{2\zeta} - \frac{1}{\zeta - \bar{\zeta}} + \sqrt{\frac{\zeta}{\bar{\zeta}}}\frac{1}{\zeta - \bar{\zeta}}\right\}$$

$$+ \sum_j{}' \frac{\mathbf{b}\mathbf{b}_j \mu}{4\pi}\left\{\frac{1}{\zeta - \zeta_j} - \frac{1}{\zeta - \bar{\zeta}_j}\right.$$

$$\left. + \sqrt{\frac{\zeta}{\zeta_j}}\frac{1}{\zeta - \zeta_j} + \sqrt{\frac{\zeta}{\bar{\zeta}_j}}\frac{1}{\zeta - \bar{\zeta}_j}\right\}. \qquad (21)$$

The second form is obviously an expansion about the near region of the crack tip, where $\mathscr{K}$ is dominant, while the first, (20), is an expansion about the outer edge of the screen, where K is dominant.

By a direct summation over the residues at the crack and the dislocation positions, using a contour surrounding the entire configuration, we can also write an expression for the total force on the entire crack-dislocation configuration. This we will term the 'dressed crack'

$$\bar{g}_{\text{total}} = \frac{\pi}{\mu}\sum_j \operatorname{Res}(\sigma^2(o) + \sigma^2(\zeta_j))$$
$$= K^2/2\mu \qquad (22)$$

where g_{total} is the force to move the entire configuration rigidly through the medium. Note it is the same as one would calculate if the dressed crack configuration was an effective bare crack with a simple K-singularity at the origin as in equation (17).

It is instructive to compare these results with those of a continuum deformation theory. In continuum theory, the stress-strain law is a nonlinear function at stresses above τ_0 (Fig. 4), but the law is single valued without hysteresis, as explained in the introduction. Under these circumstances, the J-integral is applicable, and is contour independent. The J-integral is defined by

$$J \equiv g, \qquad (23)$$

where g_1 is defined in equation (8). In the elastic region, the equation

$$\sigma = K\sqrt{2\pi z} \qquad (24)$$

holds and J can be calculated on a contour where (24) is valid. Thus

$$J = g_{\text{total}} = K^2/2\mu. \qquad (25)$$

We note that even if the stress-strain law is perfectly plastic for $2\epsilon > \tau_0\,\mu$, the J-integral is finite for an infinitesimal contour around the crack tip, because the strain is singular at the tip.

The connection between these continuum results and ours is that, unlike J, our g is not invariant as the contour is shrunk through the deformation region containing dislocations (see Fig. 5). The reason is that as the contour is shrunk, small circles are left behind surrounding each of the dislocations outside the main contour. When we draw a contour leaving out all or some portion of these singularities, the g thus calculated will be different from that using a contour in the elastic region surrounding the entire dressed crack. Contour integrations thus remain invariant when the contours are shrunk (or deformed in some other way) only in so far as they do not pass through one of the singularities of the configuration. This breakdown of the path independence theorem for the screened cracks will be seen to have important consequences.

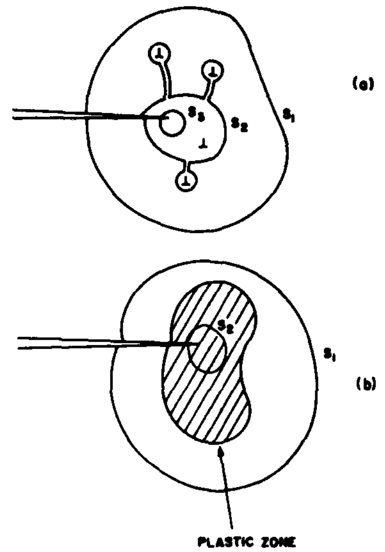

Fig. 5. In (a), the deformation is represented by actual dislocations. Thus as S_1 is shrunk to S_2 it leaves behind a wake of enclosed singularities if $\bar{g}$ is to remain invariant. S_3, since it encloses a different set of poles than S_1 and S_2, does not have the same value. In (b), the deformation in the shaded region is modeled as a nonlinear stress-strain relation, and $\bar{g}$ is an invariant; $\bar{g}(S_1) = \bar{g}(S_2)$.

4. IMPLICATIONS FOR FRACTURE MECHANICS

The mathematical theorems in the previous two sections will now be applied to some of the central problems of fracture mechanics. From our standpoint, the 'solution' of the fracture problem is obtained when one is able to find a self-consistent distribution of dislocations (perhaps time dependent) such that there is zero net force (including other material induced forces specified below) on all elements of the total configuration. This is an enormous task, of course, and is not to be attempted directly except in some very special cases. Otherwise, it will be necessary to develop suitable approximate approaches. However, there are some important rigorous results and insights which can be obtained without carrying through such a program in detail.

4.1. *Governing equations for crack equilibrium*

As explained above, a static configuration is achieved only when each singularity is in local static force equilibrium. The forces derived in [18–21] represent crack extension forces from external stresses, and dislocation-dislocation and dislocation-crack interaction forces. However, the lattice itself exerts additional forces on the crack and dislocations due to surface tension, interactions with chemical inhomogeneities, and lattice resistance from lattice trapping and Peierls forces. When these additional terms are included, we can write the basic governing equations for static equilibrium of the configuration as

$$\mathcal{K}^2/2\mu = 2\gamma + t$$

$$\tilde{g}_d(\zeta) = r_d. \tag{26}$$

The first equation represents local equilibrium for the crack in terms of the surface tension or energy, γ and the lattice trapping barriers, simply written as t. In essence, it is simply an expression of the Griffith relation for the core crack, because in true static equilibrium $t = 0$ and the t terms are only important for slow crack growth. Although it has not been observed, except in some special cases, it should be kept in mind that materials and circumstances exist for which trapping may be sufficiently large that t must be included explicitly in the Griffith equilibrium condition. For the dislocations, we have simply written the resistance terms as r_d. r_d is intended to include the complex set of material phenomena such as impurity pinning, grain boundary effects, and Peierls barriers which must be juggled by the theorist in modeling all dislocation mobility problems. The first equation in (26) is of particular interest, because it expresses a necessary fracture criterion for sharp cracks which is beyond anything derivable from continuum plasticity theories. In continuum plastic theories, the asymptotic stress dependence at the crack tip is a power law different from $-1/2$ and it has not been possible to link these theories up to the cohesive forces at the crack tip which hold the atoms of the crack tip together [1]. On the other hand, (26) follows in our theory from the fact that the dislocations are discrete, and the core crack has a K field at its tip. This result can only be self-consistent if the density of dislocations near the tip is low enough that dislocation cores and crack cores do not overlap. It must also be true that the crack cleavage surface is not blunted by the action of the dislocations, but we will discuss that separately in a later section.

Equation (26) also predicts that an overall fracture criterion will feature the intrinsic surface energy, γ, in an important way. It is often said that γ is such a small portion of the 'effective surface energy' for a moving crack that the overall criterion will be insensitive to it. However, γ is one of the important force balancing parameters driving the whole fracture process, because it specifies the essential core crack equilibrium condition and crack tip stress intensity factor, $\mathcal{K}$ to which the rest of the dislocation configuration must conform. Thus, a change in γ will generate first order changes in the overall equilibrium criterion. Attempts to calculate the critical K_{IC} for crack growth, by approximate means confirm this prediction [4–6].

Equation (26) refers to a static crack. However, brittle cracks in some materials are known to be able to sustain stable crack growth over a range of K values. In such a case, specifying the value of K specifies a velocity for the core crack. Time dependence and rate effects must then be introduced into the response of the dislocation screen. Hart [7] has constructed such a quasi-static theory on the basis of a continuum constitutive creep law for the dislocation screen. His work is limited to dislocation density distributions which lead to asymptotic $1/\sqrt{z}$ stresses at the crack tip. Our results, however, show that his approach has a larger generality provided the discrete character of the dislocation distribution near the crack tip and the core crack K field are treated consistently.

4.2. *Energy balance criteria and moving cracks*

The equilibrium conditions (26) are part of a prescription for finding the detailed distributions of the dislocations in a complete solution of the fracture problem. However, there are also integral relations which are derivable from total energy conservation of a system in which an external stress drives a crack through a material in steady state. Without asking how the state is attained, we assume the crack moves quasi-statically through the medium carrying its screen with it, but also leaving a wake of plastically worked material behind.

To describe this situation adequately requires us to distinguish between the geometric and nongeometric dislocation density. In any real material, during plastic deformation, dislocation generation is a very chaotic process in which dislocations of all allowed Burgers vectors are formed. Thus there is a large degree of cancellation of long-range stress. To be quantitative, we suppose in mode III that we define the local geometric Burgers vector by summing the Burgers vectors of the actual dislocations over an area which contains a large number of dislocations, and divide by the area of the element. Since all $\mathbf{b}_j$ are in the x_3 direction, we have

$$\mathbf{B} = \frac{\sum_A \mathbf{b}_j}{A} = \mathbf{B}(z). \tag{28}$$

Over cells of the medium containing many dislocations, $\mathbf{B}(z)$ will be a reasonably well defined and continuous function. $\mathbf{B}(z)$ is the geometric density, and gives rise to the long-range stress which we actually wish to use in all sums like equation (15) to calculate the shielding of the crack. On the other hand

$$\mathbf{B}(z) \ll \frac{N|\mathbf{b}|}{A} \tag{29}$$

where N is the actual number of dislocations threading the area in question. In general the actual work of fracture can be expected to be from one to several orders of magnitude larger than the work to form the geometric density.

In these terms, as a crack progresses through a material, in the plastic zone dislocations are generated with the necessary local geometric density, $\mathbf{B}(z)$, to shield the crack. In addition, in order for the crack and its geometric screening cloud to move uniformly, generation, motion and a large degree of annihilation of the real dislocations will be required. In order to simulate this complex system, we assume that the screening geometrical dislocation density, $\mathbf{B}(z)$, is known and is stationary in a reference system rigidly fixed to the uniformly moving crack, Fig. 6. In addition, because of the irreversibility of the deformation, a wake is generated. This wake is composed of tangled dislocations whose long-range stress is zero, many of whom ultimately annihilate, and which we will represent as a distribution of dislocation dipoles in the crystal. The wake contains a large amount of stored and thermal energy, however, and since it is produced by the deformation field, it exerts an effective drag force on the geometric screening dislocations against their motion through the crystal. The drag force on the bare crack is γ and the drag force on the geometric dislocations is given by the rate of formation of stored dislocation and thermal energy in the wake. If the wake contains only dipoles, then it exerts no elastic force on the crack and, of course, to a first approximation, the dislocation dipoles are themselves force free in the (slowly varying) stress field of the wake.

Thus the total driving force on crack plus screen is $K^2/2\mu$ and the force balance becomes

$$\frac{K^2}{2\mu} = 2\gamma + R + 2\Gamma \tag{30}$$

$2\Gamma\delta x$ is the stored dislocation dipole energy in a section δx wide perpendicular to the cleavage plane, $R\delta x$ is the energy dissipated into heat and γ is the intrinsic surface energy. This equation is, of course, simply an expression of energy conservation, but it cannot be obtained by straightforward application of the J-integral. We noted earlier that γ, even though small, is an important term in an equation like (30), which means that Γ and R depend on γ, because the size of the plastic zone depends upon γ.

We note here that there is no reason to expect $B(z)$ to be strictly zero in the wake, but this point is explored in the next section.

4.3 *Burgers vector conservation and crack blunting*

Dislocation sources in two dimensions create dislocations in pairs of dislocations with opposite sign.

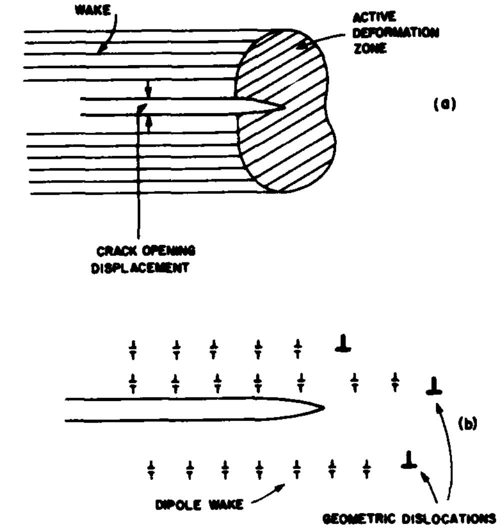

Fig. 6. A uniformly growing crack generates a screening distribution in the active deformation region surrounding the core crack tip and leaves behind a debris of plastically worked material in which the long-range stress is zero. In (b) the geometrical dislocations are assumed to move rigidly with the crack, and leave behind a wake of dislocation dipoles which become the plastically deformed wake. If the wake is composed purely of dipoles, the crack faces in the wake region are parallel and the separation is the COD.

Hence, the total dislocation distribution will possess zero net Burgers vector and a local excess of one sign must be balanced elsewhere by an excess of opposite sign, unless one class of dislocations is absorbed systematically by the cleavage surfaces of the crack. From equation (18), the dislocations which act to shield the crack from the external stress are repelled from the crack by the K-field of the crack at distances beyond the influence of the short range image terms. Their partners, by equation (18), must be anti-shielding dislocations which are attracted by the crack K-field. Thus for the dislocation screen to have a net overall shielding effect on the crack, the anti-shielding dislocations must be absorbed by the crack surface (see Fig. 7).

Since anti-shielding dislocations which are absorbed by the cleavage surface create slip steps of one predominant sign on that surface, they contribute the plastic component of the crack opening displacement. By conservation of Burgers vector, then, the total Burgers vector in the screen is equal to the crack opening displacement

$$\mathrm{COD} = \sum_j (\mathbf{b}_j)_y \tag{31}$$

(this result is limited to two dimensions, because in three dimensions, shielding can occur without COD contributions when the slip planes are skew to the crack line. Thus (31) is only an approximate law in real materials.)

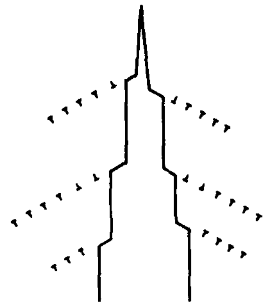

Fig. 7. Conservation of Burgers vector at a crack. Initially, the total number of shielding and anti-shielding dislocations is equal. The anti-shielding dislocations are attracted to the crack, and tend to be annihilated in the cleavage surface. The COD is equal to the total number of anti-shielding dislocations annihilated.

Although (31) has been derived assuming that the dislocations are created in pairs at internal sources, the same result follows if the dislocations are created at the crack tip, because when a dislocation is created, the crack is blunted by one Burgers vector.

Equation (31) provides the fundamental connection between the crack opening displacement and the overall toughness, because a high screening 'charge' of total Burgers vector must be roughly associated with higher toughness. The precise relation, however, depends upon the details of the distribution of the screen, i.e. far away dislocations do not screen as well as close in ones, etc.

We may make use of the continuum solution for the mode III fracture to calculate the predicted COD-toughness relation in that case. From Ref. [4], we take the approximate relation

$$\epsilon^e_{3\theta} \simeq \epsilon_0\left(\frac{K^2\cos\theta}{2\pi\tau_0^2 r}\right)^{(n/1+n)} \tag{32}$$

$\epsilon^e_{3\theta}$ is the elastic portion of the strain. The coordinate system is depicted in Fig. (8). The tip of the crack is placed at the edge of the plastic zone, as in perfect plasticity, whereas in a work hardening material with work hardening exponent, n, the tip of the crack extends a small distance into the plastic zone. See Ref. [4]. $\epsilon^e_{3\theta}$ is the elastic component of the strain, τ_0 and ϵ_0 are the critical shear stress and strain, respectively. The density of geometric Burgers vector is given by

$$\begin{aligned}\mathbf{b}_z(r,\theta) &= 2(\nabla\times\epsilon)_z \\ &= \frac{2\epsilon_0}{(1+n)r}\left(\frac{K^2\cos\theta}{\pi\tau_0^2 r}\right)^{(n/1+n)} \\ &= \frac{2\epsilon_0}{(1+n)r}\left(\frac{R_p}{R}\right)^{(n/1+n)}.\end{aligned} \tag{33}$$

(This expression for density of Burgers vector corrects equations (8) and (9) of Ref. [4]). The total Burgers vector $\Sigma\mathbf{b}_j$, is then given by

$$\begin{aligned}\mathrm{COD} &= 2\int_0^{\pi/2} d\theta \int_0^{2R_p\cos\theta} \mathbf{b}_z(r,\theta) r\,dr \\ &= 8R_p\epsilon_0 = \frac{4}{\pi}\bar{g}/\tau_0.\end{aligned} \tag{34}$$

In deriving (34) we have used the result $K^2 = 2\pi\tau_0^2 R_p$ where R_p is the plastic zone radius. Because of the approximation of placing the crack tip at the edge of the plastic circle, (34) is only valid for the limit where $n \simeq 0$. This equation, however, is in good agreement with the BCS result as quoted in equation (218) of Ref. [16], and with the various dimensional arguments connecting J with COD [16]. In spite of the relation (34), however, it is clear that the rough equivalence between COD and $\bar{g}/\tau_0$ is model dependent and for extreme dislocation distributions (34) may be poorly obeyed.

We mentioned in the previous section that the wake may contain a non-zero $\mathbf{B}(z)$. One could expect this to be the case often, because as the crack passes, some portion of the geometric component will surely be trapped by the wake dipole component. These dislocations will, of course, be attracted to the open cleavage surface, but only those very near this surface will actually be able to push their way through the work-hardened region of the wake to annihilate on the open surface. The moving crack in this case will generate net Burgers vector on a continuous basis. See Fig. 9. Thus, as the crack moves a distance δx, it leaves behind a net Burgers vector given by

$$\mathbf{B}\delta x = \int_0^{R_p} [\mathbf{B}(z)]_y\, dy\, \delta x \tag{35}$$

where R_p is the width of the wake. $\mathbf{B}$ is the Burgers vector contained in the wake per unit length of wake along one cleavage surface. By the arguments of this section, the net Burgers vector is reflected in additional crack opening, so that if $\mathbf{B}(z)$ in the wake does not depend on x in the wake, the crack shape becomes a simple wedge, with opening angle given by α,

$$\tan\alpha = \mathbf{B}. \tag{36}$$

If the wake retains some geometric component, it will also exert elastic forces on the crack and on the mobile screening dislocations whose distribution

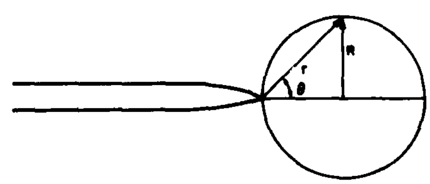

Fig. 8. In the continuum solution, the strain is a function of r. If work-hardening is present, the circle overlaps the crack tip slightly for small n, but we approximate the result by putting the crack tip on the radius of the circle.

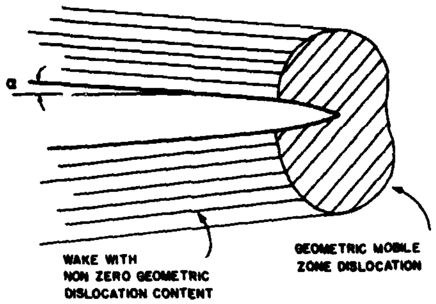

Fig. 9. When the wake retains some net Burgers vector, the crack develops a wedge shape with an angle, α, related to the net geometric dislocation density.

moves with the crack. If we assume as in (35) that R_p is the radius of the 'mobile' screen and likewise the width of the wake, then the geometric dislocations $\mathbf{B}\delta x$ in the wake at distance x from the crack tip where $x \gg R_p$ will actually appear to the crack as a dipole, because of the image term of **B** in the cleavage surface. Thus, except for the near region of the wake where $x \simeq R_p$, the elastic interaction of the wake with both the screening charge and the crack is negligible.

4.4 *Mixed continuum-discrete approaches*

As noted earlier the complete self-consistent solution of the fracture problem is not feasible in practice, and recourse must be taken to approximate methods. Clearly, deep in the plastic zone where a continuous dislocation density function can be defined, the continuum plastic approach is appropriate. The usual constitutive law between stress and plastic strain takes the place of the governing equilibrium equations for the dislocations. The difficulty comes in the matter of how to cut off the continuum solution near the crack tip, and what kind of boundary condition to set on the inner continuum boundary. Two papers have dealt with this question in different ways [4, 5]. Both have postulated an inner elastic enclave which models the fact that the crack at its tip sees a K-field, and have used the Griffith relation at the tip, (26), to obtain the fracture criterion.

Two difficulties have emerged from this work. The first relates to the appropriate boundary conditions to specify on the boundary of the elastic enclave. This problem is not straightforward, because the enclave boundary is the locus of the cut-off for the plastic solution (the dislocation density function loses its meaning, because it degenerates into sparsely spaced δ-functions), but the elastic portion of the solution remains valid down to atomic dimensions. Since elastic and plastic displacements are intrinsically tied to one another in a continuum plasticity theory, a proper procedure for separating the one from the other must be fashioned on physical grounds. So far [6], a fully satisfactory method for doing this has not been fashioned.

In these theories, however, the size of the elastic enclave is an important parameter, and this is the second difficulty referred to above. Little guidance has been given for making estimates of the enclave size. In the terms of this paper, the radius of the elastic enclave is equal to the scale of dislocation heterogeneity at the crack tip; i.e. given by the limiting density of dislocations, limiting spacing of glide planes, etc. It is precisely on this point where the discreteness of the dislocation description is crucial, but also where our knowledge is most inadequate, because so little is known about the characteristics of very dense dislocation distributions in regions of combined high stress and high stress gradient.

In addition to the expected role to be played by the dislocation-dislocation interactions and source density and placement in determining the enclave radius, it is possible that the image vs K-field competition in equation (21) plays a crucial role. Assume that a dislocation is very near the crack, and interacts with it more strongly than with all the other dislocations. Then the first term and the first brace in (21) are the dominant terms. We note that the two terms in question are of opposite sign, and that one dominates at close distances (the image term in the brace) and the other at greater distance (the K term). Thus the force on a shielding dislocation (repulsive at large distance) shows a maximum at a few atomic distances from the tip which turns out be be quite broad. That is, the repulsive term is significantly softened out to the order of several tens of atomic distances. This is far enough to be a serious contender in determining the radius of the elastic enclave. Physically, what happens is that at short distances, the crack tip loses its hard core of repulsion against the 'pile up' of shielding dislocations and this sets a limit to the maximum stress in the 'pile up' at the crack tip. Theoretical estimates have been made of the extent of the maximum in Ref. [3] and Majumdar and Burns [15] believe it has been observed in experiments in both ionic crystals and metals. In our opinion, further detailed studies of an experimental nature are badly needed on this point.

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